Applications of chiral perturbation theory: electromagnetic properties of baryons

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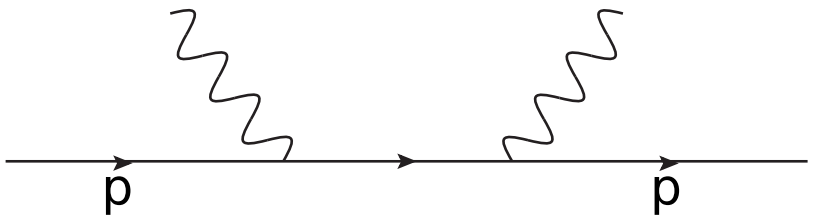
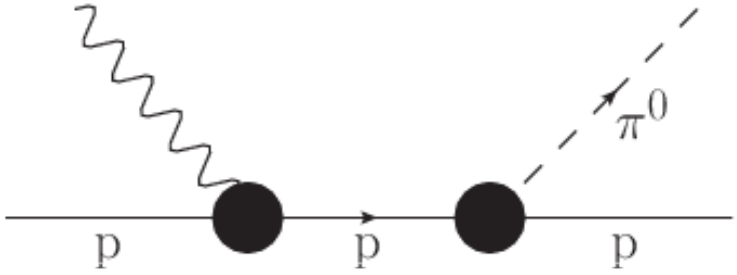
Photon beams

Electromagnetic interactions provide **clean probes** of the inner structure of hadrons

- Low photon energies ($\sim$ 100 MeV): Compton scattering



- Slightly higher ($\gtrsim$ 140 MeV): pion photoproduction



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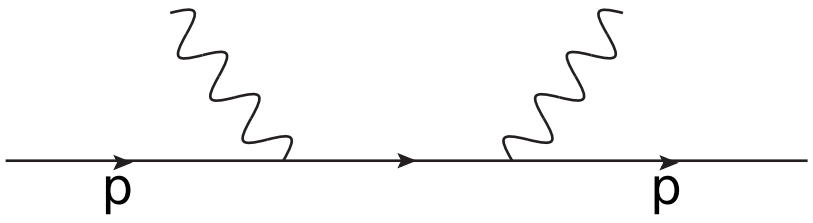
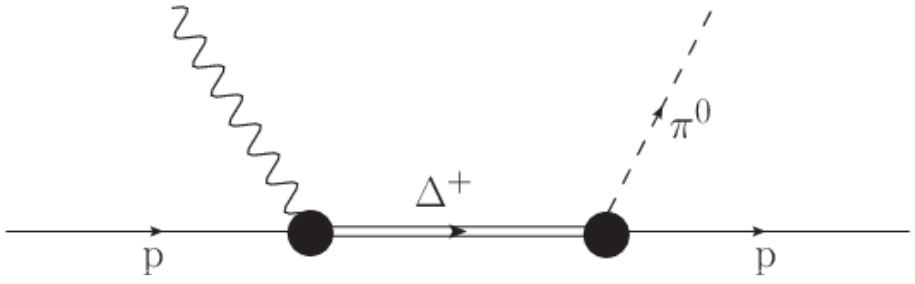
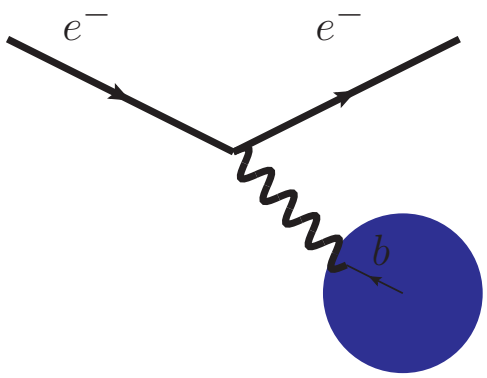


- Even higher: start feeling resonance production



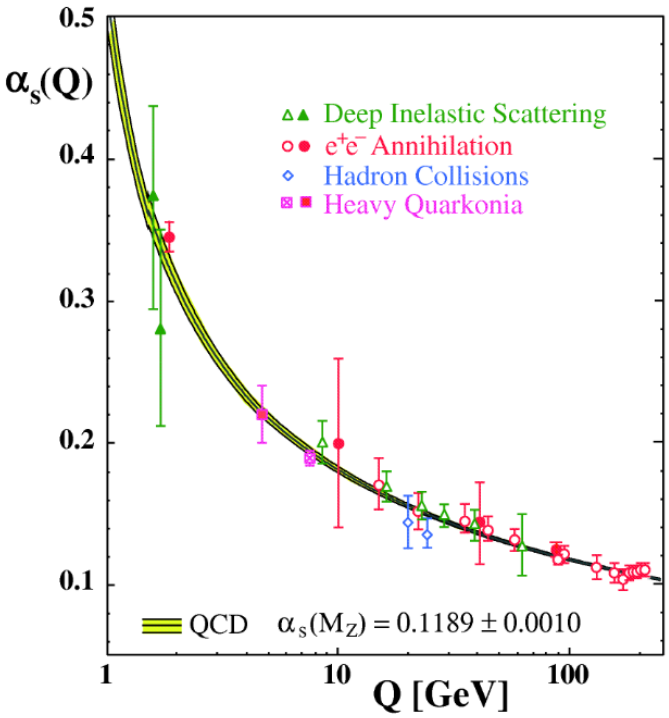
Virtual photons

- E.g. elastic electron scattering



- For all these processes we focus on:
  small external momenta/momentum transfer

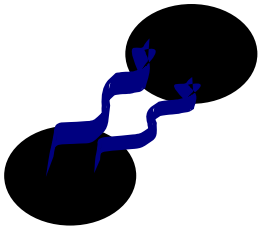
Non-perturbative QCD vs. chiral perturbation theory

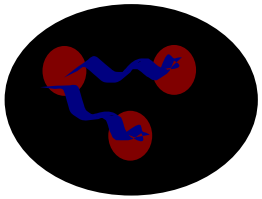


$$E_\gamma \approx \mathcal{O}(m_\pi) \Rightarrow \alpha_s = \mathcal{O}(1)$$

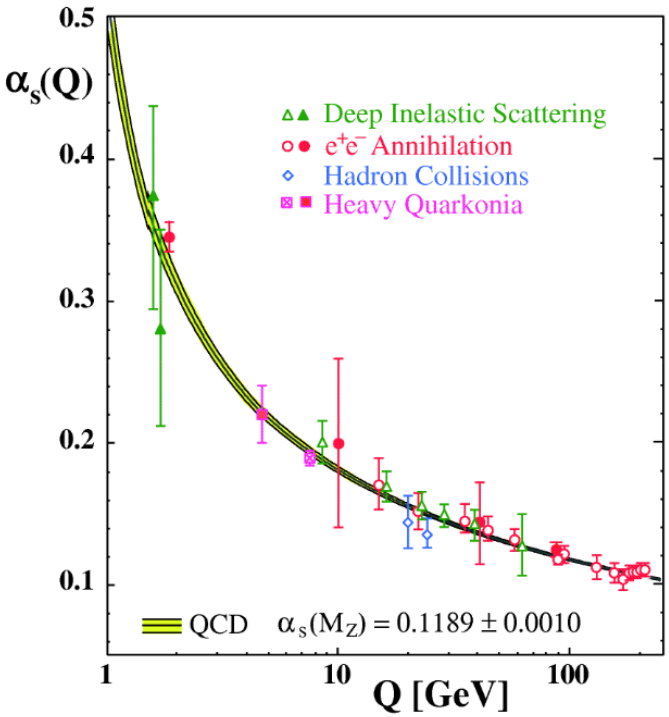
Perturbative QCD breaks down

$\Longrightarrow$ EFT: expansion around **other parameters**





Non-perturbative QCD vs. chiral perturbation theory



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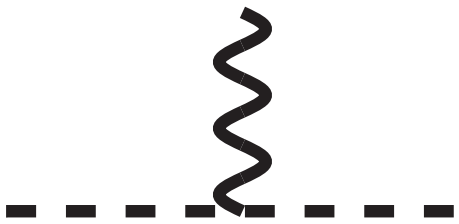
Chiral perturbation theory:

- Small masses, momenta ($\frac{m_\pi}{1\text{ GeV}}, \frac{p_{\text{ext}}}{1\text{ GeV}} \ll 1$): combined expansion
- New degrees of freedom: ~~quarks and gluons~~ $\Longrightarrow$ mesons and baryons

Lagrangians of ChPT

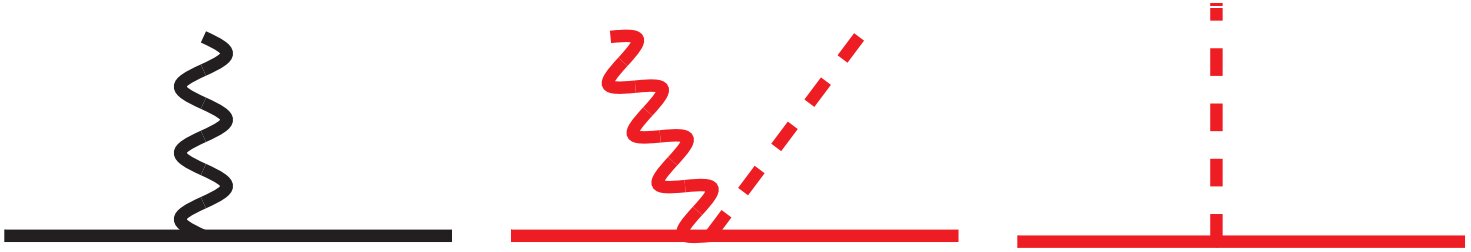
Lowest-order **meson** Lagrangian $\sim p_{\text{ext}}^2,\ m_\pi^2$



$$\mathcal{L}^{(2)}_{\phi\phi\gamma} = \frac{F_0^2}{4}\text{Tr}\left(\nabla_\mu U \nabla^\mu U^\dagger + \chi_+\right)$$

