Neutrino physics (3)

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Plan of the lectures

- Introduction.
- Brief overview of experimental results
- Weyl, Dirac and Majorana fermions
- Neutrino masses in simplest extensions of the Standard Model. The seesaw mechanism(s).
- Neutrino oscillations in vacuum
  - Same E or same p ?
  - QM uncertainties and coherence issues
  - Wave packet approach to neutrino oscillations
  - Lorentz invariance of oscillation probabilities
  - 2f and 3f neutrino mixing schemes and oscillations
  - Implications of CP, T and CPT

Plan of the lectures – contd.

- Neutrino oscillations in matter – the MSW effect
  - Evolution equation
  - Adiabaticity condition and adiabatic evolution
  - Non-adiabatic regime
  - Graphical interpretation and mechanical analogy
  - Earth matter effects on $\nu_\odot$ (day-night asymmetry)
- Neutrino oscillations in matter – parametric resonance
- Direct neutrino mass measurement experiments
- Neutrinoless double β-decay
- Neutrino electromagnetic properties
- Subtleties of the theory of neutrino oscillations
  - Do charged leptons oscillate?
  - Oscillations of Mössbauer neutrinos
- Neutrinos and the baryon asymmetry of the universe

Plan of the lectures – contd.

- Exptl. results: Solar neutrino oscillations and KamLAND
- Oscillations of atmospheric and accelerator neutrinos
- Discovery of θ_{13} in reactor and accelerator expts.
- Future: What's next?

What is left out:

– Oscillations of SN neutrinos (incl. non-linear collective effects)

– Cosmological bounds on # of neutrino species and $\sum m_\nu$

– keV sterile neutrinos as Dark Matter

– Geoneutrinos

. . .

Phenomenology of neutrino oscillations

Neutrino mixing schemes

I. Dirac case

$$-\mathcal{L}_{w+m} = \frac{g}{\sqrt{2}}(\bar{e}_L \gamma^\mu V_L^\dagger U_L \nu_L) W_\mu^- + \sum_{\alpha=1}^{n} m_{l\alpha} \bar{e}_\alpha e_\alpha + \sum_{i=1}^{n} m_i \bar{\nu}_i \nu_i + h.c.$$

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♢ $$V_L^\dagger U_L \equiv U ; \qquad \nu_{\alpha L} = \sum_{i=1}^{n} U_{\alpha i} \nu_{iL} \qquad \Rightarrow \qquad |\nu_{\alpha L}\rangle = \sum_{i=1}^{n} U_{\alpha i}^* |\nu_{iL}\rangle$$

$(\alpha = e, \mu, \tau, \quad i = 1, 2, 3$

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♢ $$P(\nu_\alpha \to \nu_\beta; L) = \left| \sum_{i=1}^{n} U_{\beta i} \, e^{-i \frac{\Delta m^2_{ij}}{2p} L} \, U^*_{\alpha i} \right|^2$$

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II. Majorana neutrinos

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Osc. probability: the same expression

Neutrino mixing schemes

III. Dirac + Majorana mass term (n LH and k RH neutrinos)

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Index a can take $n+k$ values; denote collectively the first n of them with α and the last k with σ $\Rightarrow$

D + M mass term – contd.

Active and sterile LH neutrino fields in terms of LH components of mass eigenstates:

$$\nu_{\alpha L} = \sum_{i=1}^{n+k} \mathcal{U}_{\alpha i} \chi_{iL} \,, \qquad (\nu_{\sigma R})^c = \sum_{i=1}^{n+k} \mathcal{U}_{\sigma i} \chi_{iL} \,.$$

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The usual oscillations described by the standard f-la with $U \to \mathcal{U}$ and summation over i up to $n+k$. In addition: new types of oscillations possible.

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Sterile - sterile neutrino oscillations:

$$P({\nu^c}_{\sigma L} \to {\nu^c}_{\rho L}; L) = \left| \sum_{i=1}^{n+k} \mathcal{U}_{\rho i}\, e^{-i \frac{\Delta m_{ij}^2}{2p} L}\, \mathcal{U}_{\sigma i}^* \right|^2 .$$

An important example: 2-flavour case

$$|\nu_e\rangle \;=\; \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$
$$|\nu_\mu\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

$\Rightarrow$

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \equiv \begin{pmatrix} c & s \\ -s & c \end{pmatrix}$$

***

$\diamondsuit$ $$P_{\rm tr} = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4p}L\right)$$

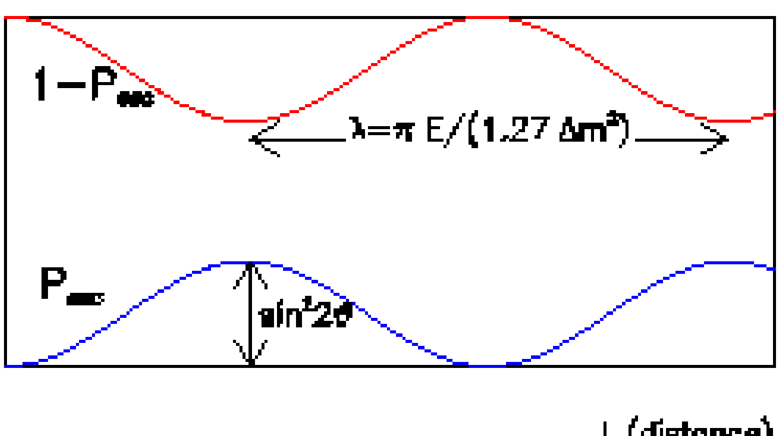
- $\diamond$ Problem: Derive this formula from the general expression for $P_{\alpha\beta}$.
- $\diamond$ Problem: Write this formula in the usual units, reinstating all factors of $\hbar$ and c. Find its classical and non-relativistic limits.

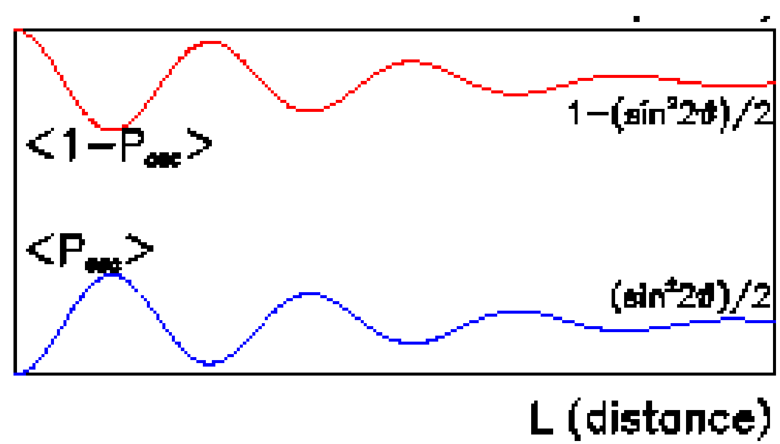
Oscillation amplitude: $\sin^2 2\theta$. Oscillation phase:

$$\frac{\Delta m^2}{4p}L = \pi\,\frac{L}{l_{\rm osc}}\,, \qquad\qquad l_{\rm osc} \equiv \frac{4\pi p}{\Delta m^2} \;\simeq\; 2.48\,{\rm m}\,\frac{p\,({\rm MeV})}{\Delta m^2\,({\rm eV}^2)}\,.$$

For large oscillation phase $\Rightarrow$ averaging regime (due to finite E-resolution of detectors and/or finite size of ν source/detector):

$$P_{\rm tr} = \sin^2 2\theta\,\sin^2\left(\frac{\Delta m^2}{4p}L\right) \quad\rightarrow\quad \frac{1}{2}\sin^2 2\theta$$





2f evolution equation in vacuum

For relativistic point-like ν's $(x \simeq t)$ the evolution equation in the flavour basis:

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = H_{\rm fl}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \left[U\begin{pmatrix}E_1 & 0\\ 0 & E_2\end{pmatrix}U^\dagger\right]\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

$E \simeq p + \frac{m^2}{2E} \quad \Rightarrow$

$$H_{\rm fl} \simeq \left[U\begin{pmatrix}p+\frac{m_1^2}{2E} & 0\\ 0 & p+\frac{m_2^2}{2E}\end{pmatrix}U^\dagger\right] \Longrightarrow \left[U\begin{pmatrix}-\frac{\Delta m_{21}^2}{4E} & 0\\ 0 & \frac{\Delta m_{21}^2}{4E}\end{pmatrix}U^\dagger\right]$$

N.B.: A term prop. to unit matrix can always be added to/subtracted from $H_{\rm fl}$. Problem: prove this!

2-flavor evolution equation:

$$\diamondsuit \quad i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

◇ Problem: find P_{tr} by solving the evolution equation with the initial contition $(1,0)^T$.

Oscillation parameters as characteristics of H_{fl}

For a 2×2 real symmetric matrix

$$\begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

the angle of rotation that diagonalizes it:

$$\tan 2\theta = \frac{2b}{c-a}.$$

Eigenvalues:

$$\lambda_{1,2} = \frac{a+c}{2} \mp \sqrt{\frac{(c-a)^2}{4} + b^2}\,.$$

$$\longleftrightarrow$$

Mixing angle θ: the angle of rotation that diagonalizes eff. Hamiltonian H_{fl}.

Eigenvalues of H_{fl}: $\mathcal{E}_{1,2} = \pm\frac{\Delta m^2}{4E}$.

Oscillation length: $l_{\text{osc}} = \frac{2\pi}{|\mathcal{E}_2 - \mathcal{E}_1|} v_g = \frac{4\pi p}{\Delta m^2}$

3f neutrino mixing and oscillations

General case of n flavours – parameter counting

$(n \times n)$ unitary mixing matrix $\tilde{U}$ $\Rightarrow$ n^2 real parameters:

$$\begin{pmatrix} n \\ 2 \end{pmatrix} = \frac{n(n-1)}{2} \quad \text{mixing angles}\,, \qquad \frac{n(n+1)}{2} \quad \text{phases}$$

For leptonic mixing matrix n phases can be absorbed into re-defenition of the phases of LH charged fields: $e_{\alpha L} \to e^{i\phi_\alpha} e_{\alpha L}$ (e.g., 1st line of $\tilde{U}$ can be made real). This can be compensated in the mass term of charged leptons by rephasing $e_{\alpha R} \to e^{i\phi_\alpha} e_{\alpha R}$, so that $\bar{e}_{\alpha L} e_{\alpha R} = inv$.

Similarly, for Dirac neutrinos phases of one column can be fixed by absorbing $n-1$ phases into a redefinition of ν_{iL} (RH neutrino fields can be rephased analogously, so that $\bar{\nu}_{iL}\nu_{iR} = inv.$) $\Rightarrow$ In Dirac ν case $n + (n-1) = 2n - 1$ phases are unphysical – can be rotated away by redefining charged lepton and neutrino fields.

N.B.: Kinetic terms of e_L, e_R and ν_L, ν_R are also invariant w.r.t. rephasing.!

Physical phases

Number of physical phases:

$$\frac{n(n+1)}{2} - (2n-1) \;=\; \frac{(n-1)(n-2)}{2}.$$

Phys. phases responsible for CP violation! $\Rightarrow$ No Dirac-type CPV for $n<3$.

In Majorana case:

$$\mathcal{L}_m \propto \nu_L^T C \nu_L \;+\; h.c.$$

Rephasing of ν_L is not possible (cannot be compensated in $\mathcal{L}_m$)

Only n phases can be removed from $\tilde{U}$ (by redefinition of $e_{\alpha L}$ fields) $\Rightarrow$ In addition to Dirac-type phases there are $(n-1)$ physical Majorana-type CP-violating phases.

Majorana phases do not affect oscillations

Majorana-type phases can be factored out in the mixing matrix:

$$\tilde{U} = UK$$

U contains Dirac-type phases, K – Majorana-type phases σ_i:

$$K = \mathrm{diag}(1\,, e^{i\sigma_1}\,, ...\,, e^{i\sigma_{n-1}})$$

Neutrino evolution equation: $i\frac{d}{dt}\,\nu = H_{\rm eff}\,\nu$

$$H_{\rm eff} \;= UK \begin{pmatrix} E_1 & & & \\ & E_2 & & \\ & & . & \\ & & & . \end{pmatrix} K^\dagger U^\dagger \;= U \begin{pmatrix} E_1 & & & \\ & E_2 & & \\ & & . & \\ & & & . \end{pmatrix} U^\dagger$$

Does not depend on the matrix of Majorana $\not{C}\not{P}$ phases K $\Rightarrow$
ν oscillations are insensitive to Majorana phases. Also true for osc. in matter.

3f oscillation parameters

Three neutrino species $(\nu_e, \nu_\mu, \nu_\tau)$ – linear superpositions of three mass eigenstates (ν_1, ν_2, ν_3). Mixing matrix U – 3×3 unitary matrix. Depends on 3 mixing angles and one Dirac-type $\not{C}\not{P}$ phase $\delta_{\rm CP}$.

Experiment: 2 mixing angles large (in the standard parameterization – θ_{12} and θ_{23}), one (θ_{13}) is relatively small.

Three neutrinos species – 2 independent mass squared differences, e.g. Δm^2_{21} and Δm^2_{31}.

$$\Delta m^2_{21} \ll \Delta m^2_{31}$$

What do we know about neutrino parameters

From atmsopheric and LBL accelerator neutrino experiments:

$$\diamondsuit \quad \Delta m_{31}^2 \simeq 2.5 \times 10^{-3}\,\mathrm{eV}^2\,, \qquad \theta_{23} \sim 45^\circ$$

From solar neutrino experiments and KamLAND:

$$\diamondsuit \quad \Delta m_{21}^2 \simeq 7.5 \times 10^{-5}\,\mathrm{eV}^2\,, \qquad \theta_{12} \simeq 33^\circ$$

From T2K + Double Chooz, Daya Bay and Reno reactor neutrino experiments:

$$\diamondsuit \quad \theta_{13} \simeq 9^\circ \quad (\text{previosly from Chooz} \lesssim 12^\circ)$$

CP-violating phase $\delta_{\rm CP}$ practically unconstrained at the moment.

Leptonic mixing and 3f osc. in vacuum

Relation between flavour and mass eigenstates:

$$\nu_\alpha = \sum_{i=1}^{3} U_{\alpha i}\, \nu_i$$

ν_α – fields of flavour eigenstates, ν_i – of mass eigenstates.

3f mixing matrix:

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Leptonic mixing and 3f osc. in vacuum

Relation btween flavour and mass eigenstates:

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U_{\alpha i}^* \, |\nu_i\rangle$$

Oscillation probability in vacuum:

$$P(\nu_\alpha \to \nu_\beta; L) \; = \; \left| \sum_{i=1}^{3} U_{\beta i} \, e^{-i \frac{\Delta m_{i1}^2}{2p} L} \, U_{\alpha i}^* \right|^2 \; = \; \left| \left[U \, e^{-i \frac{\Delta m^2}{2p} L} \, U^\dagger \right]_{\beta\alpha} \right|^2$$

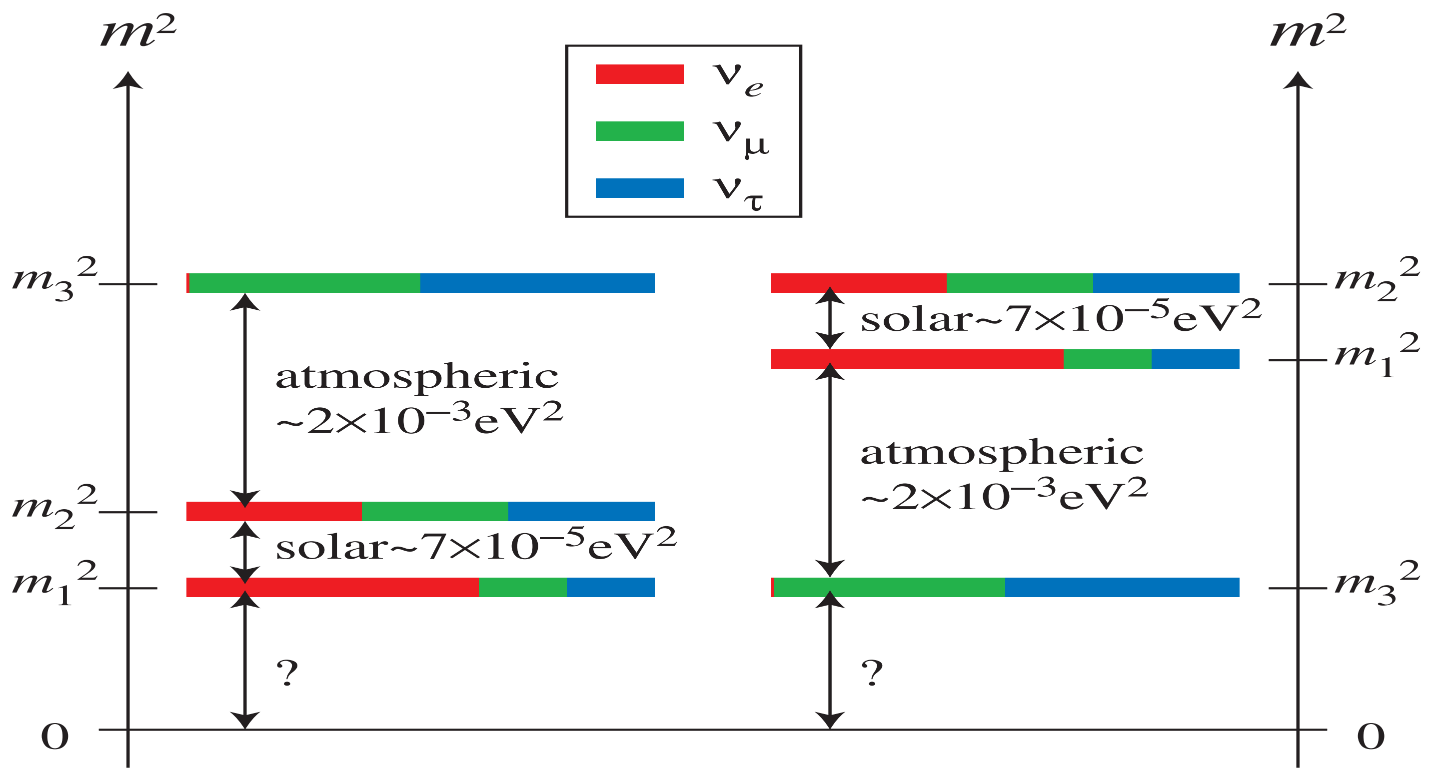
3f mixing matrix in the standard parameterization ($c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$):

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{\rm CP}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{\rm CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= O_{23} \, (\Gamma_\delta \, O_{13} \, \Gamma_\delta^\dagger) \, O_{12} \, , \qquad \qquad \Gamma_\delta \equiv {\rm diag}(1\,, \; 1\,, \; e^{i\delta_{\rm CP}})$$

3f neutrino mixing

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & c_{13}c_{23} \end{pmatrix}$$



2f oscillations: physical ranges of parameters

$$|\nu_e\rangle \;=\; \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$

$$|\nu_\mu\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

In general, $\theta \in [0, 2\pi]$. But: there are transformations that leave ν mixing formulas unchanged:

- $\theta \to \theta + \pi, \quad |\nu_1\rangle \to -|\nu_1\rangle, \quad |\nu_2\rangle \to -|\nu_2\rangle \quad \Rightarrow \quad \theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$
- $\theta \to -\theta, \quad |\nu_2\rangle \to -|\nu_2\rangle, \quad |\nu_\mu\rangle \to -|\nu_\mu\rangle \quad \Rightarrow \quad \theta \in [0, \frac{\pi}{2}]$
- $\theta \to \frac{\pi}{2} - \theta, \quad |\nu_1\rangle \leftrightarrow |\nu_2\rangle, \quad |\nu_\mu\rangle \to -|\nu_\mu\rangle \quad \Rightarrow \quad \Delta m^2 \to -\Delta m^2$

One can always choose $\Delta m^2 > 0$ by choosing appropriately θ within $[0, \frac{\pi}{2}]$.

