Nucleosynthesis $^{12}C(\alpha,\gamma)^{16}O$ at MAGIX/MESA

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MAGIX Collaboration Meeting 2017

Topics

S-Factor

Simulation

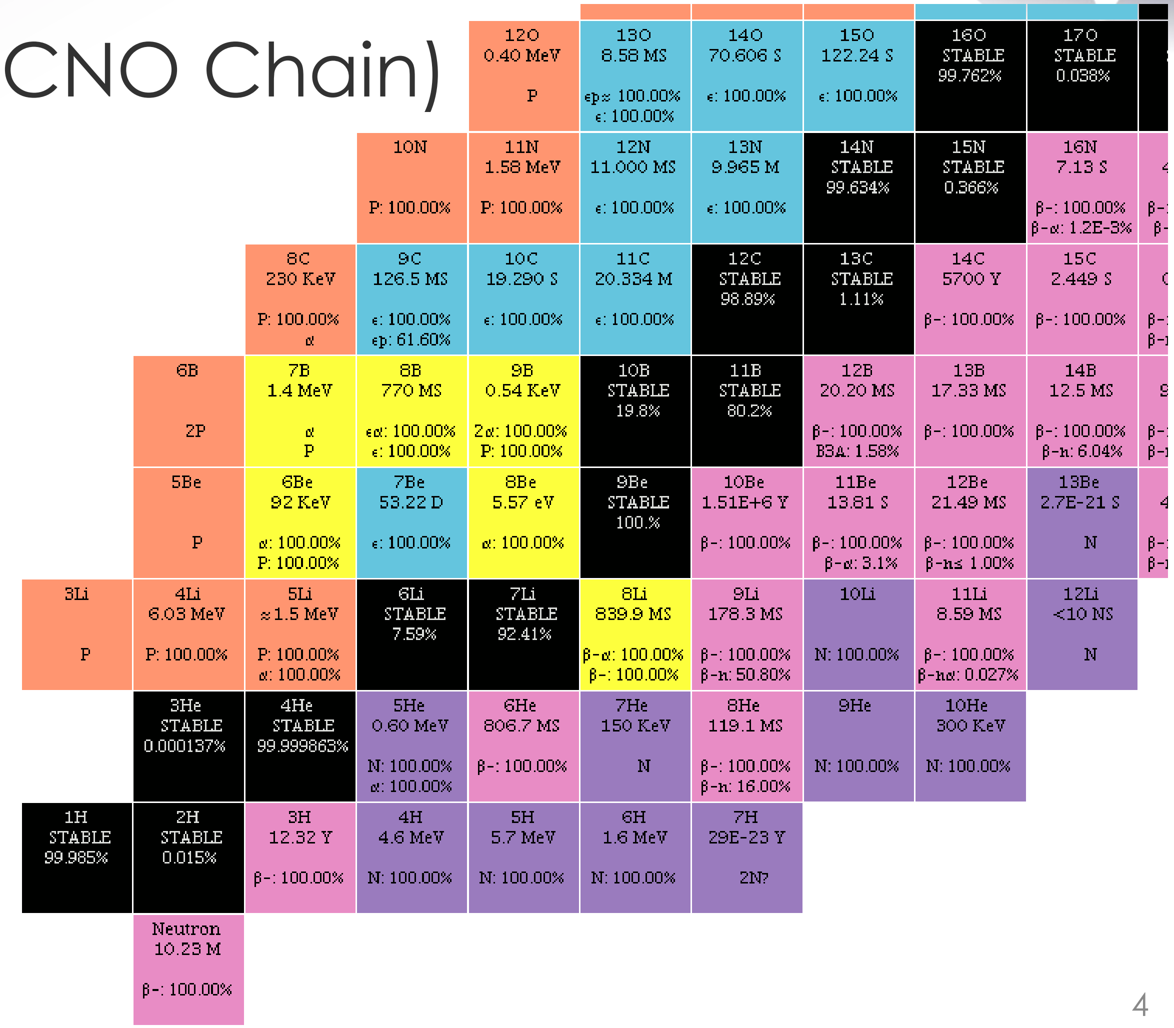
Outlook

S-Factor

Stages of stellar nucleosynthesis

- Hydrogen Burning (PPI-III & CNO Chain)
  - Fuel: proton
  - $T \approx 2 \cdot 10^7$ K
  - Main product: ^{4}He

- Helium Burning
  - Fuel: ^{4}He
  - $T \approx 2 \cdot 10^8$ K
  - Main product: ^{12}C, ^{16}O



