Modern Hadron Spectroscopy : Challenges and Opportunities

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Lecture 1: Hadrons as laboratory for QCD:

- **Introduction to QCD**
- **Bare vs effective effective quarks and gluons**
- **Phenomenology of Hadrons**

Lecture 2: Phenomenology of hadron reactions

- **Kinematics and observables**
- **Space time picture of Parton interactions and Regge phenomena**
- **Properties of reaction amplitudes**

Lecture 3: Complex analysis

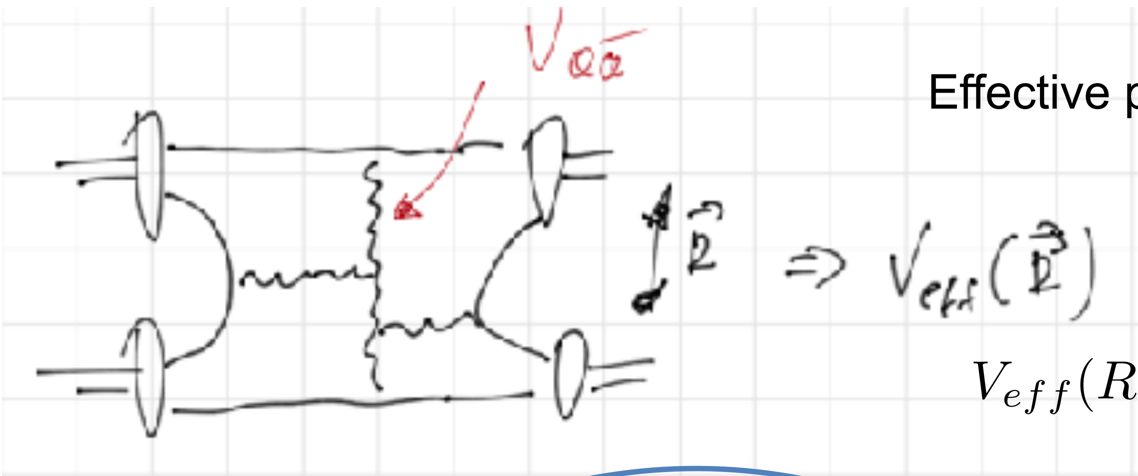
Lecture 4: How to extract resonance information from the data

- **Partial waves and resonance properties**
- **Amplitude analysis methods (spin complications)**

Constructing partial waves (from singularities)

K-matrix
K-matrix with Chew-Mandelstam "phase space"
N/D
Coupled channels
Flatte formula
Isobar model
Kinematical singularities (example)
Veneziano Amplitude

Accessing QCD resonances

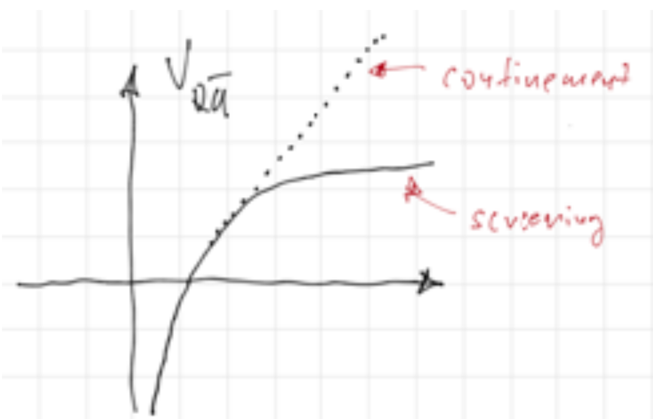


Effective potential between meson

$$V_{eff}(R) \sim \int \psi_1 \psi_2 V_{Q\bar{Q}} \psi_3 \psi_4$$

$$T = V_{eff} + \sum_n V_{eff} \frac{|n\rangle\langle n|}{E - E_n} V_{eff} + \cdots$$

High spin/mass resonances generated by confining V_{QQ} interact with multi-hadron intermediate states and screen the quark-antiquark potential

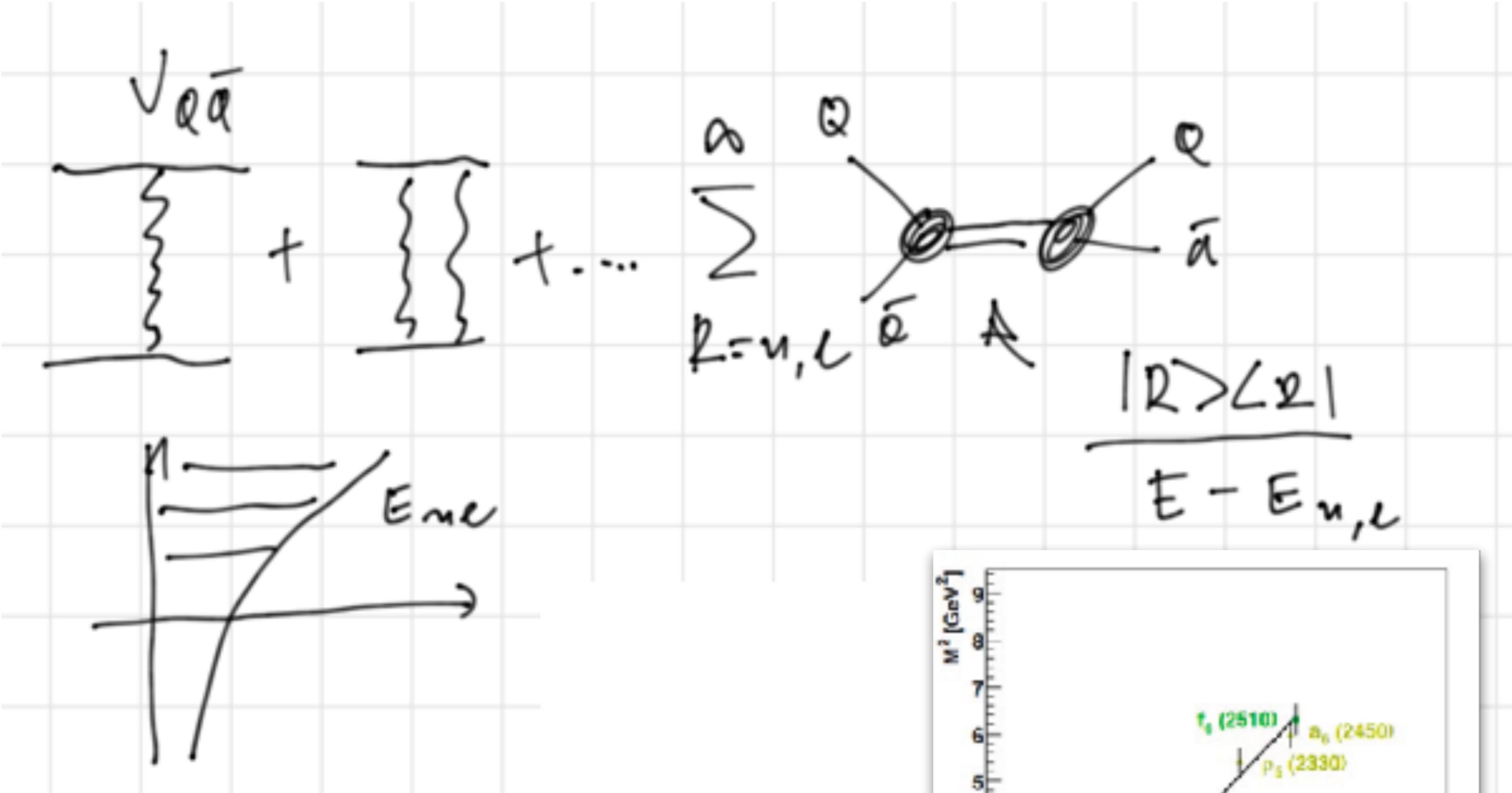


Both potentials produce ∞ number of poles ! (poles cannot disappear)

Confined : ∞ n number of bound states in the continuum. No parton production thresholds

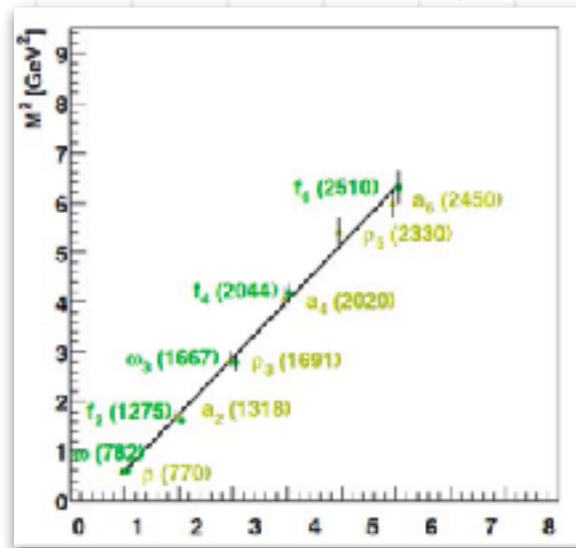
Screened : Large decay withers to mupltipartilce finals states (cover rapidity gap in inclusive production)

Accessing QCD resonances



e.g. for harmonic oscillator confinement

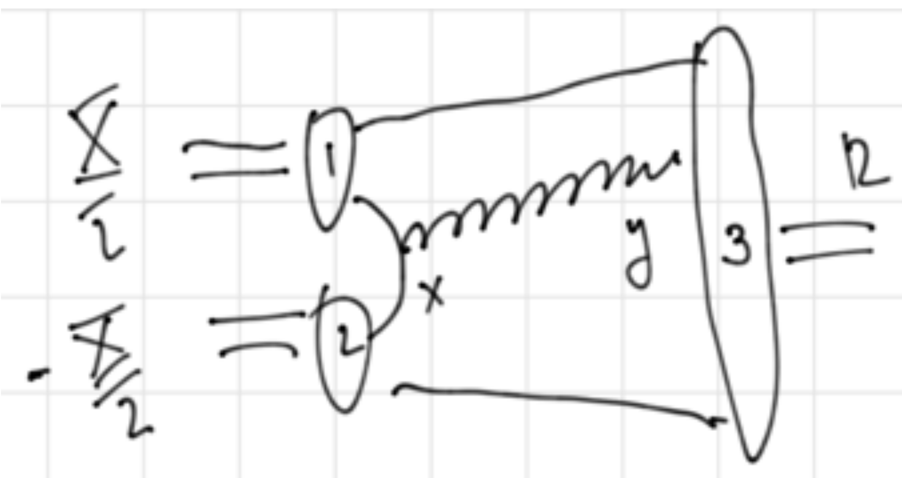
E = 2n + l

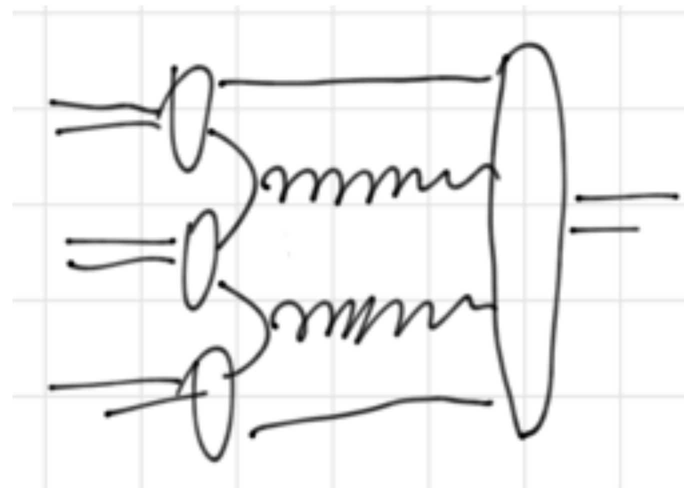


Accessing QCD resonances

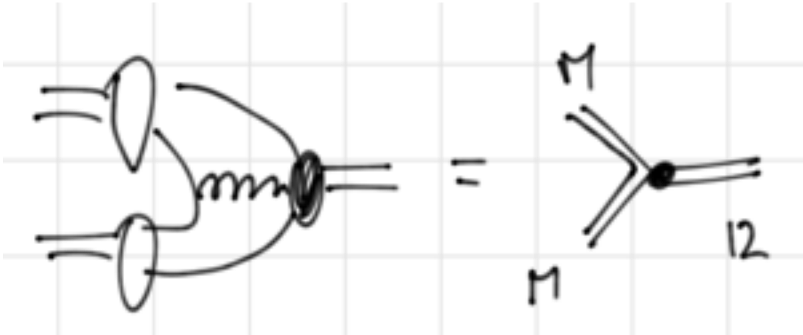
Coupling to multi-hadron asymptotic states
increases with mass/spin of the resonance

