HLbL contribution to $(g-2)_\mu$ in a dispersive approach

Vladyslav Pauk

in coll with Marc Vanderhaeghen

JLab

Newport News, USA

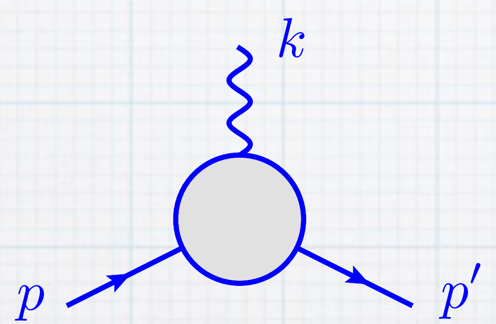
Phi-Psi 2017

Schloss Waldthausen, Mainz, Germany

Definition of a_μ

covariant expansion of the muon's electromagnetic vertex:

$$\bar{u}(p')\Gamma_\mu(p',p)u(p) = \bar{u}(p')\left[\gamma_\mu F_1(k^2) + \frac{i}{2m}\sigma_{\mu\nu}k^\nu F_2(k^2)\right]u(p)$$ Pauli form factor



$$k = p' - p$$

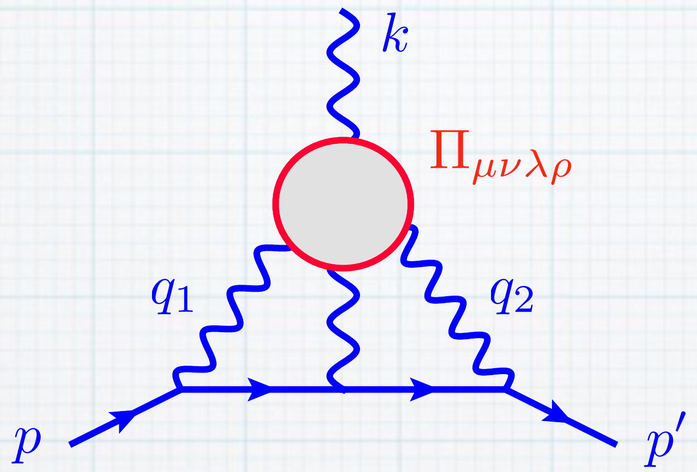
$$F_2(k^2) = \mathrm{Tr}\left[(\not{p} + m)\Lambda_\nu(p',p)(\not{p}' + m)\Gamma^\nu(p',p)\right]$$

projection operator $$\Lambda_\nu(p',p) = \frac{m^2}{k^2(4m^2-k^2)}\left[\gamma_\nu + \frac{k^2+2m^2}{m(k^2-4m^2)}(p'+p)_\nu\right]$$

$$a_\mu = F_2(0)$$

Pauli form factor
in the limit of static electromagnetic field

HLbL contribution to a_μ



HLbL contribution to the muon's electromagnetic current:

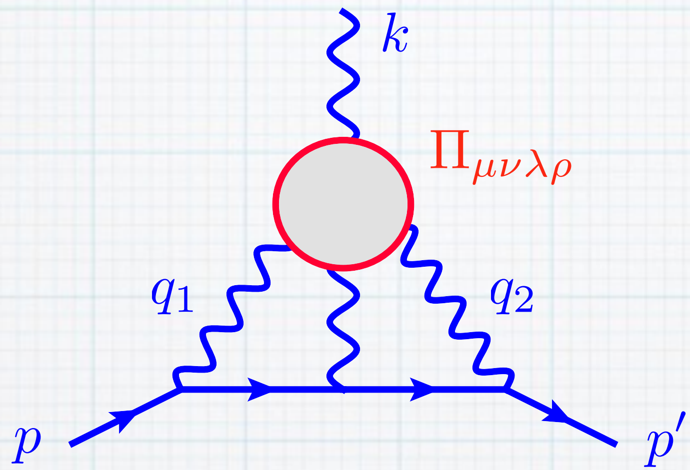
$$(-ie)\Gamma_\rho(p',p) = \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4} \frac{(-i)^3}{q_1^2 q_2^2 (k-q_1-q_2)^2} \frac{i^2}{[(p+q_1)^2-m^2]\,[(p+k-q_2)^2-m^2]}$$
$$\times (-ie)^3 \gamma^\lambda (\not{p}+\not{k}-\not{q}_2+m)\gamma^\nu(\not{p}+\not{q}_1+m)\gamma^\mu (ie)^4 \Pi_{\mu\nu\lambda\rho}(q_1, k-q_1-q_2, q_2, k)$$

HLbL contribution to the Pauli form factor:

$$F_2(k^2) = e^6 \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4} \frac{\Pi_{\mu\nu\lambda\rho}(q_1,k-q_1-q_2,q_2,k) L^{\mu\nu\lambda\rho}(p,p',q_1,k-q_1-q_2,q_2)}{q_1^2 q_2^2 (k-q_1-q_2)^2\,[(p+q_1)^2-m^2]\,[(p+k-q_2)^2-m^2]}$$

$$L^{\mu\nu\lambda\rho}(p,p',q_1,k-q_1-q_2,q_2) = \mathrm{Tr}\left[\Lambda^\rho(p,p')\gamma^\lambda(\not{p}+\not{k}-\not{q}_2+m)\gamma^\nu(\not{p}+\not{q}_1+m)\gamma^\mu\right]$$

HLbL contribution to a_μ



HLbL contribution to the muon's electromagnetic current:

$$(-ie)\Gamma_\rho(p',p) = \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4} \frac{(-i)^3}{q_1^2 q_2^2 (k-q_1-q_2)^2} \frac{i^2}{[(p+q_1)^2-m^2]\,[(p+k-q_2)^2-m^2]}$$
$$\times (-ie)^3 \gamma^\lambda (\not p + \not k - \not q_2 + m)\gamma^\nu (\not p + \not q_1 + m)\gamma^\mu (ie)^4 \Pi_{\mu\nu\lambda\rho}(q_1, k-q_1-q_2, q_2, k)$$

completeness relation

$$\sum_\lambda (-1)^\lambda \varepsilon^{\mu *}(q,\lambda)\varepsilon^\nu(q,\lambda) = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}$$

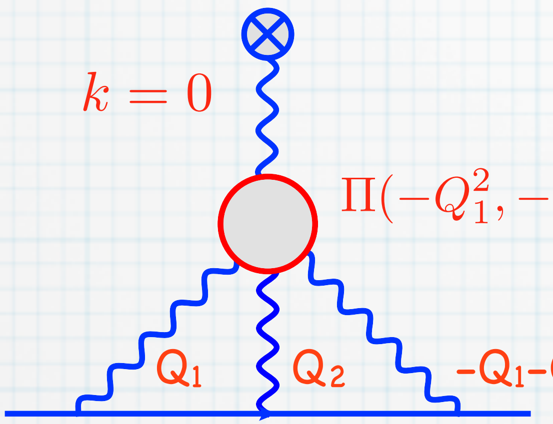
helicity amplitudes

$$\Pi_{\mu\nu\lambda\rho}L^{\mu\nu\lambda\rho} = \sum_{\lambda,\lambda_1,\lambda_2,\lambda_3} (-1)^{\lambda+\lambda_1+\lambda_2+\lambda_3} L_{\lambda_1\lambda_2\lambda_3\lambda}\Pi_{\lambda_1\lambda_2\lambda_3\lambda}$$

$$F_2(k^2) = e^6 \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda} (-1)^{\lambda+\lambda_1+\lambda_2+\lambda_3} \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4} L_{\lambda_1\lambda_2\lambda_3\lambda}(p,p',q_1,k-q_1-q_2,q_2)$$
$$\times \frac{\Pi_{\lambda_1\lambda_2\lambda_3\lambda}(q_1,k-q_1-q_2,q_2,k)}{q_1^2 q_2^2 (k-q_1-q_2)^2\,[(p+q_1)^2-m^2]\,[(p+k-q_2)^2-m^2]}$$

