Precision QED systems

Light-by-light scattering in the Lamb shift and the bound electron g factor

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Outline

- Introduction; QED bound states
- Light-by-light contribution to the Lamb shift
- Bound electron g-factor and the LBL contribution
- Conclusions

Some of the lepton-related SM puzzles

- muon g-2 anomaly
- proton radius puzzle
- theoretical challenges (mass hierarchy of SM particles, flavor structure of the theory etc.)

New, precise measurements with muons and electrons

- Energy levels of hydrogen and muonic hydrogen
- Bound electron g-factor
- Muonium hyperfine spiting
- Decay spectrum of bound muon (CLFV)

The largest corrections come from the QED.
We need precise SM predictions and cross-checks before we can claim any New Physics discovery.

How can we check SM prediction

Electron g-2 may be sensitive to the same New Physics
$\delta g_e \sim \frac{m_e^2}{m_\mu^2}\delta g_\mu$, but a new source of α is needed

- Atomic spectroscopy ($R_\infty = \frac{\alpha^2 m_e c}{4\pi\hbar}$)
- Bound electron g
  - currently the best source of m_e
  - in the future also a source of α

We need QED corrections for the Lamb shift, and bound electron g-factor!
Current relative uncertainty for $1S - 2S$ transition $\sim 10^{-15}$ and for bound g-factor $\sim 10^{-10}$; improvement expected soon.

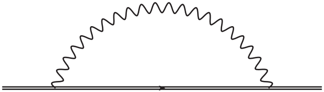
Lamb shift

70 years ago - first radiative correction in QED

PHYSICAL REVIEW VOLUME 72, NUMBER 4 AUGUST 15, 1947

The Electromagnetic Shift of Energy Levels

H. A. BETHE
Cornell University, Ithaca, New York
(Received June 27, 1947)



$$\Delta E_{2S-2P} \sim \frac{\alpha}{\pi}(Z\alpha)^4 \ln(Z\alpha) \sim 1057 \text{ MHz}$$

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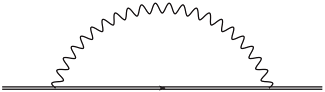


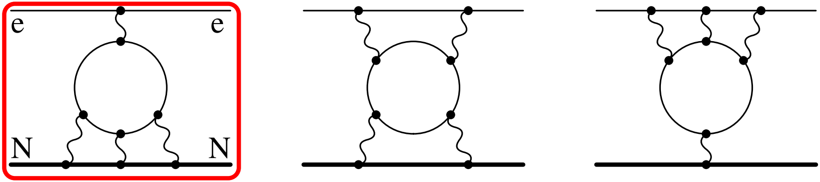
$$\Delta E_{2S-2P} \sim \frac{\alpha}{\pi}(Z\alpha)^4 \ln(Z\alpha) \sim 1057 \text{ MHz}$$

$$\Delta E = \frac{\alpha}{\pi}\left(A_{41}(Z\alpha)^4 \ln(Z\alpha)^{-2} + A_{40}(Z\alpha)^4 + A_{50}(Z\alpha)^5 + \ldots\right) +$$

$$\left(\frac{\alpha}{\pi}\right)^2 \left(B_{40}(Z\alpha)^4 + B_{50}(Z\alpha)^5 + B_{63}(Z\alpha)^6 \ln^3(Z\alpha)^{-2} + \ldots\right) + \ldots$$

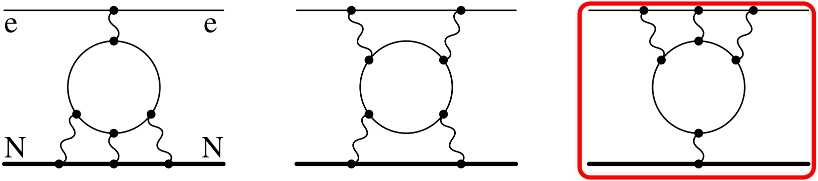
$+$ relativistic corrections, recoil corrections, finite nuclear size corrections . . .