For vacuum oscillations: $P_{\rm tr}$, $P_{\rm surv}$ depend only on $\sin^2 2\theta$ $\Rightarrow$ one can choose θ to be in $[0, \frac{\pi}{4}]$. Not true for oscillations in matter!

Similar considerations in the 3f case: all $\theta_{ij} \in [0, \frac{\pi}{2}]$; $\delta_{\rm CP} \in [0, 2\pi]$.

$\not{CP}$ and $\not{T}$ in ν osc. in vacuum

$\nu_a \to \nu_b$ oscillation probability:

$$\diamondsuit \quad P(\nu_\alpha, t_0 \to \nu_\beta; t) = \left| \sum_i U_{\beta i}\, e^{-i\frac{\Delta m_{i1}^2}{2E}(t-t_0)}\, U_{\alpha i}^* \right|^2$$

- CP: $\nu_{\alpha,\beta} \leftrightarrow \bar{\nu}_{\alpha,\beta} \quad \Rightarrow \quad U_{\alpha i} \to U_{\alpha i}^* \quad (\{\delta_{\rm CP}\} \to -\{\delta_{\rm CP}\})$
- T: $t \rightleftarrows t_0 \quad \Leftrightarrow \quad \nu_\alpha \leftrightarrow \nu_\beta$

$$\Rightarrow \quad U_{\alpha i} \to U_{\alpha i}^* \quad (\{\delta_{\rm CP}\} \to -\{\delta_{\rm CP}\})$$

T-reversed oscillations ("backwards in time") $\Leftrightarrow$ oscillations between interchanged initial and final flavours

$\diamond$ *$\not{CP}$ and $\not{T}$ – absent in 2f case, pure $N \geq 3$f effects!*

$\diamond$ No $\not{CP}$ and $\not{T}$ for survival probabilities $(b = a)$.

CP and T violation in vacuum – contd.

- CPT: $\nu_{\alpha,\beta} \leftrightarrow \bar{\nu}_{\alpha,\beta}$ & $t \rightleftarrows t_0$ $(\nu_\alpha \leftrightarrow \nu_\beta)$

  ◇ $P(\nu_\alpha \to \nu_\beta) \to P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$

The standard formula for $P_{\alpha\beta}$ in vacuum is CPT invariant!

$\not{CP} \;\Leftrightarrow\; \not{T}$ – *consequence of CPT*

Measures of $\not{CP}$ and $\not{T}$ – probability differences:

$$\Delta P^{\rm CP}_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

$$\Delta P^{\rm T}_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)$$

From CPT:

◇ $\Delta P^{\rm CP}_{\alpha\beta} = \Delta P^{\rm T}_{\alpha\beta}\,;$ $\quad\quad \Delta P^{\rm CP}_{\alpha\alpha} = 0$

3f case

One $\not{CP}$ Dirac-type phase $\delta_{\rm CP}$ *(Majorana phases do not affect ν oscillations!)* $\Rightarrow$ one $\not{CP}$ and $\not{T}$ observable:

$$\diamond \quad \Delta P_{e\mu}^{\rm CP} = \Delta P_{\mu\tau}^{\rm CP} = \Delta P_{\tau e}^{\rm CP} \equiv \Delta P$$

$$\Delta P = -\, 4 s_{12}\, c_{12}\, s_{13}\, c_{13}^2\, s_{23}\, c_{23}\, \sin\delta_{\rm CP}$$
$$\times \left[\sin\left(\frac{\Delta m_{12}^2}{2E}\,L\right) + \sin\left(\frac{\Delta m_{23}^2}{2E}\,L\right) + \sin\left(\frac{\Delta m_{31}^2}{2E}\,L\right)\right]$$

Vanishes when

- At least one $\Delta m_{ij}^2 = 0$
- At least one $\theta_{ij} = 0$ or $90°$
- $\delta_{\rm CP} = 0$ or $180°$
- In the averaging regime
- In the limit $L \to 0$ (as L^3)

Very difficult to observe!

Small parameters

Approximate formulas for probabilities can be obtained using expansions in small parameters:

$$(1) \quad \frac{\Delta m^2_\odot}{\Delta m^2_{\rm atm}} = \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \sim 1/30$$

$$(2) \quad |U_{e3}| = |\sin\theta_{13}| \sim 0.16$$

In the limits $\Delta m^2_{21} = 0$ or $U_{e3} = 0$ – probabilities take an effective 2f form.

(N.B.: $P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha)$)

Neutrino oscillations in matter

The MSW effect (Wolfenstein, 1978; Mikheyev & Smirnov, 1985)

Matter can change the pattern of neutrino oscillations drastically

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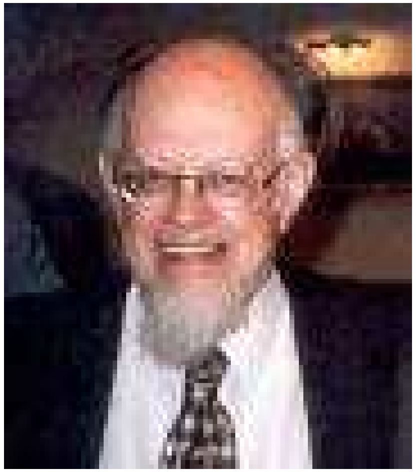


How can matter affect neutrino oscillations?

For $E \sim 1$ MeV neutrinos mean free path in lead is ~ 1 l.y. !

$$\diamondsuit \quad \text{mean free path} = \langle \sigma n v \rangle^{-1},$$

For incoherent processes (capture, finite-angle scattering)

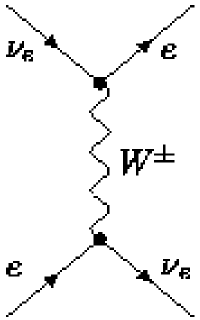
$$\diamondsuit \quad \sigma \propto (G_F)^2$$

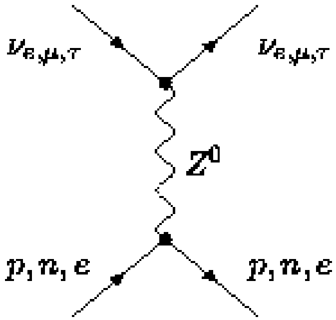
Coherent forward scattering: effects $\sim G_F$, i.e. much stronger!

Lead to effective potentials for neutrinos in matter $\sim G_F N$.

Neutrino oscillations in matter

Coherent forward scattering on the particles in matter





$$V_e^{\rm CC} \equiv V = \sqrt{2}\, G_F\, N_e$$

2f neutrino evolution equation $(x \simeq t)$:

$$\diamondsuit \quad i\frac{d}{dx}\begin{pmatrix}\nu_e \\ \nu_\mu\end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + V(x) & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}\begin{pmatrix}\nu_e \\ \nu_\mu\end{pmatrix}$$

For antineutrinos $V(x) \to -V(x)$.

Neutrino potential in matter

At low neutrino energies the effective Hamiltonian CC interactions

$$H_{\rm CC} = \frac{G_F}{\sqrt{2}}\,[\bar{e}\gamma_\mu(1-\gamma_5)\nu_e][\bar{\nu}_e\gamma^\mu(1-\gamma_5)e] = \frac{G_F}{\sqrt{2}}\,[\bar{e}\gamma_\mu(1-\gamma_5)e][\bar{\nu}_e\gamma^\mu(1-\gamma_5)\nu_e]\,,$$

(Fierz transformation used). To obtain the matter-induced potential for ν_e fix the variables corresponding to ν_e and integrate over the electron variables:

$$H_{\rm eff}(\nu_e) = \langle H_{\rm CC}\rangle_{electron} \equiv \bar{\nu}_e V_e \nu_e\,.$$

We have:

$$\langle\bar{e}\gamma_0 e\rangle = \langle e^\dagger e\rangle = N_e\,, \qquad \langle\bar{e}\boldsymbol{\gamma} e\rangle = \langle\mathbf{v}_e\rangle\,, \quad \langle\bar{e}\gamma_0\gamma_5 e\rangle = \langle\frac{\boldsymbol{\sigma}_e\mathbf{p}_e}{E_e}\rangle\,, \quad \langle\bar{e}\boldsymbol{\gamma}\gamma_5 e\rangle = \langle\boldsymbol{\sigma}_e\rangle\,,$$

For unpolarized medium of zero total momentum only the first term survives $\Rightarrow$

$$\diamondsuit \qquad \boxed{(V_e)_{\rm CC} \equiv V = \sqrt{2}\,G_F N_e}\,.$$

Oscillations in matter of constant density

$\diamondsuit$ $$P_{\rm tr} = \sin^2 2\theta_m \sin^2\left(\pi \frac{L}{l_{\rm osc}^m}\right)$$

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Osc. length: $l_{\rm osc}^m = l_{\rm osc}(\sin 2\theta_m / \sin 2\theta)$.

Oscillations in matter of constant density

♢ $$P_{\rm tr} = \sin^2 2\theta_m \sin^2\left(\pi \frac{L}{l^m_{\rm osc}}\right)$$

♢ $$\sin^2 2\theta_m = \frac{\sin^2 2\theta \cdot (\frac{\Delta m^2}{2E})^2}{[\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}G_F N_e]^2 + (\frac{\Delta m^2}{2E})^2 \sin^2 2\theta}$$

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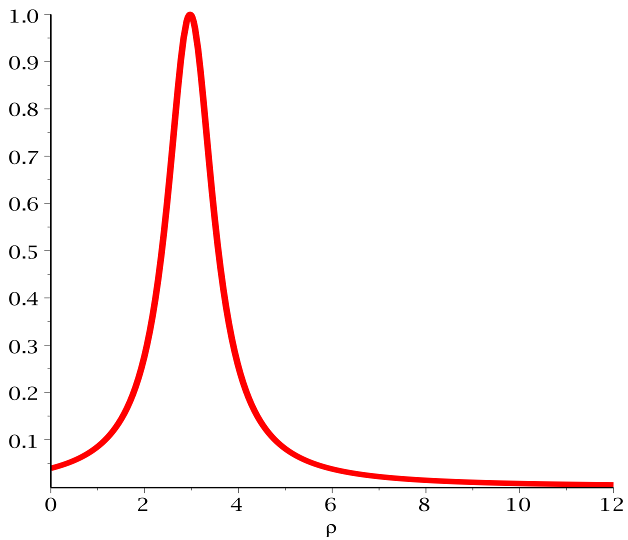

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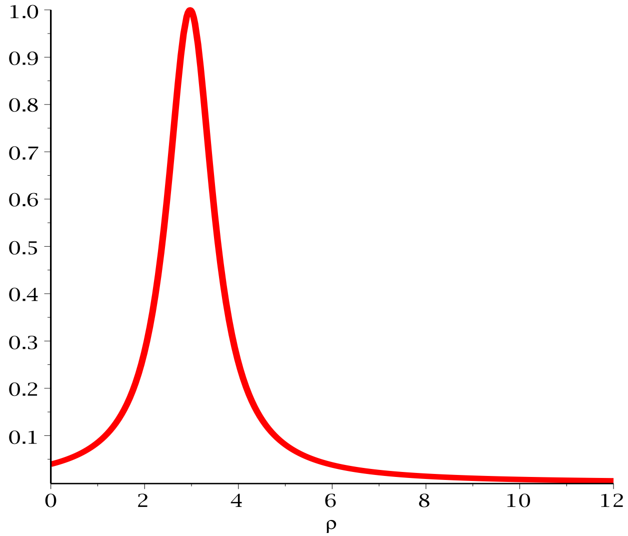


MSW resonance:

$\diamondsuit$ $$\sqrt{2}G_F N_e = \frac{\Delta m^2}{2E}\cos 2\theta \quad \Rightarrow$$

$$\theta_m = 45^\circ$$

independently of θ !

$$(l_{\rm osc}^m)_{\rm res} = l_{\rm osc}/\sin 2\theta.$$

The MSW resonance condition

$\diamondsuit$ $$\boxed{\pm\sqrt{2}G_F N_e = \frac{\Delta m^2}{2E}\cos 2\theta}$$

For given E yields $(N_e)_{res}$ (or vice versa).

For neutrinos LHS > 0 $\Rightarrow$ can only be satisfied if RHS > 0:

$$\Delta m^2 \cos 2\theta \;=\; (m_2^2 - m_1^2)(\cos^2\theta - \sin^2\theta) \;>\; 0$$

$\Rightarrow$ If ν_2 is heavier than ν_1, one needs $\cos^2\theta \;>\; \sin^2\theta$ and vice versa.

$\Leftrightarrow$ Lighter mass eigenstate must have larger ν_e component.

If one chooses $\cos 2\theta > 0$, the resonance for neutrinos occurs when

$$\Delta m_{21}^2 \;>\; 0.$$

For $\Delta m_{21}^2 \;<\; 0$ $\Rightarrow$ res. takes place for antineutrinos.

Matter of varying density

At any point x eff. Hamiltonian $H_m(x)$ can be diagonalized by unitary transf. $U_m = U_m(x)$ with the mixing angle $\theta_m = \theta_m(x)$:

$$\diamondsuit \qquad \tan 2\theta_m(x) = \frac{\sin 2\theta \cdot \frac{\Delta m^2}{2E}}{\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}G_F N_e(x)}$$

In general osc. probability cannot be found in closed form.

$|\nu_{1m}\rangle$, $|\nu_{2m}\rangle$ – local (at point x) eigenstates of H_m (matter eigenstates):

$$|\nu_{1m}\rangle = \cos\theta_m\,|\nu_e\rangle - \sin\theta_m\,|\nu_\mu\rangle$$
$$|\nu_{2m}\rangle = \sin\theta_m\,|\nu_e\rangle + \cos\theta_m\,|\nu_\mu\rangle$$

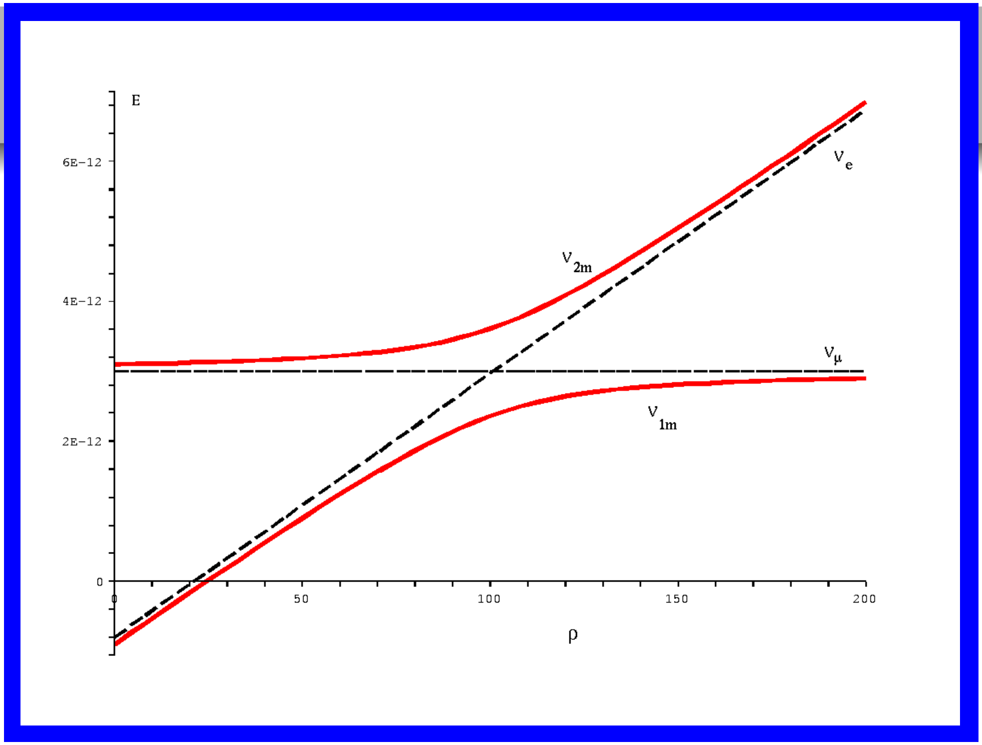
$N_e \gg (N_e)_{\rm res}: \quad \theta_m \approx 90^\circ$

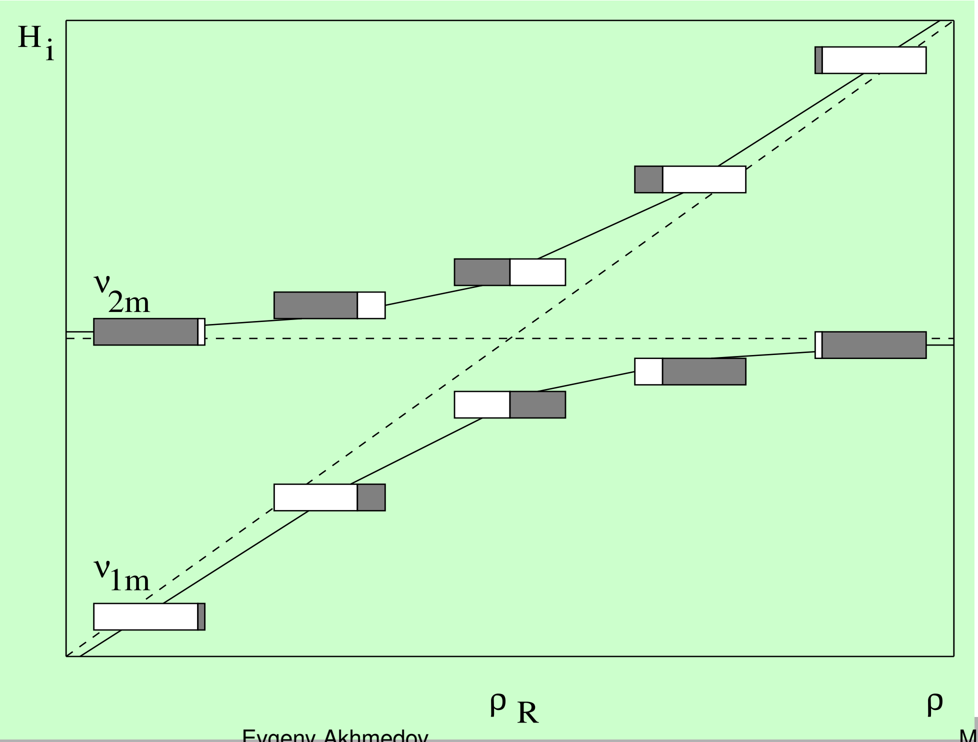
$N_e = (N_e)_{\rm res}: \quad \theta_m = 45^\circ$

$N_e \ll (N_e)_{\rm res}: \quad \theta_m \approx \theta$

In the adiabatic regime: ν_{1m} and ν_{2m} do not go into each other $\Rightarrow$ ν_e born at high density will remain ν_e at small N_e with probability $\sin^2\theta$ and go to ν_μ with probability $\cos^2\theta$ independently of L !