Evaluation of HLbL contribution to a_μ

limit of static electromagnetic field



Wick rotation

$$Q_1^2 = -q_1^2$$
$$Q_2^2 = -q_2^2$$
$$Q^2 = -(q_1 + q_2)^2$$

space-like invariants

master formula for the HLbL contribution to a_μ:

$$a_\mu = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau \sqrt{1-\tau^2} Q_1^3 Q_2^3 \sum T_i(Q_1, Q_2, \tau) \Pi_i(-Q^2, -Q_1^2, -Q_2^2)$$

$$Q^2 = Q_1^2 + Q_2^2 + 2 Q_1 Q_2 \tau$$

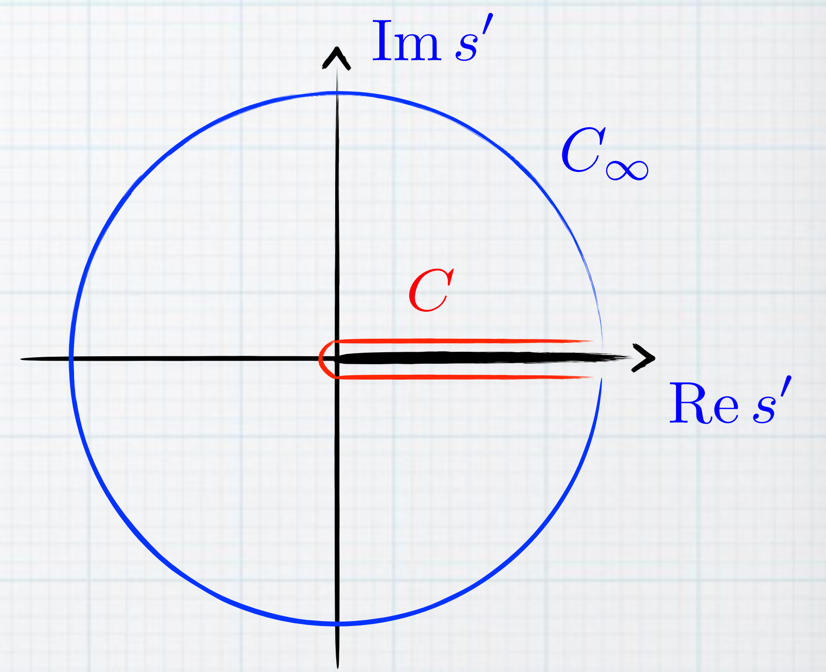
talk by G. Colangelo

integration over the unphysical region → analytical continuation

Dispersion relation for Pauli form factor

Cauchy theorem:

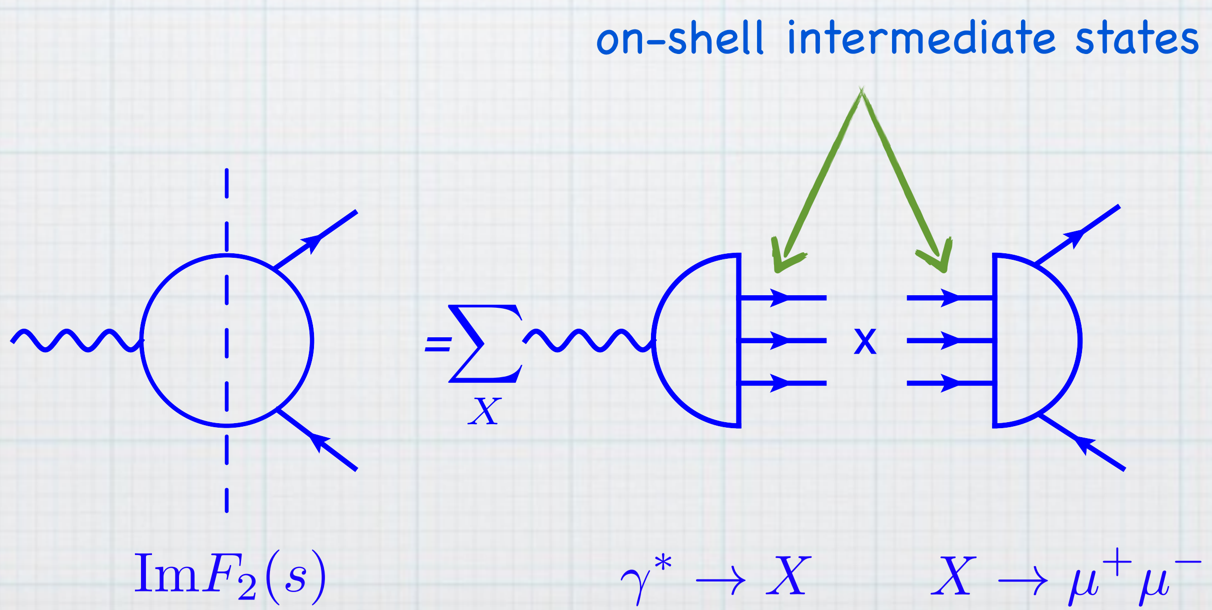
$$F_2(s) = \frac{1}{2\pi i} \oint_{C+C_\infty} \mathrm{d}s' \frac{F_2(s')}{s'-s} \qquad s \equiv k^2$$



