The Standard Model of Cosmology: ΛCDM

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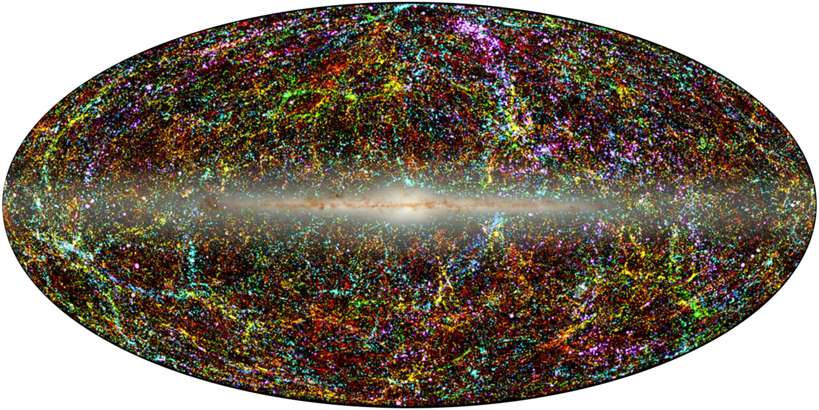
Outline

Geometry and Dynamics

Assumptions

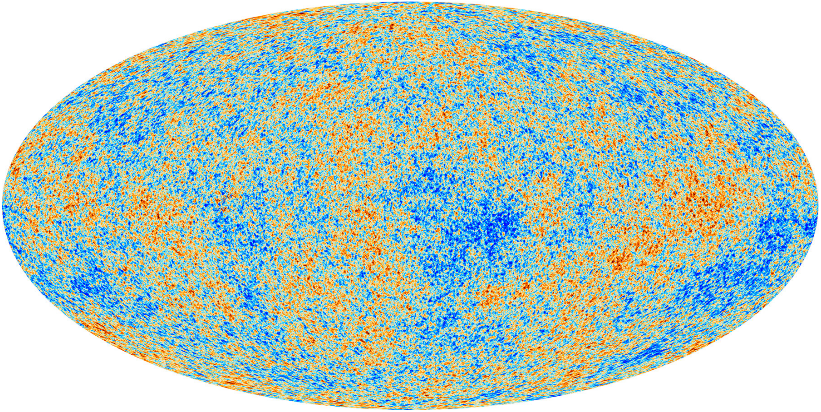
- standard cosmology makes two fundamental assumptions:
  1. observable properties of the Universe are isotropic
  2. our position in the Universe is not preferred to any other (cosmological principle);
- such a Universe is homogeneous and isotropic
- only relevant interaction is gravity: search for cosmological models in general relativity

Assumptions: Isotropy?



2-Micron All-Sky Survey, 2MASS

Assumptions: Isotropy?



CMB temperature fluctuations, measured by Planck

Metric

- metric tensor $g_{\mu\nu}$ has ten independent components: g_{00}, g_{0i}, and g_{ij}; two fundamental assumptions greatly simplify the metric

Metric

- eigentime should equal coordinate time for fundamental observers: [signature chosen: $(-,+,+,+)$]

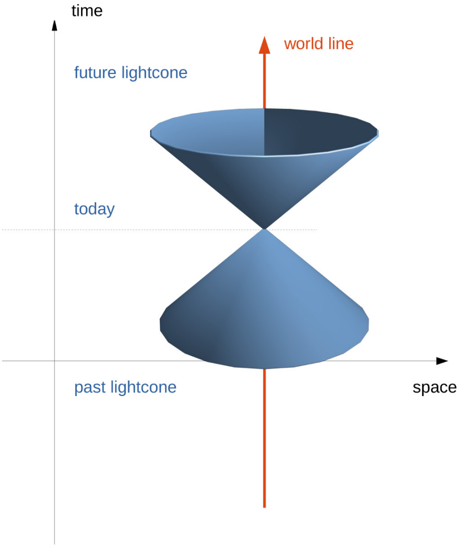
$$\mathrm{d}s^2 = g_{00}\mathrm{d}t^2 = -c^2\mathrm{d}t^2 \;\Rightarrow\; g_{00} = -c^2$$

- isotropy requires $g_{0i} = 0$ and spherical symmetry for three-space, thus

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a^2(t)\left[\mathrm{d}w^2 + f_K^2(w)\mathrm{d}\Omega^2\right] ,$$

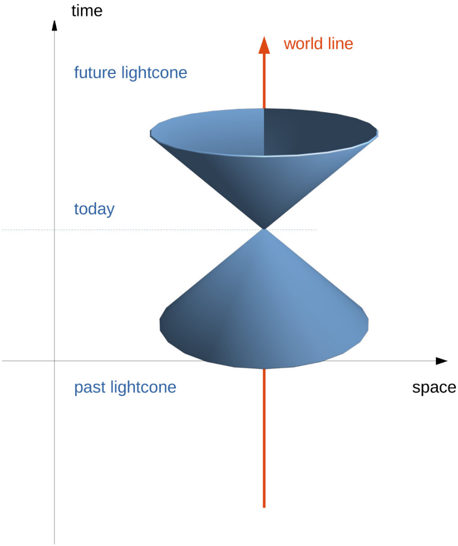
$$\text{with} \quad f_K(w) = \begin{cases} K^{-1/2}\sin(K^{1/2}w) & (K > 0) \\ w & (K = 0) \\ |K|^{-1/2}\sinh(|K|^{1/2}w) & (K < 0) \end{cases}$$

Metric: Lightcone

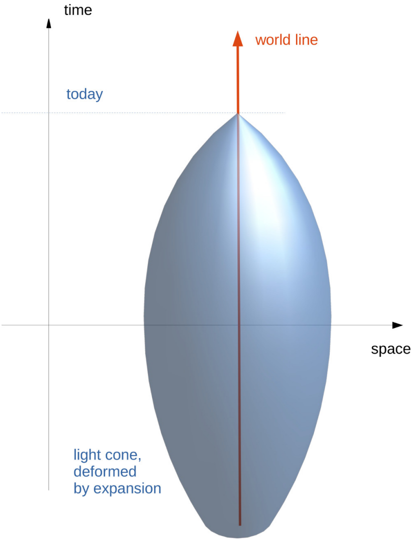


Minkowski space-time

Metric: Lightcone



Minkowski space-time



Expanding space-time

Redshift

- space can expand or shrink, leading to red- or blueshift
- propagation condition for light, $\mathrm{d}s = 0$, implies

$$\frac{\nu_\mathrm{e}}{\nu_\mathrm{o}} = \frac{\lambda_\mathrm{o}}{\lambda_\mathrm{e}} = 1 + \frac{\lambda_\mathrm{o} - \lambda_\mathrm{e}}{\lambda_\mathrm{e}} = 1 + z = \frac{a(t_\mathrm{e})}{a(t_\mathrm{o})}$$

- light is red- or blueshifted by the same amount as space expands or shrinks
- redshifts of galaxies shows that the Universe is expanding

Dynamics

- dynamics of the metric is expressed by dynamics of the scale factor $a(t)$
- Einstein's field equations reduce to Friedmann's equations

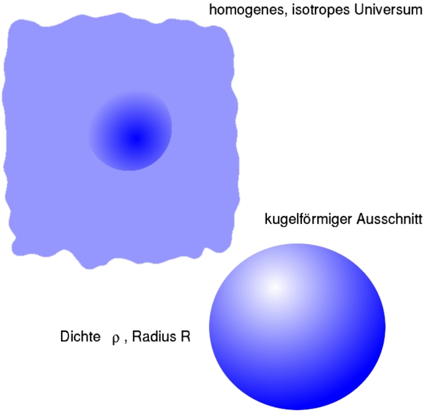
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2} + \frac{\Lambda c^2}{3}\,, \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}$$

- they can be combined to give the *adiabatic equation*

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(a^3\rho c^2\right) + p\frac{\mathrm{d}}{\mathrm{d}t}\left(a^3\right) = 0$$

expressing energy conservation

Remark on Newtonian Dynamics



Remark on Newtonian Dynamics

- Friedmann equations can also be derived from Newtonian gravity, except for the Λ term
- study homogeneous sphere of arbitrary radius r, ignore surrounding matter
- pressure term adds to the density: pressure means kinetic energy density, equivalent to a mass density; yields equation of motion

$$\ddot{r} = -\frac{4\pi G}{3} r \left(\rho + \frac{3p}{c^2} \right)$$

- integrating, using energy conservation, gives

$$\left(\frac{\dot{r}}{r} \right)^2 = \frac{8\pi G}{3} \rho + \frac{C}{r^2}$$

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Forms of Matter

- two forms of matter can broadly be distinguished, relativistic and non-relativistic; they are often called radiation and dust, respectively

- for radiation:

$$p = \frac{\rho c^2}{3}$$

which implies

$$\rho(t) = \rho_0 a^{-4} \ ,$$

($a = 1$ today)

- for dust, $p = 0$ because $p \ll \rho c^2$, and

$$\rho(t) = \rho_0 a^{-3}$$

Parameters

- Hubble parameter, relative expansion rate:

$$H(t) \equiv \frac{\dot{a}}{a}\,, \qquad H_0 \equiv H(t_0) = 100\,h\,\frac{\mathrm{km}}{\mathrm{s\,Mpc}} = 3.2 \times 10^{-18}\,h\,\mathrm{s}^{-1}$$

Parameters

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- Hubble parameter, relative expansion rate:

$$H(t) \equiv \frac{\dot{a}}{a}$$

- critical density

$$\rho_{\rm cr}(t) \equiv \frac{3H^2(t)}{8\pi G}\,, \quad \rho_{\rm cr0} \equiv \rho_{\rm cr}(t_0) = \frac{3H_0^2}{8\pi G} = 1.9 \times 10^{-29}\, h^2\, {\rm g\, cm^{-3}}$$

Parameters

- Hubble parameter, relative expansion rate:

$$H(t) \equiv \frac{\dot{a}}{a}$$

- critical density

$$\rho_{\mathrm{cr}}(t) \equiv \frac{3H^2(t)}{8\pi G}$$

- dimension-less density parameters

$$\Omega(t) \equiv \frac{\rho(t)}{\rho_{\mathrm{cr}}(t)}\,, \quad \Omega_0 \equiv \frac{\rho(t_0)}{\rho_{\mathrm{cr}0}}\,, \quad \Omega_\Lambda(t) = \frac{\Lambda c^2}{3H^2(t)}\,, \quad \Omega_{\Lambda 0} \equiv \frac{\Lambda c^2}{3H_0^2}$$

Parameters

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- Hubble parameter, relative expansion rate:

$$H(t) \equiv \frac{\dot{a}}{a}$$

- dimension-less density parameters

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- Friedmann's equation becomes

$$H^2(a) = H_0^2 \left[\Omega_{\rm r0} a^{-4} + \Omega_{\rm m0} a^{-3} + \Omega_{\Lambda 0} - \frac{Kc^2}{a^2} \right]$$

Parameters

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- Friedmann's equation becomes

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  specialising to $a = 1$ allows to solve for K,

$$-Kc^2 = 1 - \Omega_{\mathrm{r}0} - \Omega_{\mathrm{m}0} - \Omega_{\Lambda 0} \equiv \Omega_{\mathrm{K}}$$

Parameters

- final form for Friedmann's equation

$$H^2(a) = H_0^2 \left[\Omega_{\mathrm{r}0} a^{-4} + \Omega_{\mathrm{m}0} a^{-3} + \Omega_{\Lambda 0} + \Omega_{\mathrm{K}} a^{-2} \right] \equiv H_0^2 E^2(a)$$

Parameters

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- final form for Friedmann's equation

$$H^2(a) = H_0^2 \left[\Omega_{r0} a^{-4} + \Omega_{m0} a^{-3} + \Omega_{\Lambda 0} + \Omega_K a^{-2} \right] \equiv H_0^2 E^2(a)$$

