(Charged) Lepton-Flavor Violation – Overview

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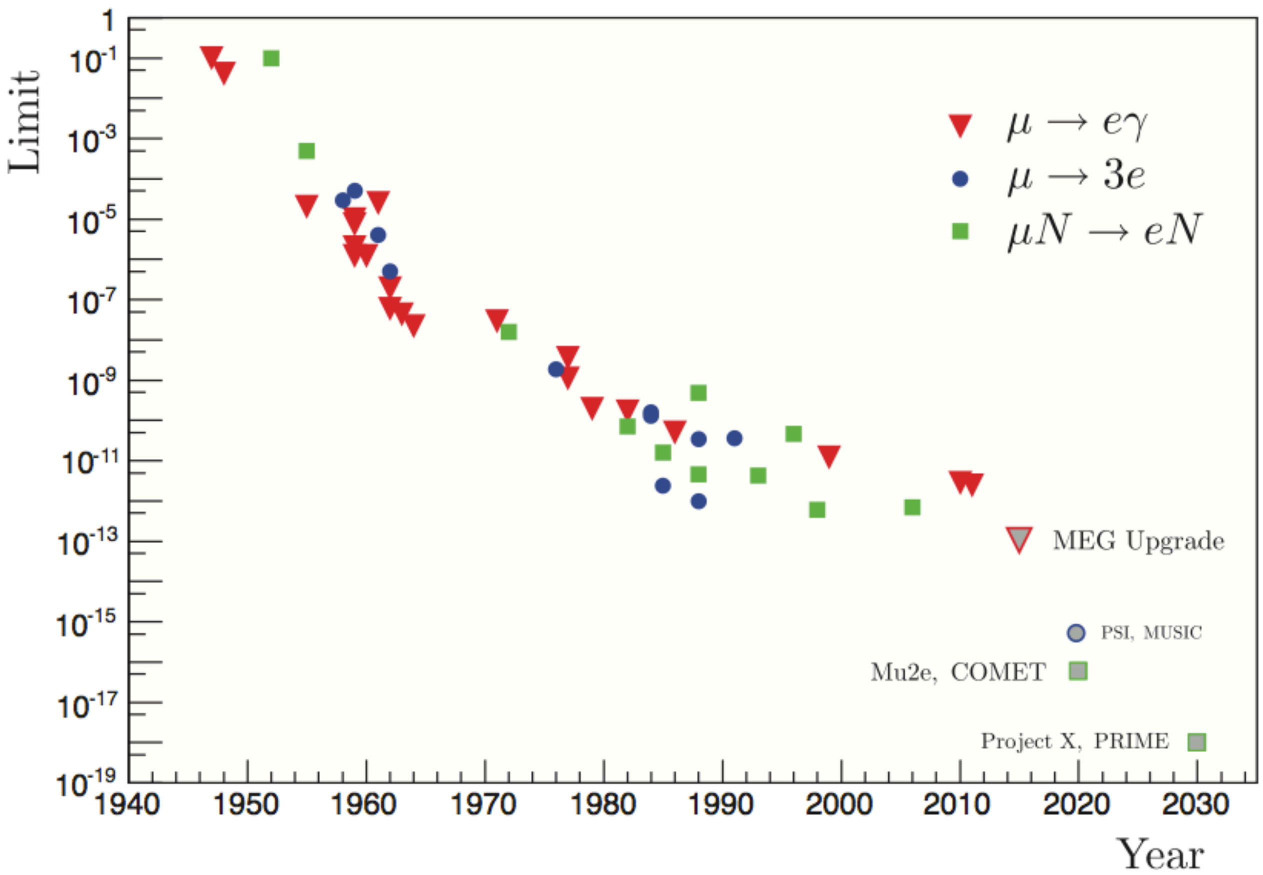
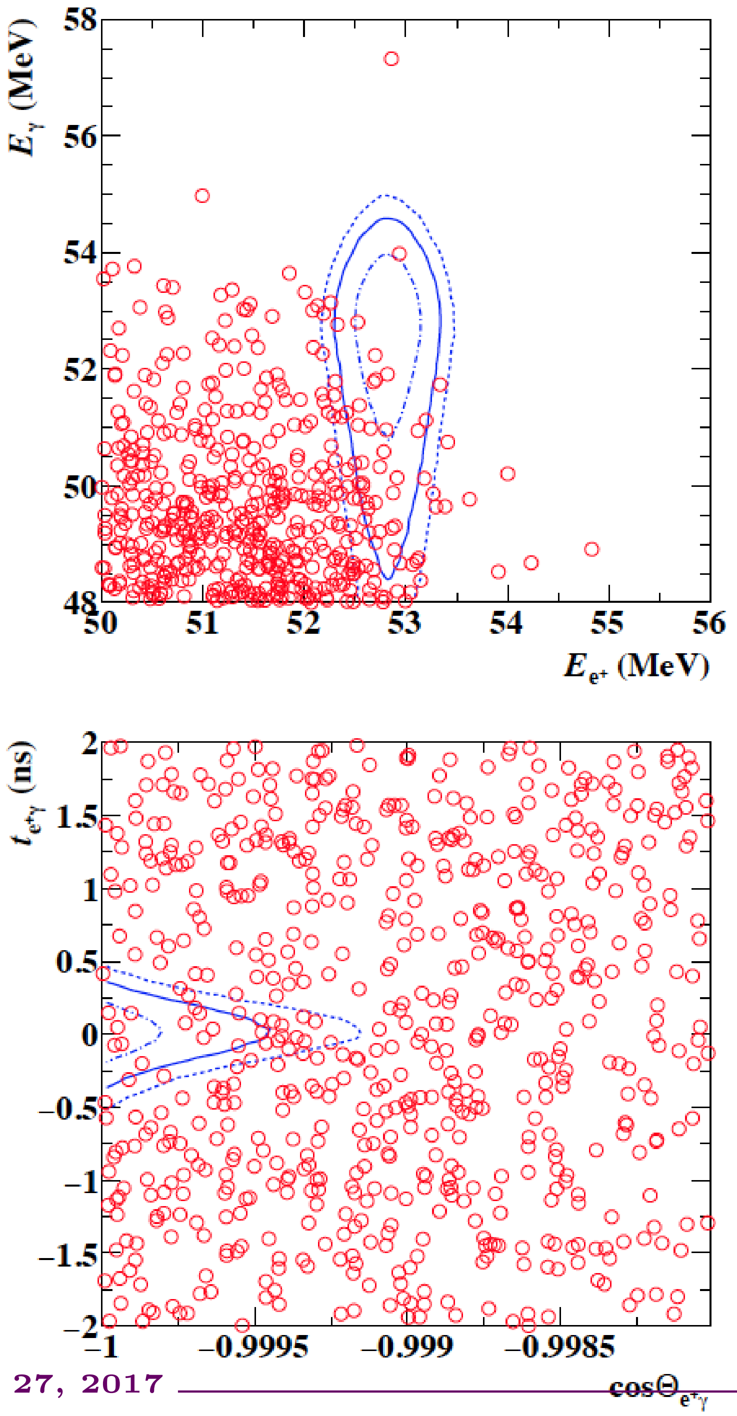
History of $\mu \to e\gamma$, $\mu N \to eN$, and $\mu \to 3e$