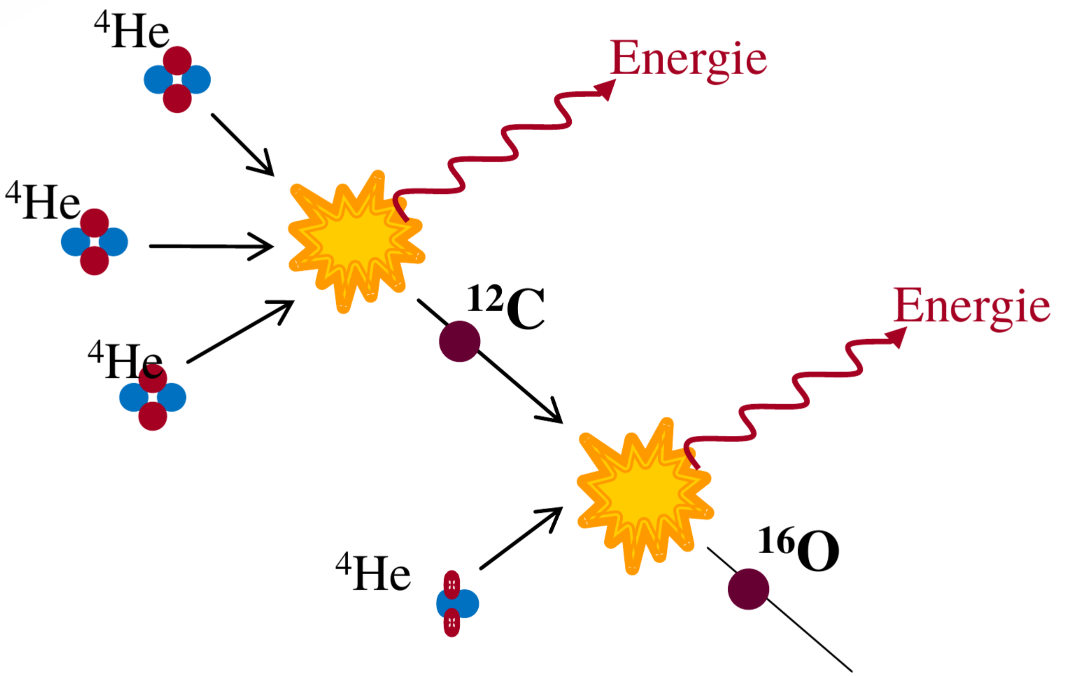
Helium Burning in red giants

- Main reactions:

$$3\alpha \rightarrow {}^{12}\mathrm{C} + \gamma$$

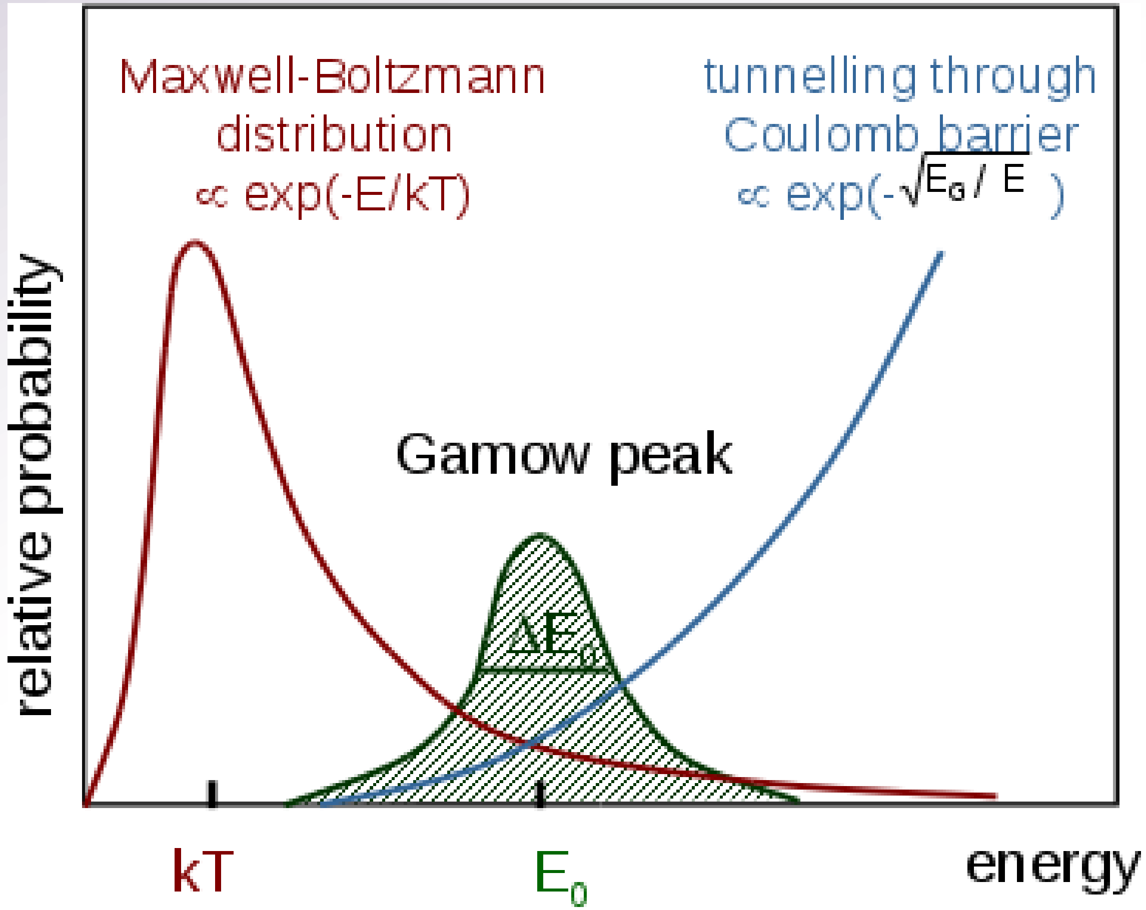
$$^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$$

- $^{12}\mathrm{C}/^{16}\mathrm{O}$ abundance ratio
  - Further burning states
  - Nucleosynthesis in massive stars



Cp. Hammache: $^{12}C(\alpha,\gamma)^{16}O$ in massive star stellar evolution

Gamow-Peak



- Fusion reaction below Coulomb barrier

  $kT \sim 15\,\text{keV} @ T = 2 \cdot 10^8\text{K}$

- Transmission probability governed by tunnel efffect

- Gamow-Peak E_0
  - Convolution of probability distribution
    - Maxwell-Boltxmann
    - QM Coulomb barrier transmission
  - Depends on reaction and temperature