Lowest-order **baryon** Lagrangian $\sim p_{\text{ext}}$

$$\mathcal{L}^{(1)}_{\phi B\gamma} = \text{Tr}\left(\bar{B}(\mathrm{i}\not{D} - m)B\right) + \frac{D}{2}\text{Tr}\left(\bar{B}\gamma^\mu\left\{u_\mu, B\right\}\gamma_5\right) + \frac{F}{2}\text{Tr}\left(\bar{B}\gamma^\mu\left[u_\mu, B\right]\gamma_5\right)$$



Inclusion of the spin-3/2 resonances

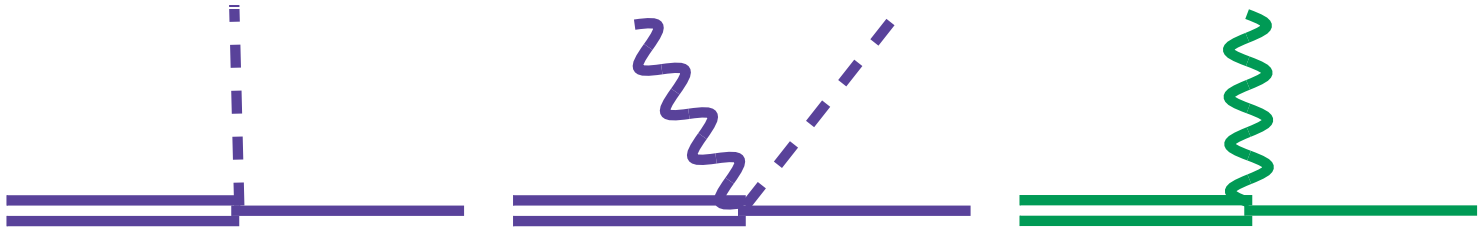
The spin-3/2 states **couple strongly** to the spin-1/2 octet baryons

Pascalutsa et al., Phys. Rept. 437 (2007) 125

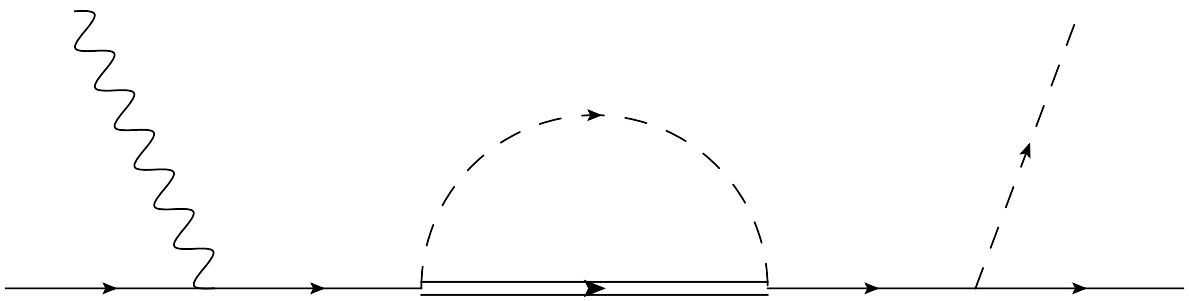
Geng et al., Phys. Lett. B 676 (2009) 63

$$\mathcal{L}^{(1)}_{\Delta\phi B} = \frac{-\mathrm{i}\sqrt{2}\,\mathcal{C}}{F_0 M_\Delta}\bar{B}^{ab}\varepsilon^{cda}\gamma^{\mu\nu\lambda}(\partial_\mu\Delta_\nu)^{dbe}(\mathrm{D}_\lambda\phi)^{ce} + \text{H.c.}$$

$$\mathcal{L}^{(2)}_{\Delta\gamma B} = -\frac{3\mathrm{i}e\,g_M}{\sqrt{2}m(m+M_\Delta)}\bar{B}^{ab}\varepsilon^{cda}Q^{ce}(\partial_\mu\Delta_\nu)^{dbe}\tilde{F}^{\mu\nu} + \text{H.c.}$$



Matching a diagram to a specific order



$$O = 4L + \sum kV_k - \mathbf{2N_\pi} - \mathbf{N_N} - N_\Delta \cdot ?$$

- Propagators: meson $\sim \mathbf{m_\pi^{-2}}$, spin-1/2 baryon $\sim \mathbf{p_{ext}^{-1}}$
- Spin-3/2 baryon: new scale $\delta = M_\Delta - m_N \approx 0.3$ GeV $> m_\pi$
- $(\delta/m_p)^2 \approx (m_\pi/m_p) \Longrightarrow$ far from resonance mass: $\mathbf{?} = \frac{1}{2}$

  Pascalutsa and Phillips, Phys. Rev. C 67 (2003) 055202
- Close to resonance mass: $p_{\text{ext}} \sim \delta \Longrightarrow \mathbf{?} = 1$

  Hemmert et al., Phys. Lett. B 395 (1997) 89

Renormalization

- Loop diagrams:
  divergences and **power counting breaking terms**

$$\frac{1}{\epsilon} = \frac{1}{4 - \text{dim}} \quad \text{and} \quad \text{e.g. terms } \propto p^2 \text{ at } \mathcal{O}(p^3)$$

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- **Low-energy constants** of these terms a priori unknwon

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- Fully analytical $\Longrightarrow$ match with **Lagrangian terms**
- **Low-energy constants** of these terms a priori unknwon
- EOMS-renormalization prescription:

  Gegelia and Japaridze, Phys. Rev. D 60 (1999) 114038

  - $\overline{MS}$ absorbs $L = \frac{2}{\epsilon} + \log(4\pi) - \gamma_E$ into LECs
  - Also subtracts PCBT by redefinition of LECs
  - Usually converges faster than other counting schemes (relativistic or not)

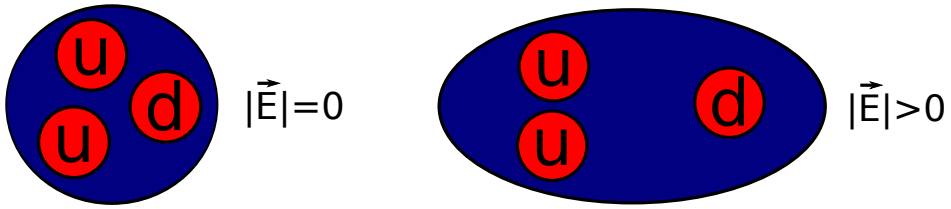
Compton scattering

and

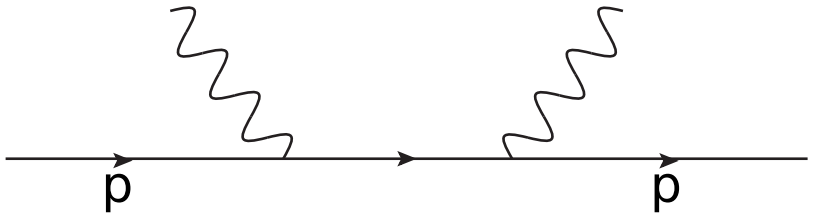
polarizabilities

Hiller Blin, Gutsche, Ledwig and Lyubovitskij
Phys. Rev. D 92 (2015) 096004
arXiv: 1509.00955 [hep-ph]

Polarizabilities



- In EM field: hadrons deformed due to charged components
- Size of deformation: related to polarizabilities



- Experiment: Compton scattering off hadron targets

Theoretical approach

- Amplitude expansion **around low photon energy** ω
  - $\mathcal{O}(\omega^0)$: total charge
  - $\mathcal{O}(\omega^1)$: anomalous magnetic moment
  - $\mathcal{O}(\omega^2)$: α_E and β_M
  - $\mathcal{O}(\omega^3)$: spin-dependent polarizabilities γ_i

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  - response to deformation relative to **spin axis**
  - photon scattering in **extreme forward direction**

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- Theory: Hemmert et al., Phys. Rev. D 57 (1998) 5746

$$\gamma_0 \left[\vec{\sigma} \cdot (\vec{\epsilon} \times \vec{\epsilon}^{\,*})\right] = -\frac{\mathrm{i}}{4\pi} \frac{\partial}{\partial \omega^2} \frac{\epsilon^\mu \mathcal{M}^{\mathrm{SD}}_{\mu\nu} \epsilon^{*\nu}}{\omega} \bigg|_{\omega=0}$$

Experimental extraction

- Sum rule: Gell-Mann et al., Phys. Rev. 95 (1954) 1612

$$\gamma_0 = -\frac{1}{4\pi^2}\int_{\omega_0}^{\infty} \mathrm{d}\omega \frac{\sigma_{3/2}(\omega) - \sigma_{1/2}(\omega)}{\omega^3}$$

$\sigma_{3/2}(\sigma_{1/2})$: photon and target helicities are (anti)parallel

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- Experiment: Pasquini et al., Phys. Lett. B 687 (2004) 160
  $\gamma_0^p = [-1.01 \pm 0.08(\text{stat}) \pm 0.10(\text{syst})] \cdot 10^{-4}\text{fm}^4$

- Dispersion relations: Drechsel et al., Phys. Rept. 378 (2003) 99
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- **First goal** is to reproduce these values theoretically

- Then extend the theoretical model to **predict** polarizabilities of not yet measured states $\Longrightarrow$ **hyperons**

