

Twisted Photons, with applications to photoexcitation in nuclear and atomic physics

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William and Mary & JGU, Mainz (Visitor)

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Topics

- History (brief)
- Twisted photon basics
- Why here?: possible applications in nuclear/hadronic resonance studies, e.g., in photoexcitation of high spin resonances.
- Examples to show it can work: atomic physics
- Theory results in atomic photoexcitation
(w/data from 2nd floor, Physics Bldg., Mainz)
- End

Some history

- Waves diffract (mostly)
- Plane waves don't: they are too bland
- 1987: Durnin points out existence of structured waves that also don't diffract.
- Structured means there are hot spots and cold spots in the wave front, and nondiffracting means the hot spots don't spread out.

History: Durnin

- Waves satisfy the Helmholtz equation

$$\left(\nabla^2 + \frac{\omega^2}{c^2} \right) \psi(t, \vec{x}) = (\nabla^2 + k^2) \psi(t, x, y, z) = 0$$

- Monochromatic: $e^{-i\omega t}$
- Traveling in z -direction, non-diff., must have $e^{i\beta z}$
- Solution

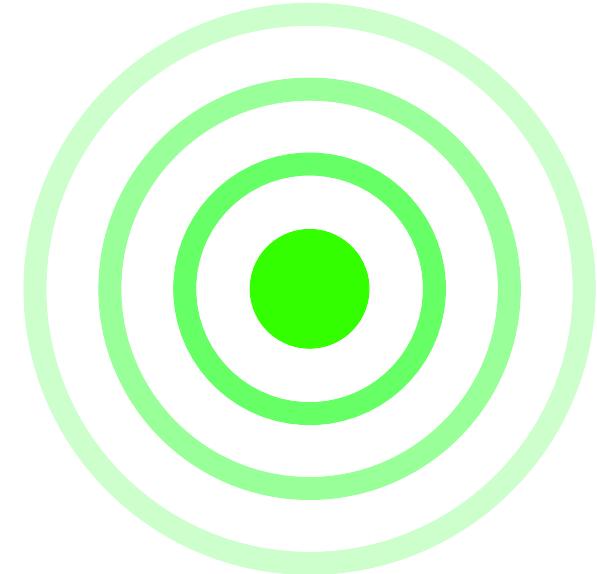
$$\psi = e^{i(\beta z - \omega t)} J_0(\alpha \rho)$$

- where

$$\alpha^2 + \beta^2 = k_{\perp}^2 + k_z^2 = k^2 ; \quad \rho = \sqrt{x^2 + y^2}$$

Bessel wave

- Wave front coming at you:



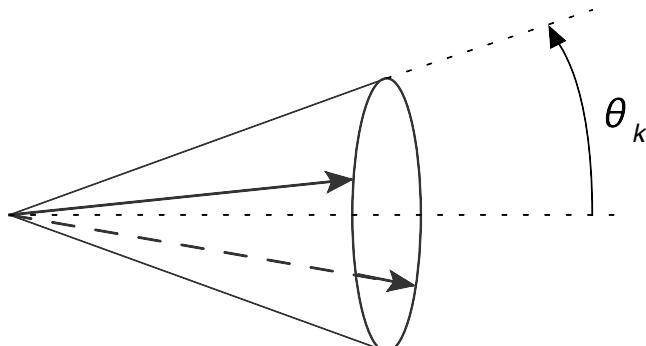
- Bullseye pattern, vortex center (vortex line)
- Hot spot in center (for J_0)

Wavenumber space

- In QM, momentum space
- Recall $\psi = e^{i(\beta z - \omega t)} J_0(\alpha \rho)$
- Fourier transform,

$$\tilde{\psi}(t, k_z, k_{\perp}, \phi_k) = \frac{(2\pi)^2}{k_{\perp}} e^{-i\omega t} \delta(k_z - \beta) \delta(k_{\perp} - \alpha)$$

- Every component in wave number space has same k_z , same k_{\perp} , but all possible azimuthal angles ϕ_k



- Component momenta form a cone, opening angle or "pitch angle" θ_k

Angular momentum

- 1992, Allen et al. show there exist photons with arbitrary angular momentum in direction of motion
- 4105 citations on Google Scholar as of 31 Aug 2017 (or 2256 on ADS and even 116 on SPIRES)
- For something called Laguerre-Gauss beams. Also works, as is being done here, for Bessel and Bessel-Gauss beams.

γ angular momentum

- Can discuss classically or QM
- Classically, for plane wave, RH polarization,

$$\text{angular momentum} = + \frac{\text{energy}}{\text{angular frequency}}$$

- QM, for single photon,

$$\text{angular momentum} = + \hbar$$

- Now (QM version),

$$\text{angular momentum} = (\text{any integer}) \times \hbar$$

- How?

Twisted photons

- Further solution to Helmholtz equation,

$$\psi(t, z, \rho, \phi_\rho) = e^{i(\beta z - \omega t)} e^{im_\gamma \phi_\rho} J_{m_\gamma}(\alpha \rho)$$

- or

$$\tilde{\psi}(t, k_z, k_\perp, \phi_k) = \frac{(2\pi)^2}{k_\perp} e^{-i\omega t} \delta(k_z - \beta) \delta(k_\perp - \alpha) i^{-m_\gamma} e^{im_\gamma \phi_k}$$

- Similar, but phase changing around edge of cone
- If

$$L_z = -i\hbar \frac{\partial}{\partial \phi_\rho}$$

have

$$L_z = m_\gamma \hbar$$

Twisted vector photons

- Can obtain or visualize L_z in other ways
- First, noting we so far have scalar photons (beloved of some theorists), will switch to vector photons.
- Easiest is to note matrix elements of fields with standardly normalized states, scalar and vector:

$$\langle 0 | \psi(0) | \vec{k} \rangle = 1$$

$$\langle 0 | A_\mu(0) | \vec{k}, \Lambda \rangle = \epsilon_\mu(k, \Lambda)$$

- These are for plane waves
- Λ is helicity of the plane wave photon state ($= \pm 1$)

