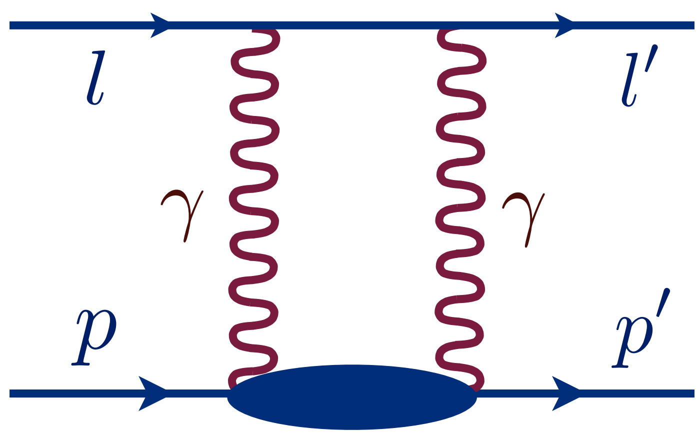
Two-photon corrections



Oleksandr Tomalak

Johannes Gutenberg University,

Mainz, Germany

1 September, 2017

Form factors measurement

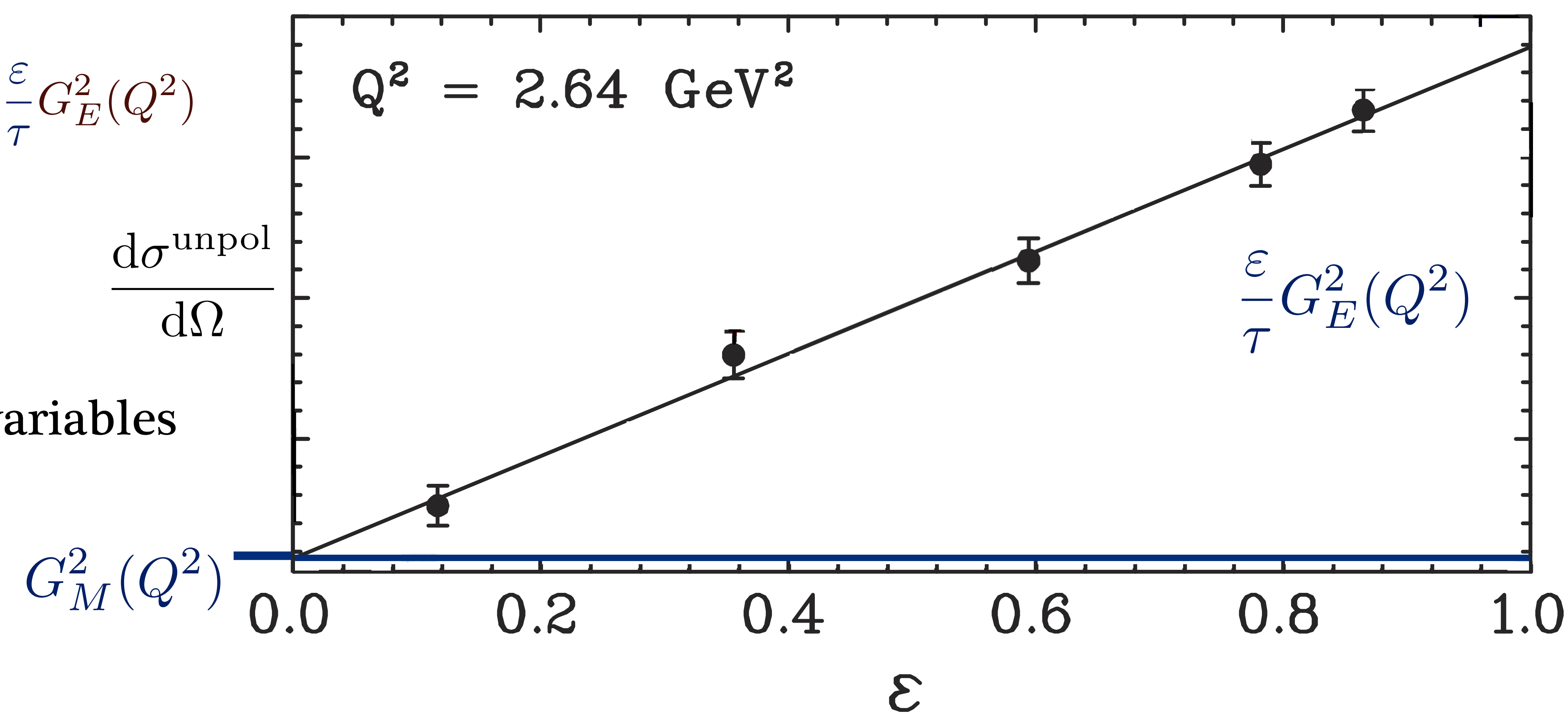
- Sachs electric and magnetic form factors:

$$G_E = F_D - \tau F_P \qquad G_M = F_D + F_P$$

- Rosenbluth separation:

$$\frac{\mathrm{d}\sigma^{\mathrm{unpol}}}{\mathrm{d}\Omega} \sim G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

τ ε kinematical variables



Qattan et al. (2005)

- Rosenbluth slope is sensitive to corrections beyond 1ɣ

Form factors measurement

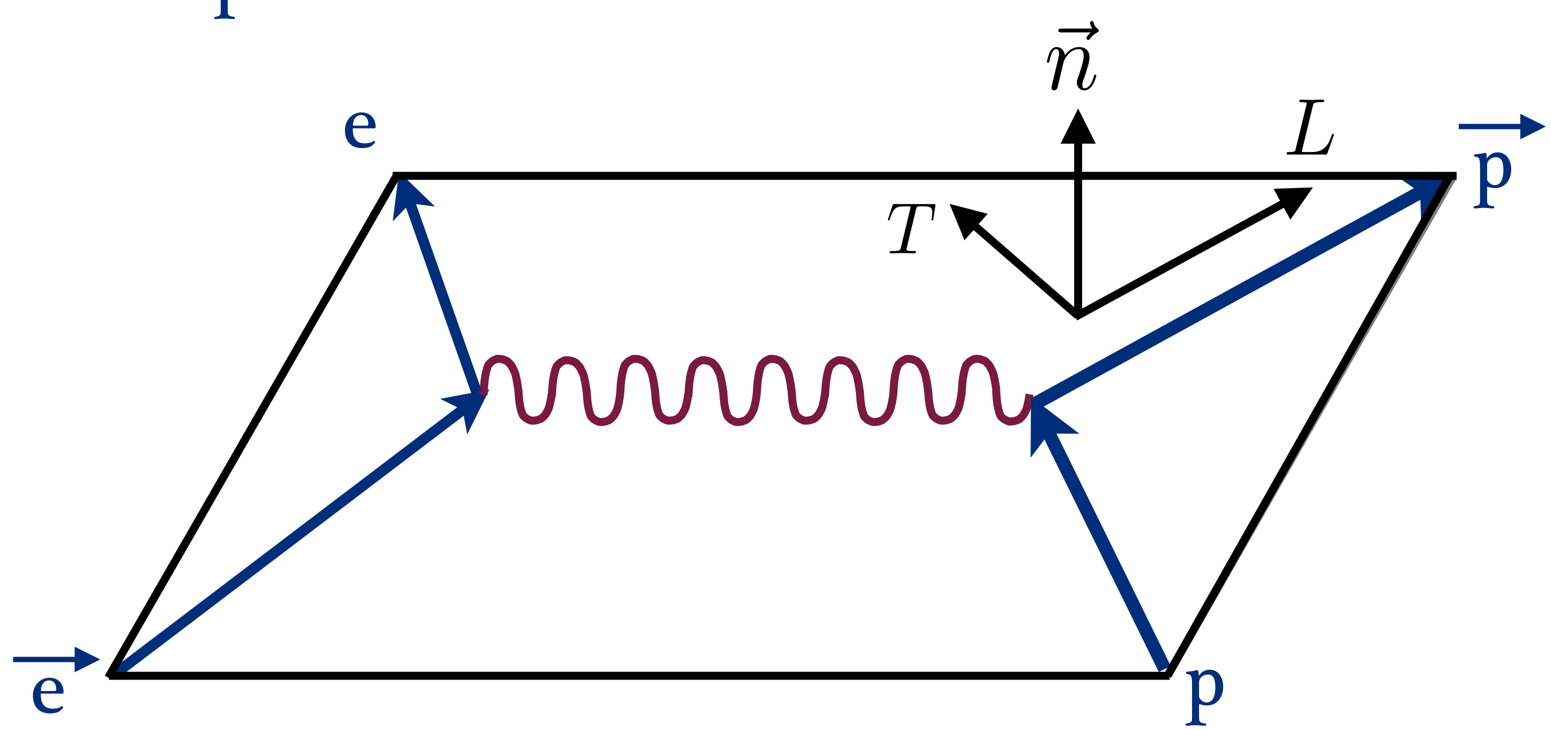
- Sachs electric and magnetic form factors:

$$G_E = F_D - \tau F_P \qquad G_M = F_D + F_P$$

- polarization transfer method:

$\vec{e} + p \rightarrow e + \vec{p}$

realized in 2000 at JLab



$$P_T \sim G_E(Q^2)G_M(Q^2)$$

$$P_L \sim G_M^2(Q^2)$$

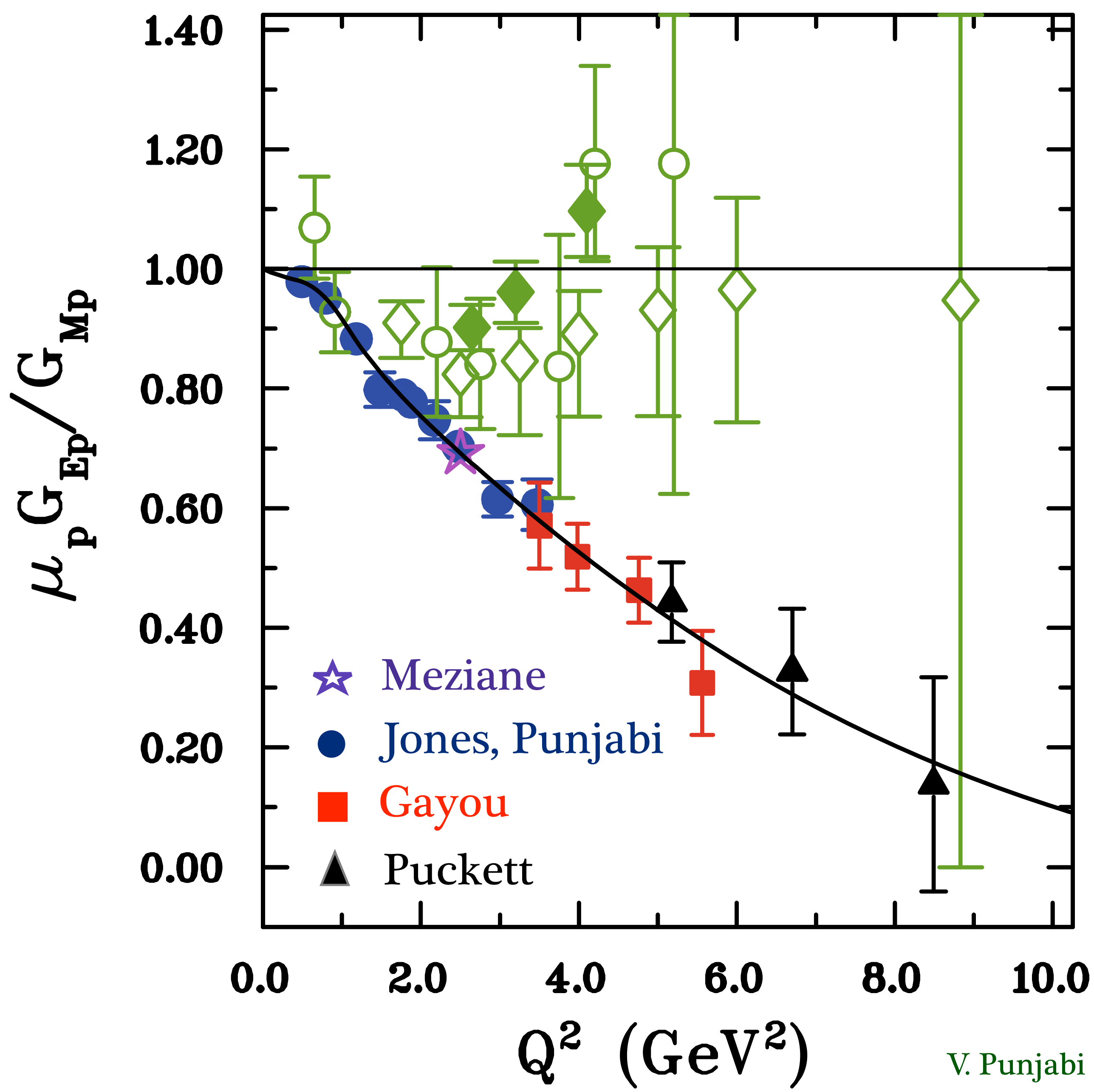
$$\frac{P_T}{P_L} \sim \frac{G_E(Q^2)}{G_M(Q^2)}$$

Proton form factors puzzle

Polarization transfer
JLab (Hall A, C)

vs.

Rosenbluth separation
SLAC, JLab (Hall A, C)



V. Punjabi et al. (2015)

Proton form factors puzzle

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vs.

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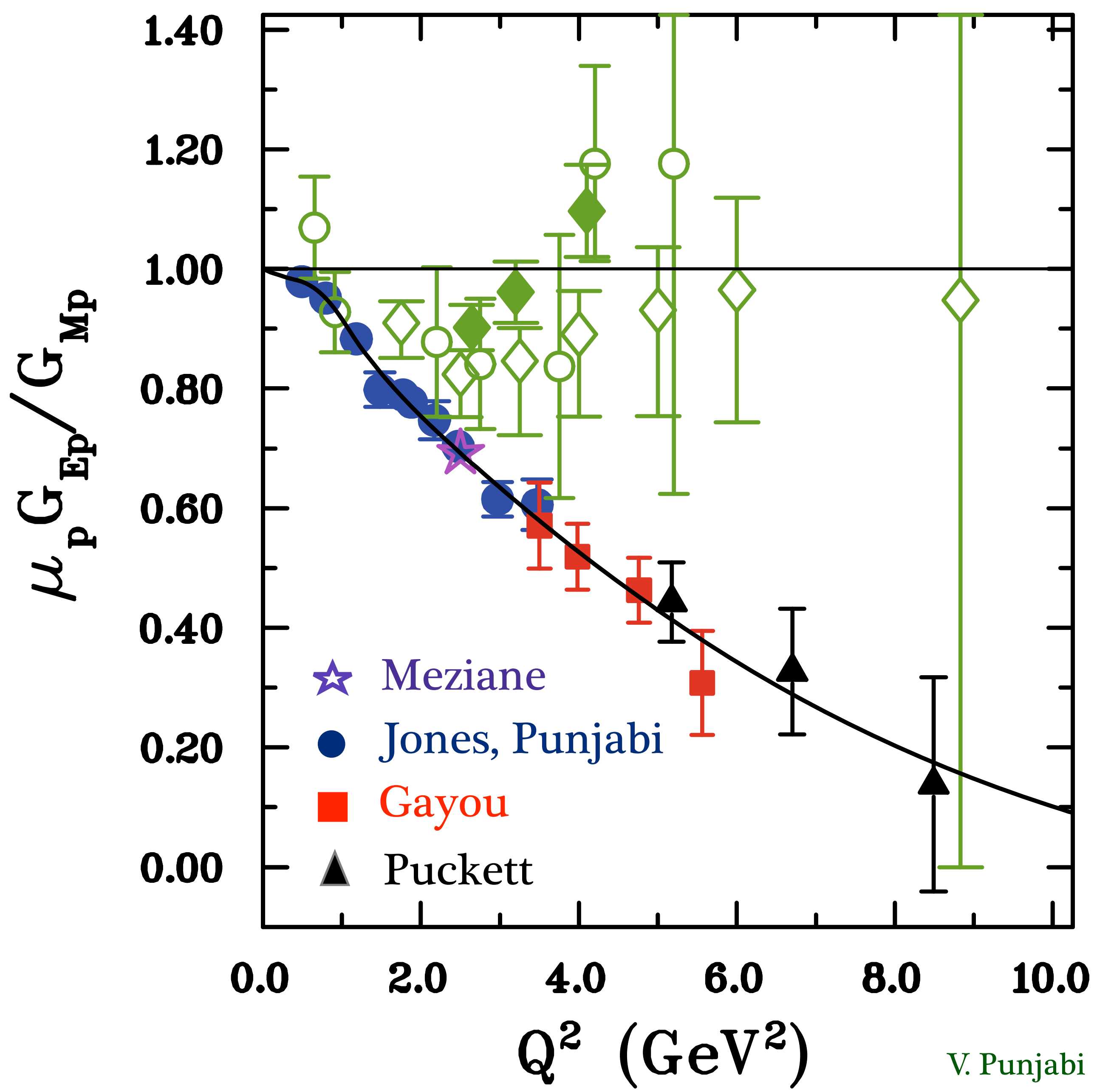