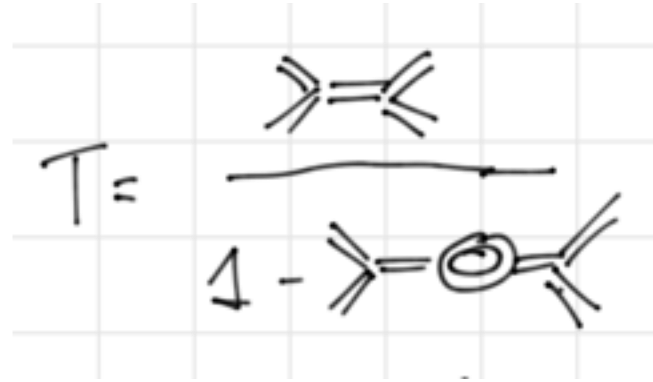
$$\int \psi_1\left(\frac{X}{2}-x\right)\psi_2\left(-\frac{X}{2}+x\right)\psi_3(y)V(x-y)$$

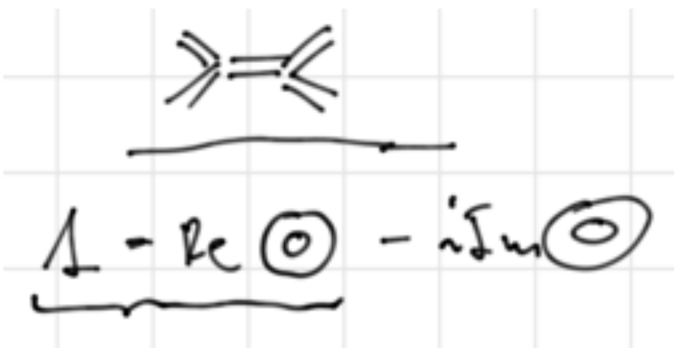




Scattering through resonances

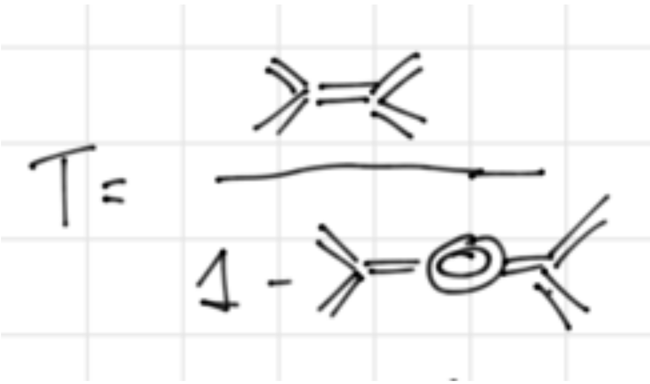






∞ number of poles

As energy increases Γ(E) receives contributions from intermediate states with an increasing number of hadrons.



$$f(E) = \frac{\Pi_r\left(-\frac{g_r^2}{E-E_r}\right)}{1 - i\Pi_r k_r\left(\frac{-g_r^2}{E-E_r}\right)}$$

One resonance, (e.g. ρ) decaying to 2π (μ=0.134)

$$\frac{g^2}{f(s)} = (0.77^2 - i0.12\sqrt{4 \times 0.134^2 - s}) - s$$

gives a pole at $\sqrt{s}$=0.767 − 0.056 (on IInd sheet)

K(s) has (∞ number of) poles

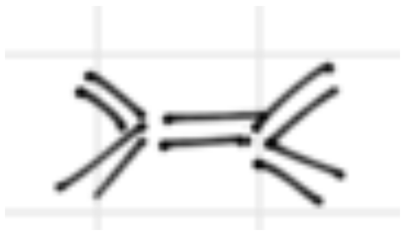
$$f(s) = \frac{1}{K^{-1}(s) - i\Gamma(s)} \qquad K^{-1} = \Pi_r(E - E_r)$$

Γ(s) Contains effects of multi particles thresholds

With ∞ number of resonances this formula doesn't make sense f(s) → 0

$K^{-1}(s)$ needs to have ∞ number of poles (K(s) needs zeros)

Example



Quadratically spaced radial trajectories

$$K(s) = \sum_{r=1}^{\infty} \frac{g_r^2}{m_r^2 - s} \to \sum_r \frac{1}{r^2 - s} \sim \frac{\cos(\pi\sqrt{s})}{\sin(\pi\sqrt{s})}$$

Linearly spaced radial trajectories (Veneziano)

$$K(s) \sim \frac{\Gamma(a-s)}{\Gamma(b-s)}$$

Unitarization

$$f(s) = \frac{K(s)}{1 - iK(s)\rho(s)}$$

K(s) has poles and zeros
Phase space, besides unitarity branch point
has spurious singularities

$$K(s) = \frac{P(s)}{Q(s)}$$

Zeros
Poles

$$f(s) = \frac{P(s)}{Q(s) - \frac{1}{\pi}\int_{s_{tr}} ds'\rho(s')\frac{P(s')}{s'-s}}$$

$$f(s) = \frac{\cos(\pi s)}{\sin(\pi s) - \frac{1}{\pi}\int_{s_{tr}} ds'\rho(s')\frac{\cos(\pi s')}{s'-s}}$$

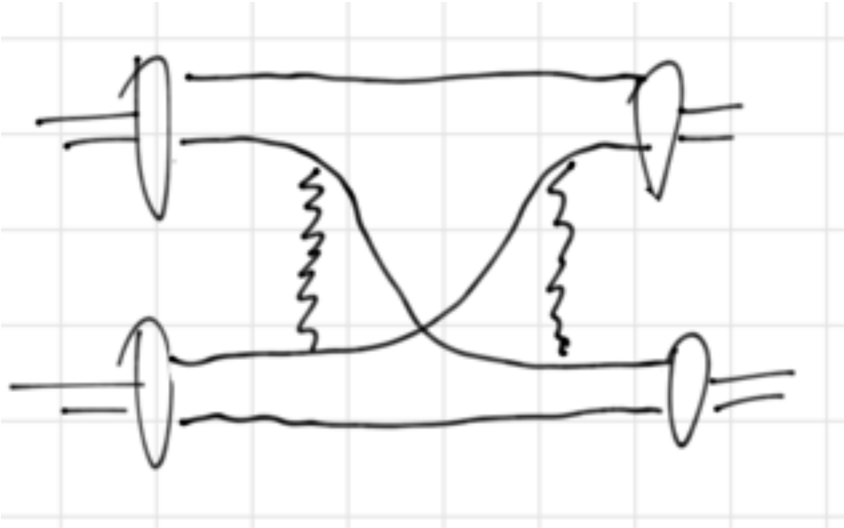
If P(s) has ∞ of zeros it is necessary to
"divide" out asymptotic behavior

$$f(s) = \frac{\frac{1}{\Gamma(b-s)}}{\frac{1}{\Gamma(a-s)} - \frac{1}{\pi}\int_{s_{tr}} ds'\rho(s')\frac{1}{\Gamma(b-s')(s'-s)}}$$

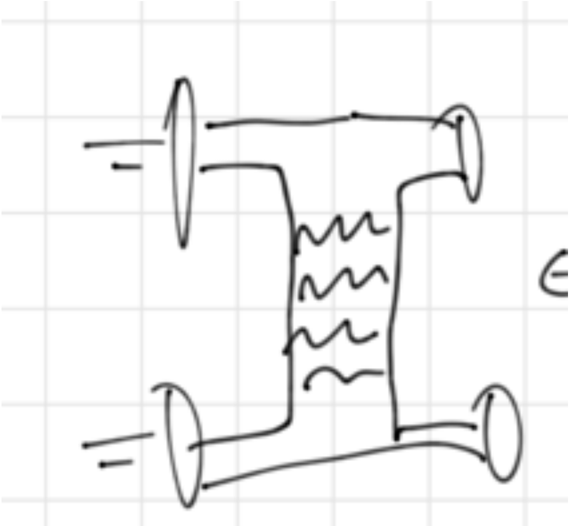
$$f(s) = \frac{\frac{1}{\Gamma(b-s)}}{\frac{1}{\Gamma(a-s)} - \frac{s^{-b+s}}{\pi}\int_{s_{tr}} ds'\rho(s')\frac{(s')^{b-s'}}{\Gamma(b-s')(s'-s)}}$$

"Almost correct", need to remove phases)

Other effects

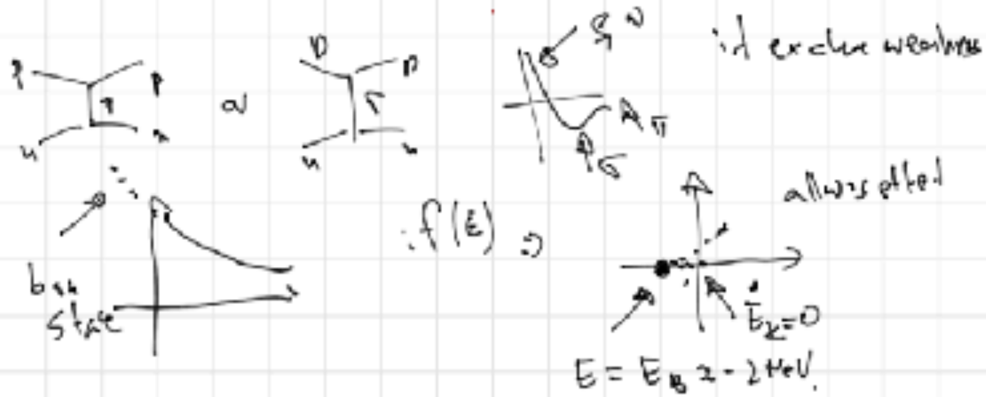


Non-quark model resonances (tetraquarks)

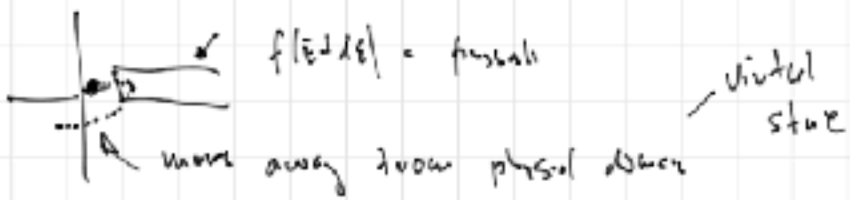


Yukawa exchange (possibly relevant of when pion exchange)

How the bound states/resonances appear in scattering amplitude

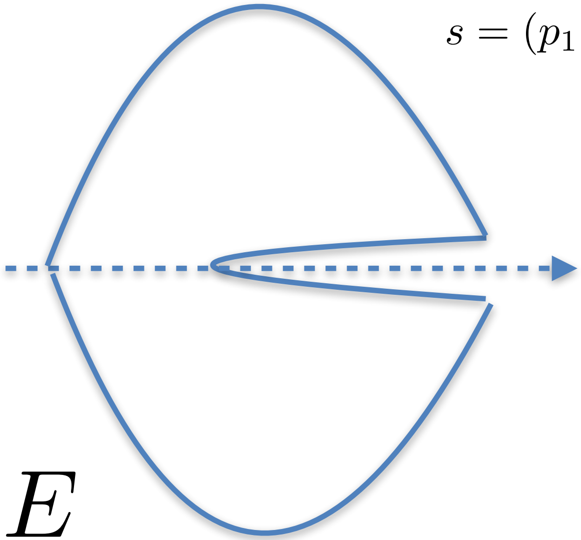


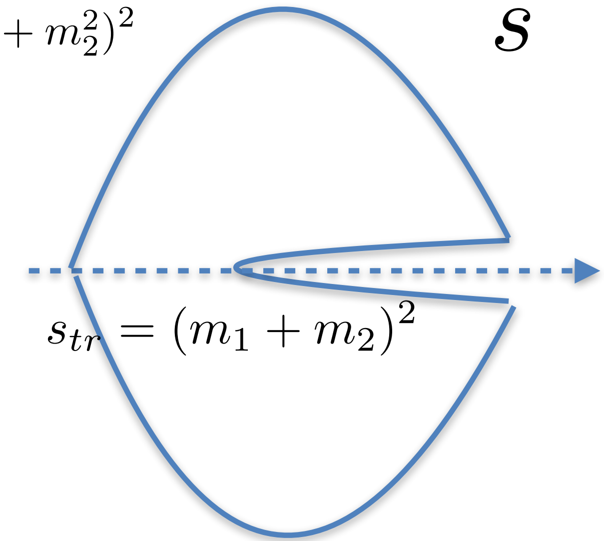
only possibility is if there is a "window" through which the pole can escape the "physical" sheet.

