Decay of η' into four pions and its impact on the doubly-virtual η' transition form factor

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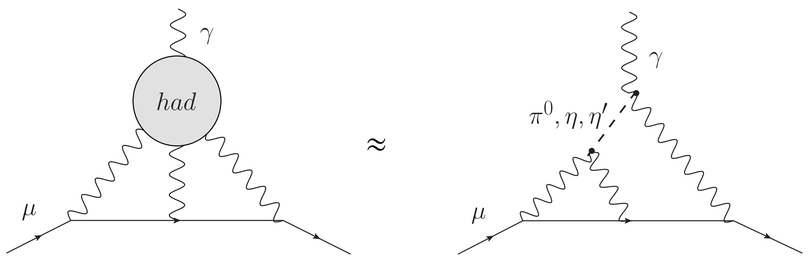
THE η' TRANSITION FORM FACTOR AND THE $g-2$ OF THE MUON

- anomalous magnetic moment of the muon [1]:
$$a_\mu^{\text{SM}} = 116591802(42)_{\text{HVP}}(26)_{\text{HLbyL}}(2)_{\text{QED+EW}}(49)_{\text{tot}} \cdot 10^{-11}$$
- uncertainty soon dominated by HLbyL scattering



- leading contributions: pseudoscalar poles (π^0, η, η')
two-meson intermediate states
- input:
doubly- and singly-virtual η' transition form factor $F_{\eta'\gamma^*\gamma^*}$

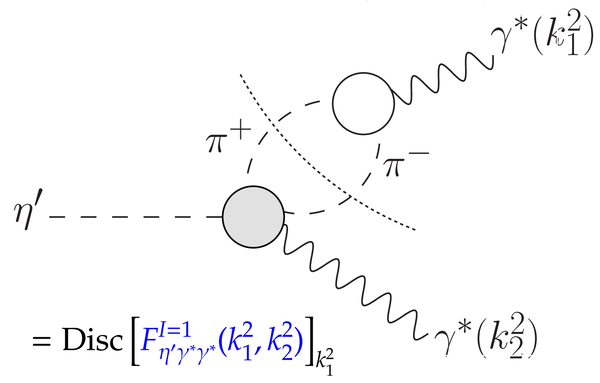
FACTORIZATION ANSATZ FOR $F_{\eta'\gamma^*\gamma^*}$

- direct measurement of the doubly-virtual form factor difficult
$$\frac{\mathcal{B}\left(\eta' \to 2(e^+e^-)\right)}{\mathcal{B}\left(\eta' \to 2(\pi^+\pi^-)\right)} \sim \alpha_{\text{QED}}^4$$
- commonly employed factorization ansatz:
$$F_{\eta'\gamma^*\gamma^*}\left(k_1^2, k_2^2\right) \approx F_{\eta'\gamma^*\gamma}\left(k_1^2\right) \cdot F_{\eta'\gamma^*\gamma}\left(k_2^2\right)$$
- incompatible with perturbative QCD prediction:
$$F_{\eta'\gamma^*\gamma^*}\left(k_1^2, k_2^2\right) \xrightarrow{k_1^2, k_2^2 \to \infty} \sim \frac{1}{k_1^2 + k_2^2}$$
- dispersion theoretical analysis $\Longrightarrow$ factorization breaking in $\eta' \to 2(\pi^+\pi^-)$ shows up in isovector part of the form factor

DISPERSION-THEORETICAL ANALYSIS OF THE FORM FACTOR

The decay amplitude $\eta(') \to \pi^+\pi^-\gamma$ has important impact on dispersion-theoretical analysis of singly-virtual $\eta(')$ transition form factor $\eta(') \to \gamma\gamma^*$ [2]

- unitarity of the S-matrix allows to compute discontinuity of amplitude through "cutting rules"
(sum over all amplitudes with on-shell intermediate states)
- singly-virtual $F_{\eta\gamma^*\gamma}$: statistical lead of hadronic input for $\eta \to \pi^-\pi^-\gamma$ over direct measurement of $\eta \to e^+e^-\gamma$



$$= \text{Disc}\left[F^{I=1}_{\eta'\gamma^*\gamma^*}(k_1^2, k_2^2)\right]_{k_1^2}$$

