Axial-Vector Nucleon-$\Delta(1232)$ Transition Form Factors in Chiral Perturbation Theory

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Outline

Chiral effective field theory

- Chiral perturbation theory (ChPT) is an effective field theory of QCD at low energies.
- Typical scale: $\Lambda_\chi \approx 1$ GeV $(4\pi F)$.
- Small scale expansion (SSE): treat the mass splitting Δ as an additional small parameter besides the external momenta and quark (meson) masses

$$\frac{p}{\Lambda_\chi}, \frac{M_\pi}{\Lambda_\chi}, \frac{\Delta}{\Lambda_\chi}$$
$$\Delta \equiv m_\Delta - m_N = 294 \text{ MeV} \approx 3F_\pi.$$

- The explicit inclusion of the $\Delta(1232)$ into an effective pion-nucleon field theory requires

$$\begin{aligned}\mathcal{L}_{\text{eff}} =& \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \ldots, \\ \mathcal{L}^{(i)} =& \mathcal{L}^{(i)}_{\pi N} + \mathcal{L}^{(i)}_{\pi N\Delta} + \mathcal{L}^{(i)}_{\pi\Delta}, \quad i = 1, 2, 3, \ldots .\end{aligned}$$

Power counting (PC)

PC scheme is to decide on relative importance of Feynman diagrams.

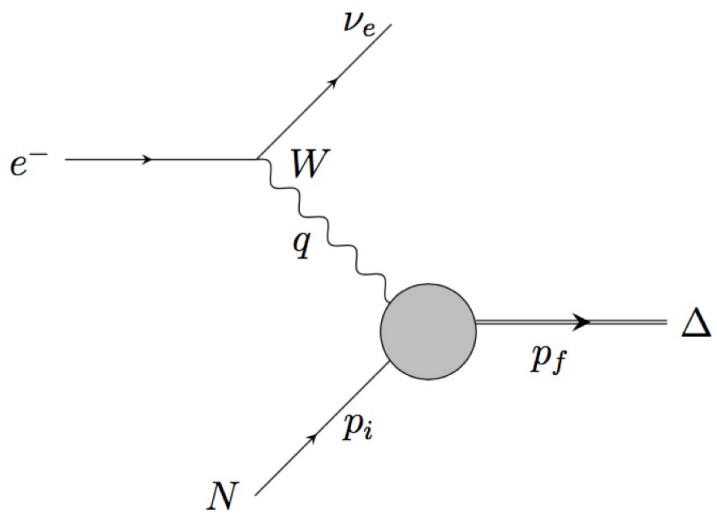
- Each diagram is assigned chiral order D.
- Renormalized diagram is of order p^D.
  - Loop integration in n dimensions $\sim \mathcal{O}(p^n)$.
  - Vertex from $\mathcal{L}^{(i)} \sim \mathcal{O}(p^i)$.
  - Nucleon and Δ propagator $\sim \mathcal{O}(p^{-1})$.
  - Pion propagator $\sim \mathcal{O}(p^{-2})$.
- Diagrams with higher order D are less important.
  - D $\geq$1.
  - Loops are suppressed.

Complex-mass renormalization scheme (CMS)

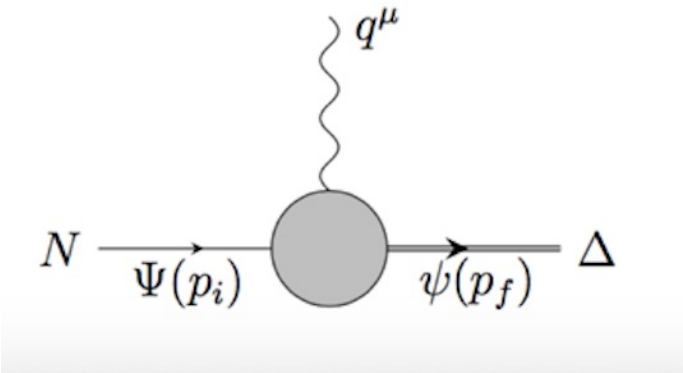
- Generalization of the on-mass-shell scheme to unstable particles.
  $\Rightarrow m_0 = (m_R + i\frac{\Gamma_R}{2}) + \delta m \equiv z_R + \delta m$.
- Renormalized mass (z_R) are chosen as complex pole of full propagator (of unstable particle) in the chiral limit.
- Complex poles for resonances are analogues of real poles for stable particles.
- Renormalized masses are included in the propagators and the counter-terms are treated perturbatively.