Adiabatic flavour conversion





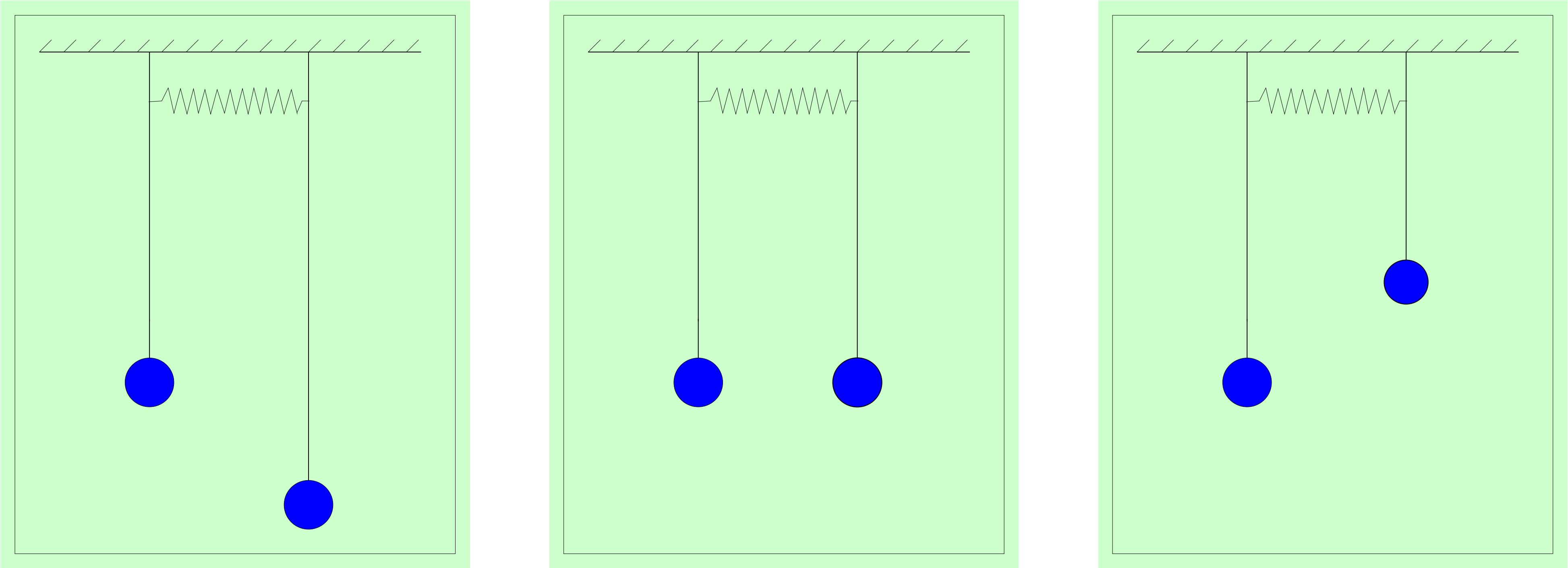
Adiabaticity: slow density change along the neutrino path

$$\frac{\sin^2 2\theta}{\cos 2\theta}\frac{\Delta m^2}{2E} L_\rho \gg 1$$

L_ρ – electron density scale hight:

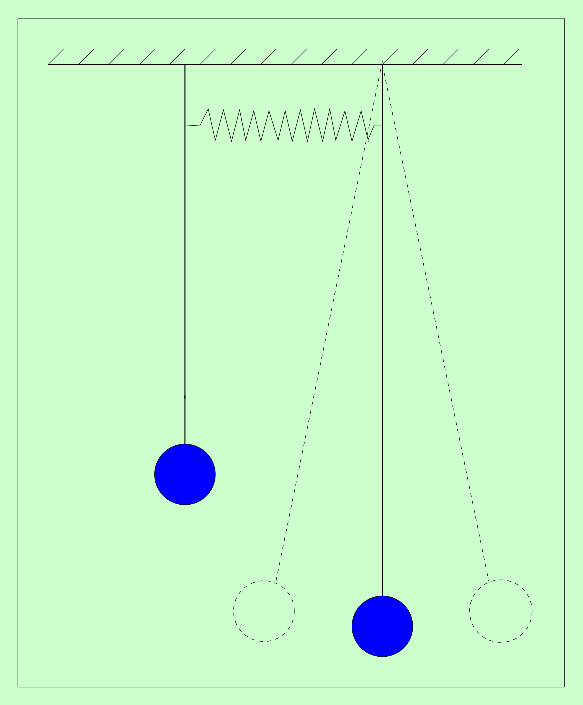
$$L_\rho = \left|\frac{1}{N_e}\frac{dN_e}{dx}\right|^{-1}$$

Analogy: Two coupled pendula



Mechanical model of the MSW effect

Analogy: Two coupled pendula



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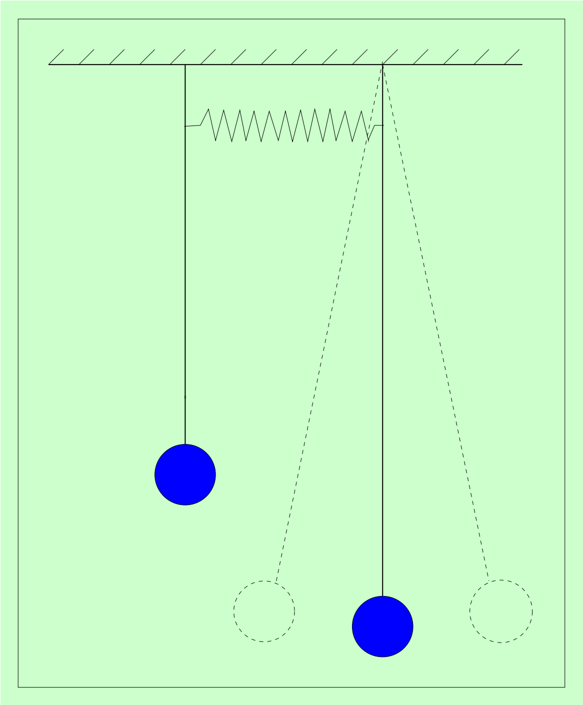

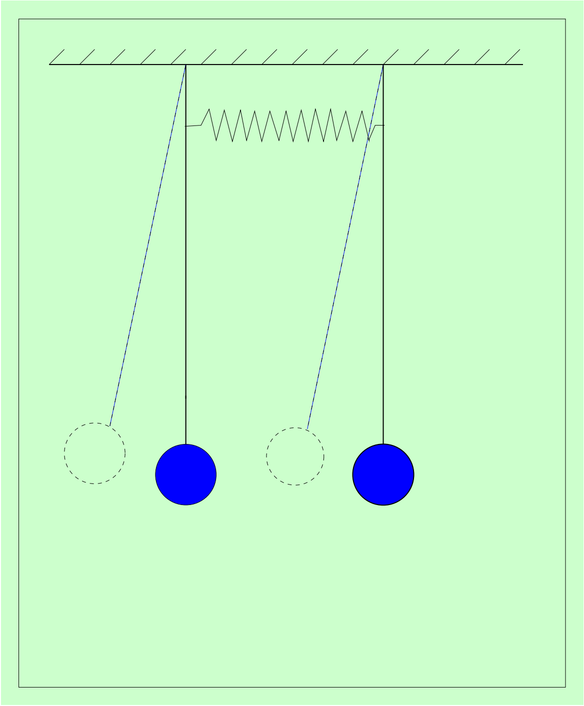

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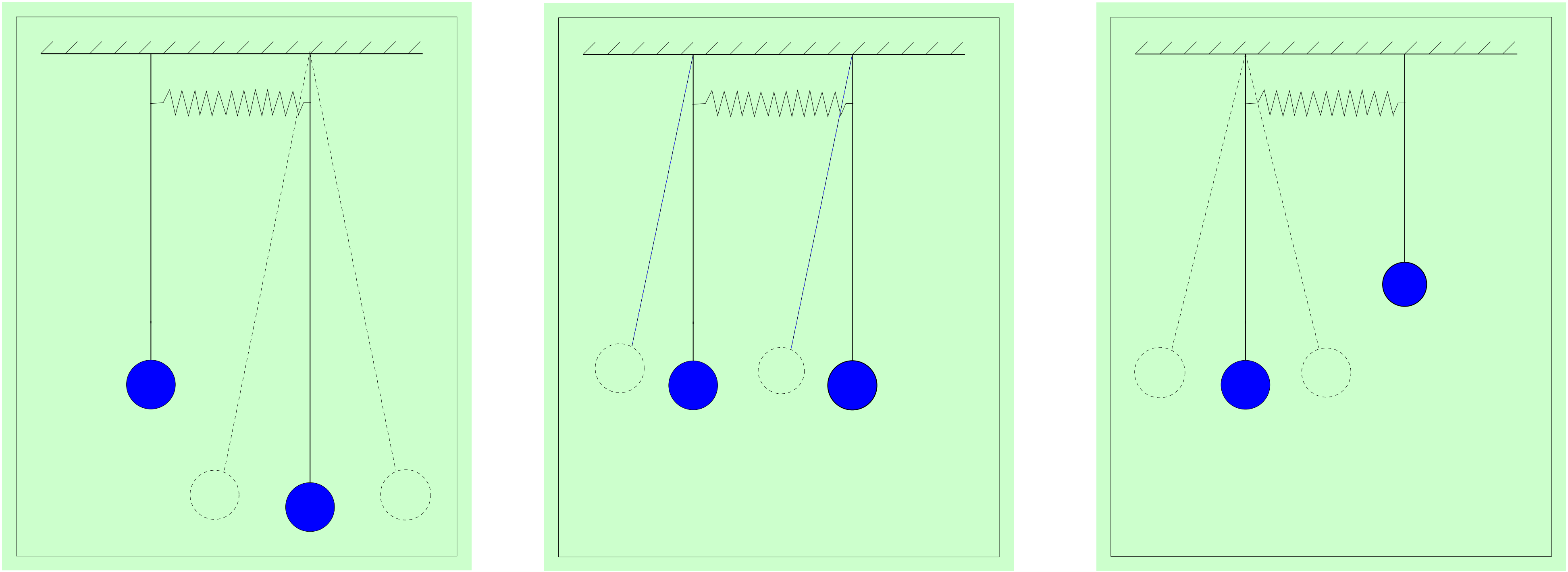


Mechanical model of the MSW effect

Evolution of matter eigenstates

Flavour states in terms of local matter eigenstates:

$$\diamondsuit \qquad |\nu_{\rm fl}\rangle \; = U_m^\dagger(x)\,|\nu_{\rm matt}\rangle$$

Evolution equation: $i\frac{d}{dx}|\nu_{\rm fl}\rangle = H_{\rm fl}^m(x)|\nu_{\rm fl}\rangle \qquad \Rightarrow$

$$\diamondsuit \qquad i\frac{d}{dx}|\nu_{\rm matt}\rangle \;=\; \left[U_m H_{\rm fl}^m U_m^\dagger \;-\; iU_m(U_m^\dagger)'\right]|\nu_{\rm matt}\rangle$$

For the 2f case: $\quad U_m \;=\; \begin{pmatrix} c_m & s_m \\ -s_m & c_m \end{pmatrix} \qquad \Rightarrow$

$$\diamondsuit \qquad i\frac{d}{dx}\begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix} = \begin{pmatrix} \mathcal{E}_1(x) & -i\theta'_m(x) \\ i\theta'_m(x) & \mathcal{E}_2(x) \end{pmatrix}\begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix}$$

$\mathcal{E}_1(x)$, $\mathcal{E}_2(x)$ – local eigenvals. of $H_{\rm fl}^m$ at a given x.

Adiabatic regime

$\diamondsuit$ $$|\mathcal{E}_2(x) - \mathcal{E}_1(x)| = \sqrt{\left[\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}G_F N_e(x)\right]^2 + \left(\frac{\Delta m^2}{2E}\right)^2 \sin^2 2\theta}$$

If $|\mathcal{E}_2 - \mathcal{E}_1| \gg 2|\theta'_m|$ (adiabatic regime) $\Rightarrow$ matter eigenstates ν_{1m} and ν_{2m} evolve independently. Adiabaticity condition:

$$\frac{|\mathcal{E}_2 - \mathcal{E}_1|_{\rm res}}{2|\theta'_m|} = \frac{\Delta m^2}{2E}\frac{\sin^2 2\theta}{\cos 2\theta} L_\rho \gg 1$$

$L_\rho \equiv |N'_e/N_e|^{-1}$ – scale height of electron number density. Let $|\nu(0)\rangle = |\nu_e\rangle$:

$$|\nu(0)\rangle = c_m(0)|\nu_{1m}\rangle + s_m(0)|\nu_{2m}\rangle$$

In the adiabatic regime:

$\diamondsuit$ $$|\nu(x)\rangle = c_m(0)\, e^{-i\int_0^x \mathcal{E}_1(x')dx'}|\nu_{1m}\rangle + s_m(0)\, e^{-i\int_0^x \mathcal{E}_2(x')dx'}|\nu_{2m}\rangle$$

Adiabatic regime

At the point x the state $|\nu_\mu\rangle$ can be expanded as

$$|\nu_\mu\rangle \;=\; -\,s_m(x)|\nu_{1m}\rangle + c_m(x)|\nu_{2m}\rangle$$

Transition probability: $P_{\rm tr} = |\langle\nu_\mu|\nu(x)\rangle|^2 \quad \Rightarrow$

$\diamondsuit$ $$\boxed{P_{\rm tr} \;=\; \frac{1}{2} - \frac{1}{2}\cos 2\theta_i \cos 2\theta_f - \frac{1}{2}\sin 2\theta_i \sin 2\theta_f \sin\Phi}$$

$$\theta_i = \theta_m(0), \qquad \theta_f = \theta_m(x), \qquad \Phi = \int_0^x (\mathcal{E}_1 - \mathcal{E}_2)\, dx'$$

◇ Problem: Derive this expression.

If $N_e(0) \gg (N_e)_{\rm res}$ or $\theta_f \ll 1$: the 3rd term can be neglected (also if $\Phi \gg 1$ and averaging is performed) $\Rightarrow$ $P_{\rm tr}$ depends only on θ_i and θ_f.

In the case $N_e(0) \gg (N_e)_{\rm res}$, $N_e(x) \ll (N_e)_{\rm res}$ (i.e. $\theta_i \simeq 90^\circ$, $\theta_f \simeq \theta$)

$\Rightarrow \quad P_{\rm tr} = \cos^2\theta, \quad P_{\rm surv} = \sin^2\theta.$

Violation of adiabaticity

Possible adiabaticty violation can be taken into account.

E.g. in the averaging regime (Parke, 1986):

$$\diamondsuit \quad \boxed{\overline{P}_{\rm tr} = \frac{1}{2} - \frac{1}{2}\cos 2\theta_i \cos 2\theta_f \,(1 - 2P')}$$

P' – probability of $\nu_{1m} \leftrightarrow \nu_{2m}$ transitions between points 0 and x. In the Landau-Zener approximation: $P' \simeq e^{-\frac{\pi}{2}\gamma}$ where γ is the adiab. parameter.

In the extreme non-adaiabatic regime:

$$\diamondsuit \quad i\frac{d}{dx}\begin{pmatrix}\nu_{1m}\\ \nu_{2m}\end{pmatrix} = \begin{pmatrix}0 & -i\theta'_m(x)\\ i\theta'_m(x) & 0\end{pmatrix}\begin{pmatrix}\nu_{1m}\\ \nu_{2m}\end{pmatrix}$$

Can be solved exactly by $x \to \tau = \theta_m(x)$, $\frac{d}{d\tau} = \frac{1}{\theta'_m(x)}\frac{d}{dx}$ $\Rightarrow$

$$\diamondsuit \quad P' = \sin^2(\theta_i - \theta_f)$$

◇ Problem: Derive this expression.