- radiation density exceeded matter density before

$$a_{eq} = \frac{\Omega_{r0}}{\Omega_{m0}}$$

Parameters

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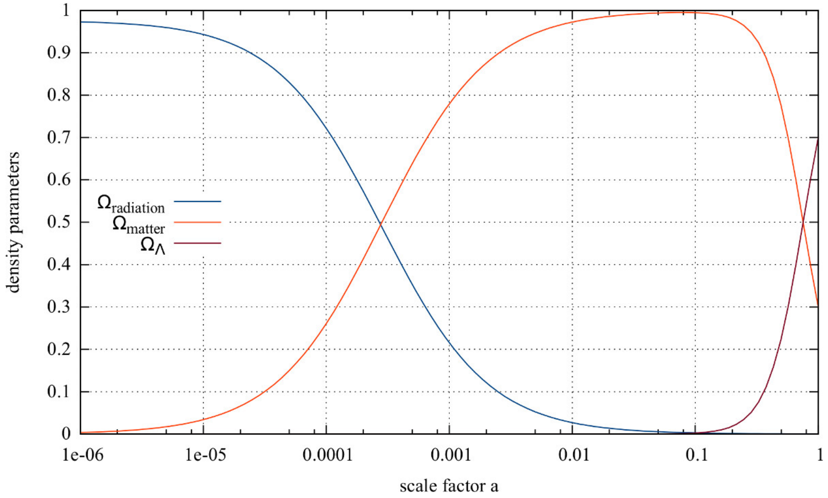
- the density parameters change with time:

$$\begin{aligned}\Omega_{\mathrm{m}}(a) &= \frac{\Omega_{\mathrm{m}0}}{a + \Omega_{\mathrm{m}0}(1-a) + \Omega_{\Lambda 0}(a^3 - a)} \,, \\ \Omega_{\Lambda}(a) &= \frac{\Omega_{\Lambda 0} a^3}{a + \Omega_{\mathrm{m}0}(1-a) + \Omega_{\Lambda 0}(a^3 - a)}\end{aligned}$$

- this implies: $\Omega_{\mathrm{m}}(a) \to 1$ and $\Omega_{\Lambda}(a) \to 0$ for $a \to 0$ regardless of their present values; if $\Omega_{\mathrm{m}0} + \Omega_{\Lambda 0} = 1$, remains so for $a < 1$

Parameters



Evolution of density parameters

Parameters: Planck 2015

Hubble constant	h	0.6727 ± 0.0066
dark-matter density	Ω_{c0}	0.2647 ± 0.0042
cosmological constant	$\Omega_{\Lambda 0}$	0.6844 ± 0.0091
baryon density	Ω_{B}	0.04917 ± 0.0006
radiation density	Ω_{r0}	$(8.51 \pm 0.050) \cdot 10^{-5}$
Hubble time age of the Universe	H_0^{-1} t_0	$14.60 \pm 0.14\,\mathrm{Gyr}$ $13.813 \pm 0.026\,\mathrm{Gyr}$
matter-radiation equality	a_{eq} z_{eq}	$(2.711 \pm 0.056) \times 10^{-4}$ 3687.2 ± 76.9
optical depth	τ	0.079 ± 0.017
fluctuation amplitude	σ_8	0.831 ± 0.013

Age and Expansion of the Universe

- since $H = \dot{a}/a$, the age of the Universe is determined by

$$\frac{\mathrm{d}a}{\mathrm{d}t} = H_0 a E(a) \;\Rightarrow\; H_0 t = \int_0^a \frac{\mathrm{d}x}{xE(x)}$$

- in a flat (late) universe with $\Omega_{\mathrm{m0}} \neq 0$ and $\Omega_\Lambda = 1 - \Omega_{\mathrm{m0}} \neq 0$:

$$H_0 t = \frac{2}{3\sqrt{1-\Omega_{\mathrm{m0}}}} \operatorname{arcsinh}\left[\sqrt{\frac{1-\Omega_{\mathrm{m0}}}{\Omega_{\mathrm{m0}}}} a^{3/2}\right]$$

the age of our universe is

$$t(a=1) = \frac{0.94}{H_0} = (13.813 \pm 0.026) \times 10^9\,\mathrm{yr}$$

Age and Expansion of the Universe: Constraints

- the Universe should be older than its oldest parts
- three ways of measuring the ages:
  1. nuclear cosmo-chronology: decay of long-lived nuclei $\approx 4.6\,\mathrm{Gyr}$ for the Earth, $7\ldots13\,\mathrm{Gyr}$ for the Galaxy;
  2. ages from stellar evolution: $\gtrsim 12\,\mathrm{Gyr}$ from globular clusters;
  3. cooling of white dwarfs: $\approx 10\,\mathrm{Gyr}$
- $t(a=1) \gtrsim 11\,\mathrm{Gyr}$ needs $H_0 \lesssim 61\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ in an Einstein-de Sitter universe ($\Omega_{\mathrm{m}0} = 1$, $\Omega_{\Lambda 0} = 0$)

Age of the Earth and the Galaxy

- uranium decay series $^{235}\mathrm{U} \to {}^{207}\mathrm{Pb}$ and $^{238}\mathrm{U} \to {}^{206}\mathrm{Pb}$ have half-lives of order Gyr
- comparison of $^{206}\mathrm{Pb}$ and $^{207}\mathrm{Pb}$ to $^{204}\mathrm{Pb}$ and present abundance ratio

$$\frac{N_{235}}{N_{238}} = 0.00725$$

yields age

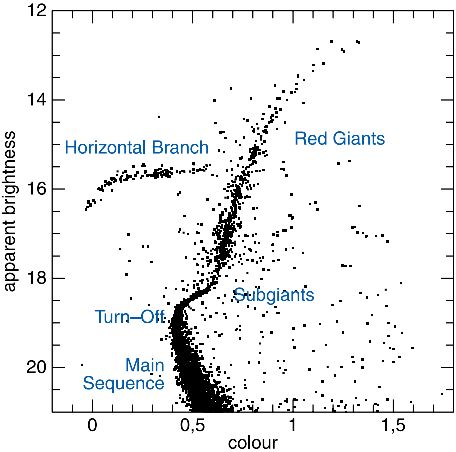
$$t_{\mathrm{Earth}} = (4.6 \pm 0.1)\,\mathrm{Gyr}$$

- variant of this method, applied to the Galaxy, gives

$$7\,\mathrm{Gyr} \lesssim t_{\mathrm{Galaxy}} \lesssim 13\,\mathrm{Gyr}$$

main uncertainty is chemical history of the Galaxy

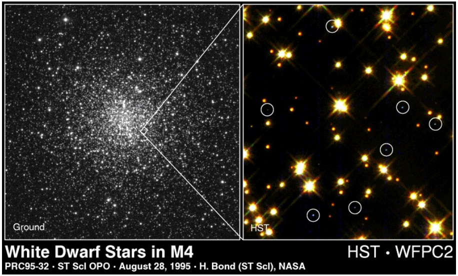
Age of Globular Clusters



- stars remain on main sequence while hydrogen burns, $\tau \propto T^{-1}$
- then move towards red giant branch
- turn-off point $(L, T) \propto (\tau^{-3/2}, \tau^{-1})$ in co-eval star populations is age indicator
- distances need to be known!
- recent applications give

$$t_{\mathrm{GC}} \gtrsim (12.5 \pm 1.3)\,\mathrm{Gyr}$$

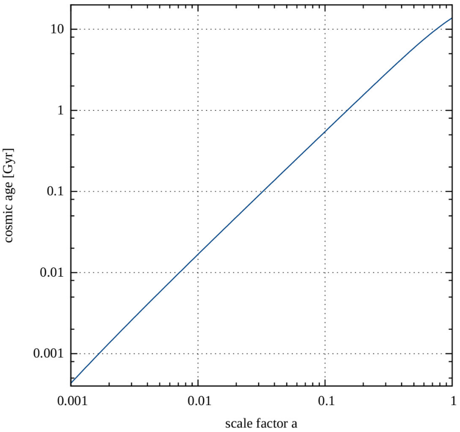
Age of White Dwarfs



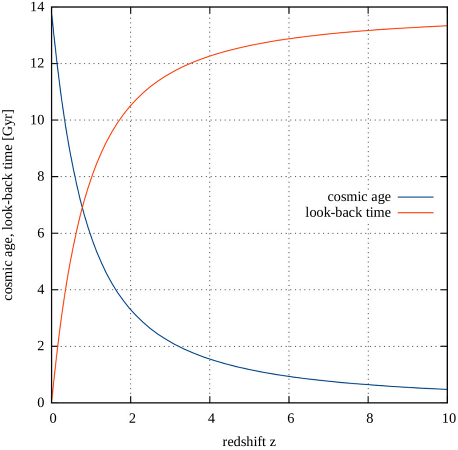
- white dwarfs are born with $M \approx (0.55 \pm 0.05)\, M_\odot$
- rapid cooling by neutrinos, then slow cooling by radiation
- population develops peak in luminosity function
- cooling models and metallicity measurements imply

$$t_{\mathrm{WD}} = (9.5 \pm 1)\,\mathrm{Gyr}$$

Cosmic Age



Age as a function of scale factor



Age as function of redshift

Distances

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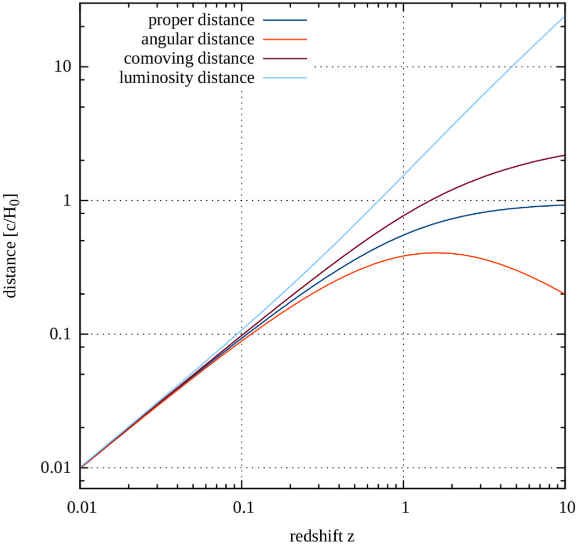
- distance measures are no longer unique in general relativity
- proper distance D_{prop}, $\mathrm{d}D_{\mathrm{prop}} = -c\mathrm{d}t = -c\mathrm{d}a/\dot{a}$
- comoving distance D_{com}, $\mathrm{d}D_{\mathrm{com}} = \mathrm{d}w$
- angular diameter distance D_{ang}

$$D_{\mathrm{ang}}(z_1, z_2) = \left(\frac{\delta A}{\delta \omega}\right)^{1/2} = a(z_2) f_K[w(z_1, z_2)]$$

- luminosity distance D_{lum},

$$D_{\mathrm{lum}}(z_1, z_2) = \left[\frac{a(z_1)}{a(z_2)}\right]^2 D_{\mathrm{ang}}(z_1, z_2)$$

Distances



Distances as functions of redshift

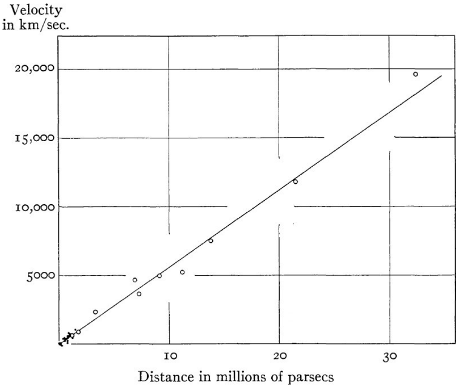
Distances: The Hubble Constant

- (nearby) galaxies move away from us with velocities proportional to their distance; Hubble's law,

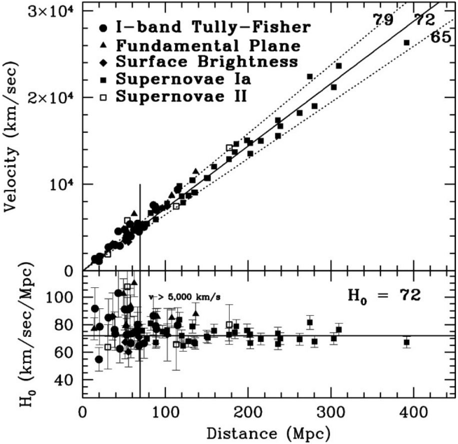
$$D = \frac{cz}{H_0} \quad \Rightarrow \quad v = cz = H_0 D \; ;$$

- local deviations due to peculiar velocities; $z \gtrsim 0.01 \ldots 0.02$ necessary
- main difficulty: accurate distance measurements to distant objects required
- "standard candles": Cepheids, supernovae, galaxy scaling relations, surface-brightness fluctuations

Distances: The Hubble Constant

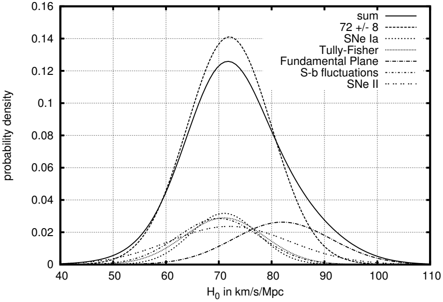


Hubble & Humason 1931



HST Key Project, Freedman et al.