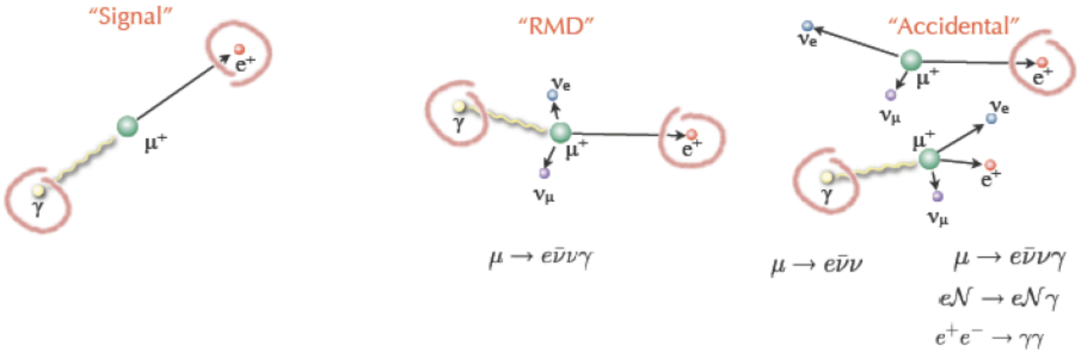


[R. Bernstein, P. Cooper, arXiv 1307.5787]

Figure 3: The history of CLFV searches in muons (not including muonium.) One sees a steady improvement in all modes and then a flattening of the rate improvement throughout the 1990s. MEG has upgrade plans for the $\mu \to e\gamma$ search. The two next generations of $\mu N \to eN$, Mu2e/COMET at FNAL and J-PARC are labeled, and possible extensions at Project X and PRIME are shown. Letters-of-intent are in process for $\mu \to 3e$ experiments at PSI and Osaka's MUSIC facility. Individual experiments are discussed in the text.



$$\mathbf{Br}(\mu \to e\gamma) < 4.2 \times 10^{-13}\ \mathbf{(90\%\ CL)}$$



[MEG Coll. arXiv:1605.05081]

Figure 27 Event distributions of observed events in the (E_{e^+}, E_γ)-

What Will/Could Happen in the Near and Far Future (my naive impressions)

- MEG: $\mu \to e\gamma$ at 10^{-13}.
- *Mu2e* (Fermilab) and COMET (J-PARC): $\mu \to e$-conversion at 10^{-16}.
- PSI: $\mu \to eee$ at 10^{-15}.
- Belle II: Rare τ processes at 10^{-10}.
- Next-generation *Mu2e*: $\mu \to e$-conversion at 10^{-18} (or precision studies).
- Muon Beams/ Storage Rings: $\mu \to e$-conversion at 10^{-20}? Revisit rare muon decays ($\mu \to e\gamma$, $\mu \to eee$) with new idea?

[see hep-ph/0109217]

SM Expectations?

In the old SM, the rate for charged lepton flavor violating processes is trivial to predict. It vanishes because individual lepton-flavor number is conserved:

- $N_\alpha(\text{in}) = N_\alpha(\text{out})$, for $\alpha = e, \mu, \tau$.

But individual lepton-flavor number are NOT conserved– ν oscillations!

Hence, in the νSM (the old Standard Model plus operators that lead to neutrino masses) $\mu \to e\gamma$ is allowed (along with all other charged lepton flavor violating processes).

These are Flavor Changing Neutral Current processes, observed in the quark sector ($b \to s\gamma$, $K^0 \leftrightarrow \bar{K}^0$, etc).

Unfortunately, we do not know the νSM expectation for charged lepton flavor violating processes $\to$ **we don't know the νSM Lagrangian !**

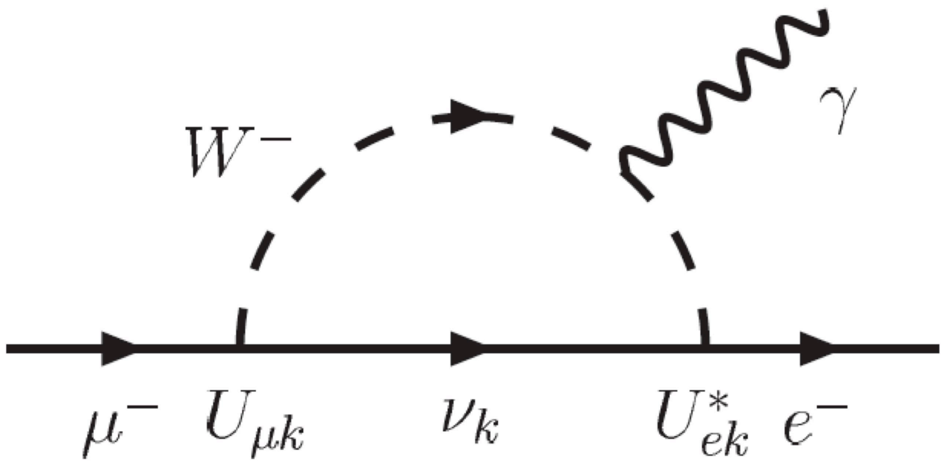
One contribution known to be there: active neutrino loops (same as quark sector).

