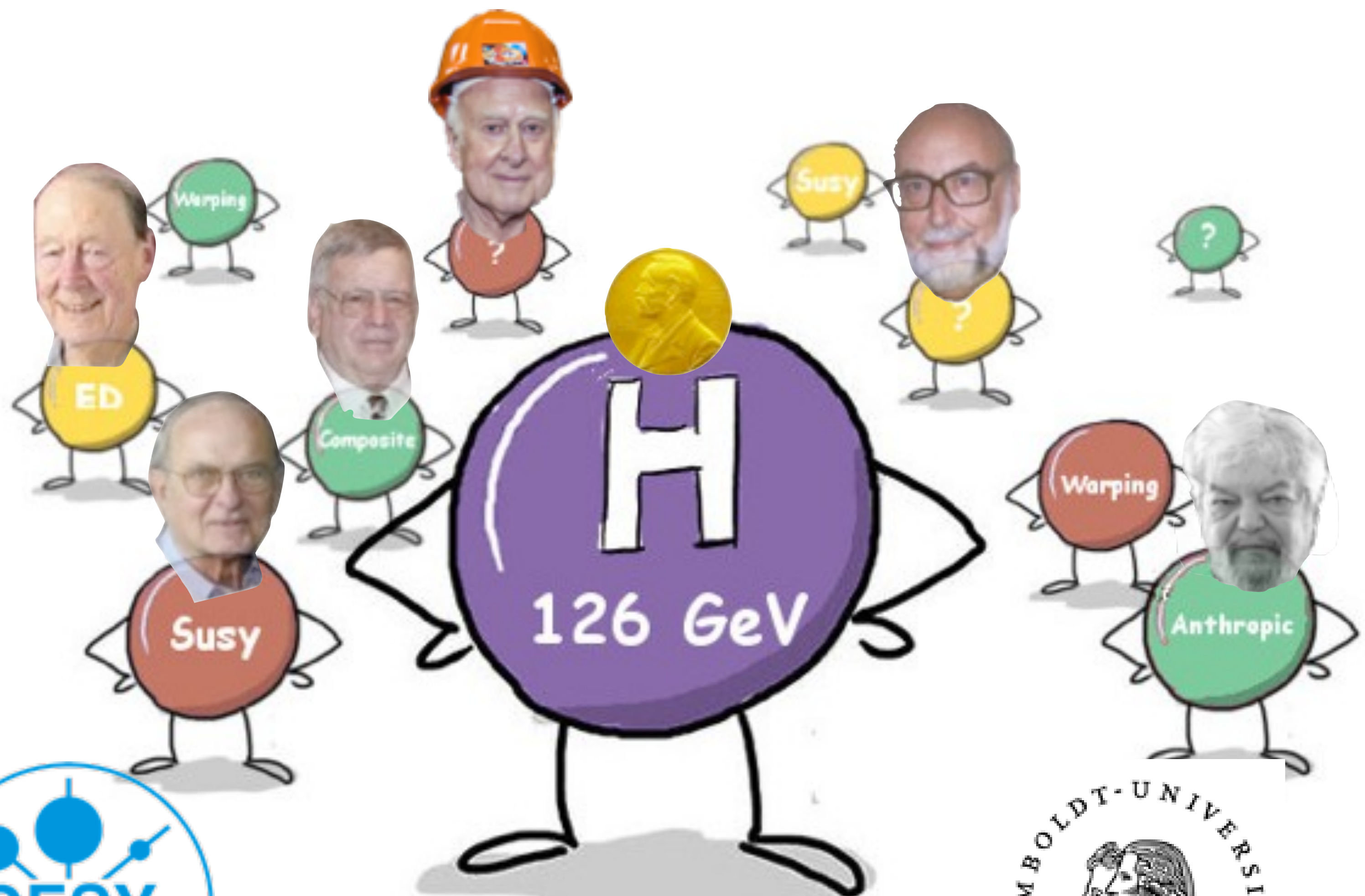
SM, EWSB & Higgs

MITP Summer School 2017

Joint Challenges for Cosmology and Colliders



Lecture 1

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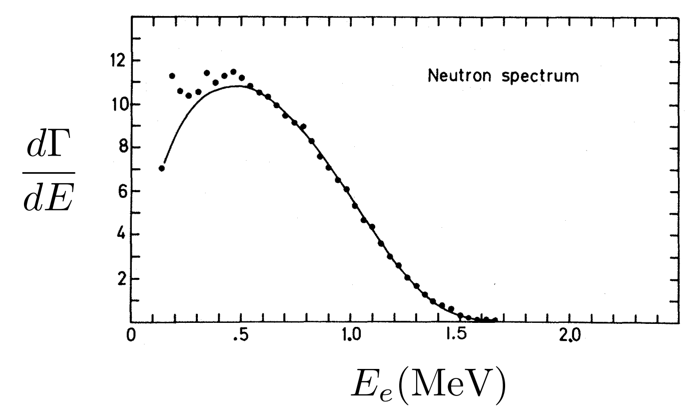
Beta decay

$${}^{40}_{19}\mathrm{K} \to {}^{40}_{20}\mathrm{Ca}^+ + e^- \qquad {}^{64}_{29}\mathrm{Cu} \to {}^{64}_{30}\mathrm{Zn}^+ + e^- \qquad {}^{3}_{1}\mathrm{H} \to {}^{3}_{2}\mathrm{He}^+ + e^-$$

- Two body decays: A➙B+C

$$E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2m_A} c^2 \qquad p = \frac{\sqrt{\lambda(m_A, m_B, m_C)}}{2m_A} c$$

$$\lambda(m_A, m_B, m_C) = (m_A + m_B + m_C)(m_A + m_B - m_C)(m_A - m_B + m_C)(m_A - m_B - m_C)$$



fixed energy of daughter particles (pure SR kinematics, independent of the dynamics)

⇛ non-conservation of energy?

Pauli '30: ∃ neutrino, very light since end-point of spectrum is close to 2-body decay limit

ν first observed in '53 by Cowan and Reines

- N-body decays: A➙B$_1$+B$_2$+...+B$_N$

$$E_{B_1}^{\min} = m_{B_1} c^2 \qquad E_{B_1}^{\max} = \frac{m_A^2 + m_{B_1}^2 - (m_{B_2} + \ldots + m_{B_N})^2}{2m_A} c^2$$

How are neutrinos produced?

$\pi \to \mu\nu$ (more about pion decay later later) $\quad \mu \to e\bar{\nu}_e\nu_\mu$ need 2 neutrino flavors and flavor conservation since $\mu \not\to e\gamma$