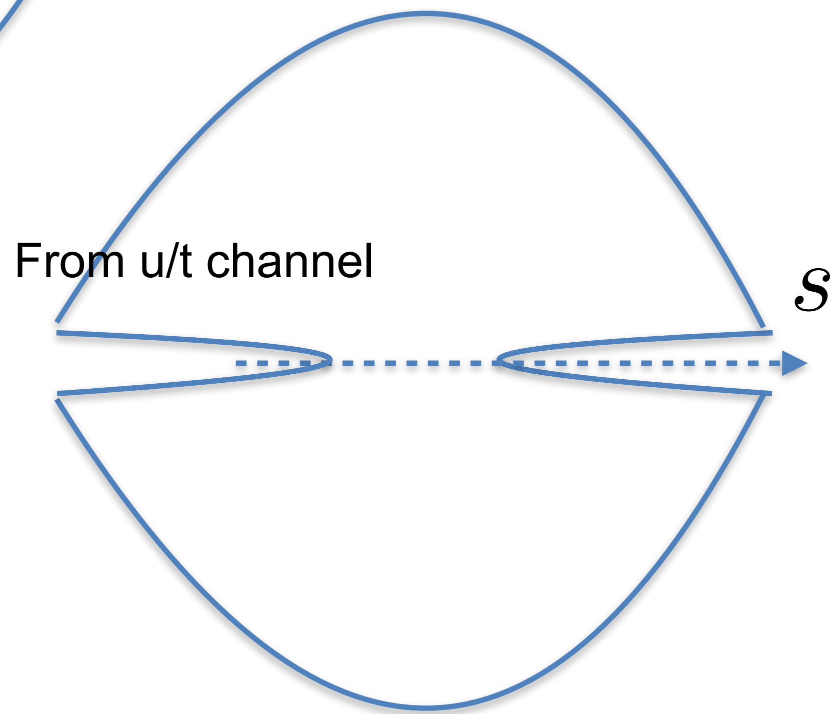


Relativistic case

$$s = (p_1 + p_2)^2 = (\sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2})^2$$







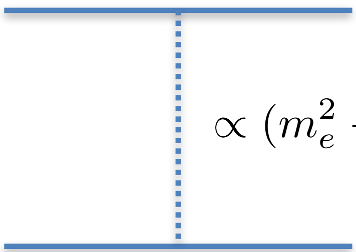
Other effects of partial wave analyticity

Scalar particle scattering 1+2 -> 3 + 4

$$A_l(s) = \int dz_s A(s, t(s, z_s), u(s, z_s)) P_l(\cos\theta)$$

Partial waves have “right hand” singularity (from s) and “left hand” (from t and u)

For example assume equal masses



$$\propto (m_e^2 - t(s, z_s))^{-1}$$

$$t = -\frac{(s - 4m^2)}{2}(1 - z_s)$$

$$A_0(s) \sim \int_{-1}^{1} dz_s \frac{1}{m_e^2 + \frac{(s-4m^2)}{2}(1 - z_s)}$$

For s>4m^2 integral is finite

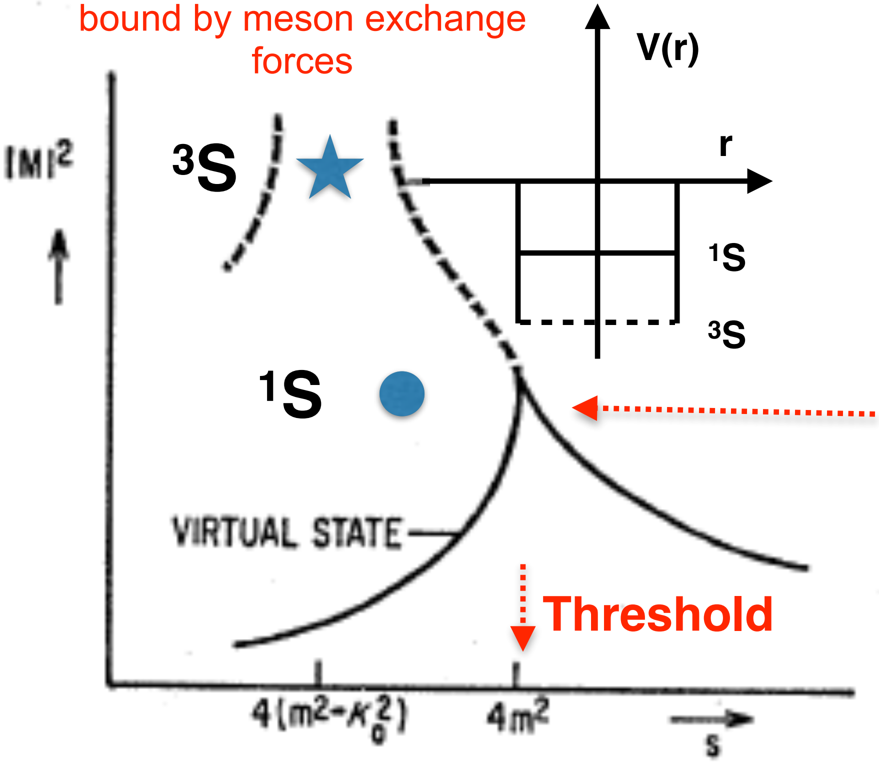
For s<4m^2 - m$_e^2$ the detonator crosses 0 within integration limi, implying A$_0$(s) has a cut for negative s

Scalar amplitudes have simple singularity structure, but partial waves a much more complicated. They also have kinematical singularities when spin and/or unequal masse are involved

Bound states and Virtual States

- f0(980),
- a0(980),
- a1(1420),
- Lambda(1405),
- XYZ,
- …

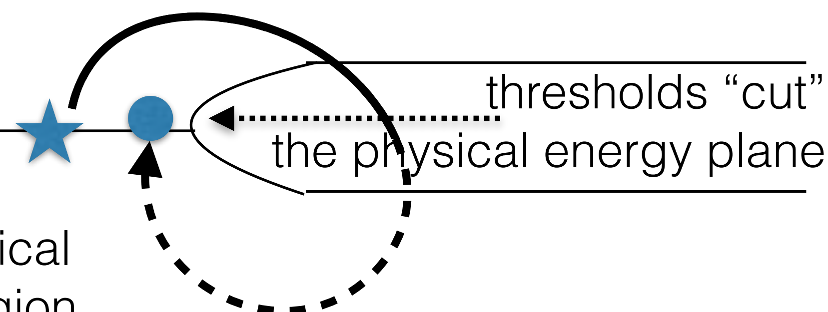
Deuteron the np molecule bound by meson exchange forces



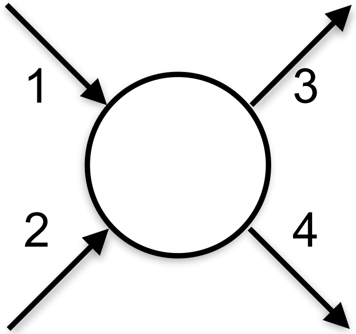
- **Thresholds are "windows" to singularities (particles, visual states, forces") located on the nearby unphysical sheet.**
- **They appear as cusps (if below threshold) or bumps (is above)**

bound state : pole on the physical energy plane

virtual state : pole on "unphysical sheet" closest the physical region



Example : B-> J/psi K pi



$J/\Psi(1)\ B(2) \to K(3)\ \pi(4)$: s-channel

$J/\Psi(1)\ K(\bar{3}) \to B(2)\ \bar{\pi}(4)$: t-channel

$J/\Psi(1)\ \pi(\bar{4}) \to K(3)\ \bar{B}(2)$: u-channel

$B(2) \to J/\Psi(\bar{1})\ K(3)\ \pi(4)$: decay

Veneziano model and application to Dalitz plot analysis

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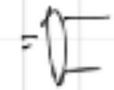
Historical role

Properties

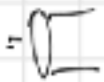
Application to $J/\psi \to 3\pi$ decays

Generalizations (dual models)

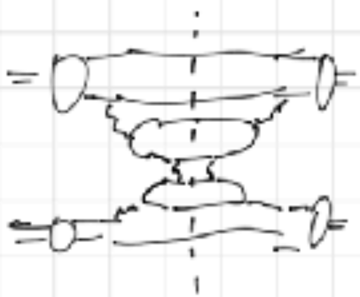
What happens as energy increases => parton showers



$\}$ $l \sim k r_0$ angular momentum barrier ~~pp~~ prevents interaction



=> make showers / like high spin resonance decay)

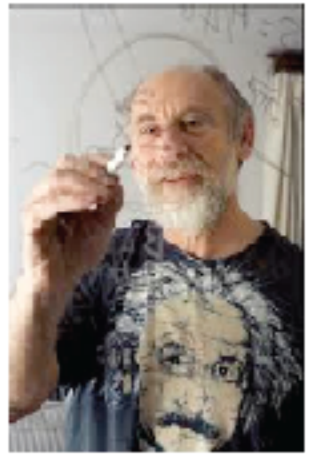


multi parton in $2 \to$ at high energy overlap

with many resonances in s-channel

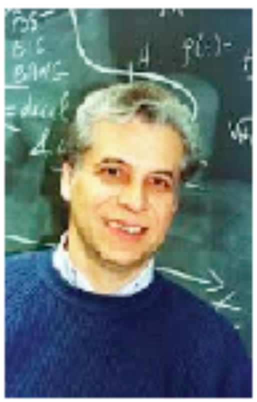
leads to specific energy in t-channel

relativistic h.o.



string of relativistic oscillators

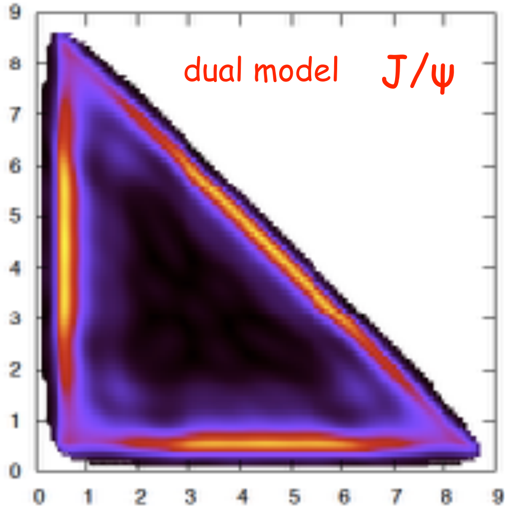
QCD, loop representation, large-N_c, AdS/CFT, ...