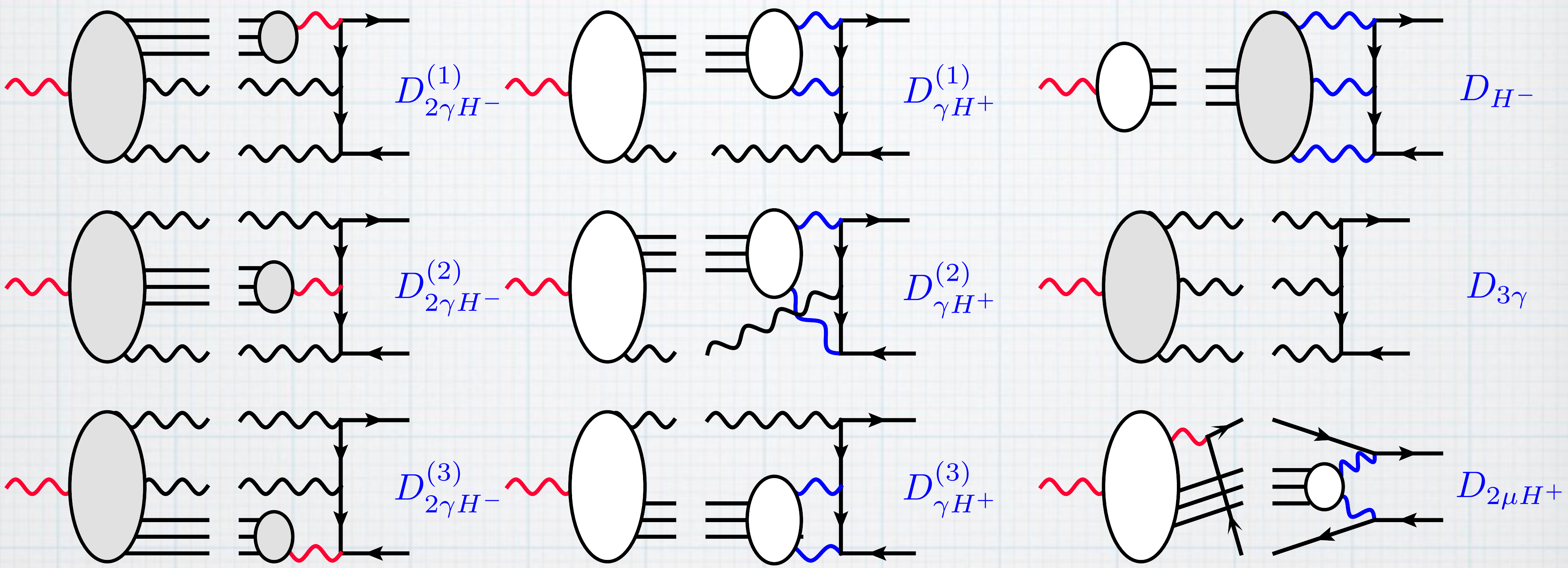
$$F_2(s) \underset{s\to\infty}{\rightarrow} 0$$

$$F_2(0) = \frac{1}{\pi} \int_{s_{th}}^{\infty} \mathrm{d}s \frac{\mathrm{Im} F_2(s)}{s - i\varepsilon}$$

Unitarity:



Unitarity contributions



$$\mathrm{Abs}F_2(k^2) = D_{3\gamma} + D_{H^-} + D_{2\mu H^+} + D^{(1)}_{\gamma H^+} + D^{(2)}_{\gamma H^+} + D^{(3)}_{\gamma H^+} + D^{(1)}_{2\gamma H^-} + D^{(2)}_{2\gamma H^-} + D^{(3)}_{2\gamma H^-}$$

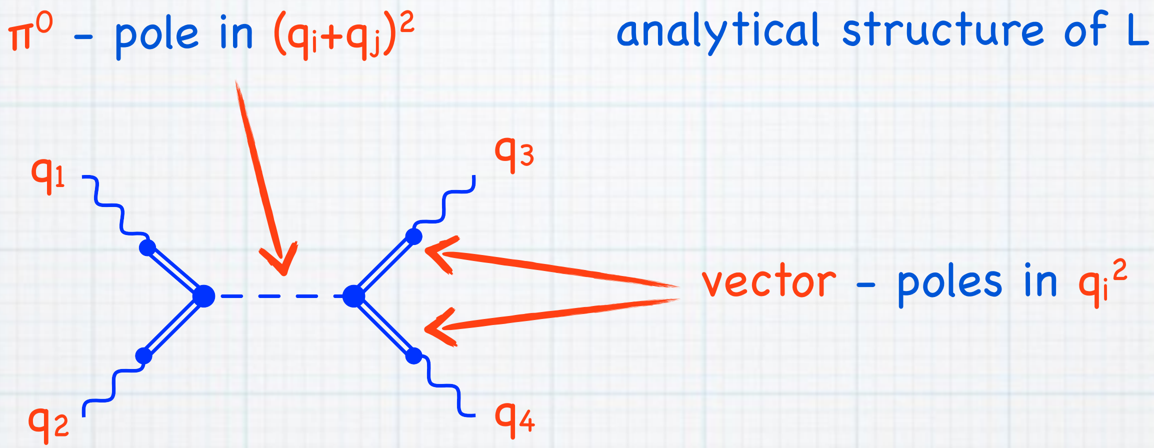
$$D_{2\mu H^+} = e^6 \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda} (-1)^{\lambda+\lambda_1+\lambda_2+\lambda_3} \int \frac{\mathrm{d}^4 q_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 q_2}{(2\pi)^4} (-2\pi i)^2 \delta\left((p+q_1)^2 - m^2\right) \delta\left((p+k-q_2)^2 - m^2\right)$$

$$\times \frac{L_{\lambda_1\lambda_2\lambda_3\lambda}(p,p',q_1,k-q_1-q_2,q_2)}{q_1^2 q_2^2 (k-q_1-q_2)^2}$$

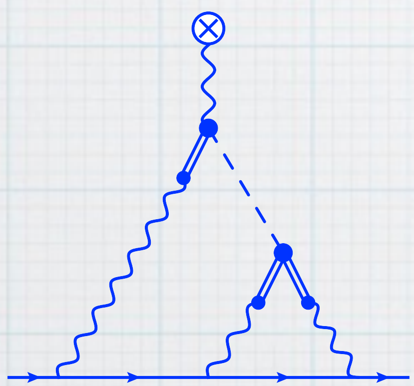
$$\times \, \mathrm{Disc}_{(q_1+q_2)^2} \Pi_{\lambda_1\lambda_2\lambda_3\lambda}(q_1, k-q_1-q_2, q_2, k)$$

Pole contributions

analytical structure of LbL amplitude

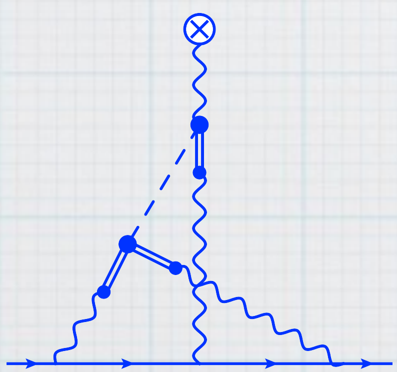


(1)



$$F_2^{(1)}(t) = e^6\Lambda^6 |F_{P\gamma^*\gamma^*}(0,0,M^2)|^2 \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4}$$
$$\times \frac{1}{q_1^2}\frac{1}{q_1^2-\Lambda^2}\frac{1}{q_2^2}\frac{1}{q_2^2-\Lambda^2}\frac{1}{(k-q_1-q_2)^2}\frac{1}{(k-q_1-q_2)^2-\Lambda^2}$$
$$\times \frac{1}{(p+q_1)^2-m^2}\frac{1}{(p+k-q_2)^2-m^2}\frac{1}{(k-q_1)^2-M^2} T_1(q_1,q_2,p,k)$$