Cp. Marialuisa Aliotta: Exotic beam studies in Nuclear Astrophyiscs

S-Factor

- Nonresonant Cross section

$$\sigma(E) = \frac{1}{E} e^{-\frac{2\pi Z_1 Z_2 \alpha c}{v}} S(E)$$

  - $e -$ Factor = probability to tunnel through Coulomb barrier

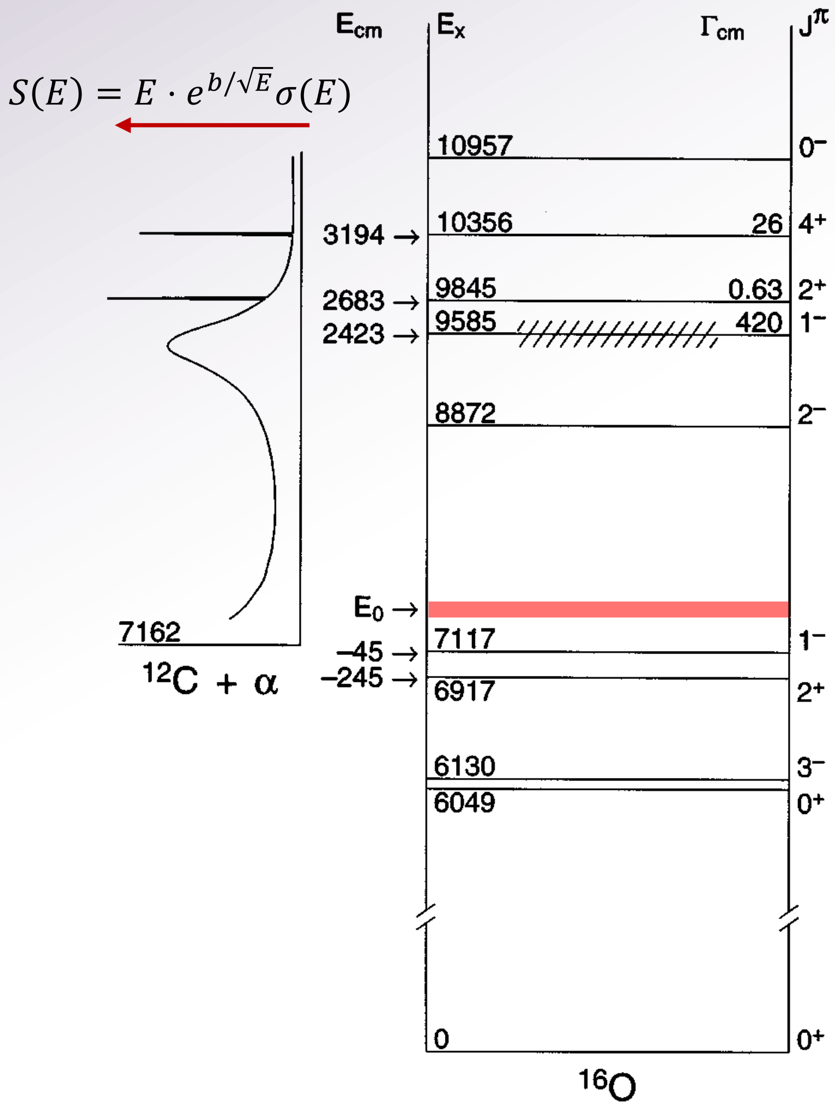
    $v =$ velocity between the two nuclei

    $\alpha =$ fine structure constant

    $Z_1, Z_2 =$ Proton number of the nuclei

  - $S(E) =$ Deviation Factor from trivial model

Gamow-Peak for $^{12}C(\alpha,\gamma)^{16}O$



- Gamow-Peak ($T \approx 2 \cdot 10^8 \mathrm{K}$)

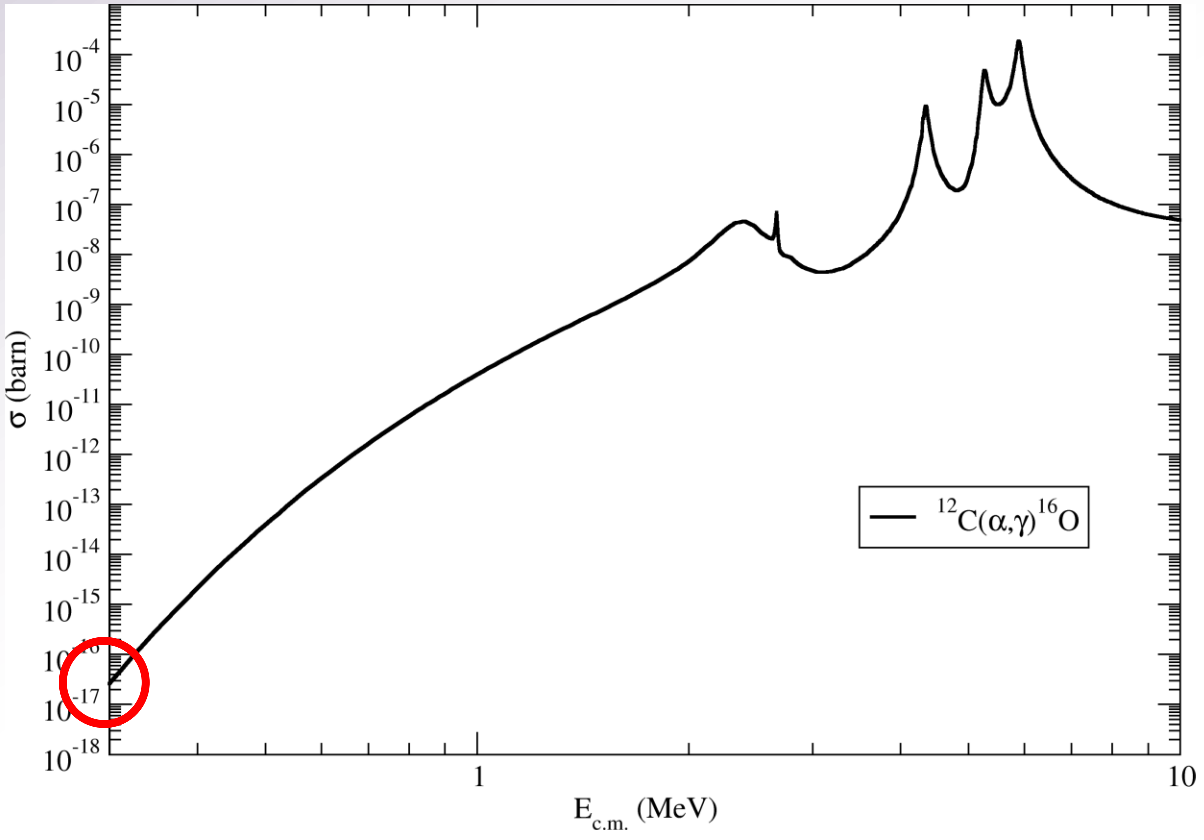
$$E_0 = \left(\frac{1}{2} b \cdot k \cdot T\right)^{\frac{2}{3}} \approx 300\ \mathrm{keV}$$