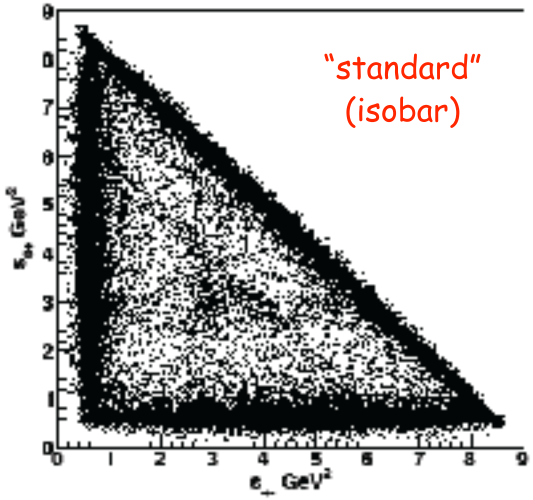


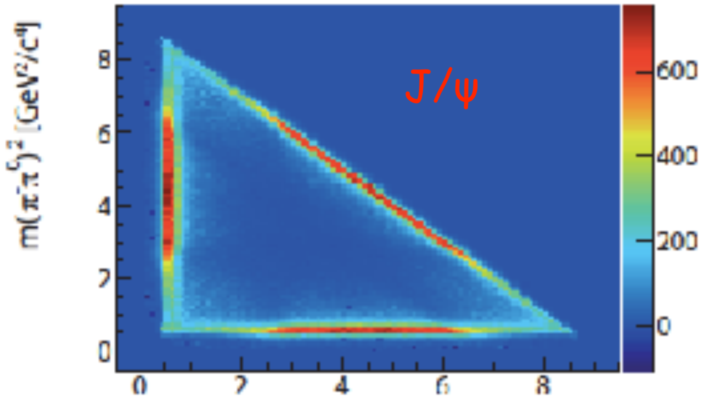
$\omega \to 3\pi$

$$\boxed{A(s,t) = \frac{\Gamma(-J(s))\Gamma(-J(t))}{\Gamma(-J(s)-J(t))}}$$

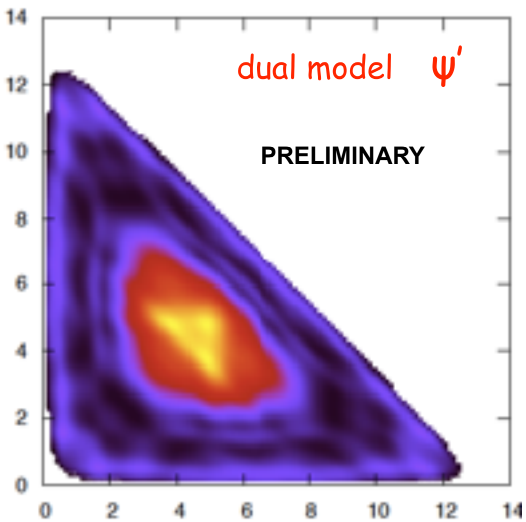
string revolution



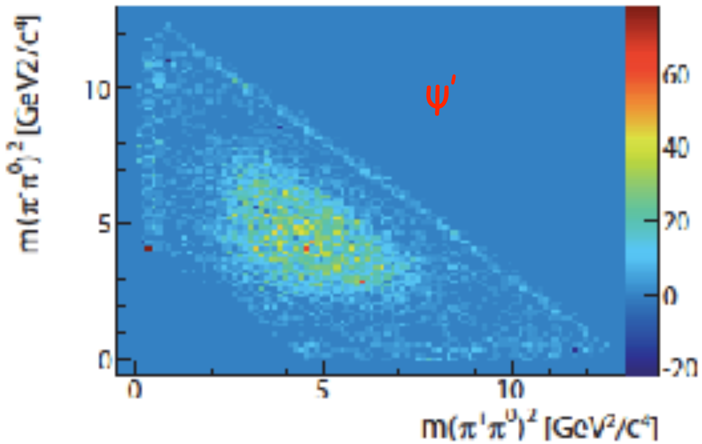




BESIII, Phys.Lett. B710 (2012) 594-599



manifestation of force - particle duality ?



$$A(s,t) = \frac{\Gamma(-J(s))\Gamma(-J(t))}{\Gamma(-J(s) - J(t))}$$

$\omega \to 3\pi$

Properties:

• Duality: resonances in direct channel dual to reggeons in cross channels and backgrounds are dual to the pomeron

• All resonances are "connected": resonances belong to Regge trajectories (reggeons)

• Asymptotics: determined by Regge poles

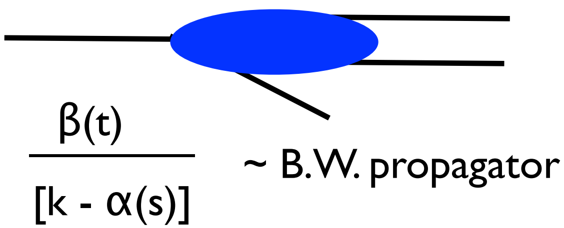
• Unitarity: imaginary parts determined by decay thresholds

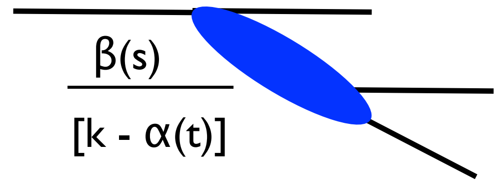
Veneziano amplitude satisfies all of the above except unitarity, which implemented in the Szczepaniak-Pennington model

Veneziano amplitude: “compact” expression for the full amplitude

$$A(s,t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s) - \alpha(t))} \qquad \alpha(s) = a + bs$$

resonance/reggeon in $s=m_{12}^2$





resonance/reggeon in $t=m_{23}^2$

A(s,t) can be written as sum over resonances in ether channel.

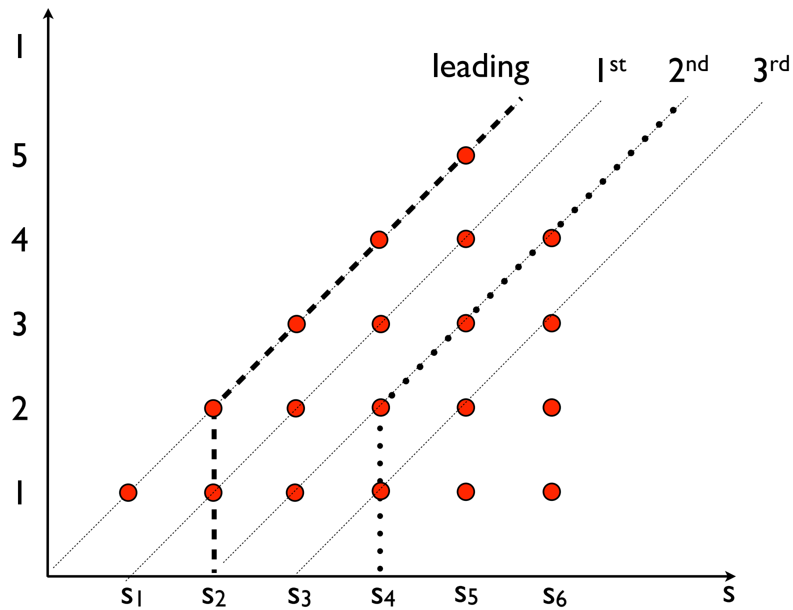
$$A(s,t) = \sum_k \frac{\beta_k(t)}{k - \alpha(s)} = \sum_k \frac{\beta_k(s)}{k - \alpha(t)}$$

Note: in V-model resonance couplings, β, are fixed!

$$\beta_k(t) \propto (1 + \alpha(t))(2 + \alpha(t)) \cdots (k + \alpha(t))$$

$$V(p,\lambda) \to \pi^i(p_1)\pi^j(p_2)\pi^k(p_3)$$

$$A(s,t,u) = \epsilon_{ijk}\epsilon_{\mu\nu\alpha\beta}\epsilon_\mu(p,\lambda)p_1^\nu p_2^\alpha p_3^\beta$$
$$\times[A_{n,m}(s,t) + A_{n,m}(s,u) + A_{n,m}(t,u)]$$



$$\alpha(s) = \alpha_0 + \alpha' s.$$

$$\alpha(s) = \frac{1}{2} + s$$

no-double poles

n ≥ m ≥ 1

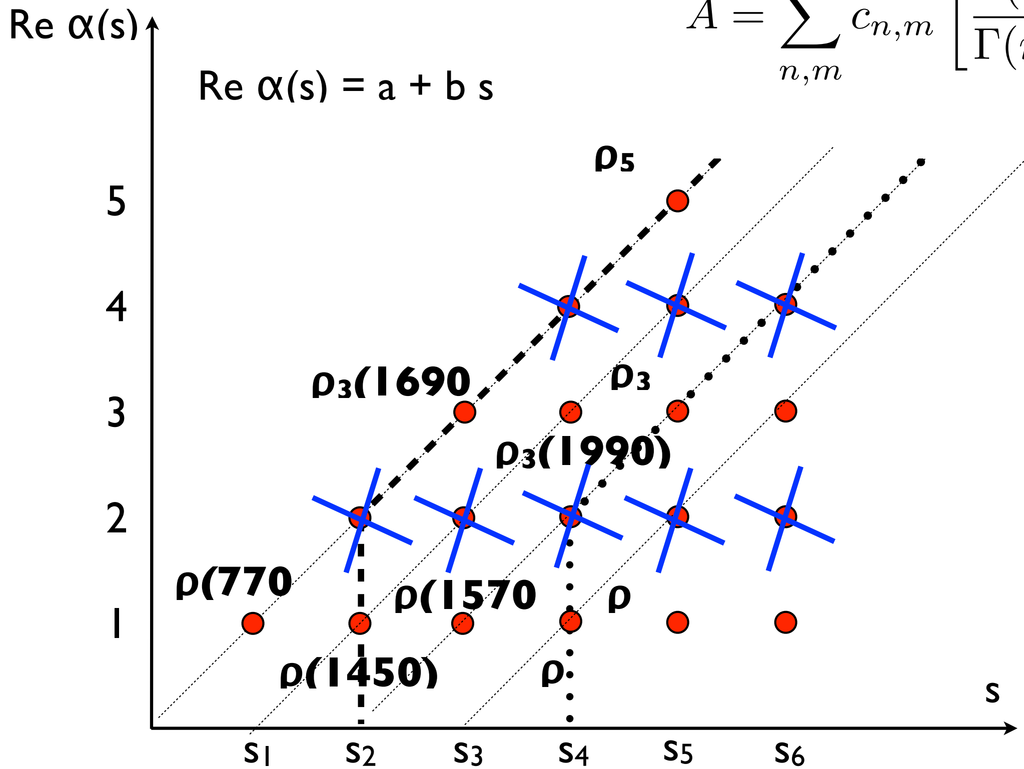
Regge limit

$$A_{n,m}(s,t) \equiv \frac{\Gamma(n-\alpha_s)\Gamma(n-\alpha_t)}{\Gamma(n+m-\alpha_s-\alpha_t)}.$$

Resonances couplings, β, should depend on final state particles: a linear superposition of Veneziano amplitudes can be used to suppress or enhance individual resonances or trajectories

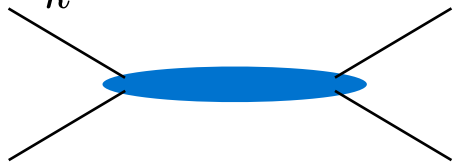
$$M = \epsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu p_3^\alpha \epsilon^\beta A(s,t,u)$$

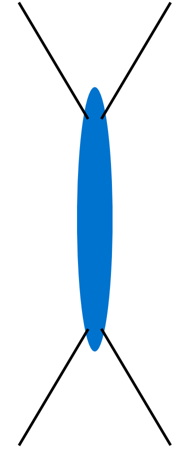
$$A = \sum_{n,m} c_{n,m} \left[\frac{\Gamma(n-\alpha(s))\Gamma(n-\alpha(t))}{\Gamma(n+m-\alpha(s)-\alpha(t))} + (s,u) + (t,u) \right]$$



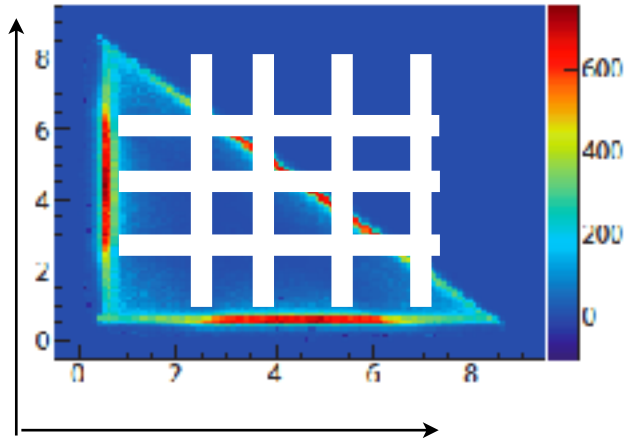
- even-spin ρ's do not couple to π π and should decouple in J/ψ→3 π
- coupling of odd-spin ρ's depend of can depend vary depending on trajectory

$$A_{n,m}(s,t) = \sum_k^{\infty} \frac{\beta(t)}{k-\alpha(s)} = \sum_k^{\infty} \frac{\beta(s)}{k-\alpha(t)}$$





how to remove (infinite) number of poles?



$n \geq m \geq 1$

$$A_{1,1} = \frac{\Gamma(1-\alpha_s)\Gamma(1-\alpha_t)}{\Gamma(2-\alpha_s-\alpha_t)}$$

has poles at α_s=1,2,3,...

$$A_{2,1} = \frac{\Gamma(2-\alpha_s)\Gamma(2-\alpha_t)}{\Gamma(3-\alpha_s-\alpha_t)}$$

$$A_{2,2} = \frac{\Gamma(2-\alpha_s)\Gamma(2-\alpha_t)}{\Gamma(4-\alpha_s-\alpha_t)}$$

have poles at α_s=2,3,4,...

