Neutrino physics

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MAX-PLANCK-INSTITUT
FÜR KERNPHYSIK

Plan of the lectures

- Introduction.
- Brief overview of experimental results
- Weyl, Dirac and Majorana fermions
- Neutrino masses in simplest extensions of the Standard Model. The seesaw mechanism(s).
- Neutrinos and the baryon asymmetry of the universe
- Neutrino oscillations in vacuum
  - Same E or same p ?
  - QM uncertainties and coherence issues
  - Wave packet approach to neutrino oscillations
  - Lorentz invariance of oscillation probabilities
  - 2f and 3f neutrino mixing schemes and oscillations
  - Implications of CP, T and CPT

Plan of the lectures – contd.

- Neutrino oscillations in matter – the MSW effect
  - Evolution equation
  - Adiabaticity condition and adiabatic evolution
  - Non-adiabatic regime
  - Graphical interpretation and mechanical analogy
  - Earth matter effects on $\nu_\odot$ (day-night asymmetry)
- Neutrino oscillations in matter – parametric resonance
- Direct neutrino mass measurement experiments
- Neutrinoless double β-decay

Plan of the lectures – contd.

- Oscillations: Exp. data and future experiments
  - Atmospheric neutrinos
  - LBL accelerator experiments
  - Solar neutrinos
  - Reactor (anti)neutrino oscillations
  - Oscillatory nature of neutrino flavour transitions
  - Discovery of θ_{13} in reactor and accelerator expts.
  - 3f global fits
  - Light sterile neutrinos?
- Future expts.: Neutrino mass ordering, CP violation, θ_{23} octant,. . .
- Coherent elastic neutrino nucleus scattering (CEvNS)
- Do charged leptons oscillate?
- Future: What's next?

What is left out:

– Neutrino electromagnetic properties

– Oscillations of SN neutrinos (incl. non-linear collective effects)

– Cosmological bounds on # of neutrino species and $\sum m_\nu$

– keV sterile neutrinos as Dark Matter

– Non-standard neutrino interactions

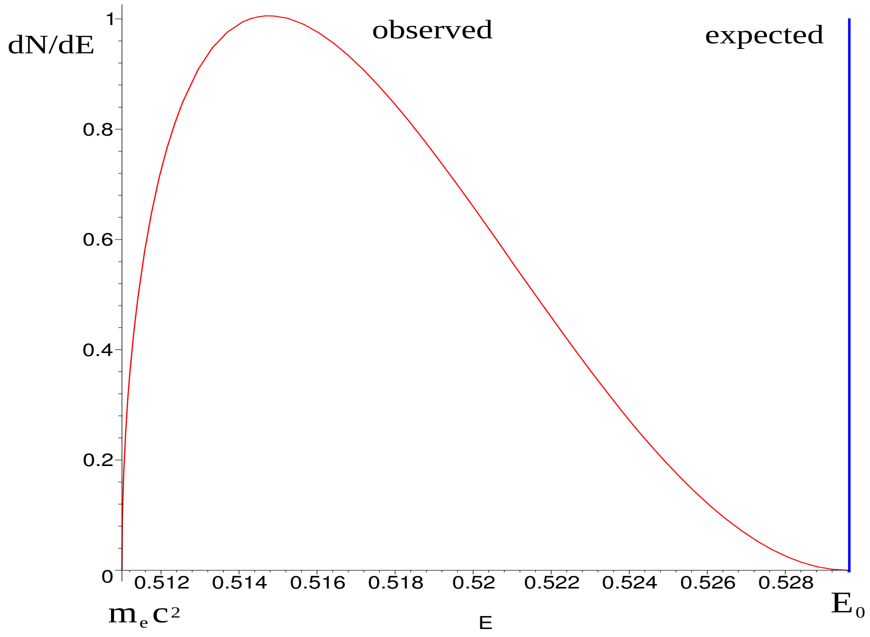
– Geoneutrinos

$\ldots$

Pauli's conjecture

Nuclear beta decay:

$$A(Z,N) \;\rightarrow\; A(Z\pm1, N\mp1) \;+\; e^{\mp}$$



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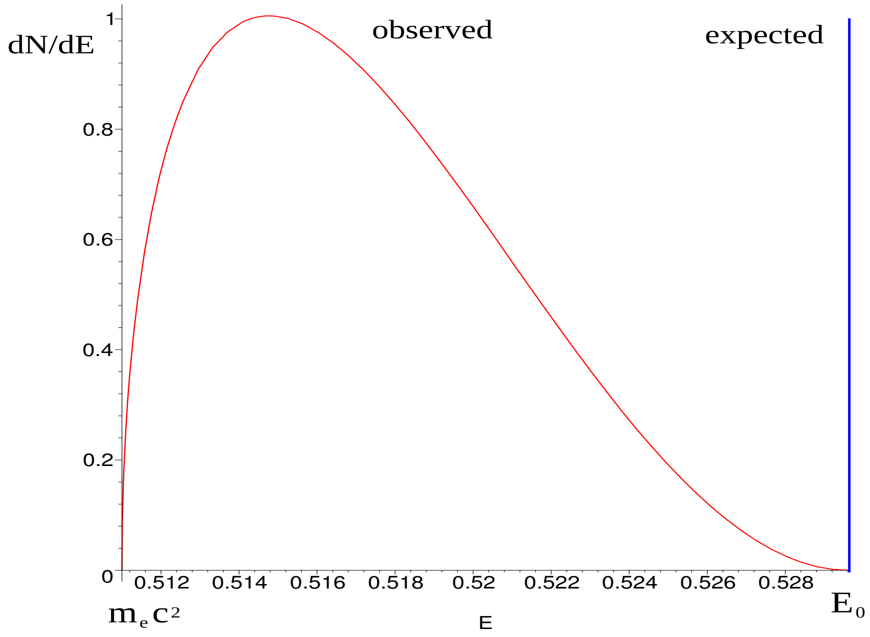

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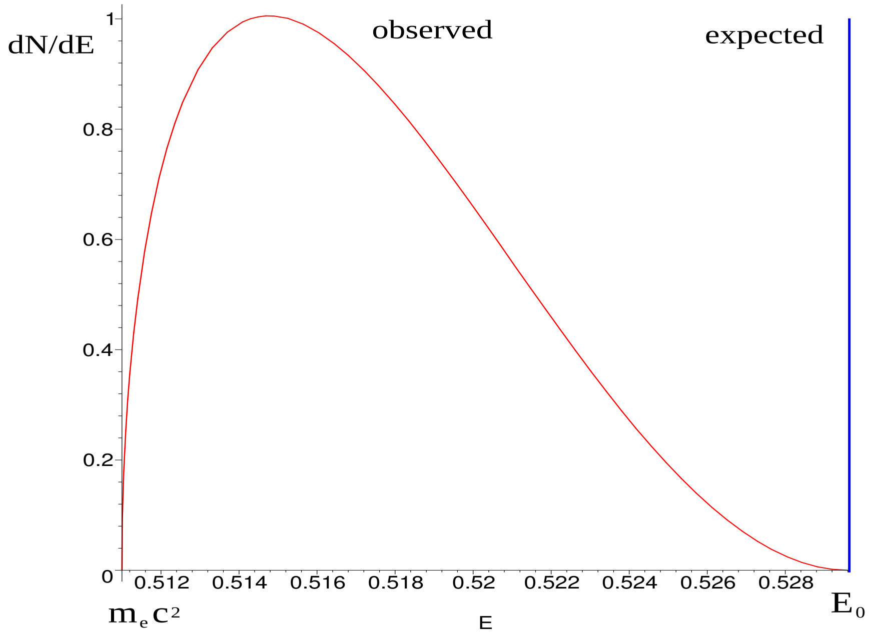

Pauli's "desperate way out" (1930):

