Neutrino physics (2)

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Plan of the lectures

- Introduction.
- Brief overview of experimental results
- Weyl, Dirac and Majorana fermions
- Neutrino masses in simplest extensions of the Standard Model. The seesaw mechanism(s).
- Neutrino oscillations in vacuum
  - Same E or same p ?
  - QM uncertainties and coherence issues
  - Wave packet approach to neutrino oscillations
  - Lorentz invariance of oscillation probabilities
  - 2f and 3f neutrino mixing schemes and oscillations
  - Implications of CP, T and CPT

Plan of the lectures – contd.

- Neutrino oscillations in matter – the MSW effect
  - Evolution equation
  - Adiabaticity condition and adiabatic evolution
  - Non-adiabatic regime
  - Graphical interpretation and mechanical analogy
  - Earth matter effects on $\nu_\odot$ (day-night asymmetry)
- Neutrino oscillations in matter – parametric resonance
- Direct neutrino mass measurement experiments
- Neutrinoless double β-decay
- Neutrino electromagnetic properties
- Subtleties of the theory of neutrino oscillations
  - Do charged leptons oscillate?
  - Oscillations of Mössbauer neutrinos
- Neutrinos and the baryon asymmetry of the universe

Plan of the lectures – contd.

- Exptl. results: Solar neutrino oscillations and KamLAND
- Oscillations of atmospheric and accelerator neutrinos
- Discovery of θ_{13} in reactor and accelerator expts.
- Future: What's next?

What is left out:

– Oscillations of SN neutrinos (incl. non-linear collective effects)

– Cosmological bounds on # of neutrino species and $\sum m_\nu$

– keV sterile neutrinos as Dark Matter

– Geoneutrinos

. . .

Neutrino oscillations

Neutrinos can oscillate !

A periodic change of neutrino flavour (identity):

$$\nu_e \to \nu_\mu \to \nu_e \to \nu_\mu \to \nu_e \ \ \ldots$$

Happens without any external influence!

Dr. Jekyll / Mr. Hyde kind of story

Neutrinos have two-sided (or even 3-sided) personality !

$$P(\nu_e \to \nu_\mu; L) = \sin^2 2\theta \cdot \sin^2\left(\frac{\Delta m^2}{4p}L\right)$$

Hints of oscillations of solar neutrinos seen since the 1960s

First unambiguous evidence – oscillations of atmospheric neutrinos (The Super-Kamiokande Collaboration, 1998)

A bit of history...

Idea of neutrino oscillations: First put forward by Pontecorvo in 1957. Suggested possibility of $\nu \leftrightarrow \bar{\nu}$ oscillations by analogy with $K^0 \bar{K}^0$ oscillations.

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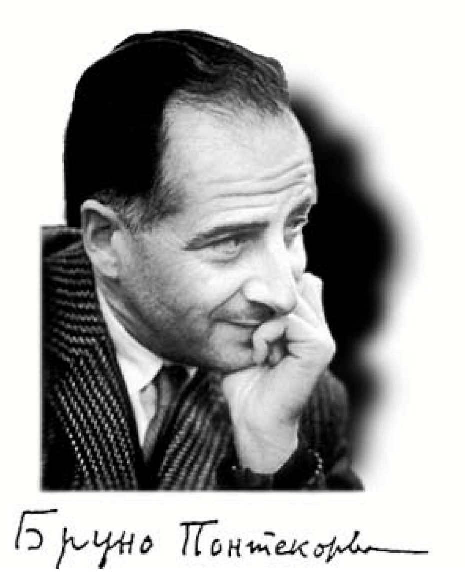
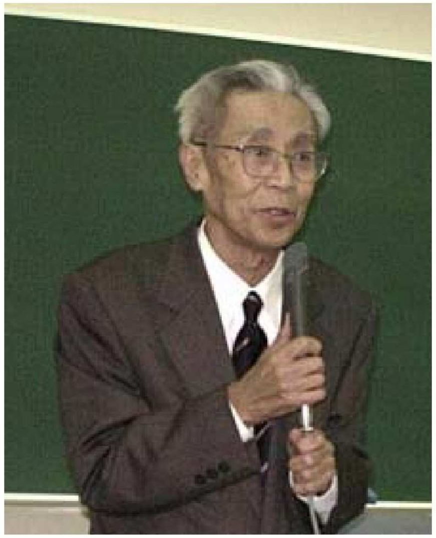
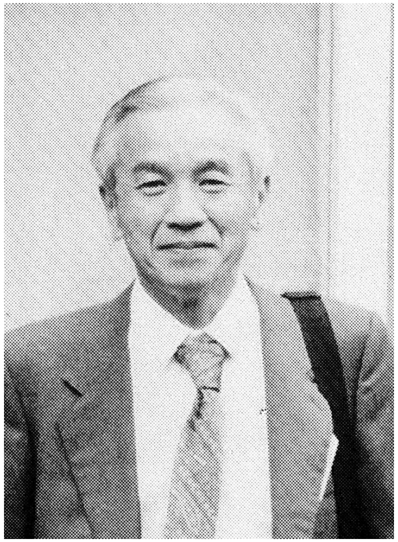