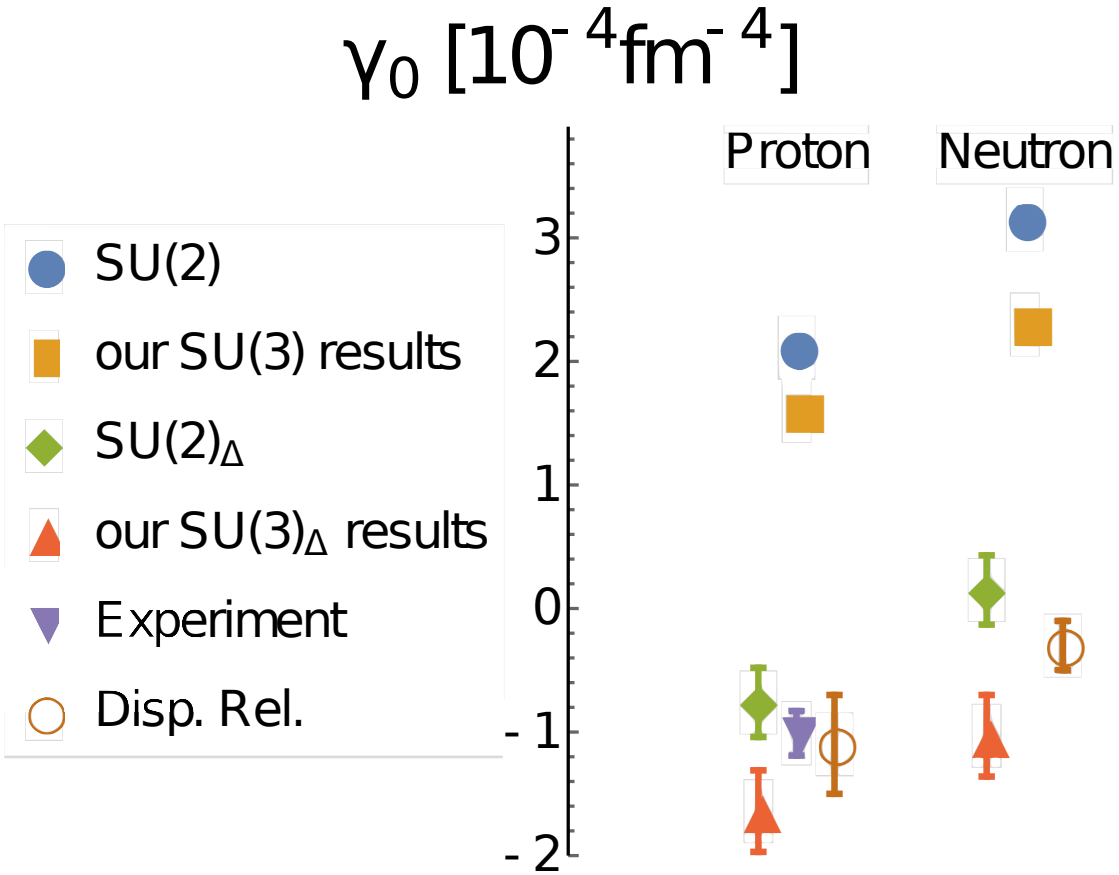
Renormalization

- ∞ and PCBT: **do not** enter pieces $\sim \omega^3$ relevant for γ_0
- Leading order for $\gamma_0 \Longrightarrow$ **no unknwon LECs**
- Results independent of renormalization or unknown LECs

$$\Downarrow$$

pure predictions of ChPT

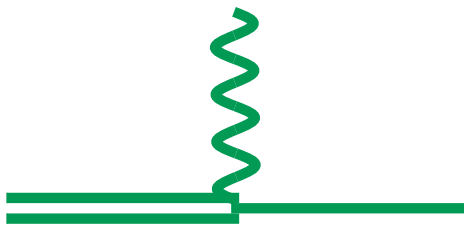
Results with different covariant ChPT models



SU(2): Bernard et al., Phys. Rev. D 87 (2013) 054032
SU(2) with Δ: Lensky et al., Eur. Phys. J. C 75 (2015) 604
Experiment: Pasquini et al., Phys. Lett. B 687 (2004) 160
Dispersion relations: Drechsel et al., Phys. Rept. 378 (2003) 99

Results for the hyperons

γ_0 [fm^{-4}10^{-4}]	Σ^+	Σ^-	Σ^0	Λ	Ξ^-	Ξ^0
Our full model	-2.30(33)	0.90	0.47(8)	-1.25(25)	0.13	-3.02(33)

- $\mathbf{g_M}$ not well known
- We estimate it from electromagnetic decay width $\Gamma_{\Delta\to\gamma N}$
-  $g_M = 3.16(16)$

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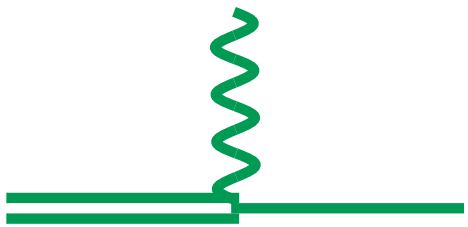
-  $g_M = 3.16(16)$

Electromagnetic transition of **negatively charged** hyperons to spin-3/2 partners $SU(3)$ **forbidden** $\Longrightarrow$ no uncertainty from g_M

Virtual photons

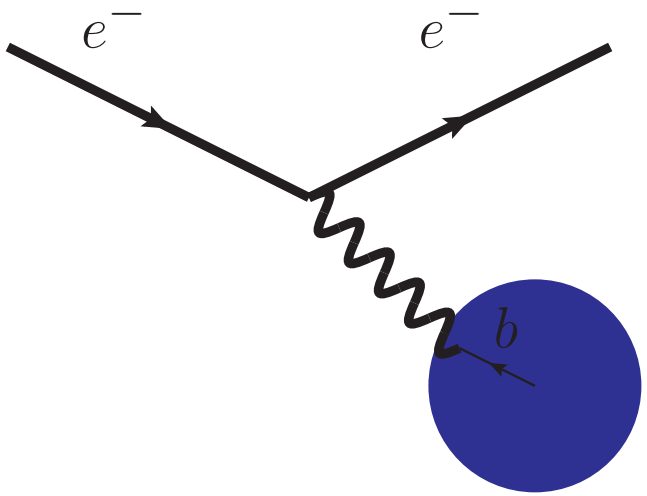
and

form factors

Alarcón, Hiller Blin, Vicente Vacas and Weiss
Nucl. Phys. A 964 (2017) 18
arXiv: 1703.04534 [hep-ph]

Form factors

- Matrix decomposition of the amplitude



$$\gamma^\mu F_1(Q^2) + \frac{\mathrm{i}\sigma^{\mu\nu} q_\nu}{2m} F_2(Q^2) \boxed{+\frac{\mathrm{i}\sigma^{\mu\nu}\gamma_5 q_\nu}{2m} F_{\mathrm{EDM}}(Q^2)}$$

***CP* violating**

Form factors

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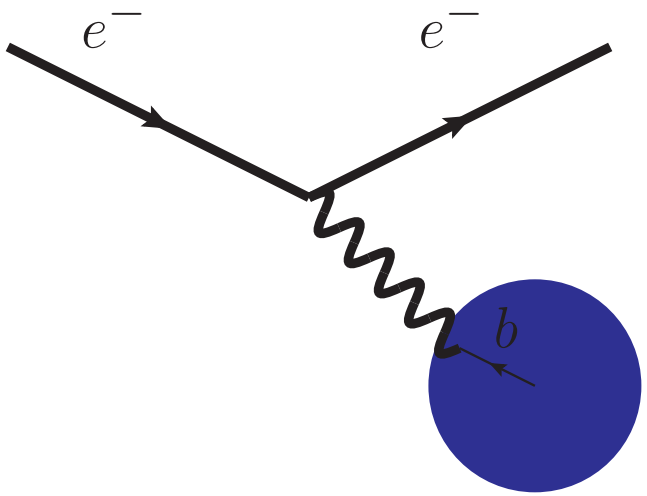
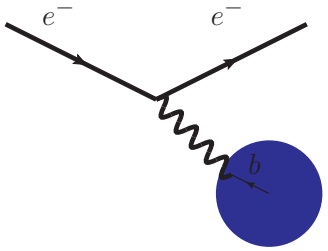


$$\gamma^\mu F_1(Q^2) + \frac{\mathrm{i}\sigma^{\mu\nu} q_\nu}{2m} F_2(Q^2)$$

- **Non-relativistic systems:**
  Fourier transforms of 3-dimensional spatial densities
- **Relativistic systems:** vacuum fluctuations!
  The number of particles in the system is not a constant

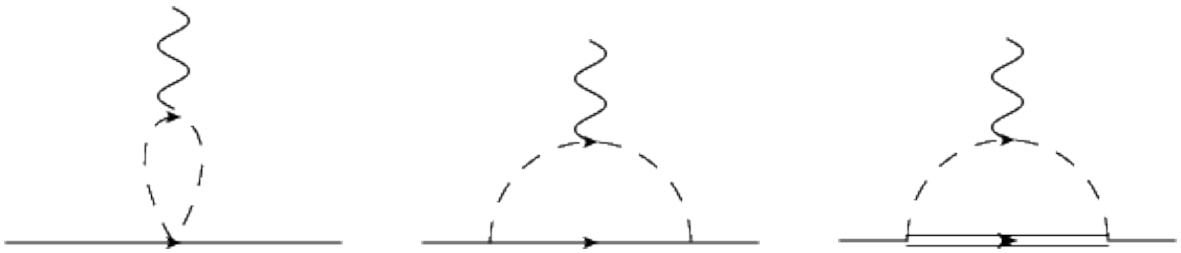
Dispersive representation of electromagnetic densities

Transverse densities decouple from vacuum fluctuations!



$$\rho_{1,2}(b) = \int_{4m_\pi^2}^{\infty} \frac{dt}{2\pi} K_0(\sqrt{t}b) \frac{\mathrm{Im}\, F_{1,2}(t)}{\pi}$$

Hohler et al., NPB114 505 (1976); Belushkin et al., PRC75 035202 (2007)



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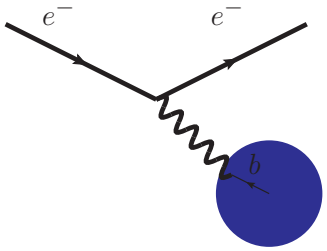