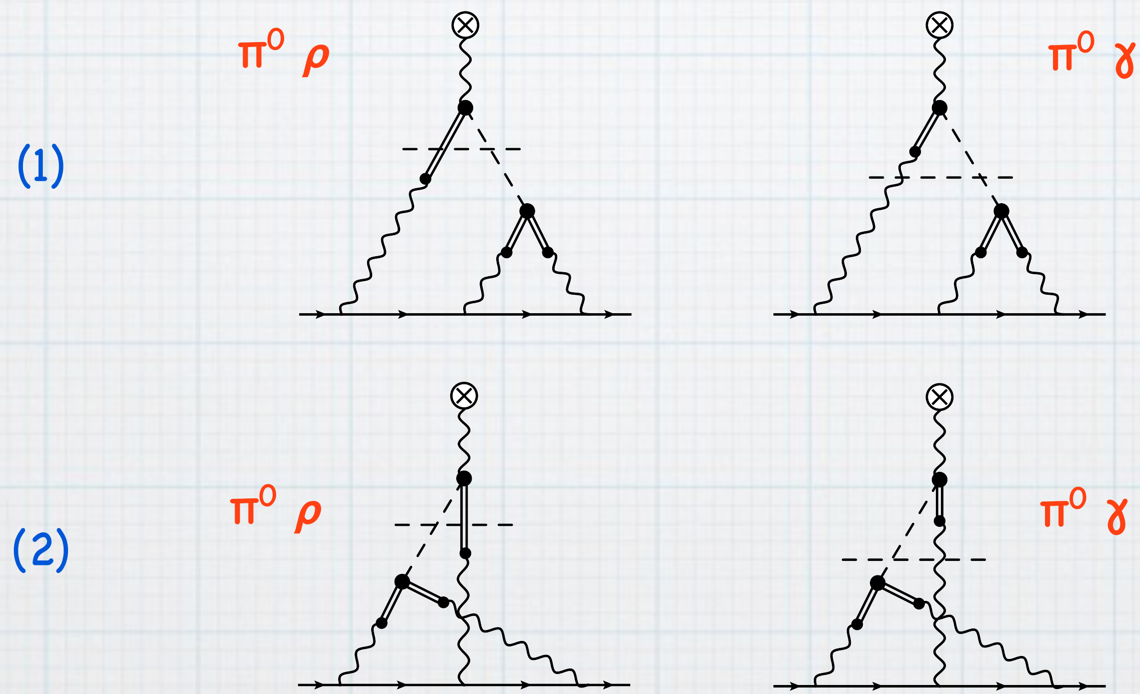
(2)



$$F_2^{(2)}(t) = e^6\Lambda^6 |F_{P\gamma^*\gamma^*}(0,0,M^2)|^2 \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4}$$
$$\times \frac{1}{q_1^2}\frac{1}{q_1^2-\Lambda^2}\frac{1}{q_2^2}\frac{1}{q_2^2-\Lambda^2}\frac{1}{(k-q_1-q_2)^2}\frac{1}{(k-q_1-q_2)^2-\Lambda^2}$$
$$\times \frac{1}{(p+q_1)^2-m^2}\frac{1}{(p+k-q_2)^2-m^2}\frac{1}{(q_1+q_2)^2-M^2} T_2(q_1,q_2,p,k)$$

2-particle discontinuities

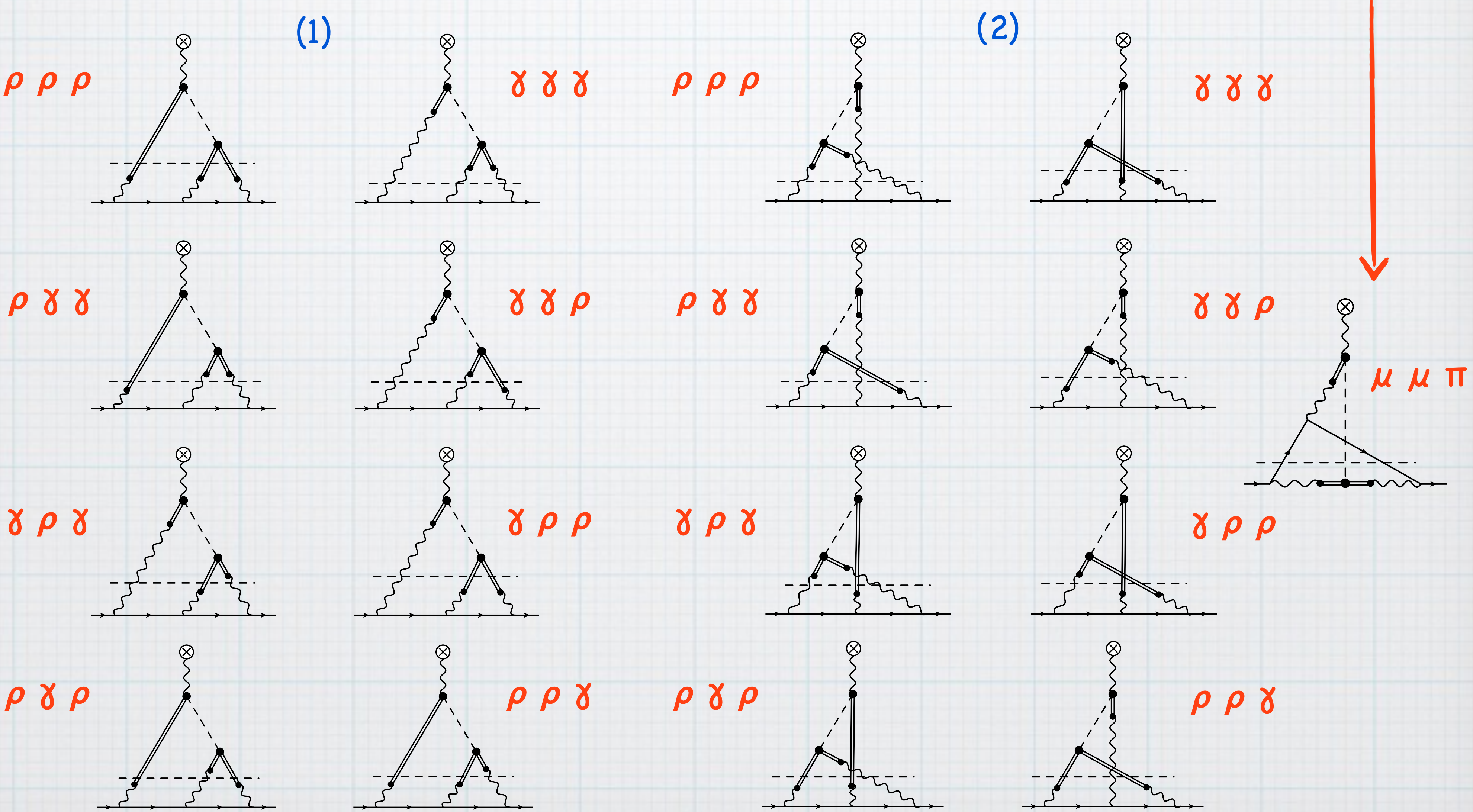
$$\mathrm{Disc}^{(2)}\Gamma_i^{\rho}(t) = \mathrm{Disc}_{\pi^0\rho}\Gamma_i^{\rho}(t) + \mathrm{Disc}_{\pi^0\gamma}\Gamma_i^{\rho}(t)$$



3-particle discontinuities

$$\mathrm{Disc}^{(3)}\Gamma_1^\rho(t) = \mathrm{Disc}_{\gamma\gamma\gamma}\Gamma_1^\rho(t) + \mathrm{Disc}_{\gamma\gamma\rho}\Gamma_1^\rho(t) + \mathrm{Disc}_{\gamma\rho\gamma}\Gamma_1^\rho(t) + \mathrm{Disc}_{\rho\gamma\gamma}\Gamma_1^\rho(t) + \mathrm{Disc}_{\gamma\rho\rho}\Gamma_1^\rho(t) + \mathrm{Disc}_{\rho\gamma\rho}\Gamma_1^\rho(t) + \mathrm{Disc}_{\rho\rho\gamma}\Gamma_1^\rho(t) + \mathrm{Disc}_{\rho\rho\rho}\Gamma_1^\rho(t)$$