B. Pontecorvo
1913 - 1993

S. Sakata
1911 – 1970

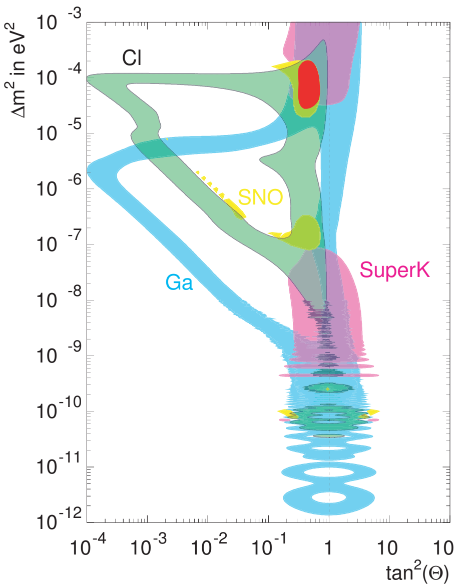


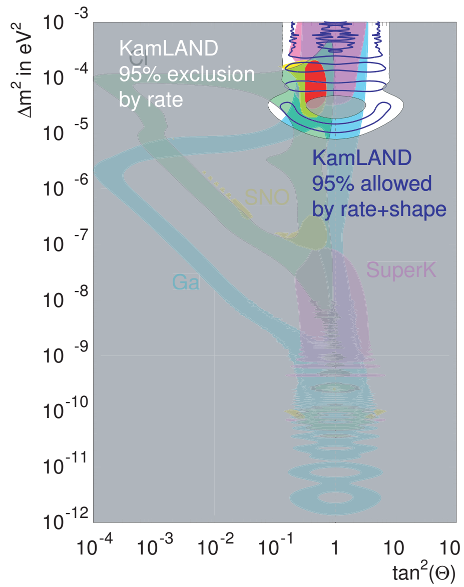
Z. Maki
1929 – 2005



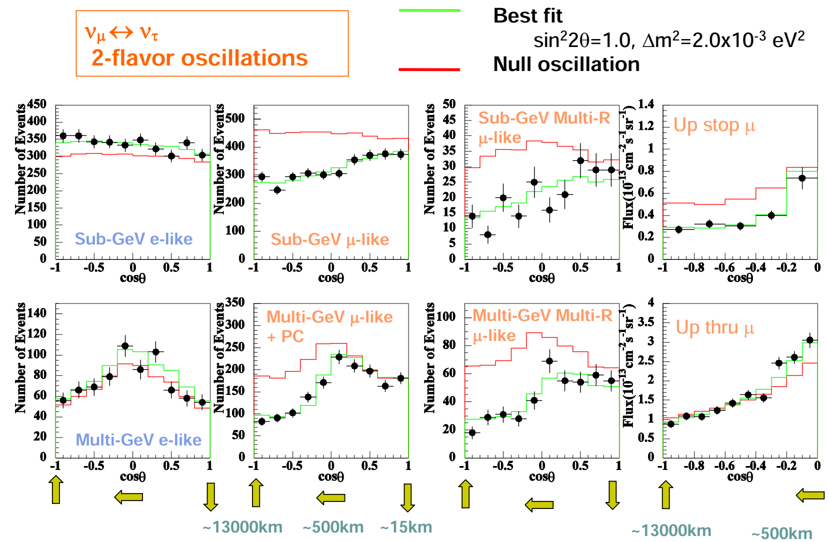
M. Nakagawa
1932 – 2001

Oscillations discovered experimentally !

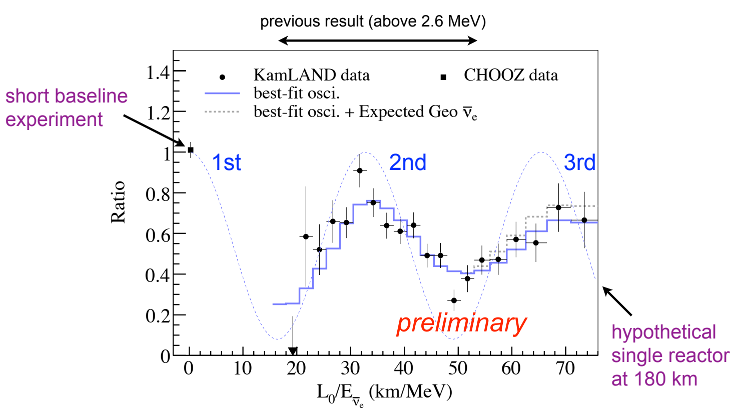




Zenith angle distributions

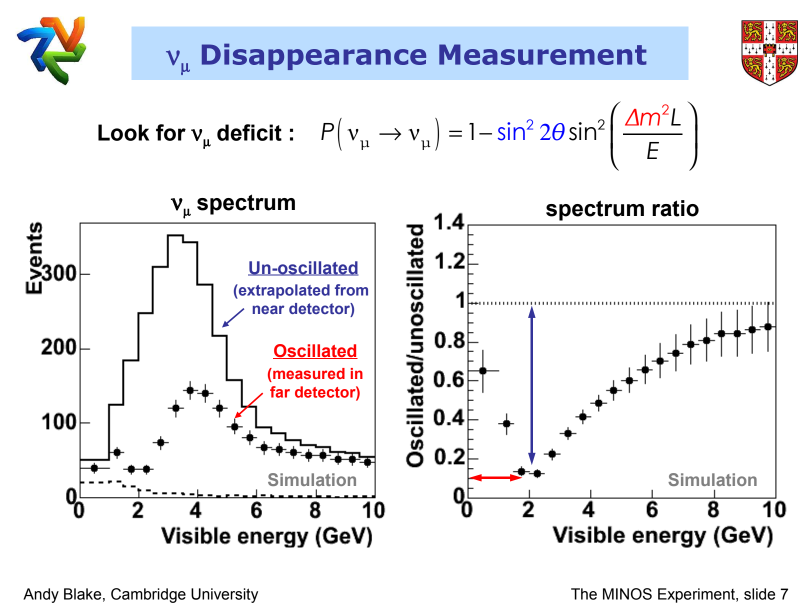


Neutrino Oscillation

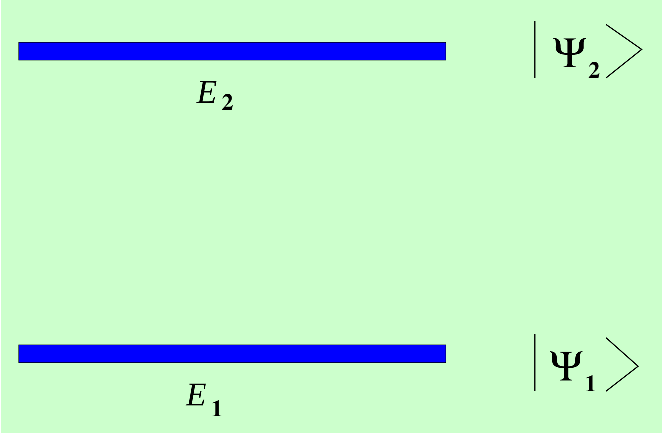


KamLAND covers the 2nd and 3rd maximum

→ **characteristic of neutrino oscillation**



Oscillations: a well known QM phenomenon



$$\Psi_1(t) = e^{-i\,E_1\,t}\,\Psi_1(0)$$

$$\Psi_2(t) = e^{-i\,E_2\,t}\,\Psi_2(0)$$

$$\Psi(0) = a\,\Psi_1(0) + b\,\Psi_2(0) \quad (|a|^2 + |b|^2 = 1)\ ; \qquad \Rightarrow$$

$$\Psi(t) = a\,e^{-i\,E_1\,t}\,\Psi_1(0) + b\,e^{-i\,E_2\,t}\,\Psi_2(0)$$

Probability to remain in the same state $|\Psi(0)\rangle$ after time t:

♢ $$P_{\rm surv} = |\langle\Psi(0)|\Psi(t)\rangle|^2 = \left||a|^2\,e^{-i\,E_1\,t} + |b|^2\,e^{-i\,E_2\,t}\right|^2$$

$$= 1 - 4|a|^2|b|^2\sin^2[(E_2 - E_1)\,t/2]$$

Neutrino oscillations: theory

Leptonic mixing

For $m_\nu \neq 0$ weak eigenstate neutrinos ν_e, ν_μ, ν_τ do not coincide with mass eigenstate neutrinos ν_1, ν_2, ν_3