Vacuum oscillation limits

1. The mixing angle and osc. length in matter θ_m, $l_{\rm osc}^m$ go to θ, $l_{\rm osc}$ when

$$V = \sqrt{2}G_F N_e \ll \frac{\Delta m^2}{2E}$$

$\Rightarrow$ $P_{\rm osc} \to P_{\rm osc}^{vac}$. In terms of convenient parameters:

$$\sqrt{2}G_F N_e \simeq 7.63\times 10^{-14}\rho({\rm g/cm^3}){\rm Y_e}\ \ {\rm eV}\,, \qquad {\rm Y_e} = \frac{\rm N_e}{\rm N_p + N_n}$$

2. In general (even in the case $V \gg \Delta m^2/2E$) the vacuum oscsill. probability is recovered in the short baseline limit. In matter of constant density:

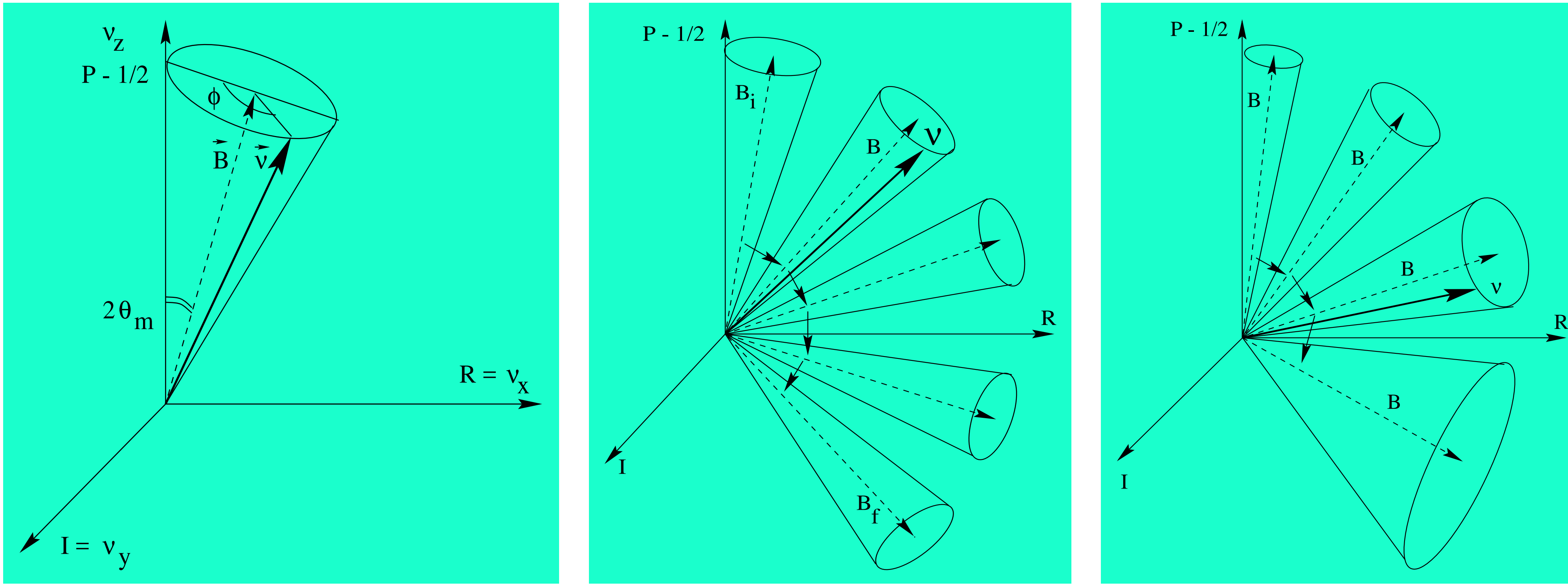
$$P_{\rm tr} = \sin^2 2\theta_m \sin^2(\omega L) = \frac{\sin^2 2\theta\cdot\left(\frac{\Delta m^2}{4E}\right)^2}{\omega^2}\sin^2(\omega L)\,, \qquad \omega \equiv \frac{1}{2}|\mathcal{E}_2 - \mathcal{E}_1|.$$

For $\omega L \ll 1$:

$$P_{\rm tr} \simeq \sin^2 2\theta\cdot\left(\frac{\Delta m^2}{4E}L\right)^2 = P_{\rm tr}^{vac} \ \ \text{in short } L \text{ limit}\,.$$

Problem (*): Does this hold also for $N_e \neq const.$?

Analogy: Spin precession in a magnetic field



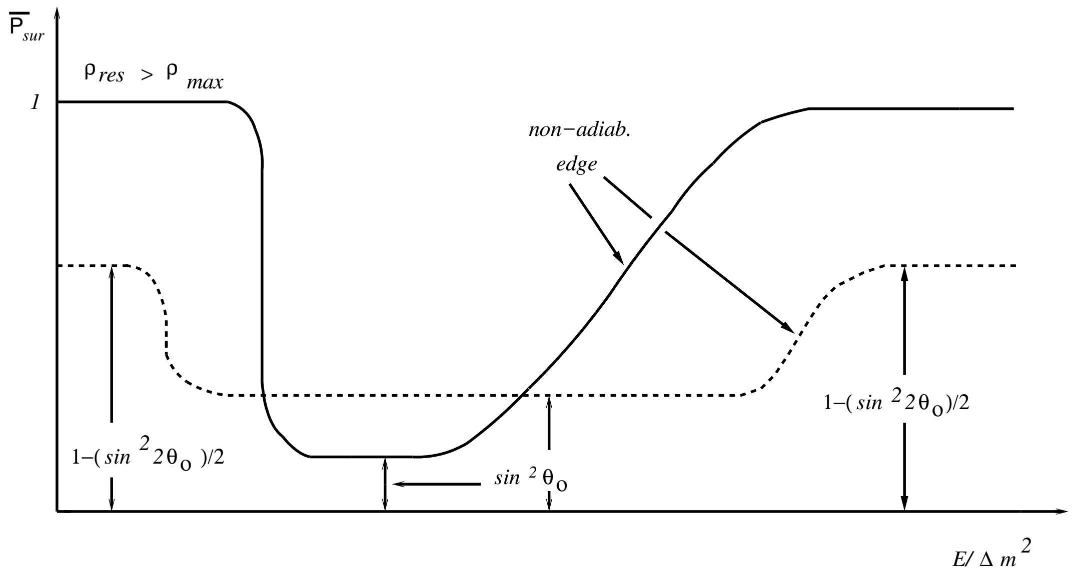
$$\frac{d\vec{S}}{dt} = 2(\vec{B} \times \vec{S})$$

$$\vec{S} = \{\mathrm{Re}(\nu_e^* \nu_\mu)\,,\ \mathrm{Im}(\nu_e^* \nu_\mu)\,,\ \nu_e^* \nu_e - 1/2\}$$

$$\vec{B} = \{(\Delta m^2/4E)\sin 2\theta\,,\ \ 0\,,\ \ V/2 - (\Delta m^2/4E)\cos 2\theta\}$$

MSW effect and solar neutrinos

The survival probability for solar ν_e:



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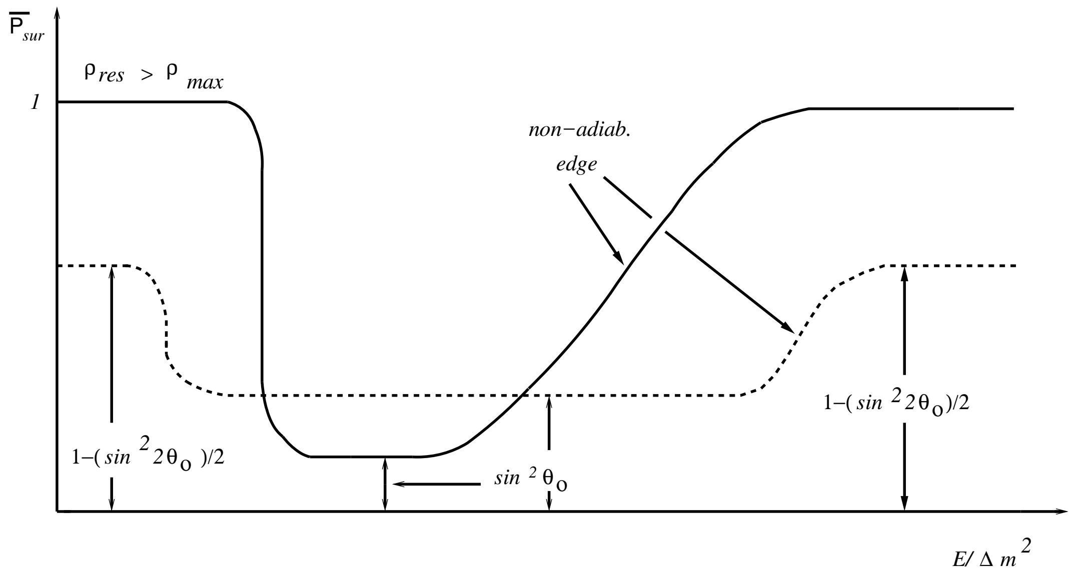

Day-night effect: the probability of finding a solar ν_e after it traverses the Earth

$$P_{SE} = \bar{P}_S + \frac{1 - 2\bar{P}_S}{\cos 2\theta_0}\,(P_{2e} - \sin^2\theta_0)\,.$$

Here: $P_{2e} = P(\nu_2 \to \nu_e)$ – probability of oscillations of the second mass eigenstate into electron neutrino inside the Earth.

How is it obtained?

Neutrino state at the surface of the Sun:

$$|\nu_\odot\rangle \;=\; a_1|\nu_1\rangle \;+\; a_2\, e^{i\phi_S}\, |\nu_2\rangle \qquad\qquad (a_{1,2} \;-\; \text{real})$$

Averaged ν_e survival probability in the Sun:

$$\overline{P}_S = \overline{|\langle\nu_e|\nu_\odot\rangle|^2} = a_1^2 \cos^2\theta + a_2^2 \sin^2\theta \quad \Rightarrow$$

$$a_2^2 \;=\; 1 - a_1^2 \;=\; \frac{\cos^2\theta - \overline{P}_S}{\cos 2\theta}$$

Solar neutrinos arrive at the Earth as an incoherent sum of ν_1 and ν_2 $\Rightarrow$

$$P_{SE} = a_1^2 P_{1e} + a_2^2 P_{2e} = a_1^2 (1 - P_{2e}) + a_2^2 P_{2e} = \bar{P}_S + \frac{1 - 2\bar{P}_S}{\cos 2\theta}\,(P_{2e} - \sin^2\theta)\,.$$

In vacuum $P_{2e} = \sin^2\theta \quad \Rightarrow \quad P_{SE} = \overline{P}_S$.

How is it obtained?

For matter of constant density:

$$\diamondsuit \quad P_{2e} - \sin^2\theta \;=\; \frac{V\delta}{4\omega^2}\sin^2 2\theta\,\sin^2(\omega L)$$

Here:

$$\delta \;\equiv\; \frac{\Delta m^2_{21}}{2E}\,, \qquad \theta \;=\; \theta_{12}\,. \qquad \omega = \sqrt{(\cos 2\theta\cdot\delta - V)^2 + \delta^2\sin^2 2\theta^2}$$

Pre-sine^2 factor in $P_{2e} - \sin^2\theta$ reaches its max. at $V = \delta$ (not at $V = \delta\cdot\cos 2\theta$ which would correspond to the MSW resonance!)

$$(P_{2e} - \sin^2\theta)_{max.\ ampl.} \;=\; \cos^2\theta\,\sin^2(\sin\theta\cdot\delta\cdot L)$$

In the (realistic) case $V \ll \delta$:

$$\diamondsuit \quad P_{2e} - \sin^2\theta \;=\; \frac{V}{\delta}\sin^2 2\theta\,\sin^2\left(\frac{1}{2}\delta\cdot L\right)$$

Another possible matter effect

Parametric resonance in neutrino oscillations

Parametric resonance in oscillating systems with varying parameters: occurs when the rate of the parameter change is correlated in a certain way with the values of the parameters themselves

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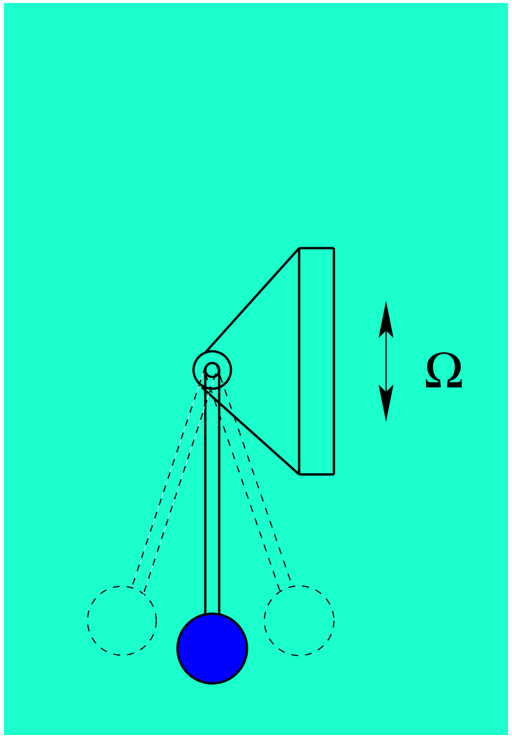
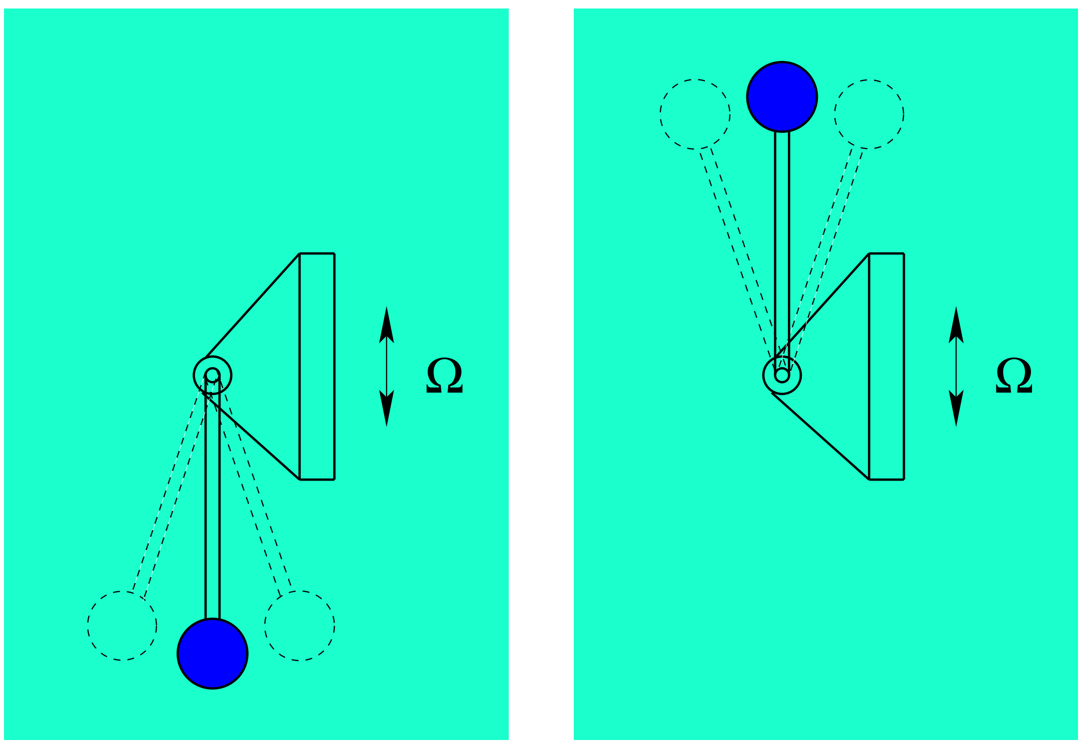


Parametric resonance in neutrino oscillations

Parametric resonance in oscillating systems with varying parameters: occurs when the rate of the parameter change is correlated in a certain way with the values of the parameters themselves



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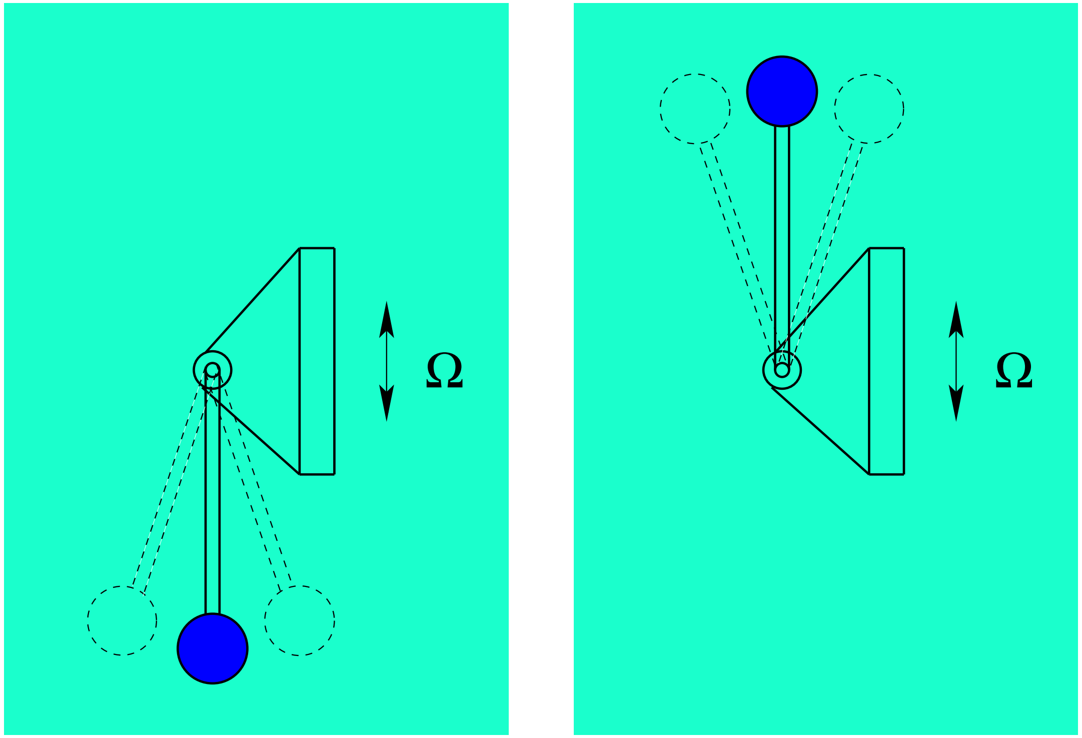


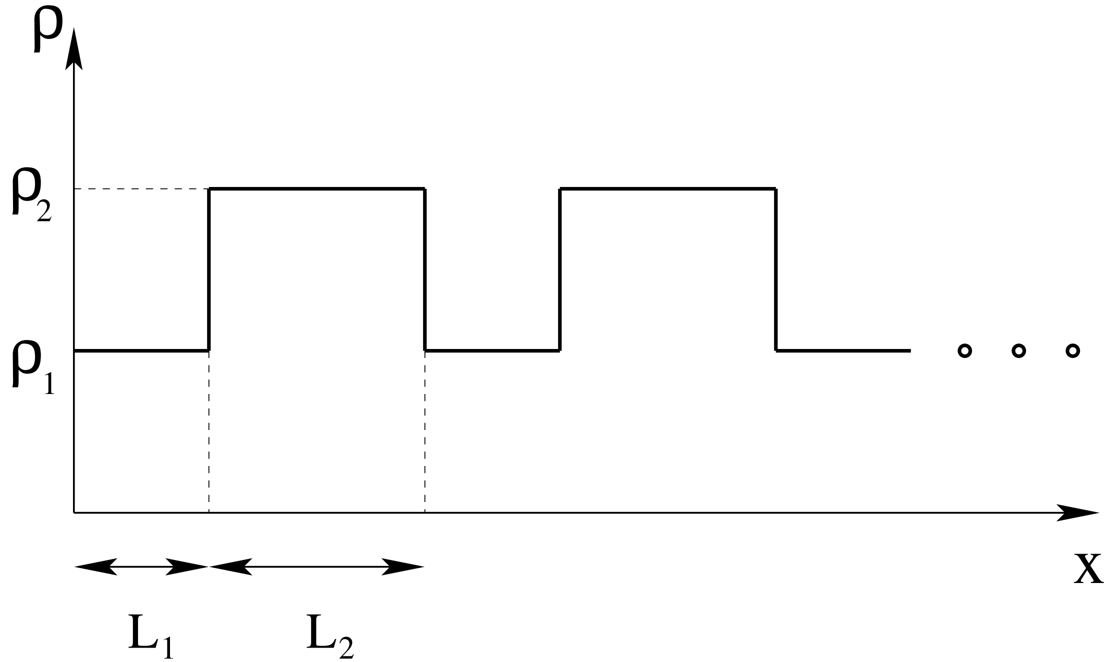
For small-ampl. osc.:

$$\Omega_{\rm res} = \frac{2\omega}{n}$$

$n = 1, 2, 3...$

Different from MSW eff. – no level crossing !