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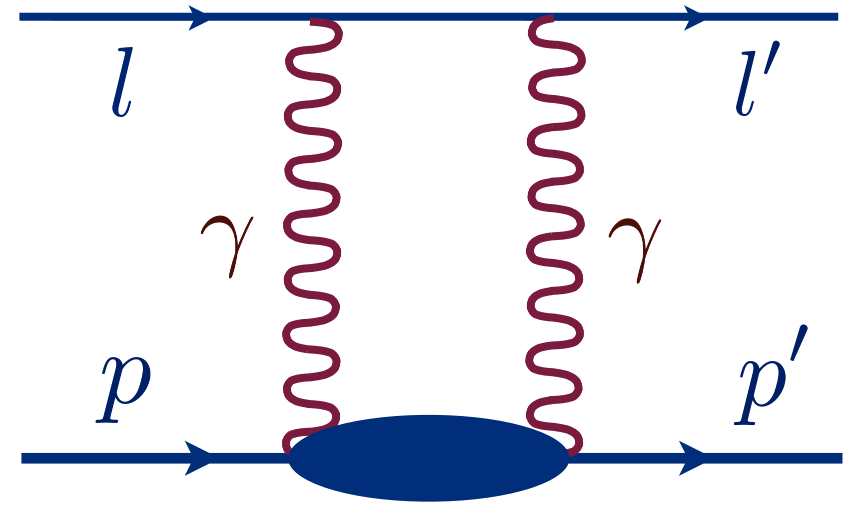

a possible explanation
two-photon exchange

2ɣ measurements
e⁺p/e⁻p cross section ratio

$$R_{2\gamma} = \frac{\sigma(e^+p)}{\sigma(e^-p)} \approx 1 - 2\delta_{2\gamma}$$

- discrepancy motivates model-independent study of 2ɣ

Scattering experiments and 2ɣ

- 2ɣ is not among standard radiative corrections

$$\sigma^{\mathrm{exp}} \equiv \sigma_{1\gamma}(1 + \delta_{\mathrm{rad}} + \delta_{\mathrm{soft}} + \delta_{2\gamma})$$

- charge radius insensitive to 2ɣ model

- magnetic radius depends on 2ɣ model

J. C. Bernauer et al. (2014)

Scattering experiments and 2ɣ

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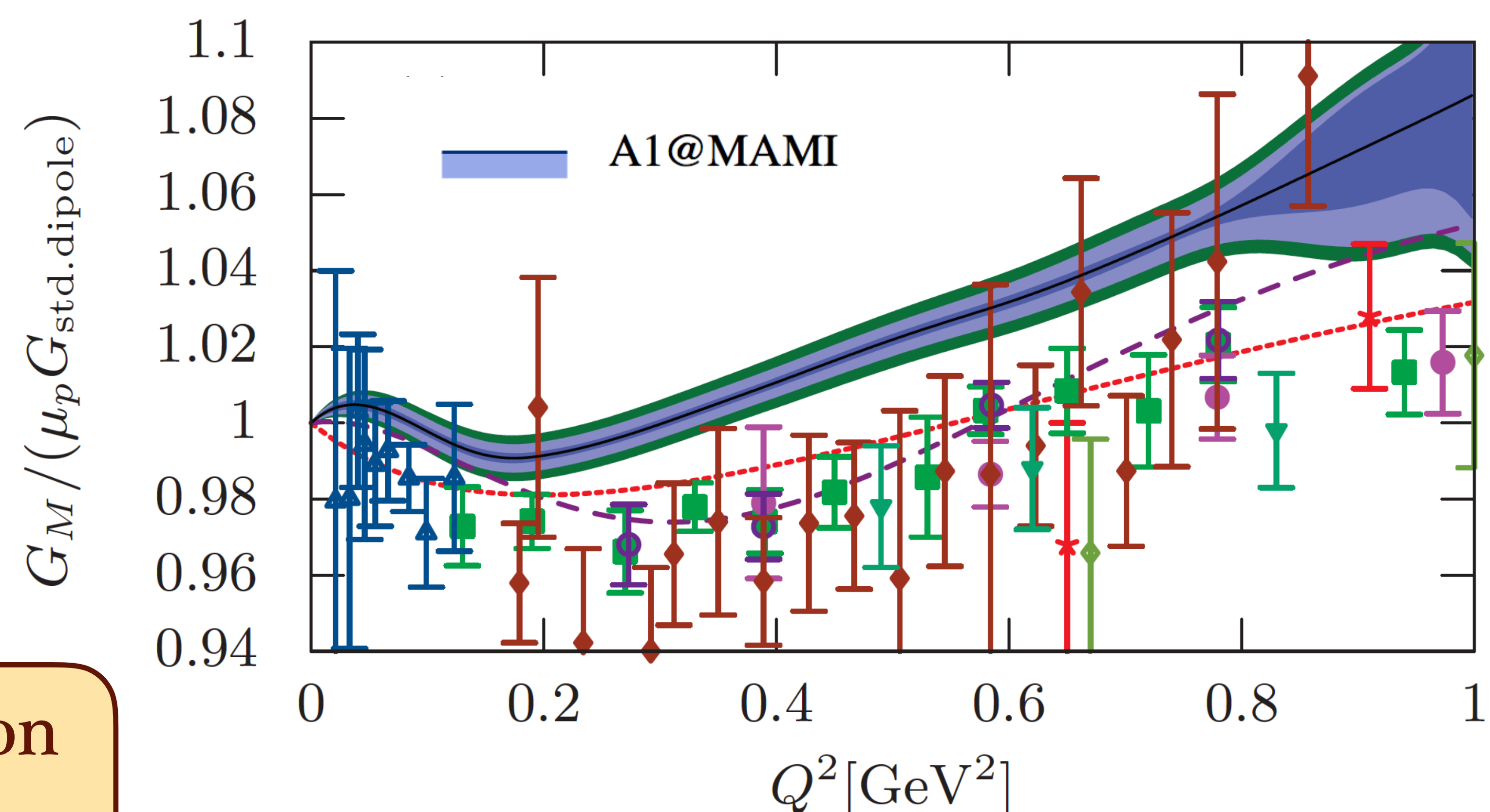
magnetic form factor

J. C. Bernauer et al. (2014)



- 2 % systematic deviation MAMI vs. world data

μH hyperfine splitting and 2ɣ

1S HFS in μH
PSI, J-PARC, RIKEN-RAL
1 ppm accuracy

R. Pohl (2016)

- leading theoretical uncertainty: 213 ppm from 2ɣ

C. Carlson, V. Nazaryan, K. Griffioen (2011)

- A1@MAMI fit allows to quantify 2ɣ uncertainty

J. C. Bernauer et al. (2014)

- radii expansion of form factors: 3 times more precise

O. Tomalak (2017)

	contribution (ppm)	uncertainty (ppm)
p (Zemach)	-7376	140→46
total, μH charge radius	-6170	98
total, scatt charge radius	-6239	104

r_E?

- magnetic radius, form factors and spin structure are important

Scattering experiments and 2γ

- charge radius extractions:

eH, eD spectroscopy	ep scattering
μH, μD spectroscopy	μp scattering ????

- μp elastic scattering is planned by MUSE@PSI(2018-19)
  measure with both electron/muon charges

- 2γ correction in MUSE ?

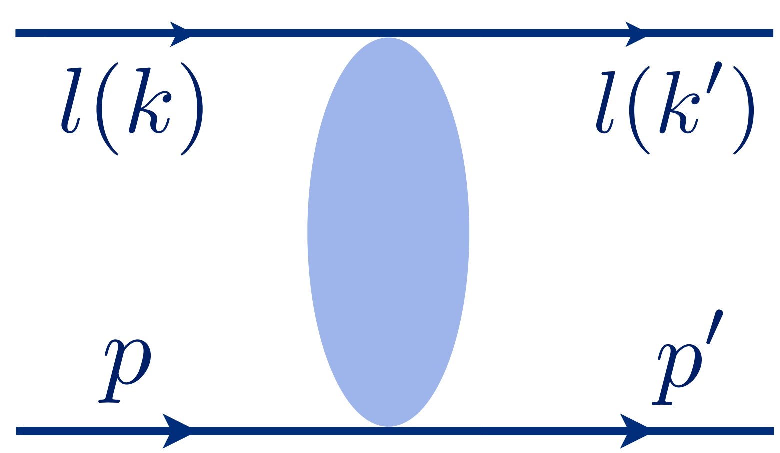
Elastic lepton-proton scattering

momentum transfer

$$Q^2 = -(k - k')^2$$

crossing-symmetric variable

ν



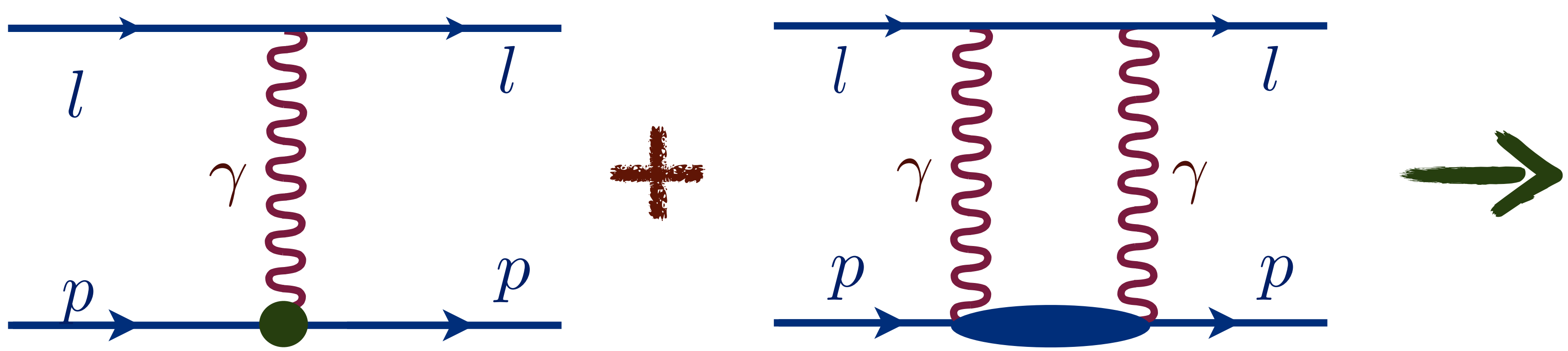
photon polarization parameter

ε

forward scattering

$\varepsilon \to 1$

- leading 2ɣ contribution: interference term

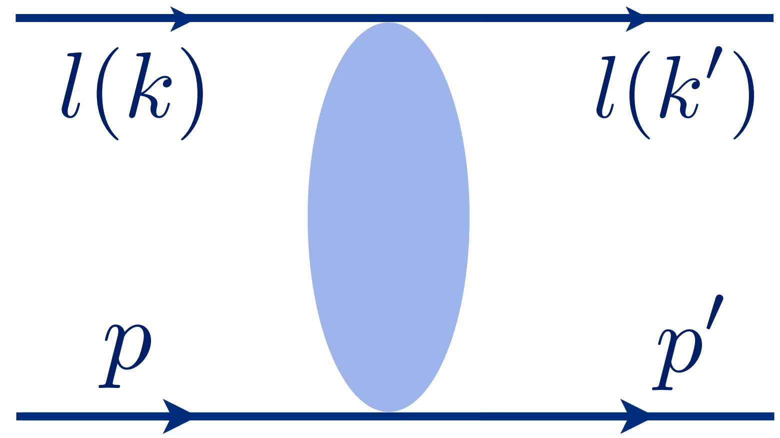


$$\delta_{2\gamma} = \frac{2 \sum\limits_{\text{spin}} T^{1\gamma} \Re T^{2\gamma}}{\sum\limits_{\text{spin}} |T^{1\gamma}|^2}$$

- 2ɣ correction to cross section is given by amplitudes real parts

Elastic lepton-proton scattering

$$K = \frac{k+k'}{2}$$



$$P = \frac{p+p'}{2}$$

- electron-proton scattering: 3 structure amplitudes

$$T^{\rm non-flip} = \frac{e^2}{Q^2}\bar{l}\gamma_\mu l \cdot \bar{N}(\mathcal{G}_M(\nu,Q^2)\gamma^\mu - \mathcal{F}_2(\nu,Q^2)\frac{P^\mu}{M} + \mathcal{F}_3(\nu,Q^2)\frac{\hat{K}P^\mu}{M^2})N$$

P.A.M. Guichon and M. Vanderhaeghen (2003)

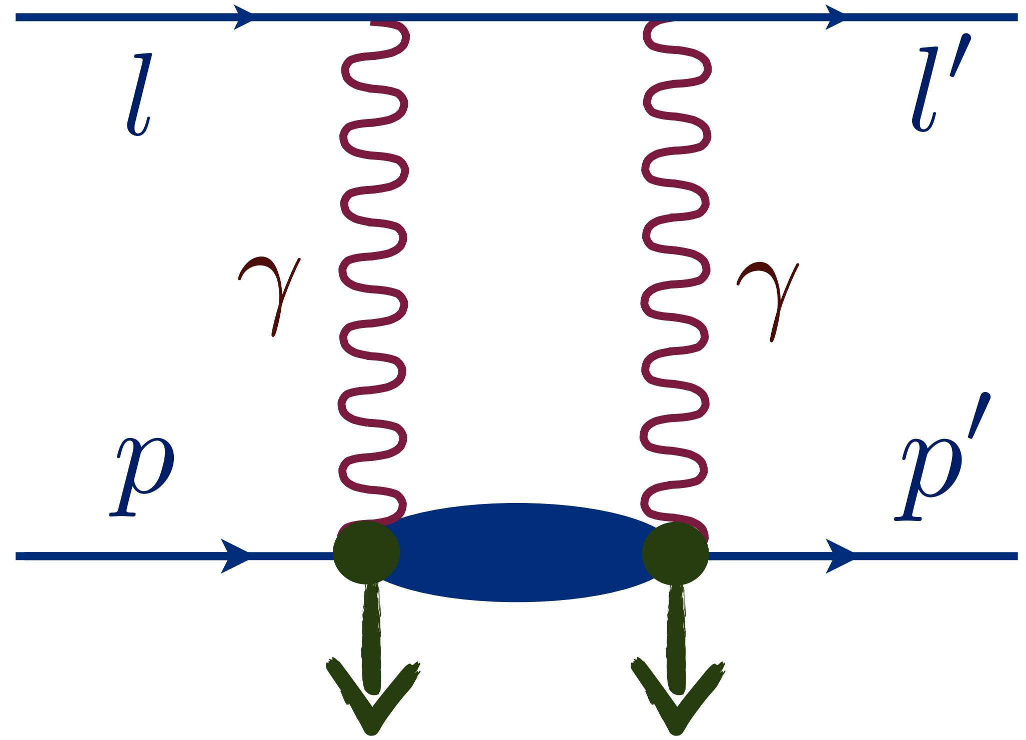
- muon-proton scattering: add helicity-flip amplitudes

$$T^{\rm flip} = \frac{e^2}{Q^2}\frac{m}{M}\bar{l}l \cdot \bar{N}(\mathcal{F}_4(\nu,Q^2) + \mathcal{F}_5(\nu,Q^2)\frac{\hat{K}}{M})N + \frac{e^2}{Q^2}\frac{m}{M}\mathcal{F}_6(\nu,Q^2)\bar{l}\gamma_5 l \cdot \bar{N}\gamma_5 N$$