Distances: The Hubble Constant



HST Key Project, Freedman et al.

- result from Hubble Key Project:

$$H_0 = (72 \pm 8)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

- result from recent Cepheid survey in the near infrared

$$H_0 = (74.2 \pm 3.6)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

Horizons

- between t_1 and $t_2 > t_1$, light can travel across comoving distance

$$\Delta w(t_1, t_2) = \int_{t_1}^{t_2} \frac{c\mathrm{d}t}{a(t)} = c\int_{a(t_1)}^{a(t_2)} \frac{\mathrm{d}a}{a\dot{a}} \propto a^{n/2-1} \quad \text{if} \quad \rho \propto \rho_0 a^{-n}$$

  if $n > 2$, light can only travel by a finite distance; there exists a particle horizon

- Hubble radius at a_{eq}, important for structure formation

$$r_{\mathrm{H,eq}} = \frac{c}{H(a_{\mathrm{eq}})} = \frac{c}{H_0}\frac{a_{\mathrm{eq}}^{3/2}}{\sqrt{2\Omega_{\mathrm{m0}}}}$$

Thermal Evolution

Assumptions

- the universe expands adiabatically – isotropy requires the universe to expand adiathermally; entropy generation is completely negligible
- thermal equilibrium can be maintained despite the expansion
- the cosmic “fluids” can be treated as ideal gases
- those assumptions are the starting point of our considerations; they need to be verified as we go along

- for relativistic boson and fermion gases in thermal equilibrium:

$$
\begin{aligned}
n_\mathrm{B} &= 10 g_\mathrm{B} \left(\frac{T}{\mathrm{K}}\right)^3 \mathrm{cm}^{-3} = 1.6 \times 10^{13} g_\mathrm{B} \left(\frac{k_\mathrm{B} T}{\mathrm{eV}}\right)^3 \mathrm{cm}^{-3}\,, \\
n_\mathrm{F} &= \frac{3}{4}\frac{g_\mathrm{F}}{g_\mathrm{B}} n_\mathrm{B} \\
u_\mathrm{B} &= 3.8 \times 10^{-15} g_\mathrm{B} \left(\frac{T}{\mathrm{K}}\right)^4 \frac{\mathrm{erg}}{\mathrm{cm}^3} = 2.35 \times 10^{-3} g_\mathrm{B} \left(\frac{k_\mathrm{B} T}{\mathrm{eV}}\right)^4 \frac{\mathrm{erg}}{\mathrm{cm}^3}\,, \\
u_\mathrm{F} &= \frac{7}{8}\frac{g_\mathrm{F}}{g_\mathrm{B}} u_\mathrm{B}\,, \quad P_\mathrm{B} = \frac{u_\mathrm{B}}{3}\,, \quad P_\mathrm{F} = \frac{u_\mathrm{F}}{3} \\
\frac{s_\mathrm{B}}{k_\mathrm{B}} &= 36 g_\mathrm{B} \left(\frac{T}{\mathrm{K}}\right)^3 \mathrm{cm}^{-3} = 5.7 \times 10^{13} g_\mathrm{B} \left(\frac{k_\mathrm{B} T}{\mathrm{eV}}\right)^3 \mathrm{cm}^{-3}\,, \\
s_\mathrm{F} &= \frac{7}{8}\frac{g_\mathrm{F}}{g_\mathrm{B}} s_\mathrm{B}
\end{aligned}
$$

Adiabatic Expansion of Ideal Gases

- for relativistic boson or fermion gases in thermal equilibrium

$$P = \frac{u}{3} = \frac{E}{3V}$$

- first law of thermodynamics implies

$$\mathrm{d}E = -P\mathrm{d}V = 3\mathrm{d}(PV) \;\Rightarrow\; P \propto V^{-4/3}$$

  i.e. $\gamma = 4/3$; for non-relativistic ideal gases, $\gamma = 5/3$

- temperature scaling:

$$\begin{aligned} T &\propto P^{1/4} \propto V^{-1/3} \propto a^{-1} \quad \text{(relativistic)} \\ T &\propto PV \propto V^{-5/3+1} \propto a^{-2} \quad \text{(non-relativistic)} \end{aligned}$$

Particle Freeze-Out

- expansion time-scale during radiation-dominated era

$$t_{\rm exp} \approx (G\rho)^{-1/2} \propto a^2$$

- collision rate and time-scale

$$\Gamma \equiv n\langle\sigma v\rangle \propto n \propto T^3 \propto a^{-3} \;, \quad t_{\rm coll} = \Gamma^{-1} \propto a^3$$

- ratio $t_{\rm exp}/t_{\rm coll} \propto a^{-1}$, thermal equilibrium can be maintained despite the expansion at early times;
- thermal equilibrium breaks down when $\Gamma \ll H$
- relativistic particle species retain their thermal-equilibrium density!

Neutrino Background

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- weak interaction
$$\nu + \bar{\nu} \leftrightarrow e^+ + e^-$$
freezes out when temperature drops to
$T_\nu \approx 10^{10.5}\,\mathrm{K} \approx 2.7\,\mathrm{MeV}$
- electron-positron pairs annihilate when temperature drops below $T \approx 2m_\mathrm{e}c^2 \approx 1\,\mathrm{MeV} \approx 10^{10}\,\mathrm{K}$
- their decay heats the photon gas, but not the neutrinos
- photon temperature is $\approx 40\%$ higher than neutrino temperature:

$$T_\gamma = \left(\frac{11}{4}\right)^{1/3} T_\nu$$

Photons and Baryons

- number density of baryons today is

$$n_\mathrm{B} = \frac{\rho_\mathrm{B}}{m_\mathrm{p}} = \frac{\Omega_\mathrm{B}}{m_\mathrm{p}} \frac{3H_0^2}{8\pi G} = 1.1 \times 10^{-5}\, \Omega_\mathrm{B} h^2\,\mathrm{cm}^{-3}$$

$$\Omega_\mathrm{B} h^2 \approx 0.02 \qquad (1)$$

- the photon number density today is

$$n_\gamma = 407\,\mathrm{cm}^{-3}$$

- their ratio is constant; about one billion photons per baryon!

$$\eta \equiv \frac{n_\mathrm{B}}{n_\gamma} = 2.7 \times 10^{-8}\, \Omega_\mathrm{B} h^2$$

Recombination Process

- approximation: Saha's equation; ionisation fraction x is

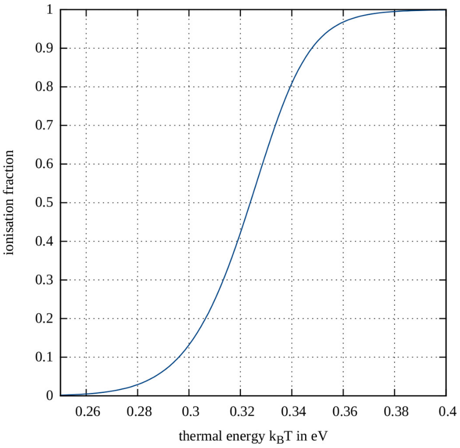
$$\frac{x^2}{1-x} = \frac{\sqrt{\pi}}{4\sqrt{2}\zeta(3)\eta}\left(\frac{m_\mathrm{e}c^2}{kT}\right)^{3/2} \mathrm{e}^{-\chi/kT} \approx \frac{0.26}{\eta}\left(\frac{m_\mathrm{e}c^2}{kT}\right)^{3/2} \mathrm{e}^{-\chi/kT}$$

- for recombination to be half-way finished, $x = 1/2 = \mathrm{l.h.s.}$
- since $\eta^{-1} \gg 1$, $kT \ll \chi$ is required
- setting $x = 1/2$ yields $kT_\mathrm{rec} = 0.32\,\mathrm{eV}$, or

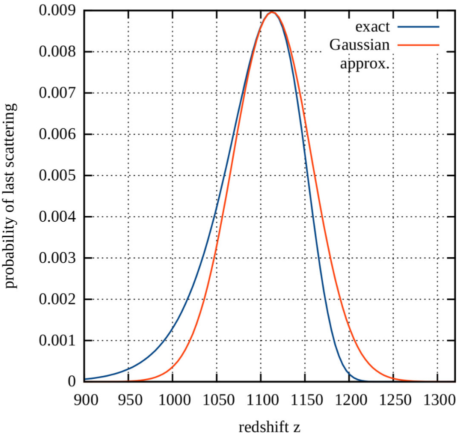
$$T_\mathrm{rec} \approx 3000\,\mathrm{K}$$

- for $\chi = 13.6\,\mathrm{eV}$, $T_\mathrm{rec} \approx 10^5\,\mathrm{K}$: very large photon-to-baryon ratio η^{-1} delays recombination considerably

Recombination Process



Ionisation as function of temperature



Probability of last photon scattering

Recombination Process: Time and Duration

- recombination time follows from expansion history,

$$t_{\mathrm{rec}} = \int_0^{a_{\mathrm{rec}}} \frac{\mathrm{d}a}{aH(a)} \approx 374\,\mathrm{kyr}$$

- width of “recombination shell” in redshift,

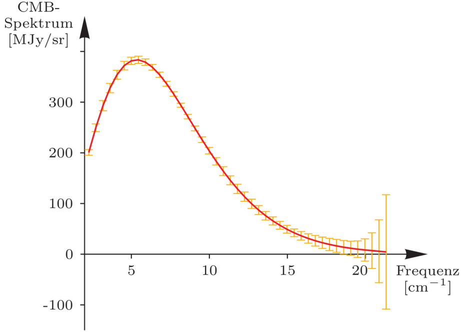
$$\delta z \approx \left.\frac{\partial z}{\partial T}\right|_{z_{\mathrm{rec}}} \delta T \approx 75$$

- corresponds to time interval

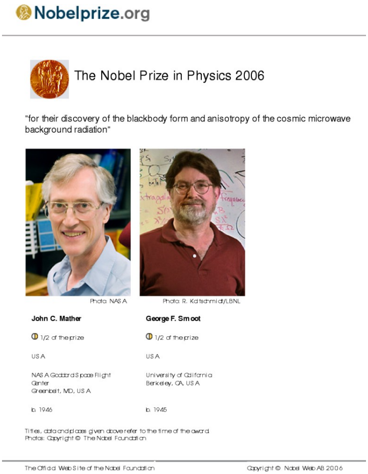
$$\delta t \approx \frac{\delta a}{a_{\mathrm{rec}} H(a_{\mathrm{rec}})} = \frac{a_{\mathrm{rec}}\delta z}{H(a_{\mathrm{rec}})} \approx 40\,\mathrm{kyr}$$