In the case of charged leptons, the **GIM suppression is very efficient**...

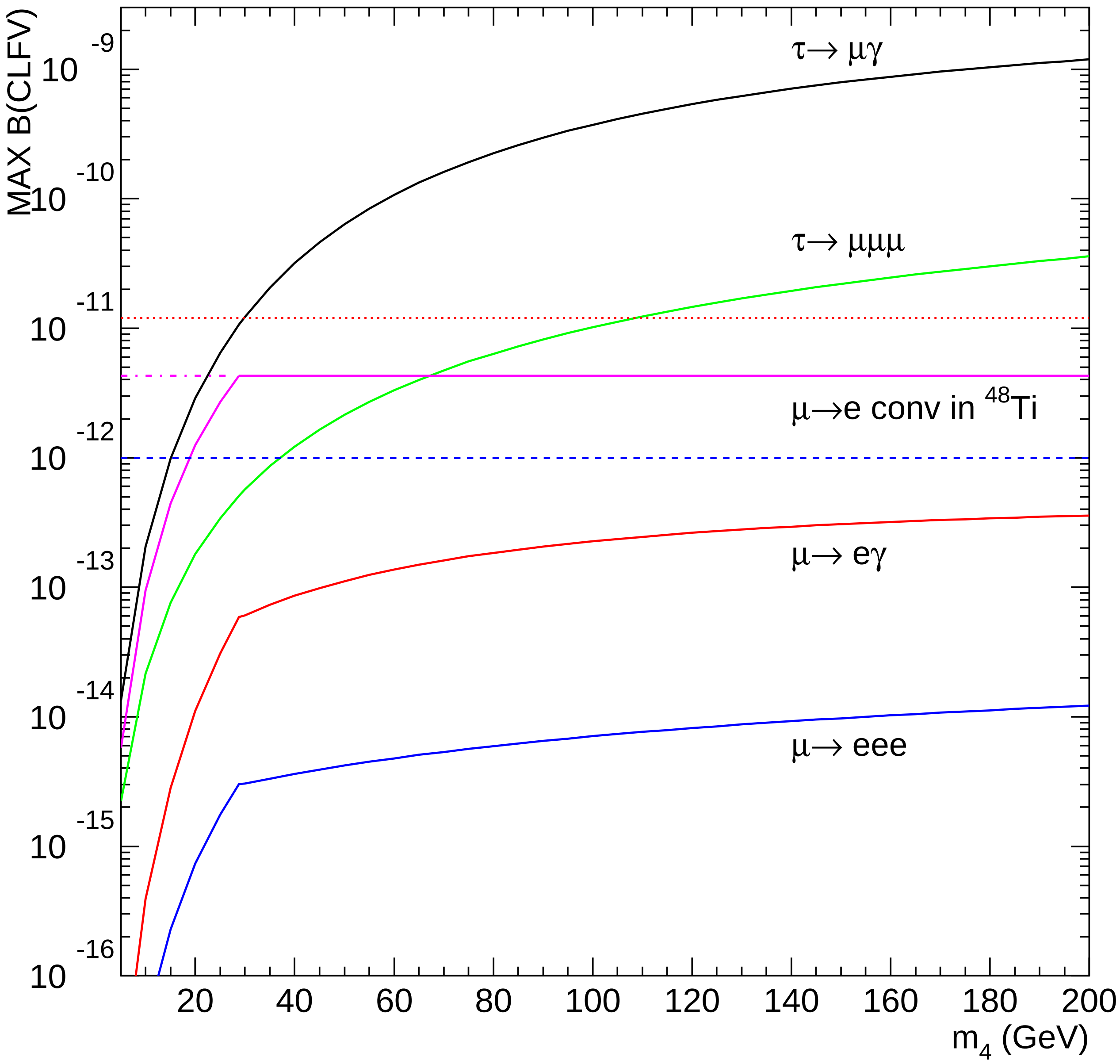
$$\text{e.g.: } Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M^2_W}\right|^2 < 10^{-54}$$

[$U_{\alpha i}$ are the elements of the leptonic mixing matrix,

$\Delta m^2_{1i} \equiv m^2_i - m^2_1$, $i = 2, 3$ are the neutrino mass-squared differences]



e.g.: SeeSaw Mechanism [minus "Theoretical Prejudice"]



arXiv:0706.1732 [hep-ph]

Independent from neutrino masses, there are strong theoretical reasons to believe that the expected rate for flavor changing violating processes is much, much larger than naive νSM predictions and that discovery is just around the corner.

Due to the lack of SM "backgrounds," searches for rare muon processes, including $\mu \to e\gamma$, $\mu \to e^+e^-e$ and $\mu + N \to e + N$ (μ-e–conversion in nuclei) are considered ideal laboratories to probe effects of new physics at or even above the electroweak scale.

Indeed, if there is new physics at the electroweak scale (as many theorists will have you believe) and if mixing in the lepton sector is large "everywhere" the question we need to address is quite different:

Why haven't we seen charged lepton flavor violation yet?

Model Independent Approach

As far as charged lepton flavor violating processes are concern, new physics effects can be parameterized via a handful of higher dimensional operators. For example, say that the following effective Lagrangian dominates CLFV phenomena:

$$\mathcal{L}_{\rm CLFV} = \frac{m_\mu}{(\kappa+1)\Lambda^2}\bar{\mu}_R\sigma_{\mu\nu}e_L F^{\mu\nu} + \frac{\kappa}{(1+\kappa)\Lambda^2}\bar{\mu}_L\gamma_\mu e_L\left(\bar{u}_L\gamma^\mu u_L + \bar{d}_L\gamma^\mu d_L\right)$$

First term: mediates $\mu \to e\gamma$ and, at order α, $\mu \to eee$ and $\mu + Z \to e + Z$

Second term: mediates $\mu + Z \to e + Z$ and, at one-loop, $\mu \to e\gamma$ and $\mu \to eee$

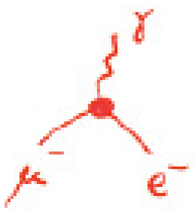
Λ is the "scale of new physics". κ interpolates between transition dipole moment and four-fermion operators.