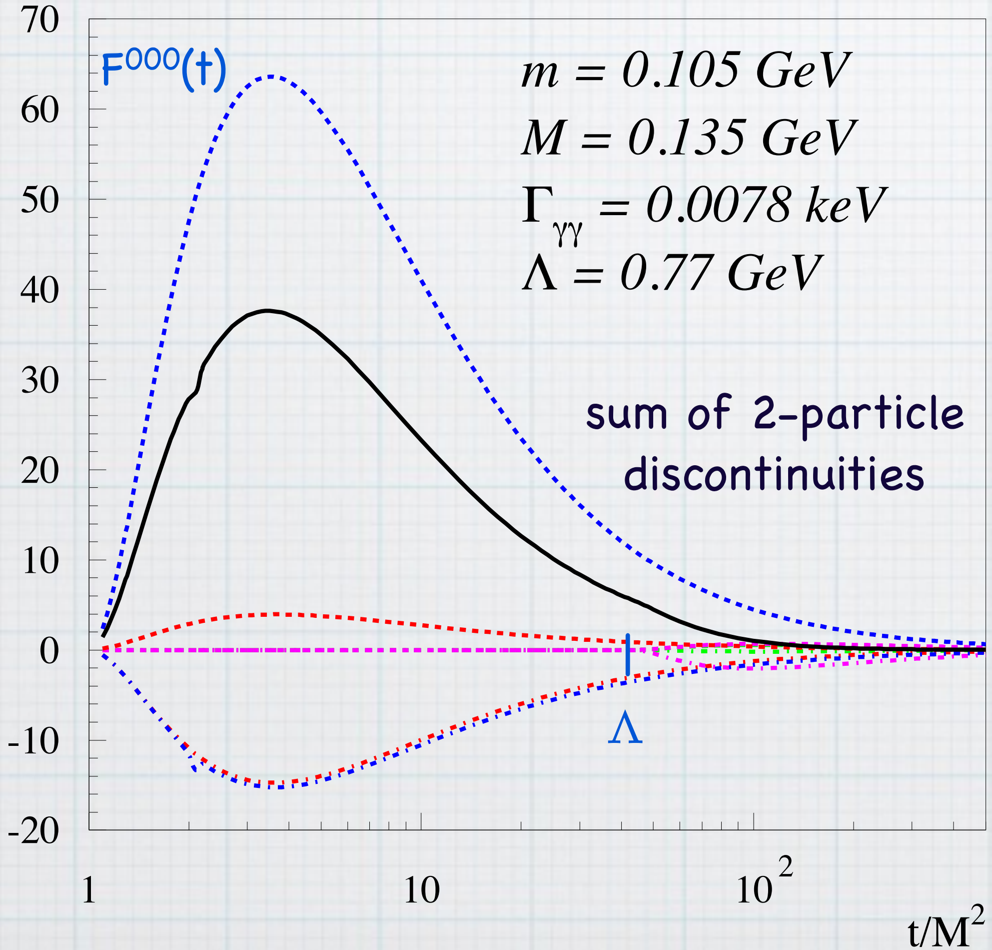
$$\mathrm{Disc}^{(3)}\Gamma_2^\rho(t) = \mathrm{Disc}_{\gamma\gamma\gamma}\Gamma_2^\rho(t) + \mathrm{Disc}_{\gamma\gamma\rho}\Gamma_2^\rho(t) + \mathrm{Disc}_{\gamma\rho\gamma}\Gamma_2^\rho(t) + \mathrm{Disc}_{\rho\gamma\gamma}\Gamma_2^\rho(t) + \mathrm{Disc}_{\gamma\rho\rho}\Gamma_2^\rho(t) + \mathrm{Disc}_{\rho\gamma\rho}\Gamma_2^\rho(t) + \mathrm{Disc}_{\rho\rho\gamma}\Gamma_2^\rho(t) + \mathrm{Disc}_{\rho\rho\rho}\Gamma_2^\rho(t) + \mathrm{Disc}_{\mu\mu\pi}\Gamma_2^\rho(t)$$

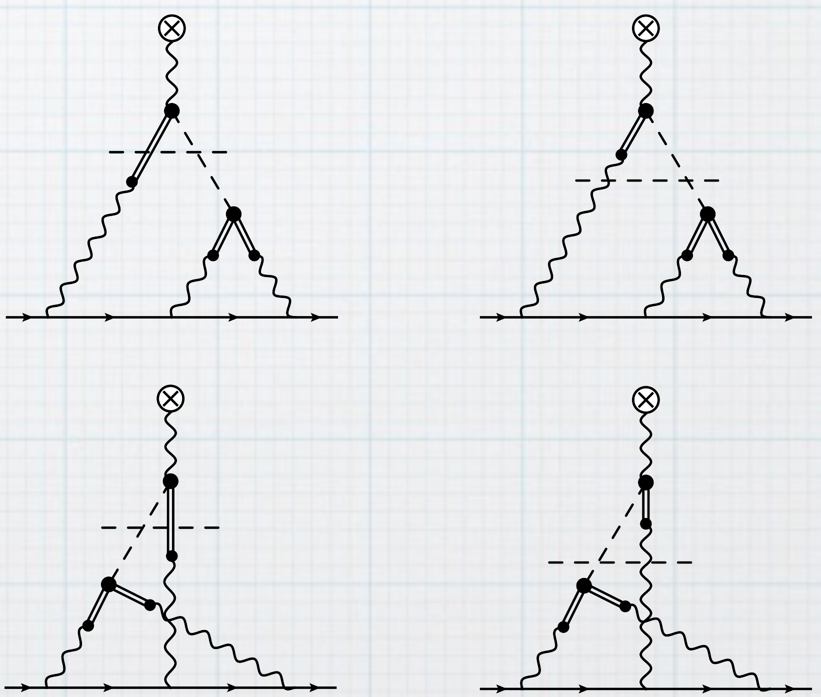


(g-2)$_\mu$: 2-particle cuts

$$\mathrm{Disc}_t^2 F_1^{\Lambda_1\Lambda_2\Lambda_3}(t) = -\frac{e^6|F(0,0,M^2)|^2}{8\pi}\beta_1 \int \mathrm{d}\cos\theta_1 \frac{N^{(1)}(q_1^2, m^2, t_1, t, \cos\theta_1)}{q_1^2 + t_1 - t - t\beta_1\beta\cos\theta_1}$$