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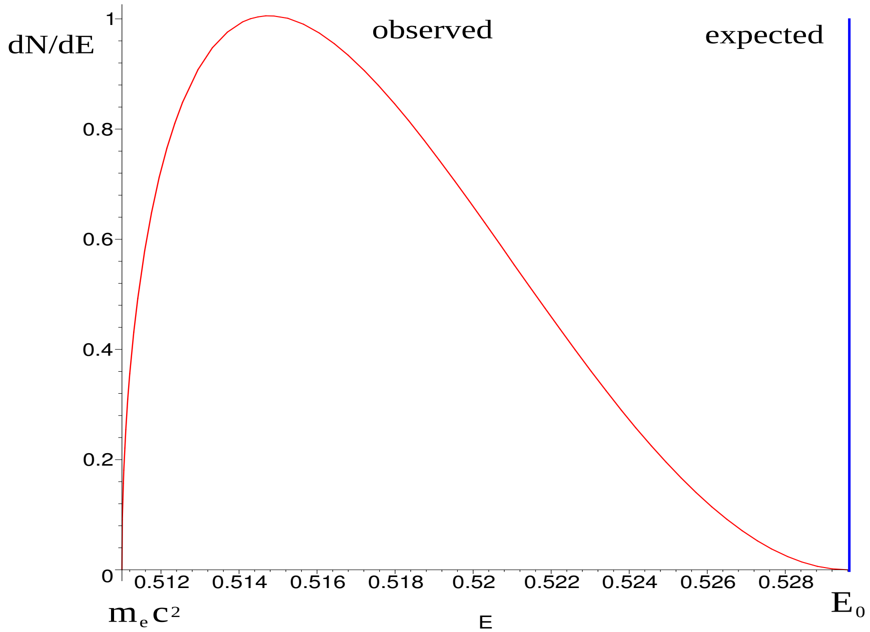

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– a very bold conjecture ! Experimentally observed 26 years later.

What do we know about neutrinos?

♢ Neutrinos: most mysterious elementary particles ever discovered

The history of their discovery and study very fascinating !

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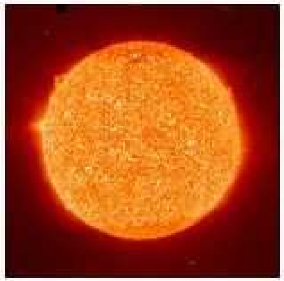
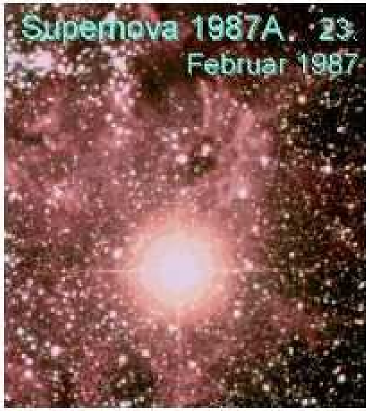
We already know a lot about neutrinos but many their properties are yet to be uncovered

Neutrinos are all-around!

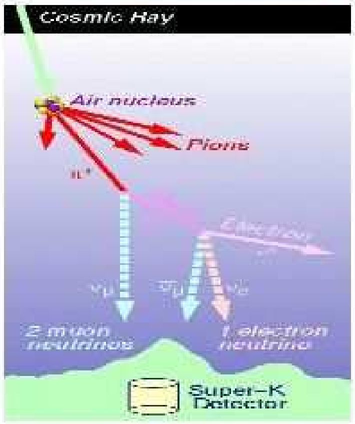


Sun

Earth

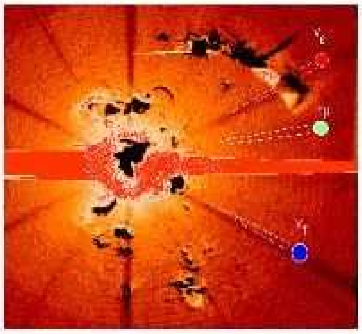


Supernovae



Atmosphere

Reactors



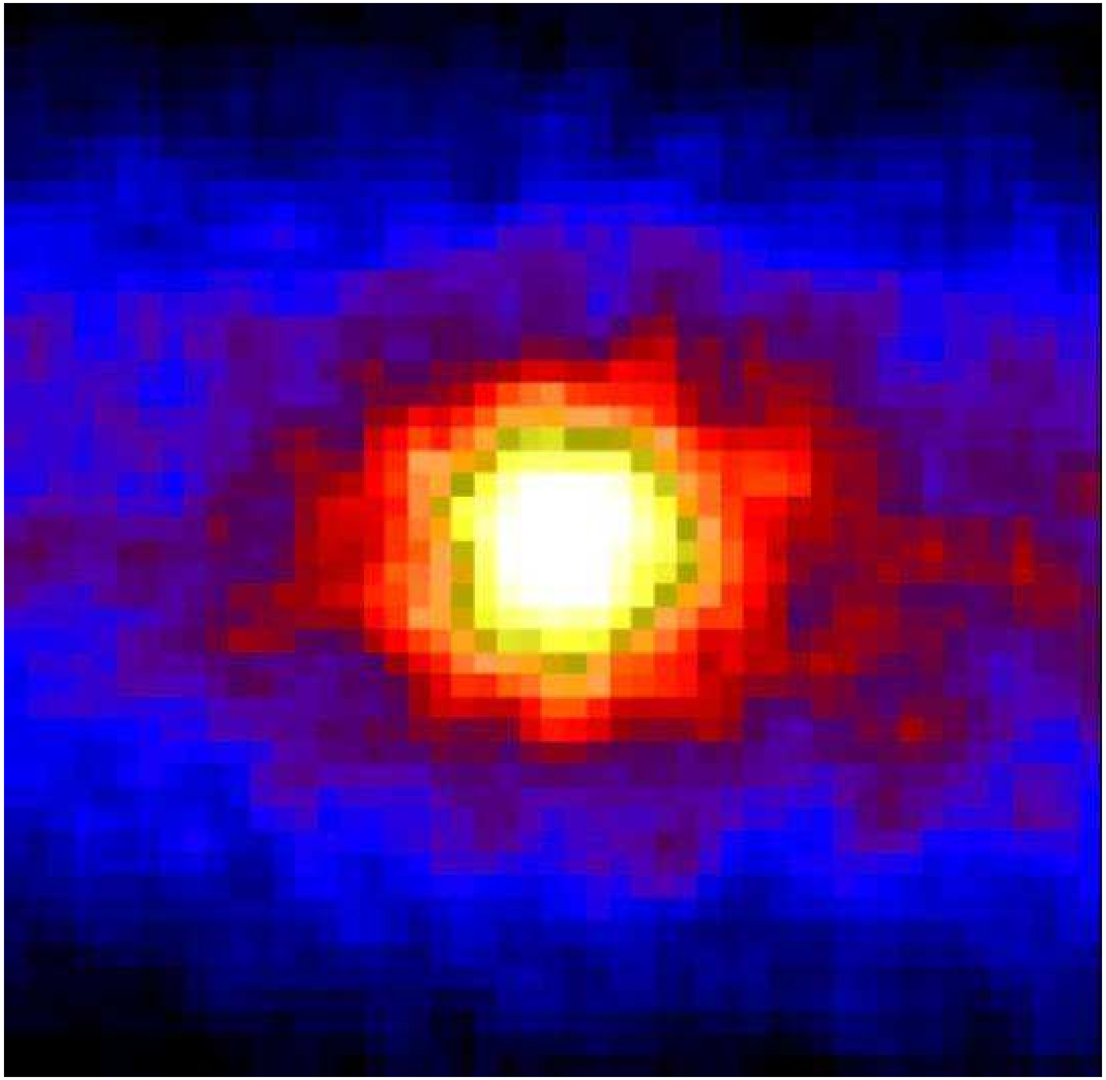
Big Bang

Accelerators &
Laboratories

$E_{SN} \sim 3 \times 10^{53}$ erg – 1000 times larger than the total energy emitted by the Sun.

◇ 99% of SN energy emitted in the form of neutrinos !

Seeing the Sun underground



The Sun still shines!