M. Gorchtein, P.A.M. Guichon and M. Vanderhaeghen (2004)

- 2ɣ correction to cross section is given by amplitudes real parts

non-forward scattering
at low momentum transfer

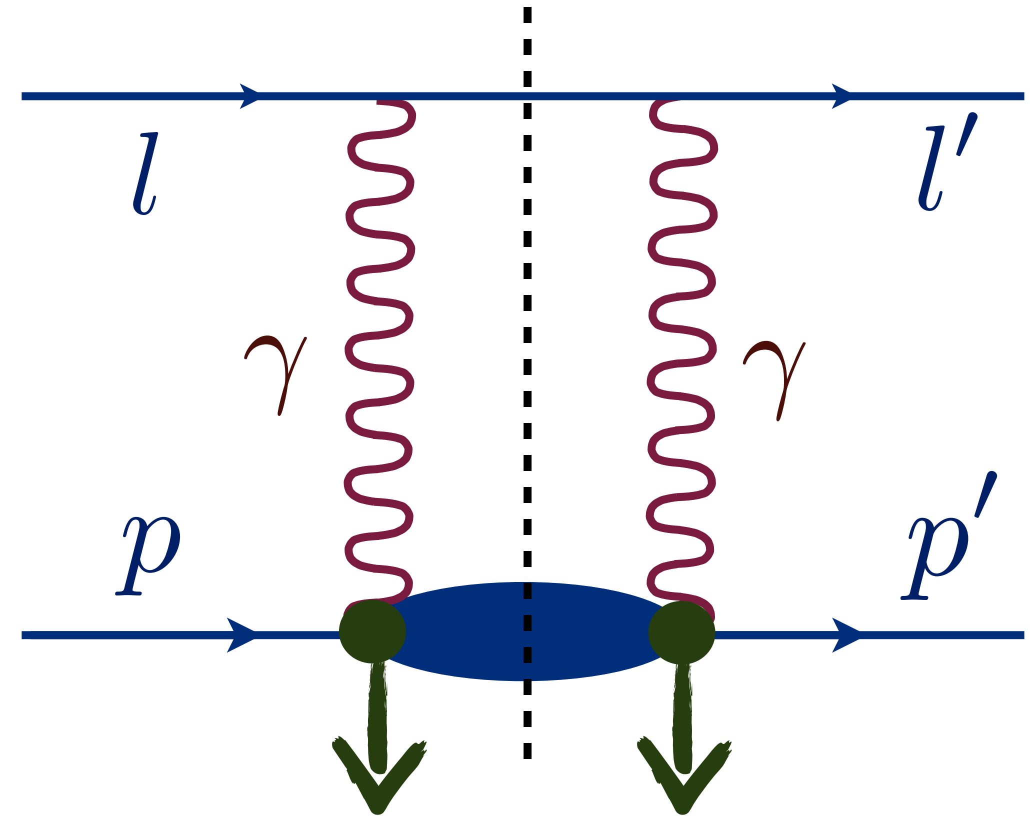


photoproduction vertex or Compton tensor

box diagram

assumption about the vertex

non-forward scattering
at low momentum transfer



photoproduction vertex or Compton tensor

box diagram

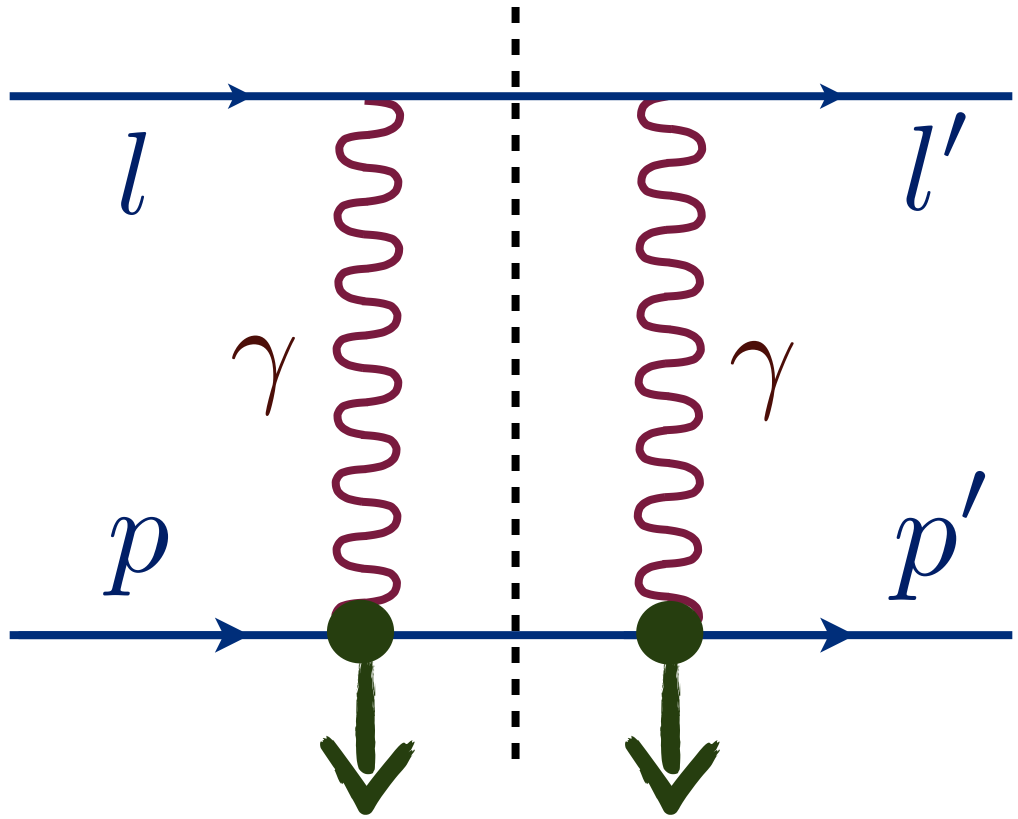
assumption about the vertex

dispersion relations

based on on-shell information

non-forward scattering

proton state



Dirac and Pauli form factors

box diagram

dispersion relations

assumption about the vertex

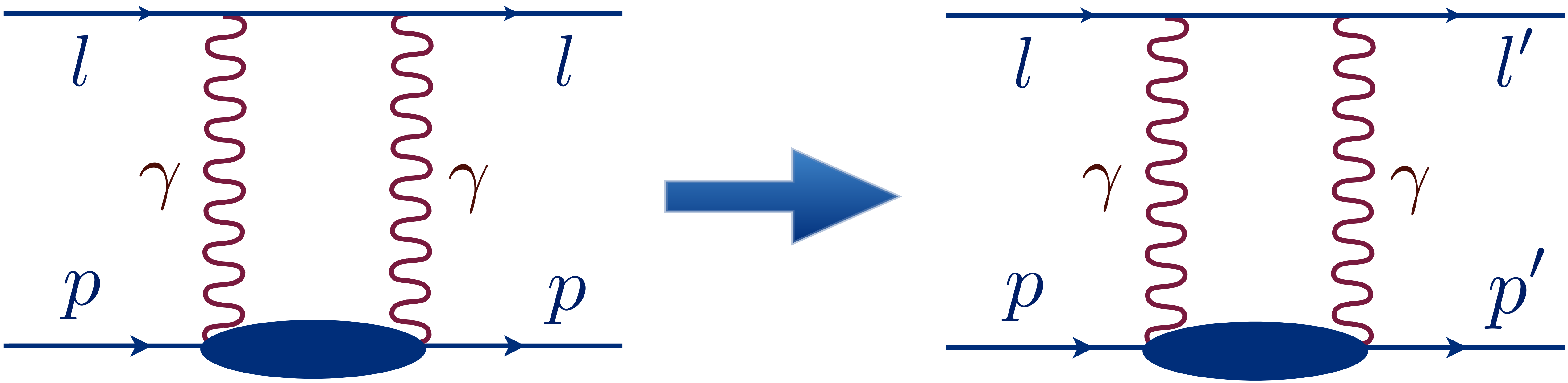
Blunden, Melnitchouk and Tjon (2003)

based on on-shell information

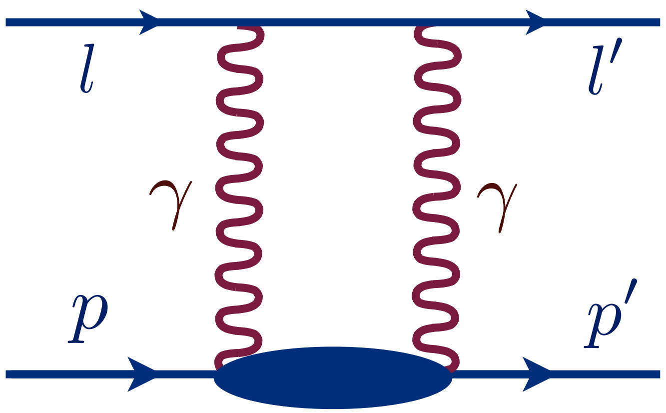
Borisyuk and Kobushkin (2008), O. T. and M. Vanderhaeghen (2014) forward scattering

near-forward scattering

account for all inelastic 2γ



Low-Q^2 inelastic 2ɣ correction (e^-p)



- 2ɣ blob: near-forward virtual Compton scattering

Feshbach inelastic elastic

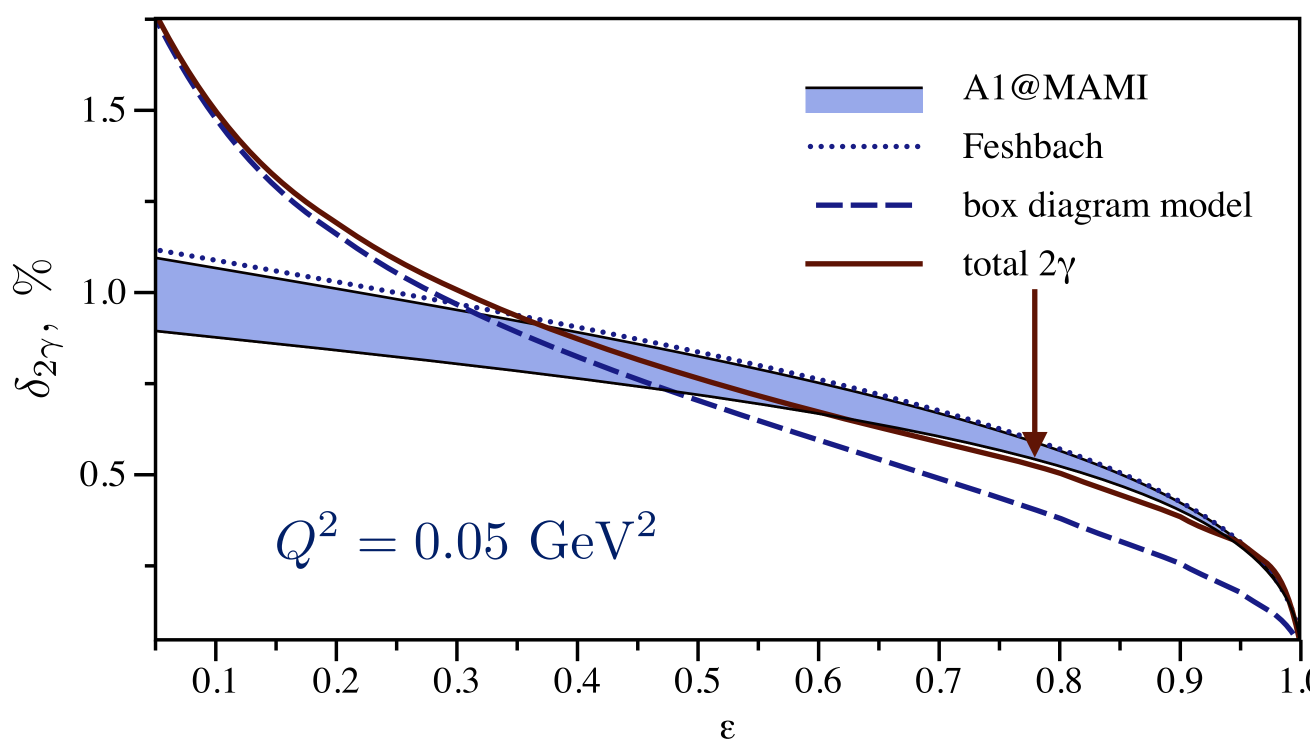
$$\delta_{2\gamma} \sim a\,\sqrt{Q^2} + b\,Q^2 \ln Q^2 + c\,Q^2 \ln^2 Q^2$$

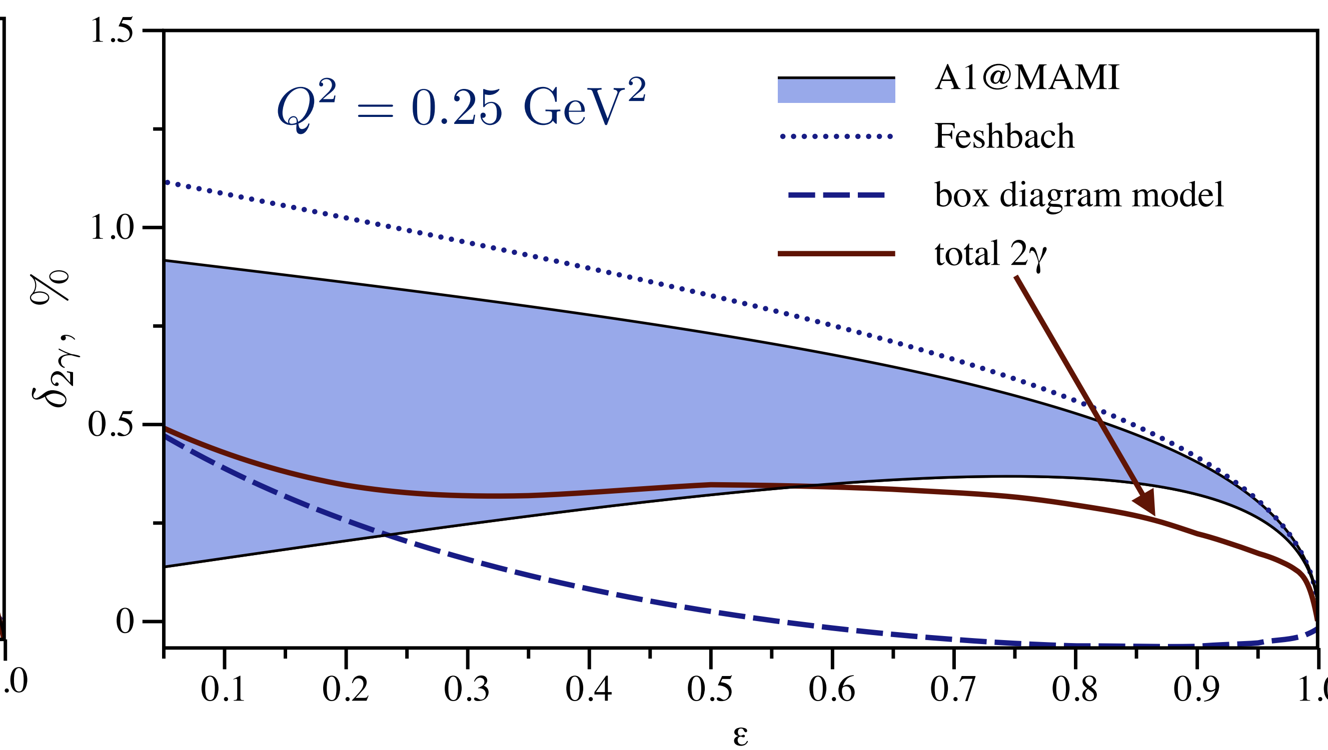
R. W. Brown (1970), M. Gorchtein (2013), O. T. and M. Vanderhaeghen (2014)

unpolarized proton structure

M. E. Christy, P. E. Bosted (2010)

$$\delta_{2\gamma} = \int \mathrm{d}\nu_\gamma \mathrm{d}\tilde{Q}^2 (\omega_1(\nu_\gamma, \tilde{Q}^2) \cdot F_1(\nu_\gamma, \tilde{Q}^2) + \omega_2(\nu_\gamma, \tilde{Q}^2) \cdot F_2(\nu_\gamma, \tilde{Q}^2))$$





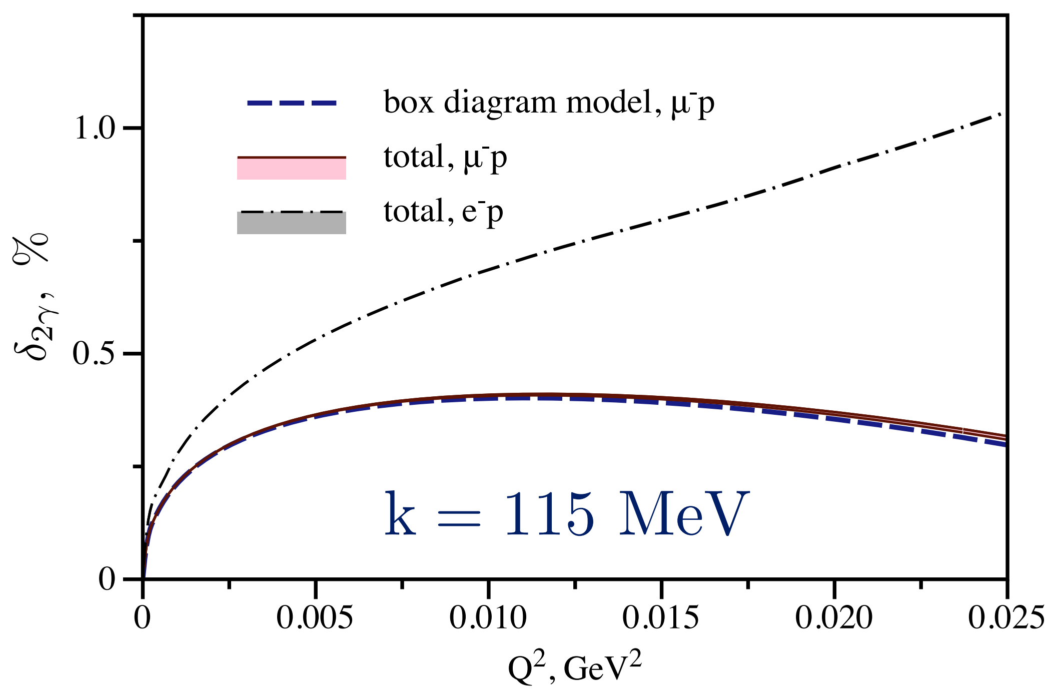
O. T. and M. Vanderhaeghen (2016)