Vector photons

- Momentum space is expansion in plane waves.
Becomes,

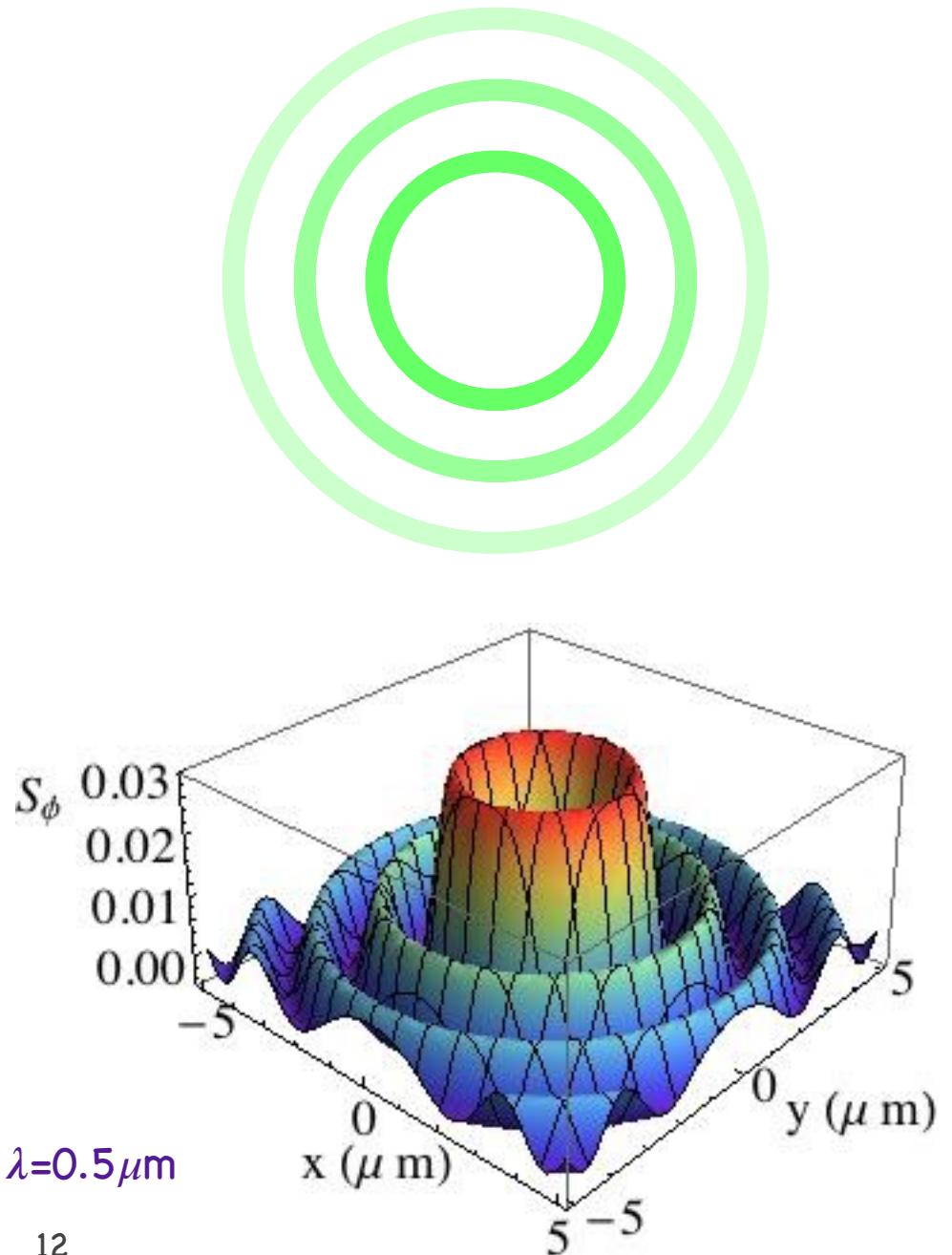
$$A_\mu^{(m_\gamma)}(t, k_z, \mathbf{k}_\perp, \phi_k, \Lambda) = \frac{(2\pi)^2}{k_\perp} e^{-i\omega t} \delta(k_z - \beta) \delta(\mathbf{k}_\perp - \alpha) i^{-m_\gamma} e^{im_\gamma \phi_k} \varepsilon_\mu(\vec{k}, \Lambda)$$

- Coordinate space expression medium long, will show on demand.
- Can work out electric and magnetic fields, and Poynting vector

Plots for Poynting vector

- Magnitude of Poynting vector: Wave front for $m_y \neq 0$ is empty in center
- Plane wave has Poynting vector only in z-direction
- Twisted photon also has azimuthal component of Poynting vector, S_ϕ

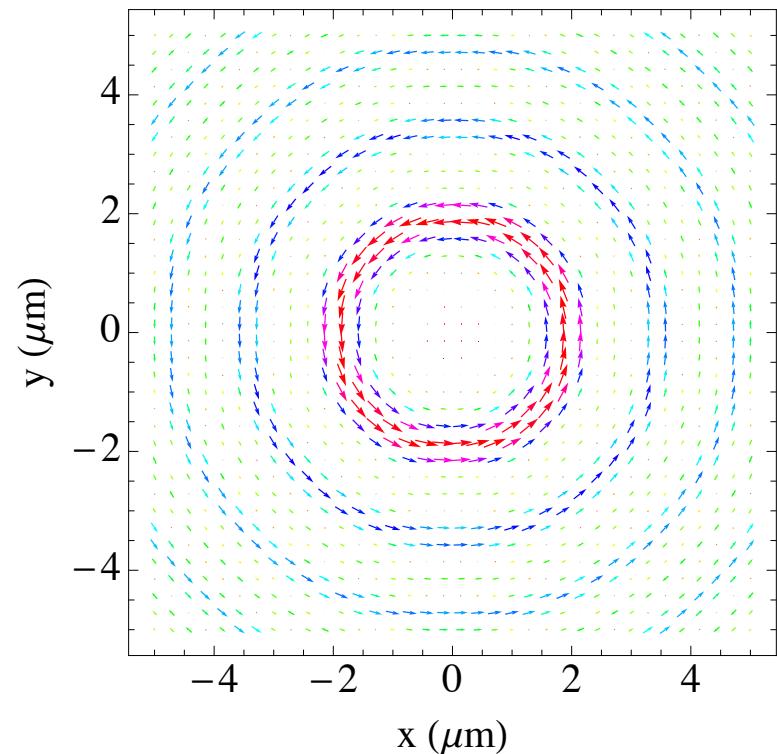
figure for $m_y=4$, $\theta_k=0.2$, $\lambda=0.5\mu\text{m}$