Neutrino fluxes at the Earth

The Sun:

$$\phi_\odot(\nu) \;\simeq\; 6 \times 10^{10}\;\; 1/(\mathrm{cm}^2\,\mathrm{s})$$

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So why do we care about them?

Why and Where are neutrinos interesting ?

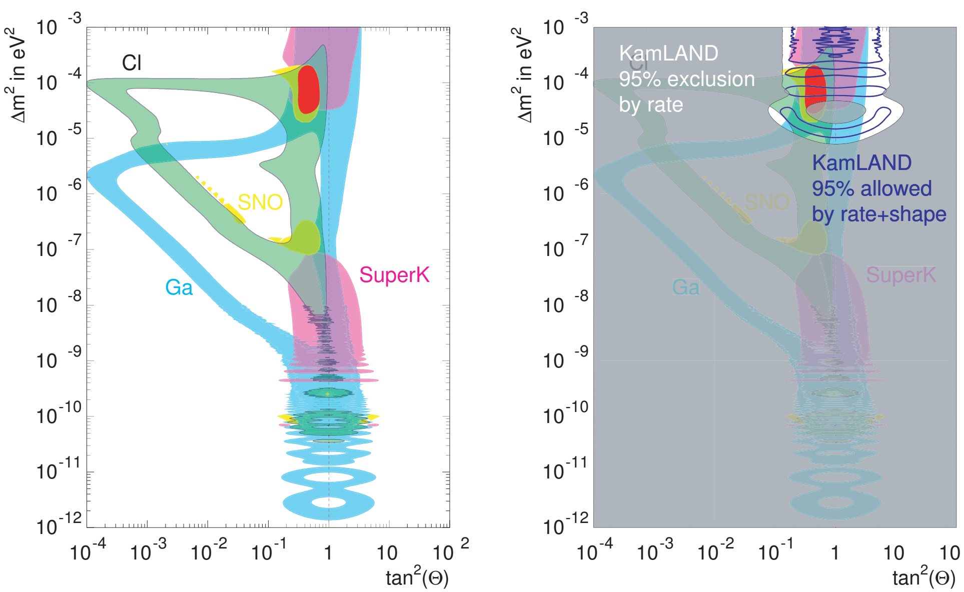
♢ Particle physics – ν's can probe very large mass (energy) scales; extra space-time dimensions; the only known particles that can be of Majorana nature

♢ Nuclear physics – clean probe of nuclear structure; cross sections important for studying neutrino properties

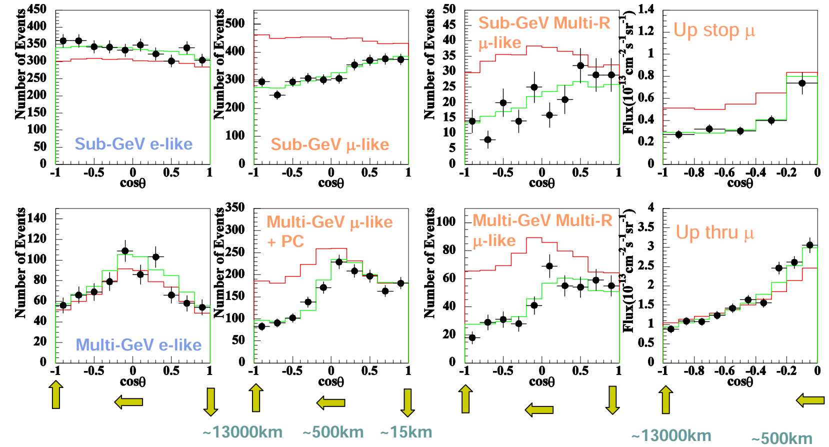
♢ Cosmology – nucleosynthesis, Dark Matter problem, baryogenesis (generation of the baryon asymmetry of the universe)

♢ Astrophysics – information on thermonuclear reactions powering our Sun; SN energetics

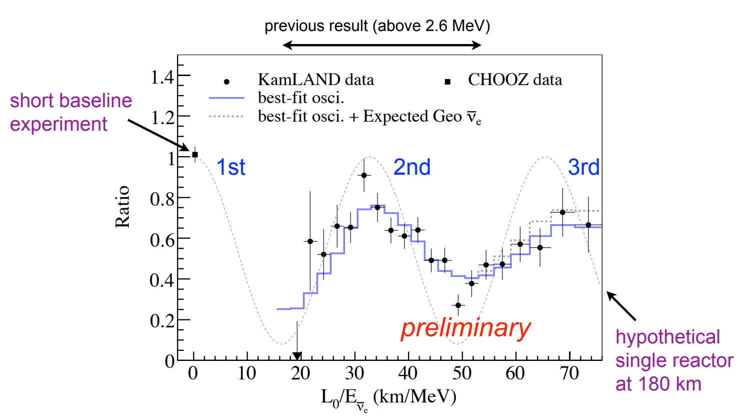
Neutrinos can oscillate !



Zenith angle distributions



Neutrino Oscillation

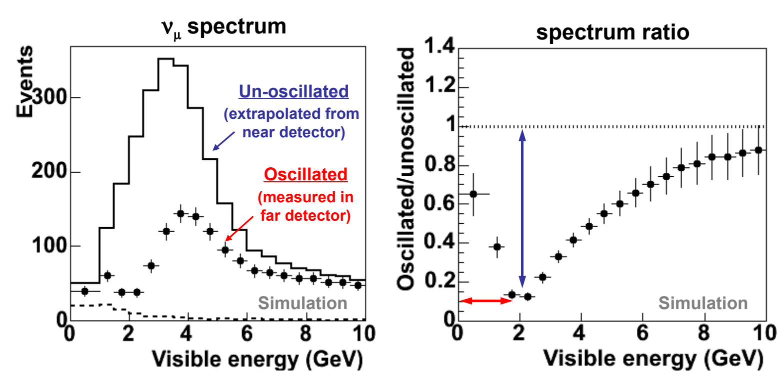


KamLAND covers the 2nd and 3rd maximum

→ **characteristic of neutrino oscillation**

ν_μ Disappearance Measurement

Look for ν_μ deficit : $P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{E}\right)$



Andy Blake, Cambridge University

The MINOS Experiment, slide 7

A brief *Curriculum Vitae* of neutrino

- Suggested by W. Pauli in 1930 to explain the continuous electron spectra in β-decay and nuclear spin/statistics
- Discovered by F. Reines and C. Cowan in 1956 in experiments with reactor $\bar{\nu}_e$ (Nobel prize to F. Reines in 1995)
- 1957 – the idea of neutrino oscillations put forward by B. Pontecorvo ($\nu \leftrightarrow \bar{\nu}$)
- 1957 – Chiral nature of ν_e established by Goldhaber, Grodzins & Sunyar
- 1962 – Discovery of the second neutrino type – ν_μ (Nobel prize to Lederman, Schwartz & Steinberger in 1988)
- 1962 – the idea of neutrino flavour oscillations put forward by Maki, Nakagawa & Sakata

- ♢ 1975 – Discovery of the third lepton flavour – τ lepton
(Nobel prize to M. Perl in 1995)

- ♢ 1985 – Theoretical discovery of resonant ν oscillations in matter by Mikheyev and Smirnov based on an earlier work of Wolfenstein
(the MSW effect)

- ♢ 1987 – First observation of neutrinos from supernova explosion (SN 1987A)

- ♢ 1998 – “Evidence for oscillations of atmospheric neutrinos” by the Super-Kamiokande Collaboration

- ♢ 2000 – Discovery of the third neutrino species – ν_τ by the DONUT Collaboration (Fermilab)

♢ 2002 – “Direct evidence for neutrino flavor transformation from neutral-current interactions in the Sudbury Neutrino Observatory” – flavor transformations of solar neutrinos confirmed

♢ 2002 – Discovery of oscillations of reactor neutrinos by KamLAND Collaboration; identification of the solution of the solar neutrino problem

♢ 2002 – Confirmation of oscillations of atmospheric neutrinos by K2K accelerator neutrino experiment

♢ 2002 – Nobel prize to R. Davis and M. Koshiba for “detection of cosmic neutrinos”

(2002 – “Annus Mirabilis” of neutrino physics)

♢ 2004 – Evidence for oscillatory nature of ν disappearance by Super-Kamiokande (atmospheric ν’s) and KamLAND.

