MITP Scientific Program

"Amplitudes, Motives and Beyond"

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The three week MITP scientific program "Amplitudes, Motives and Beyond" brought together about thirty mathematicians and physicists in the intersection from the fields of high energy physics, mathematics and string theory. The main emphasis of the program was on the interplay and similarities between physics and mathematics found in the study of scattering amplitudes on the physics side and in the theory of motives on mathematics side. The program evolved in a stimulating and productive atmosphere, with one talk usually scheduled in the morning and another talk scheduled in the afternoon. Very often the talks turned into discussions, which were continued in the coffee room. Albeit participants came from different scientific backgrounds, the common language spoken by all participants was a remarkable fact.

Scattering amplitudes in particle physics are related to the probability with which a certain scattering process occurs. Perturbative quantum field theory offers – in theory at least – a systematic way to calculate the scattering amplitudes through Feynman diagrams. However, any practitioner in the field realises soon that an approach based on Feynman diagrams is feasible only for the simplest processes. The complexity of the calculation increases with the number of external particles and with the number of internal loops. However, it is very often the case that the final answer is much shorter than any intermediate expression. This is an indication that not all structures and symmetries of the problem have been identified.

The recent years have shown great progress in this area and have given rise to an active interaction between high energy physics, mathematics and string theory.

Simplicity with respect to the number of internal loops is obtained by making use of the algebra of transcendental functions, like the algebra of multiple polylogarithms. Multiple polylogarithms are an important class of functions. In physics they are closely tied to Feynman integrals with massless propagators. Talks by D. Broadhurst, F. Dulat, J. Henn, A. von Manteuffel, O. Schnetz and

M. Sogaard addressed the problem of expressing amplitudes in terms of multiple polylogarithms.

In mathematics multiple polylogarithms are related to periods of mixed Hodge structures (or motives) of Tate type. The connection between the physics side and the mathematics side is based on representing the parametric Feynman integral as a (possibly divergent) period integral on an algebraic variety given by the complement of a graph hypersurface. Motives and varieties were discussed in the talks of J. Stienstra, A. Mellit and S. Galkin. The relation between periods, multiple polylogarithms and string amplitudes were addressed in the talks of S. Stieberger and D. Zagier.

Of particular interest are motives, which are not of mixed Tate type. This would correspond to Feynman integrals, which cannot be expressed in terms of multiple polylogarithms. The physics motivation comes from scattering amplitudes with massive particles as well as from amplitudes with sufficient many external massless particles. In going beyond the mixed Tate type one encounters elliptic integrals and elliptic multiple polylogarithms. Elliptic generalisations have been discussed in the talks by J. Brödel, N. Matthes, O. Schlotterer and Y. Zhang.

Simplicity with respect to the number of external legs is obtained by using methods associated to twistor variables, Grassmanians, cluster coordinates or Yangian symmetries. Here, ideas and concepts of string theory propagated into methods for quantum field theory. To give an example: One line of investigation originated from a new method of computing Feynman integrals in QCD based on the twistor formalism and on the positive Grassmannian. Amplitudes are computed as volumes of certain polytopes in twistor space. In the case of $N = 4$ super Yang–Mills theory this method has been further developed in terms of the *amplituhedron* generalizing the role of the positive Grassmannian. Polylogarithms appear naturally in this approach. Yangian symmetry, the amplituhedron and $N = 4$ super Yang–Mills amplitudes were discussed in the talks of B. Eden, L. Ferro, D. Nandan, G. Papathanasiou and M. Staudacher. Closely related is research in the direction of cluster algebras and quivers. Here on tries to identify natural variables which appear as arguments of polylogarithms. These ideas were exposed in the talks of J. Drummond and A. Volovich.

Quite recently it was discovered that Yang-Mills amplitudes and gravity amplitudes can be calculated from a (discrete) set of solutions to certain equations, named "scattering equations". L. Mason explained how these formulae arise from a holomorphic string theory in ambitwistor space.

A high-light of the scientific program was certainly the talk by Y. Manin on the motivic structure of quantum cohomology.

The MITP scientific program "Amplitudes, Motives and Beyond" was very successful in bringing together scientist from mathematics and physics. The discussions were very stimulating and fruitful and triggered certainly various scientific research projects.