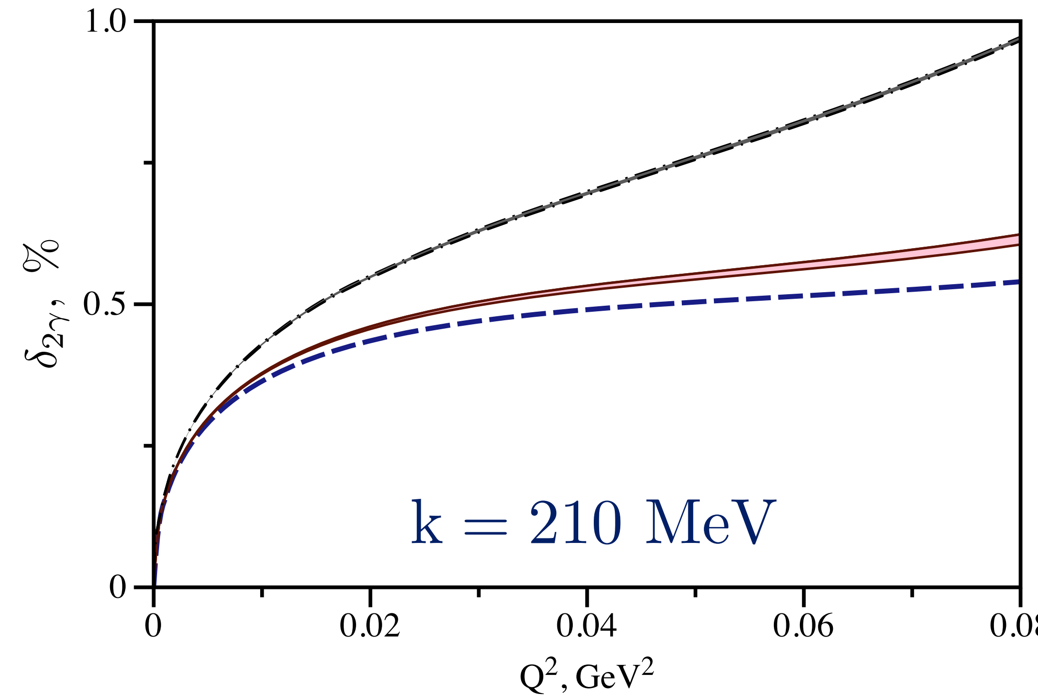
- 2ɣ at large ε agrees with empirical fit

r_E extraction ✓

MUSE@PSI (2018-19) estimates ($\mu^{-}p$)

- proton box diagram model + inelastic 2γ





O. T. and M. Vanderhaeghen (2014, 2016)

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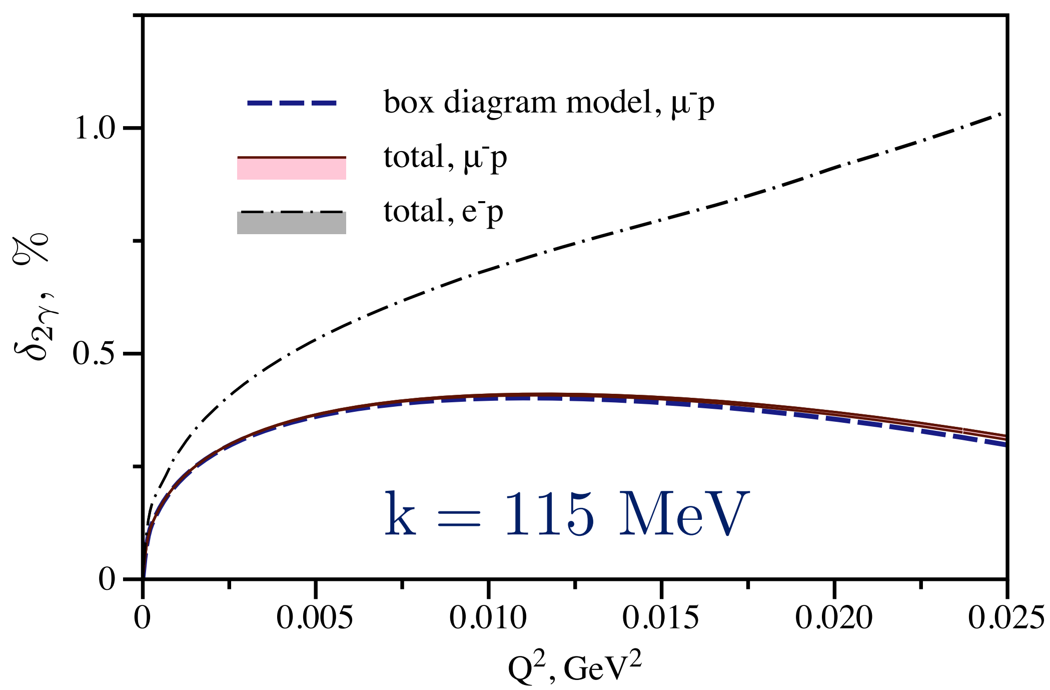
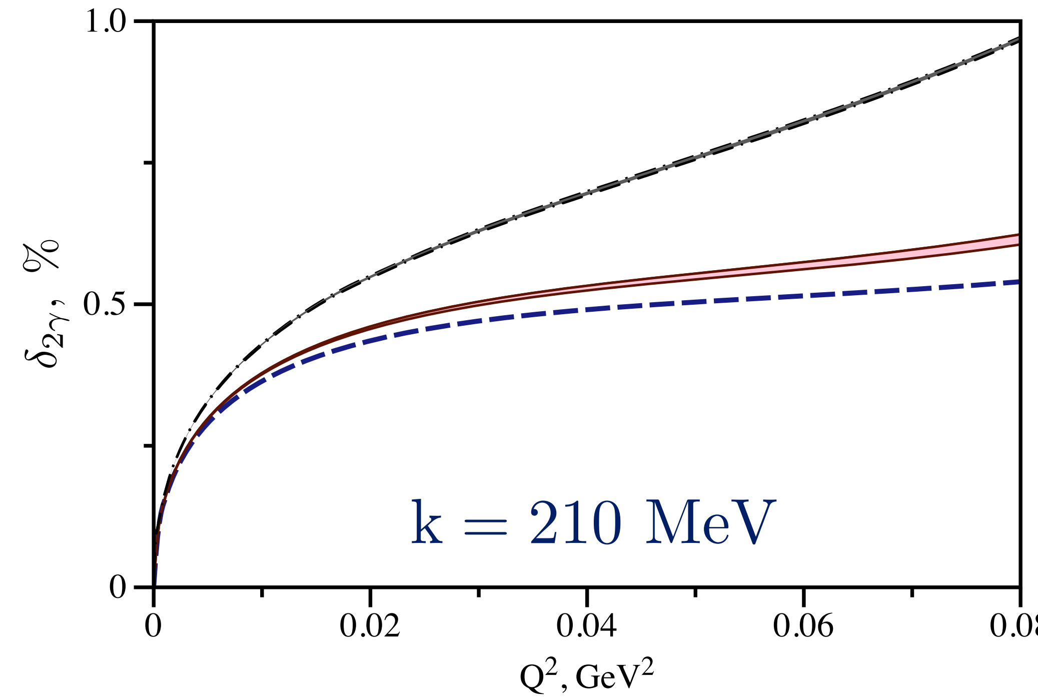




O. T. and M. Vanderhaeghen (2014, 2016)

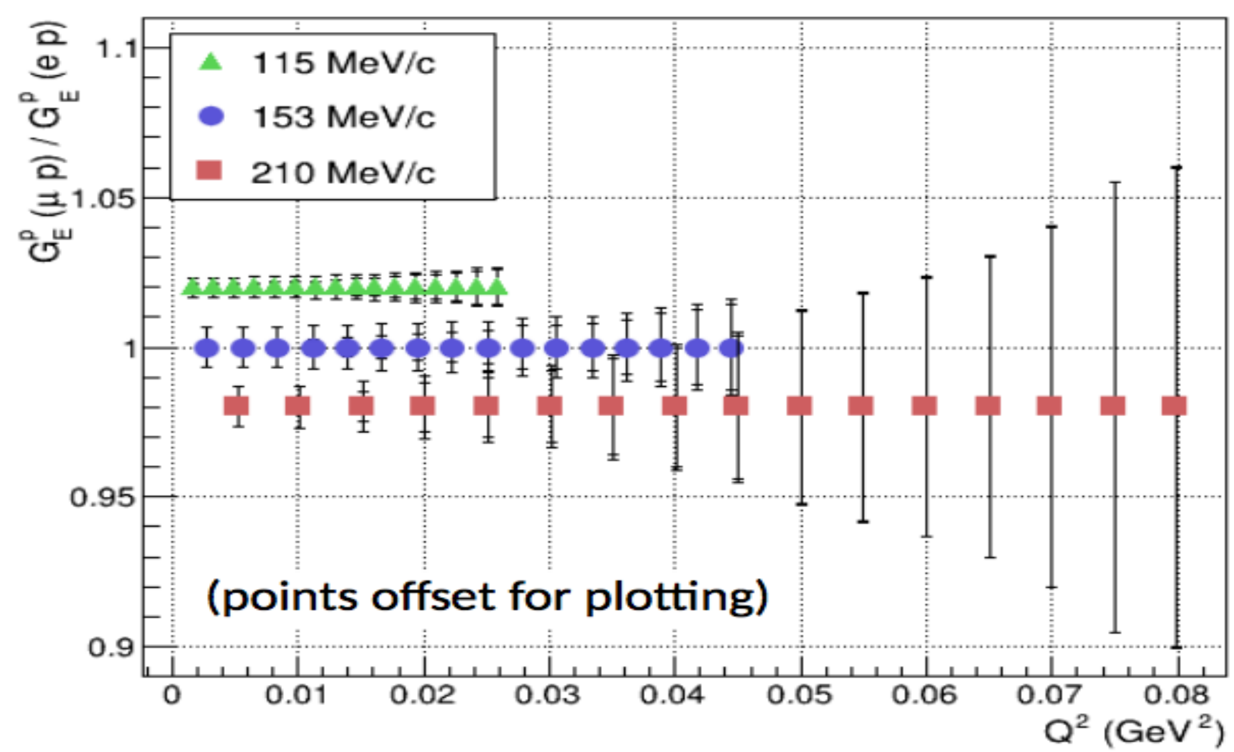
- expected muon over electron ratio

small inelastic 2γ

↓

small 2γ uncertainty

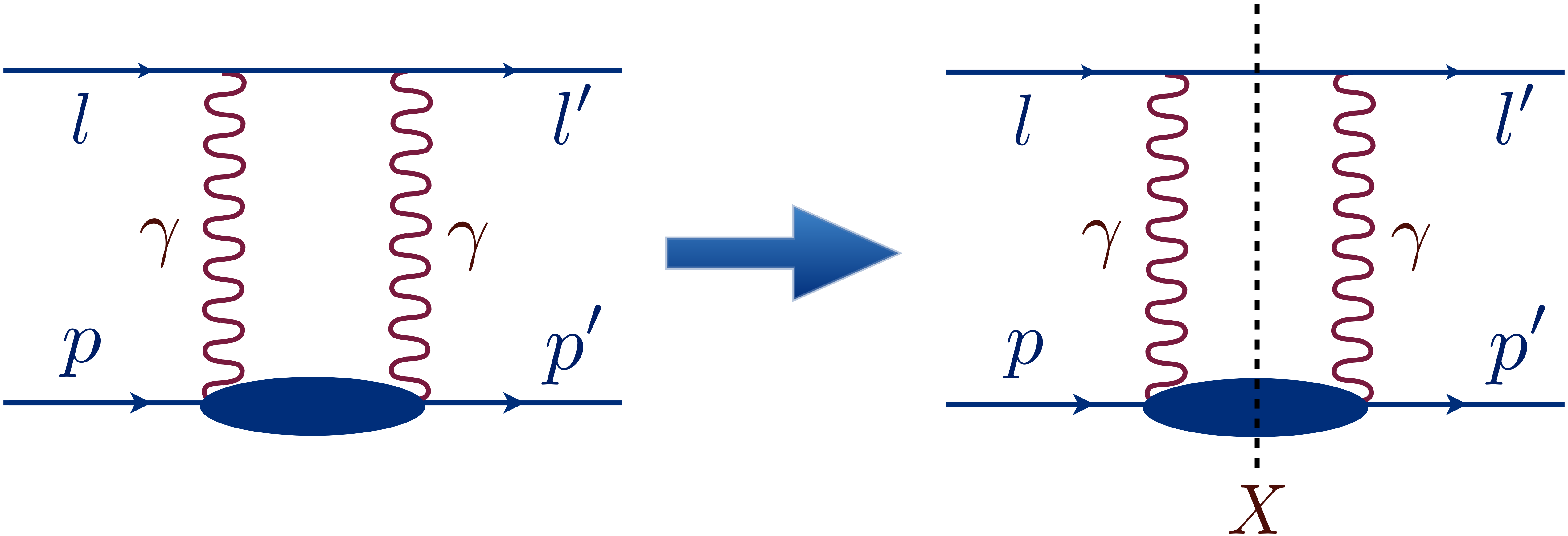
- MUSE can test r_E in one charge channel



K. Mesick talk (PAVI 2014), MUSE TDR (2016)

near-forward scattering
(large ε)

dispersion relations
(arbitrary ε)