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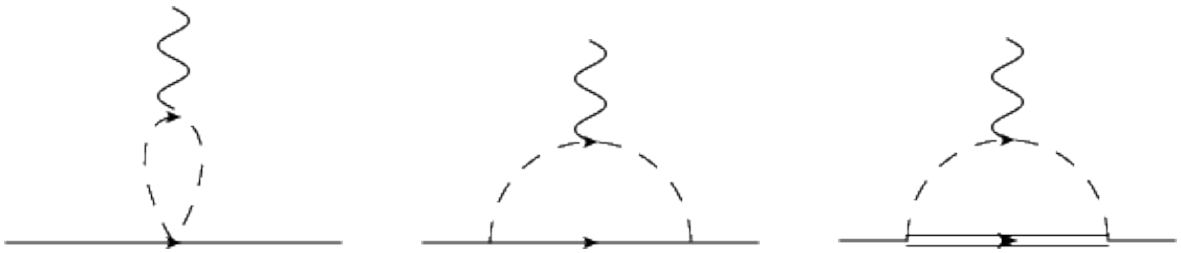


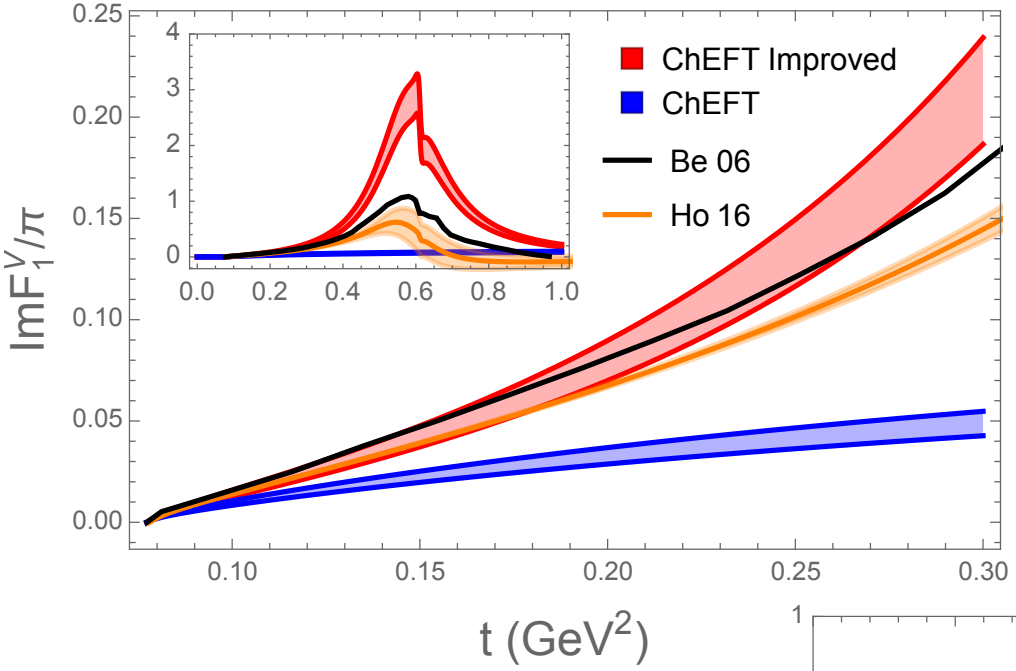
Bessel function $K_0 \sim e^{-b\sqrt{t}}$: **suppression at large t**

$\Longrightarrow$ Distance b is a filter of masses $\sqrt{t} \sim 1/b$

Strikman and Weiss PRC82 042201 (2010); Granados and Weiss JHEP1401 092 (2014)

Two-pion cut: low-mass states $\rightarrow$ **peripheral density**

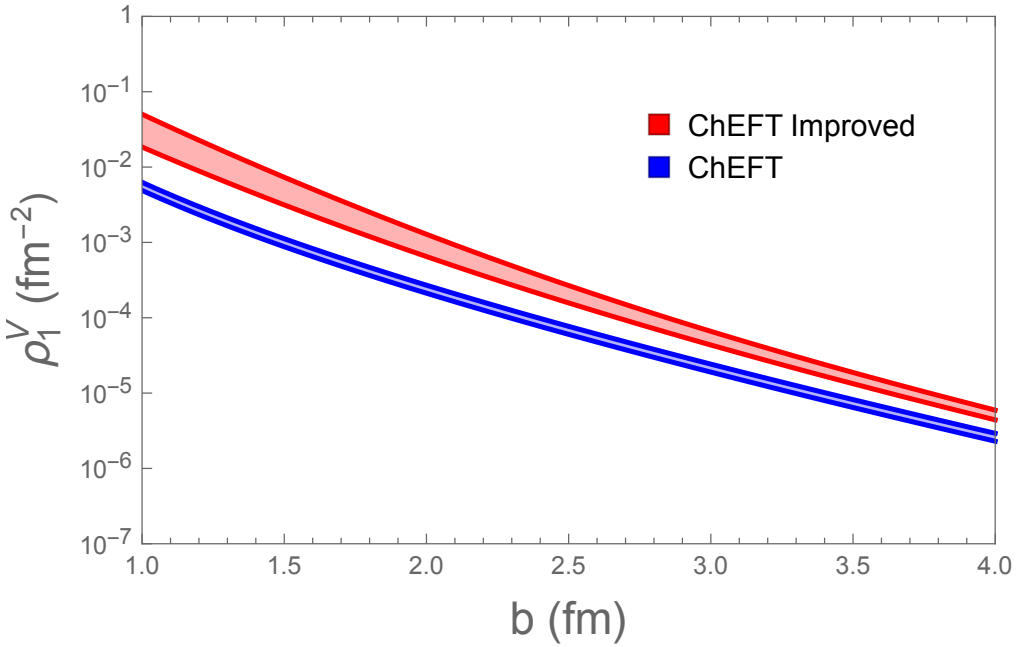
Proton form factor and spatial density



Analytic continuation

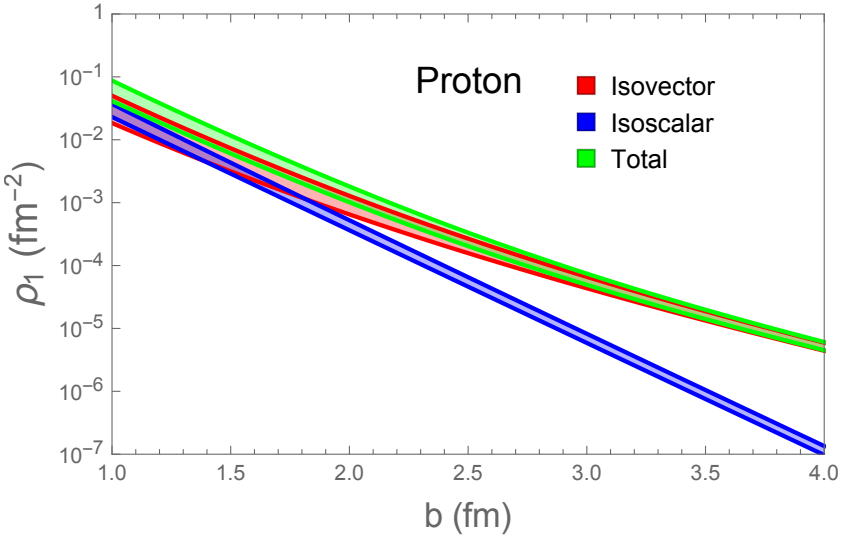
Roy-Steiner equations

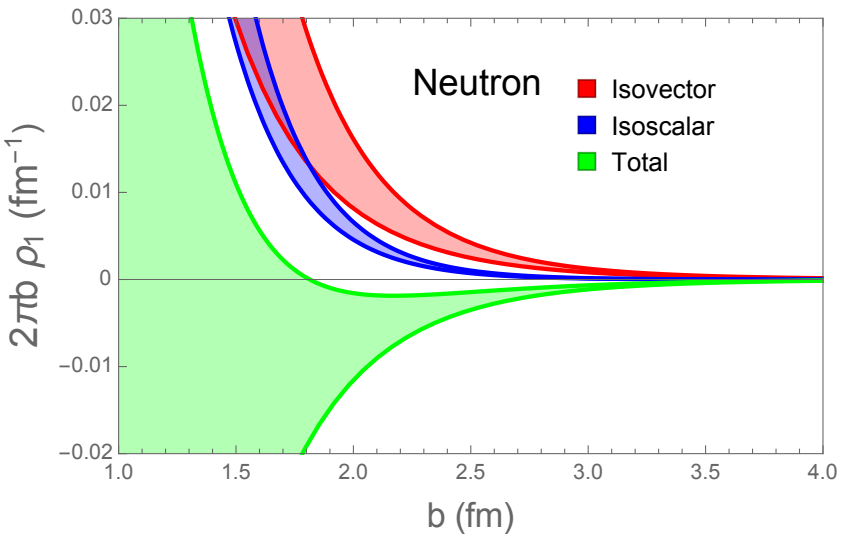
Improvement by one order of magnitude due to vector-meson inclusion