- provides indirect way of constraining relativistic particle species

Recombination Process: CMB Spectrum



COBE-FIRAS CMB spectrum, COBE



Nobelprize.org

The Nobel Prize in Physics 2006

"for their discovery of the blackbody form and anisotropy of the cosmic microwave background radiation"

Photo: NASA

Photo: R. Kaltschmidt/LBNL

John C. Mather	George F. Smoot
1/2 of the prize	1/2 of the prize
USA	USA
NASA Goddard Space Flight Center Greenbelt, MD, USA	University of California Berkeley, CA, USA
b. 1946	b. 1945

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Primordial Nucleosynthesis: Concepts

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- ^{4}He abundance is $\approx 25\,\%$ by mass, far more than stars can have produced
- Universe must have acted as a fusion reactor
- MeV energies require scale factor

$$a \lesssim \frac{\text{meV}}{\text{MeV}} \approx 10^{-9} \ll a_{\text{eq}}$$

- only radiation was important at that time
- baryon-to-photon ratio η is the only relevant parameter,

$$\eta = 10^{10}\eta_{10}\;, \quad \eta_{10} = 273\Omega_{\text{B}}h^2$$

- time scale during early radiation-dominated era

$$t \approx 0.89\left(\frac{T}{\text{MeV}}\right)^{-2}\;\text{s}$$

Primordial Nucleosynthesis: Helium Abundance

- deuterium fusion is crucial, delayed by photon background until $T_\mathrm{D} \approx 78\,\mathrm{keV}$
- further fusion builds upon two-body processes, e.g.

$$n + p \to {}^2\mathrm{H} + \gamma \,, \; {}^2\mathrm{H} + {}^2\mathrm{H} \to {}^3\mathrm{He} + n \,,$$
$${}^3\mathrm{He} + {}^2\mathrm{H} \to {}^4\mathrm{He} + p \,, \; {}^4\mathrm{H} + {}^3\mathrm{H} \to {}^7\mathrm{Li} + \gamma \qquad (2)$$

- neutrons form when weak interaction freezes out at $T_n \approx 0.87\,\mathrm{MeV}$, at $t \approx 2\mathrm{s}$
- abundance controlled by Boltzmann factor,

$$\frac{n_n}{n_p} = \exp\left(-\frac{Q}{kT_n}\right)$$

- and subsequent neutron decay with half-life

$$\tau_n = (885.7 \pm 0.8)\,\mathrm{s}$$

Primordial Nucleosynthesis: Results

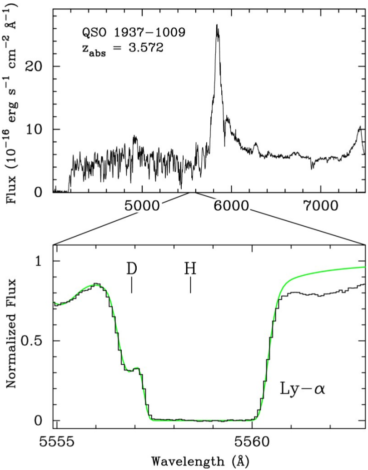
- neutron abundance $X_n \approx 0.17$ by mass at freeze-out
- neutron decay until onset of fusion at $t_\mathrm{D} \approx 150\,\mathrm{s}$ reduces this to $X_n \approx 0.14$, which implies $^4\mathrm{He}$ abundance of $Y \approx 0.28$
- D is most trustworthy baryometer; measured abundance

$$\frac{n_\mathrm{D}}{n_\mathrm{H}} = \left(2.68^{+0.27}_{-0.25}\right) \times 10^{-5}$$

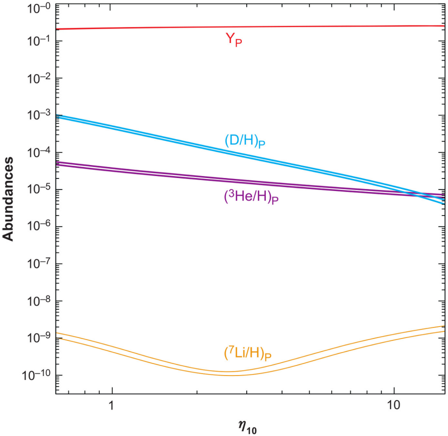
- abundances of D and $^3\mathrm{He}$ decrease with η, $^4\mathrm{He}$ increases, $^7\mathrm{Li}$ has characteristic valley
- measured element abundances imply

$$0.0207 \lesssim \Omega_\mathrm{B} h^2 \lesssim 0.0234$$

Primordial Nucleosynthesis: Results



Deuterium signal in QSO spectrum



From Steigman 2007

The Growth of Perturbations

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Newtonian Equations

- Newtonian hydrodynamics is a valid approximation (flatness, no retardation, short mean free path)
- continuity equation (mass conservation)

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

- Euler's equation (momentum conservation)

$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla}\right) \vec{v} = -\vec{\nabla}\Phi - \frac{\vec{\nabla} p}{\rho}$$

- Poisson equation

$$\nabla^2 \Phi = 4\pi G \rho$$

Density Perturbations: Equations

- split ρ and $\vec{v}$ into background and fluctuations,

$$\rho(t,\vec{x}) = \langle\rho\rangle(t) + \delta\rho(t,\vec{x}) \ , \quad \vec{v}(t,\vec{x}) = \langle\vec{v}\,\rangle(t) + \delta\vec{v}(t,\vec{x})$$

  split velocity into Hubble flow and peculiar velocity

$$\vec{v} = \dot{\vec{r}} = \dot{a}\vec{x} + a\dot{\vec{x}} = H\vec{r} + a\dot{\vec{x}} = \langle\vec{v}\rangle + \delta\vec{v}$$

- comoving coordinates $\vec{x} = \vec{r}/a$, comoving peculiar velocities $\vec{u} \equiv \delta\vec{v}/a$, density contrast $\delta = \delta\rho/\langle\rho\rangle$
- we are now left with the three equations

$$\dot{\delta} + \vec{\nabla}\cdot\vec{u} = 0 \ , \quad \dot{\vec{u}} + 2H\vec{u} = -\frac{\vec{\nabla}\delta\Phi}{a^2} - \frac{\vec{\nabla}\delta p}{a^2\langle\rho\rangle} \ , \quad \nabla^2\delta\Phi = 4\pi G\langle\rho\rangle a^2\delta$$

Density Perturbations: Equations

- combining these, decomposing δ into plane waves

$$\ddot{\delta} + 2H\dot{\delta} = \left(4\pi G\langle\rho\rangle - \frac{c_s^2 k^2}{a^2}\right)\delta$$

sound speed $c_s^2 = \delta p/\delta\rho$

Density Perturbations: Equations

- combining these, decomposing δ into plane waves

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\langle\rho\rangle\delta$$

  sound speed $c_s^2 = \delta p / \delta\rho$

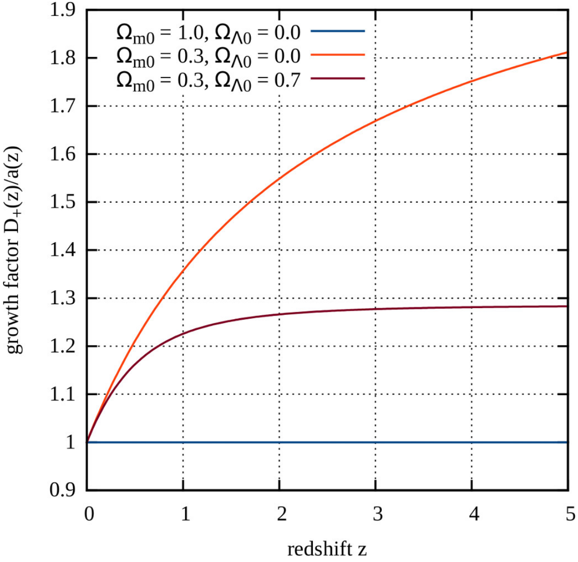
- assume large perturbations,

$$\frac{c_s^2 k^2}{a^2} \ll 4\pi G\langle\rho\rangle \quad \Rightarrow \quad k \ll a\frac{\sqrt{4\pi G\langle\rho\rangle}}{c_s}$$

- linear growth factor

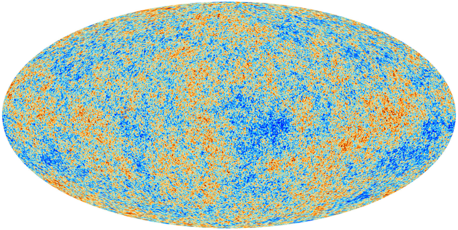
$$\delta(a) = \delta_0 D_+(a)$$

Density Perturbations: Growth

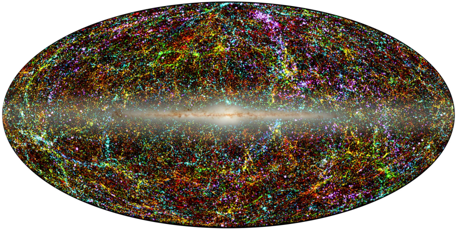


Growth factor divided by scale factor

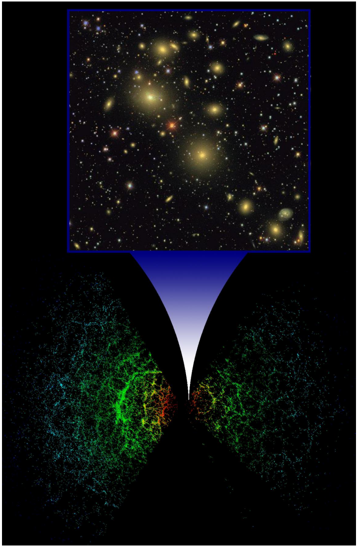
Density Perturbations: Growth



Planck CMB map



2MASS sky map



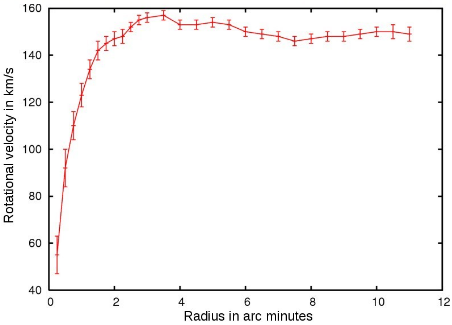
SDSS map

The Amount of Dark Matter

- stellar populations need approximately $6.4\,M_\odot/L_\odot$
- stellar kinematics in galaxies reveals substantially more than the stellar mass
- galaxy population comes up for $\Omega_{\rm g0} \approx 0.08$
- gas-to-mass ratio in galaxy clusters suggests $\Omega_{\rm m0} \approx 0.3$
- evolution of galaxy clusters (slower for lower $\Omega_{\rm m0}$) supports this
- most of the matter is dark; most of the baryons are not shining

The Amount of Dark Matter

Coma cluster, X-ray and optical



NGC 3198

Velocity Perturbations

- ignoring pressure gradients, the second equation (30) says

$$\dot{\vec{u}} + H\vec{u} = \frac{\vec{\nabla}\delta\Phi}{a^2}$$

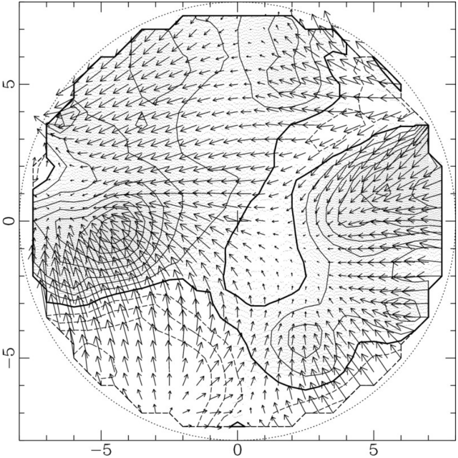
- defining

$$f(\Omega) \equiv \frac{\mathrm{d}\ln D_+(a)}{\mathrm{d}\ln a} \approx \Omega^{0.6} \ ,$$