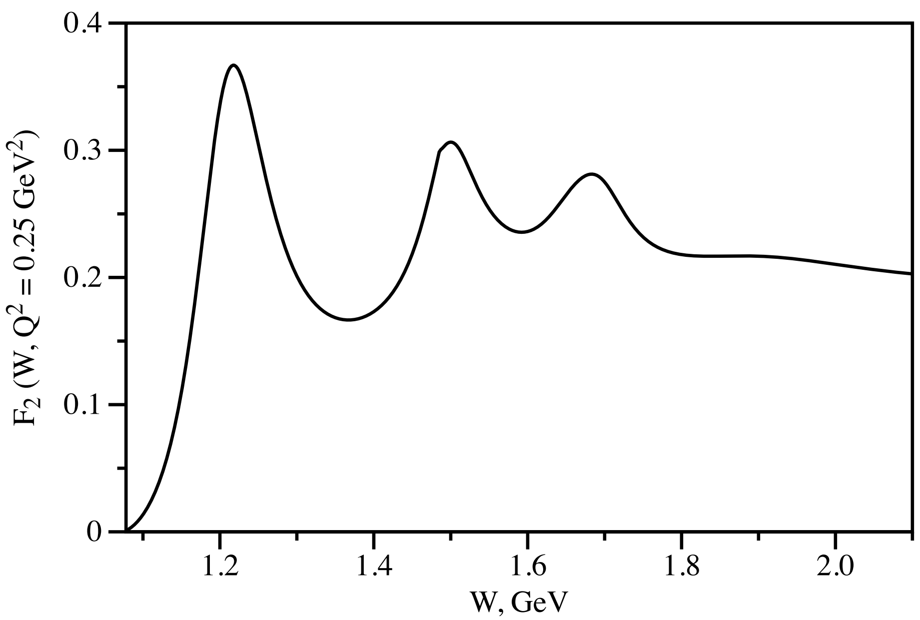


p + all inelastic

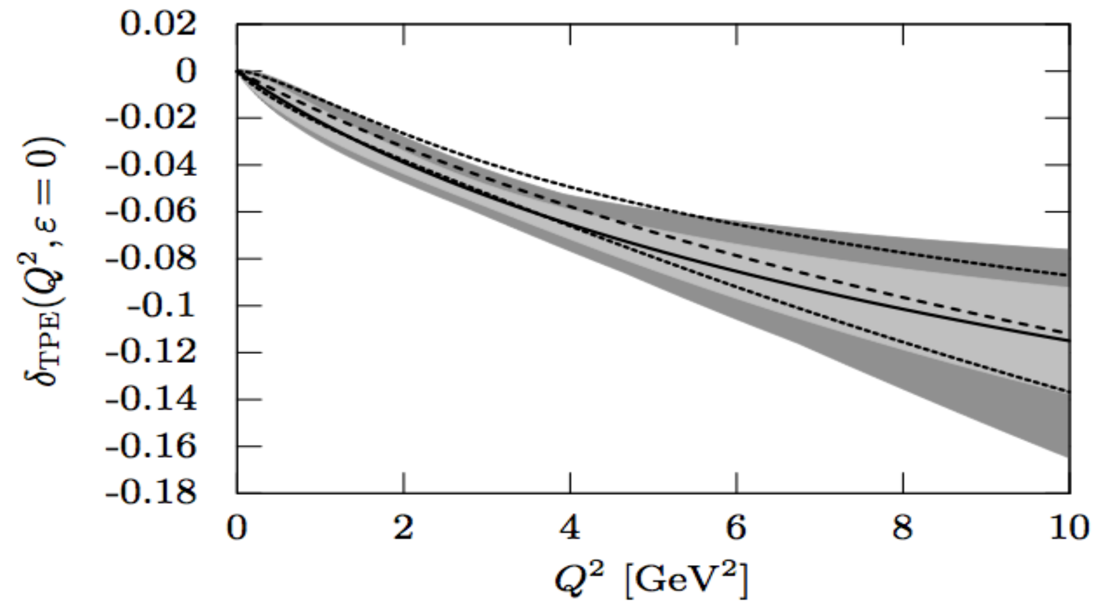
$$X = p + \pi N$$

Fixed-Q^2 dispersion relation framework

on-shell 1ɣ amplitudes



2ɣ prediction



experimental data

cross section correction

unitarity

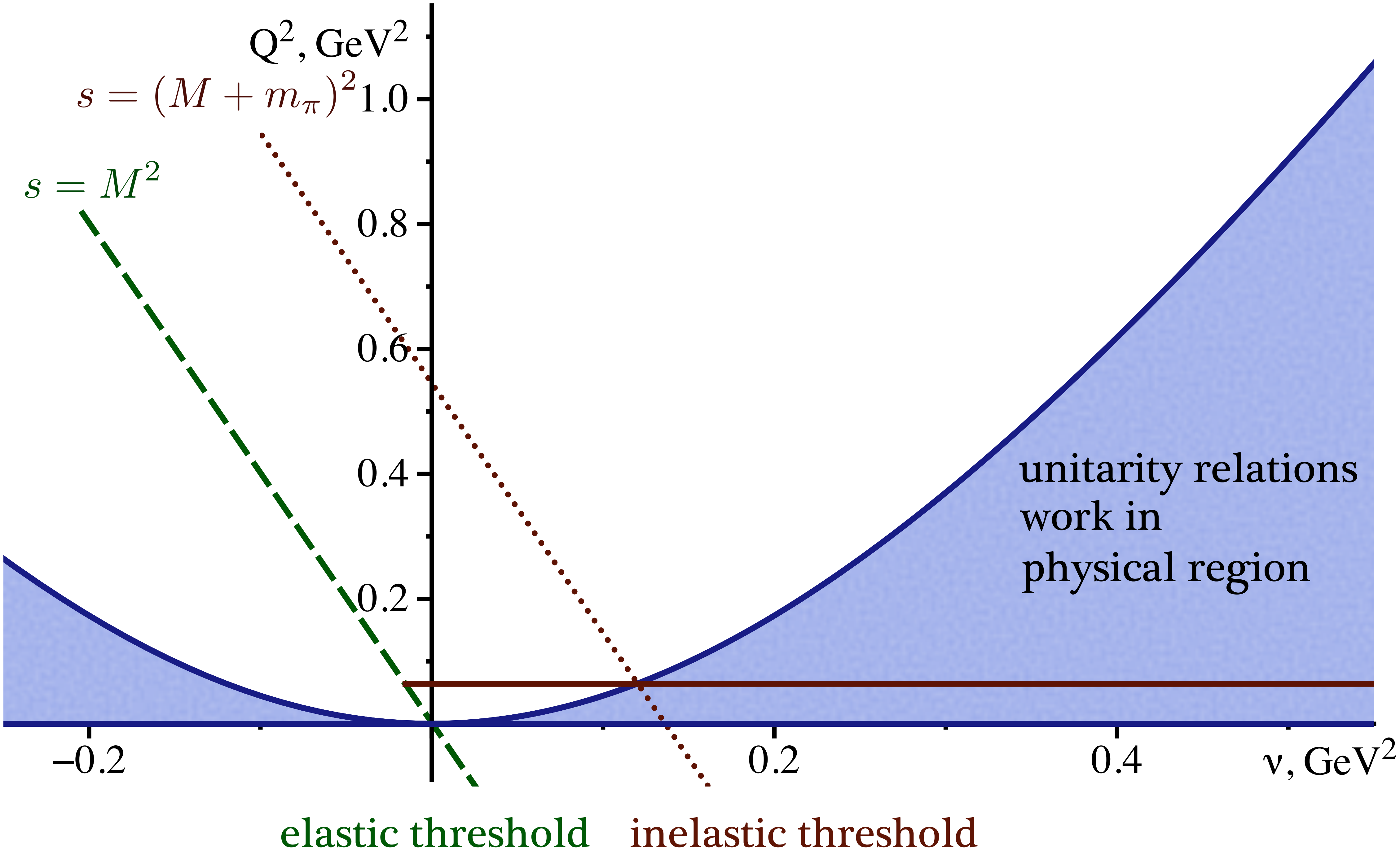
$$\Re\mathcal{F}(\nu) = \frac{2\nu}{\pi}\mathcal{P}\int_{\nu_{min}}^{\infty} \frac{\Im\mathcal{F}(\nu' + i0)}{\nu'^2 - \nu^2}\mathrm{d}\nu'$$

2ɣ imaginary parts

disp. rel.

2ɣ real parts

Mandelstam plot (ep)



- proton intermediate state is outside physical region for $Q^2 > 0$
- πN intermediate state is outside physical region for $Q^2 > 0.064$ GeV²

O. T. and M. Vanderhaeghen (2014)

Analytical continuation. Elastic state

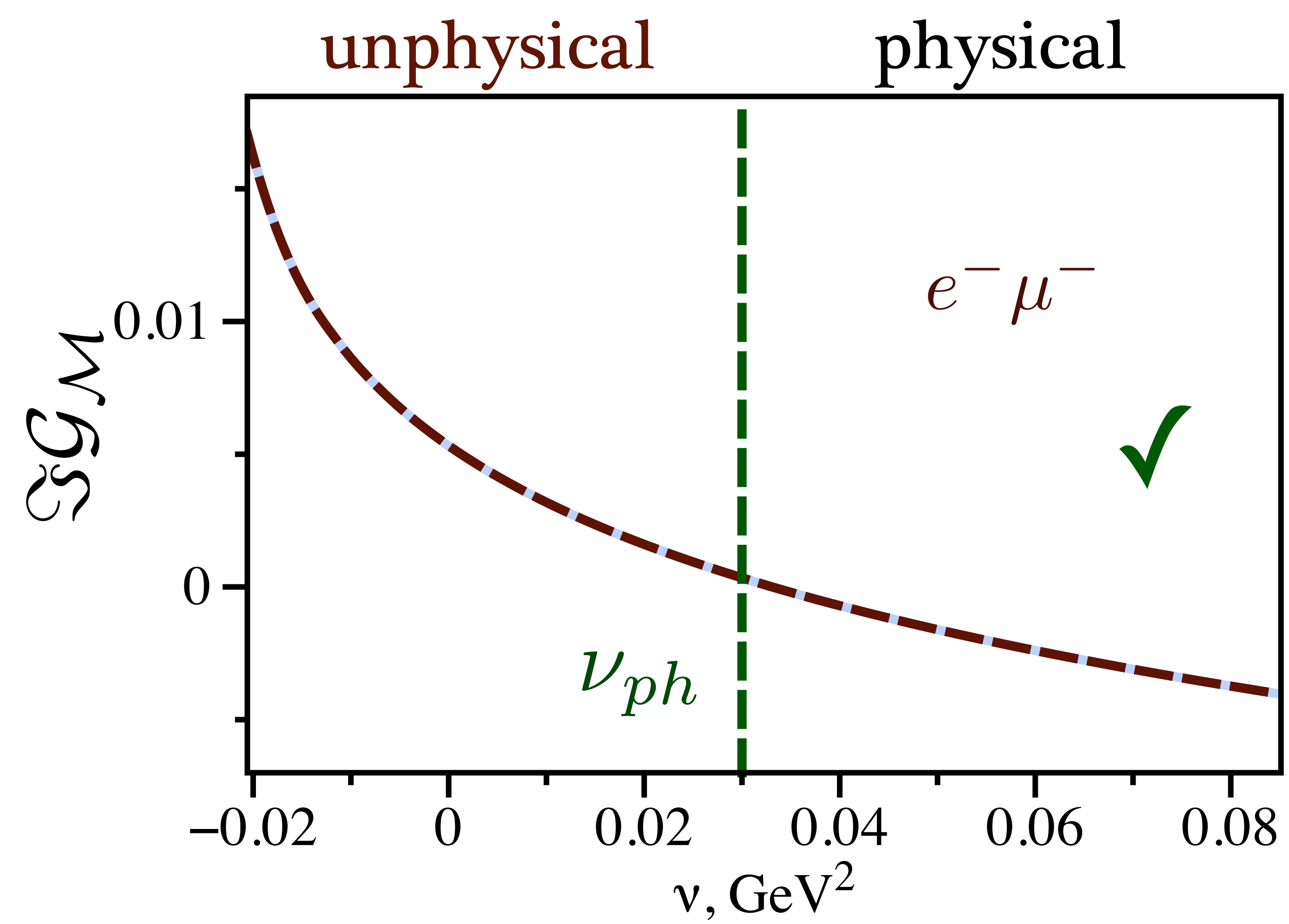
- contour deformation method:

O. T. and M. Vanderhaeghen (2014), Blunden and Melnitchouk (2017)

$\int d\Omega$ → angular integration to integration on curve in complex plane → deform integration contour keeping poles inside going to unph. region

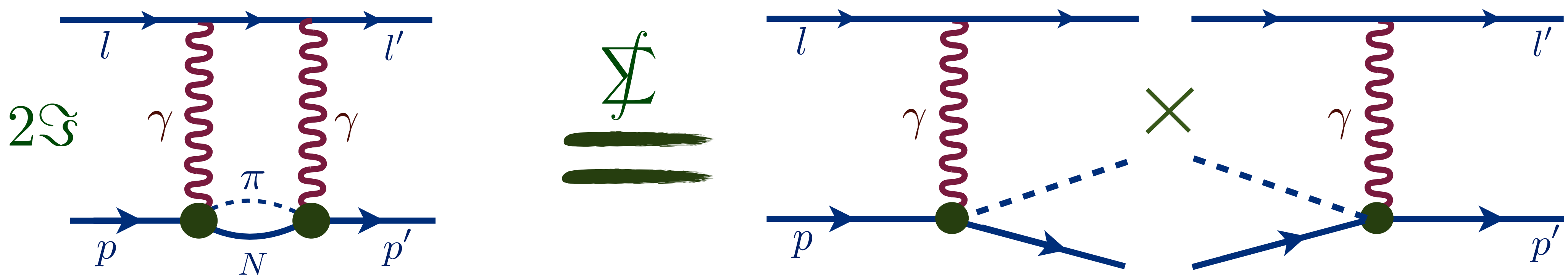
- analytical continuation reproduces results in unphysical region

$Q^2 = 0.1 \text{ GeV}^2$



- central value: form factor fit of A1@MAMI (2014)
- uncertainty: difference to 2γ with dipole form factors

Analytical continuation. πN states

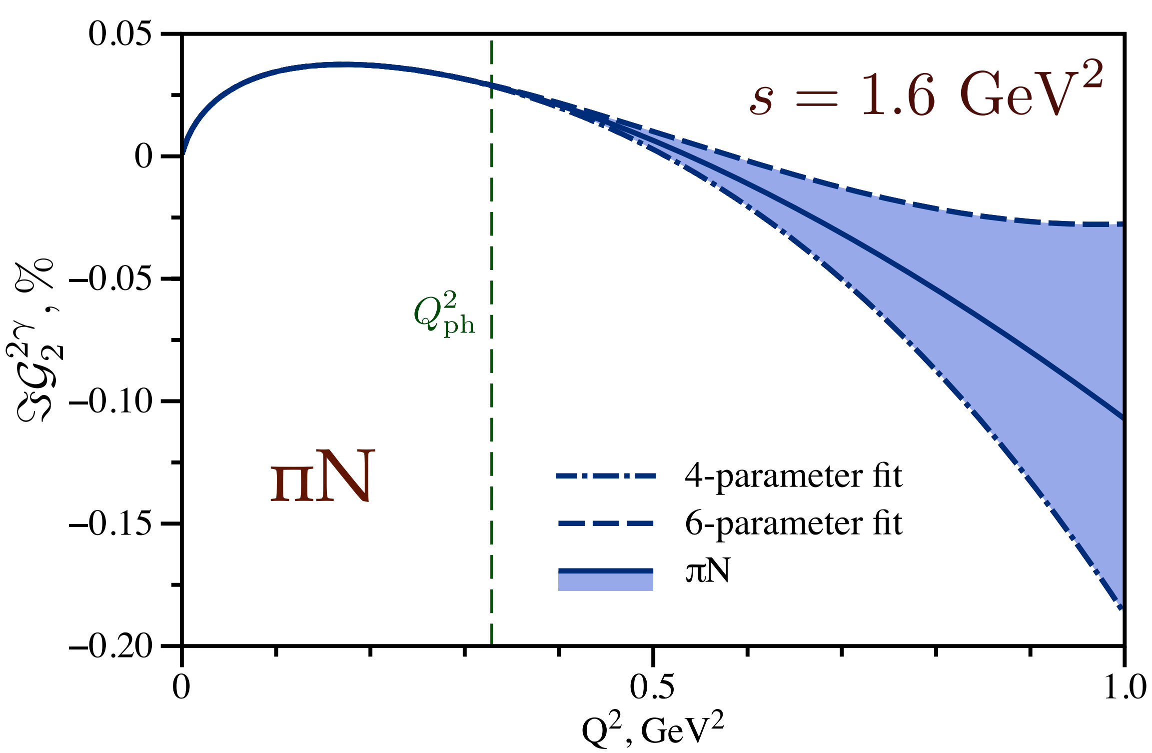


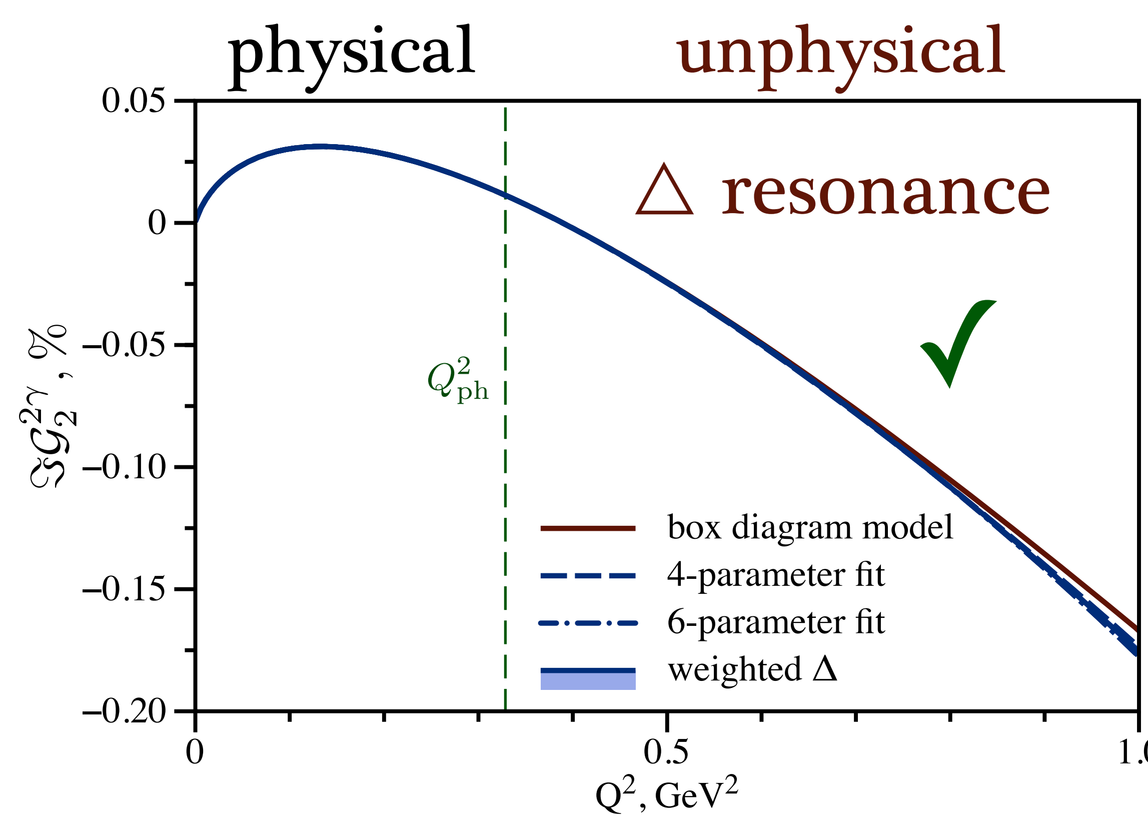
- pion electroproduction amplitudes: MAID2007