Diagonalization of leptonic mass matrices:

$$e'_L \to V_L \, e_L \,, \qquad \nu'_L \to U_L \, \nu_L \,\ldots \qquad \Rightarrow$$

$$-\mathcal{L}_{w+m} = \frac{g}{\sqrt{2}}(\bar{e}_L \gamma^\mu \, V_L^\dagger U_L \, \nu_L) \, W_\mu^- + \text{diag. mass terms} + h.c.$$

Leptonic mixing matrix: $U = V_L^\dagger U_L$

♢ $$\nu_{\alpha L} = \sum_i U_{\alpha i} \, \nu_{iL} \qquad \Rightarrow \qquad |\nu_{\alpha L}\rangle = \sum_i U_{\alpha i}^* \, |\nu_{iL}\rangle$$

$$(\alpha = e\,,\ \mu\,,\ \tau, \qquad i = 1\,,\ 2\,,\ 3)$$

Master formula for ν oscillations

The standard formula for the oscillation probability of relativistic or quasi-degenerate in mass neutrinos in vacuum:

$$\diamondsuit \qquad \boxed{P(\nu_\alpha \to \nu_\beta; L) = \left| \sum_i U_{\beta i}\, e^{-i\frac{\Delta m_{ij}^2}{2p}L}\, U_{\alpha i}^* \right|^2}$$

$$(\hbar = c = 1)$$

Problem: prove that the RHS does not depend on the index j.

Oscillation disappear when either

- $U = \mathbb{1}$, i.e. $U_{\alpha i} = \delta_{\alpha i}$ (no mixing) or
- $\Delta m_{ij}^2 = 0$ (massless or mass-degenerate neutrinos).

How is it usually derived?

Assume at time $t = 0$ and coordinate $x = 0$ a flavour eigenstate $|\nu_\alpha\rangle$ is produced:

$$|\nu(0,0)\rangle \;=\; |\nu_\alpha^{\rm fl}\rangle \;=\; \sum_i U_{\alpha i}^*\,|\nu_i^{\rm mass}\rangle$$

After time t at the position x, for plane-wave particles:

$$|\nu(t,\vec{x})\rangle \;=\; \sum_i U_{\alpha i}^*\,e^{-ip_i x}|\nu_i^{\rm mass}\rangle$$

Mass eigenstates pick up the phase factors $e^{-i\phi_i}$ with

$$\phi_i \;\equiv\; p_i\,x \;=\; Et \;-\; \vec{p}\,\vec{x}$$

$$P(\nu_\alpha \to \nu_\beta) \;=\; \left|\langle\nu_\beta^{\rm fl}|\nu(t,x)\rangle\right|^2$$

How is it usually derived?

Consider $\vec{x}\,||\,\vec{p} \quad \Rightarrow \quad \vec{p}\vec{x} = \mathrm{px} \quad (\mathrm{p} = |\vec{p}|, \quad \mathrm{x} = |\vec{x}|)$

Phase differences between different mass eigenstates:

$$\Delta\phi = \Delta E \cdot t - \Delta \mathrm{p} \cdot \mathrm{x}$$

<u>Shortcuts to the standard formula</u>

1. Assume the emitted neutrino state has a well defined momentum (same momentum prescription) $\Rightarrow \quad \Delta\mathrm{p} = 0.$

   For ultra-relativistic neutrinos $E_i = \sqrt{\mathrm{p}^2 + m_i^2} \simeq p + \frac{m_i^2}{2\mathrm{p}} \quad \Rightarrow$

$$\Delta E \simeq \frac{m_2^2 - m_1^2}{2E} \equiv \frac{\Delta m^2}{2E}\,; \qquad t \approx x \qquad (\hbar = c = 1)$$

$\Rightarrow$ The standard formula is obtained

How is it usually derived?

2. Assume the emitted neutrino state has a well defined energy (same energy prescription) $\Rightarrow \quad \Delta E = 0$.

$$\Delta\phi = \Delta E \cdot t - \Delta \mathrm{p} \cdot \mathrm{x} \quad \Rightarrow \quad - \Delta \mathrm{p} \cdot \mathrm{x}$$

For ultra-relativistic neutrinos $\mathrm{p}_i = \sqrt{E^2 - m_i^2} \simeq E - \frac{m_i^2}{2\mathrm{p}} \quad \Rightarrow$

$$-\Delta \mathrm{p} \equiv \mathrm{p}_1 - \mathrm{p}_2 \approx \frac{\Delta m^2}{2E};$$

$\Rightarrow$ The standard formula is obtained

Stand. phase $\Rightarrow$ $\boxed{(l_{\mathrm{osc}})_{ik} = \frac{4\pi E}{\Delta m_{ik}^2} \simeq 2.5\ m\ \frac{E\,(\mathrm{MeV})}{\Delta m_{ik}^2\ \mathrm{eV}^2}}$

Same E and same p approaches

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Very simple and transparent

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Allow one to quickly arrive at the desired result

Same E and same p approaches

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Allow one to quickly arrive at the desired result

Trouble: they are both wrong

Kinematic constraints

Same momentum and same energy assumptions: contradict kinematics!

Pion decay at rest $(\pi^+ \to \mu^+ + \nu_\mu, \quad \pi^- \to \mu^- + \bar{\nu}_\mu)$:
For decay with emission of a massive neutrino of mass m_i:

$$E_i^2 = \frac{m_\pi^2}{4}\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2 + \frac{m_i^2}{2}\left(1-\frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_i^4}{4m_\pi^2}$$

$$p_i^2 = \frac{m_\pi^2}{4}\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2 - \frac{m_i^2}{2}\left(1+\frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_i^4}{4m_\pi^2}$$

For massless neutrinos: $E_i = p_i = E \equiv \frac{m_\pi}{2}\left(1-\frac{m_\mu^2}{m_\pi^2}\right) \simeq 30 \ \mathrm{MeV}$

To first order in m_i^2:

$$E_i \simeq E + \xi\frac{m_i^2}{2E}, \qquad p_i \simeq E - (1-\xi)\frac{m_i^2}{2E}, \qquad \xi = \frac{1}{2}\left(1-\frac{m_\mu^2}{m_\pi^2}\right) \approx 0.2$$

Kinematic constraints

Same momentum or same energy would require
$\xi = 1$ or $\xi = 0$ – not the case!

Also: would violate Lorentz invariance of the oscillation probability

How can wrong assumptions lead to the correct oscillation formula ?

Problems with the plane-wave approach

- Same momentum $\Rightarrow$ oscillation probabilities depend only on time. Leads to a paradoxical result – no need for a far detector ! "Time-to-space conversion" (??) – assumes neutrinos to be point-like particles (notion opposite to plane waves).
- Same energy – oscillation probabilities depend only on coordinate. Does not explain how neutrinos are produced and detected at certain times. Correspponds to a stationary situation.