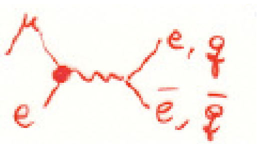
Which term wins? $\to$ Model Dependent

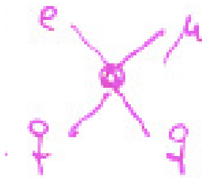
- $\mu \to e$-conv at 10^{-17} "guaranteed" deeper probe than $\mu \to e\gamma$ at 10^{-14}.

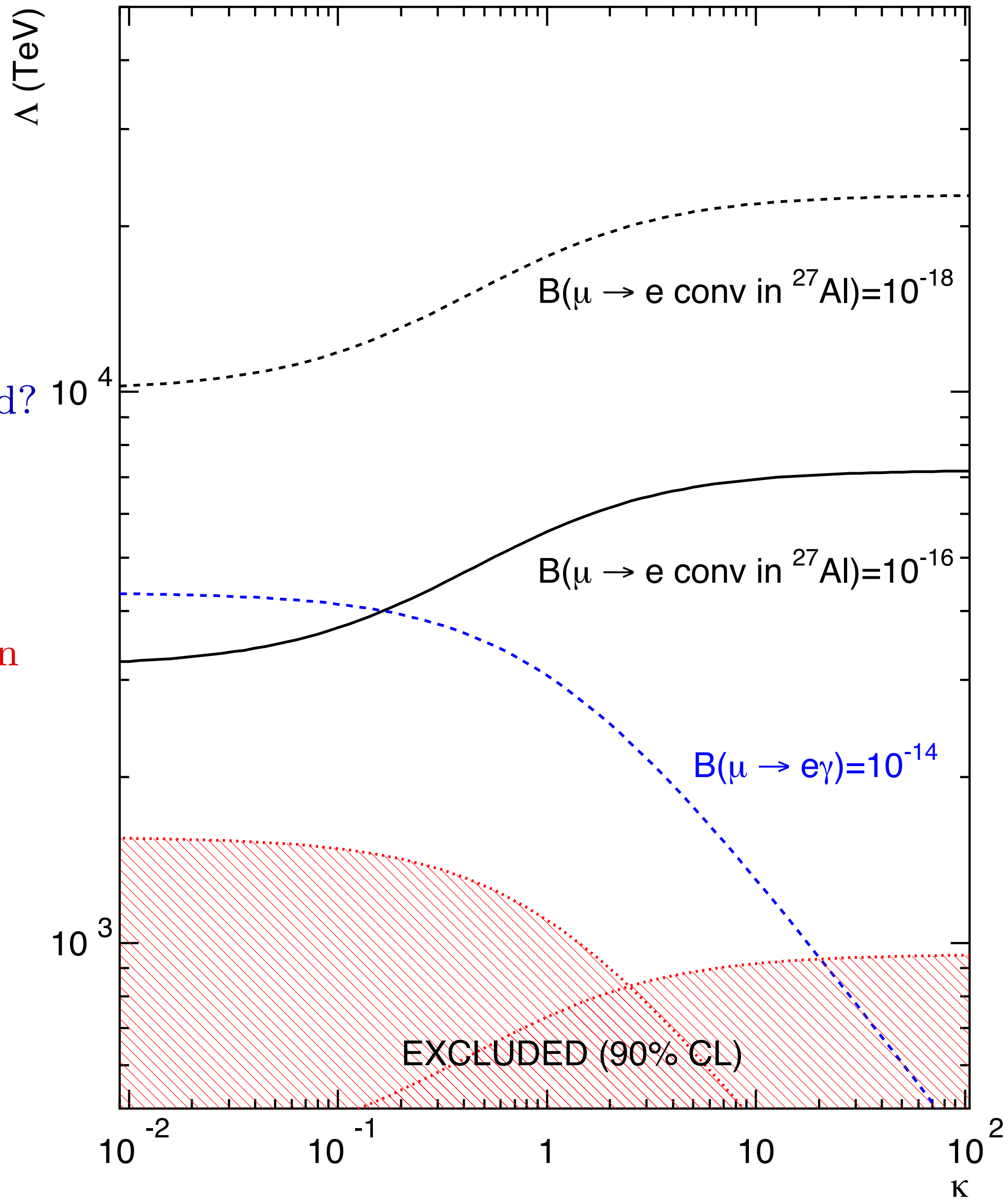
- It is really hard to do $\mu \to e\gamma$ much better than 10^{-14}. $\mu \to e$–conv "best" way forward?

- If the LHC does not discover new states $\mu \to e$-conv among very few process that can access 10,000+ TeV new physics scale: tree-level new physics: $\kappa \gg 1$, $\frac{1}{\Lambda^2} \sim \frac{g^2 \theta_{e\mu}}{M^2_{\rm new}}$.