  - k = Bolzmann constant
  - $b = \pi\alpha Z_1 Z_2 \sqrt{2\mu c^2}$
  - $\mu = \frac{M_1 M_2}{M_1 + M_2}$ reduced mass

- Gamow Width

$$\Delta = 4\sqrt{E_0 kT/3}$$

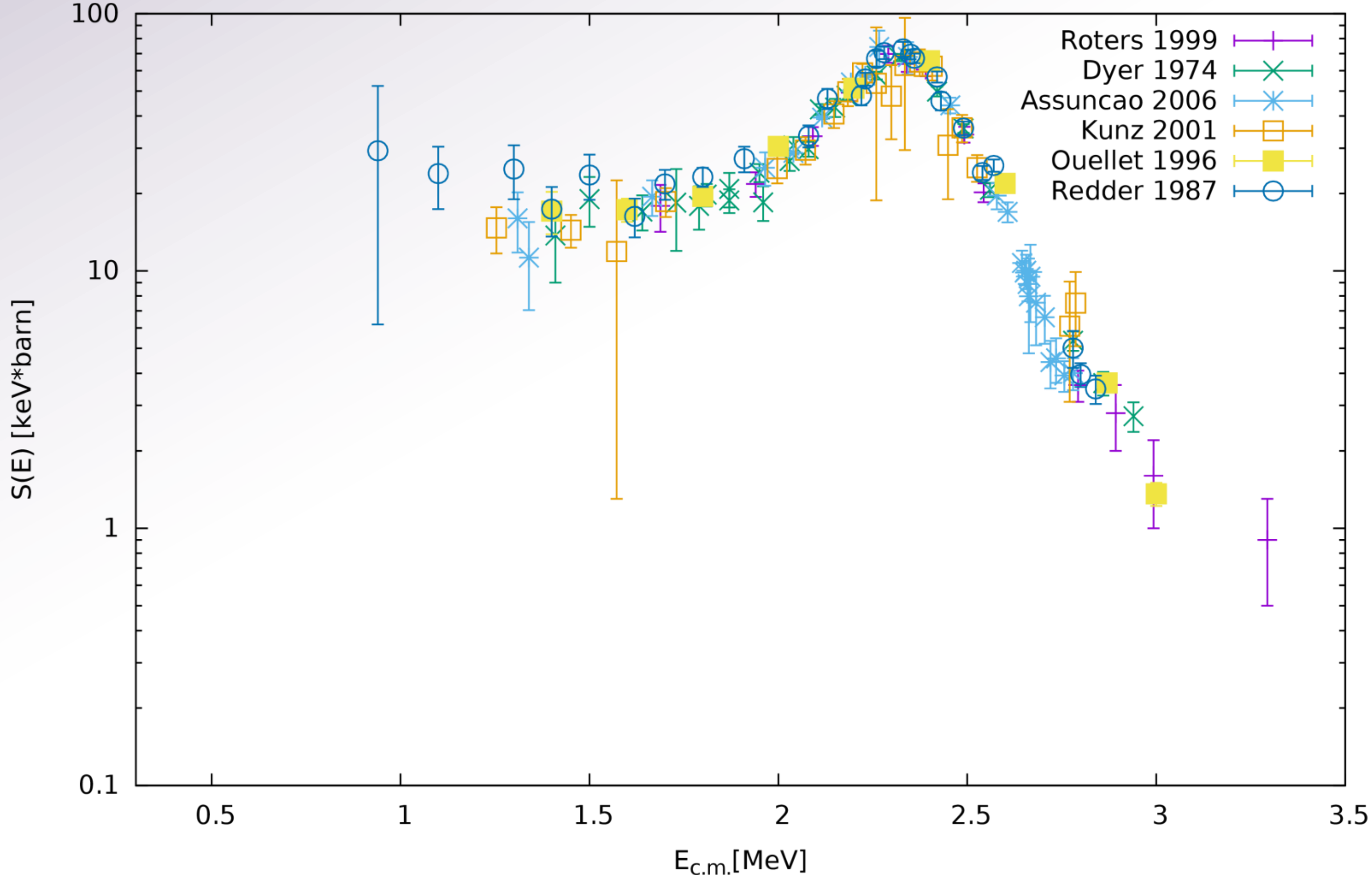
Cross section



- $\sigma(E_0) \sim 10^{-17}\text{barn}$
- Precise low-energy measurements required
  - ➢MAGIX@MESA
- Direct measurements never done @$E_{\text{cm}} < 0.9\ \text{MeV}$