Axial-vector $N \to \Delta(1232)$ transition form factor

- All scattering processes of strongly interacting particles are described by the S-matrix.
- EFT describes the S-matrix of QCD in terms of effective degrees of freedom. Resonances are represented by corresponding fields.
- Neutrino scattering $\nu N \to \mu\pi N$, $e^- N \to \Delta\nu_e$
- Axial vector with momentum q^μ.
- The nucleon and the delta are on the mass shell $p_i^2 = m_N^2, \quad p_f^2 = z_\Delta^2$.



A processes of weak pion-production.



A schematic diagram for axial $N - \Delta$ transition.

Definition of the matrix element

- The corresponding matrix element between initial nucleon (N) and final delta (Δ) states is parameterized as

$$\left\langle \Delta(p^{'}) \middle| -A^3_\mu \middle| N(p) \right\rangle = \bar{u}^\lambda(p^{'})[(\frac{C_3^A(q^2)}{m_N}\gamma^\nu + \frac{C_4^A(q^2)}{m_N^2}p^{'\nu})(g_{\lambda\mu}g_{\rho\nu} - g_{\lambda\rho}g_{\mu\nu})q^\rho + C_5^A(q^2)g_{\lambda\mu} + \frac{C_6^A(q^2)}{m_N^2}q_\lambda q_\nu]u(p).$$

A^3_μ : The physically relevant axial isovector current.
q^μ : Momentum transfer, $q^\mu = p^{'\mu} - p^\mu$.
m_N : Nucleon mass.

Definition of the matrix element

- The structure of a particle is encoded in several form factors.
- For the weak $N \to \Delta$ transition: the four form factors of the isovector axial-vector current, $C_3^A(q^2)$, $C_4^A(q^2)$, $C_5^A(q^2)$ and $C_6^A(q^2)$.
- They encode the structure of the matrix elements of the isovector axial-vector current $A^{\mu,i}(x)$ which, in the SU(2) case, is given by

$$A^{\mu,i} \equiv \bar{q}(x)\gamma^\mu\gamma_5\frac{\tau^i}{2}q(x), \quad q = \begin{pmatrix} u \\ d \end{pmatrix}, \quad i = 1, 2, 3.$$

Contributing Feynman diagrams

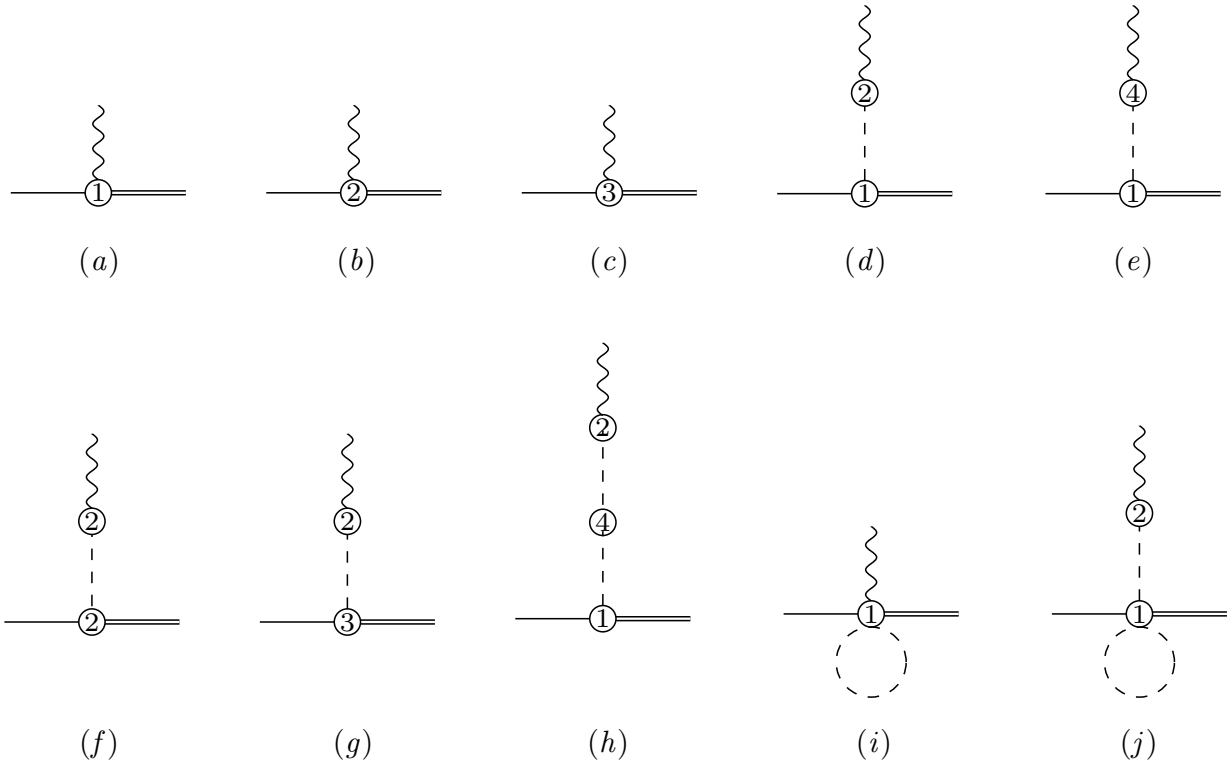


Fig. I: Tree level and one-loop contributions to the axial $N \to \Delta$ transition form factors at $\mathcal{O}(p^3)$. Single, double, dashed and wiggly lines correspond to nucleon, delta, pion and axial-vector current, respectively.