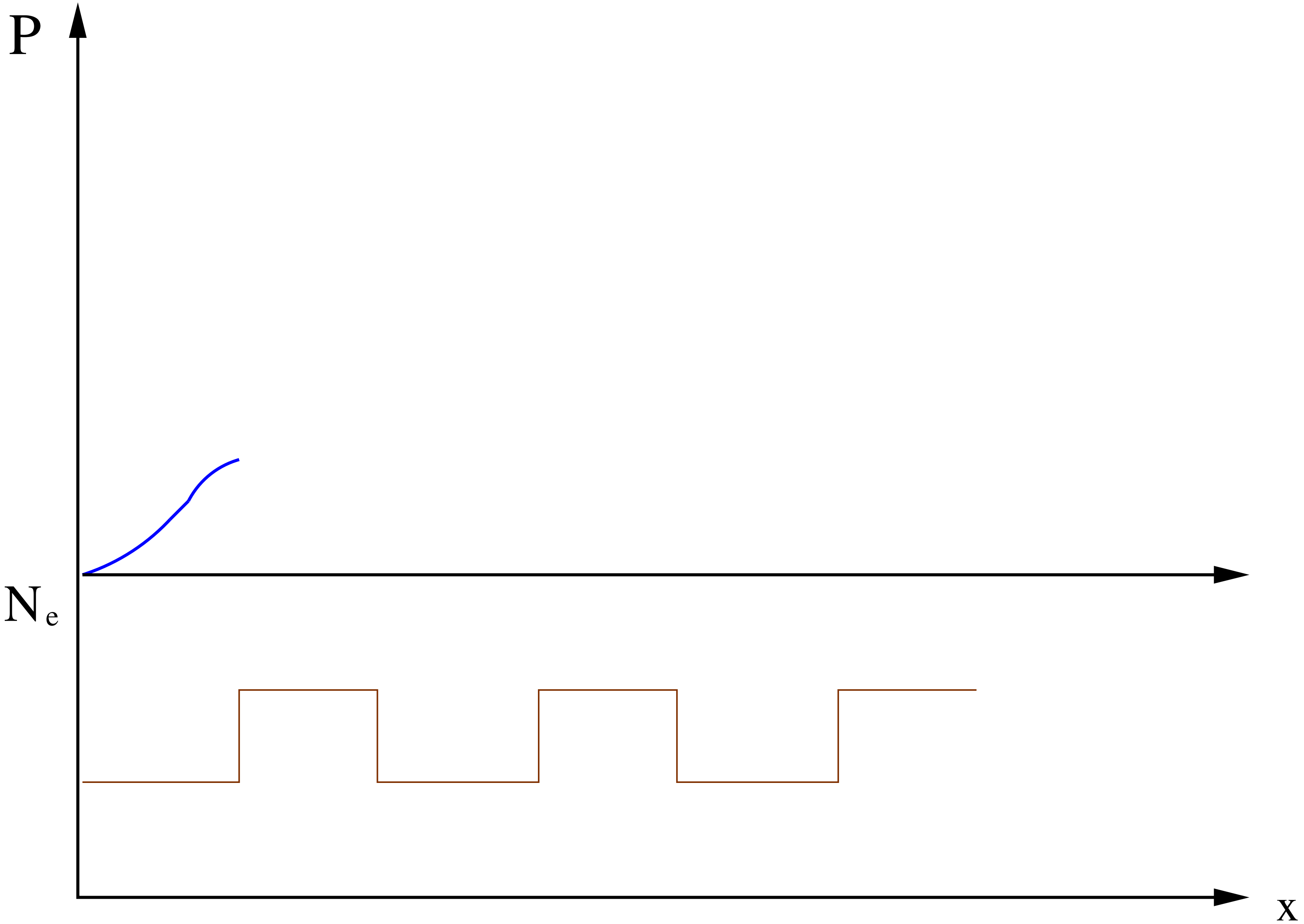
An example admitting an exact analytic solution – “castle wall” density profile (E.A., 1987, 1998):

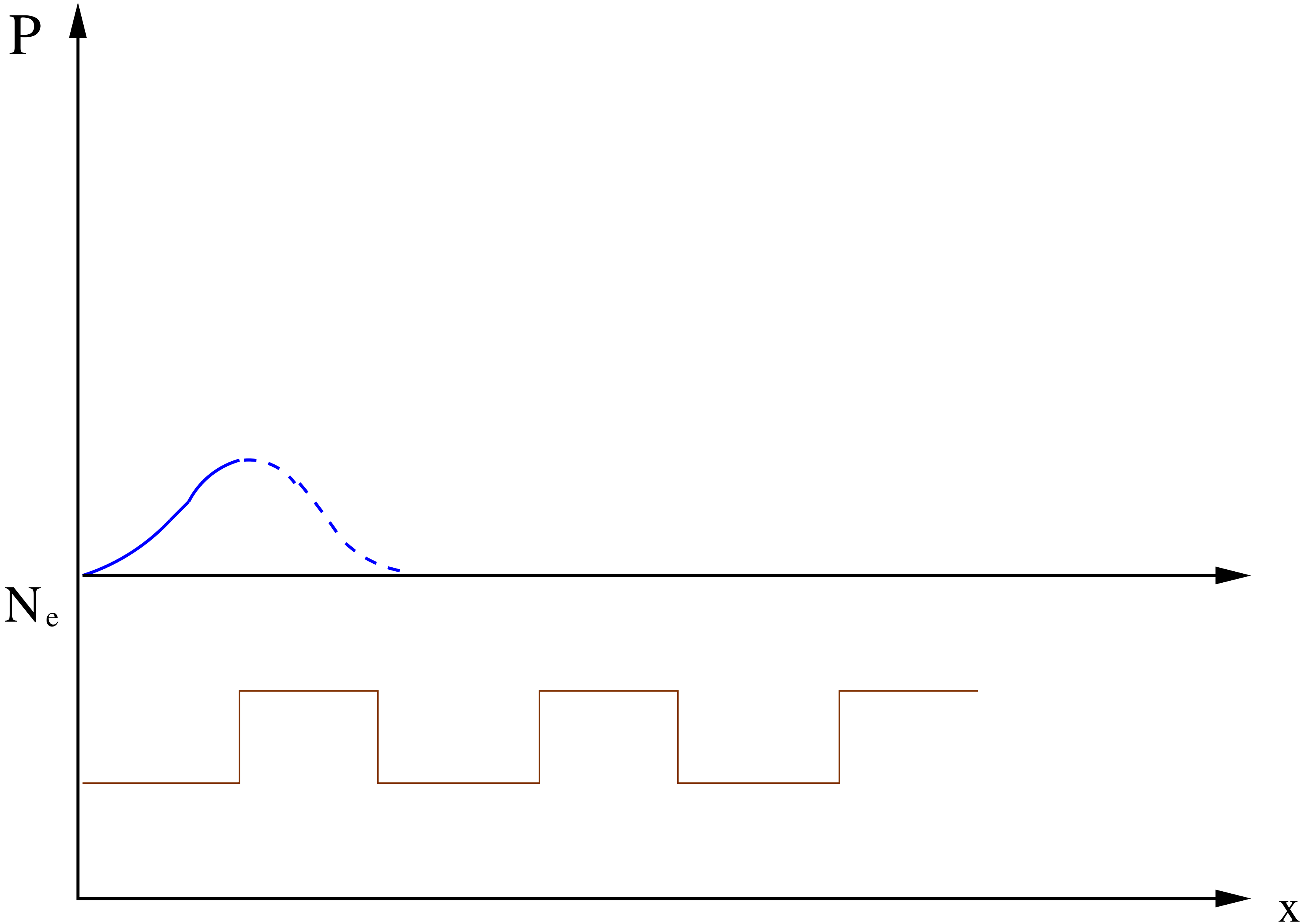


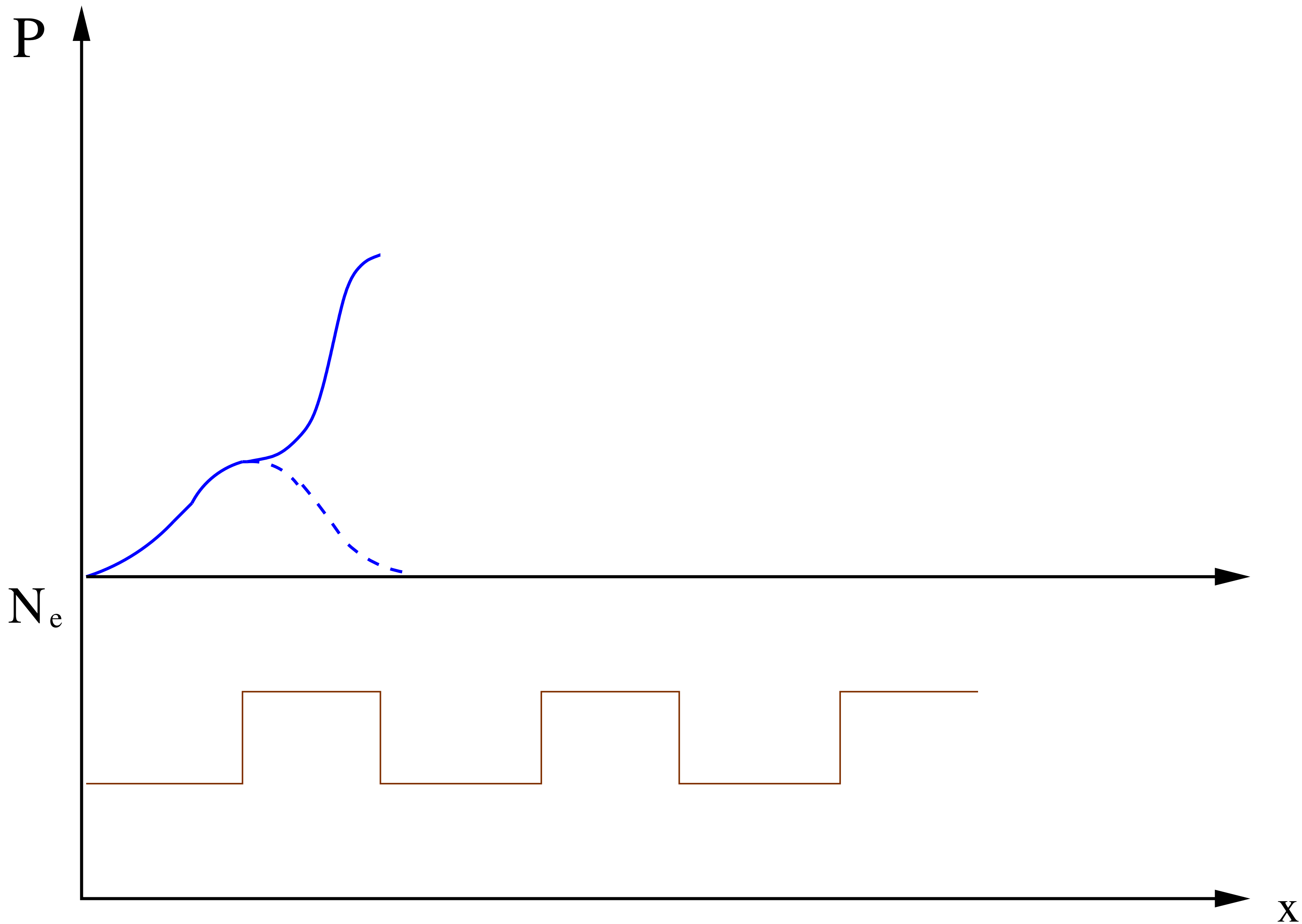
Resonance condition:

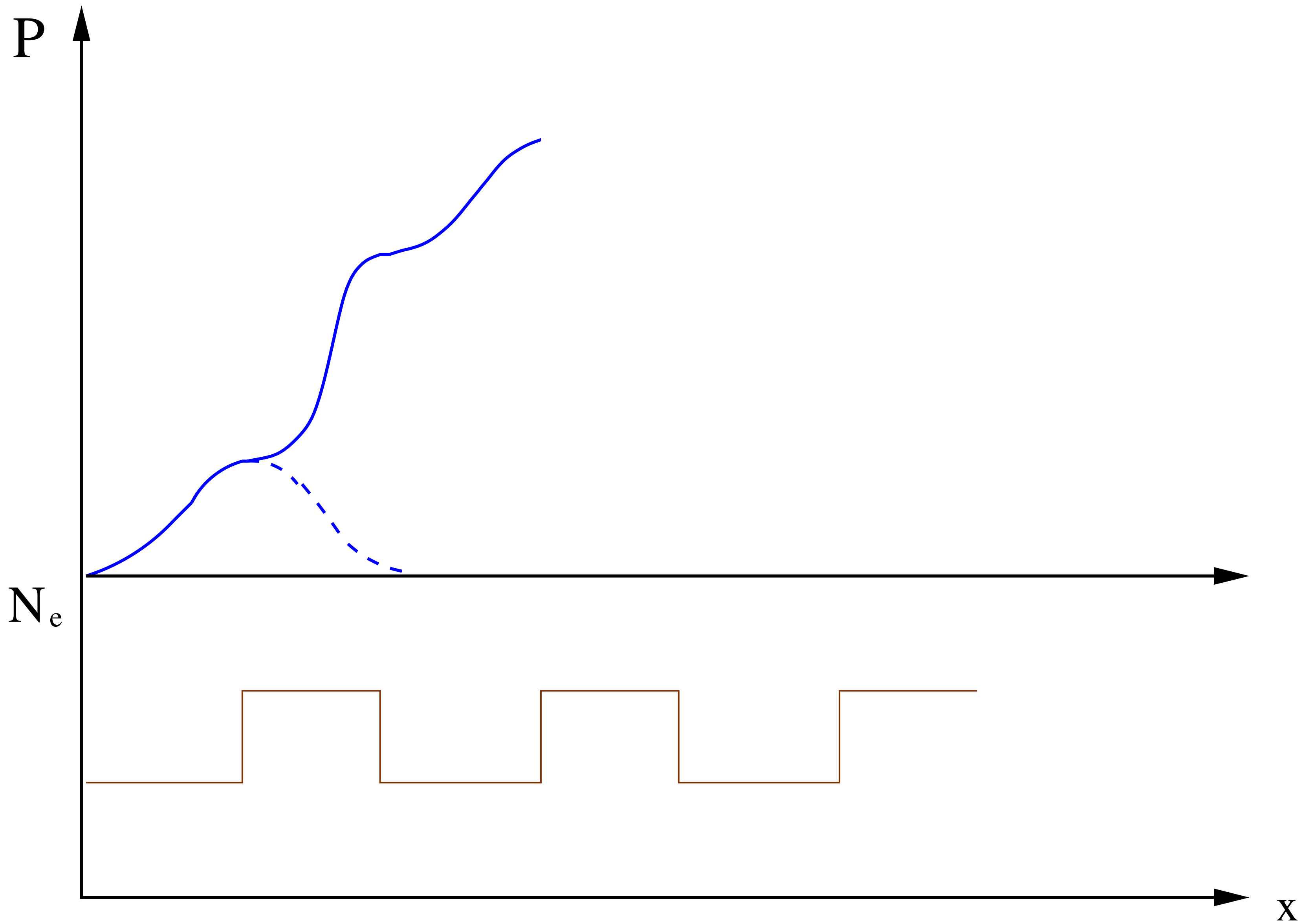
$$X_3 \equiv -(\sin\phi_1 \cos\phi_2 \cos 2\theta_{1m} + \cos\phi_1 \sin\phi_2 \cos 2\theta_{2m}) = 0$$

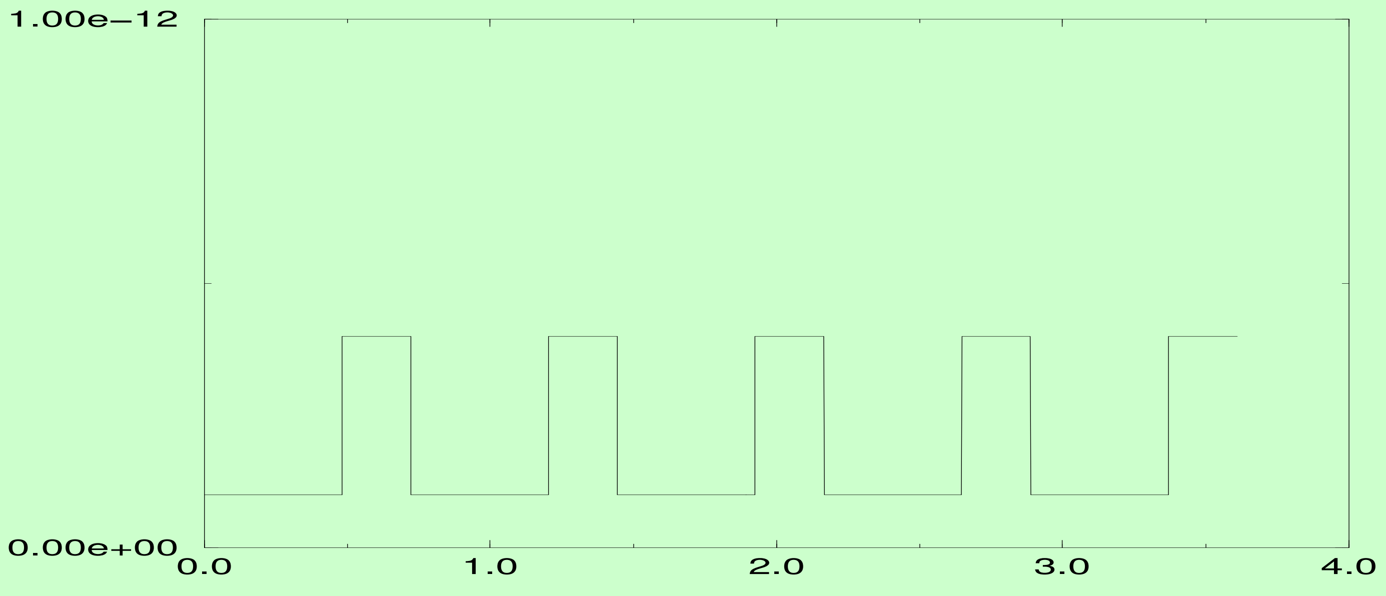
$\phi_{1,2}$ – oscillation phases acquired in layers 1, 2