D. Drechsel. S. Kamalov and L. Tiator (2007)

- analytical continuation: fit of low-Q² expansion in physical region

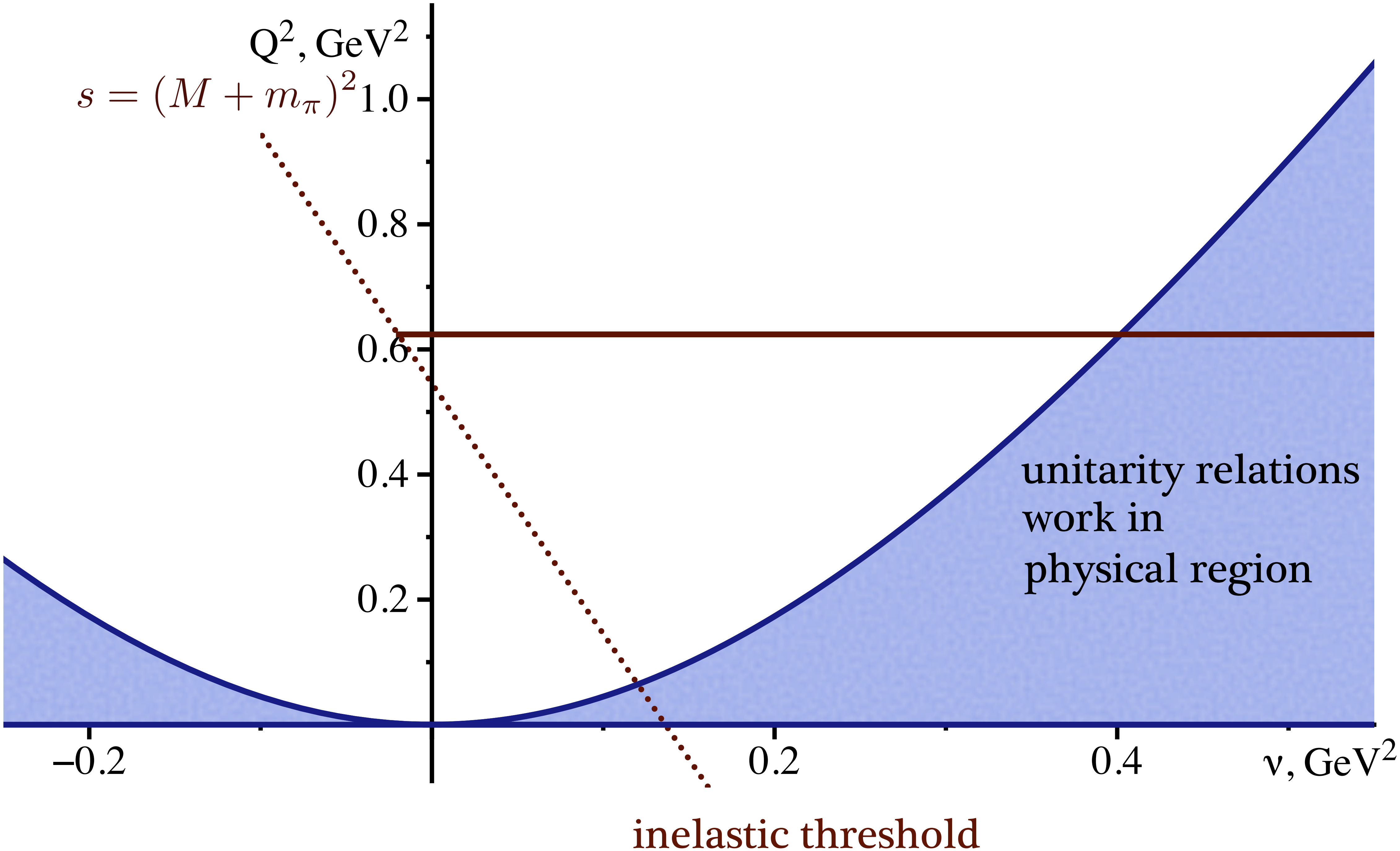
$$\mathcal{G}_{1,2}(s,\ Q^2),\ Q^2\mathcal{F}_3(s,\ Q^2) \sim a_1 Q^2 \ln Q^2 + a_2 Q^2 + a_3 Q^4 \ln Q^2 + \dots$$





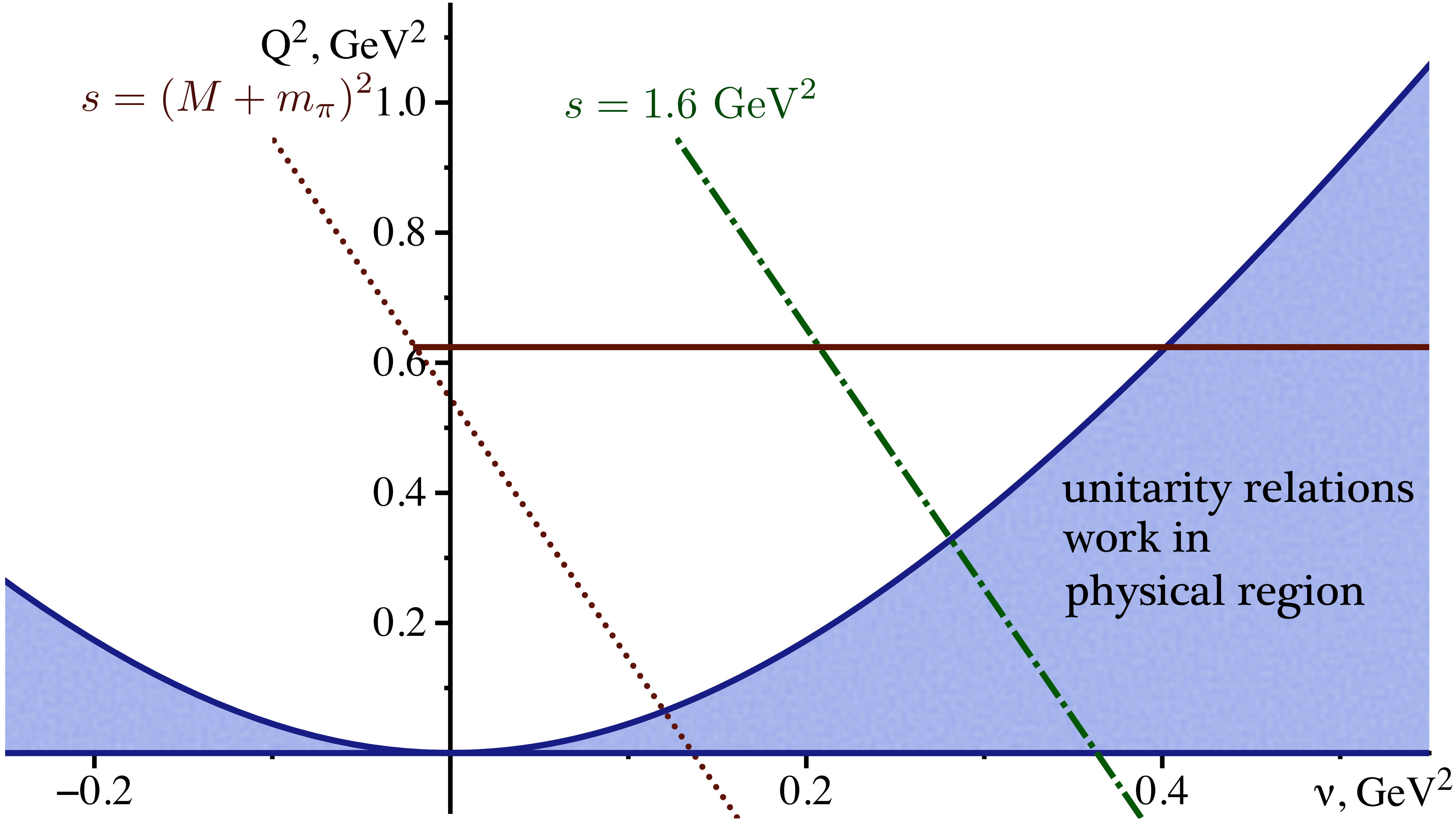
- uncertainty: extrapolation + large invariant masses

Mandelstam plot (ep)



- πN intermediate state is outside physical region for $Q^2 > 0.064$ GeV^2

Mandelstam plot (ep)



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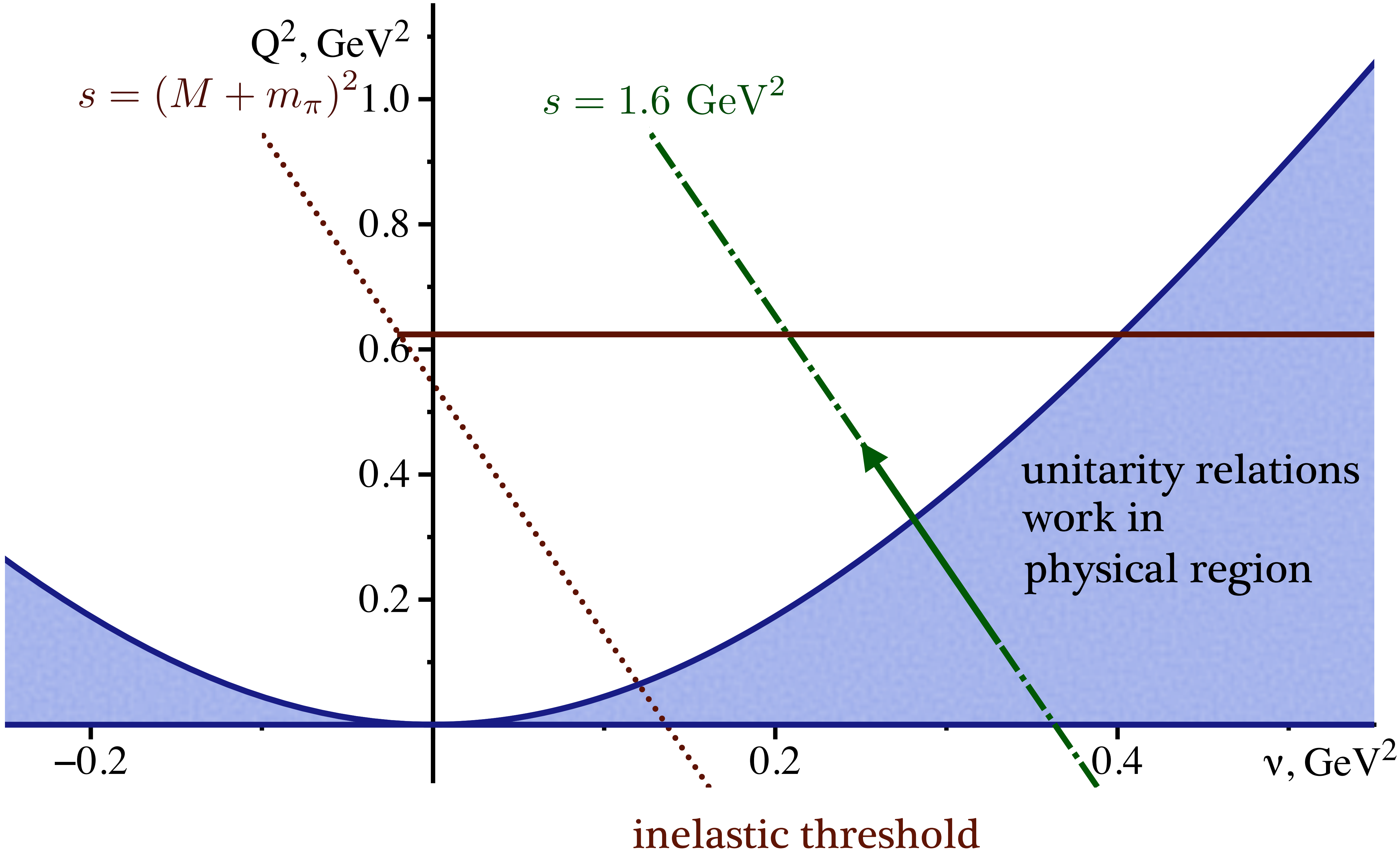

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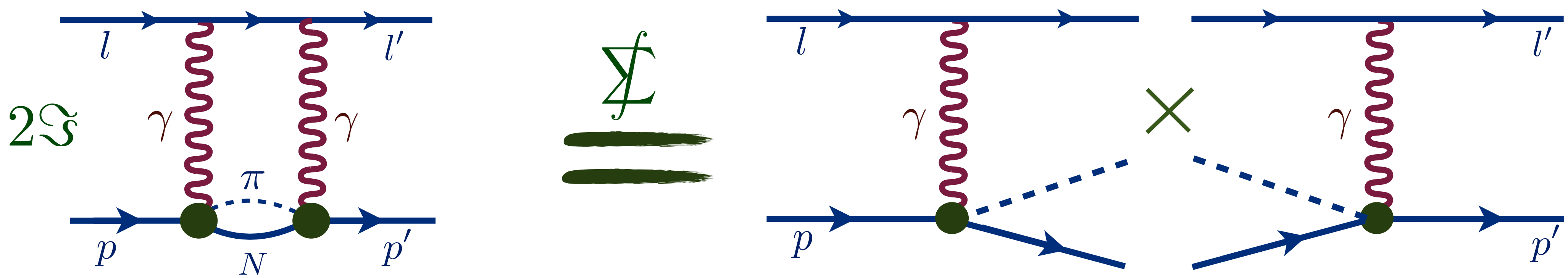
Analytical continuation. πN states