Contributing Feynman diagrams

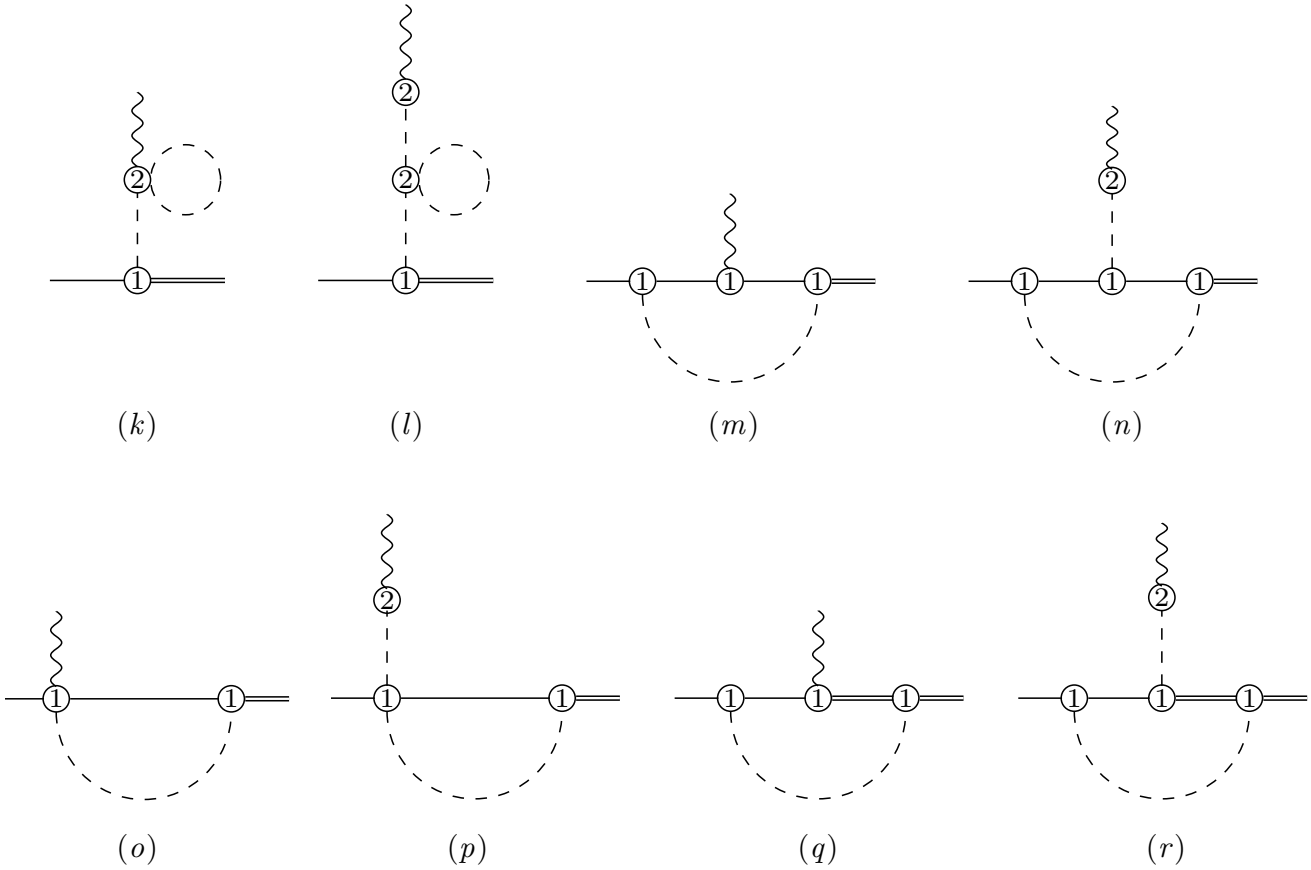


Fig. II: Tree level and one-loop contributions to the axial $N \to \Delta$ transition form factors at $\mathcal{O}(p^3)$. Single, double, dashed and wiggly lines correspond to nucleon, delta, pion and axial-vector current, respectively.