Use a linear combination of $A_{2,1}$ and $A_{2,2}$ to remove pole at α_s =2

Use a linear combination of $A_{3,1}$, $A_{3,2}$,$A_{3,3}$, to remove pole at α_s =3,

$$A_{3,1}, A_{3,2}, A_{3,3}$$

have poles at α_s=3,4,5,...

etc.

$$A_{4,1}, A_{4,2}, A_{4,3}, A_{4,4}$$

have poles at α_s=4,5,6,...

$$A_{n,m}(s,t) \to \mathcal{A}(s,t) = \sum_{n\geq 1, n\leq m\leq 1} c_{n,m} A_{n,m}(s,t)$$

remove all poles but the one at α=1

$$c_{n,1} = \frac{c_{1,1}}{\Gamma(n)},\ c_{n,2} = -\frac{c_{1,1}}{\Gamma(n-1)},\ c_{n,m} = 0 \text{ for } m > 2,$$

$$\mathcal{A}_1(s,t) = c_{1,1} \frac{2-\alpha_s-\alpha_t}{(1-\alpha_s)(1-\alpha_t)}.$$

... but the Regge limit is now lost !

remove all poles between $N \geq \alpha \geq 2$

$$\begin{aligned} \mathcal{A}_1(s,t;N) &= c_{1,1} \frac{2-\alpha_s-\alpha_t}{(1-\alpha_s)(1-\alpha_t)} \\ &\times \frac{\Gamma(N+1-\alpha_s)\Gamma(N+1-\alpha_t)}{\Gamma(N)\Gamma(N+2-\alpha_s-\alpha_t)} \end{aligned}$$

has Regge limit is for s > N

In the past this was done by choosing an arbitrary set of n,m and fitting c(n,m) to the data (e.g. Lovelace, Phys. Lett. B28, 265 (1968),Altarelli, Rubinstein, Phys. Rev. 183, 1469 (1969))

The Szczepaniak-Pennington model does this in a systematic way. In addition it allows for imaginary non-linear (and complex) trajectories without introducing "ancestors"

$$\mathcal{A}_n(s,t;N) = \frac{2n-\alpha_s-\alpha_t}{(n-\alpha_s)(n-\alpha_t)} \sum_{i=1}^{n} a_{n,i}(-\alpha_s-\alpha_t)^{i-1}$$
$$\times \frac{\Gamma(N+1-\alpha_s)\Gamma(N+1-\alpha_t)}{\Gamma(N+1-n)\Gamma(N+n+1-\alpha_s-\alpha_t)}.$$

n: number of Regge trajectories

$a_{n,i}$: determine resonance couplings

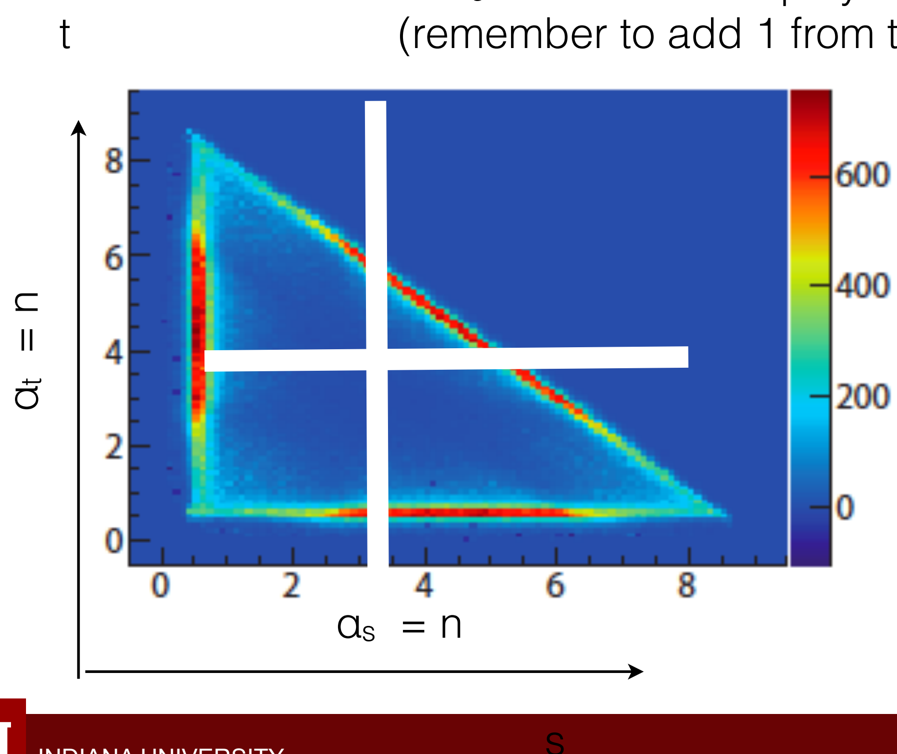
N: determines the onset of Regge behavior

$\alpha(s)$, $\alpha(t)$ = Re α + i Im α: with Im α related to resonance widths

All poles below α = N except at α = n

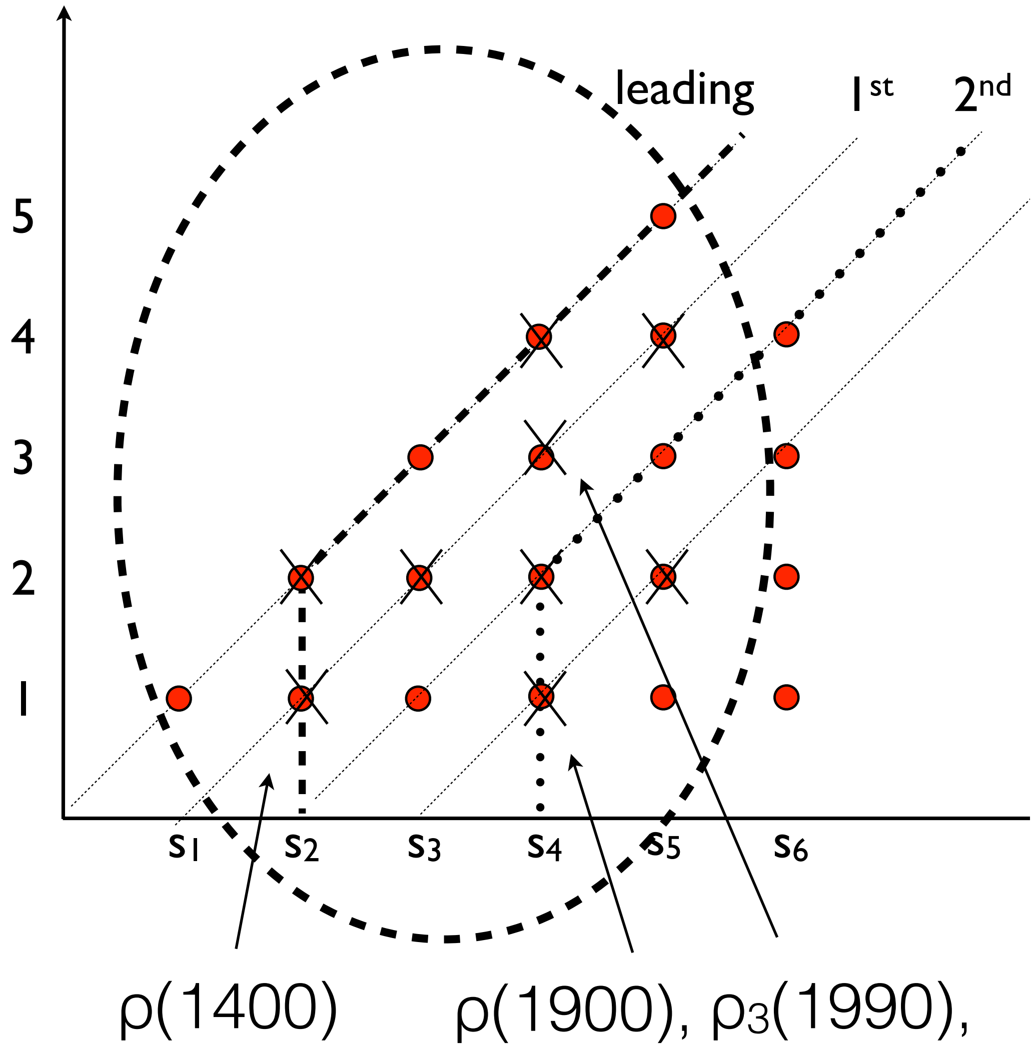
$$\mathcal{A}_n(s,t;N) = a_{n,0}\frac{2n-\alpha_s-\alpha_t}{(n-\alpha_s)(n-\alpha_t)}\left[\Pi_{i=1}^{n-1}(a_{n,i}-\alpha_s-\alpha_t)\right]$$
$$\times\ \frac{\Gamma(N+1-\alpha_s)\Gamma(N+1-\alpha_t)}{\Gamma(N+1-n)\Gamma(N+n+1-\alpha_s-\alpha_t)}$$

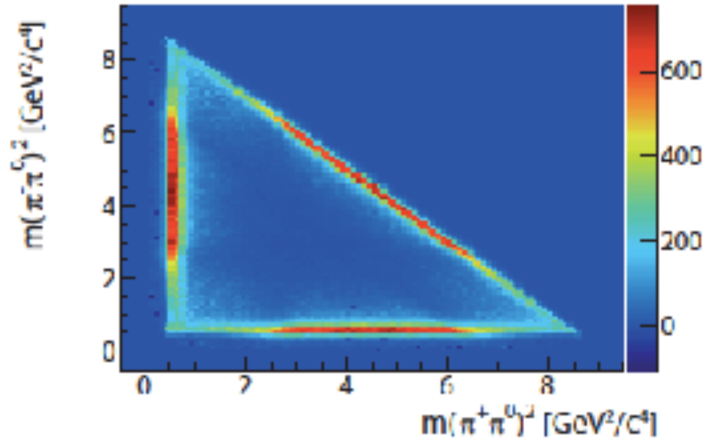
at α_s=n residue is a polynomial in t of order n-1
(remember to add 1 from the Levi-Civita tensor)

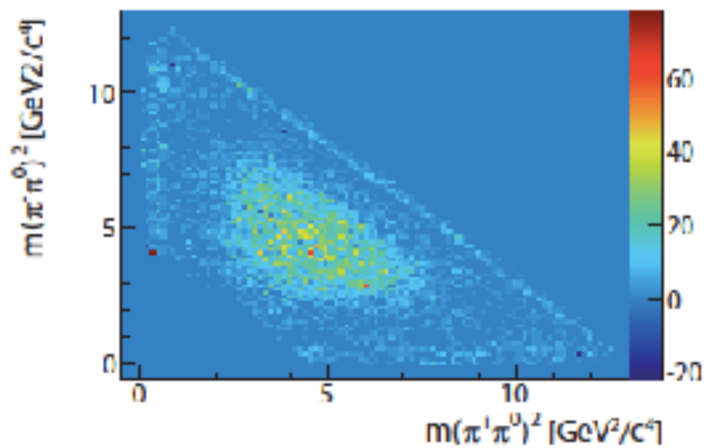


$$A(s,t,u) = \epsilon_{ijk}\epsilon_{\mu\nu\alpha\beta}\epsilon_\mu(p,\lambda)p_1^\nu p_2^\alpha p_3^\beta$$
$$\times[A_{n,m}(s,t)+A_{n,m}(s,u)+A_{n,m}(t,u)]$$

A_1 has ρ(770)
A_3 has ρ(1700), ρ_3(1690)
A_5 has ρ''(2150), ρ_3(2250), ρ_5(2350)







$$m_2 = \sqrt{s_2} = \sqrt{2 - \frac{1}{2}} = 1.23 \qquad m_4 = \sqrt{s_4} = \sqrt{4 - \frac{1}{2}} = 1.87$$

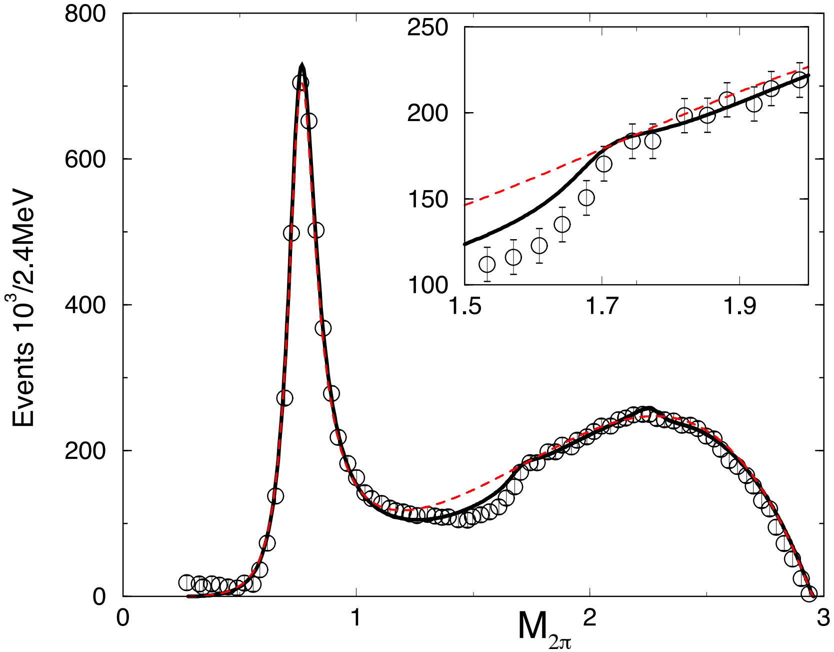


FIG. 2: Dalitz plot projection of the di-pion mass distribution from J/ψ decay. The solid is the result of the fit with three amplitudes and the dashed line with the amplitude $\mathcal{A}_1$ alone. The insert shows the mass region of the ρ_3 and its contribution from the fit with the full set of amplitudes (solid line) as compared. Absence of the structure at 1.7GeV from the fit with the $\mathcal{A}_1$ amplitude is indicated by the dashed line.