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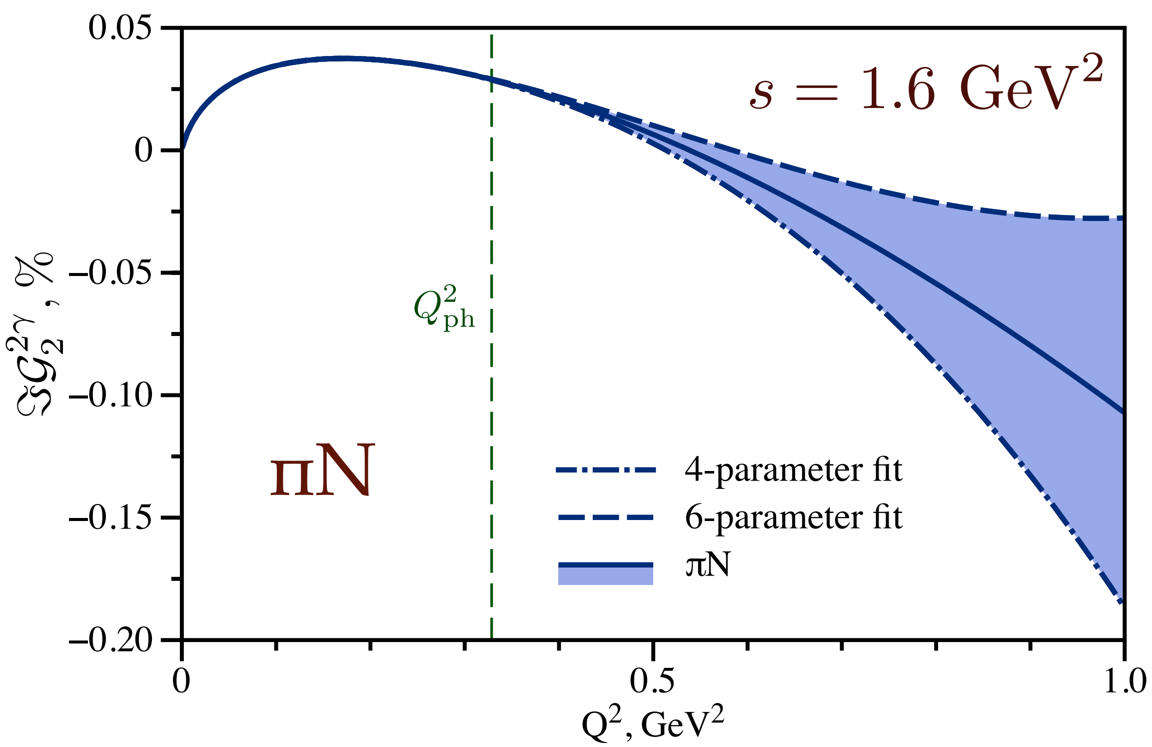
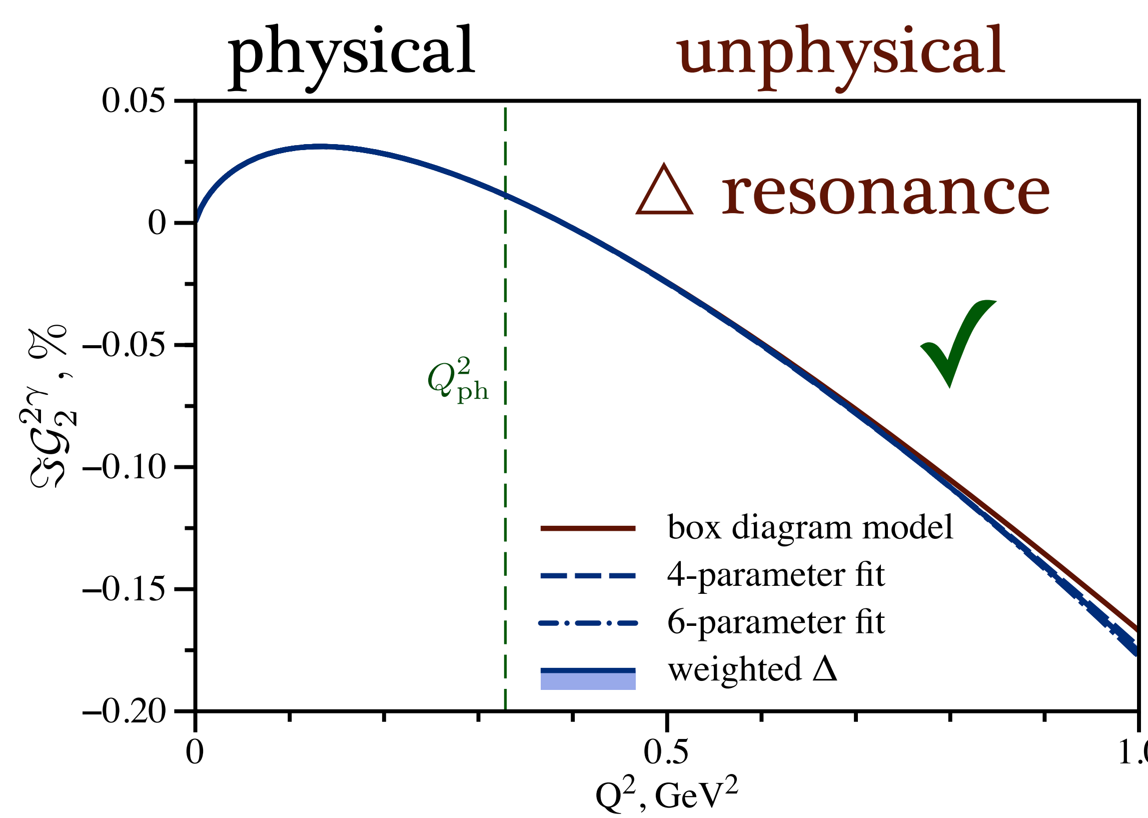
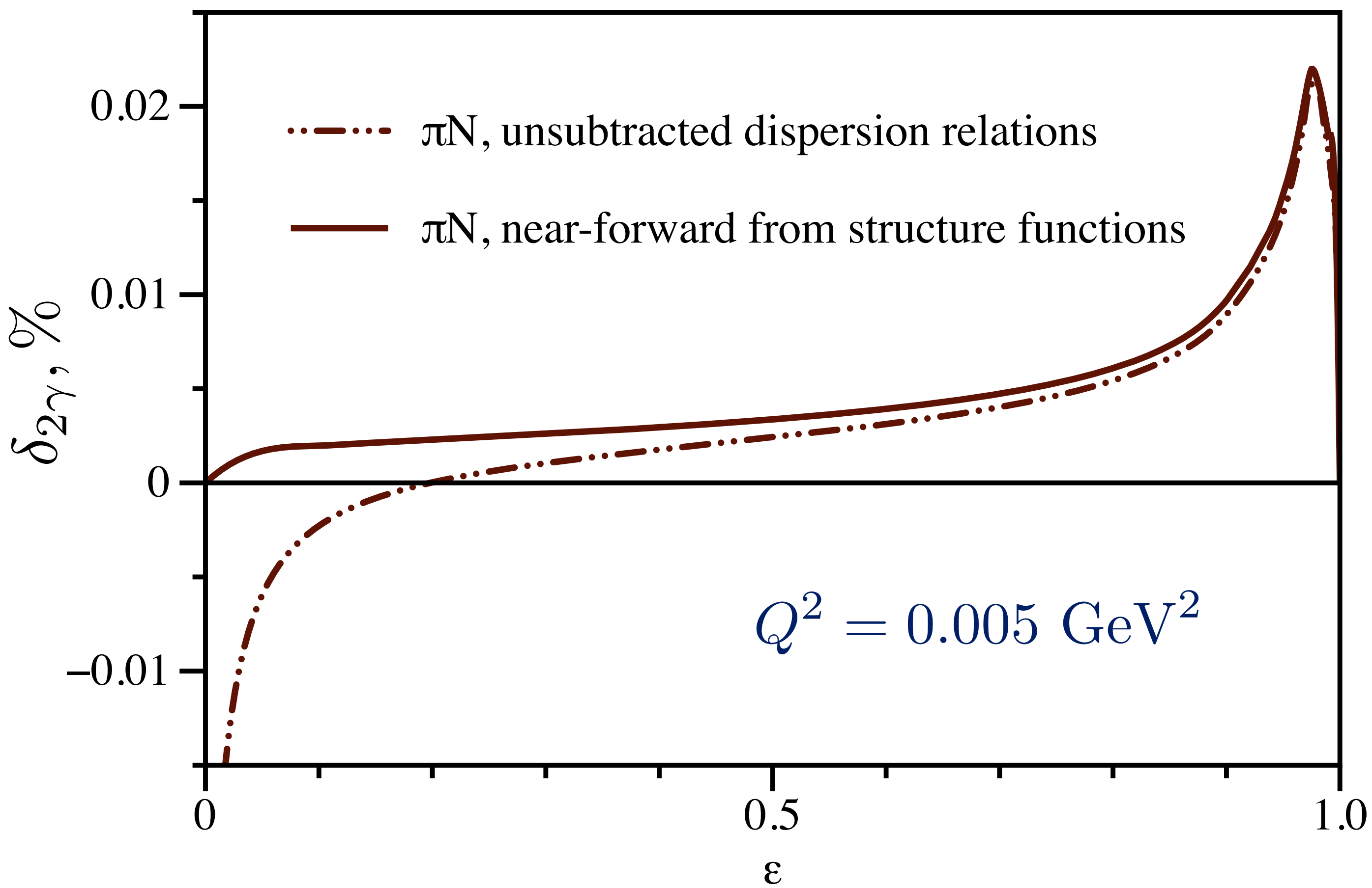




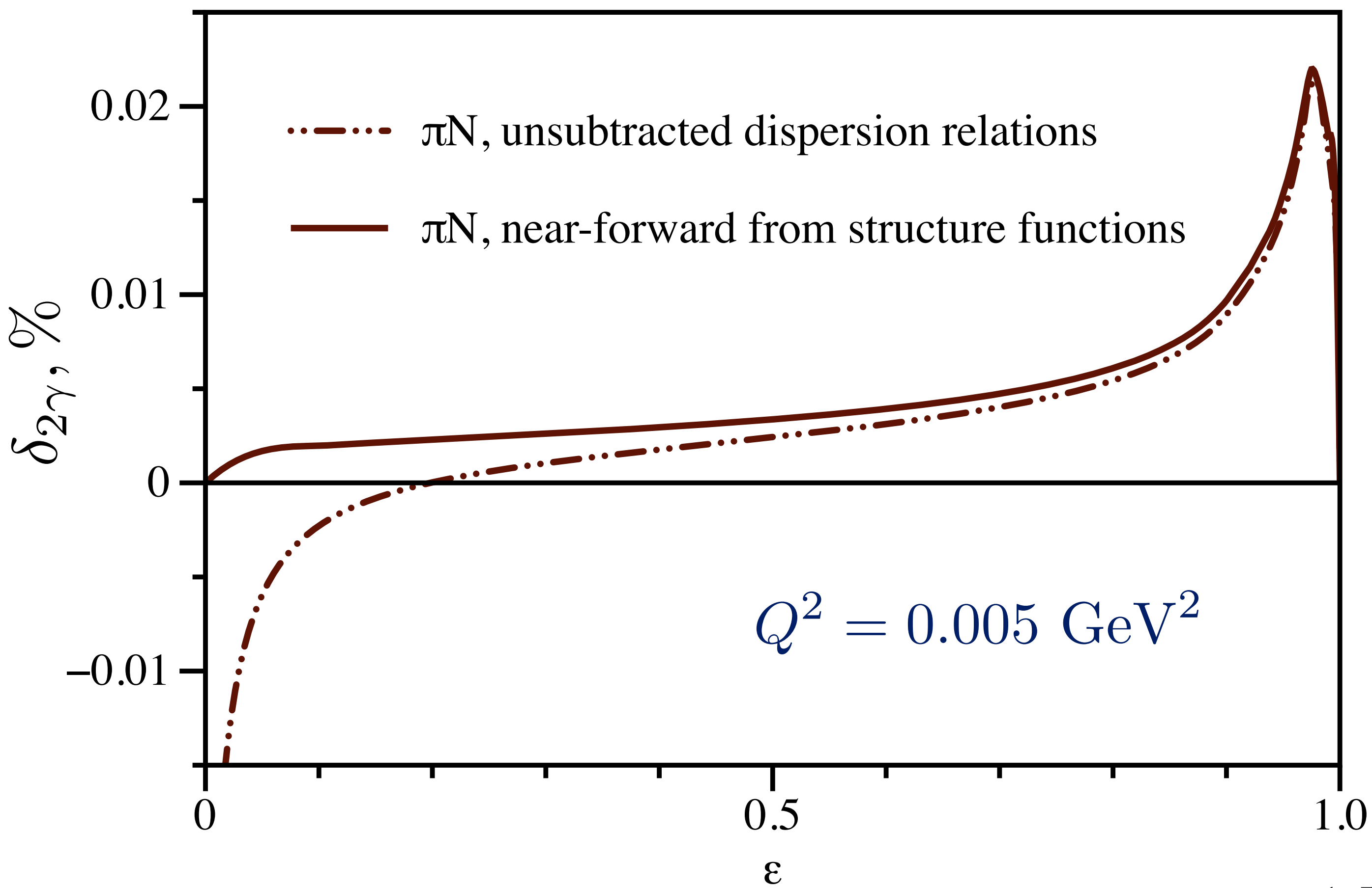
- uncertainty: extrapolation + large invariant masses

πN in dispersive framework (e⁻p)



- dispersion relations agree with near-forward at large ε

πN in dispersive framework (e⁻p)



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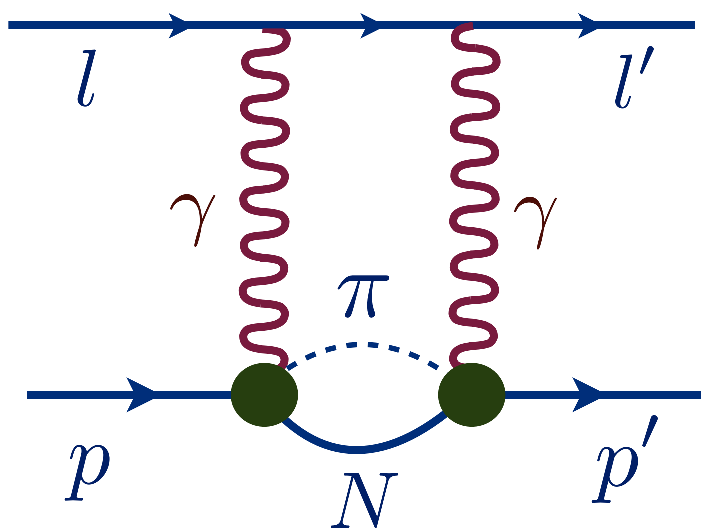
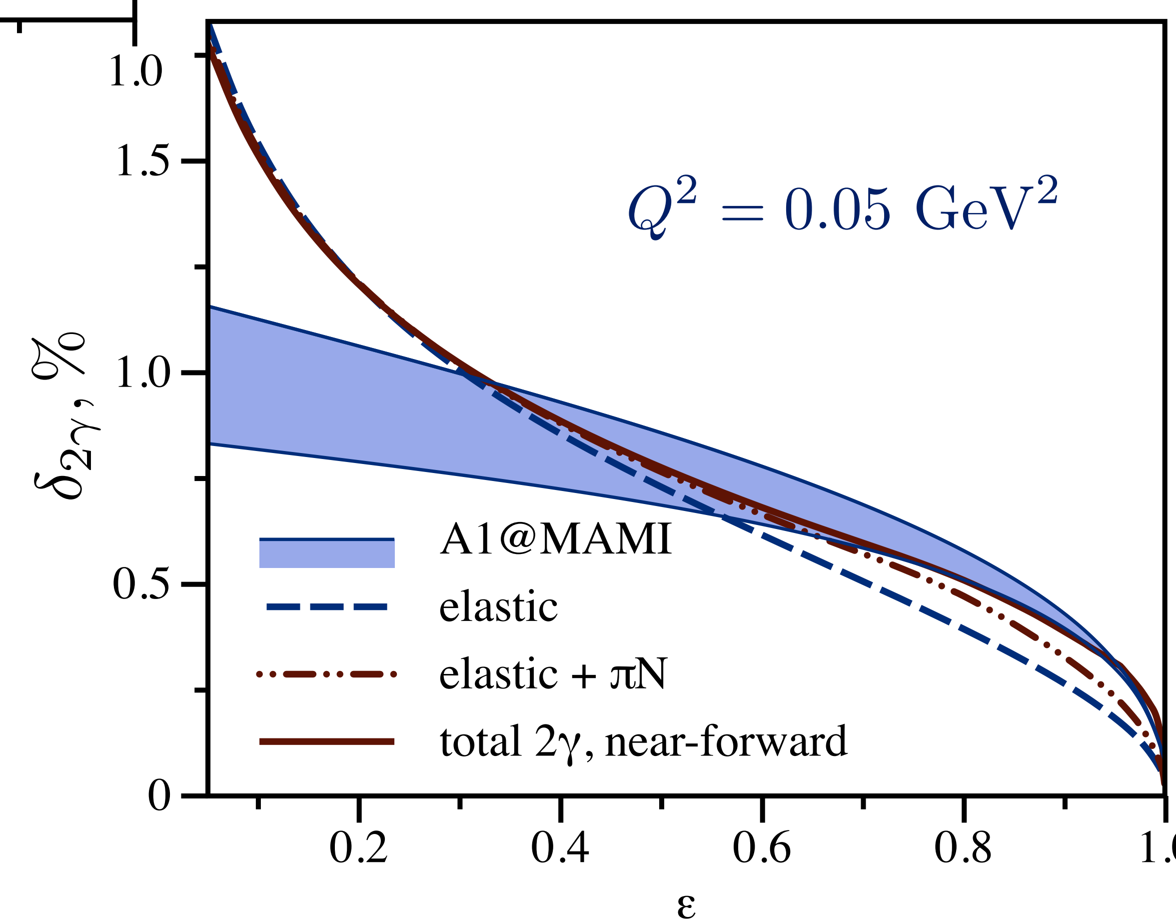
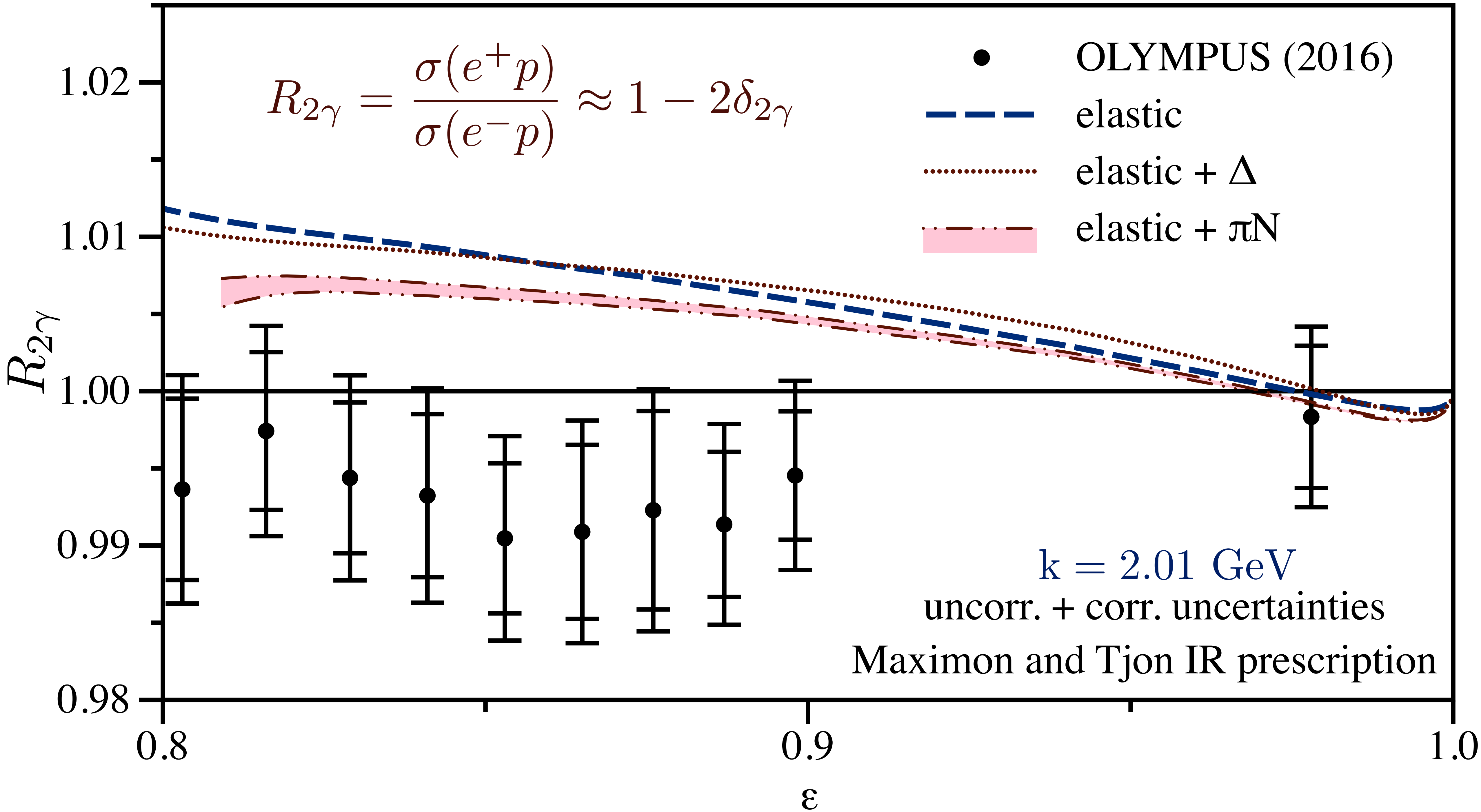