Contributing Feynman diagrams

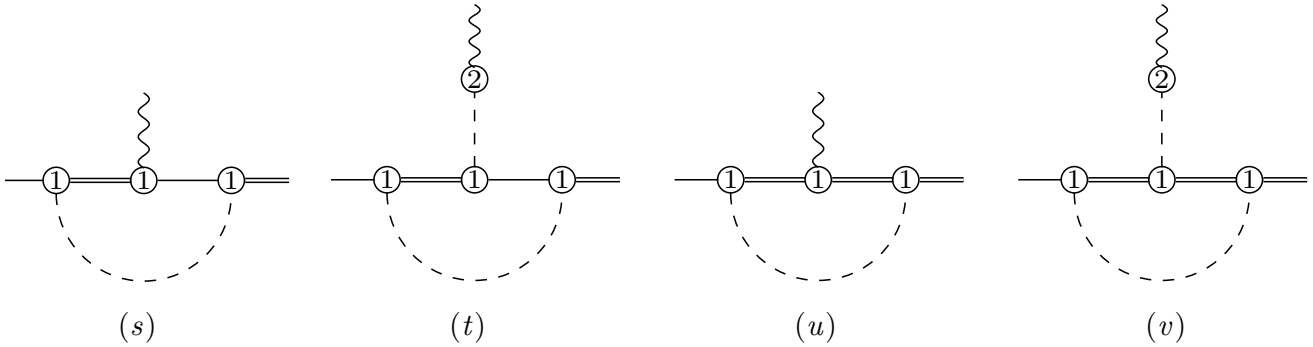


Fig. III: Tree level and one-loop contributions to the axial $N \to \Delta$ transition form factors at $\mathcal{O}(p^3)$. Single, double, dashed and wiggly lines correspond to nucleon, delta, pion and axial-vector current, respectively.

Lagrangians I

$$
\begin{aligned}
\mathcal{L}_2 =& \frac{F^2}{4}(Tr[D_\mu U(D^\mu U)^\dagger] + Tr[\chi U^\dagger + U\chi^\dagger]),\\
\mathcal{L}_4^{GL} =& \frac{l_1}{4}(Tr[D_\mu U(D^\mu U)^\dagger])^2 + \frac{l_2}{4} Tr[D_\mu U(D_\nu U)^\dagger] Tr[D^\mu U(D^\nu U)^\dagger]\\
&+ \frac{l_3}{16}(Tr[\chi U^\dagger + U\chi^\dagger])^2 + \frac{l_4}{4} Tr[D_\mu U(D^\mu \chi)^\dagger] + D_\mu \chi(D^\mu U)^\dagger] + ...,\\
\mathcal{L}_{\pi N}^{(1)} =& \bar{\Psi}(i\not{D} - m_N + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu)
\end{aligned}
$$

where

$$
\begin{aligned}
\chi =& 2B(s + ip),\\
u_\mu =& i[u^\dagger \partial_\mu u - u\partial_\mu u^\dagger - i(u^\dagger a_\mu u + u a_\mu u^\dagger)],\\
U =& u^2 = \exp\frac{i\phi}{F}, \quad \phi = \vec{\tau}\cdot\vec{\phi}.
\end{aligned}
$$

- $l_1, l_2, l_3, ...$ are known LECs.
- F is pion decay constant, g_A is known axial-vector coupling constant.

Lagrangians II

$$\mathcal{L}^{(1)}_{\pi\pi NN} = \frac{1}{4F^2}\epsilon_{abc}\bar{\Psi}\gamma^\mu\phi_a\partial\mu\phi_b\tau_c\Psi,$$

$$\begin{aligned}\mathcal{L}^{(1)}_{\pi\Delta} = &-\bar{\psi}^\mu\ P^{\frac{3}{2}}\Big\{(i\not{D} - m_\Delta)g^{\mu\nu} - i(\gamma^\mu D^\nu + \gamma^\nu D^\mu)\\ &+ i\gamma_\mu\not{D}\gamma_\nu + m_\Delta\gamma_\mu\gamma_\nu\\ &+ \frac{g_1}{2}\not{u}\gamma_5 g_{\mu\nu} + \frac{g_2}{2}(\gamma_\mu u_\nu + u_\mu\gamma_\nu)\gamma_5\\ &+ \frac{g_3}{2}\gamma_\mu\not{u}\gamma_5\gamma_\nu\Big\}\ P^{\frac{3}{2}}\psi^\nu.\end{aligned}$$

- The requirement of the consistency of the corresponding EFT in the sense of having the right # of DoF, leads to non-trivial constraints among coupling constants [1]: $g_1 = -g_2 = -g_3 = g_1$ known.

