Partial wave analysis of eta meson photoproduction using fixed-t dispersion relations

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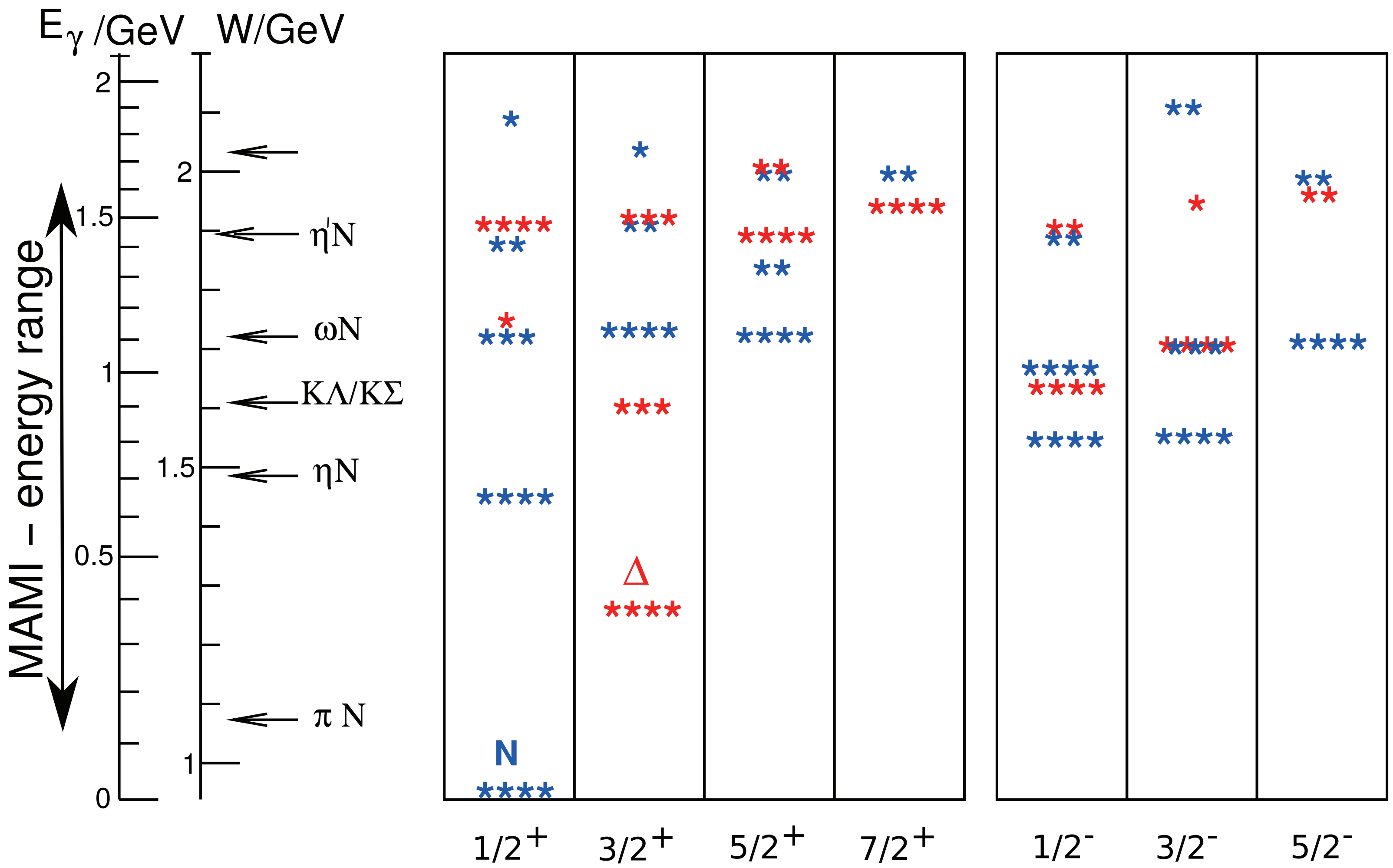
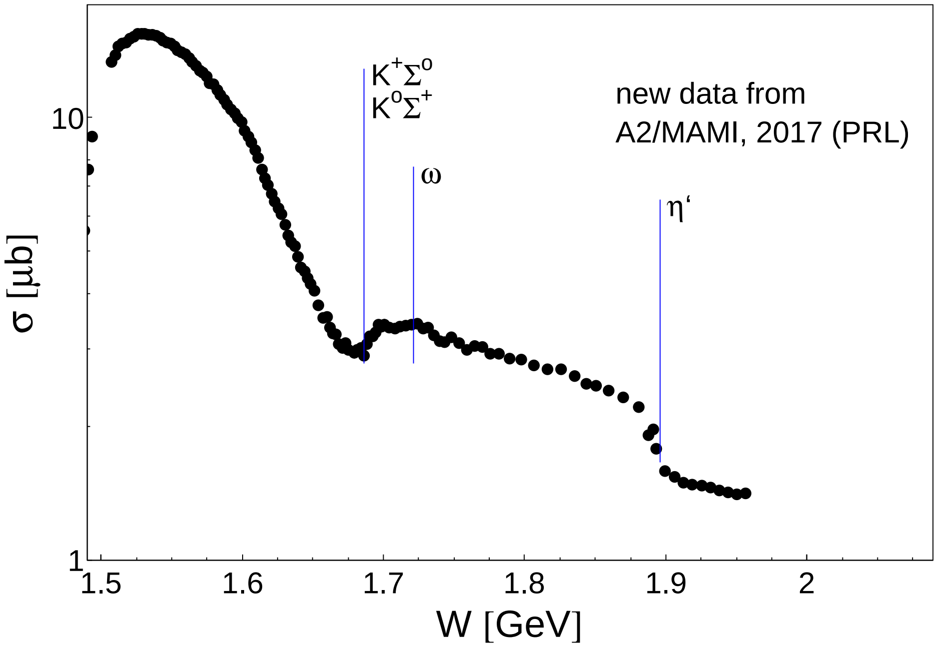
Overview