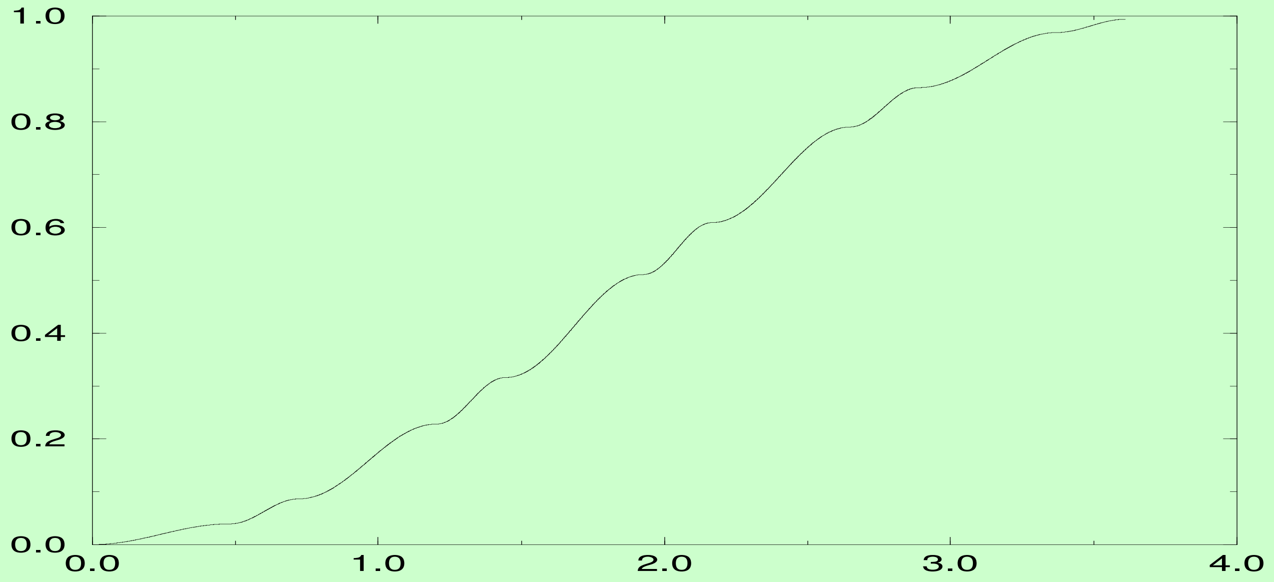

P
N_e
x


P
N_e
x

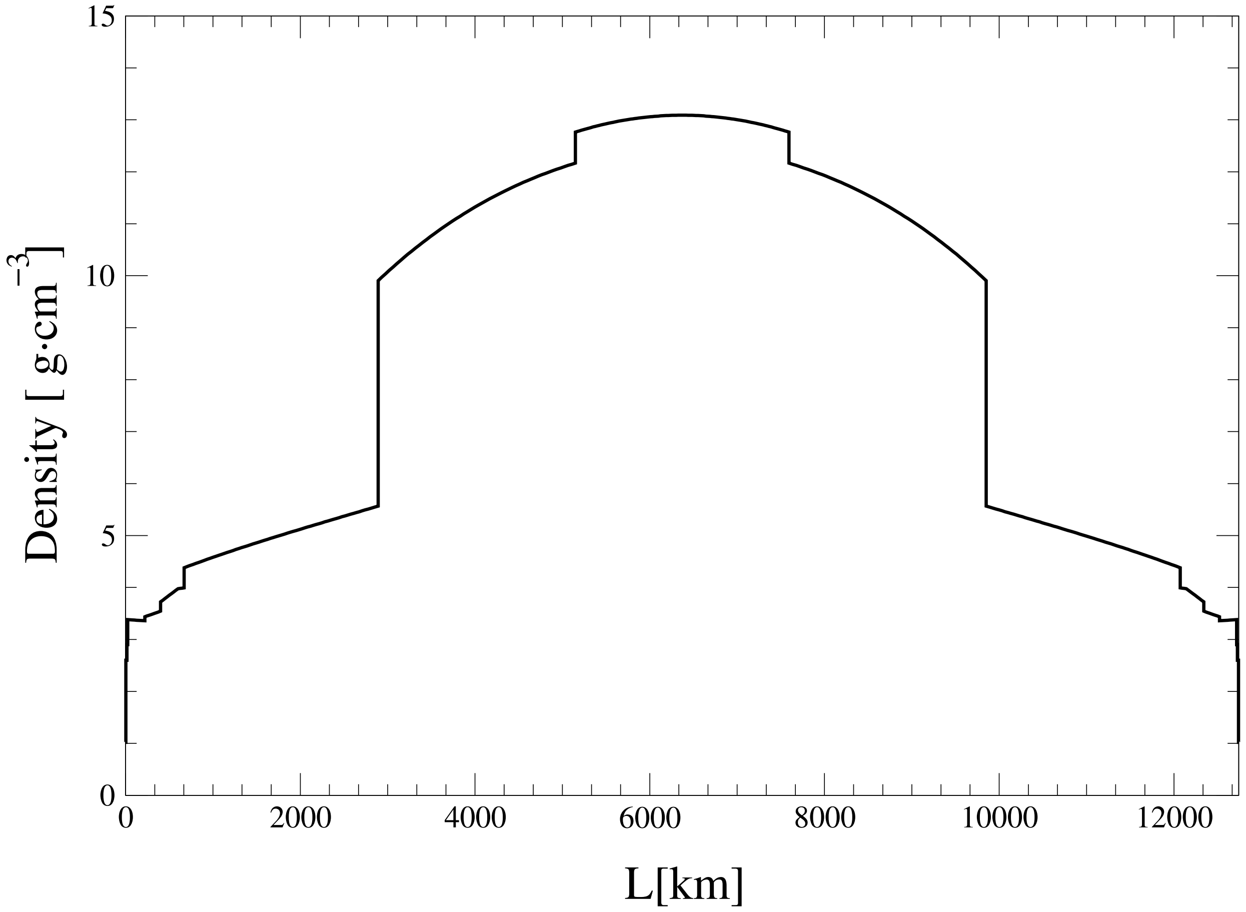

P
N_e
x


P
N_e
x

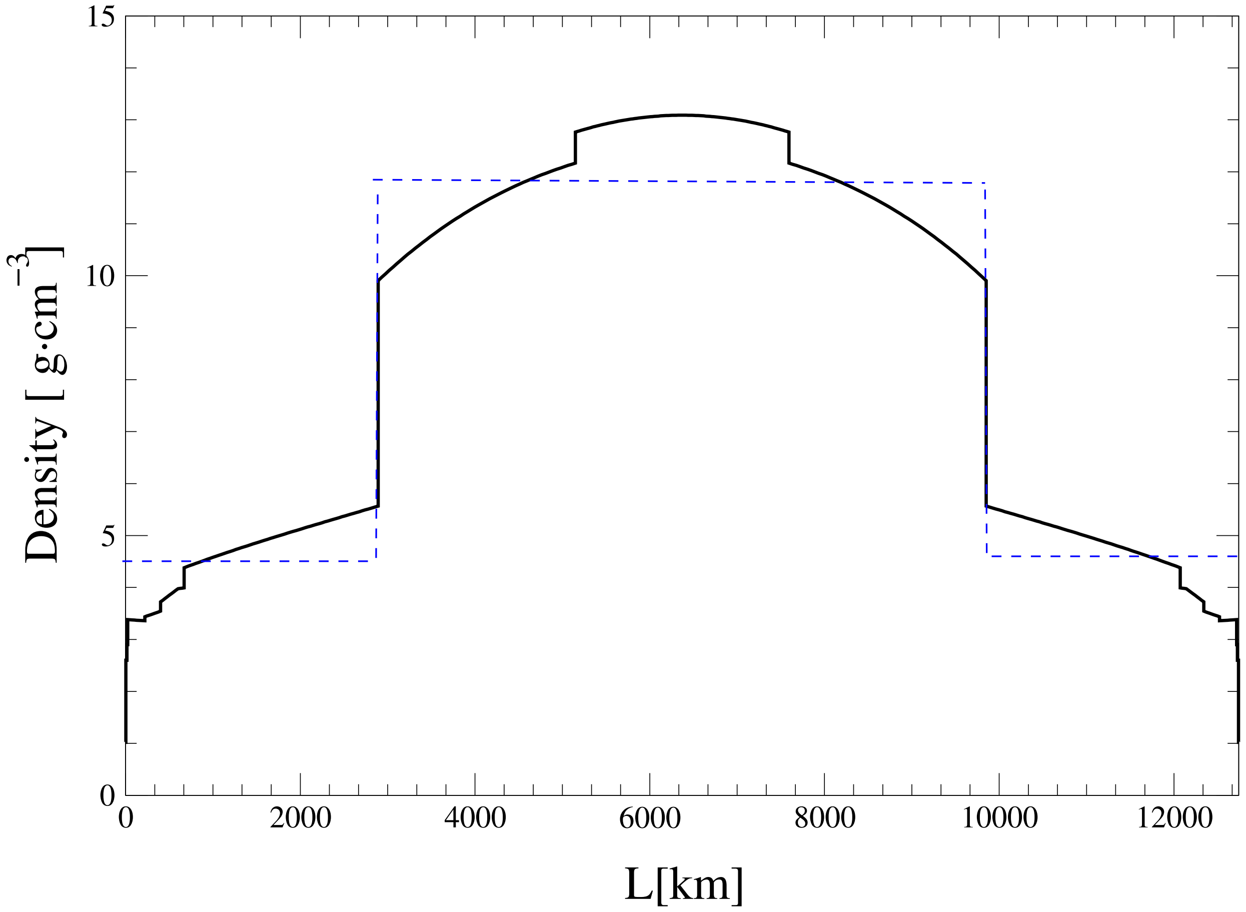

1.00e−12
0.00e+00
0.0
1.0
2.0
3.0
4.0


1.0
0.8
0.6
0.4
0.2
0.0
0.0
1.0
2.0
3.0
4.0

Earth's density profile (PREM model) :

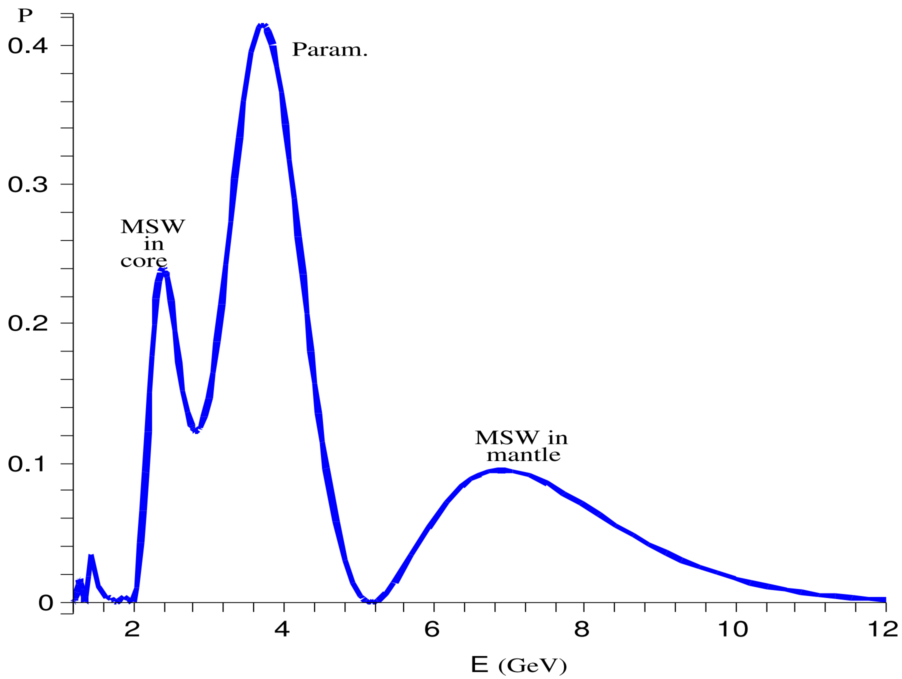


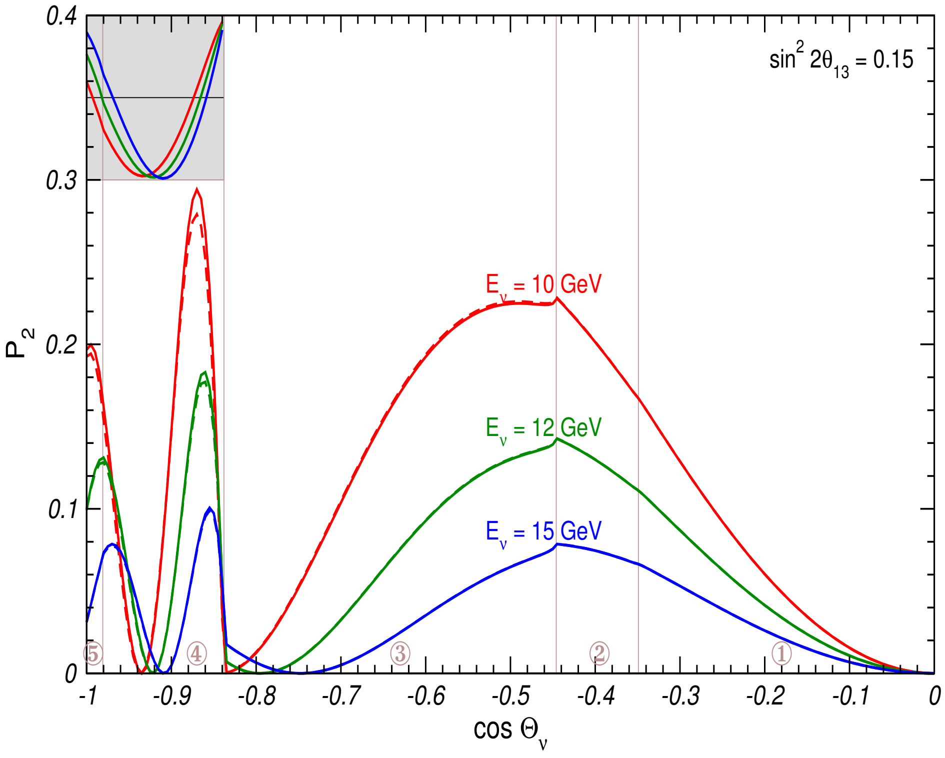
Earth's density profile (PREM model) :



Param. res. condition: $(l_{\rm osc})_{\rm matt} \simeq l_{\rm density\ mod.}$

Fulfilled for $\nu_e \leftrightarrow \nu_{\mu.\tau}$ oscillations of core-crossing ν's in the Earth for a wide range of energies and zenith angles !





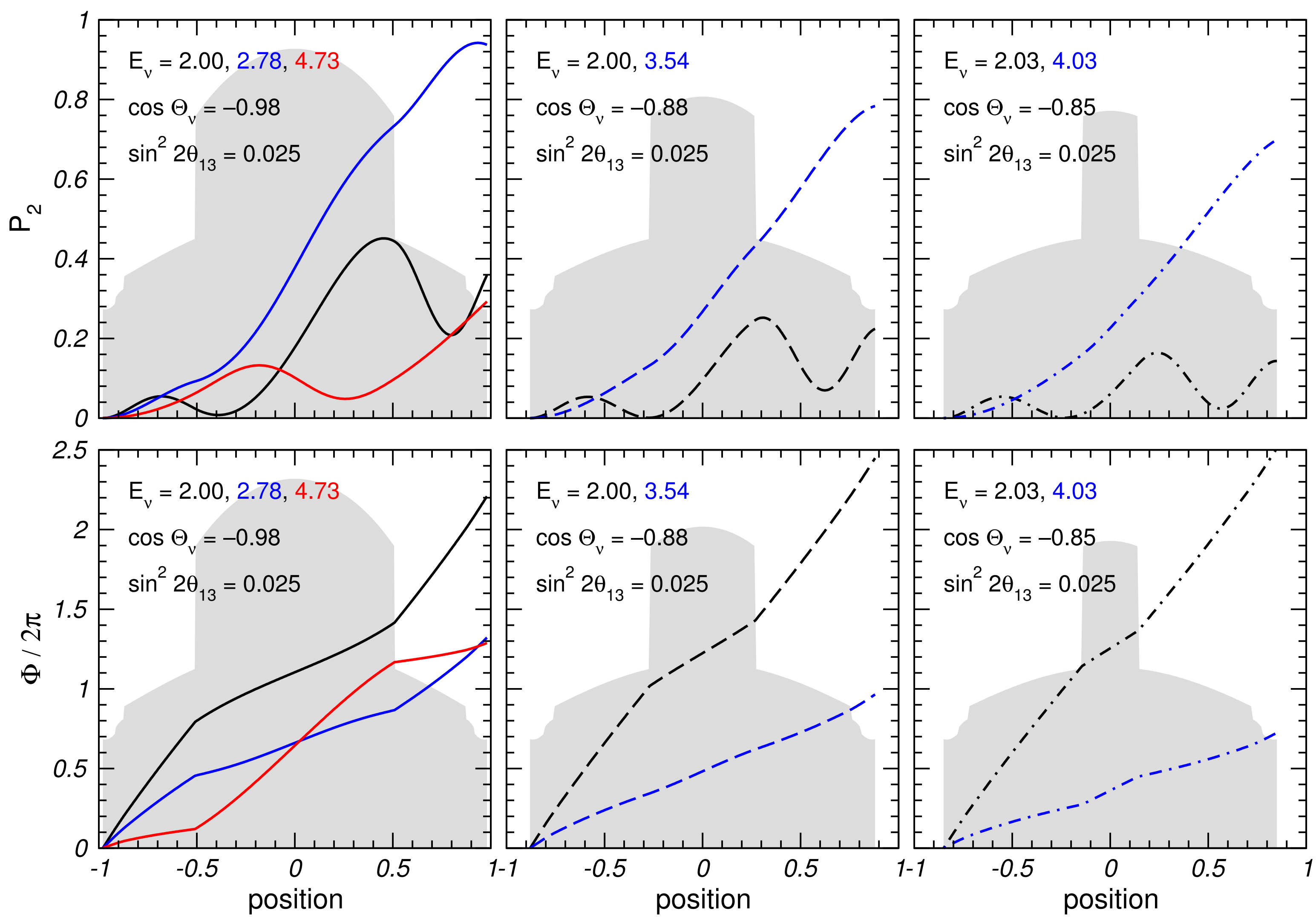
Intermed. energies

$\cos\Theta = -0.89 \qquad \sin^2 2\theta_{13} = 0.01$

(Liu, Smirnov, 1998; Petcov, 1998; E.A. 1998)