Plane wave approach $\Leftrightarrow$ exact energy-momentum conservation. Neutrino energy and momentum are fully determined by those of external particles $\Rightarrow$ only one mass eigenstate can be emitted!

- Consistent approaches:

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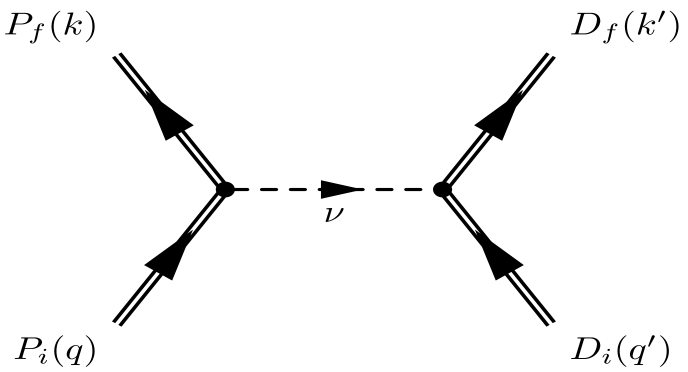
- QM wave packet approach – neutrinos described by wave packets rather than by plane waves

♢ Consistent approaches:

- QM wave packet approach – neutrinos described by wave packets rather than by plane waves
- QFT approach: neutrino production and detection explicitly taken into account. Neutrinos are intermediate particles described by propagators

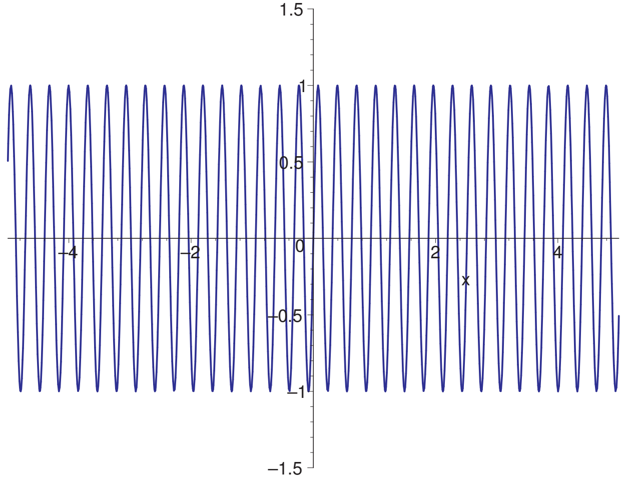


QM wave packet approach

In QM propagating particles are described by wave packets!

– Finite extensions in space and time.

Plane waves: the wave function at time $t = 0$ $\quad \Psi_{\vec{p}_0}(\vec{x}) = e^{i\vec{p}_0\vec{x}}$



Wave packets: superpositions of plane waves with momenta in an interval of width σ_p around mom. p_0 $\Rightarrow$ constructive interference in a spatial interval of width σ_x around some point x_0 and destructive interference outside it.

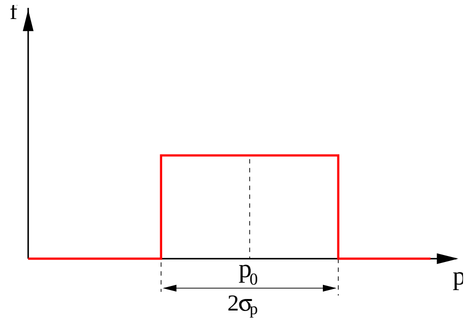
$$\sigma_x\,\sigma_p \geq 1/2 \quad \text{– QM uncertainty relation}$$

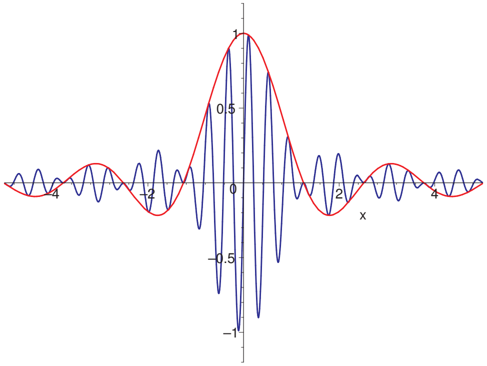
Wave packets

W. packet centered at $\vec{x}_0 = 0$ at time $t = 0$:

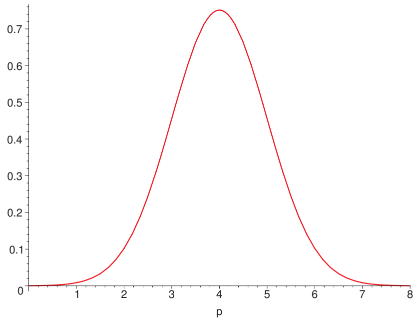
$$\Psi(\vec{x};\, \vec{p}_0, \sigma_{\vec{p}}) \;=\; \int \frac{d^3p}{(2\pi)^3}\, f(\vec{p}-\vec{p}_0)\, e^{i\vec{p}\,\vec{x}}$$

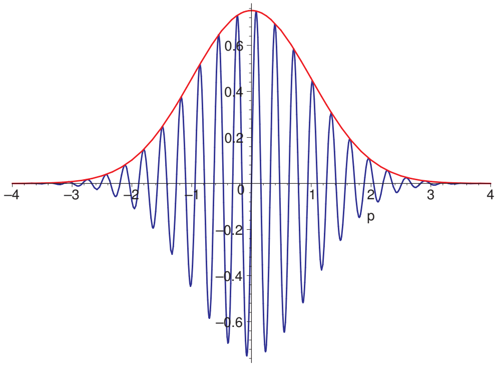
Rectangular mom. space w. packet:





Gaussian mom. space w. packet:





$\sigma_x \sigma_p \;=\; 1/2$ – minimum uncertainty packet

Propagating wave packets

Include time dependence:

$$\Psi(\vec{x},\,t) \;=\; \int \frac{d^3p}{(2\pi)^3}\, f(\vec{p}-\vec{p}_0)\, e^{i\vec{p}\vec{x}-iE(p)t}$$

Example: Gaussian wave packets

Momentum-space distribution:

$$f(\vec{p}-\vec{p}_0) \;=\; \frac{1}{(2\pi\sigma_p^2)^{3/4}}\exp\left\{-\frac{(\vec{p}-\vec{p}_0)^2}{4\sigma_p^2}\right\}$$

Momentum dispersion: $\langle\vec{p}^{\,2}\rangle - \langle\vec{p}\,\rangle^2 = \sigma_p^2$.