One more plot and comment

- Previous figure gave magnitude of S_ϕ ;
here indicate direction:

also for $m_\gamma=4$, $\theta_k=0.2$, $\lambda=0.5\mu\text{m}$



- Swirling gives photon orbital angular momentum in z-dir.
- Spin of photon projects to $\Lambda \cos(\theta_k)$ in z-dir.
- Total projected angular momentum of state is m_γ

Comments

- This is component of orbital angular momentum (OAM) in direction of motion. For plane wave, or momentum eigenstate, necessarily zero:

$$\hat{p} \cdot \vec{L} = \hat{p} \cdot \vec{r} \times \vec{p} = 0$$

- ∴ not discussing momentum eigenstates

Comments

- Freshman physics: component of OAM in direction of overall momentum motion is independent of origin of coordinate system.

$$\vec{P} = \sum_i \vec{p}_i$$

$$\vec{L} = \sum_i \vec{r}_i \times \vec{p}_i$$

$$\vec{L}_{\text{shifted}} = \sum_i (\vec{r}_i + \vec{r}_{\text{shift}}) \times \vec{p}_i = \sum_i \vec{r}_i \times \vec{p}_i + \vec{r}_{\text{shift}} \times \vec{P}$$

$$\hat{P} \cdot \vec{L}_{\text{shifted}} = \hat{P} \cdot \vec{L}$$

Comments

- Hence “intrinsic” OAM
- We might like to call it helicity, but in this area, helicity is reserved for plane wave photons

Selection rules

- Set of potential applications based on selection rules
- Consider photoexcitation.
Initial state $\{j_i, m_i\}$ goes to final state $\{j_f, m_f\}$
by absorbing photon
- Plane wave photon, $m_f - m_i = \Lambda$,
 $| j_f - j_i | = 1$ (usually)
These are E1 transitions.
- With twisted photon and direct hit (vortex line passing through atom's center), angular momentum conservation dictates $m_f - m_i = m_\gamma$ (may be $\gg 1$)
- Selectively excite higher angular momentum nuclear/nucleon resonances or higher atomic states.

For electron accelerators

- Make high energy photons by backscattering optical photons off energetic electrons
- Jentschura-Serbo (2011) showed this backscattering maintains the twistedness.
- Achievable energy, if electron energy is $E_e = \gamma m_e$, and initial energy is ω_1 ,

$$\omega_2 \approx \frac{4\gamma^2 \omega_1}{1 + 4\gamma\omega_1/m_e}$$

- For $0.5\mu\text{m}$ light (2.48 eV) in,
get 1.11 GeV photons out for 6 GeV electrons, or
get 3.75 GeV photons out for 12 GeV electrons.

electron accelerators

- Lots of energy to excite baryon resonances, with hoped for angular momentum selectivity
- There exists study group at JLab (Joe Grimes et al.)
- Claim that twisted photon beam of $10^{34} \text{ cm}^{-2}\text{sec}^{-1}$ luminosity is possible
- Beam steering may be crucial

Atomic possibilities

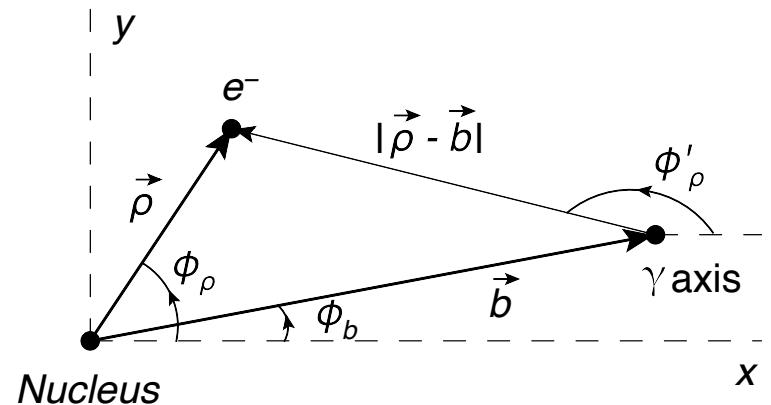
- Two possibilities:
 - Test idea that, with good beam and atom location control, on the nose strikes give quantum number changes not possible with plane wave photons
 - See effects on photoexcitation when vortex line misses the atom by measured amount

Off axis transitions

- Qualitative: Electric field swirls as seen already for Poynting vector.
- Atom smaller than structure scale of wave front
- Atom on vortex line sees circular swirling. Electron will absorb all photon's angular momentum, if transition occurs.
- Farther out, the atom being small sees a roughly spatially constant E-field. Transitions will be largely electric transitions like that produced by plane wave.

Off axis-more detail

- Transition matrix element for photon with vortex line displaced from center of atom by impact parameter \mathbf{b}
 - If atomic ground state is S-state, matrix element is
- $$\mathcal{M}_{j_z s_z \Lambda}^{(m_\gamma)}(\vec{b}) = \langle n_f j_f j_z; l_f s_f | H_{int} | n_i s_z; k_\perp k_z m_\gamma \Lambda \vec{b} \rangle$$
- where the location of the vortex line (impact parameter) is indicated for the photon state.
- Expand the photon state as collection of plane waves at polar angle θ_k . Rotate each to z-direction, also rotating atomic state.



Off axis development

- Also, expand final atomic state in angular momentum and spin parts. This give Clebsch-Gordan coefficient.
- Unrotate atomic state, using theorems like

$$\langle n_f l_f l_z | R(\phi_k, \theta_k) = e^{-il_z\phi_k} \sum_{l'_z} \langle n_f l_f l_z | R_y(\theta_k) | n_f l_f l'_z \rangle \langle n_f l_f l'_z | = e^{-il_z\phi_k} \sum_{l'_z} d_{l_z, l'_z}^{l_f}(\theta_k) \langle n_f l_f l'_f |$$

- All states are now quantized along z-axis, photons have momenta along z-axis, and are plane waves.
- Basic amplitude will be plane wave amplitude.
- There are some integrals that give Bessel functions, and general result (for initial S-states) is

$$\left| \mathcal{M}_{j_z s_z \Lambda}^{(m_\gamma)}(\vec{b}) \right| = \left| J_{j_z - s_z - m_\gamma}(k_\perp b) \mathcal{M}_{n_f n_i; l_f \Lambda \Lambda}^{(\text{pw})}(\theta_k = 0) \langle j_f j_z; l_f 1/2 | l_f, j_z - s_z; 1/2, s_z \rangle d_{j_z - s_z, \Lambda}^{l_f}(\theta_k) \right|$$