[1] N. Wies, J. Gegelia, and S. Scherer, Phys. Rev. D73, 094012 (2006).

Lagrangians III

$$\mathcal{L}^{(1)}_{\pi N\Delta} = g_\Delta \bar{\psi}^i_\mu P^{\frac{3}{2}}_{ij} (g^{\mu\nu} - \gamma^\mu\gamma^\nu) u^j_\nu \Psi + h.c.,$$

$$\mathcal{L}^{(2)}_{\pi N\Delta} = \bar{\psi}^i_\mu P^{\frac{3}{2}}_{ij} \theta^{\mu\alpha} [i b_3\, \omega^j_{\alpha\nu}\gamma^\nu - \frac{b_8}{m_N}\omega^j_{\alpha\nu} D^\nu]\Psi,$$

$$\mathcal{L}^{(3)}_{\pi N\Delta} = \bar{\psi}^i_\mu P^{\frac{3}{2}}_{ij} \theta^{\mu\nu} (\frac{f_1}{m_N}[D_\nu, \omega^j_{\alpha\beta}] i\gamma^\alpha D^\beta - \frac{f_2}{2m_N^2}[D_\nu, \omega^j_{\alpha\beta}]\{D^\alpha, D^\beta\}$$
$$+ f_4\, \omega^j_\nu \mathrm{Tr}[\chi_+] + f_5 [D_\nu, i\chi^j_-])\Psi$$

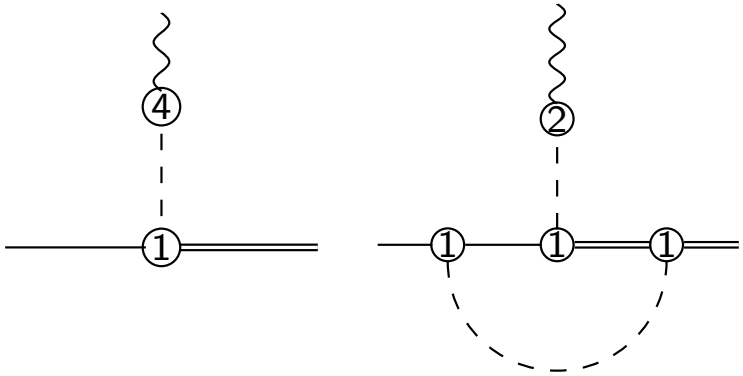
where

$$\theta^{\mu\nu} = g^{\mu\nu} - \gamma^\mu\gamma^\nu,$$
$$\omega^i_{\mu\nu} = \frac{1}{2}\mathrm{Tr}(\tau^i[\partial_\alpha, u_\beta]).$$

- b_3, b_8, f_1, f_2, f_4 and f_5 are unknown coupling constants [2,3].

[2] T. R. Hemmert, B. R. Holstein and J. Kambor, J. Phys. G24, 1831 (1998).
[3] De-Liang Yao et al., JHEP 1605, 038 (2016).

A contribution from loop and tree



$$\Gamma^{\mu\nu}_{ij} = \frac{2ig_\Delta M^2 q^\mu l_4(\gamma^\nu \not{q} - q^\nu)}{F^2(q^2 - M^2)}\delta_{ij},$$

$$\Gamma^{\mu\nu}_{ij} = \frac{5g_\Delta g_1 g_A}{12F^2}\int \frac{d^n k}{(2\pi)^n}(g^{\mu\alpha}\not{k}\gamma_5 - \gamma^\mu \not{k}\gamma_5\gamma^\alpha - \gamma^\mu k^\alpha \gamma_5 - k^\mu \gamma^\alpha \gamma_5)$$
$$\frac{\not{p}_f + \not{k} + m_\Delta}{(p_f + k)^2 - m_\Delta^2}[g_{\alpha\beta} - \frac{\gamma_\alpha\gamma_\beta}{n-1} - \frac{(n-2)(k_\alpha + p_{f,\alpha})(k_\beta + p_{f,\beta})}{(n-1)m_\Delta^2}$$
$$- \frac{(k_\beta + p_{f,\beta})\gamma^\alpha - \gamma_\beta(k_\alpha + p_{f,\alpha})}{(n-1)m_\Delta}](q^\beta - \gamma^\beta \not{q})$$
$$\frac{q^\nu \not{k}\gamma_5}{(\not{p}_i + \not{k} - m_N)(k^2 - M^2)(q^2 - M^2)}\delta_{ij}.$$

How to extract form factors?