Cp. Simulation of Ugalde 2013

Measurement of S-Factor

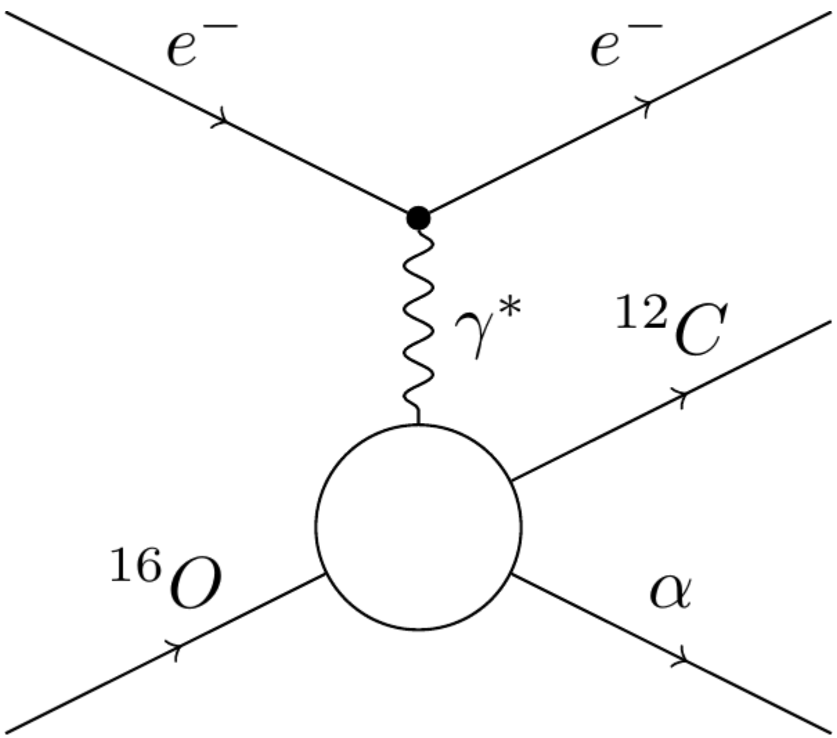


Approximate $S(300\ \mathrm{keV})$

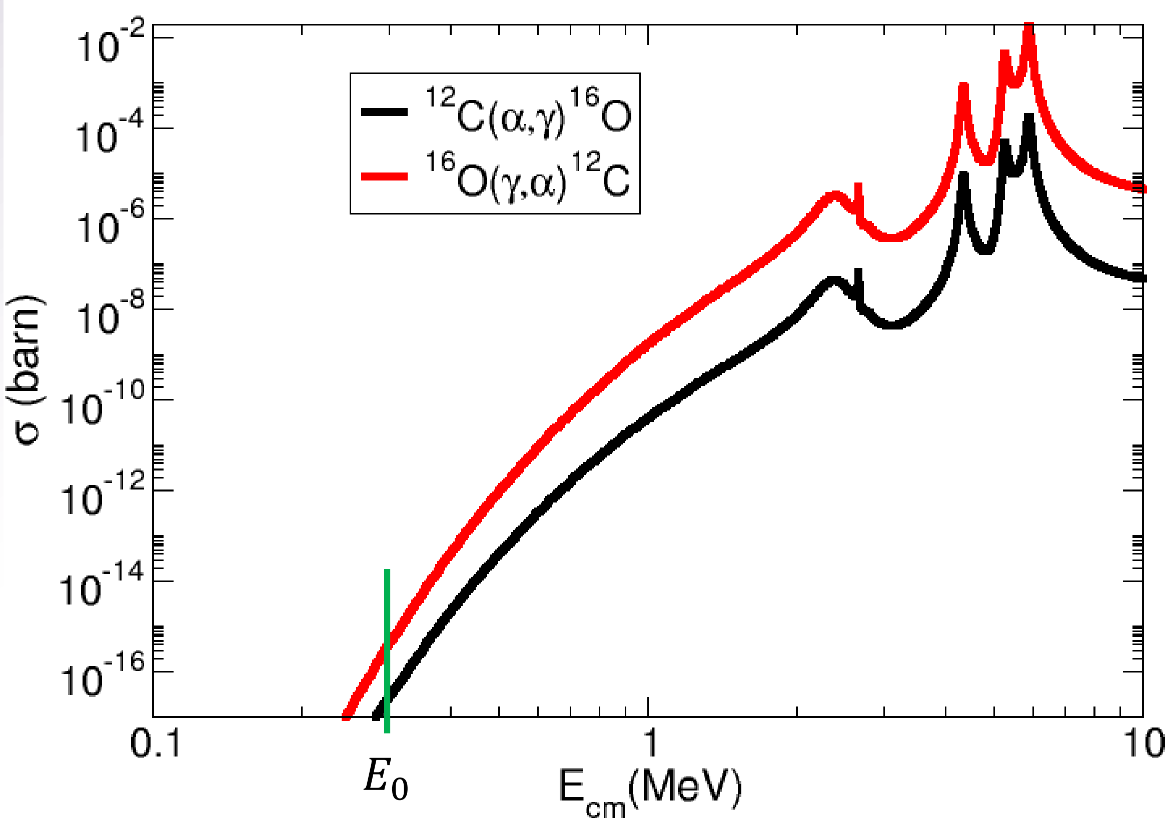
- Buchmann (2005)
  - $102 - 198\ \mathrm{keV{\cdot}b}$
- Caughlan and Fowler (1988)
  - $120 - 220\ \mathrm{keV \cdot b}$
- Hammer (2005)
  - $162 \pm 39\ \mathrm{keV{\cdot}b}$

Measurement at MAGIX@MESA

- Time reverted reaction $^{16}\mathrm{O}(\gamma,\alpha)^{12}\mathrm{C}$
  - ➢Cross section gain a factor of $\times$ 100
- Inelastic e^- scattering on oxygen gas
- Measurement of coincidence (e^-,α)
  - ➢suppress background
  - ➢α-Particle with low energy
- High Luminosity



Inverse Kinematik



- Time reversed reaction:

$$\sigma(E_0) \sim 10^{-15} \text{barn}$$

- High Energy resolution required
  - ➢ MAGIX

Cp. Simulation of Ugalde 2013

Simulation

Introduction

- MXWare *(see talk Caiazza)*
- Monte Carlo Integration
- Fix Beam Energy
- Target at Rest
- Simulation acceptance 4π

Kinematik

- Momentum transfer

$$q^2 = -4EE' \sin^2 \frac{\theta}{2}$$

- Photon Energy

$$\nu = \frac{W^2 - M^2 - q^2}{2M}$$

  with

  - $W^2 = (p_\gamma^\mu + p_O^\mu)^2$
    invariant mass of photon and oxygen
  - M = Oxygen mass

- Inelastic scattering cross section

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{q^4}\left[W_2(q^2,\nu)\cdot\cos^2\left(\frac{\theta}{2}\right) + 2W_1(q^2,\nu)\cdot\sin^2\left(\frac{\theta}{2}\right)\right]$$