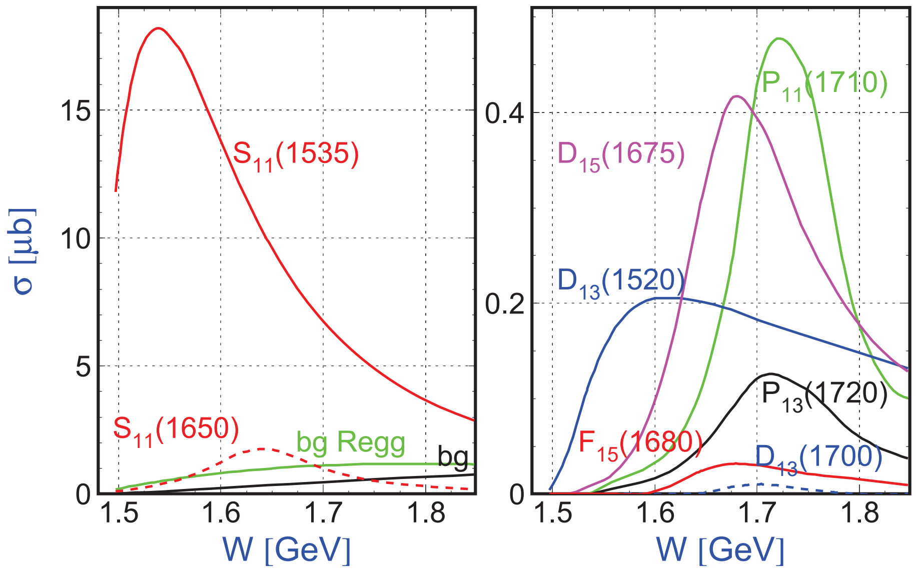
- Motivation and goals
- Theory and formalism
- ηMAID model
- Results and conclusion

Resonance spectrum in photoproduction



Total cross section data from Mainz for $\gamma p \to \eta p$. Resonance contribution





Motivation and goals

Motivation:

- Study nucleon resonances using isobar model (ηMAID).
- By construction ηMAID is non analytic.
- Apply fixed-t dispersion relations.
  I.Aznauryan in [Phys.Rev. C68 (2003) 065204]

Goals:

- Fit the new data. Resonance parameters used as fitting parameters.
- Obtain masses, widths, branching ratios, e.t.c in an improved and less model dependent way.

Kinematics of $\gamma p \to \eta p$

Consider kinematical quantities independent of the reference frame.

$$\gamma(k) \ + \ p(p_i) \to \eta(q) \ + \ p(p_f) \tag{1}$$

Variables in brackets denote the 4-momenta of the participating particles.

k - photon,

p_i , p_f - target and recoil proton, i and f denote initial and final states

q - meson (η)

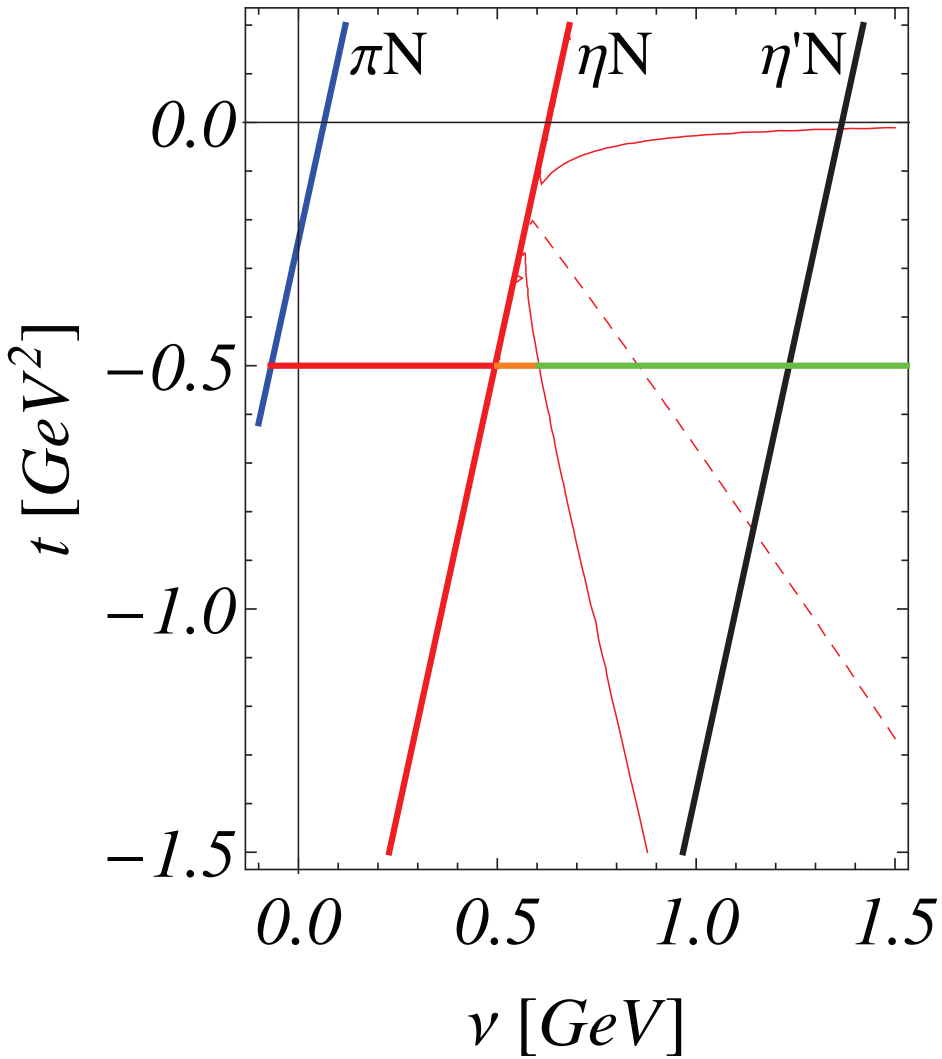
Mandelstam variables:

$$\begin{aligned} s &= (p_i + k)^2 = (q + p_f)^2, \\ t &= (q - k)^2 = (p_f - p_i)^2, \\ u &= (p_i - q)^2 = (p_i - q)^2. \end{aligned} \tag{2}$$

$$s + t + u = 2m_p + m_\eta$$

$\sqrt{s}$ - total energy, t - momentum transfer squared from the $photon$ to the $meson$.

Mandelstam plane for $\gamma p \to \eta p$



Crossing symmetrical variable.

$$\nu = \frac{s-u}{4m_p} \tag{3}$$

Crossing EVEN $f(-\nu) = f(\nu)$

Crossing ODD $f(-\nu) = -f(\nu)$

Invariant and CGLN amplitudes

Chew,Golberger,Low,Nambu - CGLN [Phys.Rev.106(1957) 1337-1344].

$\gamma p \to \eta p$ can be described by 4 amplitudes, the matrix element in c.m. takes the form:

$$t_{\gamma,\eta} = \bar{u}(p_f) \sum_{i=1}^{4} A_i(\nu,t)\, \varepsilon_\mu M_i^\mu \, u(p_i) = -\frac{4\pi W}{M_N} \chi_f^\dagger \mathcal{F} \chi_i \,, \tag{4}$$

$$\begin{aligned} M_1^\mu &= -\frac{1}{2} i\gamma_5 \left(\gamma^\mu k\!\!\!/ - k\!\!\!/ \gamma^\mu\right) \,, \\ M_2^\mu &= 2i\gamma_5 \left(P^\mu \, k\cdot(q-\frac{1}{2}k) - (q-\frac{1}{2}k)^\mu \, k\cdot P\right) \,, \\ M_3^\mu &= -i\gamma_5 \left(\gamma^\mu \, k\cdot q - k\!\!\!/ q^\mu\right) \,, \\ M_4^\mu &= -2i\gamma_5 \left(\gamma^\mu \, k\cdot P - k\!\!\!/ P^\mu\right) - 2M_N \, M_1^\mu \,, \end{aligned} \tag{5}$$

where $k\!\!\!/ = k_\mu \gamma^\mu$, $P^\mu = (p_i^\mu + p_f^\mu)/2$.

$$\mathcal{F} = i\,(\vec{\sigma}\cdot\hat{\epsilon})\, F_1 + (\vec{\sigma}\cdot\hat{q})\,(\vec{\sigma}\times\hat{k})\cdot\hat{\epsilon}\, F_2 + i\,(\hat{\epsilon}\cdot\hat{q})\,(\vec{\sigma}\cdot\hat{k}) F_3 + i(\hat{\epsilon}\cdot\hat{q})(\vec{\sigma}\cdot\hat{q})\, F_4 \tag{6}$$

where $\epsilon^\mu = (\epsilon_0, \vec{\epsilon})$ and $\vec{\epsilon}\cdot\vec{k} = 0$. Both types of amplitudes are used further.

Truncated partial wave expansion of CGLN amplitudes

CGLN amplitudes can be expanded in terms of the partial waves and an angle.

$$
\begin{aligned}
F_1(W,x) &= \sum_{\ell=0}^{\ell_{max}} \left[\left(\ell M_{\ell+} + E_{\ell+} \right) P'_{\ell+1}(x) + \left(\left(\ell+1 \right) M_{\ell-} + E_{\ell-} \right) P'_{\ell-1}(x) \right], \\
F_2(W,x) &= \sum_{\ell=1}^{\ell_{max}} \left[\left(\ell+1 \right) M_{\ell+} + \ell M_{\ell-} \right] P'_{\ell}(x), \\
F_3(W,x) &= \sum_{\ell=1}^{\ell_{max}} \left[\left(E_{\ell+} - M_{\ell+} \right) P''_{\ell+1}(x) + \left(E_{\ell-} + M_{\ell-} \right) P''_{\ell-1}(x) \right], \\
F_4(W,x) &= \sum_{\ell=2}^{\ell_{max}} \left[M_{\ell+} - E_{\ell+} - M_{\ell-} - E_{\ell-} \right] P''_{\ell}(x). \qquad (7)
\end{aligned}
$$