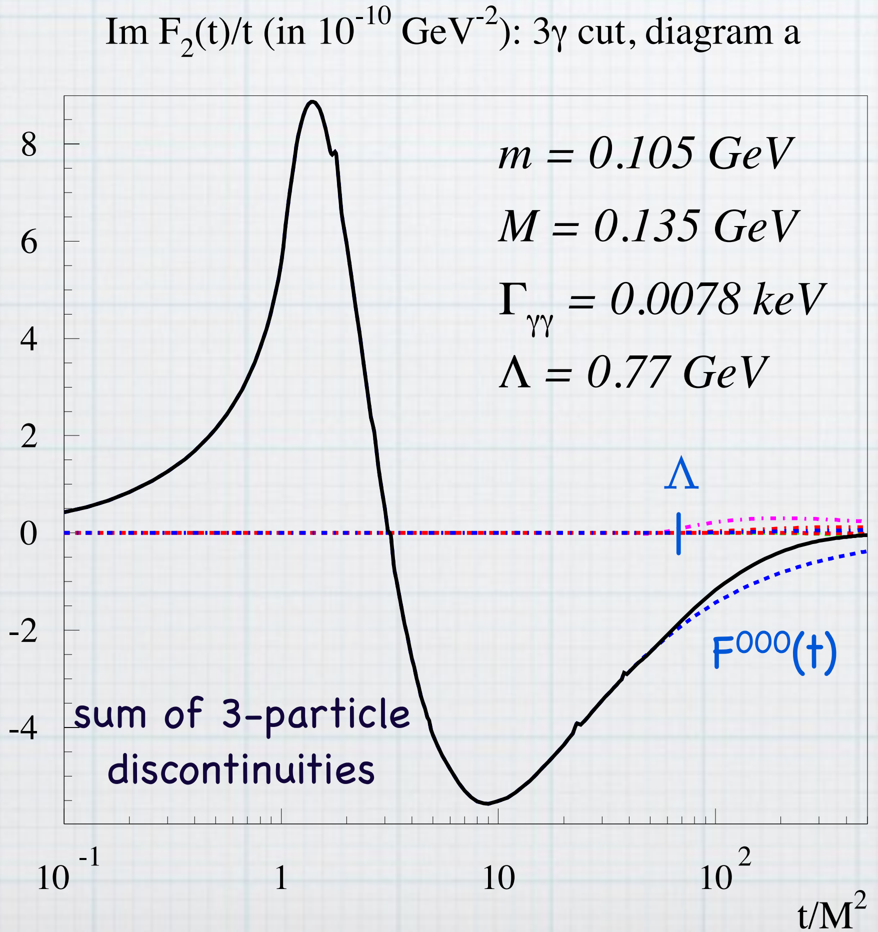
Im $F_2(t)/t$ (in 10^{-10} GeV^{-2}): πγ cut, diagram a

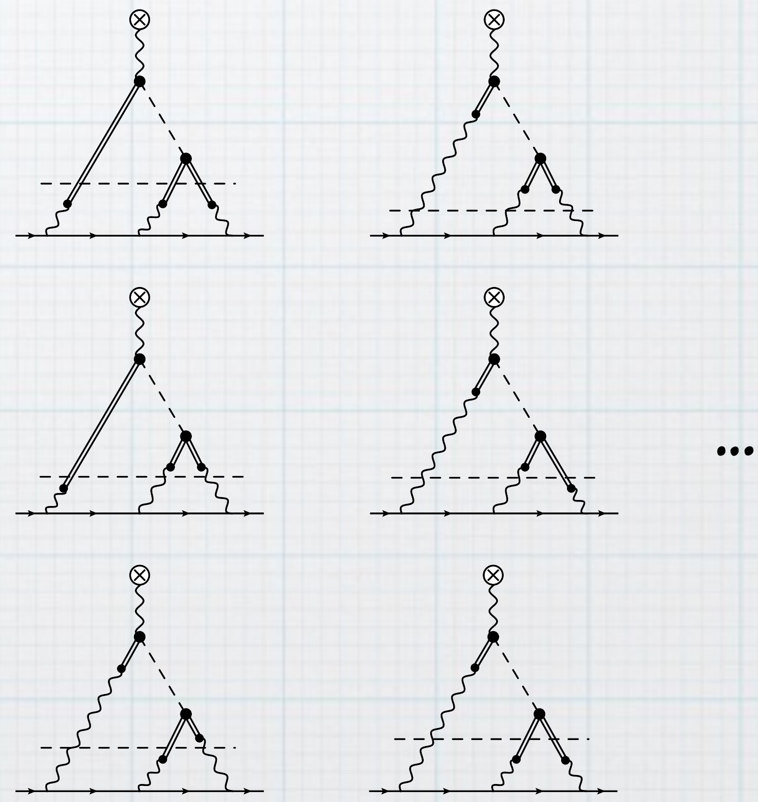




3-particle cuts

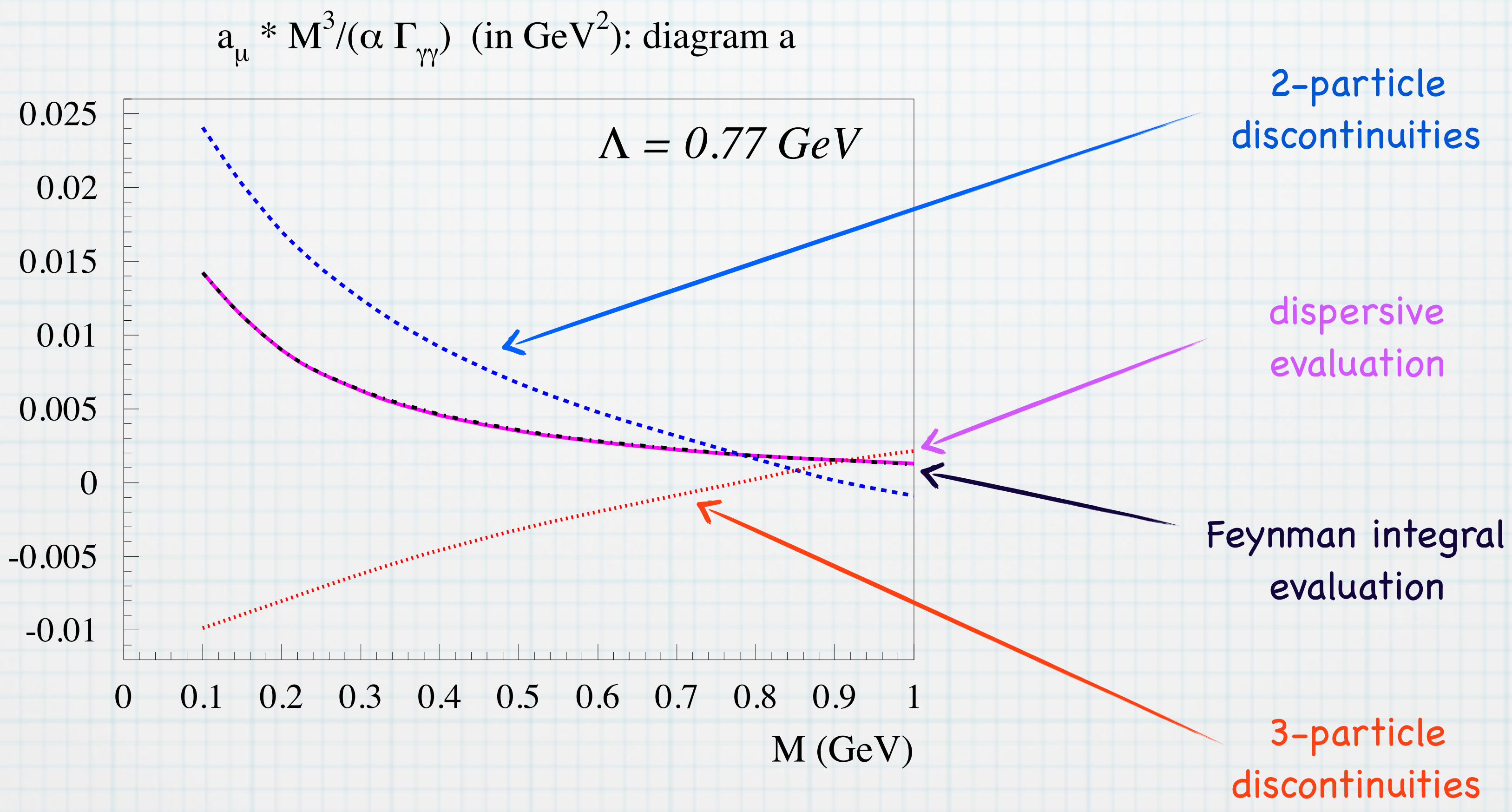
$$\mathrm{Disc}_t^3 F_1^{\Lambda_1\Lambda_2\Lambda_3}(t) = \frac{ie^6|F(0,0,M^2)|^2}{(2\pi)^4 32t}\int \mathrm{d}t_1 \int \mathrm{d}t_2 \frac{1}{t_1 - M^2}$$
$$\times \int_0^{\pi} \mathrm{d}\cos\theta_1 \int_0^{2\pi} \mathrm{d}\theta_2 \, \frac{2}{2m^2 - 2m_1^2 + q_1^2 + t_1 - t - t\beta_1\beta\cos\theta_1}$$
$$\times \frac{2}{2m^2 - 2m_2^2 + q_2^2 - t + t_2 + t\beta_2\beta(\sin\theta_1\cos\theta_2\sin\theta + \cos\theta_1\cos\theta)} L(t_1,\ldots)P(M^2,\ldots)$$