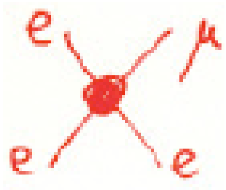
Other Example: $\mu \to ee^+e^-$

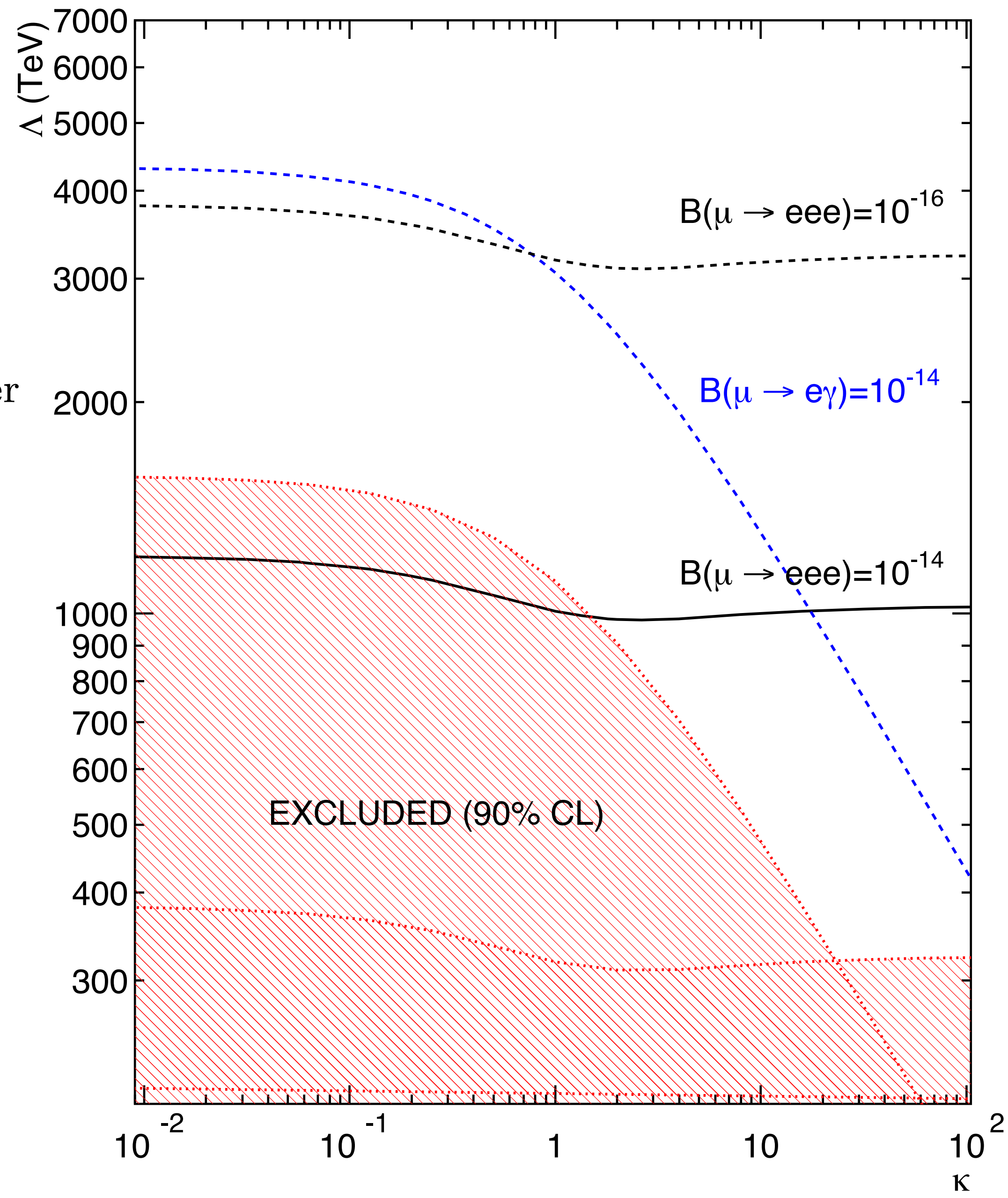
$$\mathcal{L}_{\rm CLFV} = \frac{m_\mu}{(\kappa+1)\Lambda^2}\bar{\mu}_R\sigma_{\mu\nu}e_L F^{\mu\nu} + $$
$$+ \frac{\kappa}{(1+\kappa)\Lambda^2}\bar{\mu}_L\gamma_\mu e_L \bar{e}\gamma^\mu e$$

- $\mu \to eee$-conv at 10^{-16} "guaranteed" deeper probe than $\mu \to e\gamma$ at 10^{-14}.

- $\mu \to eee$ another way forward after MEG?

- If the LHC does not discover new states $\mu \to eee$ among very few process that can access 1,000+ TeV new physics scale: tree-level new physics: $\kappa \gg 1$, $\frac{1}{\Lambda^2} \sim \frac{g^2\theta_{e\mu}}{M^2_{\rm new}}$.





Another example: the $g-2$ of the muon

If there is new ultra-violet physics, it will manifest itself, as far as a_μ is concerned, via the following effective operator (dimension 6):

$$\frac{\lambda H}{\Lambda^2}\bar{\mu}\sigma_{\mu\nu}\mu F^{\mu\nu} \to \frac{m_\mu}{\Lambda^2}\bar{\mu}\sigma_{\mu\nu}\mu F^{\mu\nu},$$

where Λ is an estimate for the new physics scale. (dependency on muon mass is characteristic of several (almost all?) models. It is NOT guaranteed)

Contribution to a_μ from operator above is

$$\delta a_\mu = \frac{4m_\mu^2}{e\Lambda^2}$$

Current experimental sensitivity: $\Lambda \sim 10$ TeV.

Note that, usually, new physics scale can be much lower due to loop-factors, gauge couplings, etc. In the SM the heavy gauge boson contribution yields

$$\frac{1}{\Lambda^2} \sim \frac{eg^2}{16\pi^2 M_W^2} \quad \longrightarrow \quad \delta a_\mu \sim \frac{m_\mu^2 G_F}{4\pi^2}$$

Not A Bad Estimate!

What does "Λ" mean?