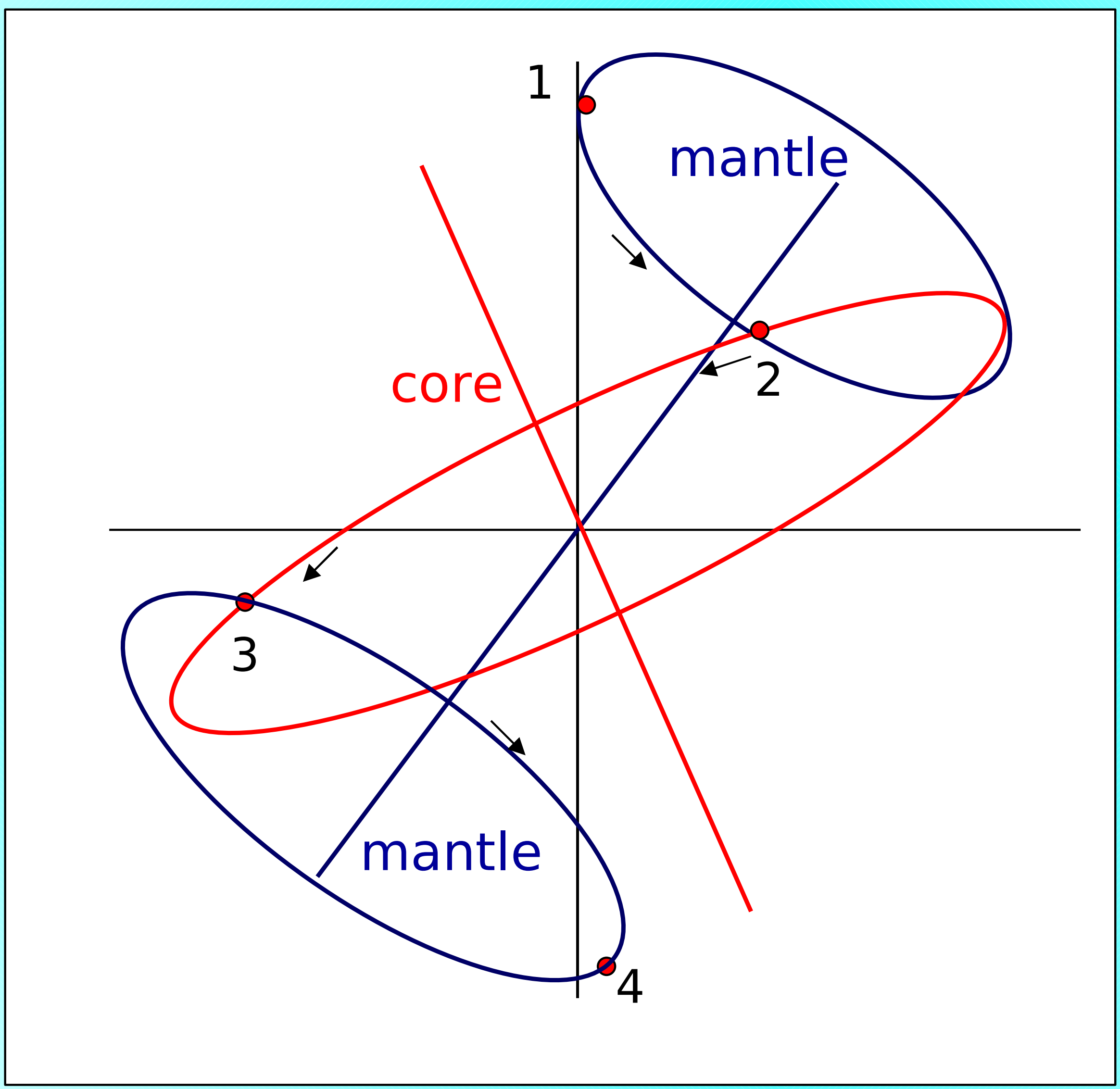
High energies, $\cos\Theta$ - dependence

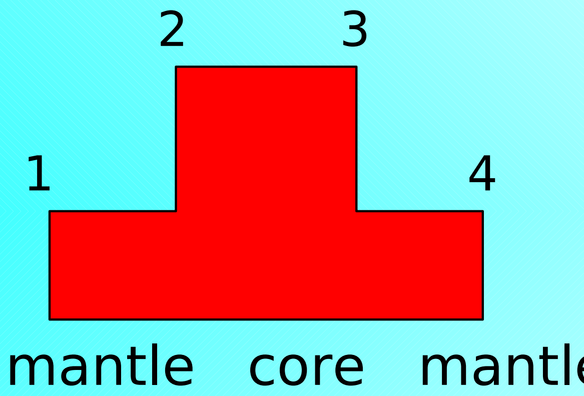
(E.A., Maltoni & Smirnov, 2005)



◇ Parametric resonance of ν oscillations in the Earth: can be observed in future atmospheric or accelerator experiments if θ_{13} is not much below its current upper limit

Parametric enhancement in the Earth





Neutrino oscillations in the Earth

A coherent description in terms of different realizations of just 2 conditions – amplitude and phase conditions

Matter with $N_e = const$:

$$\diamondsuit \quad P_{\rm tr} = \sin^2 2\theta_m \sin^2 \phi_m$$

- amplitude condition = MSW resonance condition $(\theta_m = 45°)$
- phase condition: $\phi_m = \pi/2 + \pi n$

Neutrino oscillations in the Earth

"Castle wall" density profile:

$$\diamondsuit \quad P_{tr}^{(n)} = \frac{X_1^2 + X_2^2}{X_1^2 + X_2^2 + X_3^2} \sin^2 n\Phi$$

Evolution matrix: $\nu(t) = S(t, t_0)\,\nu(0)$. For 2 layers:

$$S^{(2)}(t, t_0) = \begin{pmatrix} Y - iX_3 & -i(X_1 - iX_2) \\ -i(X_1 + iX_2) & Y + iX_3 \end{pmatrix}, \qquad Y^2 + \mathbf{X}^2 = 1$$

- amplitude condition = parametric resonance condition $(X_3 = 0)$
- phase condition: $\Phi \equiv \arccos Y = \pi/2 + \pi n$

Neutrino oscillograms of the Earth

Contours of equal osc. probabilities in (Θ_ν, E_ν) plane

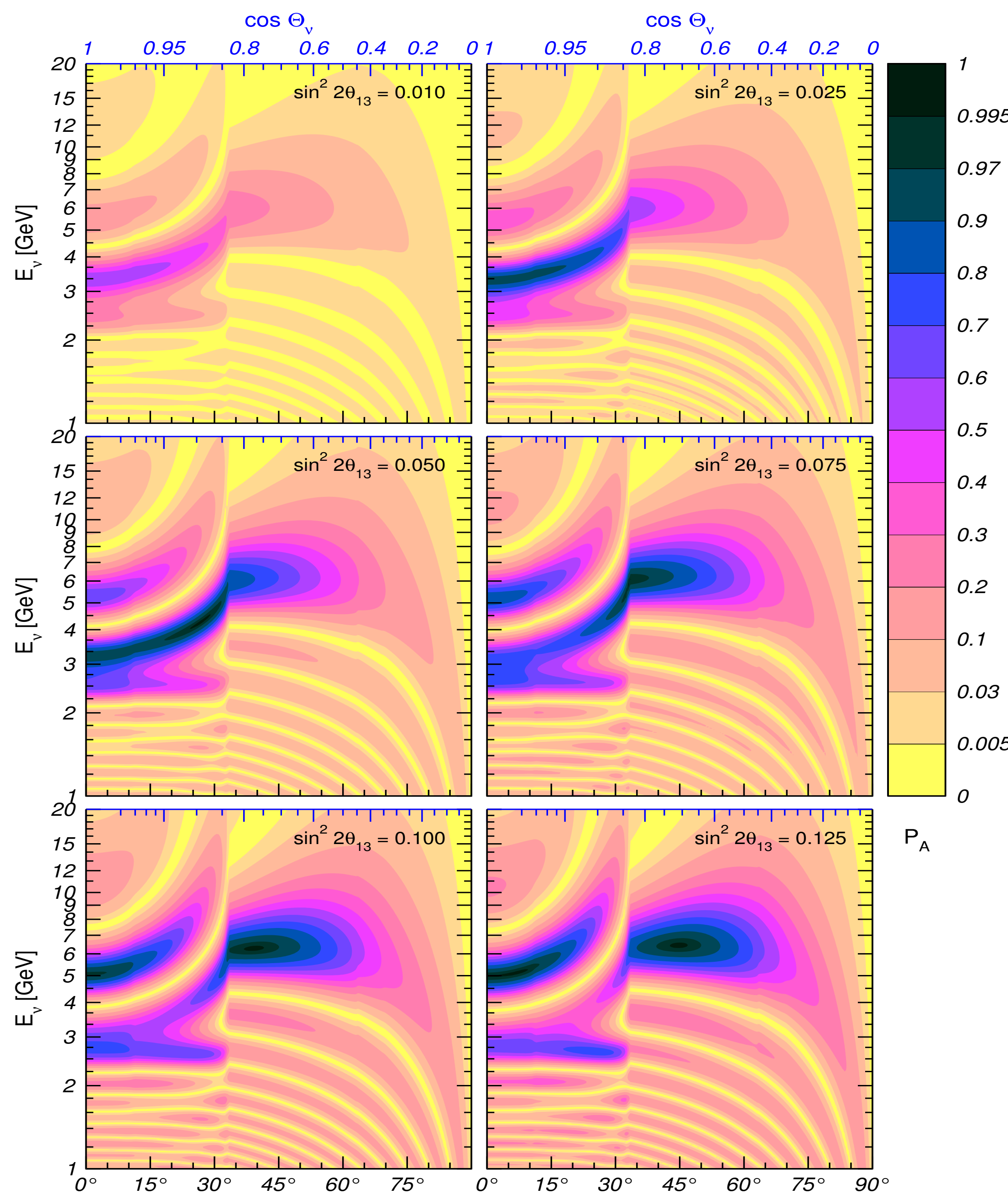
Θ_{13} - dependense of P_A $\Rightarrow$

P_A – effective 2f transition probability $(\Delta m^2_{sol} \to 0)$

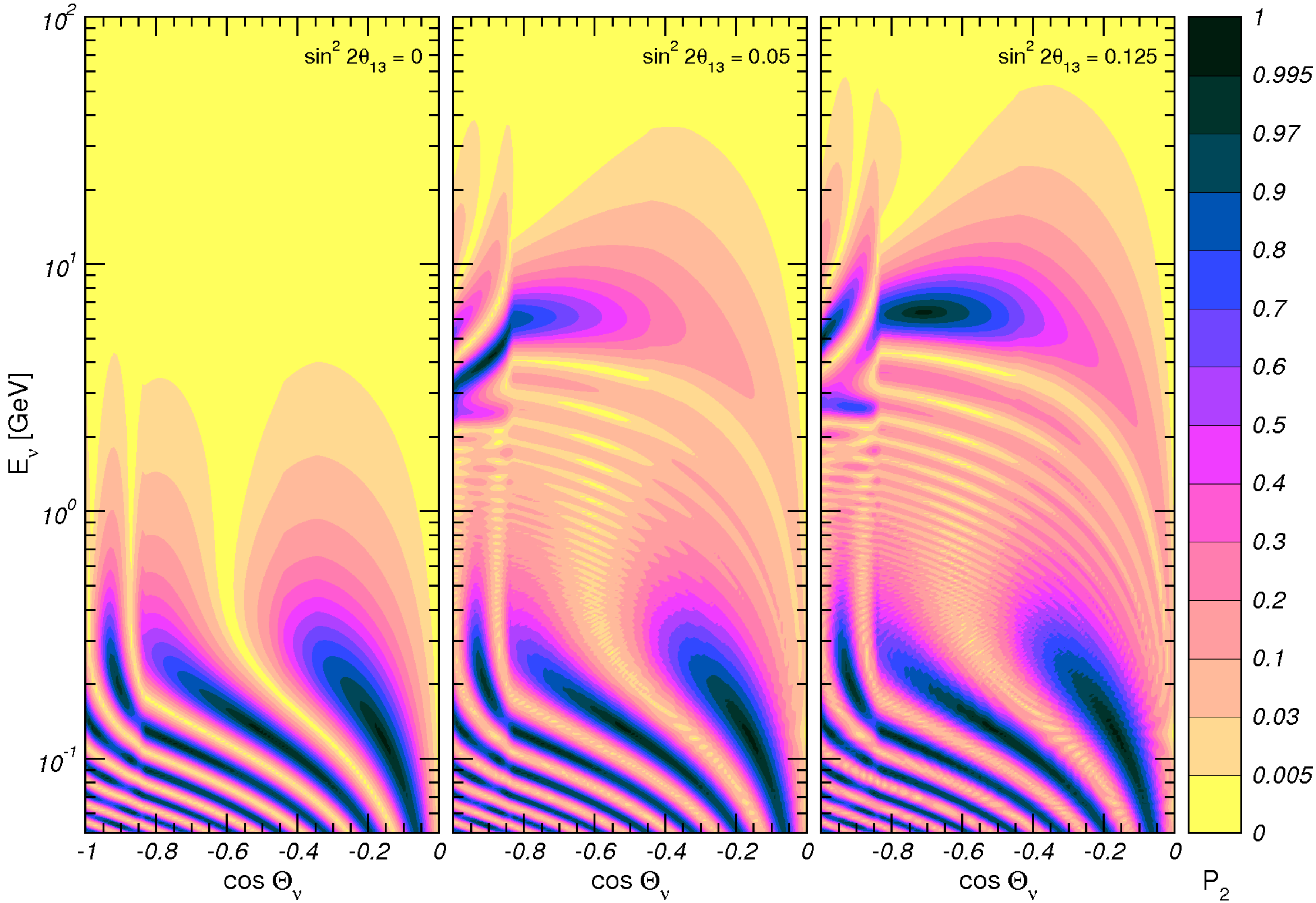
$$P_{e\mu} = s^2_{23} P_A$$

$$P_{e\tau} = c^2_{23} P_A$$

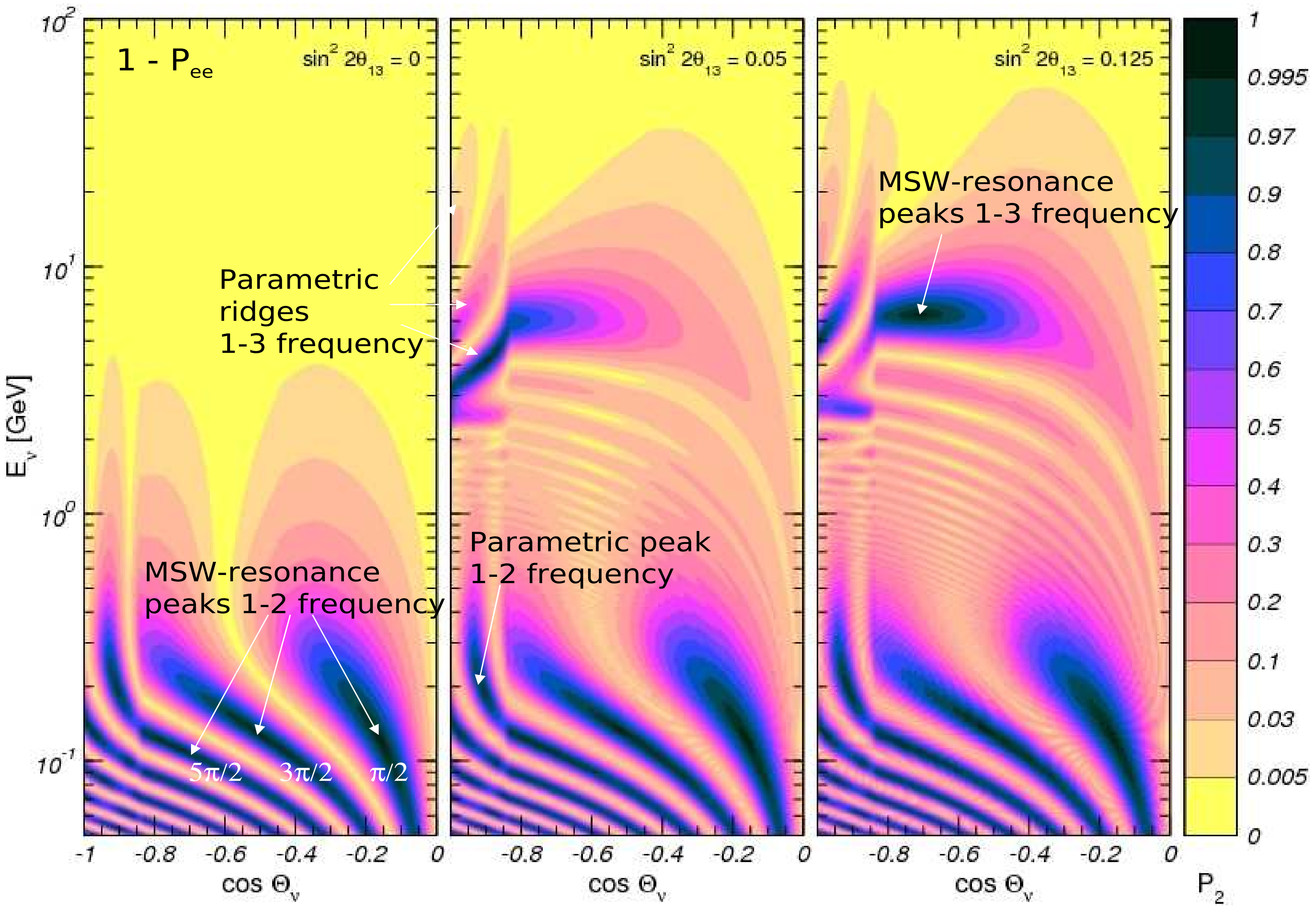
(E.A., Maltoni & Smirnov, 2006)