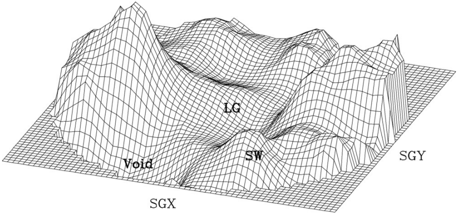
- the peculiar velocity field can be written as

$$\delta\vec{v} = a\vec{u} = \frac{2f(\Omega)}{3aH\Omega}\vec{\nabla}\delta\Phi$$

Velocity Perturbations: Our Local Neighbourhood



local velocity field



reconstructed local density field

Statistics and Non-Linear Evolution

Power Spectra

- variance of δ in Fourier space defines the power spectrum $P(k)$,

$$\left\langle \hat{\delta}(\vec{k})\hat{\delta}^*(\vec{k}') \right\rangle \equiv (2\pi)^3 P(k)\delta_{\mathrm{D}}(\vec{k}-\vec{k}')$$

- variance of δ on spatial scale R:

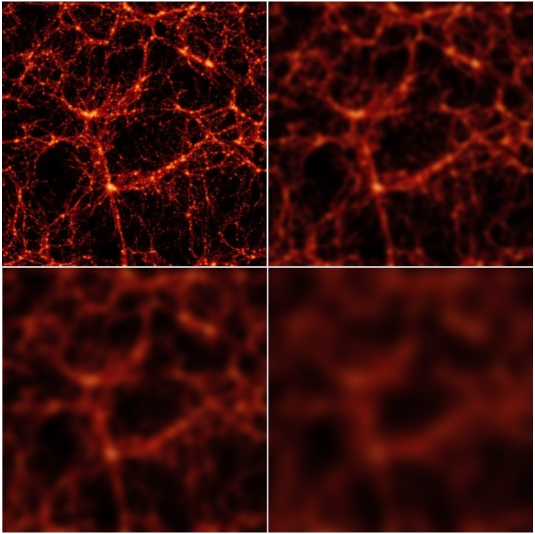
$$\bar{\delta}_R(\vec{x}) \equiv \int \mathrm{d}^3y\, \delta(\vec{x})\, W_R(|\vec{x}-\vec{y}|)$$

- the variance of the filtered density-contrast field is

$$\sigma_R^2 = 4\pi \int \frac{k^2 \mathrm{d}k}{(2\pi)^3} P(k)\hat{W}_R^2(k)$$

σ_8 is often used for normalizing the power spectrum

Power Spectra: Smoothing



Simulated density field, progressively smoothed

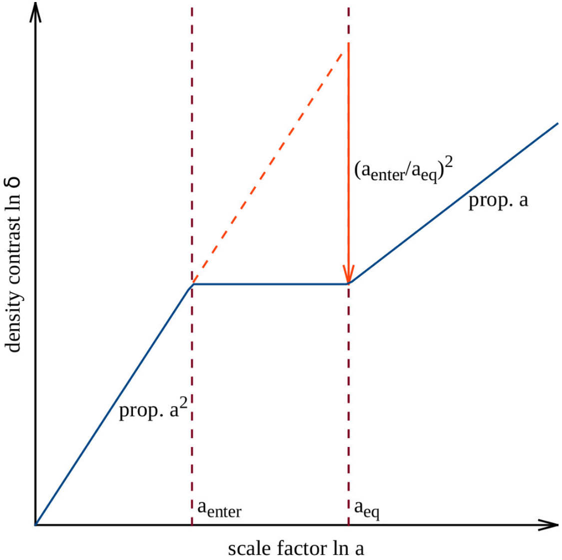
Evolution of the Power Spectrum

- modes entering the horizon (Hubble radius) while radiation dominates are relatively suppressed compared to larger modes
- assumed time-independence of fluctuation power entering the horizon, combined with suppression for $k > k_{\mathrm{eq}}$ gives

$$P(k) \propto \begin{cases} k^n & (k < k_{\mathrm{eq}}) \\ k^{n-4} & (k \gg k_{\mathrm{eq}}) \end{cases}$$

  with $n \approx 1$
- this is the shape of the spectrum for cold dark matter (CDM)
- hot dark matter (HDM) cuts off the spectrum exponentially

Evolution of the Power Spectrum: Suppression



Suppression of small perturbations

Evolution of the Power Spectrum: Nonlinear

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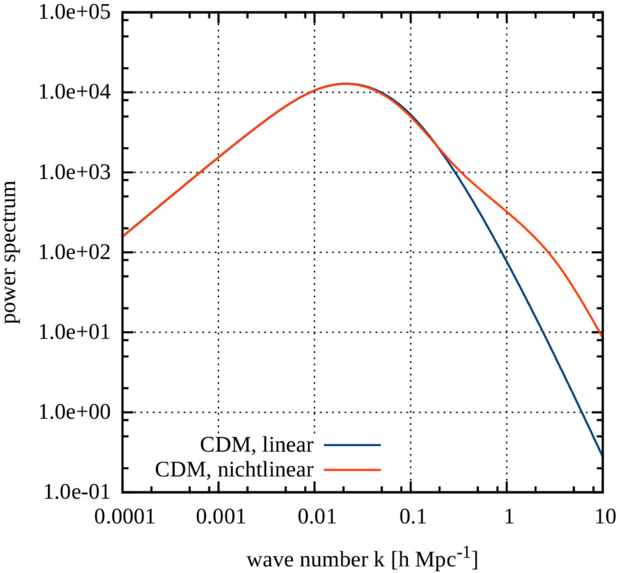
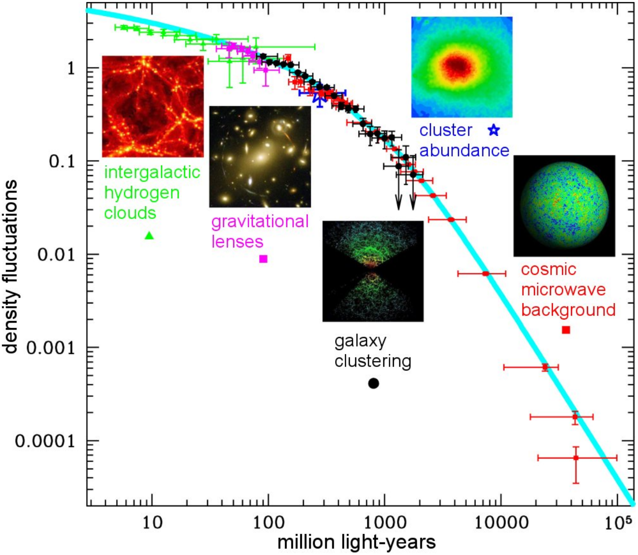


Linear and nonlinear density-fluctuation spectra

Power Spectrum: Measurements



Measured density-fluctuation power

The Observed Power Spectrum

- observable is the correlation function

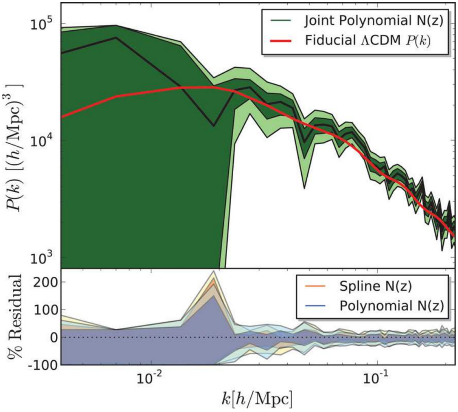
$$\xi(r) = \int_0^\infty \frac{k^2 \mathrm{d}k}{2\pi^2} P(k) \frac{\sin kx}{kx}$$

- correlation function is measured through pair counts,

$$1 + \xi(r) = \frac{\langle DD \rangle}{\langle RR \rangle}$$

- complicated by shape of the survey volume, masking, inhomogeneous survey coverage, biasing, redshift-space distortions, shot noise, Malmquist bias, ...

The Observed Power Spectrum



WiggleZ survey, Poole et al.

shape of the measured power spectrum

- supports CDM
- constrains matter-density parameter to

$$\Omega_{m0} = 0.233 \pm 0.022$$

- shows indications of baryonic acoustic oscillations (BAO)

The Zel'dovich Approximation

- Zel'dovich: approximate kinematical treatment of particle trajectories,

$$\vec{r} = a\left[\vec{x} + \frac{\vec{u}}{Hf(\Omega)}\right] \,, \quad F_{ij} \equiv \frac{\partial r_i}{\partial x_j}$$

- important consequence is the probability distribution $p(\lambda_1, \lambda_2, \lambda_3)$ for the eigenvalues of the deformation tensor F_{ij}:

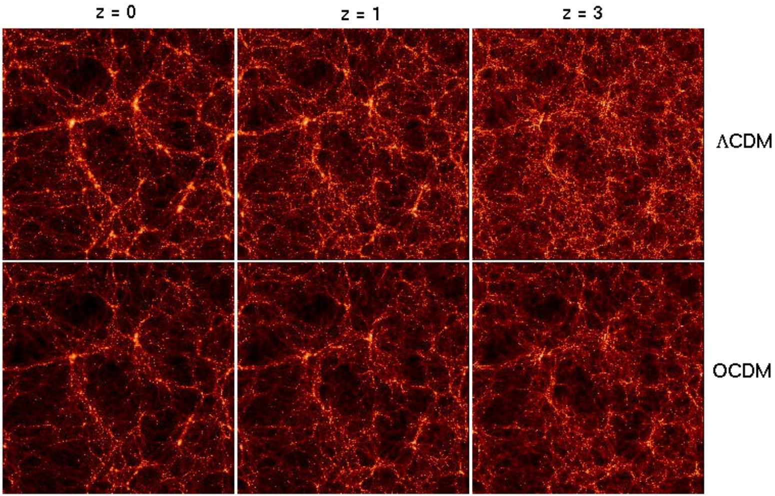
$$p(\lambda_1, \lambda_2, \lambda_3) \propto |(\lambda_3 - \lambda_2)(\lambda_3 - \lambda_1)(\lambda_2 - \lambda_1)|$$

  probability for two eigenvalues of F_{ij} to be equal is zero, implying anisotropic collapse! (starting from Gaussian random field)

Nonlinear Evolution

- non-linear evolution so far requires numerical simulations; decompose the matter distribution into particles whose equations of motion are solved
- non-linear evolution causes mode coupling: modes of different wave lengths couple, causing power transport from large to small scales as structures collapse
- even originally Gaussian density perturbation fields δ must develop non-Gaussianities during non-linear evolution
- typical behaviour seen in numerical simulations shows the formation of "pancakes", filaments and voids

Nonlinear Evolution: Simulations



Structures in the CMB

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CMB Theory: Dipole, Fluctuation Level

- Earth's motion causes temperature dipole,

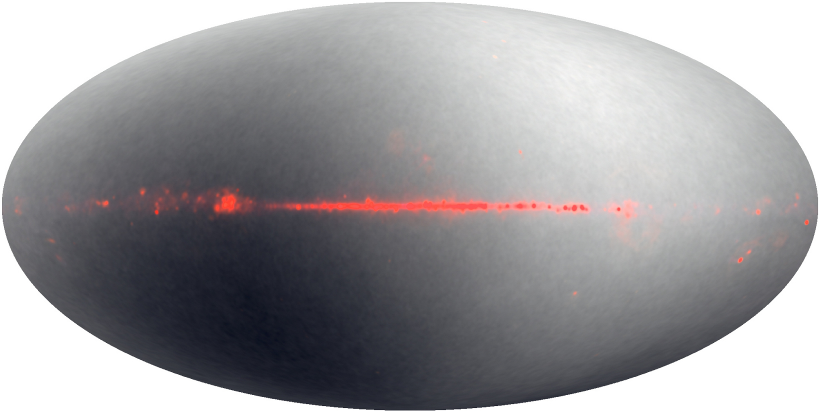
$$T(\theta) = T_0 \left(1 + \frac{v}{c} \cos\theta\right) + \mathcal{O}\left(\frac{v^2}{c^2}\right)$$