- ♢ 2006 – Independent confirmation of oscillations of atmospheric neutrinos by MINOS accelerator neutrino experiment
- ♢ 2007 – First real-time detection of solar $^7\mathrm{Be}$ neutrinos by Borexino
- ♢ 2011/12 – Measurement of the last leptonic mixing angle θ_{13} by T2K, Double Chooz, Daya Bay and Reno
- ♢ 2012/14 – Detection of solar pep and pp neutrinos by Borexino
- ♢ 2015 – Nobel prize to Takaaki Kajita and Arthur McDonald "for the discovery of neutrino oscillations, which shows that neutrinos have mass"
- ♢ 2017 – First observation of coherent neutrino scattering on nuclei by the COHERENT Collaboration

...

More to come !

Neutrino revolution

Neutrino mass had been unsuccessfully looked for for almost 40 years (several wrong discovery claims)

Since 1998 – an avalanche of discoveries :

Oscillations of atmospheric, solar, reactor and accelerator neutrinos

Neutrino oscillations imply that neutrinos are massive

In the standard model neutrinos are massless $\Rightarrow$ we have now the first compelling evidence of physics beyond the standard model !

Weyl, Dirac and Majorana neutrino femions

Dirac equation:

$$(i\gamma^\mu \partial_\mu - m)\psi(x) = 0$$

The chiral (Weyl) representation of the Dirac γ-matrices:

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix},$$

LH and RH chirality projector operators:

$$P_L = \frac{\mathbb{1} - \gamma_5}{2}, \qquad P_R = \frac{\mathbb{1} + \gamma_5}{2}.$$

They have the following properties:

$$P_L^2 = P_L\,, \quad P_R^2 = P_R\,, \quad P_L P_R = P_R P_L = 0\,, \quad P_L + P_R = \mathbb{1}$$

LH and RH spinor fields: $\Psi_{R,L} = \frac{\mathbb{1} \pm \gamma_5}{2}\Psi\,, \qquad \Psi = \Psi_L + \Psi_R\,.$

Why LH and RH chirality? For relativistic particles chirality almost coincides with helicity (projection of the spin of the particle on its momentum).

$$P_{\pm} = \frac{1}{2}\left(1 \pm \frac{\boldsymbol{\sigma}\mathbf{p}}{|\mathbf{p}|}\right).$$

At $E \gg m$ positive-energy solutions satisfy

$$\Psi_R \simeq \Psi_+\,, \qquad\qquad \Psi_L \simeq \Psi_-\,.$$

N.B.: Helicity of a free particle is conserved; chirality is not (unless $m = 0$).

Particle - antiparticle conjugation operation $\hat{C}$:

$$\hat{C}: \qquad \psi \to \psi^c = \mathcal{C}\bar{\psi}^T$$

where $\bar{\psi} \equiv \psi^\dagger\gamma^0$ and $\mathcal{C}$ satisfies

$$\mathcal{C}^{-1}\gamma_\mu\mathcal{C} = -\gamma_\mu^T\,, \qquad \mathcal{C}^\dagger = \mathcal{C}^{-1} = -\mathcal{C}^* \quad (\Rightarrow\ \mathcal{C}^T = -\mathcal{C})\,.$$

In the Weyl representation: $\mathcal{C} \;=\; i\gamma^2\gamma^0$.

Some useful relations:

◊ $(\psi^c)^c = \psi\,, \quad \overline{\psi^c} = -\psi^T \mathcal{C}^{-1}\,, \quad \overline{\psi_1}\psi_2^c = \overline{\psi_2}\psi_1^c\,, \quad \overline{\psi_1} A \psi_2 = \overline{\psi_2^c}(\mathcal{C} A^T \mathcal{C}^{-1})\psi_1^c\,.$

(A – an arbitrary 4×4 matrix).

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$$\diamondsuit \qquad (\psi_L)^c = (\psi^c)_R\,, \qquad (\psi_R)^c = (\psi^c)_L\,,$$

i.e. the antiparticle of a left-handed fermion is right-handed.

$\diamond$ Problem: Prove these relations.

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From the expression for γ_5:

$$\psi_L = \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \qquad \psi_R = \begin{pmatrix} 0 \\ \xi \end{pmatrix},$$

$\Rightarrow$ Chiral fields are 2-component rather than 4-component objects.

Dirac *vs.* Majorana neutrino masses

Dirac equation in terms of 2-spinors ϕ and ξ:

$$(i\partial_0 - i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})\phi - m\xi = 0\,,$$
$$(i\partial_0 + i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})\xi - m\phi = 0\,.$$

Fermion mass couples LH and RH components of ψ. For $m=0$ eqs. for ϕ and ξ decouple (Weyl equations; Weyl fermions).

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Dirac Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi\,.$$

The fermion mass Lagrangian:

$$-\mathcal{L}_m \;=\; m\,\bar{\psi}\psi \;=\; m\,(\bar{\psi}_L + \bar{\psi}_R)(\psi_L + \psi_R) \;=\; m\,(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R)\,,$$

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LH and RH fields are necessary to make up a fermion mass.

Dirac fermions: ψ_L and ψ_R are completely independent fields

For Majorana fermions: $\psi_R \;=\; (\psi_L)^c$, where $(\psi)^c \;\equiv\; \mathcal{C}\,\bar{\psi}^T$.

Dirac *vs.* Majorana neutrino masses

Acting on a chiral field, particle-antiparticle conjugation flips its chirality:

$$(\psi_L)^c = (\psi^c)_R\,, \qquad (\psi_R)^c = (\psi^c)_L$$

(the antiparticle of a left handed fermion is right handed) $\Rightarrow$
one can construct a massive fermion field out of ψ_L and $(\psi_L)^c$:

$$\chi \;=\; \psi_L \;+\; (\psi_L)^c$$

$\Rightarrow$ Majorana field:

$$\chi^c \;=\; \chi$$

Majorana mass term:

$$-\mathcal{L}_m^{Maj} \;=\; \frac{m}{2}\,\overline{(\psi_L)^c}\,\psi_L \;+\; h.c. \;=\; -\,\frac{m}{2}\,\psi_L^T\mathcal{C}^{-1}\,\psi_L \;+\; h.c. \;=\; \frac{m}{2}\,\bar{\chi}\chi\,.$$

Breaks all charges (electric, lepton, baryon) – can only be written for entirely neutral fermions $\Rightarrow$ Neutrinos are the only known candidates!

D. and M. fields: plane wave decomposition

Plane-wave decomposition of a Dirac field:

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}} \sum_s \left[b_s(\vec{p})u_s(\vec{p})e^{-ipx} + d_s^\dagger(\vec{p})v_s(\vec{p})e^{ipx}\right]$$

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The spinors $u_s(\vec{p})$ and $v_s(\vec{p})$ satisfy

$$\mathcal{C}\,\overline{u}^T = v\,, \qquad \mathcal{C}\,\overline{v}^T = u \qquad \Rightarrow$$

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$$\boxed{\chi^c\;\equiv\;\mathcal{C}\bar{\chi}^T\;=\;\chi}$$

◇ Majorana particles are genuinely neutral (coincide with their antiparticles).

Fermion masses in the Standard Model

Come from Yukawa interactions of fermions with the Higgs field:

$$-\mathcal{L}_Y = h^u_{ij}\overline{Q}_{Li}u_{Rj}\tilde{H} + h^d_{ij}\overline{Q}_{Li}d_{Rj}H + f^e_{ij}\bar{l}_{Li}e_{Rj}H + h.c.$$

$$Q_{Li} = \binom{u_{Li}}{d_{Li}}, \qquad l_{Li} = \binom{\nu_{Li}}{e_{Li}}, \qquad H = \binom{H^+}{H^0}, \qquad \tilde{H} \;=\; i\tau_2 H^*$$

$u_{Ri},\ d_{Ri},\ e_{Ri}$ – $SU(2)_L$ - singlets.