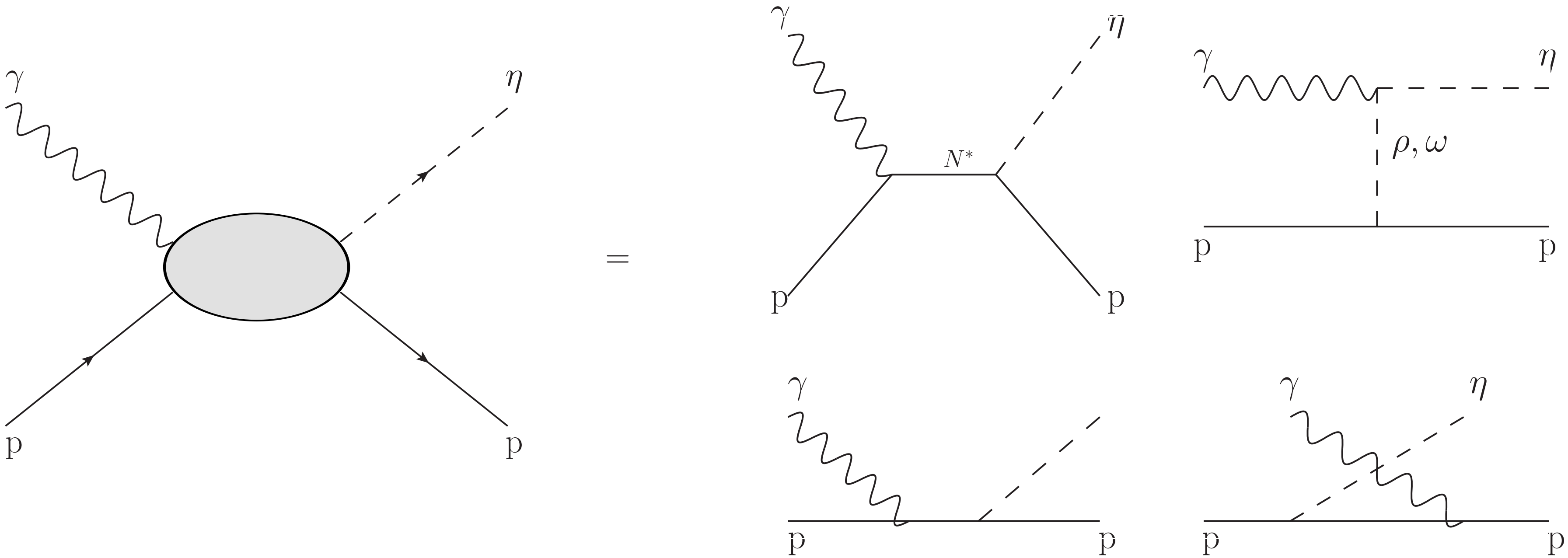
where ℓ is an orbital angular momentum of the ηN system, $x = \cos\theta$ is the cosine of the scattering angle, $M_{\ell\pm}, E_{\ell\pm}$ are multipoles, "$\pm$" $\rightarrow J = \ell \pm 1/2$.

Consider example: $E_{0+}: \; \ell = 0, \; J = 0 + 1/2, \; P = -(-1)^{\ell} = -1$

Some important resonances in η photoproduction

Resonance $N(mass)\ J^P$	ℓ	J	P	Multipole	$J = \ell \pm 1/2$
$N(1535)\ 1/2^-$	0	1/2	-	E_{0+}	$1/2 = 0 + 1/2$
$N(1650)\ 1/2^-$	0	1/2	-	E_{0+}	$1/2 = 0 + 1/2$
$N(1440)\ 1/2^+$	1	1/2	+	M_{1-}	$1/2 = 1 - 1/2$
$N(1710)\ 1/2^+$	1	1/2	+	M_{1-}	$1/2 = 1 - 1/2$
$N(1720)\ 3/2^+$	1	3/2	+	E_{1+}, M_{1+}	$3/2 = 1 + 1/2$
$N(1520)\ 3/2^-$	2	3/2	-	E_{2-}, M_{2-}	$3/2 = 2 - 1/2$
$N(1700)\ 3/2^-$	2	3/2	-	E_{2-}, M_{2-}	$3/2 = 2 - 1/2$
$N(1675)\ 5/2^-$	2	5/2	-	E_{2+}, M_{2+}	$5/2 = 2 + 1/2$
$N(1680)\ 5/2^+$	3	5/2	+	E_{3-}, M_{3-}	$5/2 = 3 - 1/2$

Constituents of the ηMAID isobar model



Resonance parametrization of the ηMAID isobar model

Multipoles $\mathcal{M}_{\ell\pm}$ ($E_{\ell\pm},\ M_{\ell\pm}$) have Breit Wigner form:

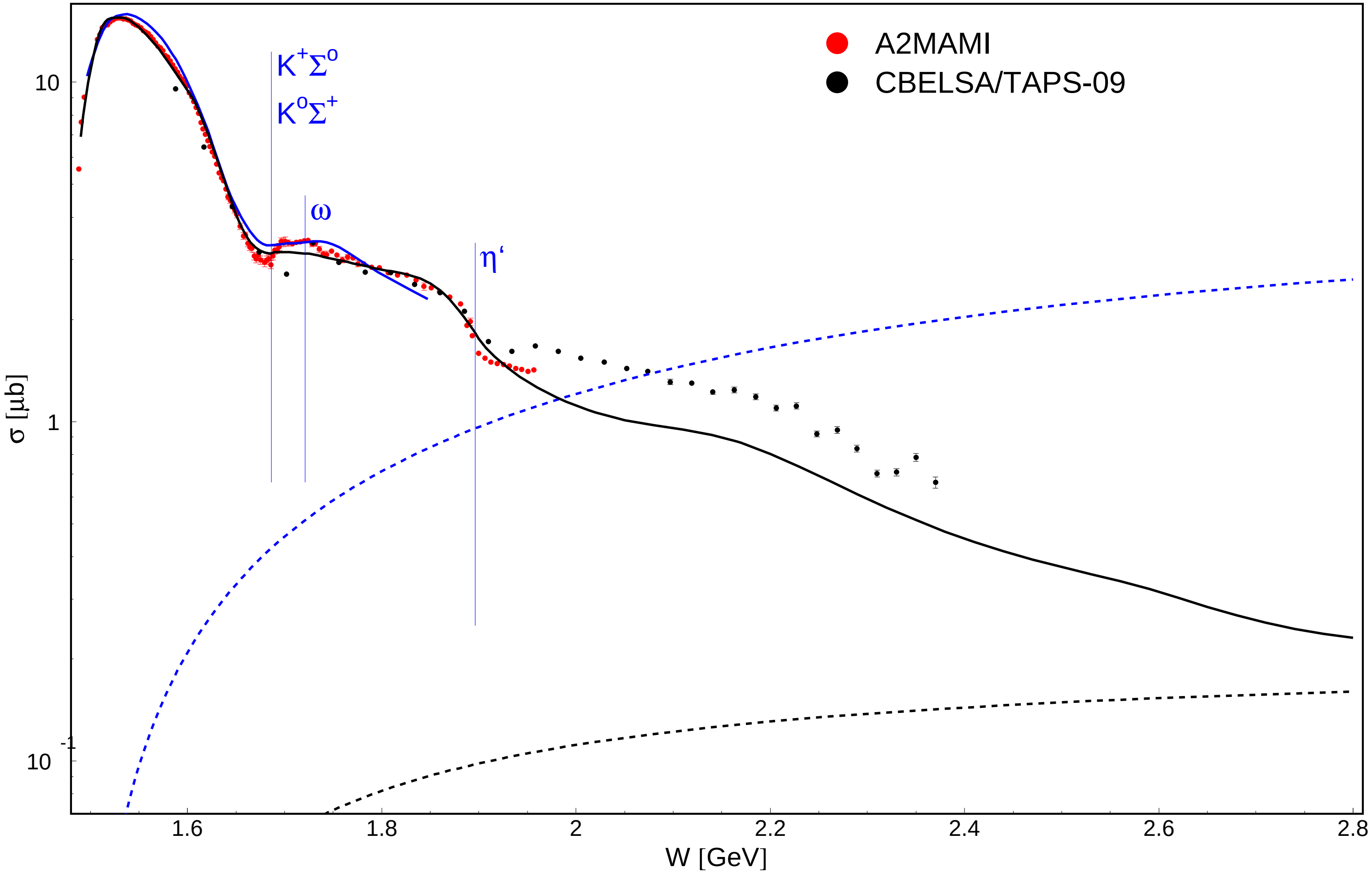
$$\mathcal{M}_{\ell\pm}(W) = \bar{\mathcal{M}}_{\ell\pm}\, f_{\gamma N}(W)\, \frac{M_R \Gamma_{\rm tot}(W)}{M_R^2 - W^2 - i M_R \Gamma_{\rm tot}(W)}\, f_{\eta N}(W)\, C_{\eta N}\,, \tag{8}$$

$\bar{\mathcal{M}}_{\ell\pm}$ is related to the photo decay amplitudes listed in PDG.

$\Gamma_{\rm tot}(W)$ is the energy dependent width.

$f_{\gamma N}(W)$ and $f_{\eta N}(W)$ are vertex functions.