Results

- (Next slides)
- Target is $^{40}\text{Ca}^+$ ions
- Ground state has S-state valence electron
- All transitions are $S \rightarrow D$, ≈ 729 nm wavelength
- Specifically, to $D_{5/2}$, with applied magnetic field Zeeman separating the different final J_z
- Figures to be shown here all have initial $S_z = -1/2$
- There is unpublished data on some figures.
Shown with permission of Christian Schmiegelow.
(Do see published data in Nature Comm., Dec. 2016)

One more item: Bessel-Gauss

- Bessel beam falls only slowly in transverse direction ($\propto 1/\rho^{1/2}$). Require unlimited energy. Can't be made.
- Real beams can be Bessel-Gauss, e.g., for scalar case

$$\psi(t, z = 0, \rho, \phi_\rho) = A e^{-i\omega t} e^{im_\gamma \phi_\rho} J_{m_\gamma}(\alpha \rho) e^{-\rho^2/w_0^2}$$

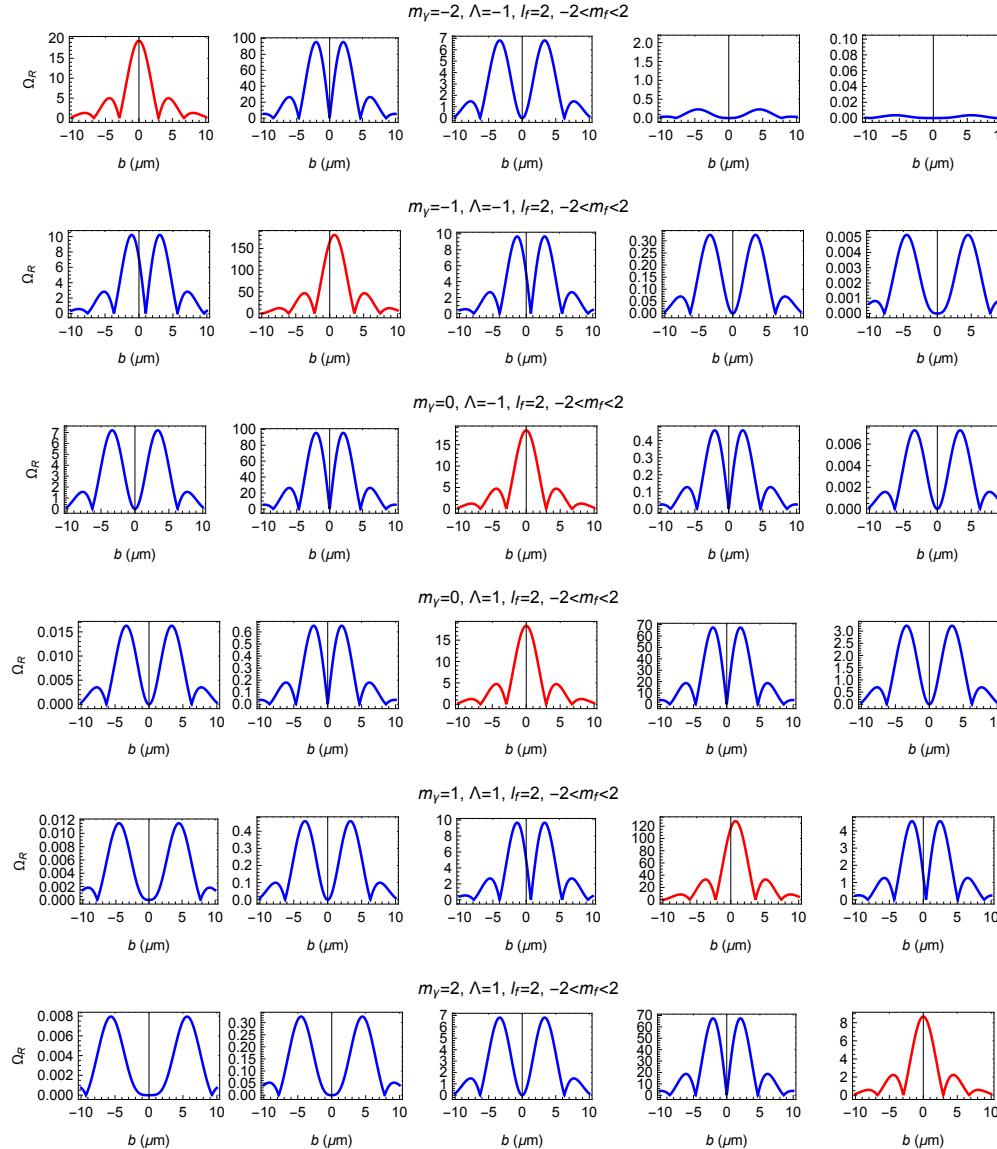
- For $w_0 \gg$ wavelength, diffraction spread slow and can be ignored under actual experimental conditions.
- Three parameters (ω given):
 - A – overall amplitude
 - θ_k – pitch angle ($\alpha = k_\perp = k \sin(\theta_k)$)
 - w_0 – width of beam

Results

Bessel–Gauss beam calculations for target electron in S-state with $s_z = -1/2$

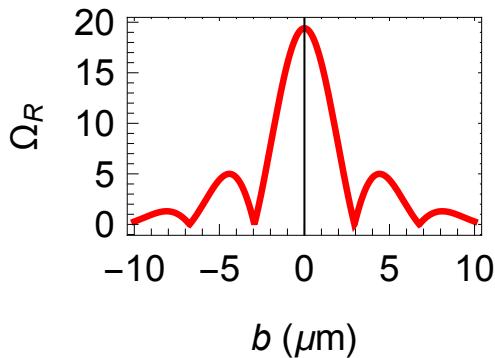
Pitch angle = 0.095 radians, Gaussian (1/e) width = $6.8 \mu\text{m}$