EWSB: $\langle H^0\rangle = v \simeq 174$ GeV $\quad\Rightarrow\quad$ fermion mass matrices are generated:

$$\diamondsuit \qquad (m_u)_{ij} = h^u_{ij}v\,, \qquad (m_d)_{ij} = h^d_{ij}v\,, \qquad (m_e)_{ij} = f^e_{ij}v\,.$$

No RH neutrinos were introduced in the SM!

Why is $m_\nu = 0$ in the Standard Model ?

- No RH neutrinos N_{Ri} – Dirac mass terms cannot be introduced
- Operators of the kind $l\,l H H$, which could could produce Majorana neutrino mass after $H \to \langle H\rangle$, are dimension 5 and so cannot be present at the Lagrangian level in a renormalizable theory
- These operators cannot be induced in higher orders either (even nonperturbatively) because they would break not only lepton number L but also $B-L$, which is exactly conserved in the SM

<u>In the Standard Model:</u>

B and L are accidental symmetries at the Lagrangian level. Get broken at 1-loop level due the axial (triangle) anomaly. <u>But:</u> their difference $B-L$ is still conserved and is an exact symmetry of the model

Diagonalization of fermion mass matrices

I. Dirac fermions (e.g. charged leptons):

$$-\mathcal{L}_m \;=\; \sum_{a,b=1}^{N_f} m'_{ab}\,\bar{\Psi}'_{aL}\Psi'_{bR} + h.c. \;=\; \bar{\Psi}'_L m' \Psi'_R + \bar{\Psi}'_R m'^{\dagger} \Psi'_L$$

Rotate Ψ'_L and Ψ'_R by unitary transformations:

$$\Psi'_L = V_L \Psi_L\,, \quad \Psi'_R = V_R \Psi_R\,; \qquad\qquad m = V_L^{\dagger} m' V_R \;=\; diag.$$

Diagonalized mass term:

$$-\mathcal{L}_m \;=\; \bar{\Psi}_L (V_L^{\dagger} m' V_R) \Psi_R + h.c. \;=\; \sum_{i=1}^{N_f} m_i \bar{\Psi}_{iL} \Psi_{Ri} + h.c.$$

Mass eigenstate fields:

$$\Psi_i \;=\; \Psi_{iL} + \Psi_{iR}; \qquad\qquad -\mathcal{L}_m = \sum_{i=1}^{N_f} m_i\,\bar{\Psi}_i \Psi_i$$

Invariant w.r.t. $U(1)$ transfs. $\Psi_i \to e^{i\alpha_i}\Psi_i$ – conservs individual ferm. numbers

Diagonalization of fermion mass matrices

II. Majorana fermions:

$$\mathcal{L}_m \;=\; -\frac{1}{2}\sum_{a,b=1}^{N_f} m'_{ab}\,\overline{(\Psi'_{aL})^c}\,\Psi'_{bL} + h.c. \;=\; \frac{1}{2}\Psi'^T_L C^{-1}\,m'\Psi'_L + h.c.$$

Matrix m' is symmetric: $m'^T = m'$. $\diamond$ Problem: prove this.

Unitary transformation of Ψ'_L:

$$\Psi'_L = U_L\Psi_L\,, \qquad\qquad m = U_L^T m'\,U_L \;=\; diag.$$

Diagonalized mass term:

$$\mathcal{L}_m \;=\; \frac{1}{2}[\Psi_L^T C^{-1}(U_L^T m'\,U_L)\Psi_L + h.c. \;=\; \frac{1}{2}\sum_{i=1}^{N_f} m_i\Psi_{Li}^T\,C^{-1}\,\Psi_{Li} + h.c.$$

Mass eigenstate fields:

$$\chi_i \;=\; \Psi_{iL} + (\Psi_{iL})^c; \qquad\qquad \mathcal{L}_m = -\frac{1}{2}\sum_{i=1}^{N_f} m_i\,\bar{\chi}_i\chi_i$$

<u>Not</u> invariant w.r.t. $U(1)$ transfs. $\Psi_{Li} \to e^{i\alpha_i}\Psi_{Li}$

Neutrino masses and lepton flavour violation

For Dirac neutrinos the relevant terms in the Lagrangian are

$$-\mathcal{L}_{w+m} \;=\; \frac{g}{\sqrt{2}}(\bar{e}'_{La}\gamma^{\mu}\,\nu'_{La})\,W^{-}_{\mu} + (m'_l)_{ab}\,\bar{e}'_{Ra}e'_{Lb} + (m'_{\nu})_{ab}\,\bar{\nu}'_{Ra}\nu'_{Lb} \;+\; h.c.$$

Diagonalization of mass matrices:

$$e'_L \;=\; V_L\,e_L\,, \quad e'_R \;=\; V_R\,e_R\,, \quad \nu'_L \;=\; U_L\,\nu_L\,, \quad \nu'_R \;=\; U_R\,\nu_R$$

$$V_L^{\dagger}m'_lV_R \;=\; m_l\,, \qquad U_L^{\dagger}m'_{\nu}U_R \;=\; m_{\nu} \qquad (m_{l,\nu} \;-\; \text{diagonal mass matrices})$$

$$-\mathcal{L}_{w+m} \;=\; \frac{g}{\sqrt{2}}(\bar{e}_L\gamma^{\mu}\,V_L^{\dagger}U_L\,\nu_L)\,W^{-}_{\mu} \;+\; \text{diag. mass terms} + h.c.$$

For $m'_{\nu}=0$: without loss of generality one can consider both CC term and m_l term diagonal $\Rightarrow$ the Lagrangian is invariant w.r.t. three separate $U(1)$ transformations:

$$\diamondsuit \quad e_{La,Ra} \to e^{i\phi_a}e_{La,Ra}\,, \quad \nu_{La,Ra} \to e^{i\phi_a}\nu_{La,Ra} \qquad (a=e,\mu,\tau)$$

Neutrino masses and lepton flavour violation

$\Rightarrow$ For massles neutrinos three individual lepton numbers (lepton flavours) $L_e,\ L_\mu,\ L_\tau$ conserved.

For massive Dirac neutrinos $L_e,\ L_\mu,\ L_\tau$ are violated $\Rightarrow$ ν oscillations and $\mu \to e\gamma,\ \ \mu \to 3e,$ etc. allowed.

But: the total lepton number $L = L_e + L_\mu + L_\tau$ is conserved.

For massive Majorana neutrinos: individual lepton flavours $L_e,\ L_\mu,\ L_\tau$ and the total lepton number L are violated.

In addition to neutrino oscillations and LFV decays $2\beta 0\nu$ decay ($\Delta L = 2$ process) is allowed.

Why are neutrinos so light ?

In the minimal SM: $m_\nu = 0$. Add 3 RH ν's N_{Ri}:

$$-\mathcal{L}_Y \supset Y_\nu \bar{l}_L N_R H + h.c., \qquad l_{Li} = \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix}$$

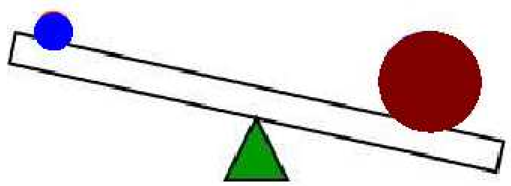
$\langle H^0 \rangle = v = 174\ \mathrm{GeV} \quad \Rightarrow \quad m_\nu = m_D = Y_\nu v$

$m_\nu < 1\ \mathrm{eV} \quad \Rightarrow \quad Y_\nu < 10^{-11}$ – Not natural !