electron energy from decay of muon at rest: $m_e c^2 \approx 511\ \mathrm{keV} \le E_e \le 53\ \mathrm{MeV} \approx \frac{m_\mu^2 + m_e^2}{2m_\mu} c^2$

Lederman, Schwartz, Steinberger '62: $p\,\bar{\nu}_\mu \to n\,\mu^+$ but $p\,\bar{\nu}_\mu \not\to n\,e^+$

Fermi theory '33

(paper rejected by Nature: declared too speculative !)

$$\mathcal{L} = G_{\mathcal{F}}(\bar{n}p)(\bar{\nu}_e e)$$

exp: G$_F$=1.166x10^{-5} GeV^{-2}

Why Gauge Theories?

How are we sure that muon and neutron decays proceed via the same interactions?

$\tau_\mu \approx 10^{-6}$s vs. $\tau_{neutron} \approx 900$s

$$\underbrace{\mathcal{L}}_{[\text{mass}]^4} = \underbrace{G_F}_{[\text{mass}]^{-2}}\, \underbrace{\psi^4}_{[\text{mass}]^{3/2\times 4}} \quad \Rightarrow \quad \Gamma \propto G_F^2 \underbrace{m}_{[\text{mass}]}{}^5$$

for the muon, the relevant mass scale is the muon mass m_μ=105MeV: $\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} \sim 10^{-19}\,\text{GeV}$

for the neutron, the relevant mass scale is $(m_n\text{-}m_p)\approx$1.29MeV: $\Gamma_n = \mathcal{O}(1)\frac{G_F^2 \Delta m^5}{\pi^3} \sim 10^{-28}\,\text{GeV}$

ex: what about $\pi^\pm$ decay $\tau_\pi \approx 10^{-8}$s? Why $\frac{\Gamma(\pi^- \to e^-\bar\nu_e)}{\Gamma(\pi^- \to \mu^-\bar\nu_\mu)} \sim 10^{-4}$?