Nucleon charge densities

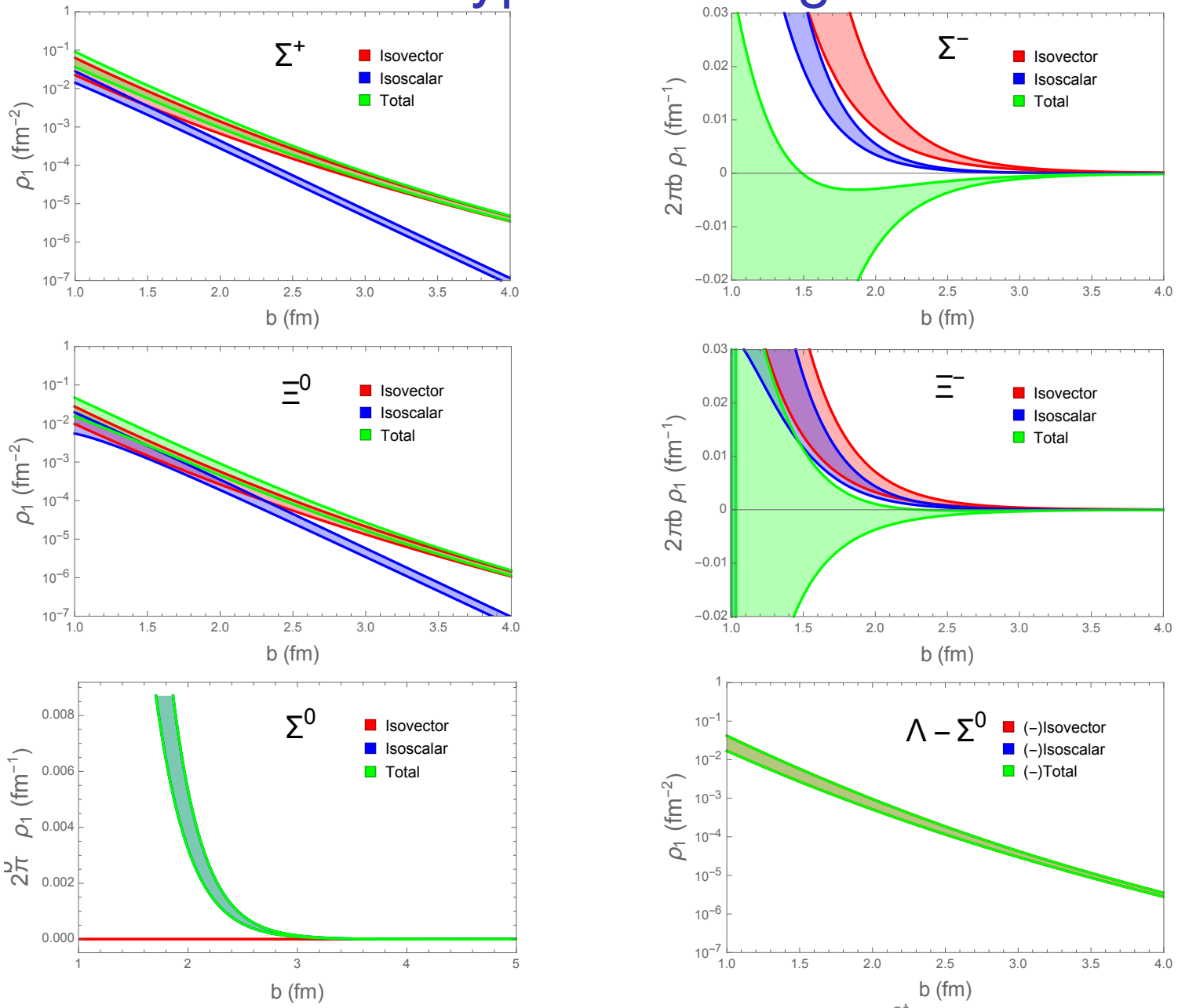
- Isovector component: 2π contributions (includes chiral piece and ρ-meson effects)
- Isoscalar component: $2K$ contributions, ω and ϕ mesons





$$\rho_1^p = \rho_1^S + \rho_1^V \qquad \rho_1^n = \rho_1^S - \rho_1^V$$

Hyperon charge densities



Summary

Framework

- Electromagnetic probes of light baryons in $SU(3)$ ChPT
- Covariant renormalization scheme: EOMS
- Explicit inclusion of the spin-3/2 resonances

Hyperon polarizabilities

- **Predictive** results for **hyperon** polarizabilities at $\mathcal{O}(p^{7/2})$
- Σ^- and Ξ^- do not depend on uncertainties from LECs
- **Outlook:** Other polarizabilities, photon virtuality, ...

Form factors

- Understanding about charge distributions in octet baryons
- **Outlook:** Δ and transition FFs, anomalous thresholds, ...

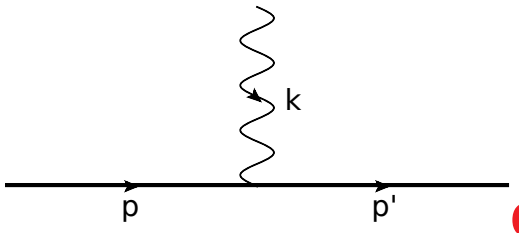
Additional material

Higher orders of the nucleonic Lagrangian

$$\mathcal{L}_N = \bar{\Psi}\Big\{ \frac{1}{8m}\left(\mathbf{c_6} f^+_{\mu\nu} + \mathbf{c_7}\,\mathrm{Tr}\left[f^+_{\mu\nu}\right]\right)\sigma^{\mu\nu}$$
$$+ \frac{\mathrm{i}}{2m}\varepsilon^{\mu\nu\alpha\beta}\left(\mathbf{d_8}\,\mathrm{Tr}\left[\tilde{f}^+_{\mu\nu}u_\alpha\right] + \mathbf{d_9}\,\mathrm{Tr}\left[f^+_{\mu\nu}\right]u_\alpha + \mathrm{H.c.}\right)\mathrm{D}_\beta$$
$$+ \frac{\gamma^\mu\gamma_5}{2}\left(\mathbf{d_{16}}\,\mathrm{Tr}\left[\chi_+\right]u_\mu + \mathrm{i}\,\mathbf{d_{18}}[\mathrm{D}_\mu, \chi_-]\right)\Big\}\Psi + \ldots$$

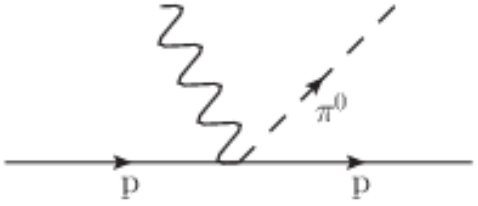
Fettes et al., Ann. Phys. 283 (2000) 273