Is it a problem? $Y_e \simeq 3 \times 10^{-6}$. But: with $m_\nu \neq 0$, huge disparity between the masses within each fermion generation !

A simple and elegant mechanism – <u>seesaw</u>
(Minkowski, 1977; Gell-Mann, Ramond & Slansky, 1979; Yanagida, 1979; Glashow, 1979; Mohapatra & Senjanović, 1980)

Heavy N_{Ri}'s make ν_{Li}'s light :



$$-\mathcal{L}_{Y+m} = Y_\nu \bar{l}_L N_R \tilde{H} + \frac{1}{2} M_R N_R N_R + h.c.,$$

In the $n_L = (\nu_L,\ (N_R)^c)^T$ basis: $-\mathcal{L}_m = \frac{1}{2} n_L^T C \mathcal{M}_\nu n_L + h.c.,$

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D^T \\ m_D & M_R \end{pmatrix}$$

N_{Ri} are EW singlets $\Rightarrow$ M_R can be $\sim M_{\rm GUT}(M_I) \gg m_D \sim v$.

Block diagonalization: $M_N \simeq M_R$,

$\diamondsuit$ $\boxed{m_{\nu_L} \simeq -m_D^T\, M_R^{-1}\, m_D}$ $\Rightarrow$ $m_\nu \sim \frac{(174\ {\rm GeV})^2}{M_R}$

For $m_\nu \lesssim 0.05\ {\rm eV}$ $\Rightarrow$ $M_R \gtrsim 10^{15}$ GeV$\sim M_{GUT} \sim 10^{16}$ GeV !

The (type I) seesaw mechanism

Consider the case of n LH and k RH neutrino fields:

$$\mathcal{L}_m = \frac{1}{2}\nu_L'^T\, \mathcal{C}^{-1}\, m_L\, \nu_L' - \overline{N_R'}\, m_D\, \nu_L' + \frac{1}{2} N_R'^T\, \mathcal{C}^{-1}\, M_R^*\, N_R' + h.c.$$

m_L and M_R – $n \times n$ and $k \times k$ symmetric matrices, m_D – an $k \times n$ matrix.

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Introduce an $n+k$ - component LH field

$$n_L = \begin{pmatrix} \nu_L' \\ (N_R')^c \end{pmatrix} = \begin{pmatrix} \nu_L' \\ N_L'^c \end{pmatrix} \qquad \Rightarrow$$

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$$\mathcal{L}_m = \frac{1}{2}\, n_L^T\, \mathcal{C}^{-1} \mathcal{M}\, n_L + h.c.\,,$$

where

$$\mathcal{M} = \begin{pmatrix} m_L & m_D^T \\ m_D & M_R \end{pmatrix} \qquad (\mathcal{M}:\ \text{matrix}\ (n+k)\times(n+k))$$

Problem: prove these formulas.

Block-diagonalization of $\mathcal{M}$

$$n_L = V \chi'_L \,, \qquad V^T \mathcal{M} V \, = \, V^T \begin{pmatrix} m_L & m_D^T \\ m_D & M_R \end{pmatrix} V \, = \begin{pmatrix} \tilde{m}_L & 0 \\ 0 & \tilde{M}_R \end{pmatrix}$$

Block-diagonalization of $\mathcal{M}$

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Look for the unitary matrix V in the form

$$V = \begin{pmatrix} \sqrt{1-\rho\rho^\dagger} & \rho \\ -\rho^\dagger & \sqrt{1-\rho^\dagger\rho} \end{pmatrix} \qquad (\rho : \text{ matrix } n \times k)$$

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Assume that characteristic scales of neutrino masses satisfy

$$m_L, m_D \ll m_R \qquad \Rightarrow \quad \rho \ll 1.$$

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Treat ρ as perturbation $\quad\Rightarrow$

$$\rho^* \simeq m_D^T M_R^{-1}\,, \qquad \tilde{M}_R \simeq M_R\,,$$

$$\boxed{\tilde{m}_L \simeq m_L - m_D^T M_R^{-1} m_D}$$

Type I seesaw mechanism – 1-gener. case

A simple 1-flavour case ($n = k = 1$). Notation change: $M_R \to m_R$, $N_R \to \nu_R$.

$$\mathcal{M} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \qquad (m_L,\, m_D,\, m_R \text{ — real positive numbers})$$

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Can be diagonalized as $O^T \mathcal{M} O = \mathcal{M}_d$ where O is real orthogonal 2×2 matrix and $\mathcal{M}_d = diag(m_1, m_2)$. Introduce the fields χ_L through $n_L = O\chi_L$:

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$$n_L = \begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \chi_{1L} \\ \chi_{2L} \end{pmatrix} \qquad (\chi_{1L},\, \chi_{2L} - \text{LH comp. of } \chi_{1,2})$$

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Rotation angle and mass eigenvalues:

$$\tan 2\theta = \frac{2m_D}{m_R - m_L}\,,$$

$$m_{1,2} = \frac{m_R + m_L}{2} \mp \sqrt{\left(\frac{m_R - m_L}{2}\right)^2 + m_D^2}\,.$$

$m_1,\, m_2$ real but can be of either sign

1-generation case – contd.

$$
\begin{aligned}
\mathcal{L}_m &= \frac{1}{2}\, n_L^T\, \mathcal{C}^{-1} \mathcal{M}\, n_L + h.c. = \frac{1}{2}\, \chi_L^T\, \mathcal{C}^{-1} \mathcal{M}_d\, \chi_L + h.c. \\
&= \frac{1}{2}(m_1\, \chi_{1L}^T\, \mathcal{C}^{-1} \chi_{1L} + m_2\, \chi_{2L}^T\, \mathcal{C}^{-1} \chi_{2L}) + h.c. = \frac{1}{2}(\, |m_1|\, \overline{\chi}_1 \chi_1 + |m_2|\, \overline{\chi}_2 \chi_2 \,)
\end{aligned}
$$

1-generation case – contd.

$$\begin{aligned}\mathcal{L}_m &= \frac{1}{2}\, n_L^T\, \mathcal{C}^{-1}\mathcal{M}\, n_L + h.c. = \frac{1}{2}\,\chi_L^T\,\mathcal{C}^{-1}\mathcal{M}_d\,\chi_L + h.c.\\ &= \frac{1}{2}(m_1\,\chi_{1L}^T\,\mathcal{C}^{-1}\chi_{1L} + m_2\,\chi_{2L}^T\,\mathcal{C}^{-1}\chi_{2L}) + h.c. = \frac{1}{2}(\,|m_1|\,\overline{\chi}_1\chi_1 + |m_2|\,\overline{\chi}_2\chi_2\,)\end{aligned}$$

Here

$$\chi_1 = \chi_{1L} + \eta_1(\chi_{1L})^c\,, \qquad \chi_2 = \chi_{2L} + \eta_2(\chi_{2L})^c\,.$$

with $\eta_i = 1$ or -1 for $m_i > 0$ or < 0 respectively.

1-generation case – contd.

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♢ Mass eigenstates $\chi_1,\ \chi_2$ are Majorana states!

1-generation case – contd.

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$$= \frac{1}{2}(m_1\, \chi_{1L}^T\, \mathcal{C}^{-1} \chi_{1L} + m_2\, \chi_{2L}^T\, \mathcal{C}^{-1} \chi_{2L}) + h.c. = \frac{1}{2}(\,|m_1|\, \overline{\chi}_1 \chi_1 + |m_2|\, \overline{\chi}_2 \chi_2\,)$$

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Interesting limiting cases:

(a) $m_R \gg m_L,\, m_D$ (seesaw limit)

$$m_1 \approx m_L - \frac{m_D^2}{m_R} \to -\frac{m_D^2}{m_R} \quad \text{for} \quad m_L = 0$$

$$m_2 \approx m_R$$

1-generation case – contd.