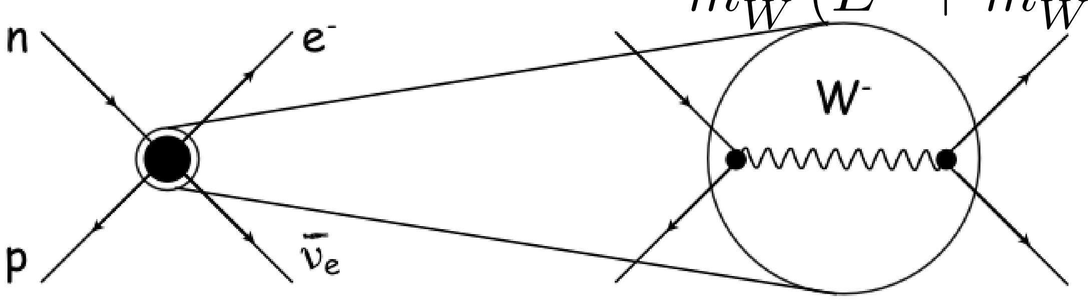
What about weak scattering process, e.g . $e\nu_e \to e\nu_e$?

$$\underbrace{\sigma}_{[\text{mass}]^{-2}} \propto \underbrace{G_F^2}_{[\text{mass}]^{-2\times 2}} \underbrace{E^2}_{[\text{mass}]^2} \quad \Rightarrow$$

non conservation of probability
(non-unitary theory)
inconsistent at energy above 300GeV

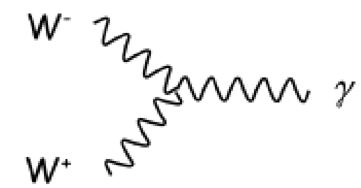
Electroweak Interactions

$\sigma \propto G_F^2 E^2$ $\qquad$ $\sigma \propto g^4 \dfrac{E^2}{m_W^2 (E^2 + m_W^2)}$



$G_F \propto \dfrac{g^2}{m_W^2}$

charged W ⇛ must couple to photon:



⇛ non-abelian gauge symmetry $[Q,T^\pm]=\pm T^\pm$

1. No additional "force": (Georgi, Glashow '72) ⇛ extra matter

$SU(2)$

$$[T^a, T^b] = i\epsilon^{abc} T^c$$

$$[T^+, T^-] = Q \qquad [Q, T^\pm] = + \pm T^\pm$$

$$T^\pm = \frac{1}{\sqrt{2}}(T^1 \pm i T^2)$$

$\mathrm{Tr}_{\mathrm{irrep}} T^3 = 0$ ⇛ extra matter $\begin{pmatrix} X_L \\ \nu_L \\ e_L \end{pmatrix} \begin{pmatrix} X_R \\ \nu_R \\ e_R \end{pmatrix}$

$SU(1,1)$

$$[T^+, T^-] = -Q$$

$$[Q, T^\pm] = + \pm T^\pm$$

non-compact

unitary rep. has dim ∞

E_2

2D Euclidean group

$$[T^+, T^-] = 0$$

$$[Q, T^\pm] = + \pm T^\pm$$

only one unitary rep.

of finite dim. = trivial rep.