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Virtual Photon flux

Relation beween structural functions and the transversal / longitudinal part of the virtual photon cross section σ_T, σ_L

$$W_1 = \frac{\kappa}{4\pi^2\alpha}\sigma_T \qquad W_2 = \frac{\kappa}{4\pi^2\alpha}\left(1-\frac{\nu^2}{q^2}\right)^{-1}(\sigma_L+\sigma_T) \qquad \text{with} \quad \kappa = \frac{W^2-M^2}{2M}$$

So we get

$$\frac{d^3\sigma}{d\Omega dE'} = \Gamma(\sigma_T + \varepsilon\sigma_L)$$

with

$$\Gamma = \frac{\alpha\kappa}{2\pi^2|q^2|}\cdot\frac{E'}{E}\cdot\frac{1}{1-\varepsilon} \qquad \varepsilon = \left(1-2\frac{\nu^2-q^2}{q^2}\tan^2\left(\frac{\theta}{2}\right)\right)^{-1}$$

For $|q^2| \to 0$: σ_L vanish and $\sigma_T \to \sigma^{\text{tot}}(\gamma^* + {}^{16}\text{O} \to X)$

$$\frac{d^5\sigma}{d\Omega_e dE' d\Omega^*} = \Gamma\frac{d\sigma_v}{d\Omega^*}$$

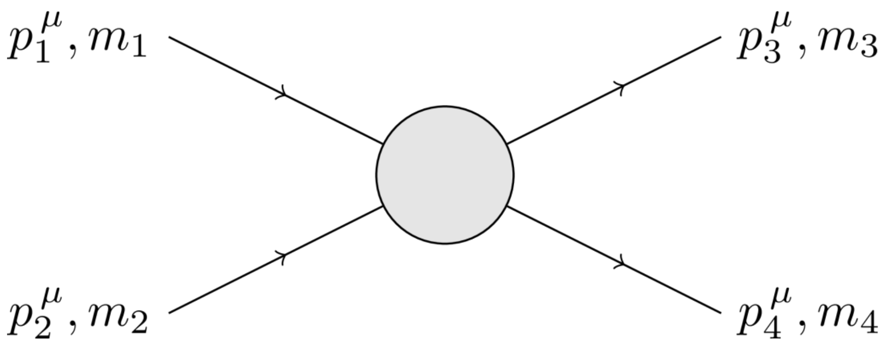
Cp. Halzen & Martin: Quarks and Leptons

Time reversal Factor

- Direct cross section -> Measurement
- Compare with inverse cross section -> extract the S-Factor
- Calculate time reversal factor

Time reversal Factor

Phase space examination under T-symmetry invariance



$$\frac{\sigma_{i\to f}}{\sigma_{f\to i}} = \frac{(2I_3+1)(2I_4+1)}{(2I_1+1)(2I_2+1)} \cdot \frac{|\vec{p}|_f^2}{|\vec{p}|_i^2}$$

Spinstatistic:

I=0 for even–even nuclides ($^4\mathrm{He}$, $^{12}\mathrm{C}$, $^{16}\mathrm{O}$) in ground state

$(2I_\gamma + 1) = 2$ for photon.