Light-by-light contribution



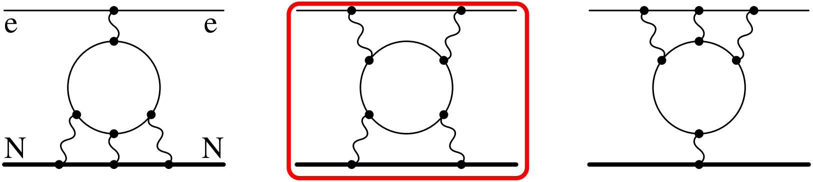
1. Wichmann-Kroll potential
   - $\mathcal{O}\left(\alpha(Z\alpha)^6\right)$: A_{60}
   - $\Delta E_{1\mathrm{S}} = 2.5\mathrm{kHz}$ (Z=1)
   - [E. Wichmann and N. M. Kroll, 1954, 1956]

Light-by-light contribution



2. Dirac form factor
   - $\mathcal{O}\left(\alpha^3(Z\alpha)^4\right)$: C_{40}
   - [K. Melnikov and T. van Ritbergen, 2000]

Light-by-light contribution



3. $\mathcal{O}\left(\alpha^2(Z\alpha)^5\right)$: B_{50}
   - $\Delta E_{1S} = -5.3\mathrm{kHz}$ (Z=1)
   - [M.I.Eides,H.Grotch,and P.Peble, 1994; K. Pachucki 1993, 1994]

A given diagram may contribute also to higher orders in $Z\alpha$. In the third case, the higher order contribution is *logarithmically enhanced* $\rightarrow B_{61}$.

Two step matching

$$\mathcal{L}_{\rm QED} \quad = \quad \overline{\Psi}\left(i\not{D} - m_e\right)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Hard-scale matching; $\mathrm{QED} \to \mathrm{NRQED}$

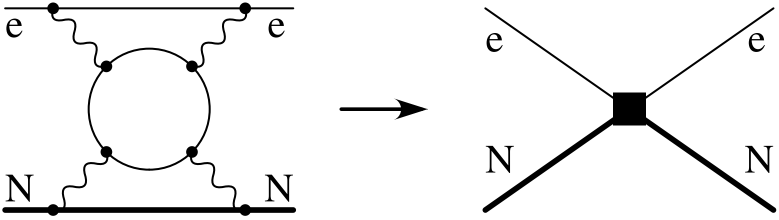
$$\mathcal{L}_{\rm NRQED} \quad \supset \quad \psi^\dagger\left(c_A\frac{\left(\vec{B}^2 - \vec{E}^2\right)}{m_e^3} - c_B\frac{\vec{E}^2}{m_e^3}\right)\psi + d\frac{\psi^\dagger\psi N N^\dagger}{m_e^2}$$

Soft-scale matching; $\mathrm{NRQED} \to \mathrm{PNRQED}$, two types of potentials

$$V_2(r) \quad \sim \quad \frac{(Z\alpha)^2}{m_e^2}\delta^3(r)$$

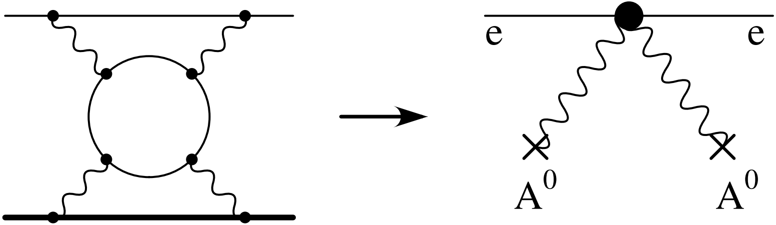
$$V_3(r) \quad \sim \quad \frac{(Z\alpha)^2}{m_e^3 r^4}$$

δ-contribution



- $\mathcal{O}\left(\alpha^2(Z\alpha)^5\right)$
- $\Delta E_{\mathrm{1S}} = -5.3\mathrm{kHz}$ (Z=1)
- [M.I.Eides,H.Grotch,and P.Peble, 1994; K. Pachucki 1993, 1994, M. Dowling, J. Mondejar, J. H. Piclum, and A. Czarnecki 2011]

Logarithmic contribution



$$\vec{E}^2 \sim \frac{(Z\alpha)^2}{r^4} \tag{1}$$

The matrix element of this operator is logarithmically divergent

$$\Delta E_{nS} = \chi_{\mathrm{LBL}} \left\langle \vec{E}^2 \right\rangle_{nS} = \frac{(Z\alpha)^6}{n^3} \ln(Z\alpha)^2 4\chi_{\mathrm{LBL}} \tag{2}$$

with the matching coefficient

$$\chi_{\mathrm{LBL}} = \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{43}{144} - \frac{133}{3456}\pi^2\right)$$

[A. Czarnecki, R.S., 2016]

Significance of the LBL correction

Total corrections at $\mathcal{O}\left(\alpha^2(Z\alpha)^6 \ln(Z\alpha)\right)$ [K. Pachucki 2001, U. D. Jentschura, A. Czarnecki, and K. Pachucki, 2005] are much larger than the LBL contribution.