- Diagram$[\lambda, \mu]$ is the result of the diagrams after one reduces the tensor integrals and simplifies the algebra,

$$\text{Diagram}[p_f, q, \mu, \nu] := k_1 p_f^\mu q^\nu + k_2 q^\mu q^\nu + k_3 \gamma^\mu q^\nu + k_4 g^{\mu\nu}.$$

- FFs$[p_f, q, \mu, \nu]$ is the structure of the transition matrix element

$$\text{FFs}[p_f, q, \mu, \nu] := [(\frac{C_3}{m_N}\gamma^\lambda + \frac{C_4}{m_N^2} p_f^\lambda)(g_{\mu\nu} g_{\rho\lambda} - g_{\nu\rho} g_{\mu\nu}) q^\rho + C_5 g^{\mu\nu} + \frac{C_6}{m_N^2} q^\mu q^\nu].$$

- $C_3 \to k_3 m_N, \quad C_4 \to k_1 m_N^2, \quad C_6 \to k_2 m_N^2,$
  $C_5 \to \frac{1}{2}(m_\Delta - m_N)(k_1(m_\Delta + m_N) + 2k_3) + k_1 q^2 + 2k_4).$
- k_1: the coefficient of the result corresponding to $p_f^\mu q^\nu$,
  $k_2 \to q^\mu q^\nu$,
  $k_3 \to q^\nu \gamma^\mu$,
  $k_4 \to g^{\mu\nu}$.

Contributions from tree level diagrams

$$
\begin{aligned}
C_3[q^2]_b &:= -\, b_3 m_N, \\
C_4[q^2]_b &:= -\, b_8 m_N, \\
C_4[q^2]_c &:= (m_N - m_\Delta)(2f_1 m_N - f_2(m_N + m_\Delta)) - f_2 q^2/2, \\
C_5[q^2]_a &:= g_\Delta, \\
C_5[q^2]_b &:= -\,(m_N - m_\Delta)(2b_3 m_N + b_8(m_N + m_\Delta)) + b_8 q^2/2m_N \\
C_5[q^2]_c &:= 32 f_4 M^2 m_N^2 + (m_N - m_\Delta)^2 (m_N + m_\Delta)(2f_1 m_N - f_2(m_N + m_\Delta)) \\
&\quad + 2f_1 m_N(-m_N + m_\Delta) q^2 + f_2 q^4/4m_N^2
\end{aligned}
$$

- Dimensions of coupling constants: $b_3(\mathrm{GeV}^{-1})$, $b_8(\mathrm{GeV}^{-1})$, $f_1(\mathrm{GeV}^{-2})$, $f_2(\mathrm{GeV}^{-2})$.
- No dimension of l_4.

Contributions from tree level diagrams

$$
\begin{aligned}
C_6[q^2]_d &:= -\, g_\Delta m_N^2/(q^2 - M^2) \\
C_6[q^2]_e &:= -\, 2 g_\Delta M^2 m_N^2 l_4/F^2(q^2 - M^2) \\
C_6[q^2]_b &:= -\, b_3 m_N \\
C_6[q^2]_c &:= (m_N - m_\Delta)(2 f_1 m_N - f_2(m_N + m_\Delta) - f_2 q^2)/2 \\
C_6[q^2]_f &:= b_3 m_N^2 (m_\Delta - m_N)/(q^2 - M^2) \\
C_6[q^2]_g &:= -\, 16 f_4 M^2 m_N^2 + 8 f_5 M^2 m_N^2 + (m_N - m_\Delta)^2 (m_N + m_\Delta) \\
&\quad (2 f_1 m_N + f_2(m_N + m_\Delta)) + 2 q^2 (m_N - m_\Delta)(f_1 m_N + f_2(m_N + m_\Delta)) - \\
&\quad f_2 q^2/4(q^2 - M^2)
\end{aligned}
$$

- Dimensions of coupling constants: $f_4(\mathrm{GeV}^{-2})$, $f_5(\mathrm{GeV}^{-2})$.