This is clearly model dependent! However, some general issues are easy to identify…

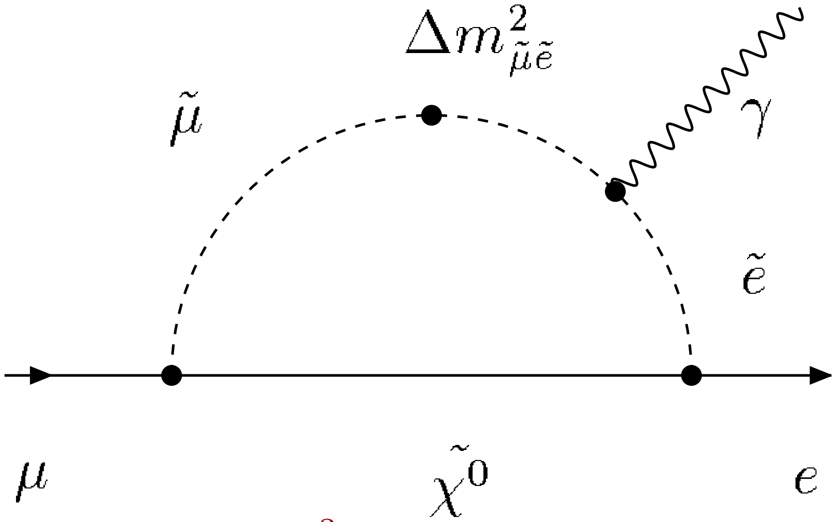
- $\mu \to e\gamma$ and a_μ always occur at the loop level, and are suppressed by E&M coupling e. Also chiral suppression (potential for "$\tan\beta$" enhancement).

$$\frac{1}{\Lambda^2} \sim \frac{e}{16\pi^2}\frac{\tan\beta}{M_{\rm new}^2}$$

- $\mu \to eee$ and $\mu \to e$-conversion in nuclei can happen at the tree-level

$$\frac{1}{\Lambda^2} \sim \frac{y_{\rm new}^2}{M_{\rm new}^2}$$

“Bread and Butter” SUSY plus High Energy Seesaw



$\rightarrow \; \theta_{\tilde{e}\tilde{\mu}} \sim \frac{\Delta m^2_{\tilde{e}\tilde{\mu}}}{\tilde{m}}$

$Br(\mu \to e\gamma) \simeq \frac{\alpha^3\pi}{G_F^2\tilde{m}^4}\theta^2_{\tilde{e}\tilde{\mu}}$, $\tilde{m}^2$ is a typical supersymmetric mass.

$\theta_{\tilde{e}\tilde{\mu}}$ measures the “amount” of flavor violation.

For $\tilde{m}$ around 1 TeV, $\theta_{\tilde{e}\tilde{\mu}}$ is severely constrained. Very big problem.

“Natural” solution: $\boxed{\boxed{\theta_{\tilde{e}\tilde{\mu}} = 0}}$ $\rightarrow$ modified by quantum corrections.

The Seesaw Mechanism

$\mathcal{L} \supset -y_{i\alpha} L^i H N^\alpha - \frac{M_N^{\alpha\beta}}{2} N_\alpha N_\beta + H.c.,$ $\Rightarrow$ N^α gauge singlet fermions,
$y_{i\alpha}$ dimensionless Yukawa couplings, $M_N^{\alpha\beta}$ (very large) mass parameters.

At low energies, integrate out the "right-handed neutrinos" N_α:

$$\mathcal{L} \supset \left(y M_N^{-1} y^t\right)_{ij} L^i H L^j H + \mathcal{O}\left(\frac{1}{M_N^2}\right) + H.c.$$

y are not diagonal $\to$ right-handed neutrino loops generate non-zero $\Delta m^2_{\tilde{e}\tilde{\mu}}$

$$\left(m^2_{\tilde{\ell}_L}\right)_{ij} \simeq -\frac{3m_0^2 + A_0^2}{8\pi^2} \sum_k (y)^*_{ki} (y)_{kj} \ln \frac{M_X}{M_{N_k}} , \qquad X = \text{Planck}, GUT, \text{etc}$$

If this is indeed the case, CLFV would serve as another channel to probe neutrino Yukawa couplings, which are not directly accessible experimentally.

Fundamentally important for "testing" the seesaw, leptogenesis, GUTs, etc

Input From/To Leptogenesis

In the case of the seesaw mechanism, the matter-antimatter asymmetry generated via leptogenesis is (yet another) function of the neutrino Yukawa couplings:

If one is to hope to ever reconstruct the seesaw Lagrangian and test leptogenesis, LFV needs to be measured.

Note that this is VERY ambitious, and we need to get lucky a few times:

- Weak scale SUSY has to exist;
- "Precision" measurement of $\mu \to e$, $\tau \to \mu$, $\tau \to e$;
- "Precision" measurement of SUSY masses;
- Very good understanding of mechanism of SUSY breaking;
- There are no other relevant degrees of freedom between the weak scale and $> 10^9$ GeV;
- etc

Other ways to do this would be much appreciated!

Type-II Seesaw: SM plus $SU(2)$ Triplet Higgs, $Y_T = 1$

$$\mathcal{L} \in \frac{\lambda_{\alpha\beta}}{2} L^\alpha L^\beta T.$$

Neutrino Majorana masses if T develops a vev ...

$$m_{\alpha\beta} = \lambda_{\alpha\beta} v_T$$

$\mu \to e\gamma$, $\mu \to e$-conversion at the loop-level. However, $\mu \to eee$ at the tree level (note direct connection to neutrino mass-matrix flavor sctructure)...