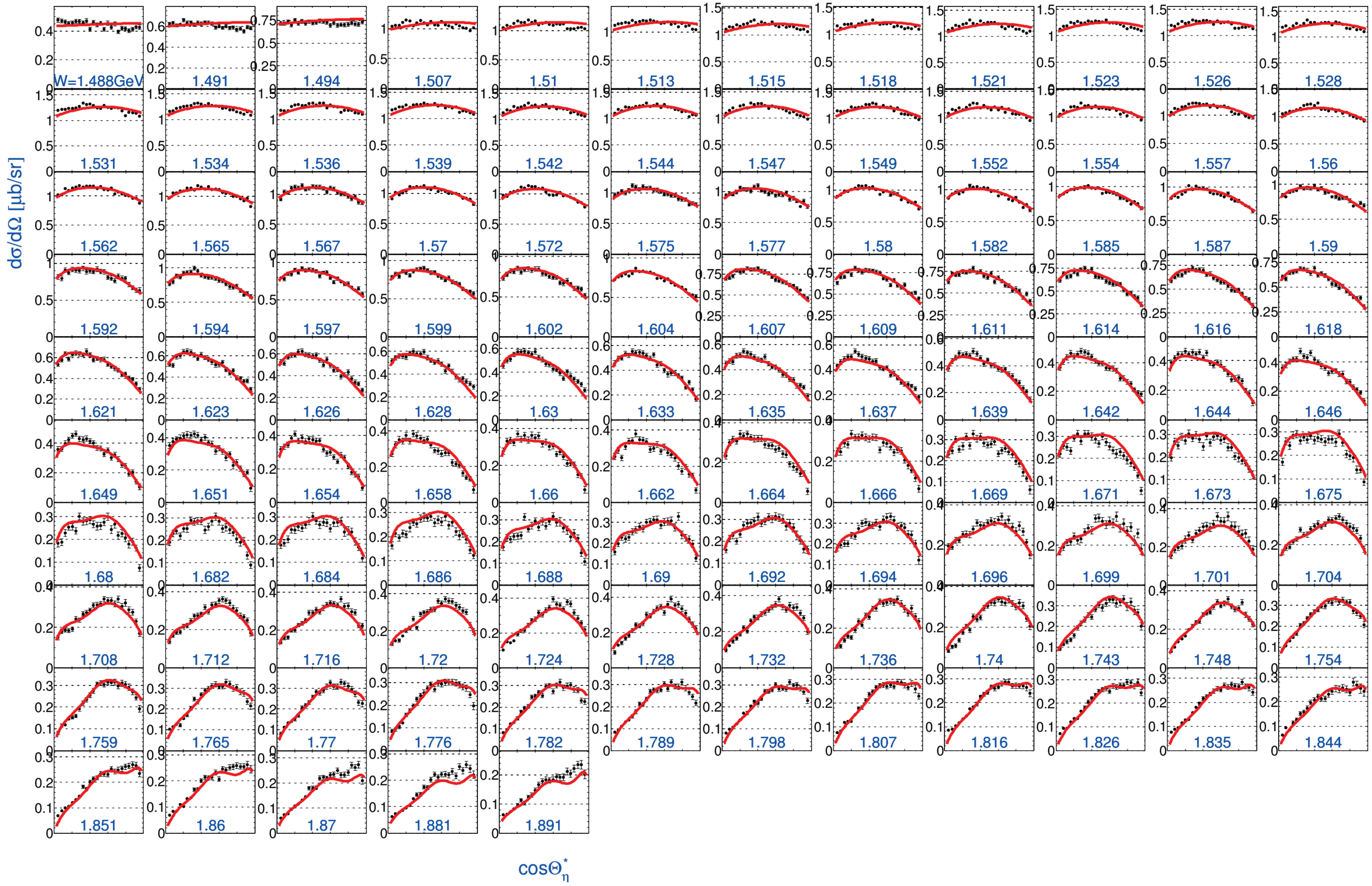
Fit results for the total cross section with an isobar model



ηMAID2003: solid - full model, dashed - background ($\rho + \omega + born$),

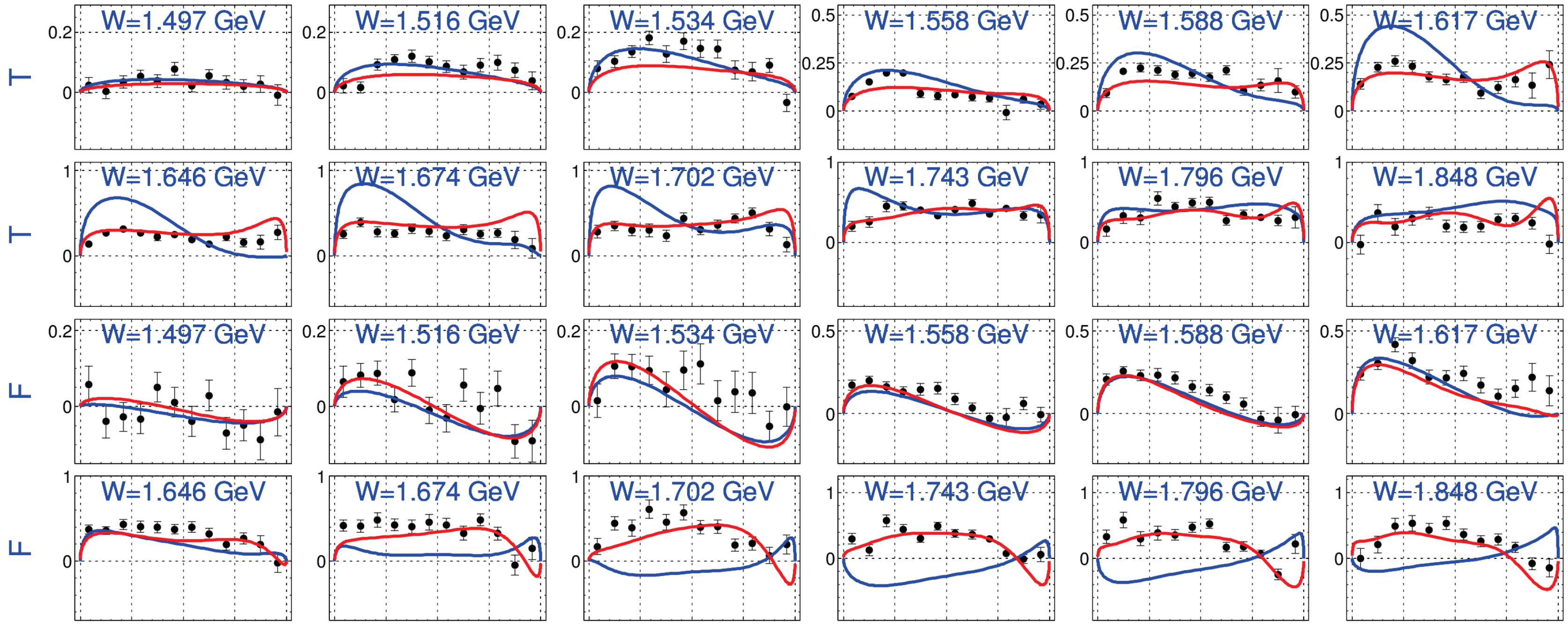
ηMAID2015: solid - full model, dashed - background ($\rho + \omega + born$)

Fit results for the differential cross section with an isobar model



black circles: A2MAMI data red: ηMAID

Fit results for T and F with an isobar model



ηMAID2003, ηMAID2015

T - polarized target,

F - polarized beam and target.