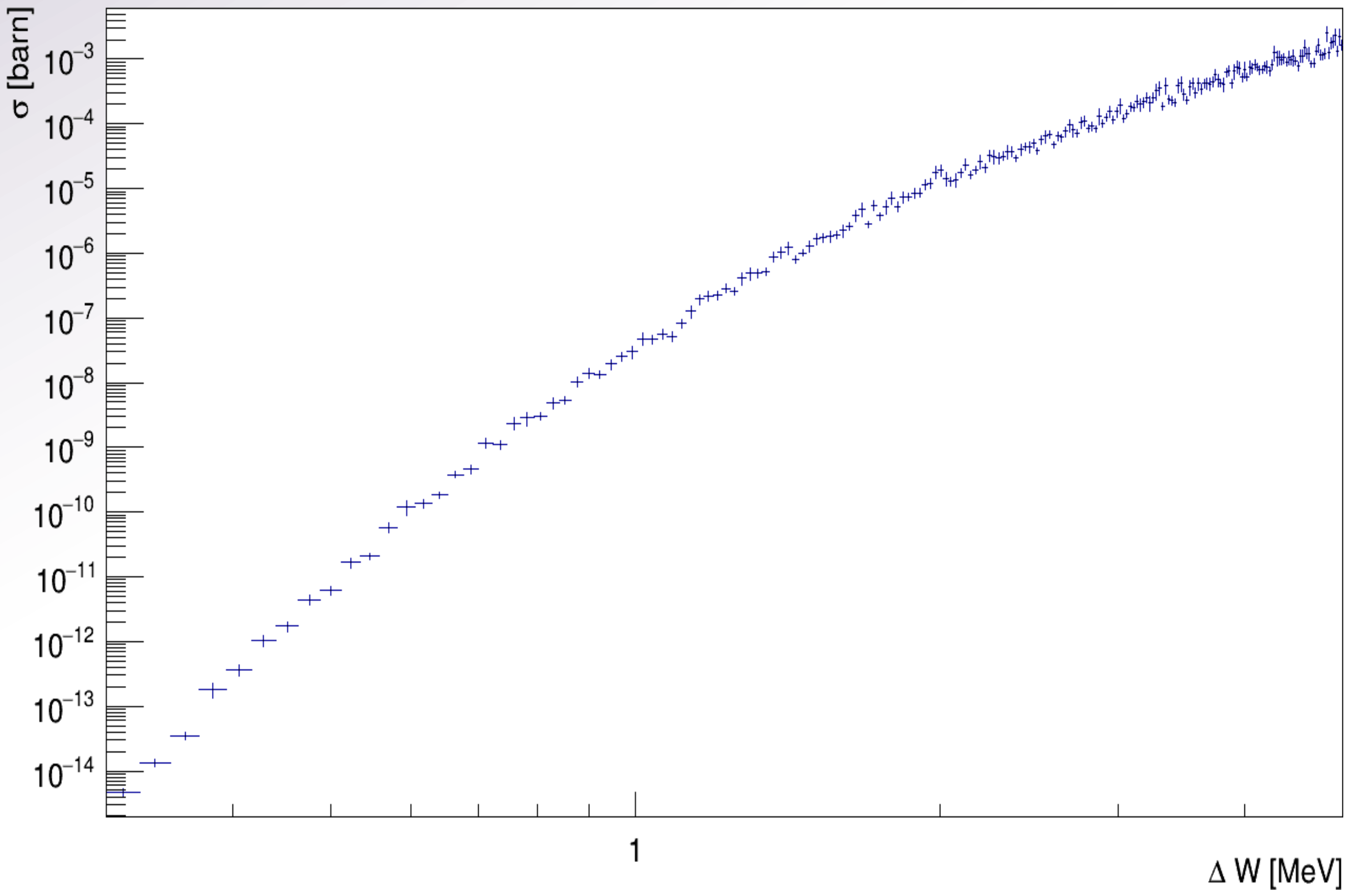
So we get

$$\sigma(^{16}\mathrm{O}(\gamma,\alpha)^{12}\mathrm{C}) = \frac{1}{2}\frac{\left(W^2 - (m_{\mathrm{He}} + m_{\mathrm{C}})^2\right)\left(W^2 - (m_{\mathrm{He}} - m_{\mathrm{C}})^2\right)}{(W^2 - m_{\mathrm{O}}^2)(W^2 - m_{\mathrm{O}}^2)} \cdot \sigma(^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O})$$

Cp. Mayer-Kuckuk Kernphysik: Chapter 7.3

Result of first simulations

Nonresonant cross section $\sigma(^{16}O(\gamma,\alpha)^{12}C)$



- Simulation correlate to the results of Ugalde
- 4π – Simulation
- $\sim 0.1\ \mathrm{mHz}$ Reaction Rate by E_0 with

$$L \sim 10^{34}\ cm^{-2}s^{-1}$$

  - ➢ Worst case Luminosity (see later talks)
- Now simulation with e^-, α –Acceptance needed.

Outlook

Simulation

- Finish simulation
  - electron acceptance
  - α-Particle acceptance
- Preliminary results
  - Need measurement on angles smaller than Spectrometer coverage
  - 0 degree scattering -> New Theoretic calculations

α-Detection

- Low kinetic energy
  - $\sim$ 20 MeV
- Needs specialized detector
  - Silicon-Strip-Detector
- Choose and Test Silicon-Strip-Detectors in the Lab

THANK YOU FOR YOUR ATTENTION!

http://magix.kph.uni-mainz.de

SFB 1044

PRISMA

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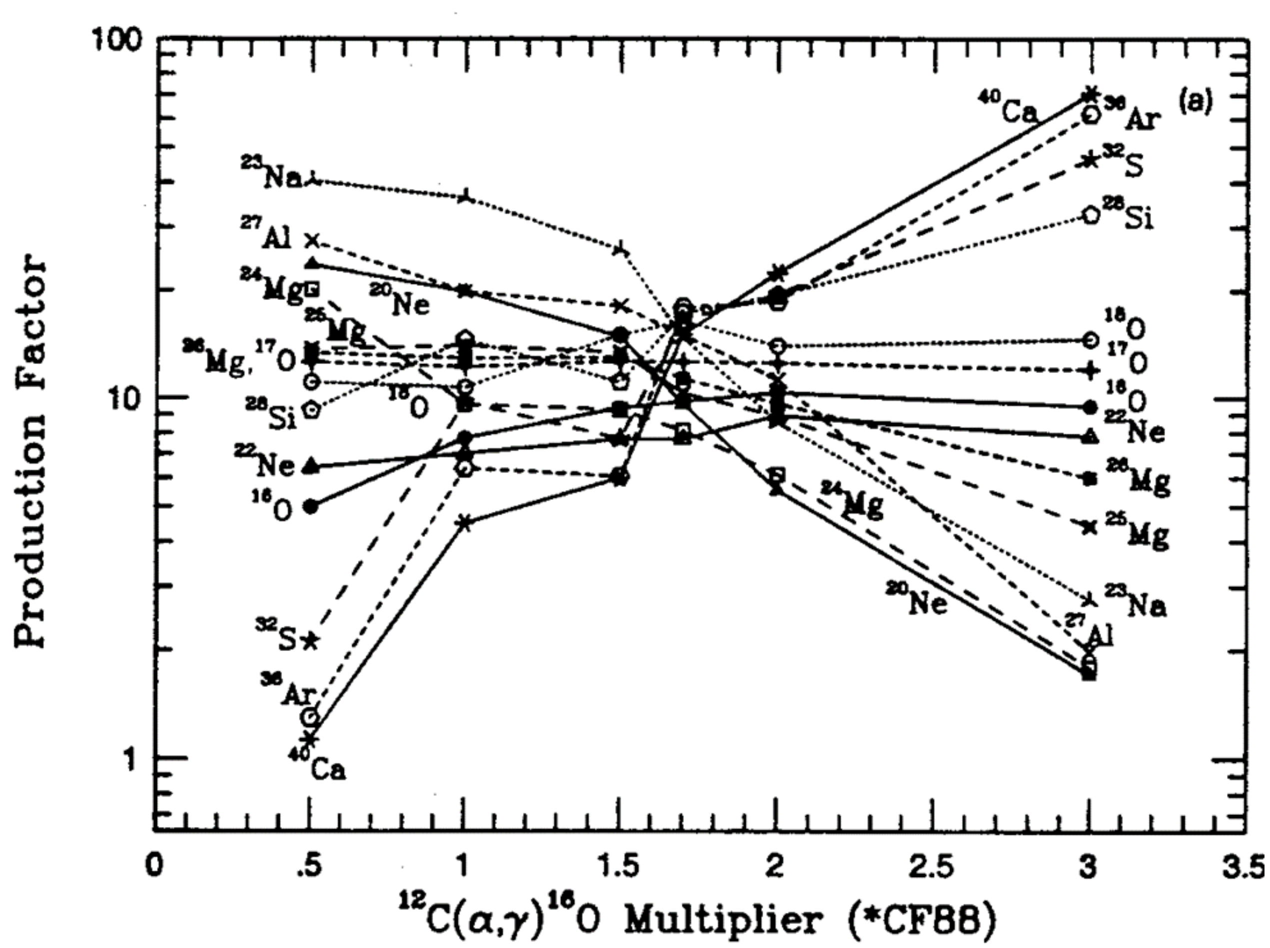
BACKUP