$\mathcal{O}(\mathbf{p^2})$

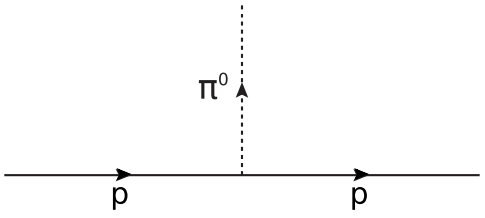


$\mathbf{c_6}$, $\mathbf{c_7}$

$\mathcal{O}(\mathbf{p^3})$

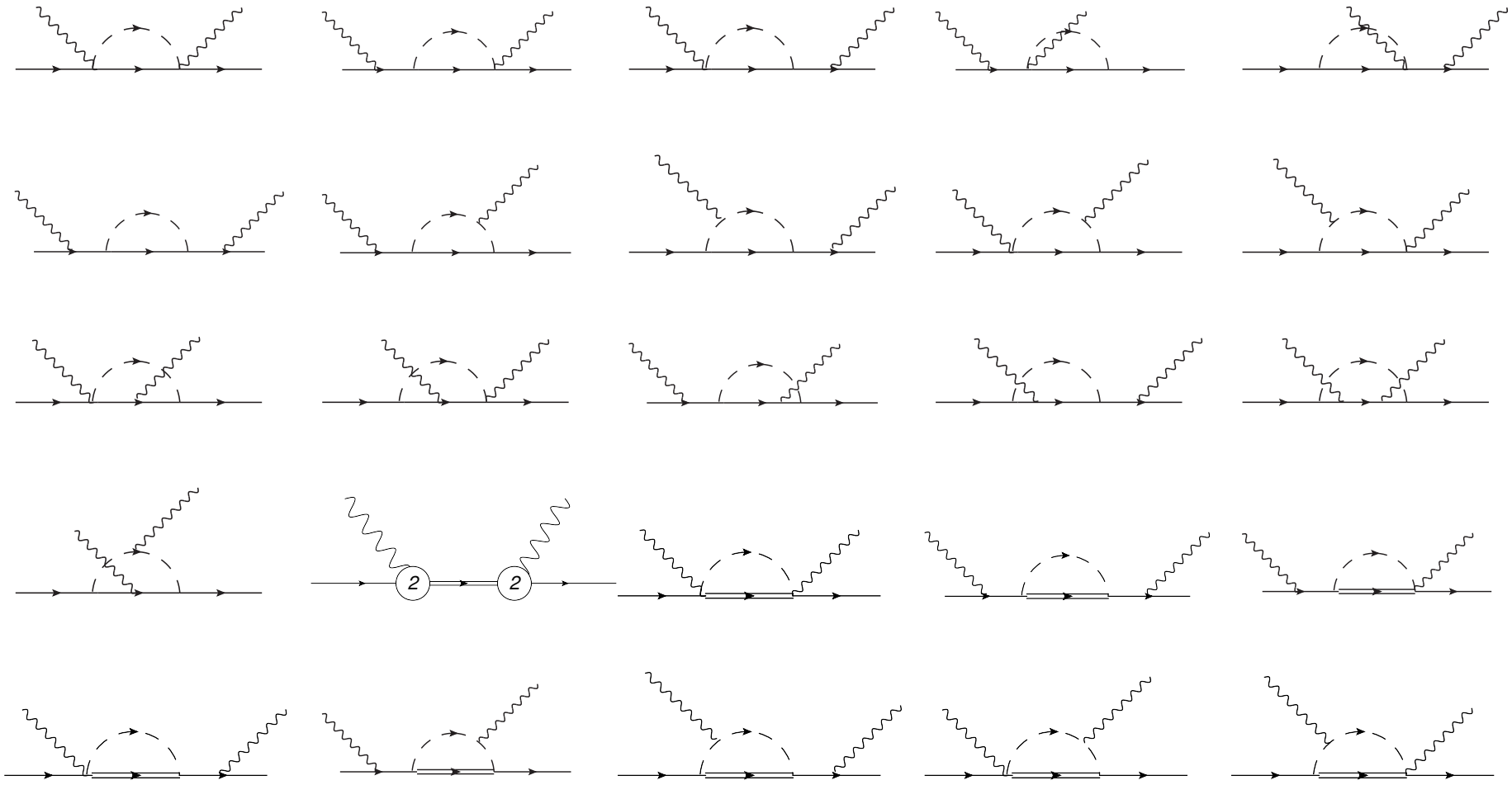


$\mathbf{d_8}$, $\mathbf{d_9}$

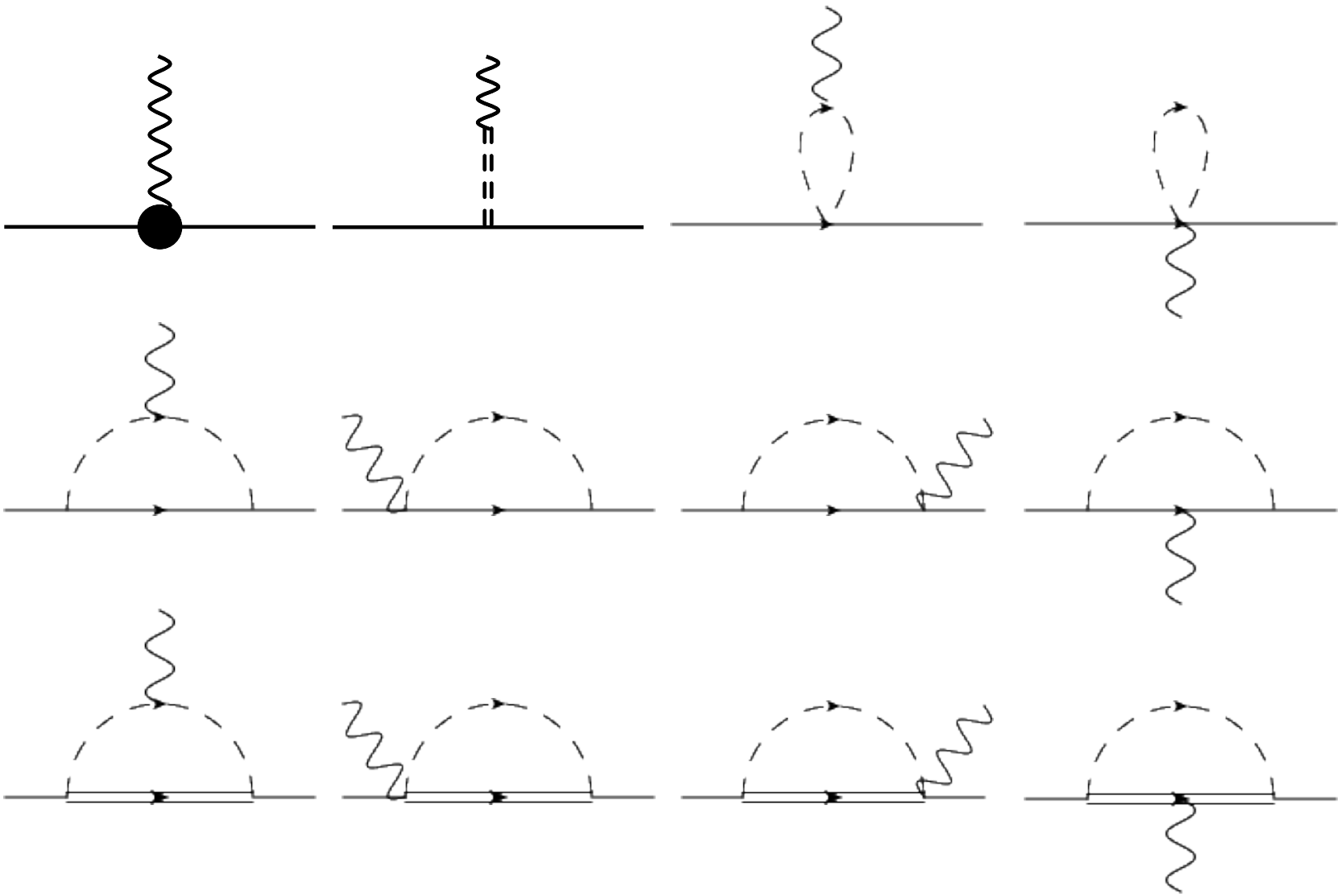


$\mathbf{d_{16}}$, $\mathbf{d_{18}}$

Diagrams contributing to γ_0 at leading loop order



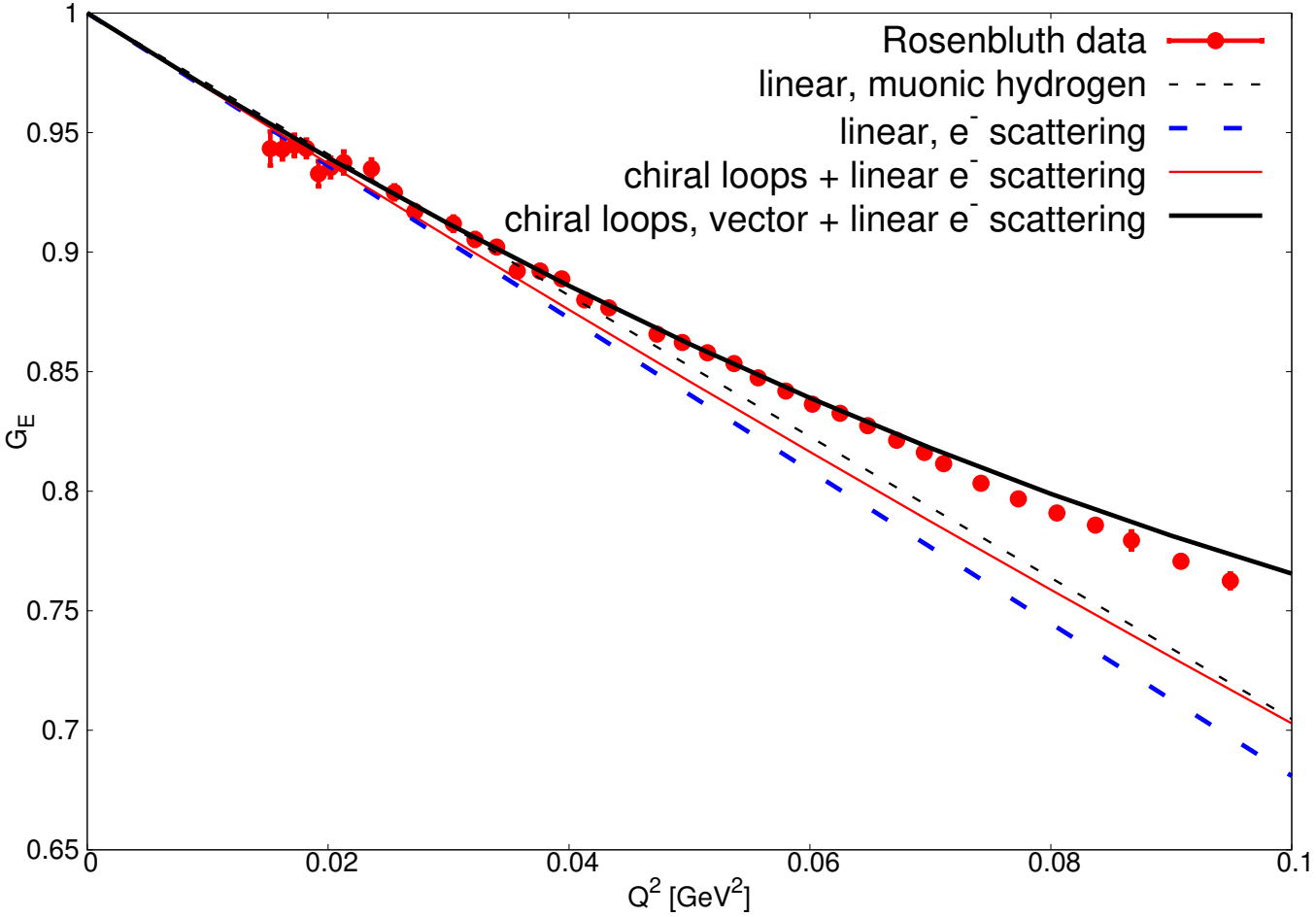
Contributions to form factors up to first loop order



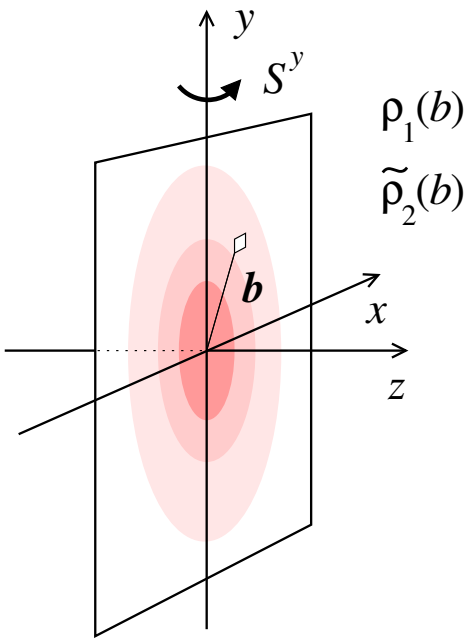
Electromagnetic form factors

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_{B0}^2} F_2(q^2)$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$



Transverse densities



- **Fixed light-front time:** $x^+ = x^0 + x^3$

  Soper, PRD15 1141 (1977); Burkardt, PRD62 071503 (2000); Miller, PRC76 065209 (2007)

  For momentum transfer $\Delta^+ = \Delta^0 + \Delta^3 = 0$ **current not affected by vacuum fluctuations!**

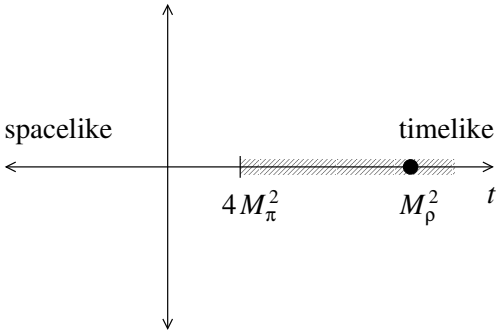
- Connection with general parton distributions