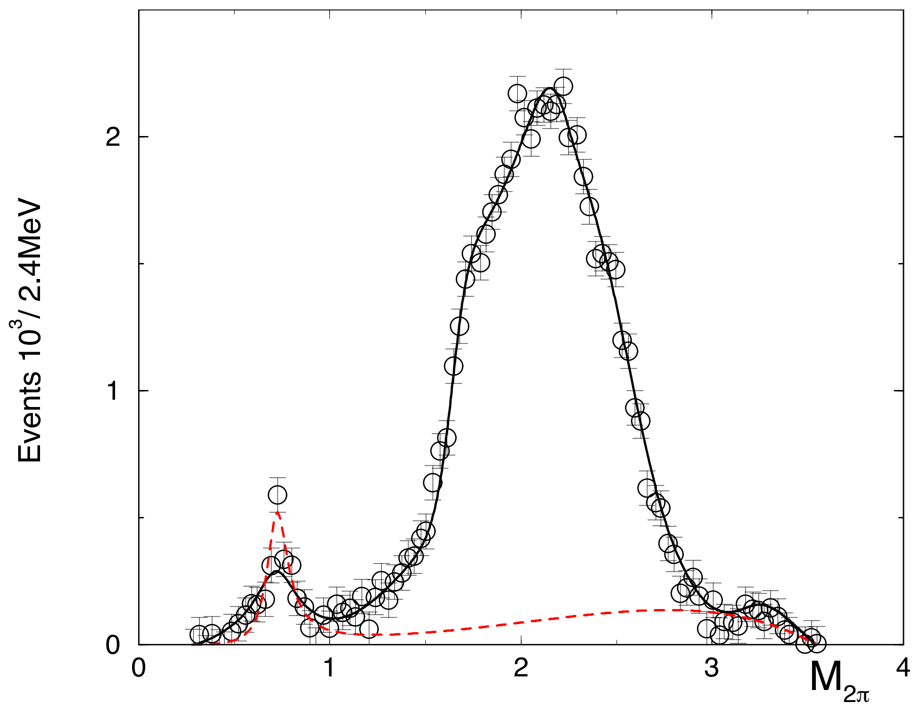
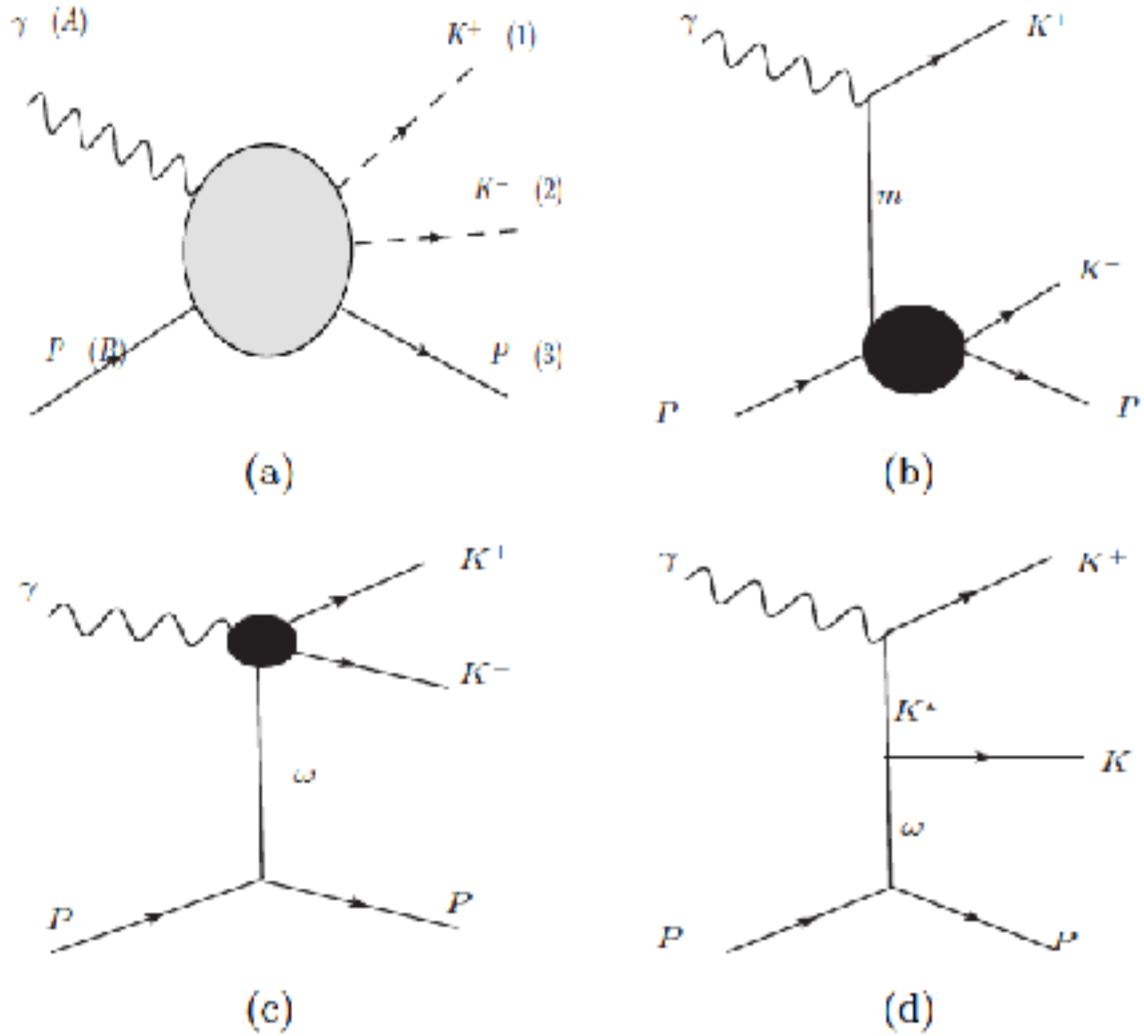


FIG. 3: Dalitz plot projection of the di-pion mass distribution from ψ' decay. The solid is the result of the fit with three amplitudes and the dashed line is the fit with $\mathcal{A}_1$ alone.

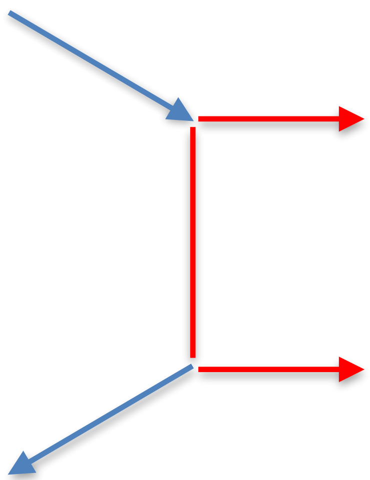
B_5 amplitude:

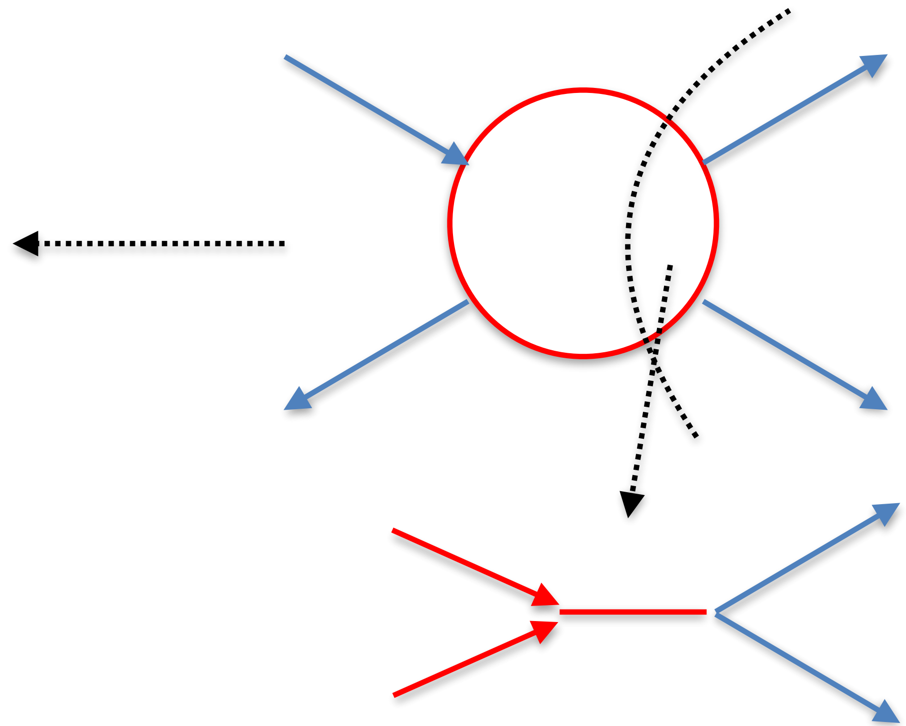
Reggeons/
Resonances in
all 5 channels



Amplitude analysis of the $Z_c(3900)$

- Reactions Y(4260) -> J/Ψππ and Y(4260) -> DD*π
- Amplitude is an analytical function of s, the J/Ψπ mass^2
- Two coupled channels -> 4 Riemann sheets of A(s) (S-wave partial wave)

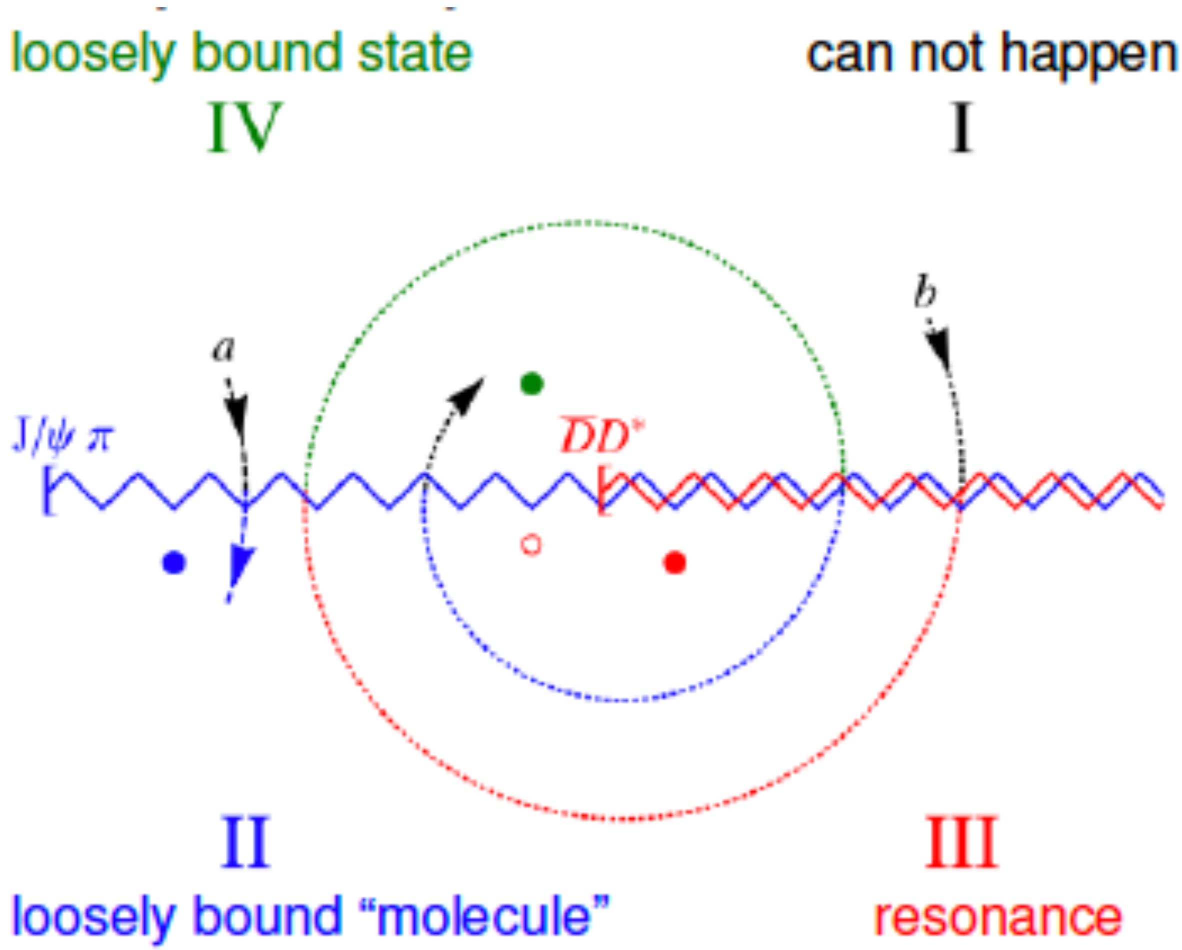




t/u channel exchange close to the s-channel physical region
(Cusps in the s-channel)