2. No additional "matter" (SM: Glashow '61, Weinberg '67, Salam '68): SU(2)xU(1)

⇛ extra force

$Q = T^3?$

as Georgi-Glashow

⇛ extra matter

$Q = Y?$

$Q(e_L) = Q(\nu_L)$

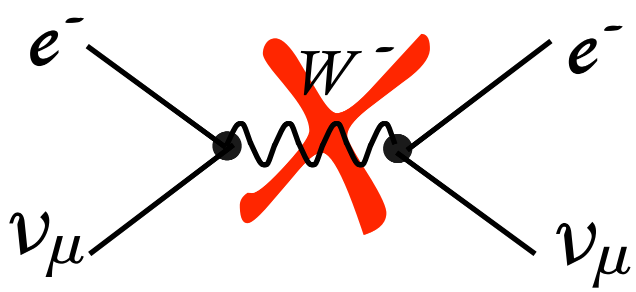
$Q = T^3 + Y!$

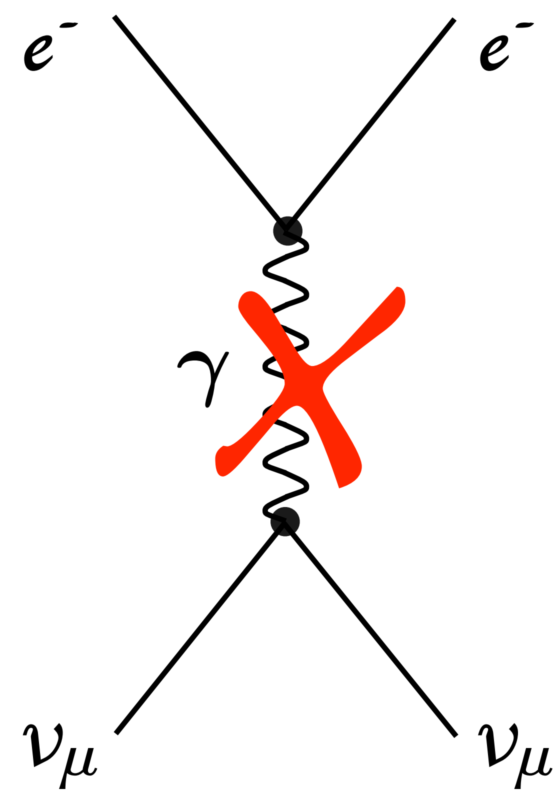
Gell-Mann '56, Nishijima-Nakano '53

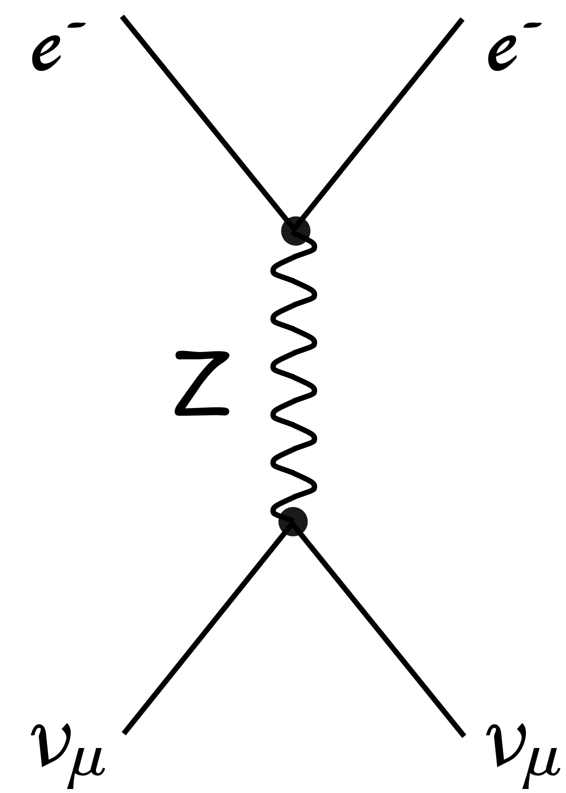
Electroweak Interactions

Gargamelle experiment ’73 first established the S(2)xU(1) structure

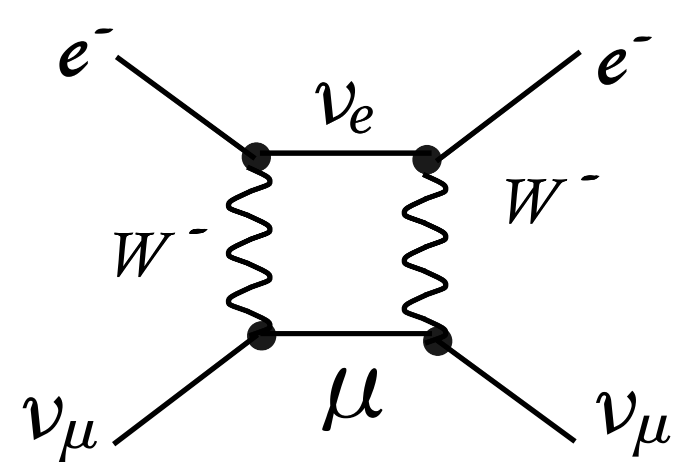
idea: rely on a particle that doesn’t interact with photon and a NC process! $\nu_\mu e^- \to \nu_\mu e^-$







loop-suppressed contribution from W:



The SM particle content

Field	$SU(3)$	$SU(2)_L$	T^3	Y	$Q = T^3 + Y$
g_μ^a (gluons)	$\mathbf{8}$	$\mathbf{1}$	0	0	0
$(W_\mu^\pm, W_\mu^0)$	$\mathbf{1}$	$\mathbf{3}$	$(\pm 1, 0)$	0	$(\pm 1, 0)$
B_μ^0	$\mathbf{1}$	$\mathbf{1}$	0	0	0
$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\mathbf{3}$	$\mathbf{2}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$\frac{1}{6}$	$\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$
u_R	$\mathbf{3}$	$\mathbf{1}$	0	$\frac{2}{3}$	$\frac{2}{3}$
d_R	$\mathbf{3}$	$\mathbf{1}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$E_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
e_R	$\mathbf{1}$	$\mathbf{1}$	0	-1	-1
$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$\frac{1}{2}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Interactions Fermions-Gauge Bosons

Gauge invariance says:

$$\mathcal{L} = gW_\mu^3 \left(\sum_i T_{3L\,i}\ \bar\psi_i \bar\sigma^\mu \psi_i \right) + g' B_\mu \left(\sum_i y_i\ \bar\psi_i \bar\sigma^\mu \psi_i \right)$$

Going to the mass eigenstate basis:

$$Z_\mu = cW_\mu^3 - sB_\mu \qquad \gamma_\mu = sW_\mu^3 + cB_\mu$$

with

$$c = \frac{g}{\sqrt{g^2+g'^2}} \qquad s = \frac{g'}{\sqrt{g^2+g'^2}}$$

$$Q = T_{3L} + Y$$

$$\mathcal{L} = \sqrt{g^2+g'^2} Z_\mu \left(\sum_i (T_{3L\,i} - s^2 Q_i)\ \bar\psi_i \bar\sigma^\mu \psi_i \right) + \frac{gg'}{\sqrt{g^2+g'^2}} \gamma_\mu \left(\sum_i Q_i\ \bar\psi_i \bar\sigma^\mu \psi_i \right)$$

not protected by gauge invariance
corrected by radiative corrections + new physics

protected by U(1)$_{em}$ gauge invariance
$\Rrightarrow$ no correction

electric charge

$$e = \frac{gg'}{\sqrt{g^2+g'^2}} = sg = cg'$$

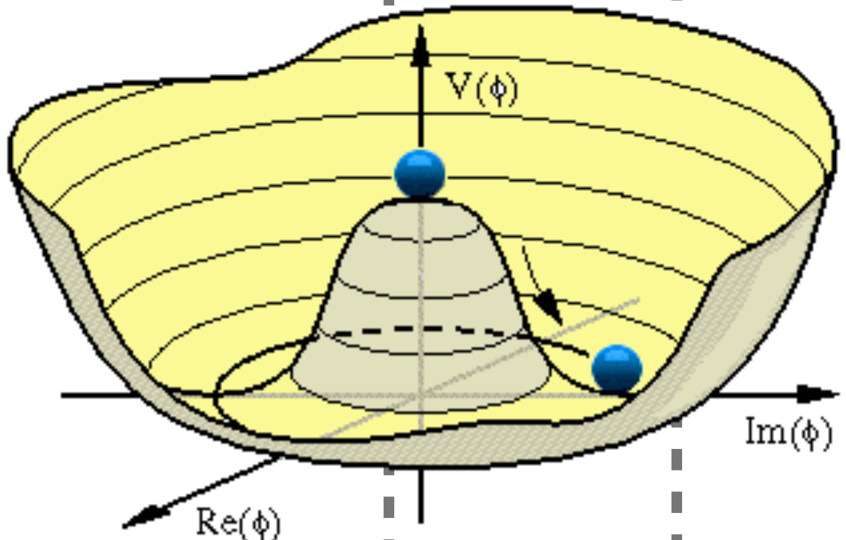
Higgs Mechanism

Symmetry of the Lagrangian

$$SU(2)_L \times U(1)_Y$$

Higgs Doublet

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$$



Symmetry of the Vacuum

$$U(1)_{e.m.}$$

Vacuum Expectation Value

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad \text{with} \quad v \approx 246 \text{ GeV}$$