Coordinate-space wave packet (neglecting spreading):

$$\Psi(\vec{x},\,t) \;=\; e^{i\vec{p}_0\vec{x}-iE(p_0)t}\;\frac{1}{(2\pi\sigma_x^2)^{3/4}}\exp\left\{-\frac{(\vec{x}-\vec{v}_g t)^2}{4\sigma_x^2}\right\}, \qquad \sigma_x^2 = 1/(4\sigma_p^2)$$

$$\langle\vec{x}\,\rangle = \vec{v}_g t\,; \qquad \langle\vec{x}^{\,2}\rangle - \langle\vec{x}\,\rangle^2 = \sigma_x^2\,.$$

QM wave packet approach

The evolved produced state:

$$|\nu_\alpha^{\rm fl}(\vec{x},t)\rangle \;=\; \sum_i U_{\alpha i}^* \,|\nu_i^{\rm mass}(\vec{x},t)\rangle \;=\; \sum_i U_{\alpha i}^* \,\Psi_i^S(\vec{x},t)|\nu_i^{\rm mass}\rangle$$

The coordinate-space wave function of the ith mass eigenstate (w. packet):

$$\Psi_i^S(\vec{x},t) \;=\; \int \frac{d^3p}{(2\pi)^3}\, f_i^S(\vec{p})\, e^{i\vec{p}\vec{x}-iE_i(p)t}$$

Momentum distribution function $f_i^S(\vec{p})$: sharp maximum at $\vec{p}=\vec{P}$ (width of the peak $\sigma_{pP} \ll P$).

$$E_i(p) \;=\; E_i(P) \;+\; \frac{\partial E_i(p)}{\partial \vec{p}}\bigg|_{\vec{P}} (\vec{p}-\vec{P}) \;+\; \frac{1}{2}\frac{\partial^2 E_i(p)}{\partial \vec{p}^2}\bigg|_{\vec{p}_0} (\vec{p}-\vec{P})^2 \;+\; \ldots$$

$$\vec{v}_i \;=\; \frac{\partial E_i(p)}{\partial \vec{p}} \;=\; \frac{\vec{p}}{E_i}\,, \qquad\qquad \alpha \;\equiv\; \frac{\partial^2 E_i(p)}{\partial \vec{p}^2} \;=\; \frac{m_i^2}{E_i^2}$$

Evolved neutrino state

$$\Psi_i^S(\vec{x},\, t) \;\simeq\; e^{-iE_i(P)t+i\vec{P}\vec{x}}\, g_i^S(\vec{x}-\vec{v}_i t) \qquad (\alpha \;\to\; 0)$$

$$g_i^S(\vec{x}-\vec{v}_i t) \;\equiv\; \int \frac{d^3q}{(2\pi)^3}\, f_i^S(\vec{q}+\vec{P})\, e^{i\vec{q}(\vec{x}-\vec{v}_g t)}$$

Problem: derive this result

Center of the wave packet: $\vec{x}-\vec{v}_i t = 0$. Spatial length: $\sigma_{xP} \;\sim\; 1/\sigma_{pP}$
(g_i^S decreases quickly for $|\vec{x}-\vec{v}_i t| \gtrsim \sigma_{xP}$).

Detected state (centered at $\vec{x}=\vec{L}$):

$$|\nu_\beta^{\rm fl}(\vec{x})\rangle \;=\; \sum_k U_{\beta k}^*\, \Psi_k^D(\vec{x})|\nu_k^{\rm mass}\rangle$$

The coordinate-space wave function of the ith mass eigenstate (w. packet):

$$\Psi_i^D(\vec{x}) \;=\; \int \frac{d^3p}{(2\pi)^3}\, f_i^D(\vec{p})\, e^{i\vec{p}(\vec{x}-\vec{L})}$$

Oscillation probability

Transition amplitude:

$$\mathcal{A}_{\alpha\beta}(T,\vec{L}) \;=\; \langle\nu_\beta^{\rm fl}|\nu_\alpha^{\rm fl}(T,\vec{L})\rangle \;=\; \sum_i U_{\alpha i}^* U_{\beta i}\,\mathcal{A}_i(T,\vec{L})$$

$$\boxed{\mathcal{A}_i(T,\vec{L}) \;=\; \int\frac{d^3p}{(2\pi)^3}\,f_i^S(\vec{p})\,f_i^{D*}(\vec{p})\,e^{-iE_i(p)T+i\vec{p}\vec{L}}}$$

Strongly suppressed unless $|\vec{L}-\vec{v}_i T|\lesssim\sigma_x$. E.g., for Gaussian wave packets:

$$\mathcal{A}_i(T,\vec{L})\;\propto\;\exp\left[-\frac{(\vec{L}-\vec{v}_i T)^2}{4\sigma_x^2}\right]\,, \qquad \sigma_x^2\;\equiv\;\sigma_{xP}^2+\sigma_{xD}^2$$

Oscillation probability:

♢ $P(\nu_\alpha\to\nu_\beta;T,\vec{L}) \;=\; |\mathcal{A}_{\alpha\beta}|^2 = \sum_{i,k} U_{\alpha i}^* U_{\beta i} U_{\alpha k} U_{\beta k}^*\,\mathcal{A}_i(T,\vec{L})\mathcal{A}_k^*(T,\vec{L})$

Phase difference

Oscillations are due to phase differences of different mass eigenstates:

$$\Delta\phi = \Delta E \cdot T - \Delta p \cdot L \qquad (E_i = \sqrt{p_i^2 + m_i^2})$$

Consider the case $\Delta E \ll E$ (relativistic or quasi-degenerate neutrinos) $\Rightarrow$

$$\Delta E = \frac{\partial E}{\partial p}\Delta p + \frac{\partial E}{\partial m^2}\Delta m^2 = v_g\,\Delta p + \frac{1}{2E}\,\Delta m^2$$

$$\Delta\phi = (v_g\,\Delta p + \frac{1}{2E}\Delta m^2)\,T - \Delta p \cdot L$$

$$= -\,(L - v_g\,T)\Delta p + \frac{\Delta m^2}{2E}T$$

In the center of wave packet $(L - v_g\,T) = 0$! In general, $|L - v_g\,T| \lesssim \sigma_x$;
if $\sigma_x \ll l_{\rm osc}$, $\quad |L - v_g\,T|\Delta p \ll 1 \qquad \Rightarrow$

$$\Delta\phi \;=\; \frac{\Delta m^2}{2E}\,T\,, \qquad\qquad L \;\simeq\; v_g T \;\simeq T$$

– the result of the “same momentum” approach recovered!

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Now instead of expressing ΔE through Δp and Δm^2 express Δp through ΔE and Δm^2:

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♢ $$\Delta\phi = -\frac{1}{v_g}(L - v_g\,T)\Delta E + \frac{\Delta m^2}{2p}L \quad \Rightarrow \quad \frac{\Delta m^2}{2p}L$$

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The reasons why wrong assumptions give the correct result:

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- Neutrinos are relativistic or quasi-degenerate with $\Delta E \ll E$

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– the result of the “same energy” approach recovered!