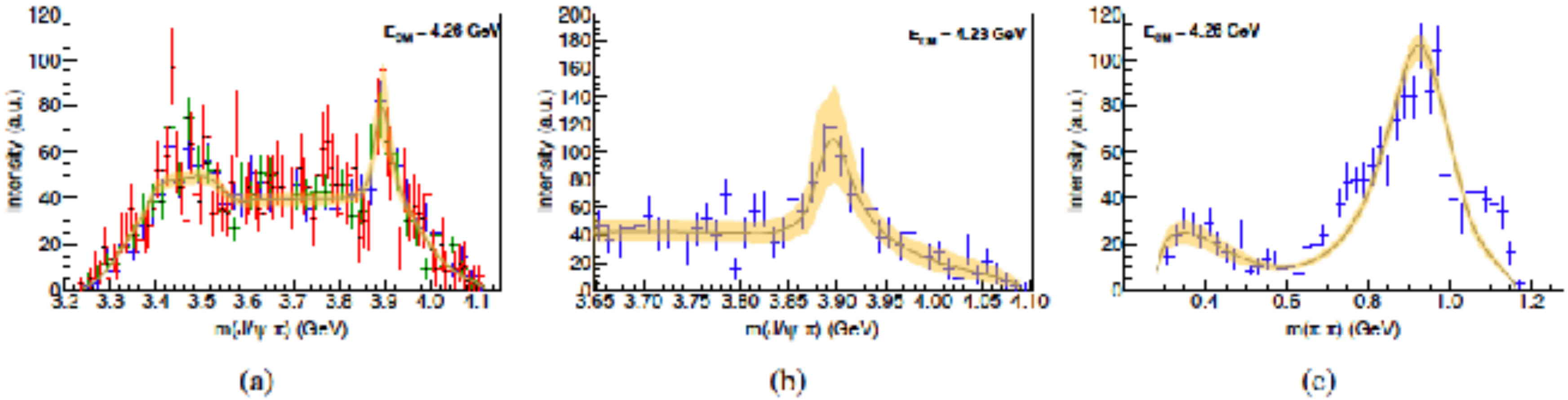
s channel exchange close to the s-channel physical region
(Resonance bumps in the s-channel)

Singularity structure



The fit I

- no t/u channel singularities + s-channel pole



III sheet
resonance
pole

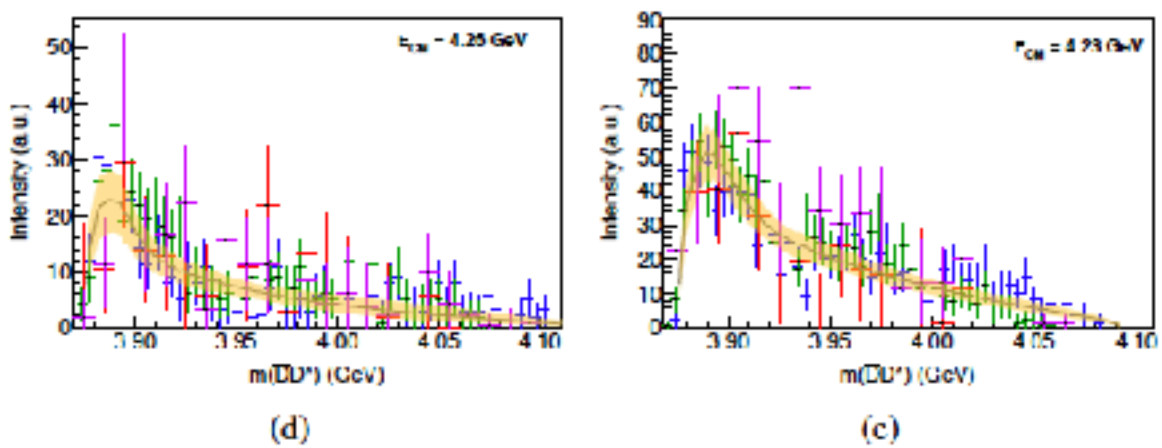
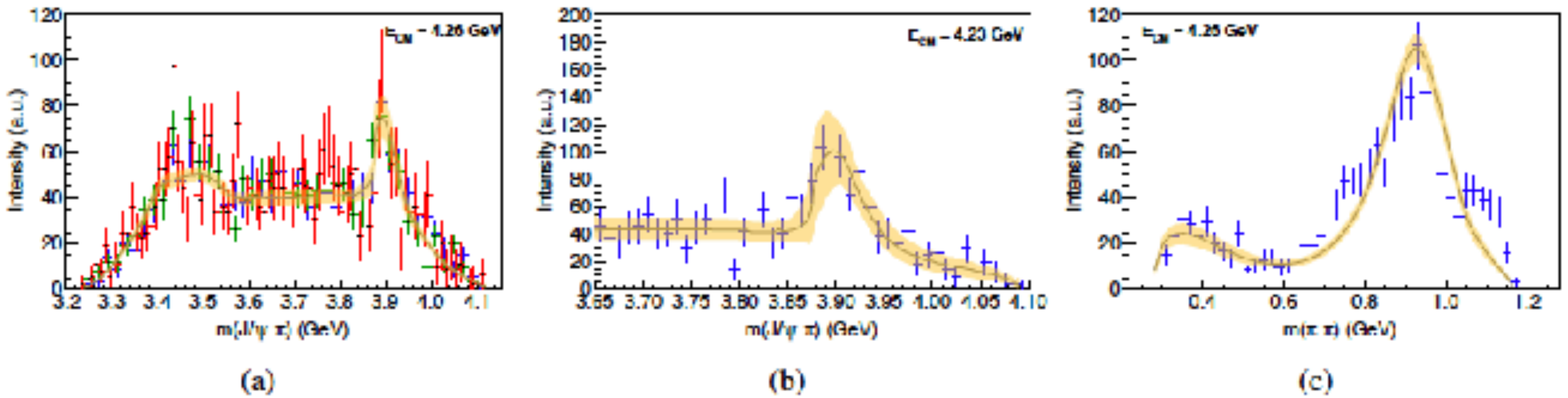


Figure 3: Result of the fit for the scenario III (Flatté K-matrix, without triangle singularity).

The fit II

- u-channel cusp + s-channel pole ((III sheet)



III sheet resonance pole enhanced by the cusp

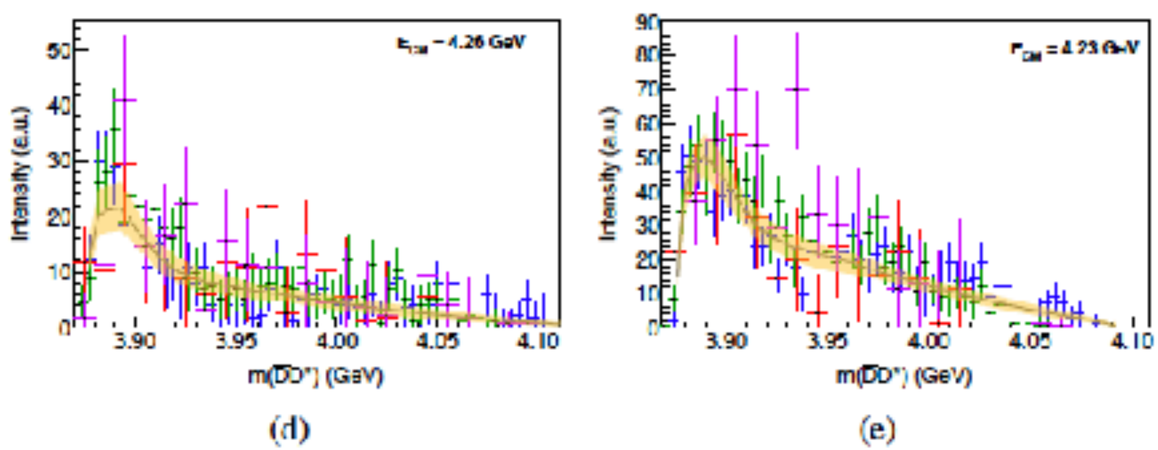
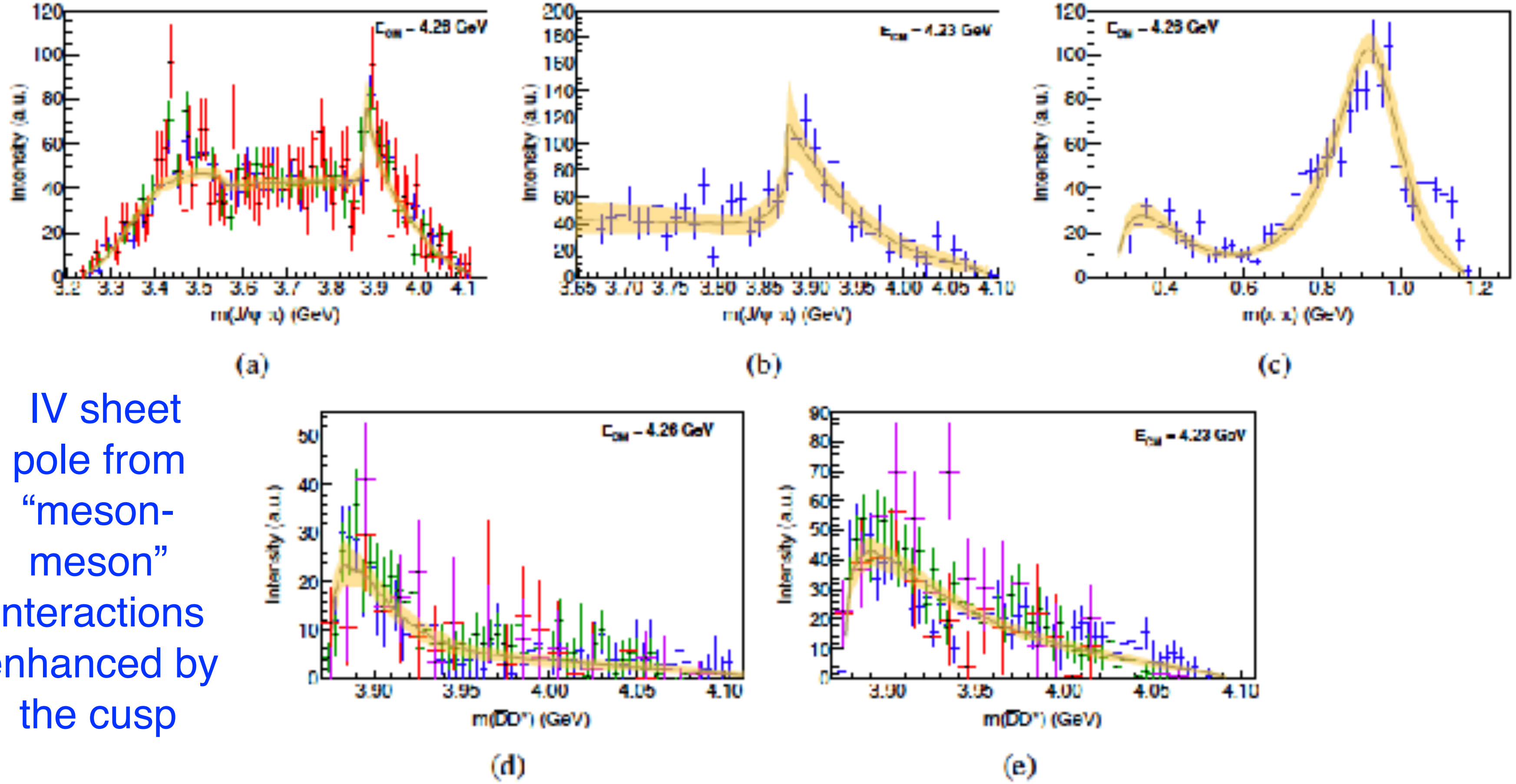


Figure 4: Result of the fit for the scenario III+tr. (Flatté K-matrix, with triangle singularity).