- $\delta \gtrsim 1$ today implies

$$\delta(a_{\mathrm{CMB}}) = \frac{\delta(a=1)}{D_+(a_{\mathrm{CMB}})} \gtrsim a_{\mathrm{CMB}}^{-1} \approx 10^{-3}$$

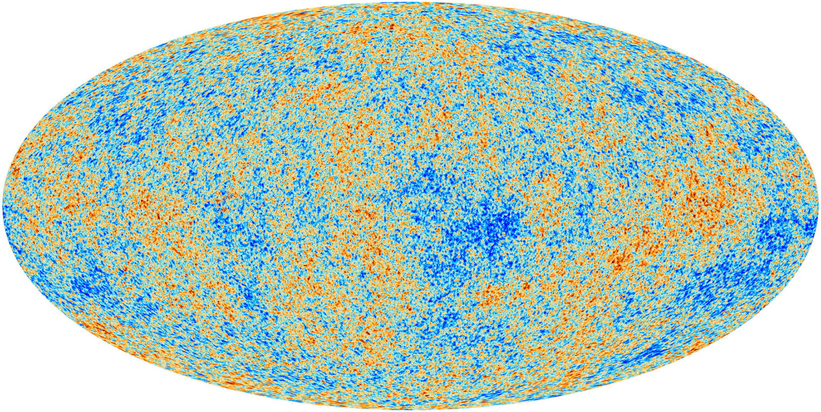
  and similar temperature fluctuations in the CMB
- such fluctuations are not found
- assuming dark matter, temperature fluctuations are expected to be $\delta T/T \approx 10^{-5}$
- detected by COBE in 1992; strongest argument for dark matter

CMB Theory: Dipole



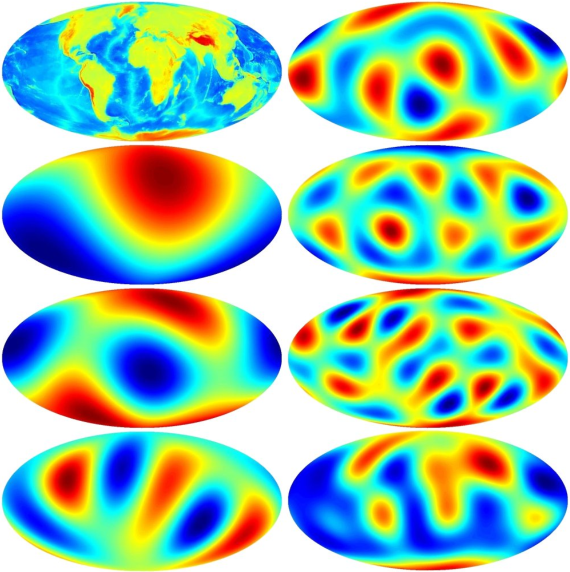
CMB dipole measured by WMAP

CMB Theory: Temperature Fluctuations



CMB temperature fluctuations measured by Planck

CMB Theory: Multipoles



Low-order multipoles of Earth map (ETOPO-5)

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- perturbation equation for relative temperature fluctuation $\Theta \equiv \delta T / T_0$:

$$\ddot{\hat{\Theta}} + \frac{c^2 k^2}{3} \hat{\Theta} - \frac{k^2}{3} \delta\hat{\Phi} - \frac{\delta\ddot{\hat{\Phi}}}{c^2} = 0$$

- for small k: Sachs-Wolfe-effect, $\hat{\Theta} \propto \delta\hat{\Phi}/c^2$
- otherwise, oscillator equation for $\hat{\Theta} - \delta\hat{\Phi}/c^2 \equiv \hat{\theta}$; solution assuming $\dot{\hat{\theta}} = 0$ at $t = 0$

$$\hat{\theta}(t_{\rm rec}) = \hat{\theta}(0) \cos\left[\frac{ck}{\sqrt{3}} t_{\rm rec}\right]$$

$c/\sqrt{3}\, t_{\rm rec} \equiv r_{\rm s}$: sound horizon; oscillations for $k > 2\pi/r_{\rm s}$

CMB Theory: Silk Damping

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- damping occurs due to photon diffusion; diffusion scale:

$$\lambda_{\mathrm{D}} = \sqrt{N}\lambda\,, \quad \lambda = \frac{1}{n_{\mathrm{e}}\sigma_{\mathrm{T}}}$$

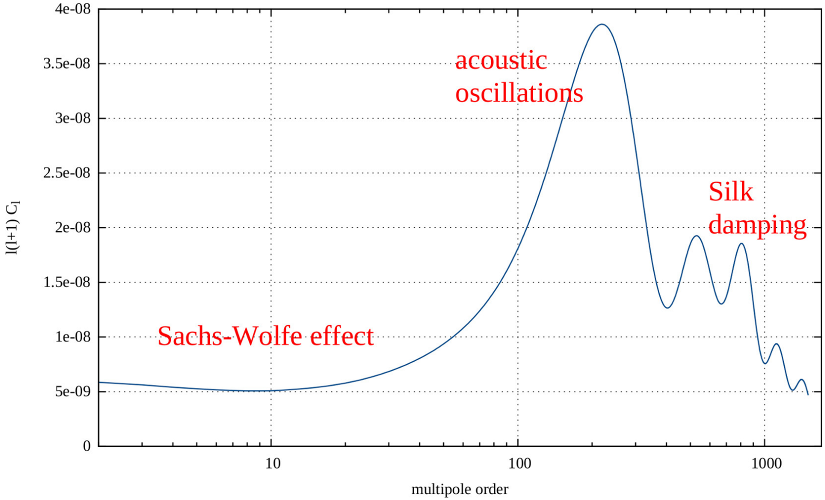
- number of collisions per unit time is $\mathrm{d}N = n_{\mathrm{e}}\sigma_{\mathrm{T}}c\mathrm{d}t$; thus,

$$\lambda_{\mathrm{D}}^2 = \int_0^{t_{\mathrm{rec}}} \frac{c\mathrm{d}t}{n_{\mathrm{e}}\sigma_{\mathrm{T}}}$$

- structures smaller than the diffusion length are damped, hence damping sets in for wave numbers

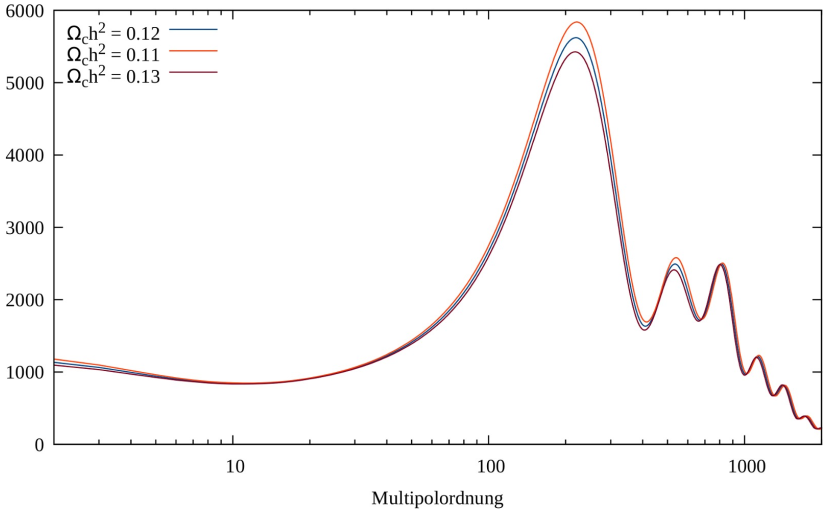
$$k > k_{\mathrm{D}} = \frac{2\pi}{\lambda_{\mathrm{D}}}$$

CMB Spectra: Principal Effects



Principal physical effects on the CMB spectrum

CMB Spectra: Cosmological Parameters



Effect of dark-matter density

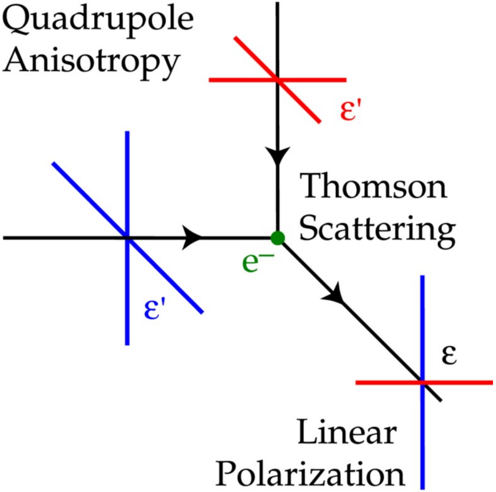
Polarization

- Thomson scattering (of CMB photons) is anisotropic:

$$\frac{d\sigma}{d\Omega} = \frac{3\sigma_{\mathrm{T}}}{8\pi} \left| \vec{e}' \cdot \vec{e} \right|^2$$

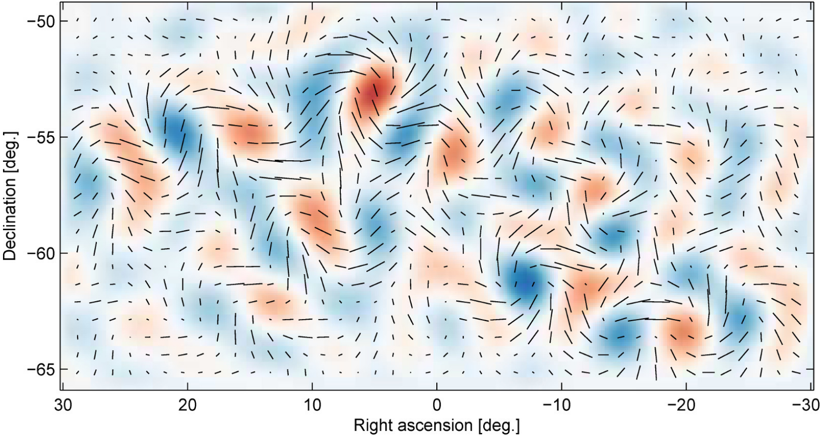
  $\vec{e}'$ and $\vec{e}$: polarization directions of incoming and scattered light
- quadrupolar intensity anisotropy of infalling radiation causes scattered radiation to be polarized
- CMB is expected to be linearly polarized
- intensity of polarized light should be $\approx 10\%$ that of the unpolarized light, amplitude of order $10^{-6}\,\mathrm{K}$