- Pure **transverse** momentum transfer $\boldsymbol{\Delta}_T = (\Delta^1, \Delta^2)$

$$F_{1,2}(t) = \int \mathrm{d}^2 b e^{\mathrm{i}\boldsymbol{\Delta}_T \cdot \boldsymbol{b}} \rho_{1,2}(b), \quad t = -|\boldsymbol{\Delta}_T|^2$$

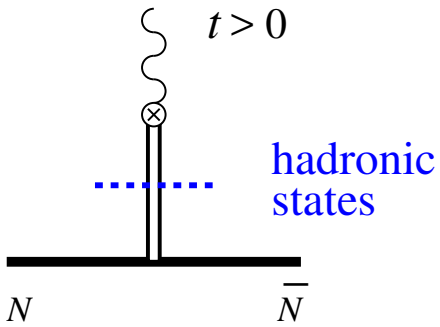
$$\langle J^+(\boldsymbol{b})\rangle_{y\text{-pol}} \sim \underbrace{\rho_1(b)}_{\text{spin-independent}} + \underbrace{(2S^y)\cos\phi \overbrace{\frac{\mathrm{d}}{\mathrm{d}b}\left[\frac{\rho_2(b)}{2M_N}\right]}^{\tilde{\rho}_2(b)}}_{\text{spin-dependent}}$$

Dispersive representation



$$F_{1,2}(t) = \int_{4m_\pi^2}^{\infty} \frac{dt'}{t' - t - i0} \frac{\mathrm{Im}\, F_{1,2}(t')}{\pi}$$

$\mathrm{Im}\, F_1(t) =$



Unphysical region from theory

Hohler et al., NPB114 505 (1976);

Belushkin et al., PRC75 035202 (2007)

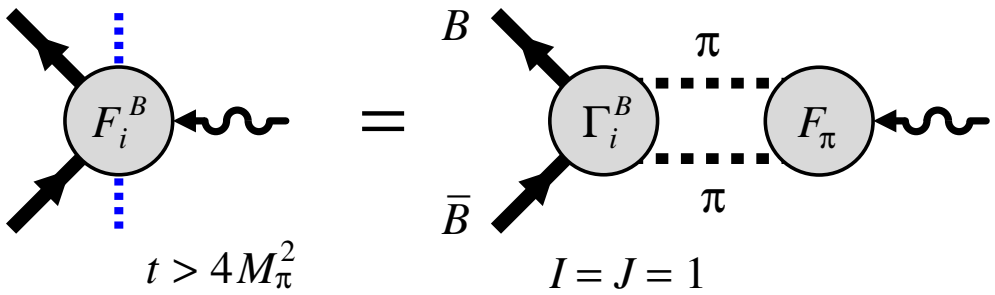
$$\rho_{1,2}(b) = \int_{4m_\pi^2}^{\infty} \frac{dt}{2\pi} K_0(\sqrt{t}b) \frac{\mathrm{Im}\, F_{1,2}(t)}{\pi}$$

Bessel function $K_0 \sim e^{-b\sqrt{t}}$: **suppression at large *t***

Distance b as filter of masses $\sqrt{t} \sim 1/b$

Dispersive improvement

- Chiral EFT works well for densities down to distances of 3 fm
- We want a good description down to 1 fm
- Include $\pi\pi$-rescattering effects — manifest in ρ resonance



$$\mathrm{Im} F_i^B(t) = \frac{k_{\mathrm{cm}}^3}{\sqrt{t}} \Gamma_i^B(t) F_\pi^*(t) = \frac{k_{\mathrm{cm}}^3}{\sqrt{t}} \frac{\Gamma_i^B(t)}{F_\pi(t)} |F_\pi(t)|^2$$

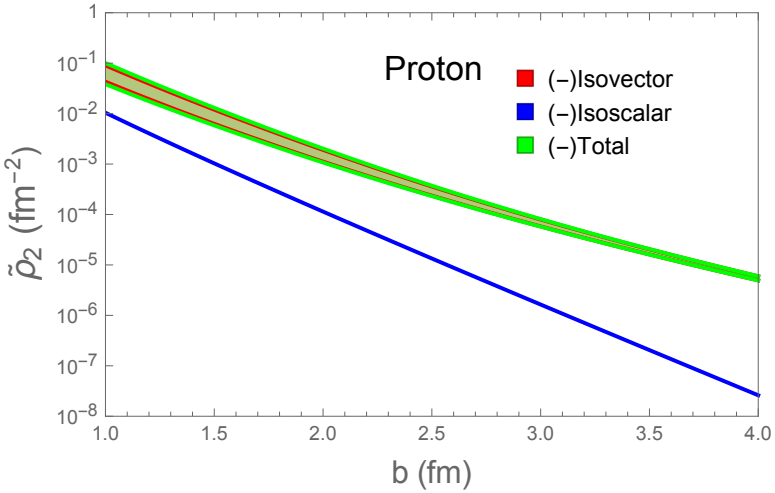
Computed with χEFT

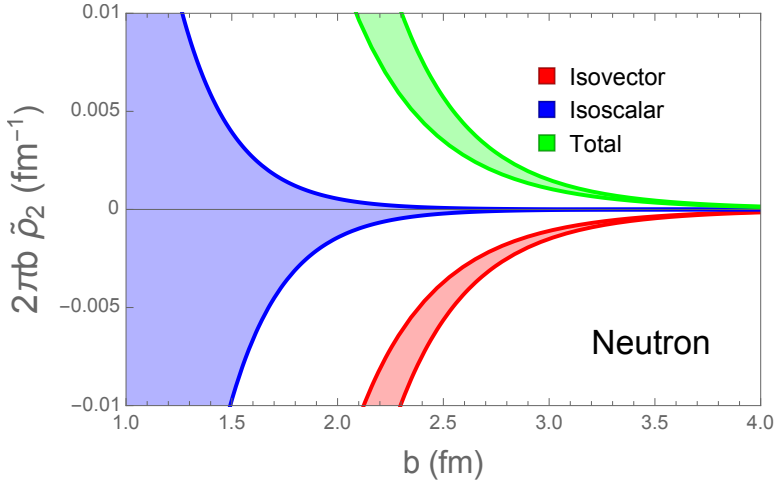
Empirical pion form factor

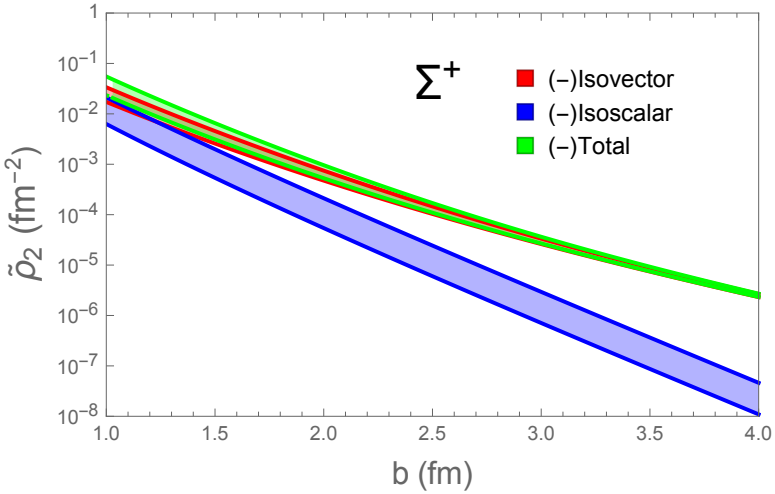
Gounaris and Sakurai, Phys. Rev. Lett. 21 (1968) 244

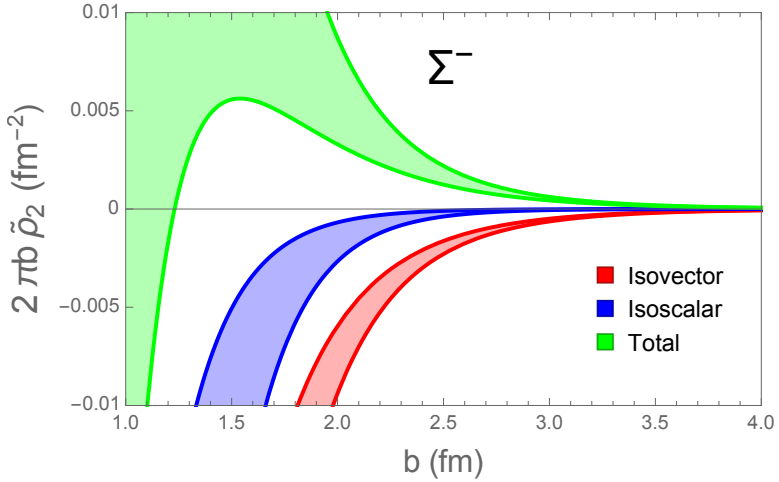
Similar approach for Λ-Σ^0 transition FF: Granados et al., arXiv:1701.09130 [hep-ph] (2017)