Fixed-t dispersion relations

Q:How to improve?

Problem of an isobar model:non analytic.

BUT: Invariant amplitudes are analytic functions of complex variables, one can derive dispersion relations at a fixed value of t.

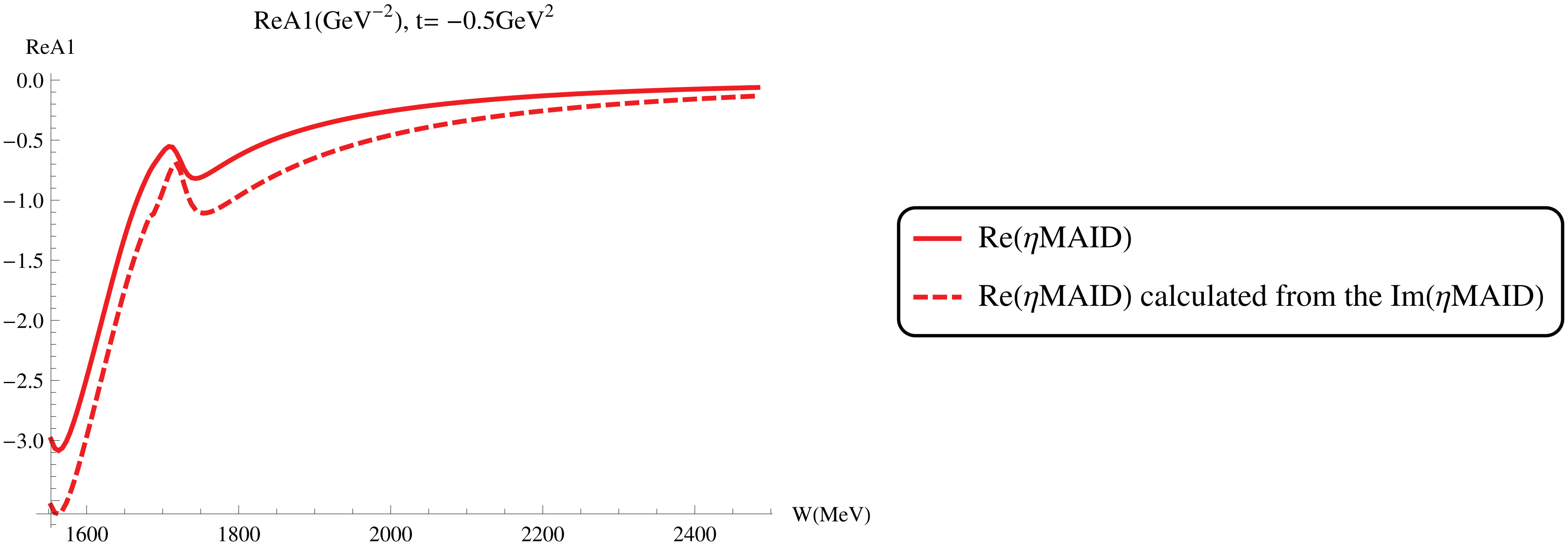
Crossing even:

$$\text{Re}A_i(\nu,t) = A_i^{pole}(\nu,t) + \frac{2}{\pi}\,\mathcal{P}\int_{\nu_{thr}(t)}^{\infty} d\nu'\,\frac{\nu'\ \text{Im}A_i(\nu',t)}{\nu'^2-\nu^2}\,, \quad \text{for } i=1,2,4 \tag{9}$$