(b) $m_L = m_R = 0$ (Dirac case)

$$\mathcal{M} = \begin{pmatrix} 0 & m \\ m & 0 \end{pmatrix} \quad \rightarrow \quad \mathcal{M}_d = \begin{pmatrix} -m & 0 \\ 0 & m \end{pmatrix} .$$

1-generation case – contd.

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Diagonalized by rotation with angle $\theta = 45^\circ$. We have $\eta_2 = -\eta_1 = 1$;

$$\chi_1 + \chi_2 = \sqrt{2}(\nu_L + \nu_R)\,, \quad \chi_1 - \chi_2 = -\sqrt{2}(\nu_L^c + \nu_R^c) = -(\chi_1 + \chi_2)^c.$$

$$\Downarrow$$

1-generation case – contd.

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$$\Downarrow$$

$$\frac{1}{2}\,m\,(\overline{\chi}_1\chi_1 + \overline{\chi}_2\chi_2) = \frac{1}{4}\,m\,[\overline{(\chi_1 + \chi_2)}(\chi_1 + \chi_2) + [\overline{(\chi_1 - \chi_2)}(\chi_1 - \chi_2)] = m\,\bar{\nu}_D\nu_D\,,$$

where

$$\nu_D \equiv \nu_L + \nu_R\,.$$

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$$\Downarrow$$

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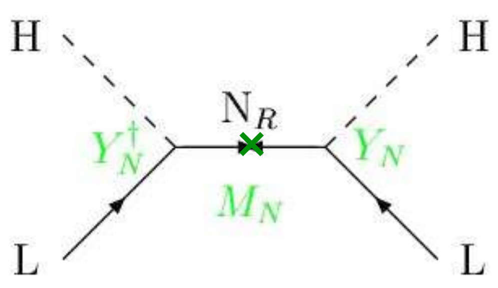
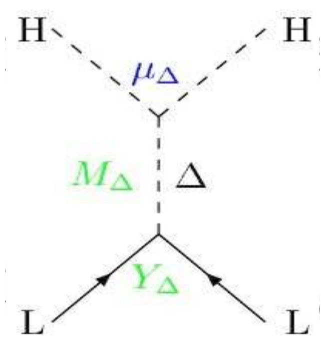
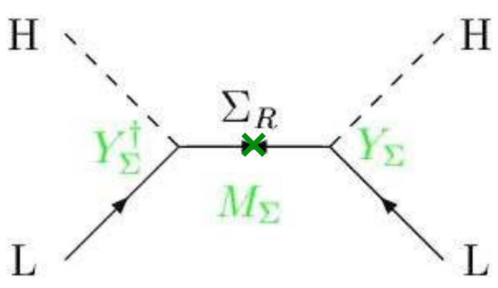
where

$$\nu_D \equiv \nu_L + \nu_R\,.$$

(c) $m_L, m_R \ll m_D$ (quasi-Dirac neutrino): $|m_{1,2}| \approx m_D \pm \frac{m_L + m_R}{2}$.

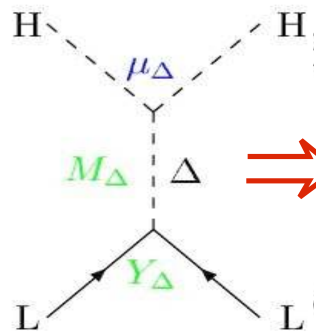
The 3 basic seesaw models

i.e. tree level ways to generate the dim 5 $\frac{\lambda}{M}LLHH$ operator

Right-handed singlet: (type-I seesaw)	Scalar triplet: (type-II seesaw)	Fermion triplet: (type-III seesaw)
		
$m_\nu = Y_N^T \frac{1}{M_N} Y_N v^2$	$m_\nu = Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$	$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$
m_ν small if M_N large (or if Y_ν small)	m_ν small if M_Δ large (or if Y_Δ, μ small)	m_ν small if M_Σ large (or if Y_Σ small)

Access to the seesaw parameters from ν mass matrix data

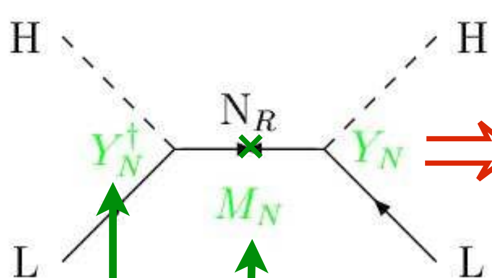
- Type II seesaw:



$$\Longrightarrow\quad m_{\nu ij} = Y_{\Delta ij}\frac{\mu_\Delta}{M_\Delta^2}v^2 \quad\Longrightarrow$$

ν mass matrix data gives full access to type II flavour structure

- Type I or III seesaw model:



$$\Longrightarrow\quad m_{\nu ij} = Y^T_{Nik}\frac{1}{M_{Nk}}Y_{Nkj}v^2 \quad\Longrightarrow$$

ν mass matrix data: gives access to 9 parameter combinations of Y_N and M_N

3 masses of the N

15 parameters in Yukawa matrix $\Rightarrow$ 9 real parameters, 6 phases

$\left.\begin{array}{l}\text{9 real parameters}\\ \text{6 phases}\end{array}\right\}$ 18 parameters

Sterile neutrinos – extra implications

- keV sterile neutrinos as dark matter
- Baryogenesis via leptogenesis
  - Decay of heavy sterile neutrinos
  - Baryogenesis via neutrino oscillations

Baryogenesis via leptogenesis

Baryogenesis via leptogenesis

(Kuzmin, Rubakov & Shaposhnikov, 1985; Fukugita & Yanagida, 1986; Luty, 1992; Covi et al., 1996; Buchmüller & Plümacher, 1996; ...)

◇ Seesaw has a built-in mechanism for generating the baryon asymmetry of the Universe! Observations:

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.04 \pm 0.08) \times 10^{-10}$$

Three Sakharov's conditions for generating an asymmetric Universe starting from a $B = 0$ state:

- Baryon number non-conservation (i)
- C and CP violation (ii)
- Deviation from thermal equilibrium (iii)

Baryogenesis via leptogenesis satisfies all of them !

Sakharov's conditions – an illustration

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X – an initial state with $B = 0$, Y – a set of final-state particles with net $B = 0$, b are the produced excess baryons.

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If the system is in thermal equilibrium, $X \to Y + b$ and $Y + b \to X$ occur at the same rate (also true for $\bar{X} \to \bar{Y} + \bar{b}$ and $\bar{Y} + \bar{b} \to \bar{X}$, of course) $\Rightarrow$ the baryon asymmetry produced in direct processes is washed out by the inverse ones.

Baryogenesis via leptogenesis – contd.

(1) Out-of-equilibrium CP and L violating decay of $N_1 \Rightarrow$
a net $L \neq 0$ is produced

L violation (due to Majorana nature of N_i):

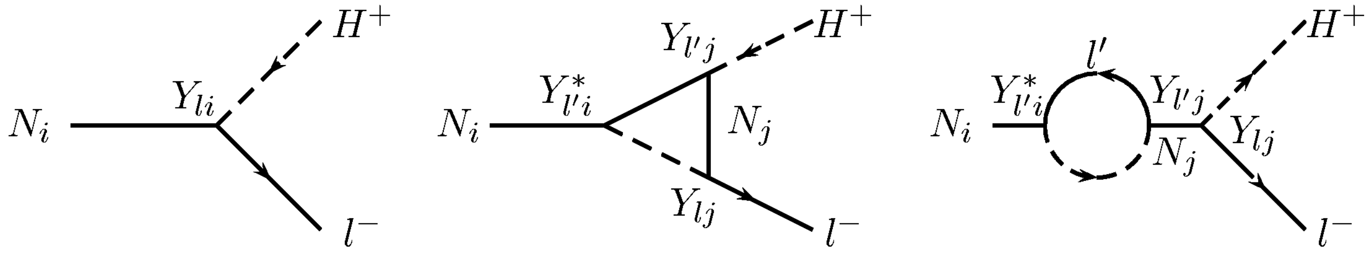
$$N_i \to l\,H\,, \qquad N_i \to \bar{l}\,\bar{H}$$

CP violation:

$$\Gamma(N_i \to lH) \neq \Gamma(N_i \to \bar{l}\bar{H})$$

at least 2 N_i needed

$N_i \to l^- H^+$:



$$\epsilon_1 = \sum_\alpha \frac{\Gamma(N_1 \to l_\alpha H) - \Gamma(N_1 \to \bar{l}_\alpha \bar{H})}{\Gamma(N_1 \to l\alpha H) + \Gamma(N_1 \to \bar{l}_\alpha \bar{H})} = \frac{1}{8\pi}\frac{1}{(Y_\nu^\dagger Y_\nu)_{11}} \sum_{i\neq 1} \mathrm{Im}[(Y_\nu^\dagger Y_\nu)^2_{1i}]\, g(M_i^2/M_1^2)$$

Baryogenesis via leptogenesis – contd.