Baryon magnetic densities I

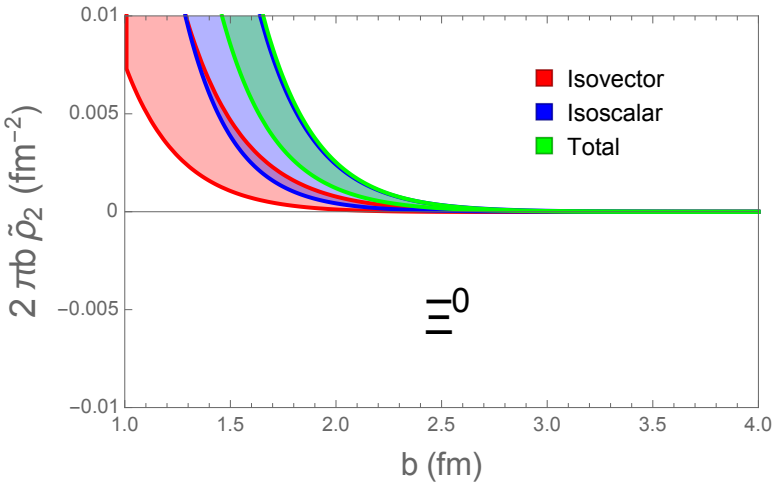


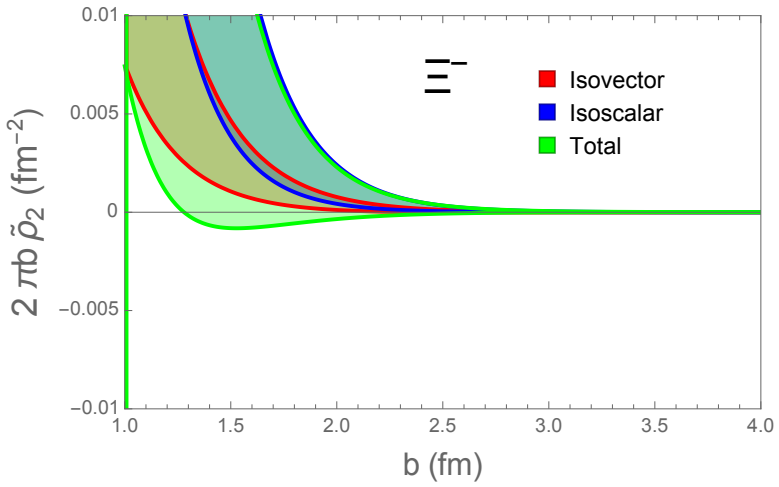


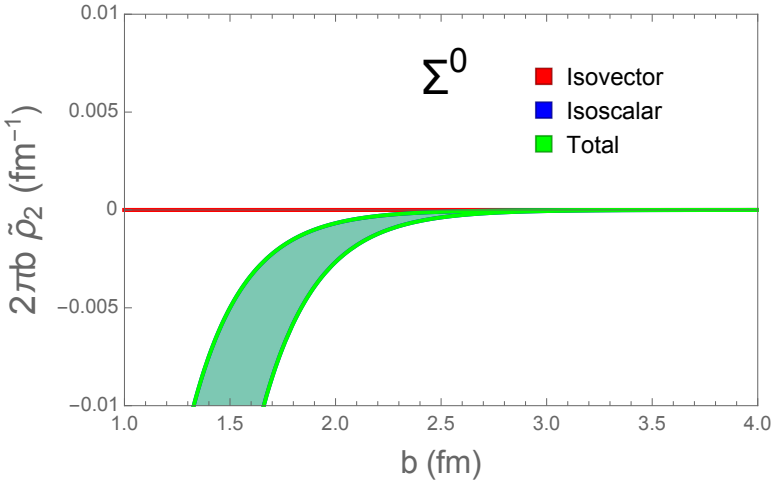


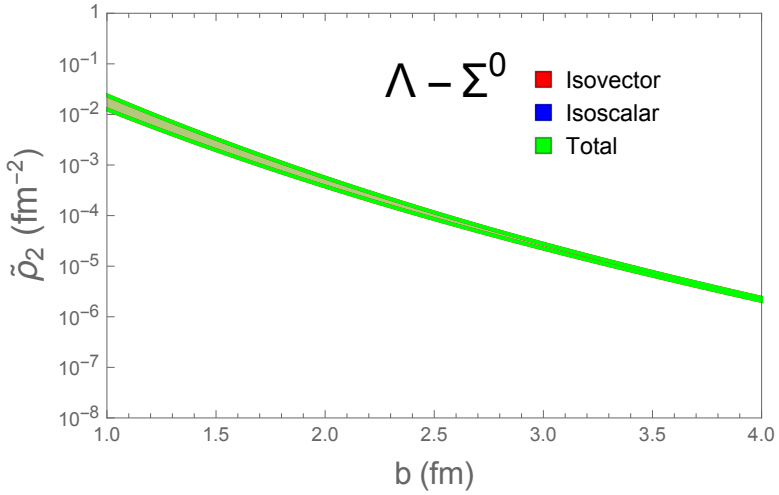


Baryon magnetic densities II





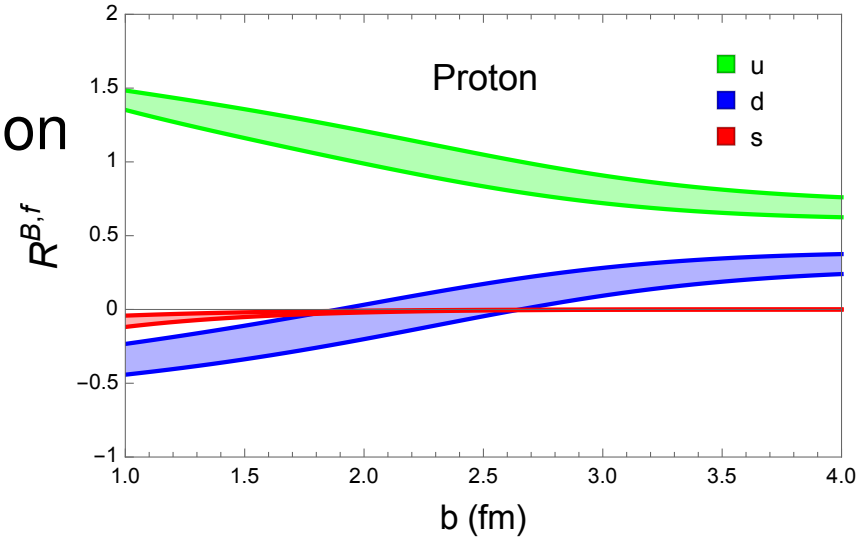


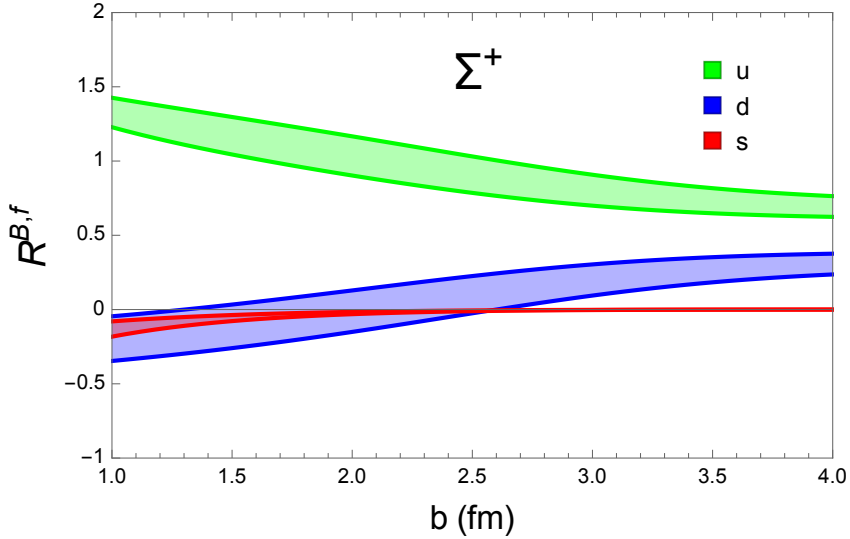


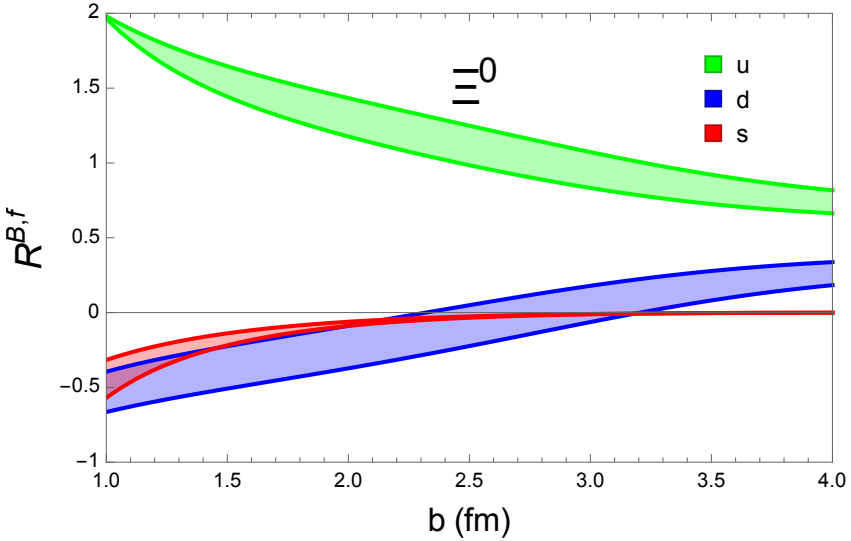
Quark contributions

Charge-weighted contribution of quark flavor to $\rho(b)$

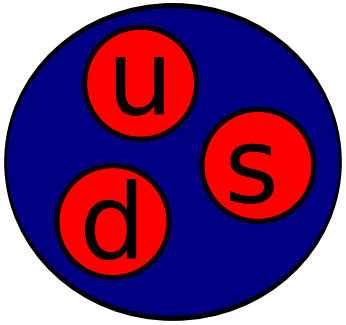
$$\sum_f R^{B,f} = 1$$







Hyperons



$$SU(2) \longrightarrow SU(3)$$

$$\begin{pmatrix} p \\ n \end{pmatrix} \longrightarrow \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

$$\pi^\pm, \pi^0 \longrightarrow \pi^\pm, \pi^0, K^\pm, K^0, \eta$$

$$\Delta(1232) \longrightarrow \Delta^\pm, \Delta^{++}, \Delta^0, \Sigma^{*\pm}, \Sigma^{*0}, \Xi^{*-}, \Xi^{*0}, \Omega$$

- Hyperons: baryons with **strangeness** $\mathcal{S} \neq 0$
- Short lifetimes $\Longrightarrow$ properties computed on the lattice
- Gives space for theoretical predictions