Preliminary results

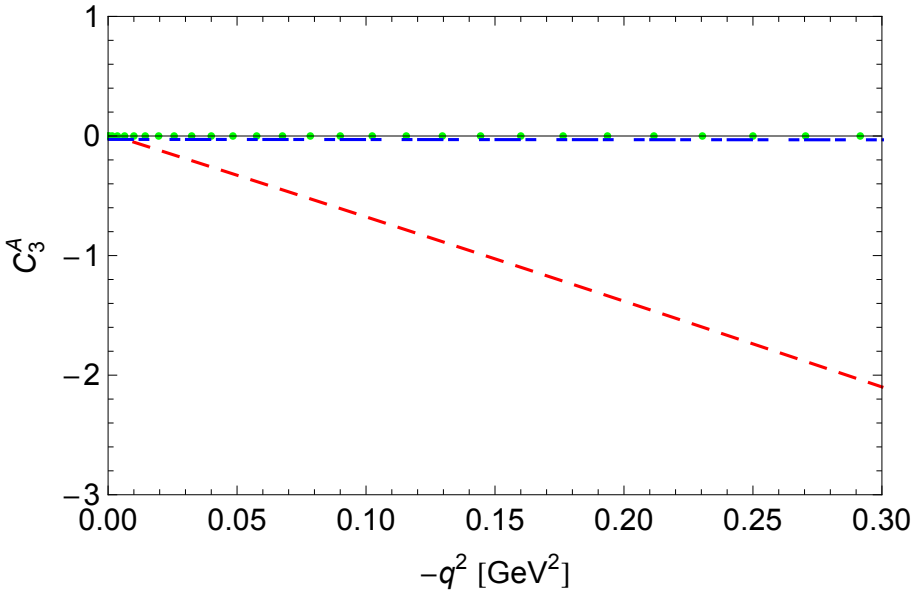


Fig. IV: q^2 dependence of C_3^A form factor.

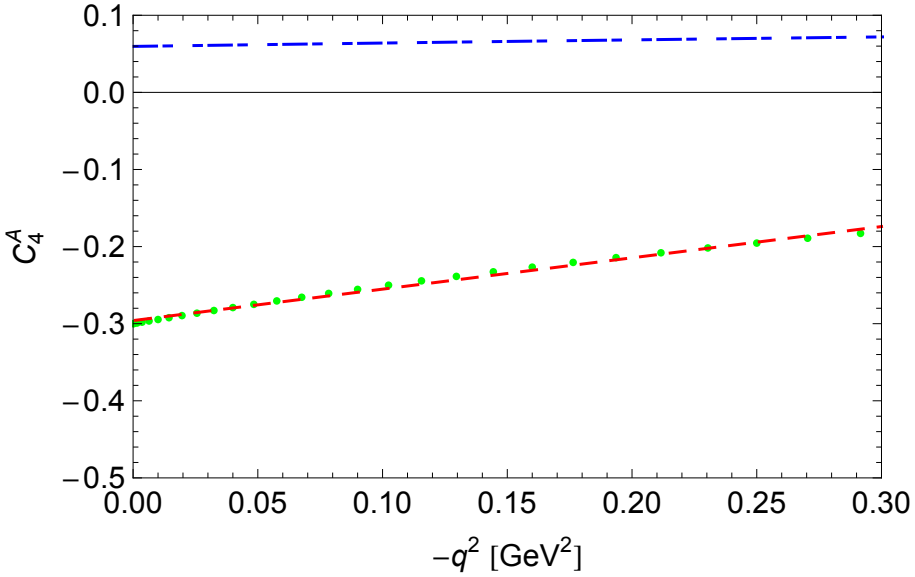


Fig. V: q^2 dependence of C_4^A form factor.

Red, blue and green lines correspond to fitted results, imaginary parts and data.

- Fit the data[4] by fixing six unknown b_3, b_8, f_1, f_2, f_4 and f_5 parameters.

$$F_i(Q^2)^{A)} = \frac{c_i(0)[1+a_i(Q^2)/(b_i+Q^2)]}{(1+Q^2/M_A^2)^2}$$

$c_i(0), a_i, b_i$ are model-dependent axial-vector FF parameters determined for the Adler model and $M_A = 1.28$ GeV is axial-vector meson mass.

[3] T. Kitagaki et al., Phys. Rev. D42, 5 (1990).

Preliminary results

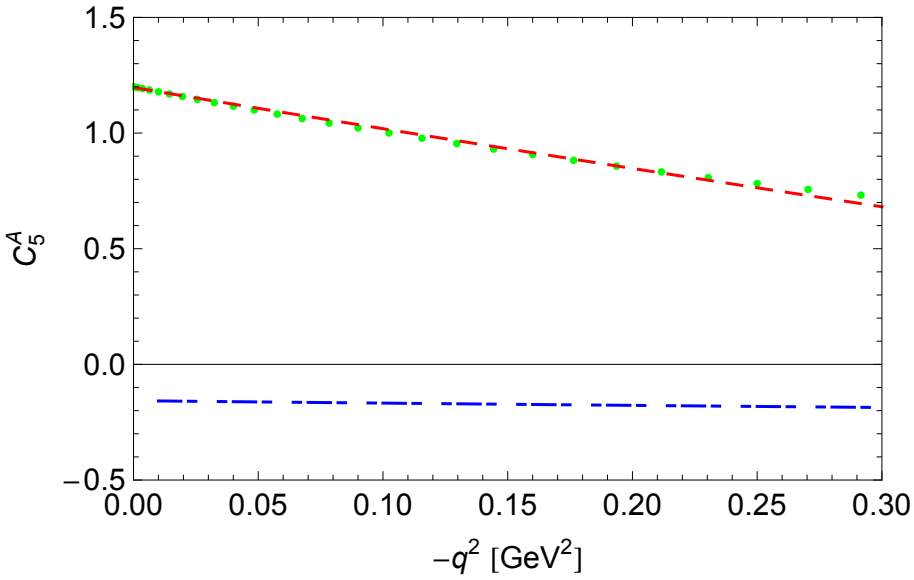


Fig. VI: q^2 dependence of C_5^A form factor.