In the standard model:

$$g(x) = \sqrt{x}\left[\frac{2-x}{1-x} - (1+x)\ln\left(\frac{1+x}{x}\right)\right]$$

N.B.:

- In the formula for ϵ_1 for simplicity summation over the flavours of final-state leptons performed, but flavour effects may actually be important
- The expression for ϵ_1 is valid only when $|M_j - M_i| \gg \Gamma_i + \Gamma_j$; the opposite case (resonant leptogenesis, Pilaftsis & Underwood, 2004, 2005) requires a special consideration.

Out-of-equilibrium decay condition:

$$\Gamma_1 = \frac{(Y_\nu^\dagger Y_\nu)_{11}}{8\pi} M_1 \; < \; H(T = M_1)$$

Hubble parameter:

$$H(T) \;=\; \frac{2\pi^{3/2}}{3\sqrt{5}}\, g_*^{1/2} \frac{T^2}{M_{\rm Pl}} \;\simeq\; 1.66\, g_*^{1/2} \frac{T^2}{M_{\rm Pl}}$$

g_*– eff. number of degrees of freedom of particles in equilibrium
For SM +1 RH singlet neutrino: $g_* = 434/4 = 108.5$.

Introduce

$$\tilde{m}_1 \;\equiv\; \frac{(m_D^\dagger m_D)_{11}}{M_1} \;=\; 8\pi \frac{v^2}{M_1^2}\Gamma_1$$

Condition $\Gamma_1 \; < \; H(T = M_1) \;\; \Rightarrow \;\; \boxed{\tilde{m}_1 \; < \; m_*}$, where

$$m_* \;\equiv\; \frac{16\pi^{5/2}}{3\sqrt{5}}\, g_*^{1/2} \frac{v^2}{M_{\rm Pl}} \;\simeq\; 1.1 \times 10^{-3} \;\; \text{eV}$$

Sphaleron mechanism: $L \to B$ reprocessing

Not too strong washout $\Rightarrow$ upper limit on m_ν:

$$\boxed{\bar{m} \equiv (m_1^2 + m_2^2 + m_3^2)^{1/2} \lesssim 0.1 \text{ eV}}$$

(2) Reprocessing of the produced L into B by electroweak sphalerons

SM: At tree level, B and L are conserved. Broken at 1-loop level by triangle anomalies. But: $\Delta B = \Delta L \Rightarrow$

$B - L$ is conserved!

Non-perturbative EW field configurations – sphalerons:
conserve $B - L$ but efficiently wash out $B + L$ for

$$10^2 \text{ GeV} \lesssim T \lesssim 10^{12} \text{ GeV}.$$

Because

$$B = \frac{1}{2}[(B+L) + (B-L)], \qquad L = \frac{1}{2}[(B+L) - (B-L)],$$

$L \to B$ reprocessing – contd.

$$B(t) \;=\; \frac{1}{2}(B+L)_0\, e^{-\Gamma_{\rm sph} t} \;+\; \frac{1}{2}(B-L)_0$$

$$L(t) \;=\; \frac{1}{2}(B+L)_0\, e^{-\Gamma_{\rm sph} t} \;-\; \frac{1}{2}(B-L)_0$$

Initially $(t=0)$: $B=0, \quad L=L_0 \neq 0$

At $t \gg \Gamma_{\rm sph}^{-1}$: $\quad B = \frac{1}{2}(B-L)_0 = -\frac{1}{2}\,L_0$!

More accurate calculation: $B = \frac{28}{79}(B-L)_0$.

The produced baryon asymmetry: $\boxed{\eta_B \;\simeq\; 10^{-2}\,\epsilon_1\,\kappa}$

κ – washout factor. Very approximately:

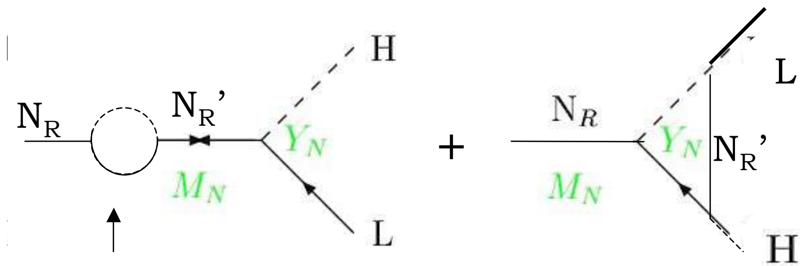
$$\kappa \;\sim\; \frac{0.3}{K(\ln K)^{0.6}} \qquad\qquad K \;\equiv\; \frac{\tilde{m}_1}{m_*}$$

◇ Viable η_B produced for $M_1 \;\gtrsim\; 10^9$ GeV (for non-degenerate N_i's)

Matter-antimatter asymmetry of the universe

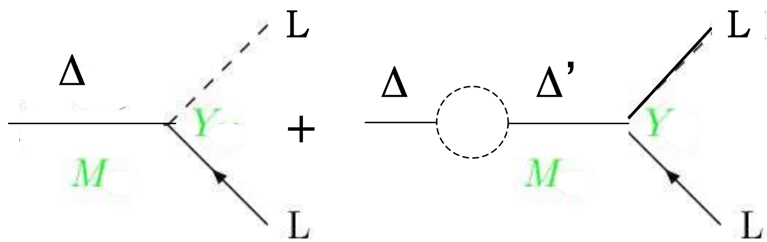
from the seesaw interactions responsible for neutrino masses one can also explain baryogenesis via leptogenesis

- type-I:



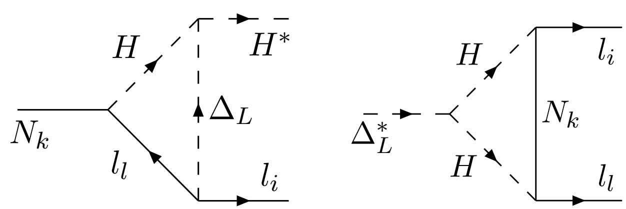
Fukugita, Yanagida 86',
Buchmüller, Plumacher 97', ...

- type-II:



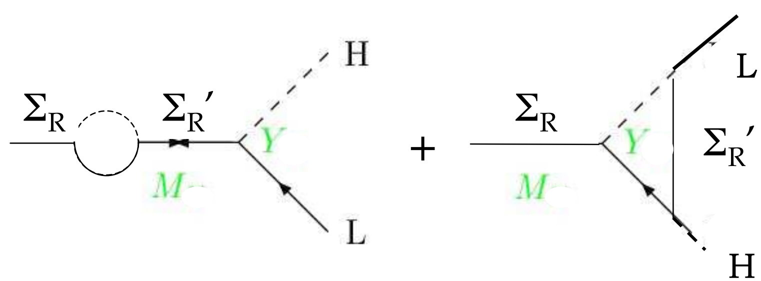
Ma, Sarkar 98', ...

- type-I+ type II:



O'Donnell, Sarkar 94',
TH, Senjanovic 04';
Antusch, King 04', ...

- type-III:



TH, Lin, Notari, Papucci, Strumia 04'; ...

Backup slides