$$\frac{1}{\Lambda^2} = \frac{m_{ee} m_{\mu e}}{v_T^2 M_T^2}$$

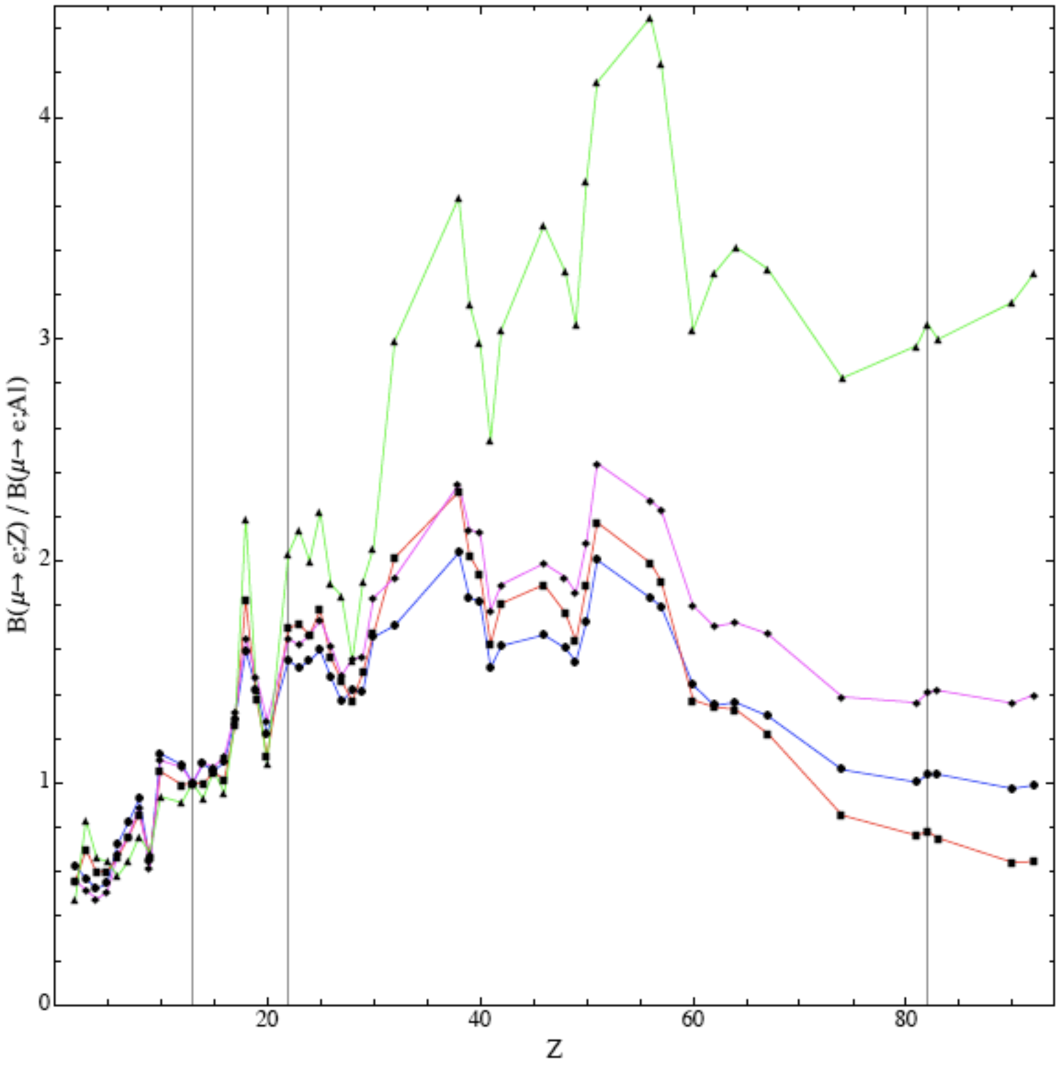
Key issue: are neutrino masses small because λ are small or because v_T is small (or both)? EWPD already push v_T below ~ 1 GeV...

What is This Really Good For?

While specific models (see last slides) provide estimates for the rates for CLFV processes, the observation of one specific CLFV process cannot determine the underlying physics mechanism (this is always true when all you measure is the coefficient of an effective operator).

Real strength lies in combinations of different measurements, including:

- kinematical observables (e.g. angular distributions in $\mu \to eee$);
- other CLFV channels;
- neutrino oscillations;
- measurements of $g-2$ and EDMs;
- collider searches for new, heavy states;
- etc.



Vector 4-Fermion Interaction (Z)

$\propto (\bar{\mu}\gamma_{\alpha}e)(\bar{q}\gamma^{\alpha}q)$

Vector 4-Fermion Interaction (γ)

Dipole ($\propto \bar{\mu}\sigma_{\alpha\beta}eF^{\alpha\beta}$)

Scalar 4-Fermion Interaction

$\propto (\bar{\mu}e)(\bar{q}q)$

Figure 3: Target dependence of the $\mu \to e$ conversion rate in different single-operator dominance models. We plot the conversion rates normalized to the rate in Aluminum ($Z = 13$) versus the atomic number Z for the four theoretical models described in the text: D (blue), S (red), $V^{(\gamma)}$ (magenta), $V^{(Z)}$ (green). The vertical lines correspond to $Z = 13$ (Al), $Z = 22$ (Ti), and $Z = 83$ (Pb).

[Cirigliano, Kitano, Okada, Tuzon, 0904.0957]

Model Independent Comparison Between $g-2$ and CLFV:

The dipole effective operators that mediate $\mu \to e\gamma$ and contribute to a_μ are virtually the same:

$$\frac{m_\mu}{\Lambda^2}\bar{\mu}\sigma^{\mu\nu}\mu F_{\mu\nu} \qquad \times \qquad \theta_{e\mu}\frac{m_\mu}{\Lambda^2}\bar{\mu}\sigma^{\mu\nu}eF_{\mu\nu}$$

$\theta_{e\mu}$ measures how much flavor is violated. $\theta_{e\mu}=1$ in a flavor indifferent theory, $\theta_{e\mu}=0$ in a theory where indiviadual lepton flavor number is exactly conserved.

If $\theta_{e\mu}\sim 1$, $\mu \to e\gamma$ is a much more stringent probe of Λ.

On the other hand, if the current discrepancy in a_μ is due to new physics, $\theta_{e\mu} \ll 1$ ($\theta_{e\mu} < 10^{-4}$). [Hisano, Tobe, hep-ph/0102315]

e.g., in SUSY models, $Br(\mu \to e\gamma) \simeq 3\times 10^{-5}\left(\frac{10^{-9}}{\delta a_\mu}\right)\left(\frac{\Delta m^2_{\tilde{e}\tilde{\mu}}}{\tilde{m}^2}\right)^2$

[Comparison restricted to dipole operator. If four-fermion operators are relevant, they will typically enhance rate for CLFV with respect to expectations from $g-2$.]