With 0% by amplitude of opposite photon helicity added in

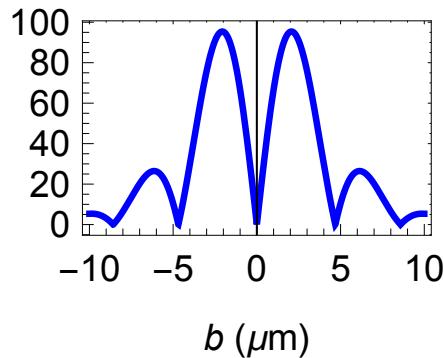


Part of top row

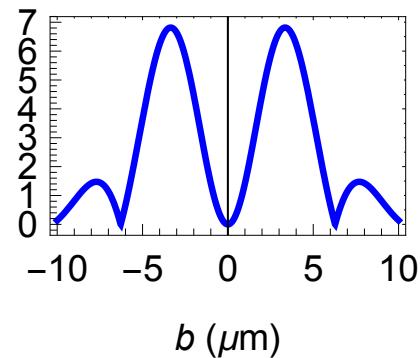
$$m_\gamma = -2, \Lambda = -1, l_f = 2, -2 < m_f < 2$$



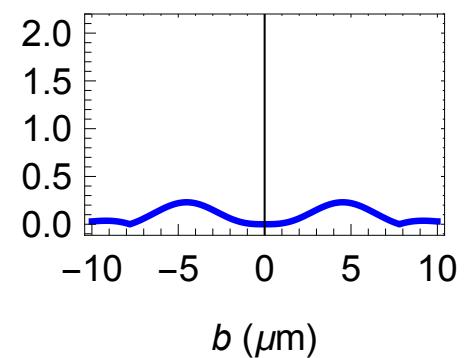
$$L_{zf} = -2$$



$$L_{zf} = -1$$



$$L_{zf} = 0$$



$$L_{zf} = +1$$

- Code: red is most interesting curve for twisted photon
- $L_{zf} = \Lambda$ is only case possible for untwisted photons
- $L_{zf} = 1$ result rather low.

... one more thing

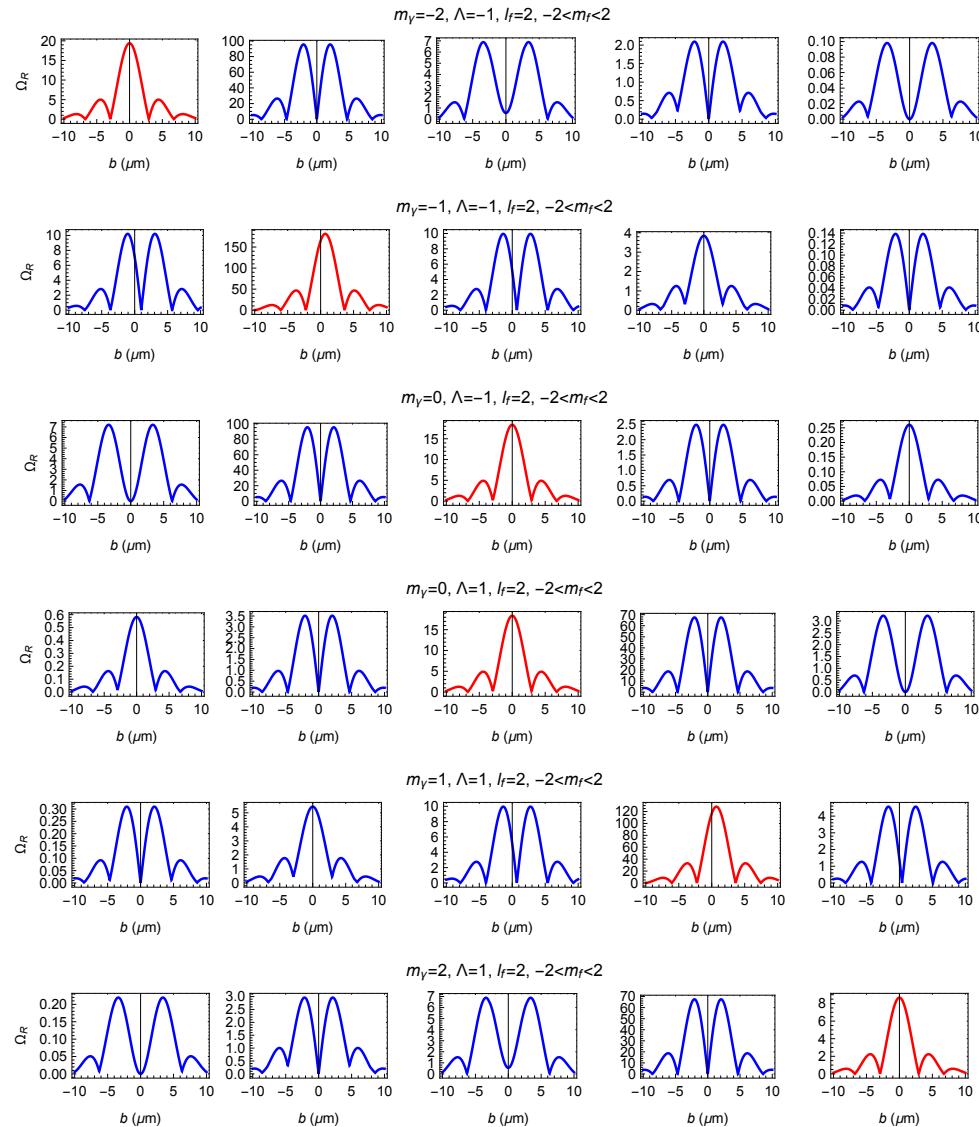
- There can be some leakage of opposite helicity photons into beam. See effect of 3% by amplitude (9×10^{-4} by intensity) of opposite helicity photon.