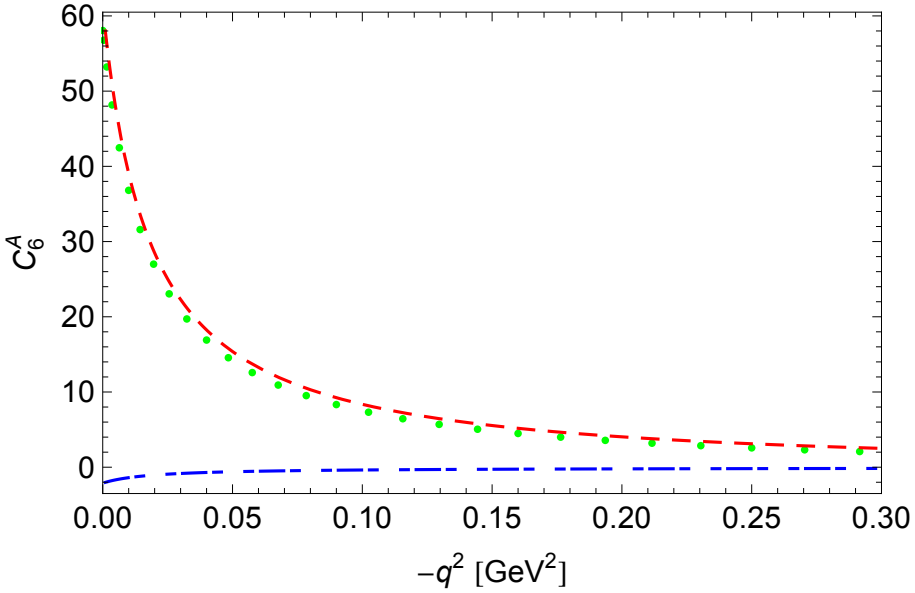


Fig. VII: q^2 dependence of C_6^A form factor.

- Because we do ChPT calculation its applicability is restricted to small q^2.
- C_5^A gives the dominant contribution to the axial transition.
- C_6^A behaves like $1/(q^2 - M^2)$ because it has a pion-pole contribution.

Axial charges

Works	$C_3^A(0)$	$C_4^A(0)$	$C_5^A(0)$	$C_6^A(0)$
Geng et al.	0	-0.29	1.16	≈-2 (non-pole)
Alexandrou et al.	0	0	0.9	3.5
Hernandez et al.	0	-0.27	1.08	—
Barquilla et al.	0.035	-0.26	0.93	52 (pion-pole)
Kucukarslan et al.	0.12	0.31	1.13	-1.61
Graczyk et al.	—	—	1.19 ± 0.08	—
Our work	0.014	-0.29	1.19	15.13

- Geng et all.: (0.0 - 0.3) GeV^2, (ChPT).
- Alexandrou et al.: (0.0 - 2.0) GeV^2, M_π= 350 - 580 MeV, (LatQCD).
- Hernandez et al.: (0.0 - 1.0) GeV^2, (ANL&BNL).
- Barquilla et al.: (0.0 - 2.0) GeV^2, (QModel).
- Kucukarslan et al.: (0.0 -10) GeV^2, (LcQCD).
- Graczyk et al.: (0.0 - 1.0) GeV^2,(ANL&BNL)

Conclusion

- The axial Nucleon to $\Delta(1232)$ transition form factors up to one-loop order in relativistic baryon chiral perturbation theory.
- Since Δ is an unstable particle $\Rightarrow$ CMS as a renormalization scheme.
- Fit the results to Kitagaki-Adler phenomenological form factors for the unknown free parameters.
  - Obtained the q^2 dependence of $C_3^A(q^2)$ should exist at $\mathcal{O}(p)^3$ conradicting with the model.
  - For $C_4^A(q^2)$, the q^2 dependence is weak compared to $C_5^A(q^2)$.
  - The contribution of $C_5^A(q^2)$ exhibits rich structure.
  - The dominant contribution comes from $C_6^A(q^2)$ because of the pion-pole.
- Our results have a reasonable agreement with other theoretical approaches.