$$D_\mu H = \partial_\mu H - \frac{i}{2} \begin{pmatrix} g W^3_\mu + g' B_\mu & \sqrt{2} g W^+_\mu \\ \sqrt{2} g W^-_\mu & -g W^3_\mu + g' B_\mu \end{pmatrix} H \quad \text{with} \quad \mathrm{W}^\pm_\mu = \frac{1}{\sqrt{2}} (\mathrm{W}^1_\mu \mp \mathrm{W}^2_\mu)$$

$$|D_\mu H|^2 = \frac{1}{4} g^2 v^2 W^+_\mu W^{-\,\mu} + \frac{1}{8} (W^3_\mu \; B_\mu) \begin{pmatrix} g^2 v^2 & -g g' v^2 \\ -g g' v^2 & g'^2 v^2 \end{pmatrix} \begin{pmatrix} W^{3\,\mu} \\ B^\mu \end{pmatrix}$$

- Gauge boson spectrum
  - electrically charged bosons $\quad M_W^2 = \frac{1}{4} g^2 v^2$
  - electrically neutral bosons

$$Z_\mu = c W^3_\mu - s B_\mu$$
$$\gamma_\mu = s W^3_\mu + c B_\mu$$

Weak mixing angle

$$c = \frac{g}{\sqrt{g^2 + g'^2}} \qquad s = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$M_Z^2 = \frac{1}{4} (g^2 + g'^2) v^2$$

$$M_\gamma = 0$$

Fermion Masses

SM is a chiral theory ($\neq$ QED that is vector-like)

$m_e \bar{e}_L e_R + h.c.$ is not gauge invariant

The SM Lagrangian doesn't not contain fermion mass terms
fermion masses are emergent quantities
that originate from interactions with Higgs vev

$$y_{ij} \bar{f}_{L_i} H f_{R_j} = \frac{y_{ij} v}{\sqrt{2}} \bar{f}_{L_i} f_{R_j} + \frac{y_{ij}}{\sqrt{2}} h \bar{f}_{L_i} f_{R_j}$$

Fermion Masses

In SM, the Yukawa interactions are the only source of the fermion masses

$$y_{ij}\bar{f}_{L_i}Hf_{R_j} = \frac{y_{ij}v}{\sqrt{2}}\bar{f}_{L_i}f_{R_j} + \frac{y_{ij}}{\sqrt{2}}h\bar{f}_{L_i}f_{R_j}$$

mass

higgs-fermion interactions

both matrices are simultaneously diagonalizable

no tree-level Flavor Changing Current induced by the Higgs

Not true anymore if the SM fermions mix with vector-like partners or for non-SM Yukawa

$$y_{ij}\left(1+c_{ij}\frac{|H|^2}{f^2}\right)\bar{f}_{L_i}Hf_{R_j} = \frac{y_{ij}v}{\sqrt{2}}\left(1+c_{ij}\frac{v^2}{2f^2}\right)\bar{f}_{L_i}f_{R_j} + \left(1+3c_{ij}\frac{v^2}{2f^2}\right)\frac{y_{ij}}{\sqrt{2}}h\bar{f}_{L_i}f_{R_j}$$

Look for SM forbidden Flavor Violating decays h → $\mu\tau$ and h → $e\tau$

(look also at t→hc ATLAS '14)

- weak indirect constrained by flavor data ($\mu\to e\gamma$): BR<10%
- ATLAS and CMS have the sensitivity to set bounds O(1%)
- ILC/CLIC/FCC-ee can certainly do much better

Fermion Masses

In SM, the Yukawa interactions are the only source of the fermion masses

$$y_{ij}\bar{f}_{L_i} H f_{R_j} = \underbrace{\frac{y_{ij}v}{\sqrt{2}}\bar{f}_{L_i} f_{R_j}}_{\text{mass}} + \underbrace{\frac{y_{ij}}{\sqrt{2}} h \bar{f}_{L_i} f_{R_j}}_{\text{higgs-fermion interactions}}$$