Results

Bessel–Gauss beam calculations for target electron in S-state with $s_z = -1/2$

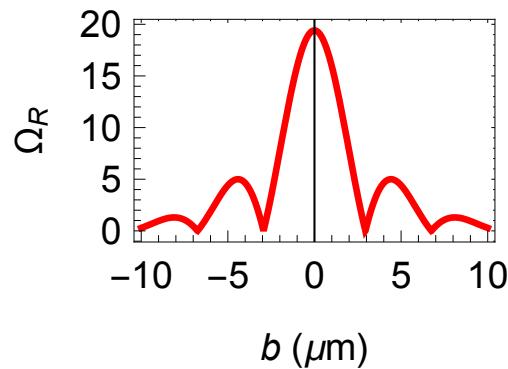
Pitch angle = 0.095 radians, Gaussian (1/e) width = 6.8 μm

With 3% by amplitude of opposite photon helicity added in

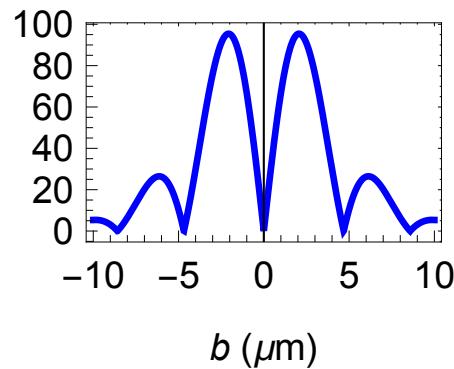


Similar part of top row

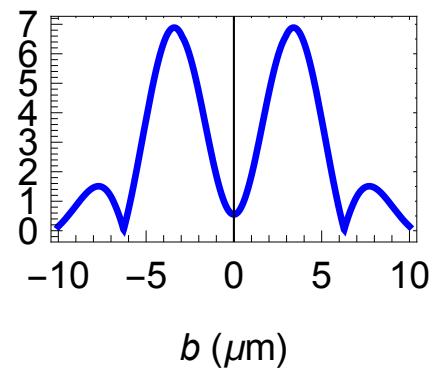
$$m_\gamma = -2, \Lambda = -1, l_f = 2, -2 < m_f < 2$$



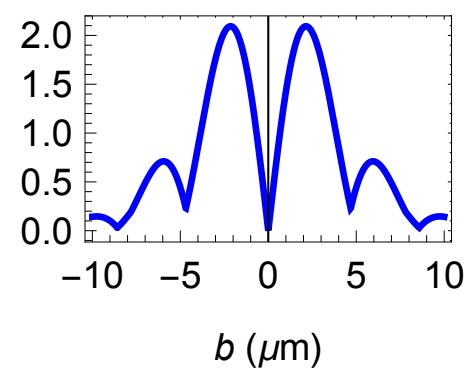
$$L_{zf} = -2$$



$$L_{zf} = -1$$



$$L_{zf} = 0$$



$$L_{zf} = +1$$

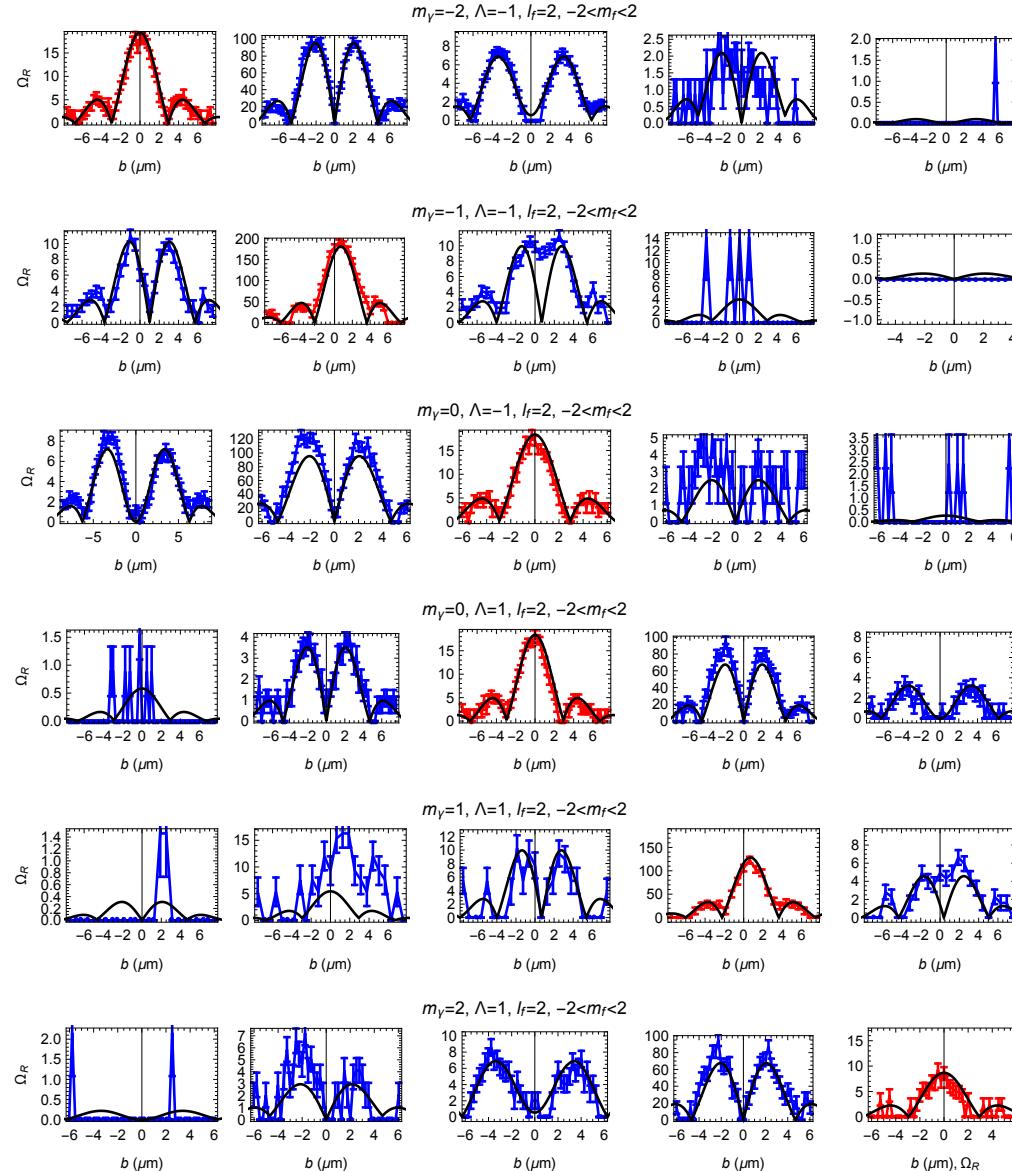
- $L_{zf} = 1$ much changed. Change in rest not noticeable.