LBL correction decreases $1S - 2S$ by $280\mathrm{Hz}$; experimental accuracy is $10\mathrm{Hz}$. Other transitions are measured with accuracy $\sim \mathrm{kHz}$.

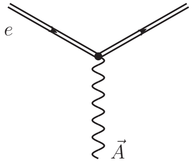
Theory of hydrogen spectrum has to be further checked!

Measurements of $1S - 2S$ transition in He^+ can provide a test of bound-state QED. [M. Herrmann et al. 2009]

Bound electron g-factor

Leading effect [Breit, 1928]

$$g_e = \frac{2}{3}\left(1 + 2\sqrt{1-(Z\alpha)^2}\right) \approx 2 - \frac{2}{3}(Z\alpha)^2 \qquad (3)$$



Radiative corrections

- $\mathcal{O}\left(\alpha^n (Z\alpha)^2\right)$: universal corrections related to free-electron $g-2$ [H. Grotch, 1970]
- $\mathcal{O}\left(\alpha (Z\alpha)^4\right)$: [K. Pachucki, U. Jentschura, and V. A. Yerokhin, 2004]
- $\mathcal{O}\left(\alpha^2 (Z\alpha)^4\right)$: [K. Pachucki, A. Czarnecki, U. Jentschura, and V.A. Yerokhin, 2005]
- $\mathcal{O}\left(\alpha (Z\alpha)^5\right)$: LBL [S.G. Karshenboim and A.I. Milstein, 2002]

Why do we measure the bound electron g-factor

- Currently it is used to determine m_e
  Larmor frequency: $\omega_L = \frac{g}{2}\frac{e}{m_e}B$
  Cyclotron frequency of the ion: $\omega_c = \frac{Q}{M}B$

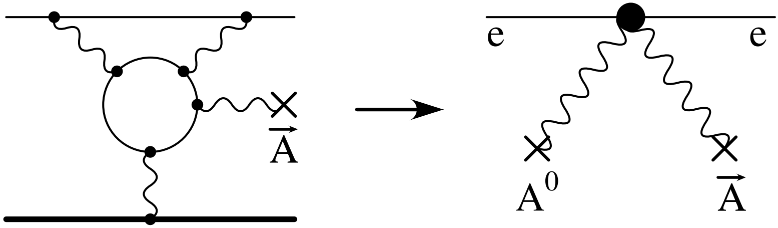
$$m_e = \frac{g}{2}\frac{e}{Q}\frac{\omega_c}{\omega_L}M \tag{4}$$

- Future plans
  - Determination of $^4He^+$ mass
  - New measurement of fine structure constant – g rather than $g-2$ is measured
    – large reduction of relativistic shifts compared to free electron, nucleus acts as an anchor
    – combination of measurements for different energy levels allows to cancel leading nuclear effects
  - Some proposals suggest measurement that will be independent of the free $g-2$ contribution

[Yerokhin et al., 2016; Shabaev et al. 2006]

LBL correction

Calculation of the LBL correction to the bound electron g-2 is similar to Lamb



$$\mathcal{L}_{\mathrm{NRQED}} \supset \frac{\psi^\dagger(\vec{\sigma}\cdot\vec{B})(\vec{\nabla}\cdot\vec{E})\psi}{m_e^3}$$

The LBL correction (not included in previous evaluation of $(Z\alpha)^4\left(\frac{\alpha}{\pi}\right)^2$ terms)

$$\delta g_e = (Z\alpha)^4\left(\frac{\alpha}{\pi}\right)^2\frac{16-19\pi^2}{108}$$

[A. Czarnecki, R.S., 2016]

LBL correction to bound electron g factor

LBL loop changes the non-Logarithmic part of the correction

$$\left(\frac{\alpha}{\pi}\right)^2 (Z\alpha)^4 \left(\frac{28}{9}\ln Z\alpha - 16.4\right) \to \left(\frac{\alpha}{\pi}\right)^2 (Z\alpha)^4 \left(\frac{28}{9}\ln Z\alpha - 18.0\right)$$

This shifts the value of electron mass by 0.3σ [J. Zatorski et al. 2017]
The sensitivity is reduced because unknown corrections $\alpha^2(Z\alpha)^5$ are treated as a fit parameter [S. Sturm et al. 2014].

Experiments designed to provide tests of bound state QED

- Mainz g-factor experiment
- ALPHATRAP (MPI-K Heidelberg)
- HITRAP (GSI Darmstadt)

Conclusions

- Spectroscopic measurements serve as the most precise source of fundamental constants and they can also facilitate discovery of new physics
- Theory of hydrogen energy levels has to be further scrutinized
- Bound electron g-factor opens exciting opportunities for progress both on theoretical and experimental side