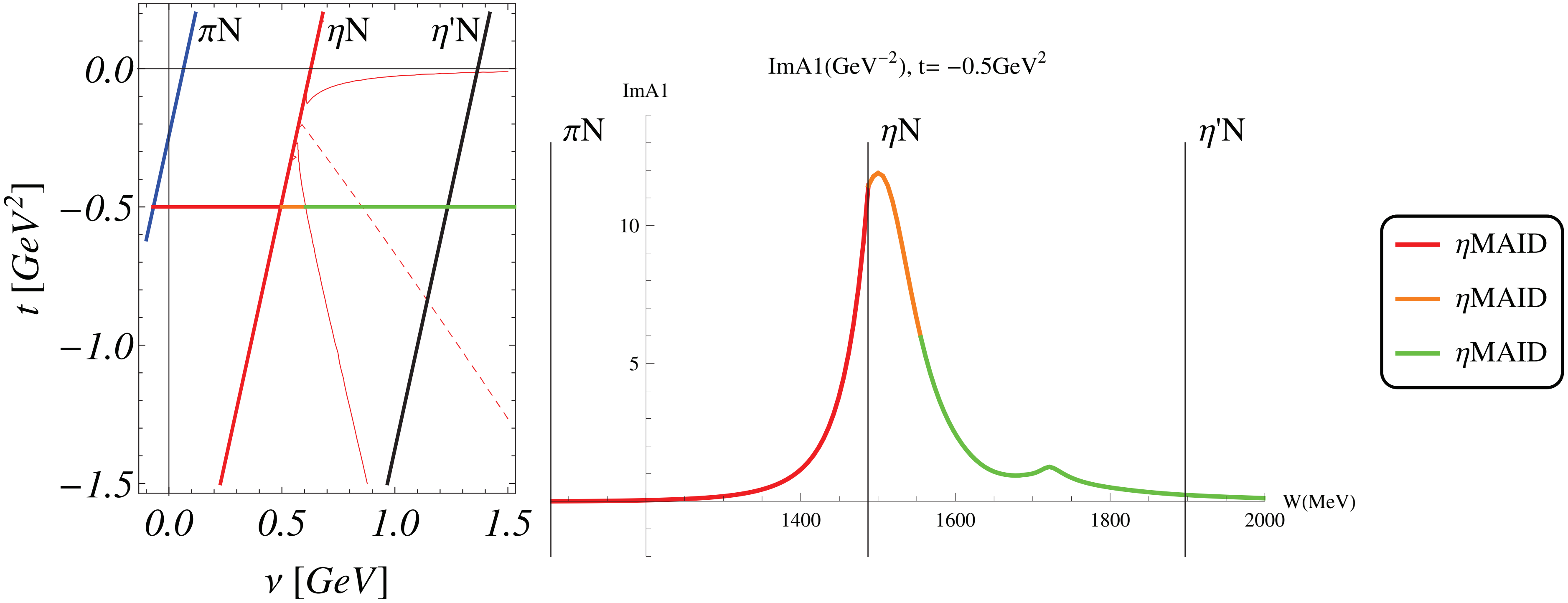
Crossing odd:

$$\text{Re}A_i(\nu,t) = A_i^{pole}(\nu,t) + \frac{2\nu}{\pi}\,\mathcal{P}\int_{\nu_{thr}(t)}^{\infty} d\nu'\,\frac{\text{Im}A_i(\nu',t)}{\nu'^2-\nu^2}\,, \quad \text{for } i=3 \tag{10}$$

Real part of A1



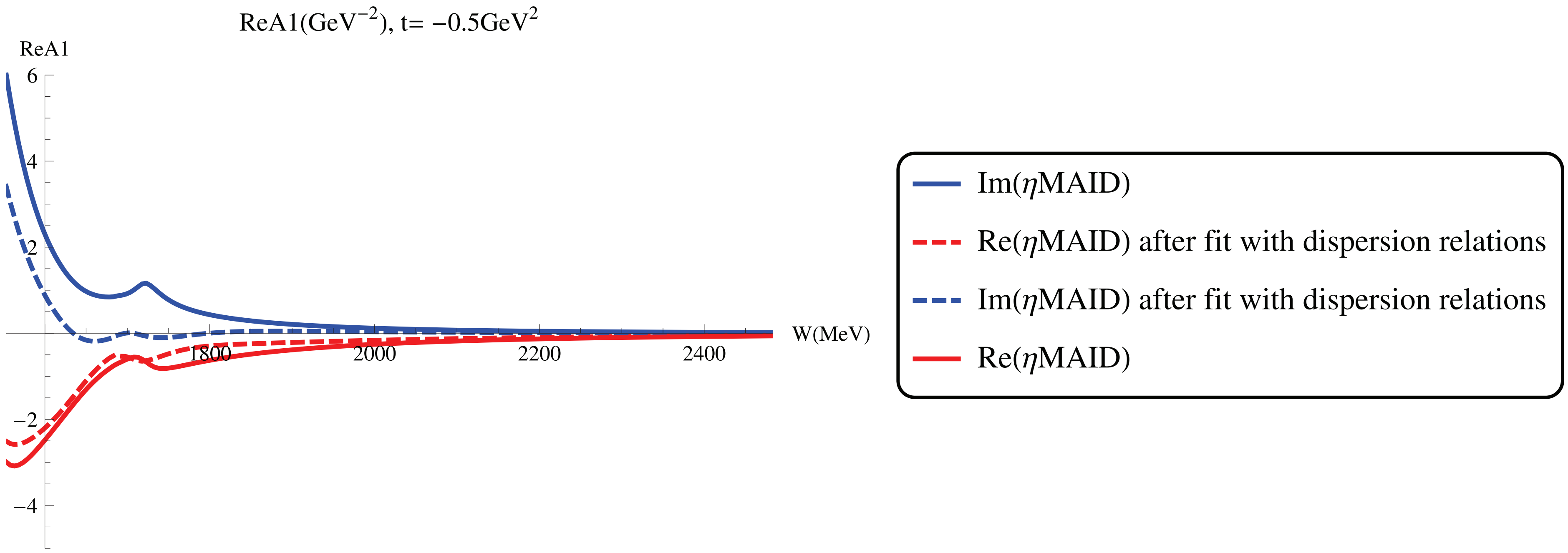
Integrating regions



Fitting procedure.

- Data: $\frac{d\sigma}{d\Omega}$,T,F (A2) and Σ (Graal) up to 1700 MeV.
- Model: Resonances + born + ρ-meson + ω-meson.
- 9 resonances.
- Constrain from dispersion relations is implemented in Minuit.

Working progress. Fit results for A1



Conclusion

- Systematic cross checks.
- Multipole extraction.
- Resonance parameters.