Real results

Bessel–Gauss beam calculations for target electron in S–state with $s_z = -1/2$

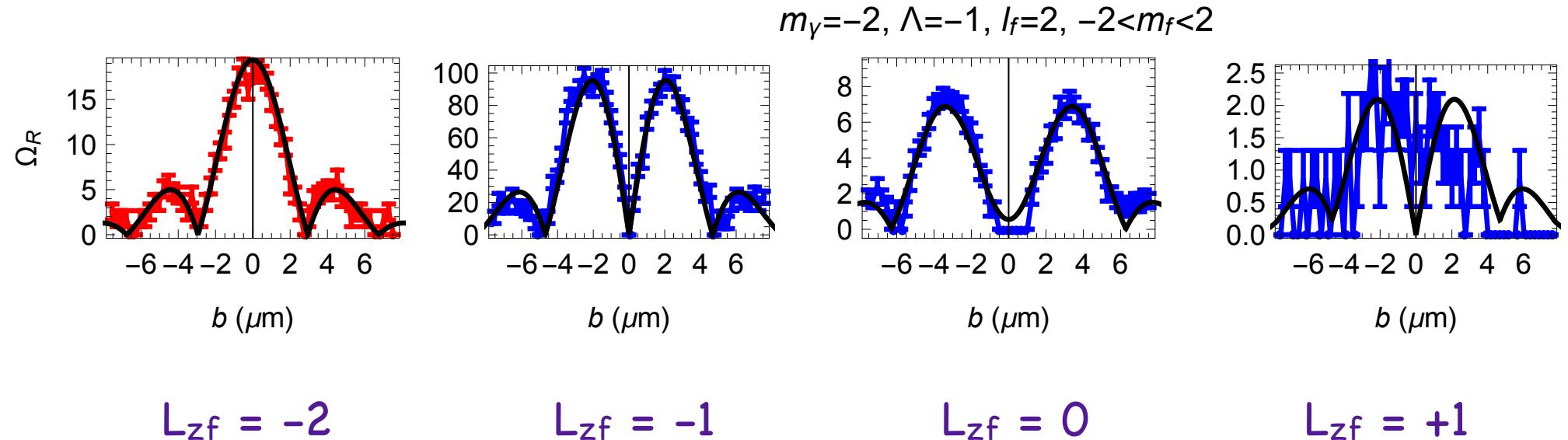
Pitch angle = 0.095 radians, Gaussian (1/e) width = 6.8 μm

With 3% by amplitude of opposite photon helicity added in



more focused

With 3% by amplitude of opposite photon helicity added in

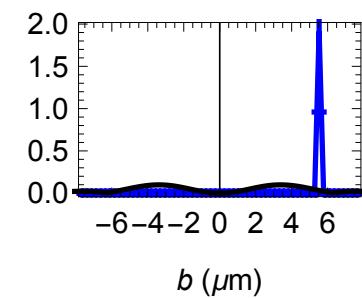
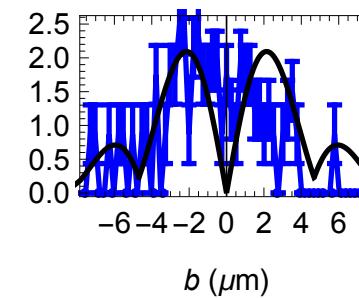
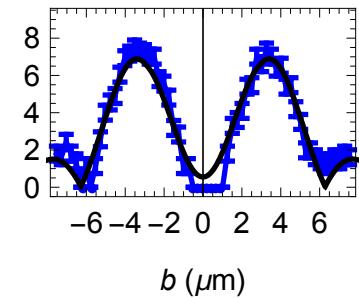
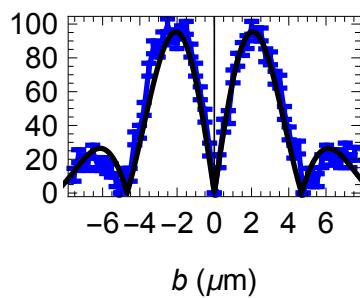
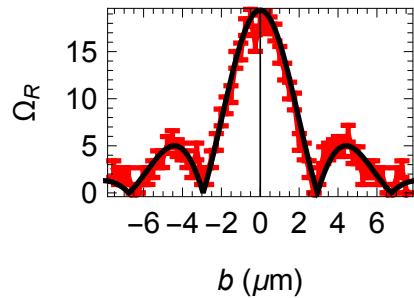


- First graph used to set A, θ_k, w_0
- $L_{zf} = 1$ used to set admixture of opposite helicity γ
- Other 34 graphs follow

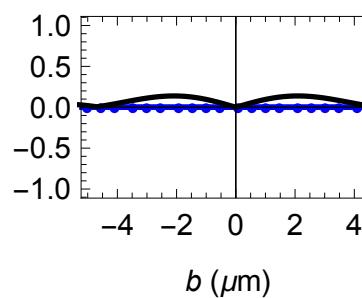
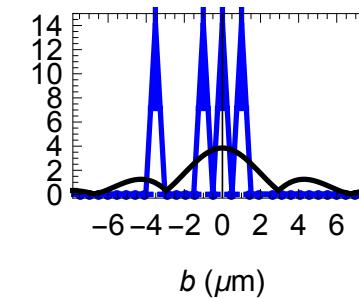
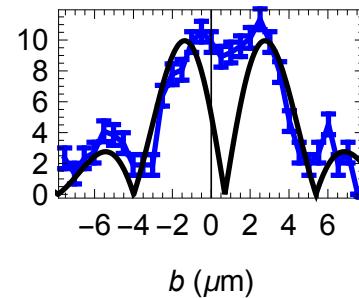
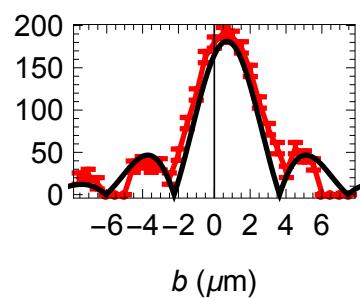
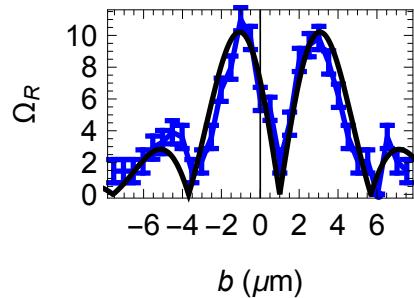
the upper half