The reasons why wrong assumptions give the correct result:

- Neutrinos are relativistic or quasi-degenerate with $\Delta E \ll E$
- The size of the neutrino wave packet is small compared to the oscillation length: $\sigma_x \ll l_{\rm osc}$ (more precisely: energy uncertainty $\sigma_E \gg \Delta E$)

Oscillation probability in WP approach

$$P(\nu_\alpha \to \nu_\beta; T, \vec{L}) \;=\; |\mathcal{A}_{\alpha\beta}|^2 = \sum_{i,k} U_{\alpha i}^* U_{\beta i} U_{\alpha k} U_{\beta k}^* \, \mathcal{A}_i(T, \vec{L}) \mathcal{A}_k^*(T, \vec{L})$$

$$\mathcal{A}_i(T, \vec{L}) \;=\; \int \frac{d^3 p}{(2\pi)^3} \, f_i^S(\vec{p}) \, f_i^{D*}(\vec{p}) \, e^{-iE_i(p)T + i\vec{p}\vec{L}}$$

Oscillation probability in WP approach

Neutrino emission and detection times are not measured (or not accurately measured) in most experiments $\Rightarrow$ integration over T:

$$P(\nu_\alpha \to \nu_\beta; L) = \int dT\, P(\nu_\alpha \to \nu_\beta; T, L) = \sum_{i,k} U_{\alpha i}^* U_{\beta i} U_{\alpha k} U_{\beta k}^*\, e^{-i\frac{\Delta m_{ik}^2}{2\bar{P}}L}\, \tilde{I}_{ik}$$

$$\tilde{I}_{ik} = N \int \frac{dq}{2\pi} f_i^S(r_k q - \Delta E_{ik}/2v + P_i) f_i^{D*}(r_k q - \Delta E_{ik}/2v + P_i)$$

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Here: $v \equiv \frac{v_i + v_k}{2}$, $\Delta v \equiv v_k - v_i$, $r_{i,k} \equiv \frac{v_{i,k}}{v}$, $N \equiv 1/[2E_i(P)2E_k(P)v]$,

Problem: derive this result. Hint: use $\Delta E_{ik} \simeq v \Delta p_{ik} + \Delta m_{ik}^2/2E$ and go to the shifted integration variable $q \equiv p - P$ where $P \equiv (P_i + P_k)/2$.

When are neutrino oscillations observable?

Keyword: *Coherence*

Neutrino flavour eigenstates ν_e, ν_μ and ν_τ are coherent superpositions of mass eigenstates ν_1, ν_2 and ν_3 $\Rightarrow$ oscillations are only observable if

- neutrino production and detection are coherent
- coherence is not (irreversibly) lost during neutrino propagation.

Possible decoherence at production (detection): If by accurate E and p measurements one can tell (through $E = \sqrt{p^2 + m^2}$) which mass eigenstate is emitted, the coherence is lost and oscillations disappear!

Full analogy with electron interference in double slit experiments: if one can establish which slit the detected electron has passed through, the interference fringes are washed out.

When are neutrino oscillations observable?

Another source of decoherence: wave packet separation due to the difference of group velocities Δv of different mass eigenstates.

If coherence is lost: Flavour transition can still occur, but in a non-oscillatory way. E.g. for $\pi \to \mu\nu_i$ decay with a subsequent detection of ν_i with the emission of e:

$$P \;\propto\; \sum_i P_{\rm prod}(\mu\,\nu_i)P_{\rm det}(e\,\nu_i) \;\propto\; \sum_i |U_{\mu i}|^2 |U_{ei}|^2$$

– the same result as for averaged oscillations.

How are the oscillations destroyed? Suppose by measuring momenta and energies of particles at neutrino production (or detection) we can determine its energy E and momentum p with uncertainties σ_E and σ_p. From $E_i = \sqrt{p_i^2 + m_i^2}$:

$$\sigma_{m^2} \;=\; \left[(2E\sigma_E)^2 \,+\, (2p\sigma_p)^2\right]^{1/2}$$

When are neutrino oscillations observable?

If $\sigma_{m^2} < \Delta m^2 = |m_i^2 - m_k^2|$ – one can tell which mass eigenstate is emitted.

$\sigma_{m^2} < \Delta m^2$ implies $2p\sigma_p < \Delta m^2$, or $\sigma_p < \Delta m^2/2p \simeq l_{\rm osc}^{-1}$.

But: To measure p with the accuracy σ_p one needs to measure the momenta of particles at production with (at least) the same accuracy $\Rightarrow$ uncertainty of their coordinates (and the coordinate of ν production point) will be

$$\sigma_{\rm x,\,prod} \gtrsim \sigma_p^{-1} > l_{\rm osc}$$

$\Rightarrow$ Oscillations washed out. Similarly for neutrino detection.

Natural necessary condition for coherence (observability of oscillations):

$$L_{\rm source} \ll l_{\rm osc}\,, \qquad L_{\rm det} \ll l_{\rm osc}$$

No averaging of oscillations in the source and detector

Satisfied with very large margins in most cases of practical interest

Wave packet separation

Wave packets representing different mass eigenstate components have different group velocities v_{gi} $\Rightarrow$ after time $t_{\rm coh}$ (coherence time) they separate $\Rightarrow$ Neutrinos stop oscillating! (Only averaged effect observable).

Coherence time and length:

$$\Delta v \cdot t_{\rm coh} \simeq \sigma_x\,; \qquad l_{\rm coh} \simeq v t_{\rm coh}$$

$$\Delta v = \frac{p_i}{E_i} - \frac{p_k}{E_k} \simeq \frac{\Delta m^2}{2E^2}$$

$$\boxed{l_{\rm coh} \simeq \frac{v}{\Delta v}\sigma_x = \frac{2E^2}{\Delta m^2}\, v\sigma_x}$$

The standard formula for $P_{\rm osc}$ is obtained when the decoherence effects are negligible.

A manifestation of neutrino coherence

Even non-observation of neutrino oscillations at distances $L \ll l_{\rm osc}$ is a consequence of and an evidence for coherence of neutrino emission and detection! Two-flavour example (e.g. for ν_e emission and detection):

$$A_{\rm prod/det}(\nu_1) \sim \cos\theta\,, \qquad A_{\rm prod/det}(\nu_2) \sim \sin\theta \qquad \Rightarrow$$

$$A(\nu_e \to \nu_e) = \sum_{i=1,2} A_{\rm prod}(\nu_i) A_{\rm det}(\nu_i) \sim \cos^2\theta + e^{-i\Delta\phi}\sin^2\theta$$

Phase difference $\Delta\phi$ vanishes at short L $\Rightarrow$

$$P(\nu_e \to \nu_e) = (\cos^2\theta + \sin^2\theta)^2 = 1$$

If ν_1 and ν_2 were emitted and absorbed incoherently) $\Rightarrow$ one would have to sum probabilities rather than amplitudes:

$$P(\nu_e \to \nu_e) \sim \sum_{i=1,2} |A_{\rm prod}(\nu_i) A_{\rm det}(\nu_i)|^2 \sim \cos^4\theta + \sin^4\theta \;<\; 1$$

Are coherence constraints compatible?