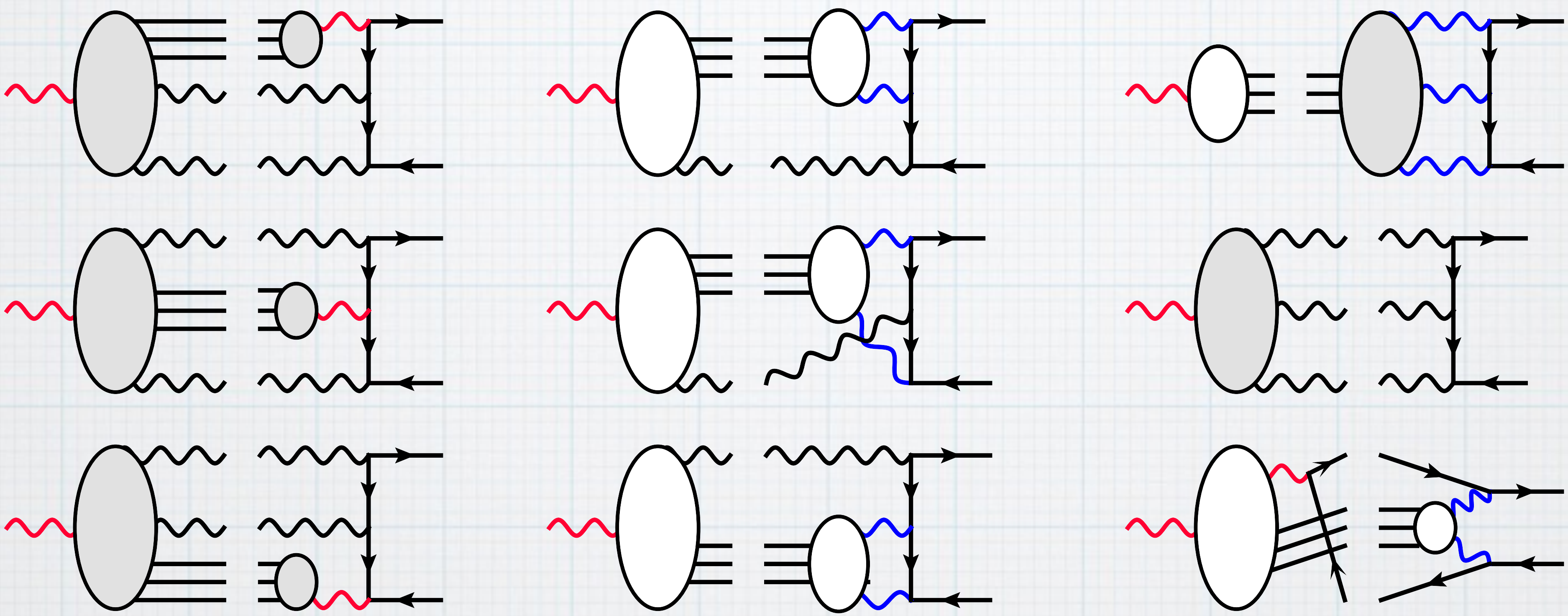
Pole contributions : real parts

$$F_2^{(i)}(0) = \frac{1}{2\pi i}\int_{M^2}^{\infty}\frac{\mathrm{d}t}{t}\,\mathrm{Disc}_{\mathrm{t}}^{(2)}F_2^{(i)}(t) + \frac{1}{2\pi i}\int_{0}^{\infty}\frac{\mathrm{d}t}{t}\,\mathrm{Disc}_{\mathrm{t}}^{(3)}F_2^{(i)}(t)$$