With 3% by amplitude of opposite photon helicity added in

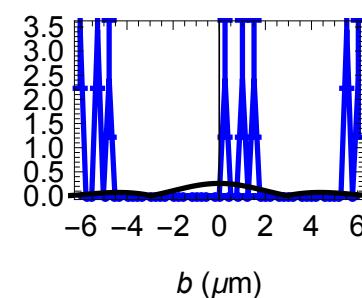
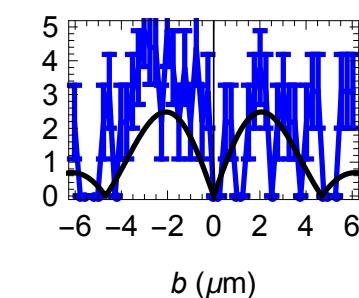
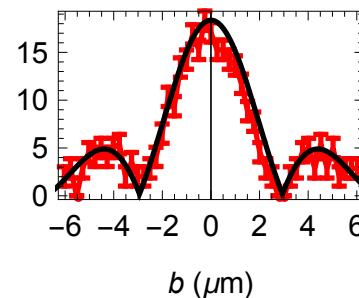
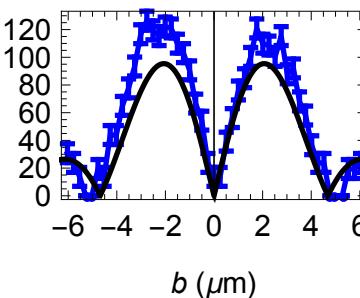
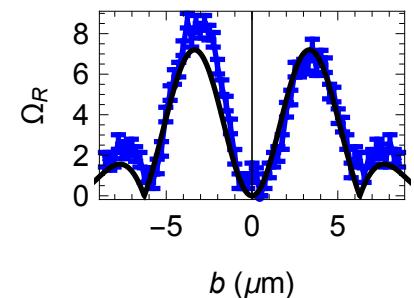
$$m_\gamma = -2, \Lambda = -1, l_f = 2, -2 < m_f < 2$$



$$m_\gamma = -1, \Lambda = -1, l_f = 2, -2 < m_f < 2$$



$$m_\gamma = 0, \Lambda = -1, l_f = 2, -2 < m_f < 2$$



Penultimate comments

- Twisted photons work
 - Significant rates for transitions with large angular momentum transfer, in situations where plane wave gives zero.
First shown on 2nd floor, Staudingerweg 7
- Detailed theory works
 - Few parameters fit to one data set
 - Obtain accurate predictions for remaining data
 - allows determining beam characteristics

Ultimate comments

- Note on scales in atomic case
 - atom ca. 10^{-1} nm
 - location of single atom to ca. 10 nm
 - wavelength ca. 10^3 nm
 - hole in wavefront several wavelengths wide
- Hope for nuclear/nucleon analogs?
 - nucleon ca. 1 fm (or 0.84087(39) fm)
nucleus ca. 10 fm
 - location placement ?
 - 1 GeV photon \rightarrow wavelength ca. 1 fm
 - width of hole is large

Thanks for coming

- Omitted topics
 - atomic recoil
 - radiation pressure and Poynting vector
 - circular dichroism on spherical targets
 - twisted light on $N \rightarrow \Delta(1232)$ transition
 - not to mention Laguerre-Gauss beams, and long distance propagation of Bessel-Gauss.

Beyond the end

Twisted γ in coordinate space

$$\mathcal{A}_{k_\perp k_z m_\gamma \Lambda}^\mu(x) = -i\Lambda e^{i(k_z z - \omega t + m_\gamma \phi_\rho)} \left\{ e^{-i\Lambda \phi_\rho} \cos^2 \frac{\theta_k}{2} J_{m_\gamma - \Lambda}(k_\perp \rho) \eta_\Lambda^\mu + \frac{i}{\sqrt{2}} \sin \theta_k J_{m_\gamma}(k_\perp \rho) \eta_0^\mu - e^{i\Lambda \phi_\rho} \sin^2 \frac{\theta_k}{2} J_{m_\gamma + \Lambda}(k_\perp \rho) \eta_{-\Lambda}^\mu \right\}$$

where $\eta_{\pm 1} = \frac{1}{\sqrt{2}} (0, \mp 1, -i, 0) = \frac{1}{\sqrt{2}} e^{\pm i \phi_\rho} (\mp \hat{\rho} - i \hat{\phi})$, $\eta_0 = (0, 0, 0, 1) = \hat{z}$

fields:

$$E_\rho = -\omega e^{i(k_z z - \omega t + m_\gamma \phi_\rho)} \left[\cos^2 \frac{\theta_k}{2} J_{m_\gamma - \Lambda}(k_\perp \rho) + \sin^2 \frac{\theta_k}{2} J_{m_\gamma + \Lambda}(k_\perp \rho) \right],$$

$$E_\phi = -i\Lambda \omega e^{i(k_z z - \omega t + m_\gamma \phi_\rho)} \left[\cos^2 \frac{\theta_k}{2} J_{m_\gamma - \Lambda}(k_\perp \rho) - \sin^2 \frac{\theta_k}{2} J_{m_\gamma + \Lambda}(k_\perp \rho) \right],$$

$$E_z = i\Lambda \omega e^{i(k_z z - \omega t + m_\gamma \phi_\rho)} \sin \theta_k J_{m_\gamma}(k_\perp \rho).$$

and

$$\vec{B} = -i\Lambda \vec{E}$$

Poynting vector (either Λ)

$$S_\rho = 0$$

$$S_\phi = \omega^2 \sin \theta_k J_{m_\gamma}(k_\perp \rho) \left(\cos^2 \frac{\theta_k}{2} J_{m_\gamma - \Lambda}(k_\perp \rho) + \sin^2 \frac{\theta_k}{2} J_{m_\gamma + \Lambda}(k_\perp \rho) \right)$$

$$S_z = \omega^2 \left(\cos^4 \frac{\theta_k}{2} J_{m_\gamma - \Lambda}^2(k_\perp \rho) - \sin^4 \frac{\theta_k}{2} J_{m_\gamma + \Lambda}^2(k_\perp \rho) \right)$$