Observability conditions for ν oscillations:

- Coherence of ν production and detection
- Coherence of ν propagation

Both conditions put upper limits on neutrino mass squared differences Δm^2 :

$$(1) \quad \Delta E_{jk} \sim \frac{\Delta m_{jk}^2}{2E} \ll \sigma_E; \qquad (2) \quad \frac{\Delta m_{jk}^2}{2E^2} L \ll \sigma_x \simeq v_g / \sigma_E$$

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But: The constraints on σ_E work in opposite directions:

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Are they compatible? – Yes, if LHS $\ll$ RHS $\Rightarrow$

$$\boxed{2\pi \frac{L}{l_{\rm osc}} \ll \frac{v_g}{\Delta v_g} \; (\gg 1)}$$

– fulfilled in all cases of practical interest

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Sterile neutrinos: hints from SBL accelerator experiments (LSND, MiniBooNE), reactor neutrino anomaly, keV sterile neutrinos, pulsar kicks, leptogenesis via ν oscillations, SN r-process nucleosynthesis, unconventional contributions to $2\beta0\nu$ decay ...

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Production/detection coherence has to be re-checked – important implications for some neutrino experiments!

Neutrino oscillations: *Coherence at macroscopic distances –*
$L >$ 10,000 km in atmospheric neutrino experiments !

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- For $(\Delta v/v)\sigma_p L \ll 1$ (i.e. $L \ll l_{\rm coh} = (v/\Delta v)\sigma_x$) $\tilde{I}_{ik}$ is approximately independent of L; in the opposite case $\tilde{I}_{ik}$ is strongly suppressed

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- $\tilde{I}_{ik}$ is also strongly suppressed unless $\Delta E_{ik}/v \ll \sigma_p$, i.e. $\Delta E_{ik} \ll \sigma_E$ – coherent production/detection condition

The standard osc. probability?

The standard formula for the oscillation probability corresponds to $\tilde{I}_{ik} = 1$.

If the two above conditions are satisfied, $\tilde{I}_{ik}$ is not suppressed and is L-, E-and i, k-independent (i.e. a constant).

The standard probability is obtained when this constant is 1 (normalization necessary!)

Normaliz. condition:

$$\int \frac{d^3p}{(2\pi)^3} |f_i^S(\vec{p})|^2 |f_i^D(\vec{p})|^2 = 1$$

The normalization prescription

Oscillation probability calculated in QM w. packet approach is not automatically normalized ! Can be normalized "by hand" by imposing the unitarity condition:

$$\sum_\beta P_{\alpha\beta}(L) \;=\; 1\,.$$

This gives

$$\int dT |\mathcal{A}_i(L,T)|^2 = 1 \quad \Rightarrow \quad \tilde{I}_{ii} = N_1 \int \frac{dp}{2\pi v} |f_i^S(p)|^2\, |f_i^D(p)|^2 \;=\; 1$$

– important for proving Lorentz invariance of the oscillation probability.

Depends on the overlap of $f_i^S(p)$ and $f_i^S(p)$ $\Rightarrow$ no independent normalization of the produced and detected neutrino wave function would do!

In QFT approach the correctly normalized $P_{\alpha\beta}(L)$ is automatically obtained and the meaning of the normalization procedure adopted in the w. packet approach clarified

Oscillations and QM uncertainty relations

Neutrino oscillations – a QM interference phenomenon, owe their existence to QM uncertainty relations

Neutrino energy and momentum are characterized by uncertainties σ_E and σ_p related to the spatial localization and time scale of the production and detection processes. These uncertainties

- allow the emitted/absorbed neutrino state to be a coherent superposition of different mass eigenstates
- determine the size of the neutrino wave packets $\Rightarrow$ govern decoherence due to wave packet separation

σ_E – the effective energy uncertainty, dominated by the smaller one between the energy uncertainties at production and detection. Similarly for σ_p.

The paradox of σ_E and σ_p

QM uncertainty relations: σ_p is related to the spatial localization of the production (detection) process, while σ_E to its time scale $\Rightarrow$ independent quantities.

On the other hand: Neutrinos propagating macroscopic distances are on the mass shell. For on-shell mass eigenstates $E^2 = p^2 + m_i^2$ means

$$E\sigma_E = p\sigma_p$$

How can this be understood?

The solution: At production, neutrinos are *not* on the mass shell. They go on shell only after they propagate $x \sim$ (a few)$\times$ De Broglie wavelengths. After that their energy and momentum get related by $E^2 = p^2 + m_i^2 \quad \Rightarrow$ the larger uncertainty shrinks towards the smaller one to satisfy $E\sigma_E = p\sigma_p$.

On-shell relation between E and p allows to determine the less certain of the two through the more certain one, reducing the error of the former.

What determines the length of ν w. packets?

The length of ν w. packets: $\sigma_x \sim 1/\sigma_p$. For propagating on-shell neutrinos:

$$\sigma_p \simeq \min\{\sigma_p^{\rm prod},\, (E/p)\sigma_E^{\rm prod}\} = \min\{\sigma_p^{\rm prod},\, (1/v_g)\sigma_E^{\rm prod}\}$$

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  But: $L_S = v_S T_S \quad \Rightarrow \quad \sigma_E < \sigma_p$ (a consequence of $v_S < 1$)

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- If $T_S > \tau$ (quasi-free parent particle) $\Rightarrow$ $\sigma_E \simeq \tau^{-1} = \Gamma$.

  $\sigma_p \simeq [(p/E)\tau]^{-1} \simeq [(p/E)\sigma_E]^{-1}$, i.e. $\sigma_E \simeq (p/E)\sigma_p < \sigma_p$.

The length of ν w. packets – contd.

In both cases $\boxed{\sigma_E^{\rm prod} < \sigma_p^{\rm prod}}$ $\Leftarrow$ also when $\nu' s$ are produced in collisions.

$$\Longrightarrow \qquad \sigma_{p\ \rm eff} \simeq \frac{\sigma_E}{v_g}\,, \qquad\qquad \boxed{\sigma_x \simeq \frac{v_g}{\sigma_E}}$$

In the stationary limit $(\sigma_E \to 0)$ one has $\sigma_{p\ \rm eff} \to 0$ even though σ_p is finite! Therefore $\sigma_x \to \infty$ and so the coherence length $l_{\rm coh} \to \infty$

– a well known result.

Universal oscillation formula?