- πN is dominant inelastic 2γ



O. T., B. Pasquini and M. Vanderhaeghen (2017)

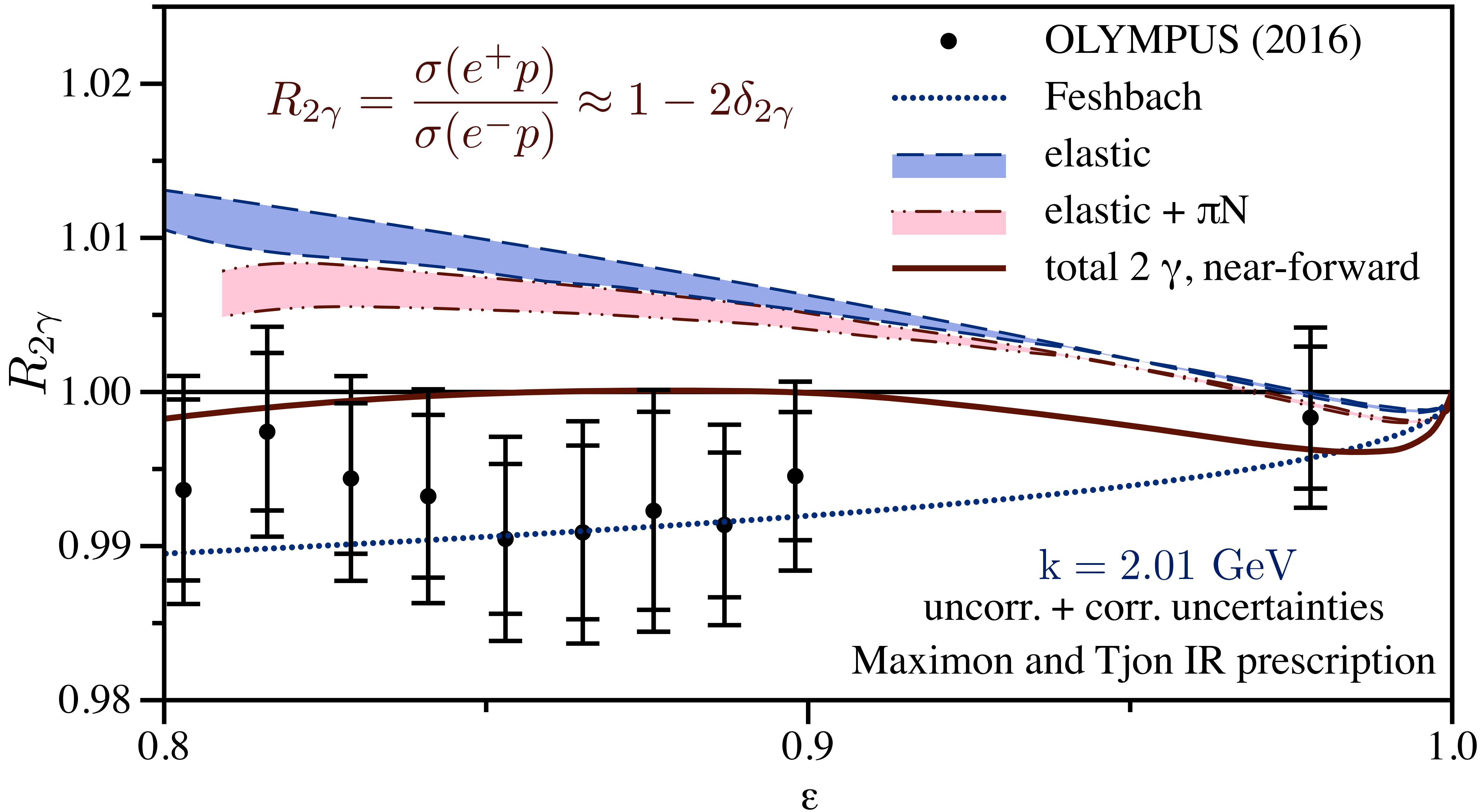
Comparison with data



- weighted △ is similar to narrow one
- πN contribution is closer to data than △ only

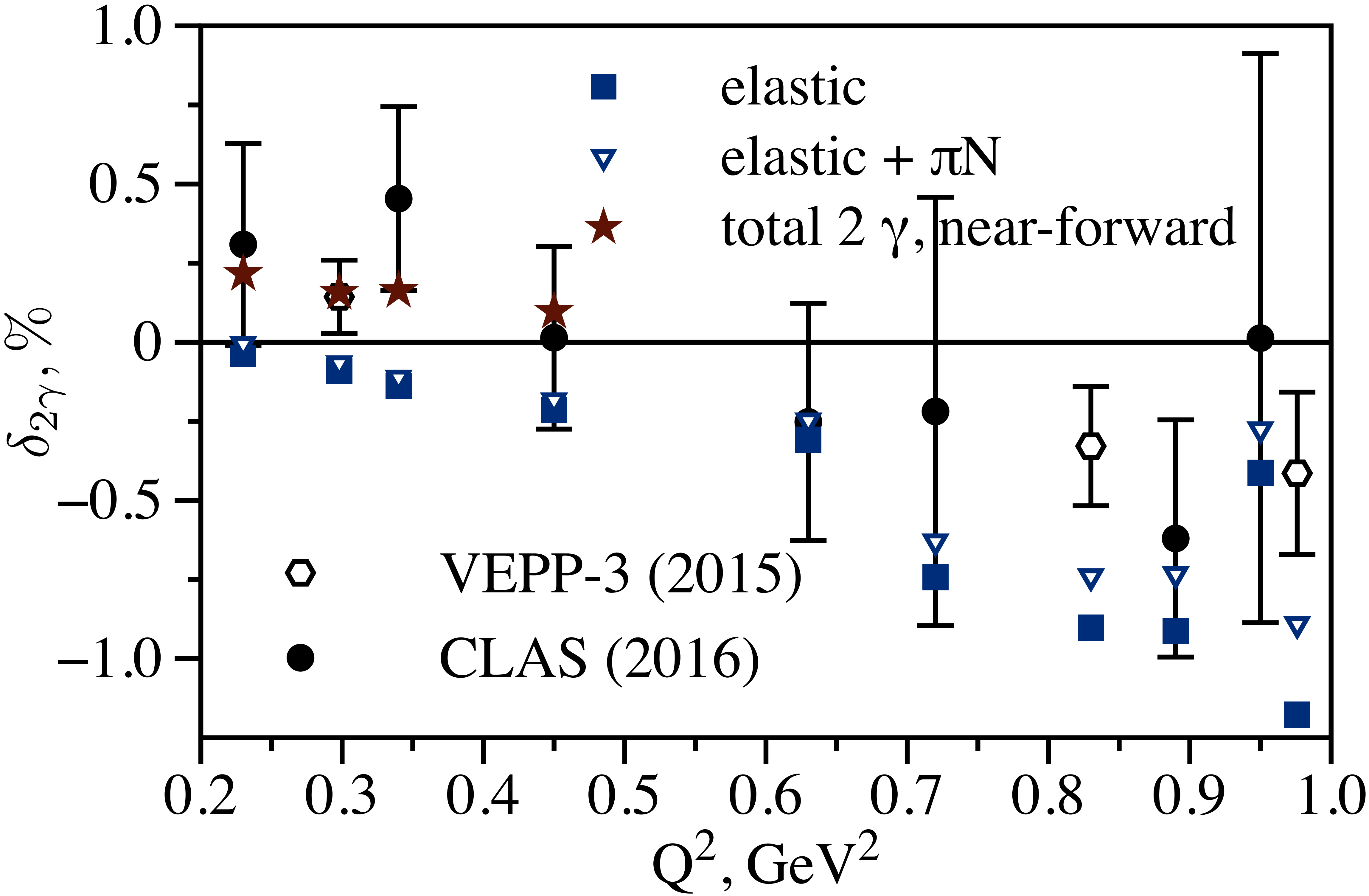
Blunden and Melnitchouk (2017)

Comparison with data



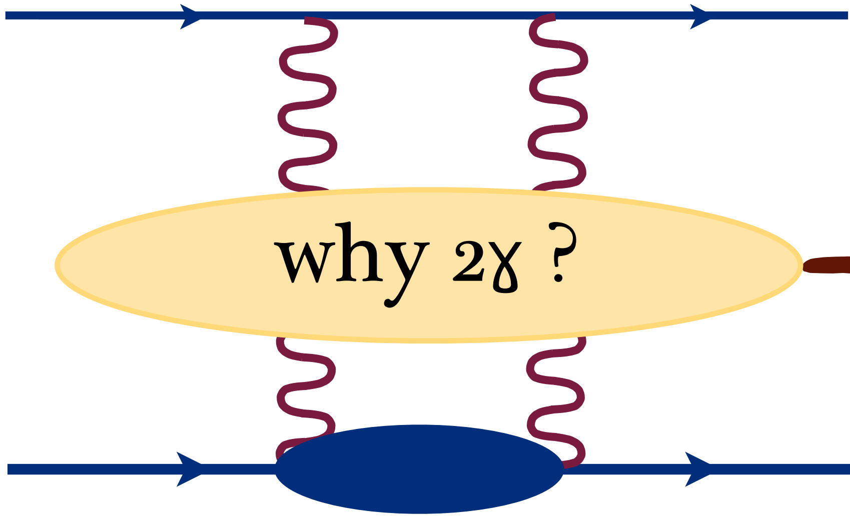
- near-forward 2ɣ agree with data
- multi-particle 2ɣ, e.g. ππN, is important

Comparison with data



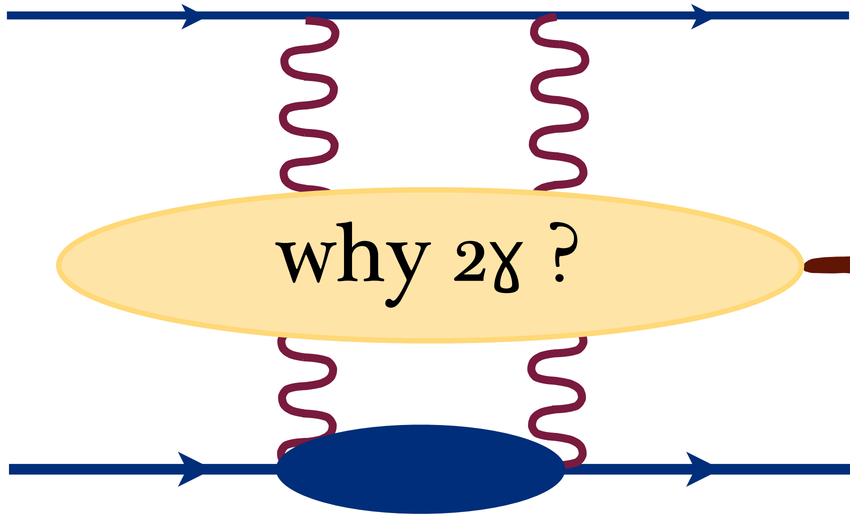
- dispersion relations agree with CLAS data

Conclusions



largest theoretical uncertainty
in low-energy proton structure

Conclusions



largest theoretical uncertainty
in low-energy proton structure

how to study ?

box diagram

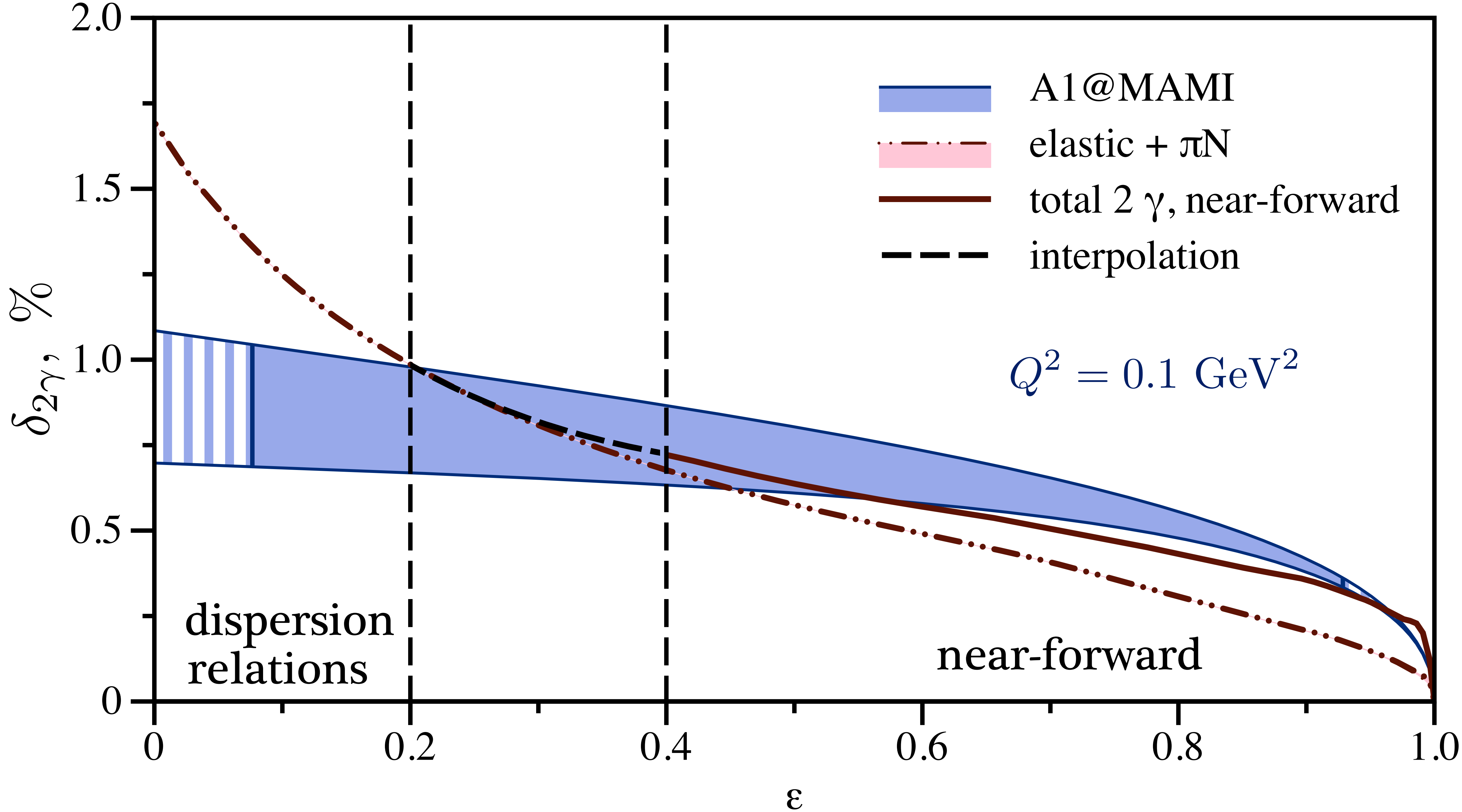
small scatt. angles
ep, μp (all states)

dispersion relations

all scatt. angles
ep (p + πN states)

- multi-particle 2γ, e.g. ππN, within dispersion relations is important

Our best 2γ knowledge



- small Q²: near-forward at large ε, all inelastic states
- Q²≲1 GeV²: elastic+πN within dispersion relations
- intermediate range: interpolation

Outlook

theoretical 2γ $\xrightarrow{ep}$ magnetic radius extraction

$\downarrow$

JLAB data (g_1, g_2) $\longrightarrow$ application to 1S HFS exp

MUSE data $\xleftrightarrow{\mu p}$ dispersive 2γ evaluation

Thanks for your attention !!!