Quark mixings

$$\mathcal{L}_{Yuk} \;\; = \;\; \lambda^L_{ij}(\bar{L}^i_L \phi^c) l^j_R + \lambda^U_{ij}(\bar{Q}^i_{L,\alpha}\phi) u^j_{R,\alpha} + \lambda^D_{ij}(\bar{Q}^i_{L,\alpha}\phi^c) d^j_{R,\alpha} + cc$$

$$\mathcal{L}_L^\dagger\left(\frac{v}{\sqrt{2}}\lambda^L\right)\mathcal{L}_R = \begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix}$$

$$\mathcal{U}_L^\dagger\left(\frac{-v}{\sqrt{2}}\lambda^U\right)\mathcal{U}_R = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}$$

$$\mathcal{D}_L^\dagger\left(\frac{v}{\sqrt{2}}\lambda^D\right)\mathcal{D}_R = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}$$

$$\begin{aligned}\mathcal{L}_{Yuk\,quad} = & - \left(\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L\right)\begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix}\begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \\ & - \left(\bar{u}_{L,\alpha}, \bar{c}_{L,\alpha}, \bar{t}_{L,\alpha}\right)\begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}\begin{pmatrix} u_{R,\alpha} \\ c_{R,\alpha} \\ t_{R,\alpha} \end{pmatrix} \\ & - \left(\bar{d}_{L,\alpha}, \bar{s}_{L,\alpha}, \bar{b}_{L,\alpha}\right)\mathcal{V}_{KM}^\dagger\begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}\begin{pmatrix} d_{R,\alpha} \\ s_{R,\alpha} \\ b_{R,\alpha} \end{pmatrix} \\ & + \; cc\end{aligned}$$

$$\mathcal{V}_{KM} = \mathcal{D}_L^\dagger \mathcal{U}_L$$

Custodial Symmetry

- Rho parameter

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_w} = \frac{\frac{1}{4}g^2v^2}{\frac{1}{4}(g^2+g'^2)v^2\frac{g^2}{g^2+g'^2}} = 1$$

- Consequence of an approximate global symmetry of the Higgs sector

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$$ Higgs doublet = 4 real scalar fields

$$V(H) = \lambda\left(H^\dagger H - \frac{v^2}{2}\right)^2$$ is invariant under the rotation of the four real components

$$SO(4) \sim SU(2)_L \times SU(2)_R$$

$$SU(2)_R$$

$$SU(2)_L \Rightarrow \left(i\sigma^2 H^\star \quad H\right) = \Phi$$

2x2 matrix

$$\Phi^\dagger\Phi = H^\dagger H \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

$$V(H) = \frac{\lambda}{4}\left(\mathrm{tr}\Phi^\dagger\Phi - \mathrm{v}^2\right)^2$$

explicitly invariant under $SU(2)_L \times SU(2)_R$

Custodial Symmetry

Higgs vev

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \qquad \langle \Phi \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

$$SU(2)_L \times SU(2)_R \to SU(2)_V$$

unbroken symmetry in the broken phase

$$(W^1_\mu, W^2_\mu, W^3_\mu) \quad \text{transforms as a triplet}$$

$$(Z_\mu \, \gamma_\mu) \begin{pmatrix} M_Z^2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} Z^\mu \\ \gamma^\mu \end{pmatrix} = (W^3_\mu \, B_\mu) \begin{pmatrix} c^2 M_Z^2 & -csM_Z^2 \\ -csM_Z^2 & s^2 M_Z^2 \end{pmatrix} \begin{pmatrix} W^{3\,\mu} \\ B^\mu \end{pmatrix}$$

The $SU(2)_V$ symmetry imposes the same mass term for all W^i thus $c^2 M_Z^2 = M_W^2$

ρ = 1

The hypercharge gauge coupling and the Yukawa couplings break the custodial SU(2)$_V$, which will generate a (small) deviation to ρ = 1 at the quantum level.