The complete process: production – propagation – detection: factorization

$$P_{\rm tot} \;=\; P_{\rm prod}\, P_{\rm prop}\, P_{\rm det}$$

with a universal $P_{\rm prop}$ is only possible when all 3 processes are independent

In general not true, and production – propagation – detection should be considered as a single inseparable process!

To get the standard formula one assumes for the emitted and absorbed states

$$|\nu_a^{\rm fl}\rangle \;=\; \sum_i U_{ai}^* \, |\nu_i^{\rm mass}\rangle$$

The weights of the mass eigenstaes are just U_{ai}^* – do not depend on the masses of ν_i $\Rightarrow$ only true when the phase space volumes at production and detection do not depend on the mass of ν_i.

$\Rightarrow$

Universal oscillation formula?

This is only true if the charact. energy E at production (and detection) is large compared to all m_i (relativistic neutrinos), or compared to all $|m_i - m_k|$ (quasi-degenerate neutrinos).

$\Rightarrow$ Neutrino oscillations can be described by a universal probability only when neutrinos are relativistic or quasi-degenerate

Also: loss of coherence of propagating neutrino state depends on the coherence of the production and detection processes

$\Rightarrow$ The standard formula for the oscillation probability is only valid when all decoherence effects are negligible !

Lorentz invariance of oscillation probability

1. “Paradox” of neutrino w. packet length

For neutrino production in decays of unstable particles at rest (e.g. $\pi \to \mu\nu_\mu$):

$$\sigma_E \simeq \tau^{-1} = \Gamma_\pi \,, \qquad\qquad \sigma_x \simeq \frac{v_g}{\sigma_E} \simeq \frac{v_g}{\Gamma_\pi} \;\; (= v_g \tau)$$

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For decay in flight: $\Gamma'_\pi = (m_\pi/E_\pi)\Gamma_\pi$. One might expect

$$\sigma'_x \simeq \frac{E_\pi}{m_\pi}\sigma_x > \sigma_x\,.$$

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<u>The solution:</u> pion decay takes finite time. During the decay time the pion moves over distance $l = u\tau'$ ("chases" the neutrino if $u > 0$).

$$\sigma'_x \simeq v'_g/\Gamma' - l = v'_g\tau' - u\tau' = (v'_g - u)\gamma_u\tau = \frac{v_g\tau}{\gamma_u(1 + v_g u)} ,$$

[the relativ. law of addition of velocities: $v'_g = (v_g + u)/(1 + v_g u)$].

Lorentz invariance issues – contd.

That is

$$\sigma'_x = \frac{\sigma_x}{\gamma_u(1+v_g u)}$$

For relativistic neutrinos $v_g \approx v'_g \approx 1 \quad \Rightarrow$

$$\sigma'_x = \sigma_x \sqrt{\frac{1-u}{1+u}}$$

$\Rightarrow$ when the pion is boosted in the direction of neutrino emission $(u > 0)$ the neutrino wave packet gets contracted; when it is boosted in the opposite direction $(u < 0)$ – the wave packet gets dilated.

Lorentz invariance issues – contd.

The oscillation probability must be Lorentz invariant ! But: L. invariance is not obvious in QM w. packet approach which (unlike QFT) is not manifestly Lorentz covariant.

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$$L' = \gamma_u(L + ut)\,, \qquad t' = \gamma_u(t + uL)\,,$$

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The stand. osc. formula results when (i) production and detection and (ii) propagation are coherent; for neutrinos from conventional sources (i) implies $\sigma_x \ll l_{\rm osc}$ $\Rightarrow$ one can consider neutrinos pointlike and set $L = v_g t$.
$\Rightarrow$ $L' = \gamma_u L(1 + u/v_g)$.

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$\Rightarrow$ $L' = \gamma_u L(1+u/v_g)$. On the other hand: $v_g = p/E$

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Lorentz invariance issues – contd.

The oscillation probability must be Lorentz invariant ! But: L. invariance is not obvious in QM w. packet approach which (unlike QFT) is not manifestly Lorentz covariant.

How can we see Lorentz invariance of the standard formula for the oscillation probability ? P_{ab} depends on L/p (contains factors $\exp[-i\frac{\Delta m_{ik}^2}{2p}L]$). Is L/p Lorentz invariant? Lorentz transformations:

$$L' = \gamma_u(L+ut)\,, \qquad t' = \gamma_u(t+uL)\,,$$

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$\Rightarrow$ $$\boxed{L'/p' \,=\, L/p}$$

Lorentz invariance issues – contd.

A more general argument (applies also to Mössbauer neutrinos which are not pointlike): Consider the phase difference

$$\diamondsuit \qquad \Delta\phi = -\frac{1}{v_g}(L - v_g\, t)\Delta E + \frac{\Delta m^2}{2p} L$$

– a Lorentz invariant quantity, though the two terms are in not in general separately Lorentz invariant.

But: If the 1st term is negligible in all Lorentz frames, the second term is Lorentz invariant by itself $\Rightarrow$ L/p is Lorentz invariant.

The 1st term can be neglected when the production/detection coherence conditions are satisfied. In particular, it vanishes in the limit of pointlike neutrinos $L = v_g t$. N.B.:

$$L' - v_g' t' = \gamma_u \left[(L + ut) - \frac{v_g + u}{1 + v_g u}(t + uL)\right] = \frac{L - v_g t}{\gamma_u (1 + v_g u)}\,,$$

i.e. the condition $L = v_g t$ is Lorentz invariant. MB neutrinos: $\Delta E \simeq 0$.

Lorentz invariance issues – contd.

The oscillation probability must be Lorentz invariant even when the coherence conditions are not satisfied !
Lorentz invariance is enforced by the normalization condition.

$$P_{ab}(L) \;=\; \sum_{i,k} U_{ai}U_{bi}^{*}U_{ak}^{*}U_{bk}\, I_{ik}(L)\,, \qquad \text{where}$$

$$I_{ik}(L) \;\equiv\; \int dT\, \mathcal{A}_i(L,T)\mathcal{A}_k^{*}(L,T)e^{-i\Delta\phi_{ik}}$$

From the norm. cond. $\int dT\,|\mathcal{A}_i(L,T)|^2 \;=\; 1 \qquad \Rightarrow$

$$|\mathcal{A}_i|^2 dT \;=\; inv. \quad\Rightarrow\quad |\mathcal{A}_i||\mathcal{A}_k|dT \;=\; inv. \quad\Rightarrow\quad \mathcal{A}_i\mathcal{A}_k^{*}dT \;=\; inv.$$

The phase difference $\Delta\phi_{ik} = \Delta E_{ik}T - \Delta p_{ik}L$ is also Lorentz invariant $\quad\Rightarrow$
so is $I_{ik}(L)$, and consequently $P_{ab}(L)$.

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They may still be realized if relatively heavy sterile neutrinos exist