Polarization



Linear polarization by Thomson scattering

Polarization



BICEP-2 E-mode polarization map

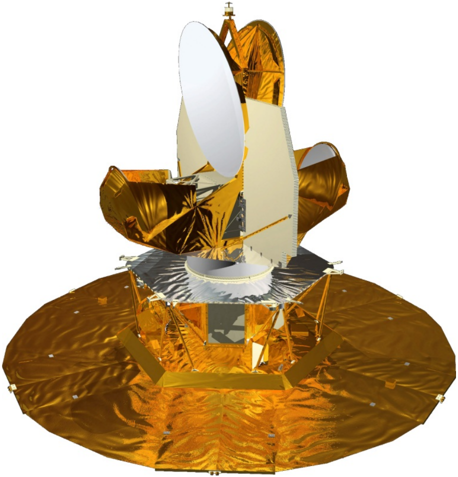
Instruments and Measurements

COBE

COBE

Boomerang, Maxima

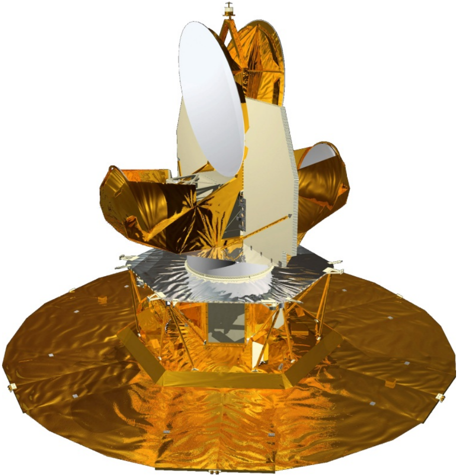
Instruments and Measurements



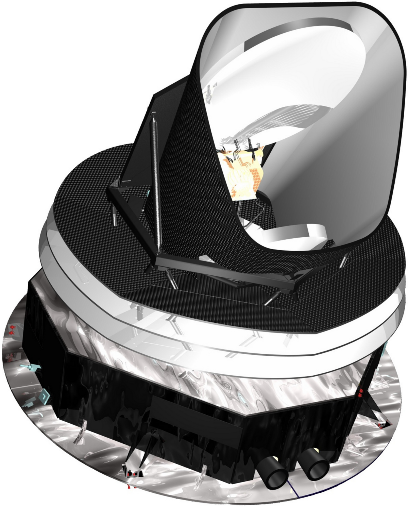
WMAP

Boomerang, Maxima

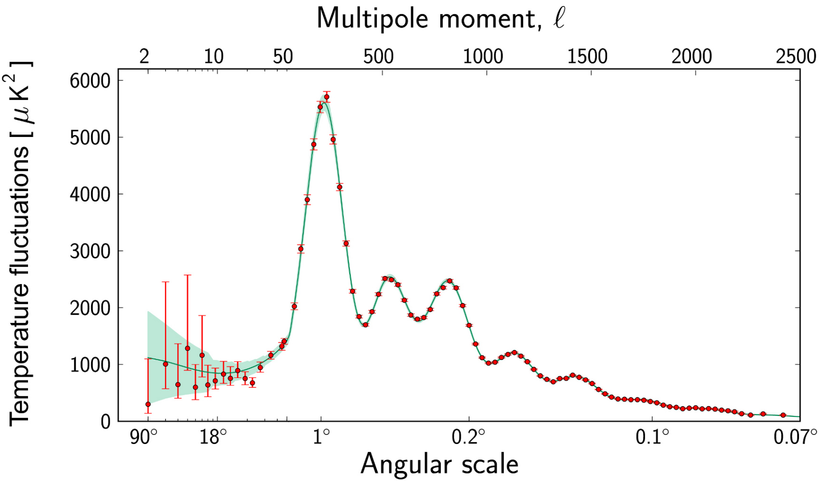
Instruments and Measurements



WMAP

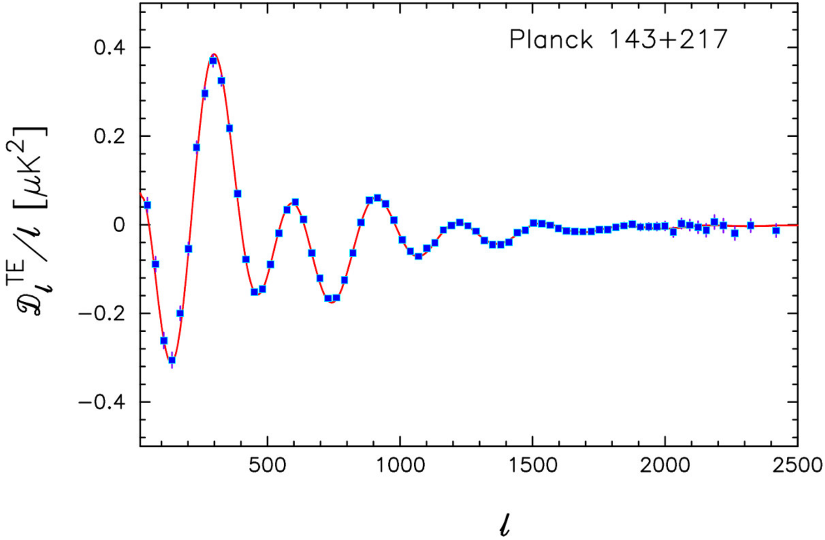


Planck



T power spectrum, measured by Planck

Instruments and Measurements



T-E power spectrum, measured by Planck

Foregrounds

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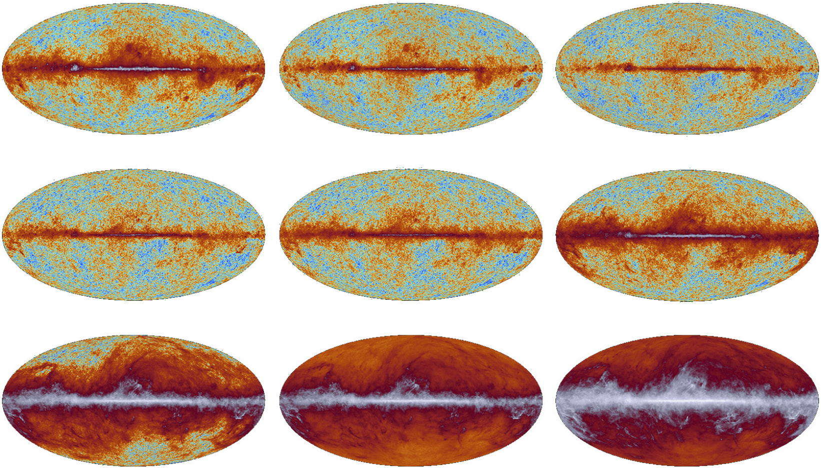
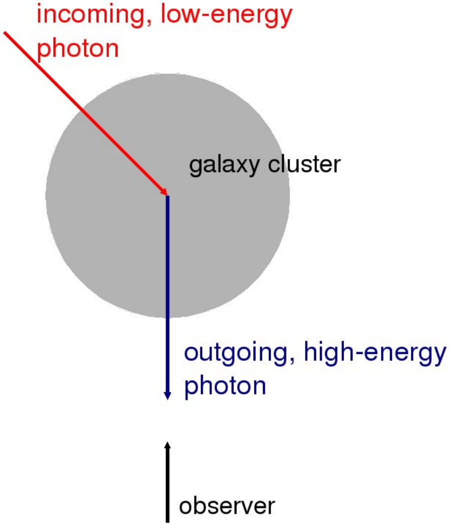
- CMB shines through the entire visible universe on its way to us
- microwave emission from our own Galaxy: warm dust in the plane of the Milky Way with a temperature near $20\,\mathrm{K}$; synchrotron emission from electrons gyrating in the Galactic magnetic field; thermal bremsstrahlung from ionised hydrogen; line emission from molecules like CO
- hot plasma in galaxy clusters inverse-Compton scatters microwave background photons to higher energies: Sunyaev-Zel'dovich effect
- point sources appearing in the microwave background, such as high-redshift galaxies, planets, asteroids, possibly comets in the Solar System, dust in the plane of the Solar System

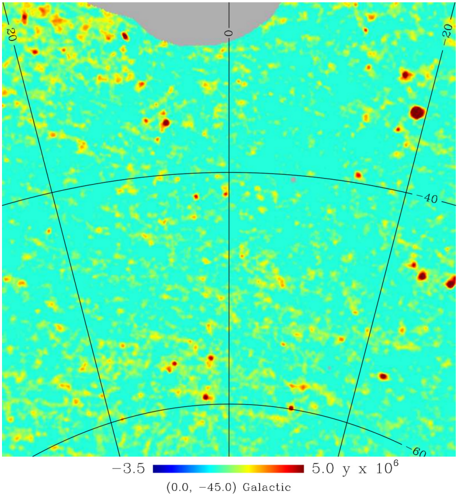
Foregrounds



microwave sky in Planck frequency bands

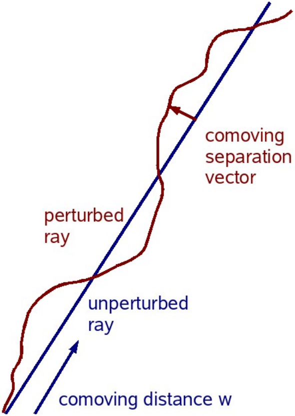
Foregrounds





Planck map with thermal SZ signal

Light Deflection



- density inhomogeneities deflect light: gravitational lensing
- astigmatism causes coherent distortions with power spectrum

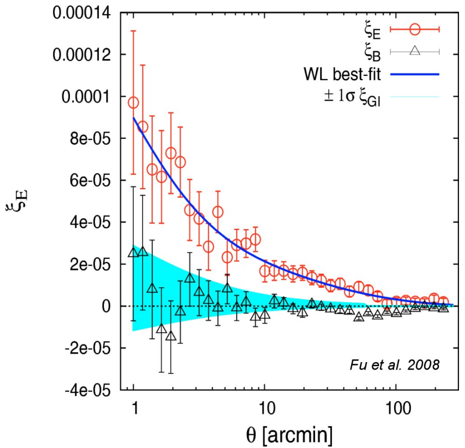
$$P_\gamma(l) = \Omega_{\mathrm{m}0}^2 \int_0^w \mathrm{d}w' \, W^2 P_\delta\left(\frac{l}{f_K(w')}\right)$$

- correlation functions

$$\xi_\gamma(\phi) = \int_0^\infty \frac{l\mathrm{d}l}{2\pi} P_\gamma(l) \mathrm{J}_0(l\phi)$$

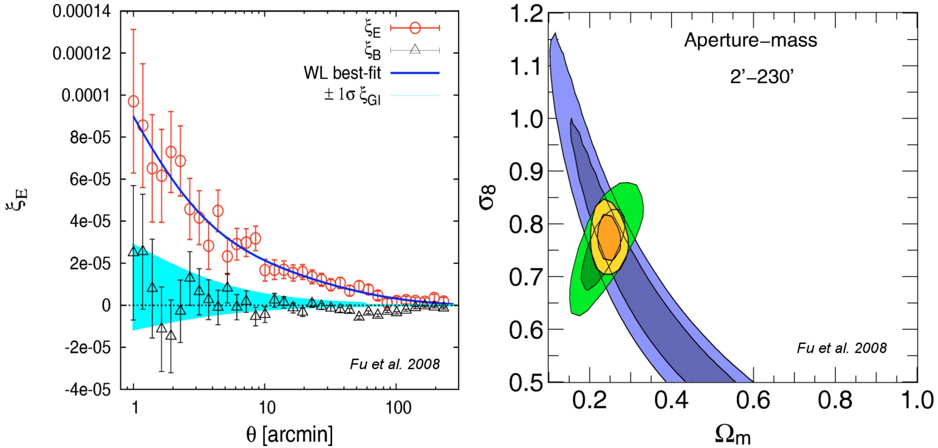
are measurable through coherent distortions of distant galaxy images

Measurements

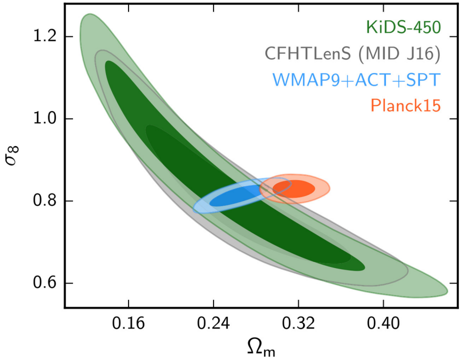


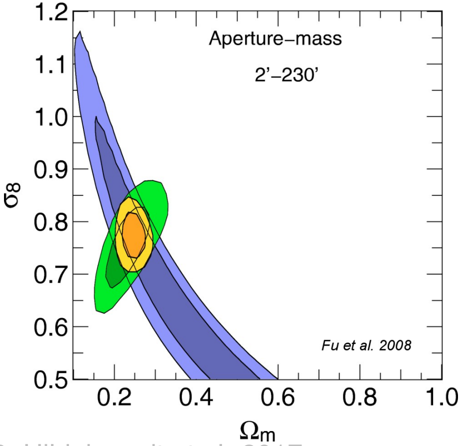
constraints from

Measurements



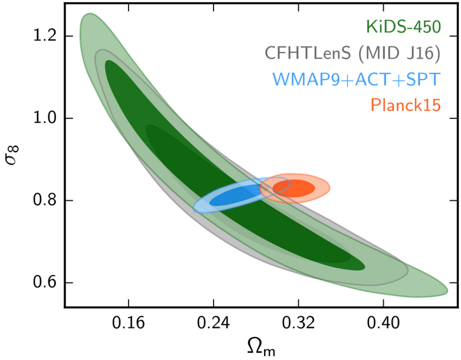
constraints from CFHTLenS, Fu et al. 2008