Production factor



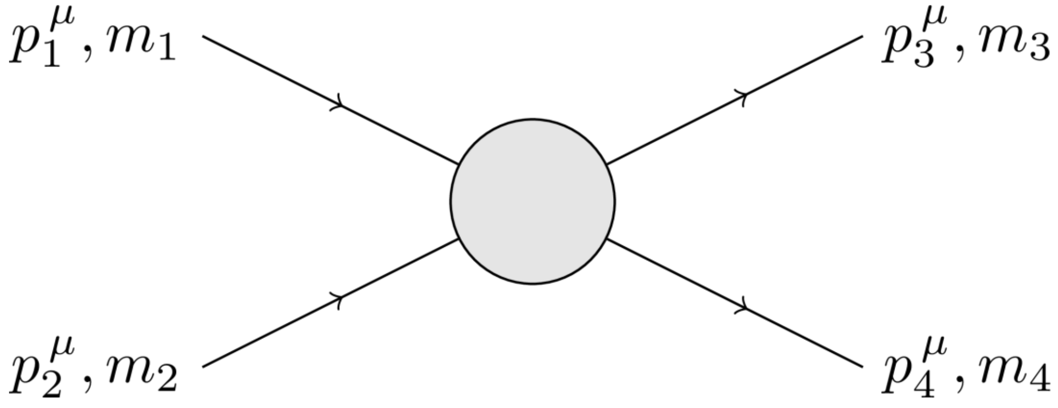
Waver and Woosley Phys Rep 227 (1993) 65

Two-Body Reaction

In the center of mass frame $^{16}O(\gamma^*,\alpha)^{12}C$

$$E_3 = \frac{W^2+m_3^2-m_4^2}{2W} \qquad E_4 = \frac{W^2+m_4^2-m_3^2}{2W}$$

$$|\vec{p}| = \sqrt{E^2-m^2} = \frac{\sqrt{(W^2-(m_3+m_4)^2)(W^2-(m_3-m_4)^2)}}{2W}$$



Electron scattering

Cross section inelastic scattering *(cp. Chapter 7.2)*

$$\frac{d^2\sigma}{d\Omega dE'} = \left(\frac{d\sigma}{d\Omega}\right)^*_{\text{Mott}} \left[W_2(q^2,\nu) + 2W_1(q^2,\nu)\tan^2\left(\frac{\theta}{2}\right)\right]$$

With structural functions W_1, W_2

And Mott crossection (in this case)

$$\left(\frac{d\sigma}{d\Omega}\right)^*_{\text{Mott}} = \frac{4\alpha^2 E'^2}{q^4}\cos^2\left(\frac{\theta}{2}\right)$$

We get *(cp. Halzen & Martin Chapter 8)*

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{q^4}\left[W_2(q^2,\nu)\cdot\cos^2\left(\frac{\theta}{2}\right) + 2W_1(q^2,\nu)\cdot\sin^2\left(\frac{\theta}{2}\right)\right]$$

Basic of Simulation

Connection between count rate and cross section

$$N = \int_{\Omega} A(\Omega) \frac{\mathrm{d}\sigma}{d\Omega} d\Omega \cdot \int L dt + N_{\mathrm{BG}}$$

With

L : Luminosity

N : Number of counts

$A(\Omega)$: Acceptance (1 full accepted, 0 not detected)

Monte Carlo Integration

Definition of mean value in volume V:

$$\langle f \rangle = \frac{1}{V} \int_V f(x) d^n x$$

Estimator for mean value:

$$\langle f \rangle \approx \frac{1}{N} \sum_{i=1}^{N} f(x_i)$$

Monte-Carlo Integration:

$$\int_V f(x) d^n x = \langle f \rangle \approx \frac{V}{N} \sum_{i=1}^{N} f(x_i) \pm \frac{V}{\sqrt{N}} \sqrt{\langle f^2 \rangle - \langle f \rangle^2}$$

Strategies for numerical improvements:

- Improve convergence $1/\sqrt{N}$
- Improve variance $\sqrt{\langle f^2 \rangle - \langle f \rangle^2}$

Cross section simulation

$$\int \frac{d\sigma}{d\Omega_e dE_e d\Omega^*} d\Omega_e dE_e d\Omega^*$$

Transform $(\Omega, E\ ) \rightarrow (W, 1/q^2, \phi)$ with $\det J = q^4 \frac{W}{2MEE'}$

With Monte-Carlo Integration:

$$\int \frac{d\sigma}{d\Omega_e dE_e d\Omega^*} d\Omega_e dE_e d\Omega^* = \frac{\mathrm{V}}{\mathrm{N}} \sum_i \det J \cdot \frac{d\sigma}{d\Omega_e dE_e d\Omega^*} (W, 1/q^2, \phi, \Omega^*)$$

Define $\omega_i = V \cdot \det J \cdot \frac{d\sigma}{d\Omega_e dE_e d\Omega^*}$

So we get

$$\omega_i = q^4 \frac{W}{2MEE'} \cdot v \cdot \Delta\phi \cdot \Delta W \cdot \Delta \cos\theta^* \cdot \Delta\phi^* \cdot \Gamma \cdot \frac{d\sigma_v}{d\Omega^*}$$