The fit III

- t/u channel singularities + no s-channel pole



IV sheet pole from "meson-meson" interactions enhanced by the cusp

Figure 5: Result of the fit for the scenario IV+tr. (constant K-matrix, with triangle singularity).

The fit IV

- t/u channel singularities + no s-channel pole

Pure III sheet branch point from u-channel exchanges

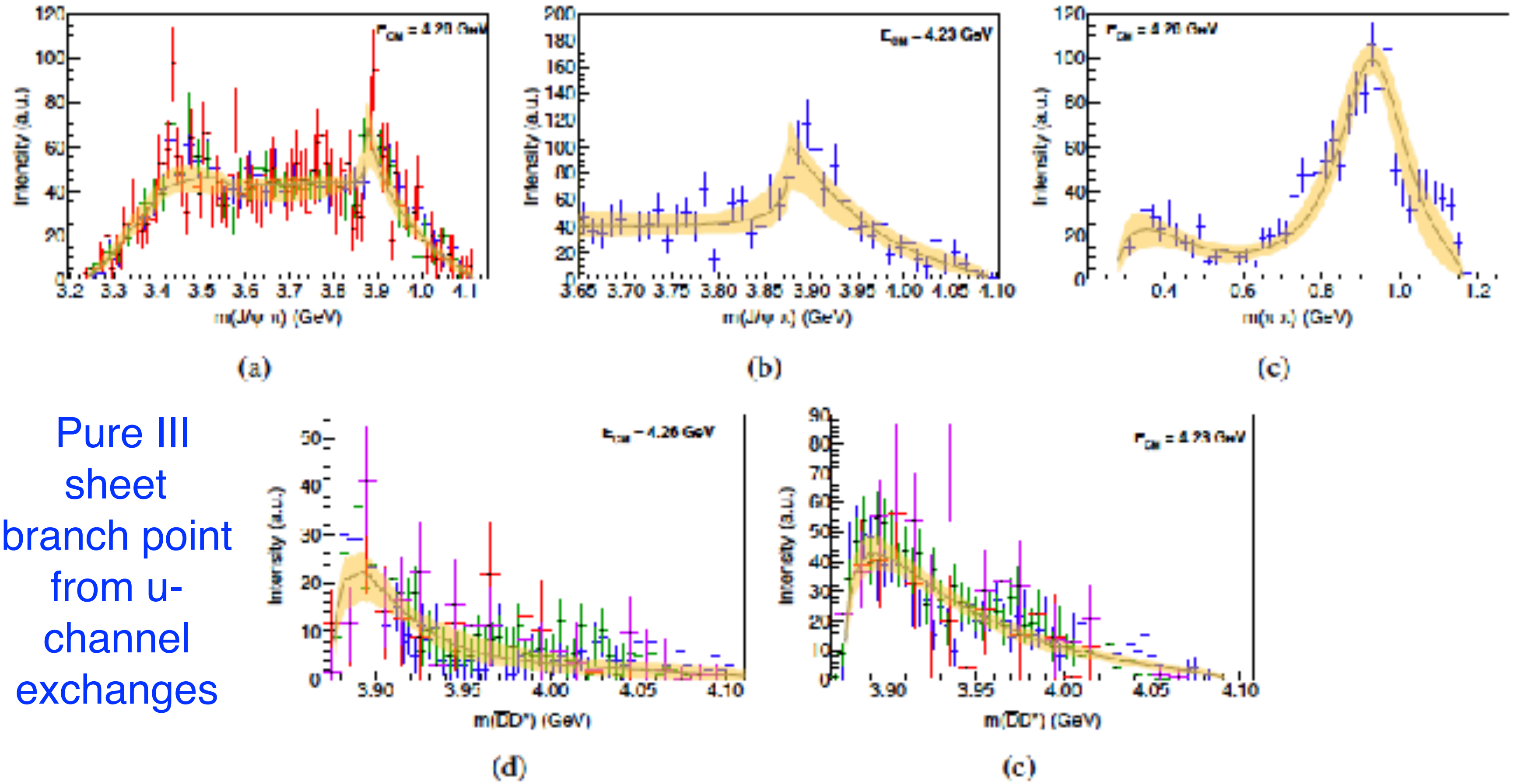
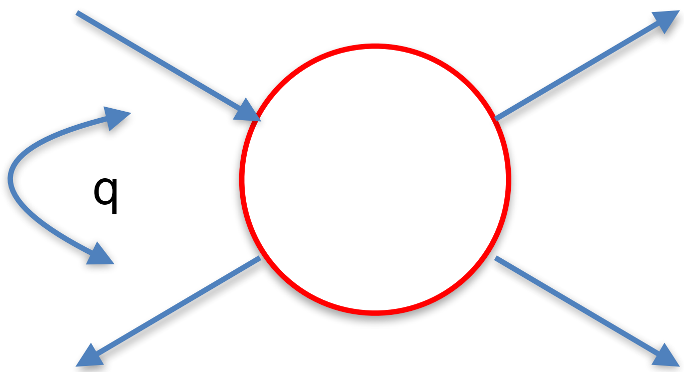


Figure 6: Result of the fit for the scenario iv (triangle singularity only).

Where do cusp come from

- Possible amplitude singularities are determined by analyticity and unitarity
  - No singularities on the physical (I^{st} sheet)
  - At fixed, physical t, growth in s is limited in all directions in the complex plane
  - Poles, sqrt, of log-branch points



A popular Quark Model motivated(*) amplitude behaves like $\exp(-q^2)$

q= I relative 3-momentum I

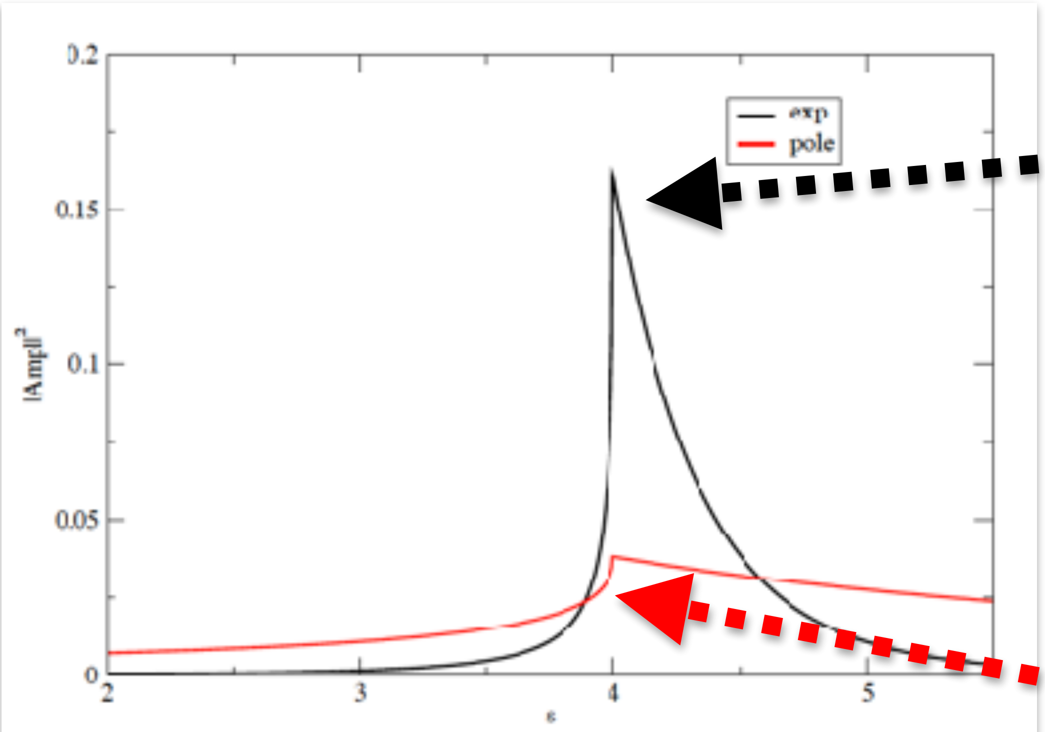
or $A(s) \sim \exp(-s)$

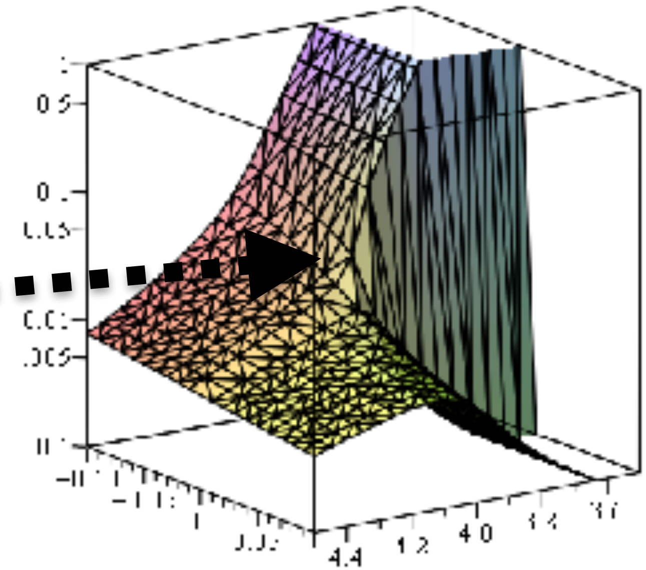
Such amplitude violates analyticity

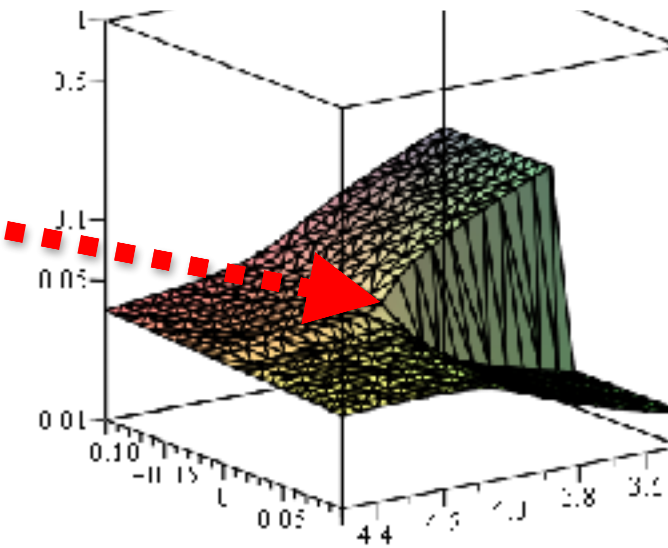
(*) From overlap of four Gaussian wave functions representing confined quark-anit quark pairs

An unphysical cusp effect

$$\text{Ampl} \propto \int_4 ds' \sqrt{1 - \frac{4}{s'}} \frac{N(s')}{s' - s}$$





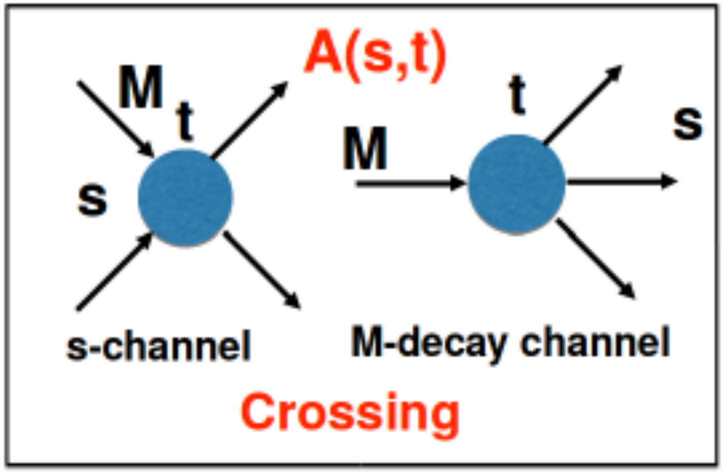


$$N(s) \propto \exp(-s)$$

Identification of Resonances

Comment on spin formalism

- Analyticity means to explore singularities. There are kinematical and dynamical singularities
- Kinematical singularities are due to spin and mass differences (e.g. pseudo thresholds)
- Helicity, Spin-Oribt, Zemach tensors, Chung's relativistic corrections, covariant LS by Fillipini et al, Bonn-Gatchina, … : a confusing state of affairs



- Introduce model dependent s,t,u factors
- Do not isolate kinematical singularities
- Do not satisfy crossing
- …..