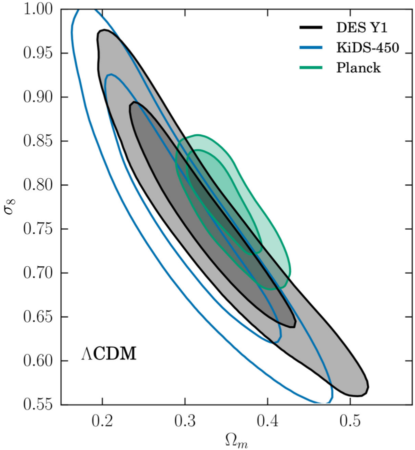




constraints from KiDS-450, Hildebrandt et al. 2017

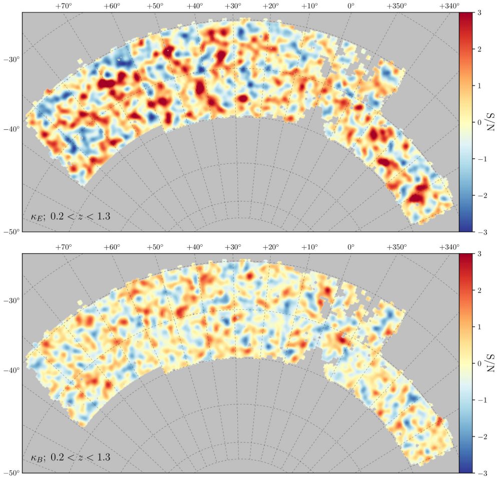
Measurements





constraints from DES, Troxel et al. 2017

Measurements



Sky maps by DES, Chang et al. 2017

Classification and Principle

Supernova 1994d

- SN-Ia: white dwarfs in binaries, driven over Chandrasekhar limit
- "standardisable" candles; measured flux gives distance; spectrum gives redshift
- spectra required for classification: no hydrogen, but silicon lines
- expansion history of the Universe can be recovered

Measurements

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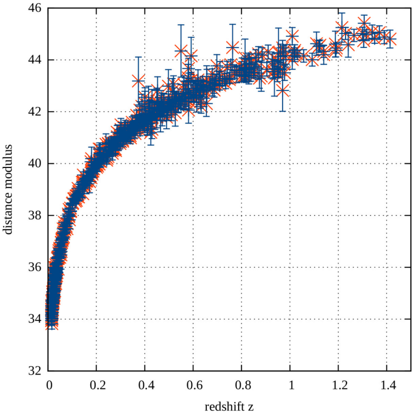
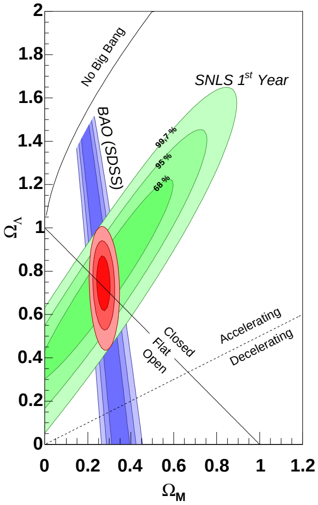


Union data set



type-Ia supernovae require

$$\Omega_{m0} = 0.263 \pm 0.037$$

Astier et al.

Problems

- angular size of the particle horizon at recombination is

$$\theta_{\text{rec}} = \frac{a_{\text{rec}} \Delta w(0, a_{\text{rec}})}{D_{\text{ang}}(0, z_{\text{rec}})} \approx \sqrt{\Omega_0 a_{\text{rec}}} \approx 1.7^\circ \sqrt{\Omega_0}$$

  causal connection? horizon problem

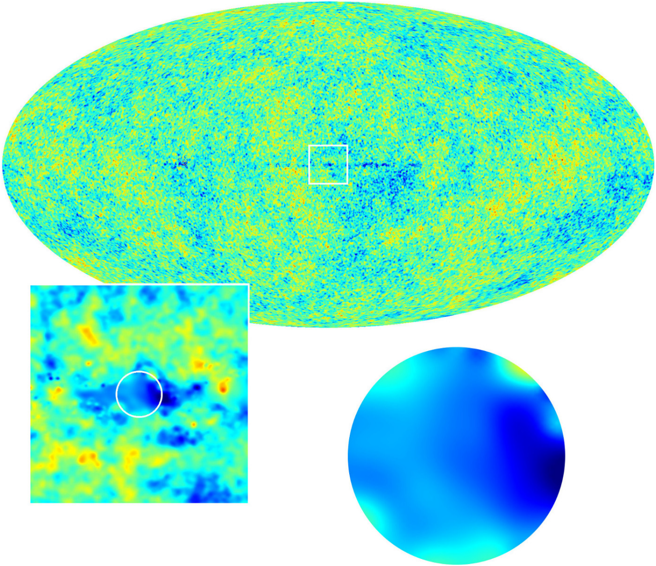
- evolution of flatness:

$$|\Omega_{\text{total}} - 1| \propto \begin{cases} t & \text{radiation-dominated era} \\ t^{2/3} & \text{early matter-dominated era} \end{cases}$$

  tiny deviations of Ω_{total} from unity grow rapidly! flatness problem

- where do structures originate from in the first place?

Problems: Causality Problem



Size of the causal horizon

Inflation: Idea

- accelerated expansion, $\ddot{a} > 0$, can drive universe towards flatness; this seems incompatible with gravity
- Friedmann's equation: accelerated expansion if

$$\rho c^2 + 3p < 0 \,, \quad p < -\frac{\rho c^2}{3}$$

- simple scalar field with Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi)$$

has negative pressure if

$$\dot{\phi}^2 < V(\phi)$$

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- slow-roll conditions:

$$\epsilon \equiv \frac{1}{24\pi G}\left(\frac{V'}{V}\right)^2 \ll 1 \;,$$

$$\eta \equiv \frac{1}{8\pi G}\left(\frac{V''}{V}\right) \ll 1$$

Inflation: Slow Roll, Measurements

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- slow-roll conditions: $\epsilon, \eta \ll 1$
- flatness requires increase in scale factor by $\approx e^{60}$
- this would also solve the horizon (or causality) problem
- inflaton field must decay through some coupling to “ordinary” matter: reheating

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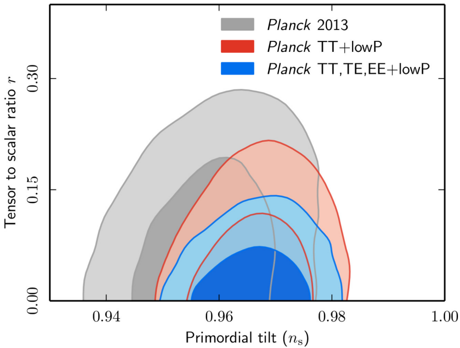


Planck 2015

$$n_s = 0.9677 \pm 0.0060$$

$$\epsilon < 0.011$$

$$\eta = -0.0092^{+0.0074}_{-0.0127}$$

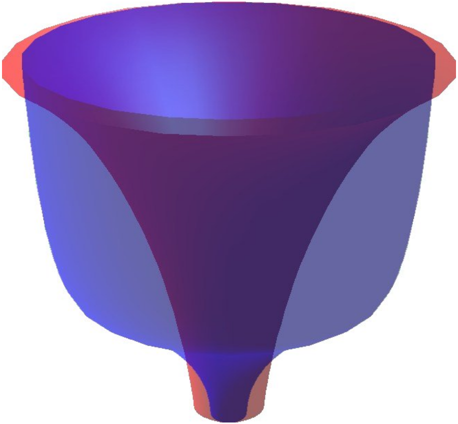
Inflation: Structure Formation

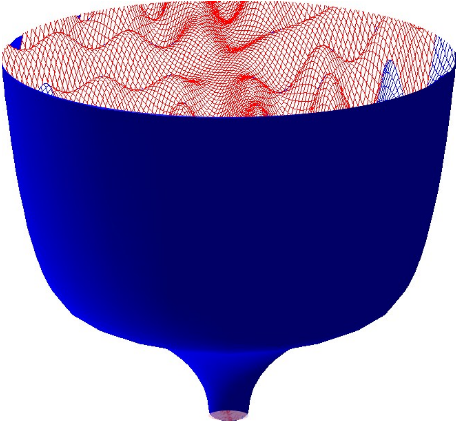
- inflaton field must have undergone vacuum fluctuations
- inflation quickly drives them out of the horizon, “freeze in” because they lack causal contact
- the (primordial) density power spectrum predicted by inflation is

$$P_{\mathrm{i}}(k) \propto k^n \ , \quad n \lesssim 1$$

- density fluctuations are expected to be Gaussian because of the central limit theorem
- inflation provides a possibility for solving the horizon and flatness problems and provides a natural explanation for the origin of structures in the universe

Inflation: Causality and Structure Formation





Expansion of the Universe

- CMB: Universe is spatially flat, i.e. its total energy density equals the critical density
- dark matter contributes $\lesssim 30\%$ to the total energy density; light-element abundances requires the baryon density to be much lower
- type-Ia supernovae reveal need for cosmological constant or accelerated expansion
- high-z supernovae show transition from decelaration to acceleration near $z \sim 1$

Modified Equation of State

- cosmological constant may be dissatisfactory
- as for inflation, assume scalar field (“cosmon”, “quintessence”) with negative pressure,

$$p = w\rho c^2 \,, \quad w < -\frac{1}{3}$$

- for constant w,

$$\rho_{\mathrm{Q}} = \rho_{\mathrm{Q0}} a^{-3(1+w)}$$

- Friedmann equation becomes

$$H^2(a) = H_0^2 \left[\Omega_{\mathrm{m0}} a^{-3} + (1 - \Omega_{\mathrm{m0}} - \Omega_{\mathrm{Q0}}) a^{-2} + \Omega_{\mathrm{Q0}} a^{-3(1+w)}\right]$$

Effects on Cosmology

- early expansion is tightly constrained by light-element abundances
- effects on the CMB: width of the recombination shell, amount of Silk damping
- modified angular-diameter and luminosity distances affect supernovae of type Ia, apparent size of CMB fluctuations, cosmic volume, overall geometry of the universe, gravitational lensing
- growth factor is modified; structures form earlier in quintessence models
- dark-matter haloes tend to be denser, which may have strong effects on their appearance

Effects on Cosmology: Examples

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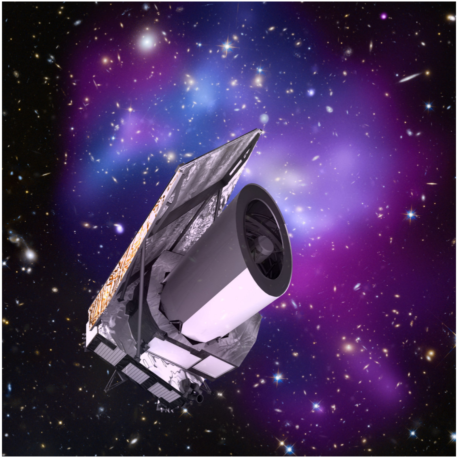
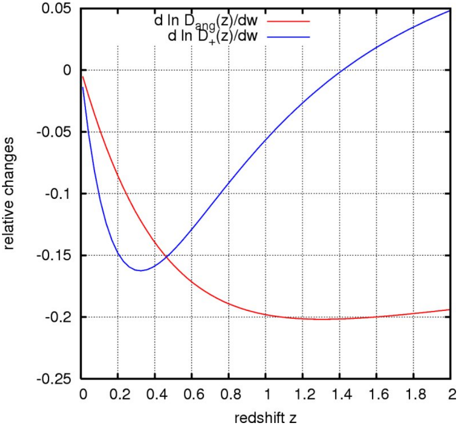


Euclid satellite



Sensitivity of observables