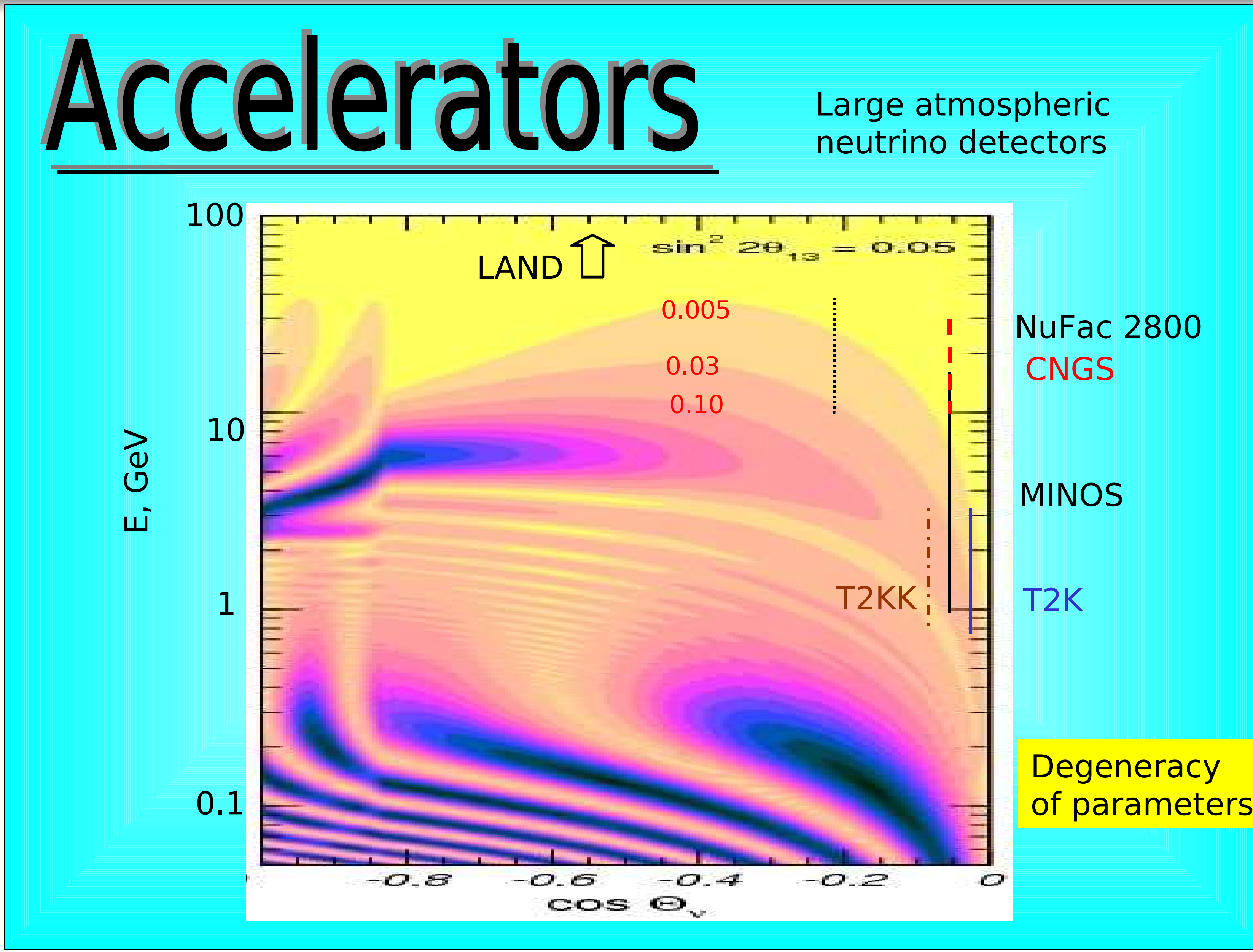
Including the effects of $\Delta m^2_{\rm sol}$: $(1 - P_{ee})$



Including the effects of $\Delta m^2_{\rm sol}$: $(1 - P_{ee})$



Producing the oscillograms



A. Smirnov, UCLA seminar

3f oscillations in matter

3f neutrino oscillations in matter

Evolution equation:

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}=\left[U\begin{pmatrix}E_1 & 0 & 0\\ 0 & E_2 & 0\\ 0 & 0 & E_3\end{pmatrix}U^\dagger+\begin{pmatrix}V(t) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

$$E_i=\sqrt{p^2+m_i^2}\simeq p+\frac{m_i^2}{2p}\,; \qquad t\simeq r$$

$$V(t)=[V(\nu_e)]_{CC}=\sqrt{2}G_F N_e(t)$$

$[V(\nu_e)]_{NC}=[V(\nu_\mu)]_{NC}=[V(\nu_\tau)]_{NC}$ – do not contribute

(Modulo tiny radiative corrections)

Evolution in the rotated basis

Evolution matrix $S(t,\, t_0)$: $\nu(t) \;=\; S(t,\, t_0)\, \nu(t_0)$. Satisfies

$$\diamondsuit \quad i\frac{d}{dt}S(t,\, t_0) \;=\; H_{\rm fl}\, S(t,\, t_0) \qquad \text{with} \quad S(t_0,\, t_0) \;=\; \mathbb{1}\,.$$

$$H_{\rm fl} \;=\; (O_{23}\,\Gamma_\delta\, O_{13}\,\Gamma_\delta^\dagger\, O_{12}\,)\,{\rm diag}(0,\, \delta,\, \Delta)\; (O_{12}^T\,\Gamma_\delta O_{13}^T\,\Gamma_\delta^\dagger\, O_{23}^T) \,+\, {\rm diag}(V(t),\, 0,\, 0)$$

$$=\; (O_{23}\,\Gamma_\delta\, O_{13}\; O_{12}\,)\,{\rm diag}(0,\, \delta,\, \Delta)\, (O_{12}^T\, O_{13}^T\,\Gamma_\delta^\dagger\, O_{23}^T) \,+\, {\rm diag}(V(t),\, 0,\, 0)$$

where

$$\delta \;\equiv\; \frac{\Delta m_{21}^2}{2E}\,, \qquad \Delta \;\equiv\; \frac{\Delta m_{31}^2}{2E}$$

Oscillation probabilities:

$$P_{\alpha\beta} \;=\; |S_{\beta\alpha}|^2$$

Define

$$O'_{23} = O_{23}\,\Gamma_\delta$$

The matrix ${\rm diag}(V(t), 0, 0)$ commutes with O'_{23} $\Rightarrow$ go to the rotated basis

Evolution in the rotated basis – contd.

$$\nu \;=\; O'_{23}\,\nu'\,, \qquad \text{or} \qquad S(t,\,t_0) \;=\; O'_{23}\,S'(t,\,t_0)\,O'_{23}{}^{\dagger}\,,$$

In the rotated basis $H' \;=\; O'_{23}\,H_{\text{fl}}\,O'_{23}{}^{\dagger}$. Explicitly:

$$H'(t) = \begin{pmatrix} s_{12}^2 c_{13}^2\delta + s_{13}^2\Delta + V(t) & s_{12}c_{12}c_{13}\delta & s_{13}c_{13}\left(\Delta - s_{12}^2\delta\right) \\ s_{12}c_{12}c_{13}\delta & c_{12}^2\delta & -s_{12}c_{12}s_{13}\delta \\ s_{13}c_{13}\left(\Delta - s_{12}^2\delta\right) & -s_{12}c_{12}s_{13}\delta & c_{13}^2\Delta + s_{12}^2 s_{13}^2\delta \end{pmatrix}$$

Dependence on θ_{23} and δ_{CP} can be obtained in the general case by rotating back to the original flavour basis. Also: easy to apply PT approximations

- If $\frac{\Delta m_{21}^2}{2E}L \ll 1$ – neglect $\delta \;=\; \frac{\Delta m_{21}^2}{2E}$
- As θ_{13} is relatively small – neglect s_{13}

or use expansion in these small parameters

General properties of $P_{\alpha\beta}$ and CP, T and CPT

General properties of $P_{\alpha\beta}$

3 flavours $\Rightarrow 3\times 3 = 9$ probabilities

$$P_{\alpha\beta} = P(\nu_\alpha \to \nu_\beta),$$

plus 9 probabilities for antineutrinos $P_{\bar{\alpha}\bar{\beta}}$.
Unitarity conditions (probability conservation):

$$\sum_\beta P_{\alpha\beta} = \sum_\alpha P_{\alpha\beta} = 1 \qquad (\alpha, \beta = e, \mu, \tau)$$

5 indep. conditions $\Rightarrow 9 - 5 = 4$ indep. probabilities left.
Additional symmetry: the matrix of matter-induced potentials $\mathrm{diag}(V(t), 0, 0)$ commutes with O_{23} $\Rightarrow$ additional relations between probabilities.

Dependence on θ_{23} and # of indep. $P_{\alpha\beta}$

Define

$$\tilde{P}_{\alpha\beta} = P_{\alpha\beta}(s_{23}^2 \leftrightarrow c_{23}^2,\, \sin 2\theta_{23} \to -\sin 2\theta_{23})$$

(e.g., $\theta_{23} \to \theta_{23} + \pi/2$). Then

$$P_{e\tau} = \tilde{P}_{e\mu} \qquad P_{\tau\mu} = \tilde{P}_{\mu\tau} \qquad P_{\tau\tau} = \tilde{P}_{\mu\mu}$$

2 out of 3 conditions are independent $\Rightarrow$ $4 - 2 = 2$ indep. probabilities (e.g., $P_{e\mu}$ and $P_{\mu\tau}$) $\Rightarrow$

$\Diamond$ *All 9 neutrino ocillation probabilities can be expressed through just two!*

$$P_{\bar{\alpha}\bar{\beta}} = P_{\alpha\beta}(\delta_{\rm CP} \to -\delta_{\rm CP},\, V \to -V)$$

$\Rightarrow$ *All 18 ν and $\bar{\nu}$ probab. can be expressed through just two*

General dependence on $\delta_{\rm CP}$

Another use of essentially the same symmetry: rotate by

$$O'_{23} = O_{23} \times {\rm diag}(1, 1, e^{i\delta_{\rm CP}})$$

From commutativity of ${\rm diag}(V(t), 0, 0)$ with O'_{23} $\Rightarrow$
General dependence of probabilities on $\delta_{\rm CP}$:

$$P_{e\mu} = A_{e\mu}\cos\delta_{\rm CP} + B_{e\mu}\sin\delta_{\rm CP} + C_{e\mu}$$

$$P_{\mu\tau} = A_{\mu\tau}\cos\delta_{\rm CP} + B_{\mu\tau}\sin\delta_{\rm CP} + C_{\mu\tau}$$

$$+ D_{\mu\tau}\cos 2\delta_{\rm CP} + E_{\mu\tau}\sin 2\delta_{\rm CP}$$

$\not{C}\not{P}$ and $\not{T}$ in ν oscillations in matter

- CP: $\nu_{\alpha,\beta} \leftrightarrow \bar{\nu}_{\alpha,\beta} \quad \Rightarrow \quad U_{\alpha i} \to U^*_{\alpha i} \quad (\{\delta_{\rm CP}\} \to -\{\delta_{\rm CP}\})$
  $V(r) \to -V(r)$

- T: $t \rightleftarrows t_0 \quad \Leftrightarrow \quad \nu_\alpha \leftrightarrow \nu_\beta$
  $\Rightarrow \quad U_{\alpha i} \to U^*_{\alpha i} \quad (\{\delta_{\rm CP}\} \to -\{\delta_{\rm CP}\})$
  $V(r) \to \tilde{V}(r)$

$$\tilde{V}(r) = \sqrt{2} G_F \tilde{N}(r)$$

$\tilde{N}(r)$: corresponds to interchanged positions of ν source and detector.
Symmetric density profiles: $\tilde{N}(r) = N(r)$

◇ *The very presence of matter [with (# of particles) $\neq$ (# of antiparticles)] violates C, CP and CPT!*

$\Rightarrow$ Fake (extrinsic) $\not{C}\not{P}$ which may complicate the study of fundamental (intrinsic) $\not{C}\not{P}$

$\not{CP}$ in matter

- Exists even in 2f case (in $\geq$ 3f case exists even when all $\{\delta_{\rm CP}\} = 0$) due to matter effects:

$$P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

E.g., MSW effect can enhance $\nu_e \leftrightarrow \nu_\mu$ and suppress $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ or vice versa.

- Survival probabilities are not CP-invariant:

$$P(\nu_\alpha \to \nu_\alpha) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\alpha)$$

To disentangle fundamental $\not{CP}$ from the matter induced one in LBL experiments – need to measure energy dependence of oscillated signal or signal at two baselines

Alternatives:

- Low-E experiments ($E \sim$ 0.1 – 1 GeV) with $L \sim$ 100 – 1000 km
- Indirect measurements: CP-even terms $\sim \cos\delta_{\rm CP}$ or area of leptonic unitarity triangle

$\not{T}$ in matter

CPT not conserved in matter $\Rightarrow$ $\not{CP}$ and $\not{T}$ are not directly related!

- Matter does not necessarily induce $\not{T}$ (only asymmetric matter with $\tilde{N}(r) \neq N(r)$ does)
- There is no $\not{T}$ (either fundamental or matter induced) in 2f case – a consequence of unitarity:

$$P_{ee} + P_{e\mu} = 1$$

$$P_{ee} + P_{\mu e} = 1$$

$$\Downarrow$$

$$P_{e\mu} = P_{\mu e}$$

• In 3f case – only one T-odd probability difference for ν's (and one for $\bar{\nu}$'s) irrespective of matter density profile – a consequence of unitarity in 3f case

$$\Delta P^T_{e\mu} = \Delta P^T_{\mu\tau} = \Delta P^T_{\tau e}$$

Matter-induced $\not{T}$:

◇ An interesting, pure 3f matter effect; absent in the case of symmetric density profiles (e.g., $N(r) = const$)

◇ Does not vanish in the regime of complete averaging

◇ May fake fundamental $\not{T}$ and complicate its study (extraction of δ_{CP} from the experiment)

◇ Vanishes when either $U_{e3} = 0$ or $\Delta m^2_{21} = 0$ (2f limits) $\Rightarrow$ doubly suppressed by both these small parameters

$\Rightarrow$ *Perturbation theory can be used to get analytic expressions*

Is T reversal in matter equivalent to $\nu_a \leftrightarrow \nu_b$?

No explicit closed form solution in general.

Still, easy to answer !

T reversal: $t \rightleftarrows t_0 \quad \Leftrightarrow \quad S(t,\, t_0) \;\Rightarrow\; S(t_0,\, t)$

One has:

$$S(t_0,\, t) \;=\; S(t,\, t_0)^{-1} \;=\; S(t,\, t_0)^{\dagger} \;=\; [S(t,\, t_0)^T]^*$$

Therefore

$$|[S(t_0,\, t)]_{\alpha\beta}|^2 \;=\; |[S(t,\, t_0)]_{\beta\alpha}|^2$$

$\Rightarrow$ In matter with arbitrary density profile, as well as in vacuum, time reversal is equivalent to interchanging the initial and final neutrino flavours

To extract fundamental $\not{T}$ need to measure:

$$\Delta P_{\alpha\beta} \equiv P_{\rm dir}(\nu_\alpha \to \nu_\beta) - P_{\rm rev}(\nu_\beta \to \nu_\alpha) \propto \sin\delta_{\rm CP}$$

Even survival probabilities $P_{\alpha\alpha}$ $(\alpha = \mu, \tau)$ can be used!

$$P_{\rm dir}(\nu_\alpha \to \nu_\alpha) - P_{\rm rev}(\nu_\alpha \to \nu_\alpha) \sim \sin\delta_{\rm CP} \quad (\alpha \neq e)$$

In 3f case P_{ee} does not depend on $\delta_{\rm CP}$ – not true if $\nu_{\rm sterile}$ is present!

Matter-induced $\not{T}$ in LBL experiments due to imperfect sphericity of the Earth density distribution cannot spoil the determination of $\delta_{\rm CP}$ if the error in $\delta_{\rm CP}$ is $> 1\%$ at 99% C.L.

$\Rightarrow$ *No need to interchange positions of ν source and detector!*

Experimental study of $\not{T}$ difficult because of problems with detection of $e^\pm$

General structure of T-odd probability diff.

$$\Delta P_{e\mu}^{T} = \underbrace{\sin\delta_{\rm CP}\cdot Y}_{\text{fundam. } \not{T}} + \underbrace{\cos\delta_{\rm CP}\cdot X}_{\text{matter-ind. } \not{T}}$$

In adiabatic approximation: $X = J_{\rm eff}\cdot$ (oscillating terms) ,

$$\diamondsuit \quad J_{\rm eff} = s_{12}\, c_{12}\, s_{13}\, c_{13}^2\, s_{23}\, c_{23}\, \frac{\sin(2\theta_1 - 2\theta_2)}{\sin 2\theta_{12}}$$

Compare with the vacuum Jarlskog invariant:

$$J = s_{12}\, c_{12}\, s_{13}\, c_{13}^2\, s_{23}\, c_{23} \sin\delta_{\rm CP}$$

$$\Rightarrow \qquad \sin\delta_{\rm CP} \Leftrightarrow \frac{\sin(2\theta_1 - 2\theta_2)}{\sin 2\theta_{12}}$$

Matter-induced $T\!\!\!/$:

◊ Negligible effects in terrestrial experiments

◊ Cannot be observed in supernova ν oscillations due to experimental indistinguishability of low-energy ν_μ and ν_τ

◊ Can affect the signal from ∼GeV neutrinos produced in annihilations of WIMPs inside the Sun

"CPT in matter"

Is there a relation between $\not{CP}$ and $\not{T}$ in matter?

For symmetric density profiles (i.e. $\tilde{V}(r) = V(r)$)

$$P(\nu_\alpha \to \nu_\beta;\, \delta_{\rm CP},\, V(r)) \;=\; P(\bar{\nu}_\beta \to \bar{\nu}_\alpha;\, \delta_{\rm CP},\, -V(r))$$

(Minakata, Nunokawa & Parke, 2002)

Easy to generalize to the case of an arbitrary density profile:

$$P(\nu_\alpha \to \nu_\beta;\, \delta_{\rm CP},\, V(r)) \;=\; P(\bar{\nu}_\beta \to \bar{\nu}_\alpha;\, \delta_{\rm CP},\, -\tilde{V}(r))$$

Unlike CPT in vacuum, does not directly relate observables

Can be useful for cross-checking theoreticl calculations

Summary – 3f effects in ν oscillations

- ♢ Two types of 3f effects – “trivial” (existence of new channels, their inter-dependence through unitarity) and nontrivial (interference of different parameter channels, qualitatively new effects – fundamental CP and T-violation, and matter - induced T violation
- ♢ 3f corrections to probabilities of oscillations of solar, atmospheric, reactor and acceler. neutrinos depend on $|U_{e3}| = |\sin\theta_{13}|$; can reach $\sim (5-10)\%$
- ♢ Possible interesting 3f effects for SN neutrinos – depend significantly on the value U_{e3} (known now to be not too small)

Summary – contd.

$\diamondsuit$ Manifestations of ≥ 3 flavours in neutrino oscillations:

- Fundamental $\not{CP}$ and $\not{T}$
- Matter-induced $\not{T}$
- Matter effects in $\nu_\mu \leftrightarrow \nu_\tau$ oscillations
- Specific CP and T conserving interference terms in oscillation probabilities

$\diamondsuit$ U_{e3} plays a very special role