On CLFV Processes Involving τ Leptons (Brief Comment)

Current Bound On Selected τ CLFV Processes (All from the B-Factories):

- $B(\tau \to e\gamma) < 1.1 \times 10^{-7}$; $B(\tau \to \mu\gamma) < 4.5 \times 10^{-8}$. $(\mu \to e\gamma)$
- $B(\tau \to e\pi) < 8.0 \times 10^{-8}$; $B(\tau \to \mu\pi) < 1.1 \times 10^{-7}$. ($\mu \to e$–conversion)
- $B(\tau \to eee) < 3.6 \times 10^{-8}$; $B(\tau \to ee\mu) < 2.0 \times 10^{-8}$, $(\mu \to eee)$
- $B(\tau \to e\mu\mu) < 2.3 \times 10^{-8}$; $B(\tau \to \mu\mu\mu) < 3.2 \times 10^{-8}$. $(\mu \to eee)$

Relation to $\mu \to e$ violating processes is model dependent. Typical enhancements, at the amplitude-level, include:

- Chirality flipping: $m_\tau \gg m_\mu$;
- Lepton mixing effects: $U_{\tau 3} \gg U_{e3}$;
- Mass-Squared Difference effects: $\Delta m^2_{13} \gg \Delta m^2_{12}$;
- etc

Future: Modest improvements expected from LHCb (?). Belle II will get to 10^{-9} level and beyond!

ASIDE: Lepton-Flavor and Lepton-Number Violation: $\mu^- \to e^+$-conversion

Experimental Sensitivities

KamLAND-Zen: $T_{0\nu\beta\beta} > 1.07 \times 10^{26}$ yr (90% CL; ^{136}Xe)

arXiv:1605.02889; KamLAND-ZEN Collaboration

SINDRUM II:

- $\mu^- \to e^-$ conversion: $R^{\rm Au}_{\mu^- e^-} \equiv \frac{\Gamma(\mu^- + {\rm Au} \to e^- + {\rm Au})}{\Gamma(\mu^- + {\rm Au} \to \nu_\mu + {\rm Pt})} < 7 \times 10^{-13}$ (90% CL)
- $\mu^- \to e^+$ conversion: $R^{\rm Ti}_{\mu^- e^+} \equiv \frac{\Gamma(\mu^- + {\rm Ti} \to e^+ + {\rm Ca})}{\Gamma(\mu^- + {\rm Ti} \to \nu_\mu + {\rm Sc})} < \begin{cases} 1.7 \times 10^{-12} & \text{(GS, 90\% CL)} \\ 3.6 \times 10^{-11} & \text{(GDR, 90\% CL)} \end{cases}$

Eur. Phys. J. ***C47****, 337 (2006); SINDRUM II Collaboration*
Phys. Lett. ***B422****, 334 (1998); SINDRUM II Collaboration*

Apples–to–apples comparison of $\mu^- \to e^-$ and $\mu^- \to e^+$?

- 1993 – simultaneous analysis!
- Apply this to future experiments

$$R^{\rm Ti}_{\mu^- e^-} < 4.3 \times 10^{-12} \text{ (90\% CL)}$$
$$R^{\rm Ti}_{\mu^- e^+} < 4.3 \times 10^{-12} \text{ (90\% CL)}$$

Phys. Lett. ***B317****, 631 (1993); SINDRUM II Collaboration*

[estimates from Berryman et al, arXiv:1611.00032]

Experimental Sensitivities

Upcoming experiments:

$$\begin{aligned} \text{DeeMe:} \quad & R^{\rm SiC}_{\mu^- e^-} > 5\times 10^{-14}\ (90\%\ \text{CL}),\\ \text{Mu2e:} \quad & R^{\rm Al}_{\mu^- e^-} > 6.6\times 10^{-17}\ (90\%\ \text{CL}),\\ \text{COMET Phase-I:} \quad & R^{\rm Al}_{\mu^- e^-} > 7.2\times 10^{-15}\ (90\%\ \text{CL}),\\ \text{COMET Phase-II:} \quad & R^{\rm Al}_{\mu^- e^-} > 6\times 10^{-17}\ (90\%\ \text{CL}),\\ \text{PRISM:} \quad & R^{\rm Al}_{\mu^- e^-} > 5\times 10^{-19}\ (90\%\ \text{CL}). \end{aligned}$$

Who could do this measurement?

- *Possibly* Mu2e and COMET Phase-I – similar to SINDRUM II
- *Probably* not DeeMe, COMET Phase-II or PRISM

$$\boxed{\begin{aligned} \text{Mu2e:} \quad & R^{\rm Al}_{\mu^- e^+} \gtrsim 10^{-16}\\ \text{COMET Phase-I:} \quad & R^{\rm Al}_{\mu^- e^+} \gtrsim 10^{-14} \end{aligned}}$$

What we can learn from CLFV and other searches for new physics at the TeV scale (a_μ and Colliders):

$g-2$	CLFV	What Does it Mean?
YES	YES	New Physics at the TeV Scale; Some Flavor Violation
YES	NO	New Physics at the TeV Scale; Tiny Flavor Violation
NO	YES	New Physics Above TeV Scale; Some Flavor Violation – How Large?
NO	NO	No New Physics at the TeV Scale; CLFV only way forward?

Colliders	CLFV	What Does it Mean?
YES	YES	New Physics at the TeV Scale; Info on Flavor Sector!
YES	NO	New Physics at the TeV Scale; New Physics Very Flavor Blind. Why?
NO	YES	New Physics "Leptonic" or Above TeV Scale; Which one?
NO	NO	No New Physics at the TeV Scale; CLFV only way forward?

Summary and Conclusions

- Low-energy muon processes constitute a powerful (often unique) probe of new physics around the electroweak scale, not unlike high-energy collider experiments (similar sensitivity to new physics energy scale).
- We know that charged lepton flavor violation must occur. Effects are, however, really tiny in the νSM (neutrino masses too small).
- If there is new physics at the electroweak scale, there is every reason to believe that CLFV is well within the reach of next generation experiments. Indeed, it is fair to ask: 'Why haven't we seen it yet?'
- It is fundamental to probe all CLFV channels. While in many scenarios $\mu \to e\gamma$ is the "largest" channel, there is no theorem that guarantees this (and many exceptions).
- CLFV may be intimately related to new physics unveiled with the discovery of non-zero neutrino masses. It may play a fundamental role in our understanding of the seesaw mechanism, GUTs, the baryon-antibaryon asymmetry of the Universe. We won't know for sure until we see it!