Counting the degrees of freedom

$$SU(2)_L \times U(1)_Y \to U(1)_{\rm EM}$$

3 broken gauge directions = 3 eaten Goldstone bosons

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$$

Higgs doublet = 4 real scalar fields

3 eaten Goldstone bosons

One physical degree of freedom the Higgs boson

unbroken phase

4 massless gauge bosons 8 dof
2 complex scalar doublet 4 dof

broken phase

3 massive gauge bosons 9 dof
1 massless gauge boson 2 dof
1 real scalar field 1 dof

Playing with the Goldstone's

U(1)

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - \lambda \left(|\phi|^2 - \frac{1}{2} f^2 \right)^2 \qquad \phi \to e^{i\alpha}\phi$$

$$\phi = \frac{1}{\sqrt{2}} \left(f + h(x)\right) e^{i\theta(x)/f} \qquad h \to h \qquad \theta \to \theta + \alpha f$$

U(1) non-linearly realized

shift symmetry forbids any mass term for θ

$$\mathcal{L} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\left(\frac{f+h}{f}\right) \partial_\mu\theta\partial^\mu\theta - \lambda\left(f^2h^2 + fh^3 + \frac{1}{4}h^4\right)$$

if the U(1) symmetry is gauged

the Goldstone boson is eaten and becomes the long. pola. of the massive gauge boson

SU(N)→SU(N-1)

$$\left(N^2-1\right) - \left(\left(N-1\right)^2 - 1\right) = 2N-1$$ Goldstone bosons

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ f \end{pmatrix} \qquad \phi = \exp\left(\frac{i}{f}\left(\begin{array}{ccc|c} -\pi_0 & & & \pi_1 \\ & \ddots & & \vdots \\ & & -\pi_0 & \pi_{N-1} \\ \hline \pi_1^\star & \dots & \pi_{N-1}^\star & (N-1)\pi_0 \end{array}\right)\right)\begin{pmatrix} 0 \\ \vdots \\ 0 \\ f \end{pmatrix}$$

(N-1) complex $\vec{\pi}$ and 1 real π_0 scalars

SU(N-1) $$\phi \to U_{N-1}\phi = U_{N-1}e^{i\pi}U_{N-1}^\dagger U_{N-1}\phi_0 = e^{iU_{N-1}\pi U_{N-1}^\dagger}\phi_0 \quad \pi \to \left(\begin{array}{c|c} U_{N-1} & \\ \hline & 1 \end{array}\right)\left(\begin{array}{c|c} \pi_0 & \pi \\ \hline \pi^\dagger & \pi_0 \end{array}\right)\left(\begin{array}{c|c} U_{N-1}^\dagger & \\ \hline & 1 \end{array}\right) = \left(\begin{array}{c|c} \pi_0 & U_{N-1}\pi \\ \hline \pi^\dagger U_{N-1}^\dagger & \pi_0 \end{array}\right)$$

linear transformation of the Goldstone under the unbroken sym.

SU(N)/SU(N-1) $$\phi \to \exp i\left(\begin{array}{c|c} & \vec{\alpha} \\ \hline \vec{\alpha}^\dagger & \end{array}\right)\exp i\left(\begin{array}{c|c} & \vec{\pi} \\ \hline \vec{\pi}^\dagger & \end{array}\right)\phi_0 \approx \exp i\left(\begin{array}{c|c} & \vec{\pi}+\vec{\alpha} \\ \hline \vec{\pi}^\dagger + \vec{\alpha}^\dagger & \end{array}\right)\phi_0$$

non-linear transformation of the Goldstone

Playing with the Goldstone's

$$\frac{SU(2)_L \times SU(2)_R}{SU(2)_V} \qquad \Sigma \to g_R \Sigma g_L^\dagger \qquad \langle \Sigma \rangle = \mathbb{1}_2$$

$$\mathcal{L} = \frac{v^2}{4} \mathrm{TR}\left(D_\mu \Sigma^\dagger D^\mu \Sigma\right)$$

$$D_\mu \Sigma = \partial_\mu \Sigma - i g W_\mu^a \frac{\sigma^2}{2} \Sigma + i g' \Sigma \frac{\sigma^3}{2} B_\mu$$

unitary gauge

$$\Sigma = \mathbb{1}_2$$

$$m_W^2 W_\mu^+ W^{-\,\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$

gauge-less limit

$$g = g' = 0$$

$$\frac{1}{2}\left(\partial_\mu \pi^a\right)^2 + \frac{1}{6v^2}\left(\pi^a \partial_\mu \pi^a\right)^2 - \frac{1}{6v^2}\left(\pi^a\right)^2\left(\partial_\mu \pi^a\right)^2$$

non-linear sigma model with a cutoff 4πv ≈ 3TeV
effective theory valid in the energy domain
(thanks to g≈0.6)

$$m_W \ll E \ll 4\pi v = 8\pi m_W / g$$