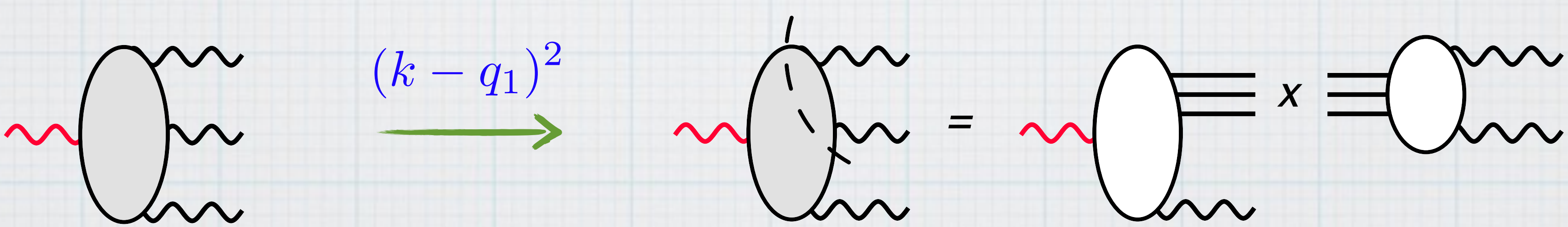


Experimental input

Unitarity contributions



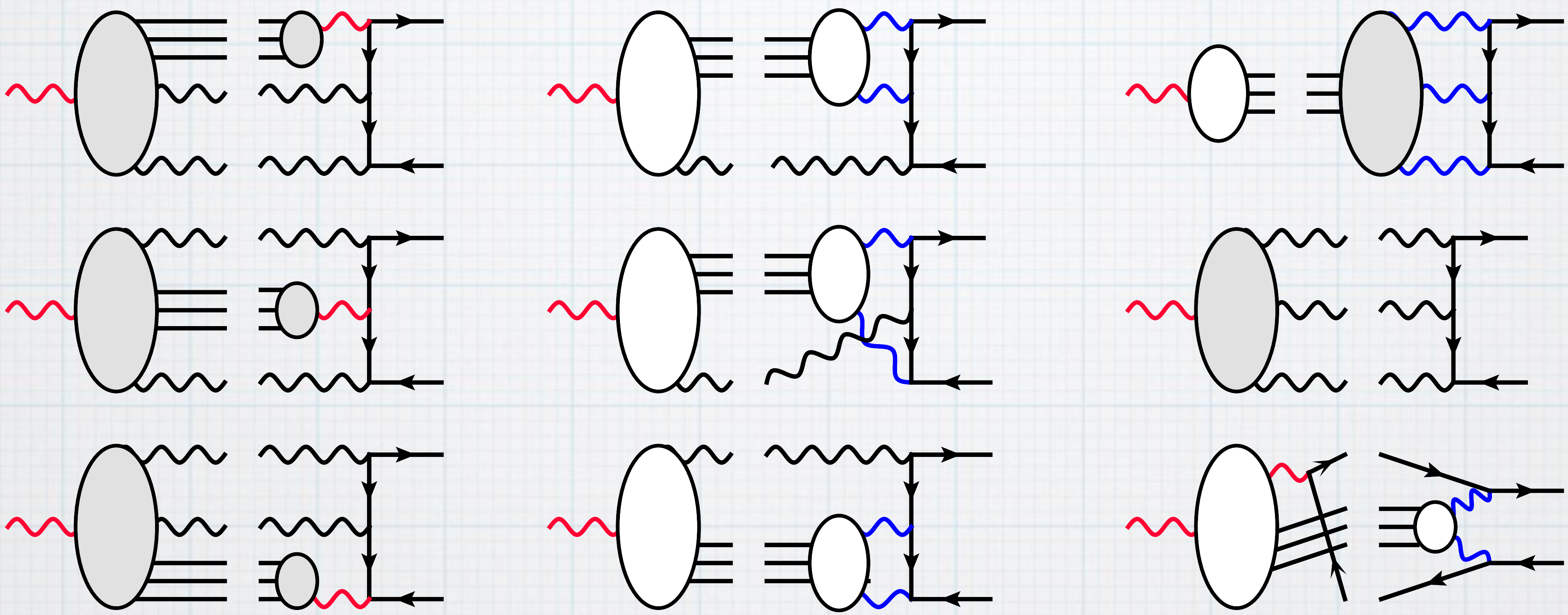
Reiterate dispersion representation in two-particle invariant masses



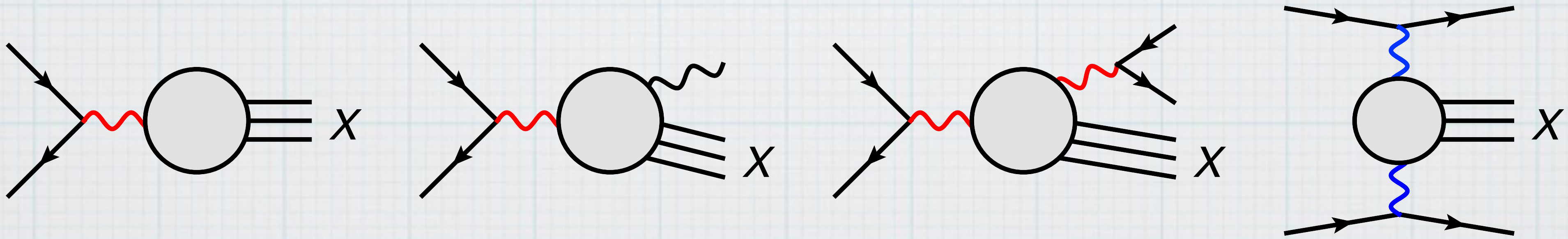
talks by I. Danilkin
B. Kubis

Experimental input

Unitarity contributions

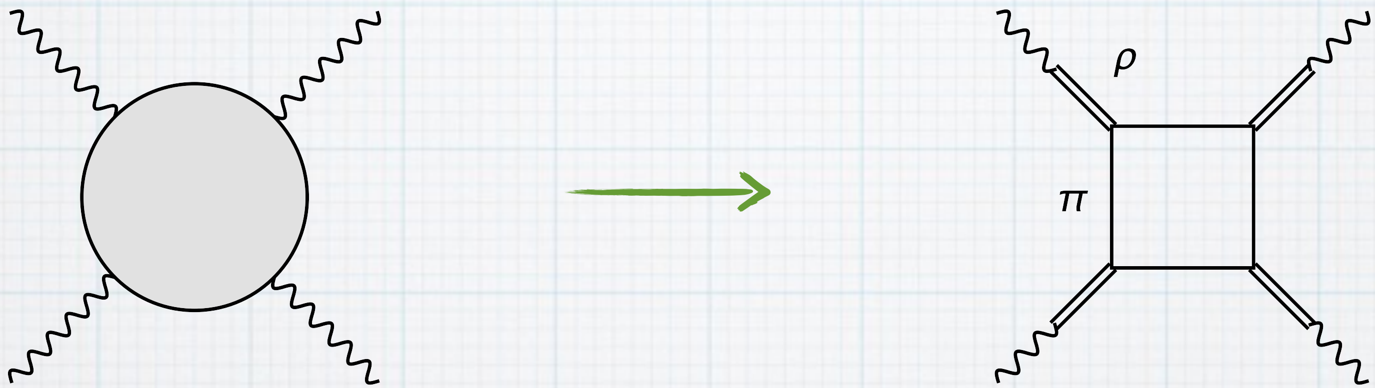


Experimental input



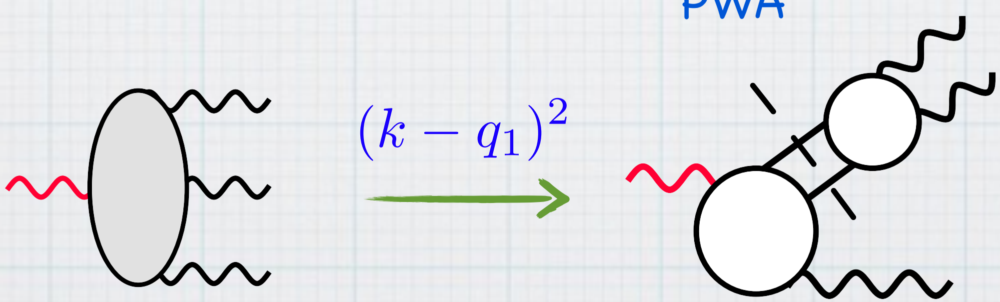
Two-pion threshold. First step: pion loop

pion loop with vector meson form factors



- phase-space and dispersive integrals are the same as for the general case
- data for S and D partial waves, pion loop for higher waves

PWA



ππ intermediate state

Conclusions & Outlook

- the first-principle calculations are currently not able to give a desired result
- resorting to data via dispersive approach is highly non-trivial, requires analytical continuation
- alternative way: analytical continuation of the muon's electromagnetic vertex, hadronic matrix elements in the physical region of hadron production processes
- requires a substantial input of experimental information from e^+e^- - annihilation to hadrons and two-photon production processes
- angular distributions and partial-wave analysis in hadronic sub-channels