for singly-virtual case $k_2^2 = 0$

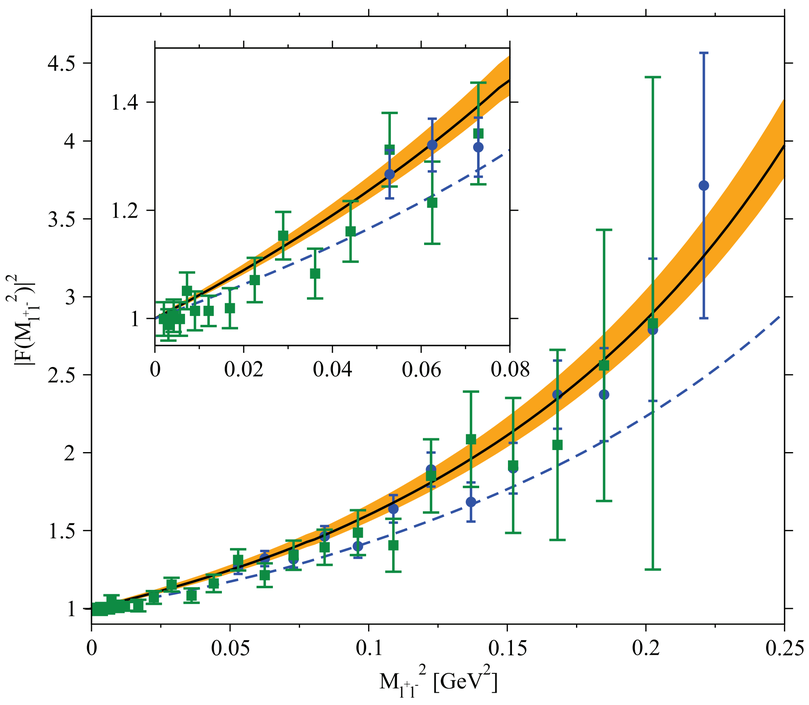


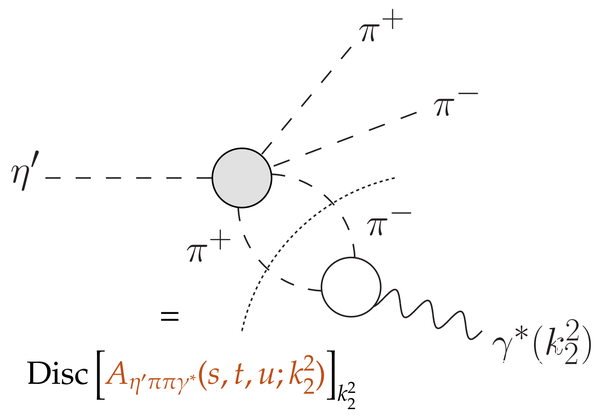
Figure courtesy of C. Hanhart. data: NA60 2011, A2 2014

Doubly-virtual form factor can be understood in more detail, starting from analysis of $\eta' \to 2(\pi^+\pi^-)$

$$F^{I=1}_{\eta'\gamma^*\gamma^*}(k_1^2, k_2^2) = \frac{1}{2\pi i}\int_{4m_\pi^2}^{\infty} \text{d}s_{\pi\pi} \frac{\text{Disc}\left[F^{I=1}_{\eta'\gamma^*\gamma^*}(s_{\pi\pi}, k_2^2)\right]_{s_{\pi\pi}}}{s_{\pi\pi} - k_1^2}$$

- inelastic intermediate states (4π, $K\bar{K}$) suppressed
- input for $F_{\eta'\gamma^*\gamma^*}$: $A_{\eta'\pi\pi\gamma^*}$
- input for $A_{\eta'\pi\pi\gamma^*}$: $A_{\eta'\to 4\pi}$

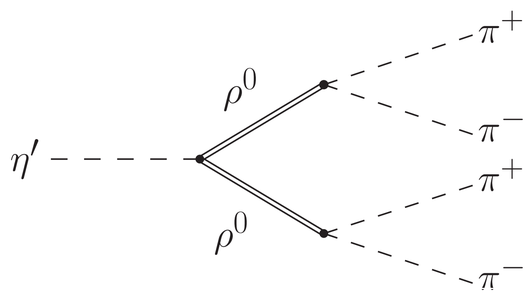
$$A_{\eta'\pi\pi\gamma^*}(s,t,u;k_2^2) = \frac{1}{2\pi i}\int_{4m_\pi^2}^{\infty} \text{d}s_{\pi\pi} \frac{\text{Disc}\left[A_{\eta'\pi\pi\gamma^*}(s,t,u;s_{\pi\pi})\right]_{s_{\pi\pi}}}{s_{\pi\pi} - k_2^2}$$



$$\text{Disc}\left[A_{\eta'\pi\pi\gamma^*}(s,t,u;k_2^2)\right]_{k_2^2}$$

$\eta' \to \pi^+\pi^-\pi^+\pi^-$: THE STATUS QUO

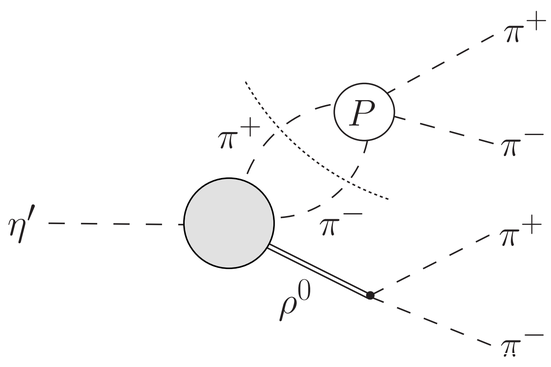
- the decay is of odd intrinsic parity
$\Longrightarrow$ driven by the chiral anomaly.
- pions have to be at least in a relative P wave
- amplitude is clearly dominated by vector-meson contributions [3]



- branching ratio predicted through the pure ρ analysis [3] has recently been confirmed by measurement of the BESIII collaboration [4]
$$\mathcal{B}_{\text{theory}} = (10 \pm 3) \cdot 10^{-5}$$
$$\mathcal{B}_{\text{experiment}} = (8.53 \pm 0.94) \cdot 10^{-5}$$
- a_2 corrections change the branching ratio by $\sim 20\%$

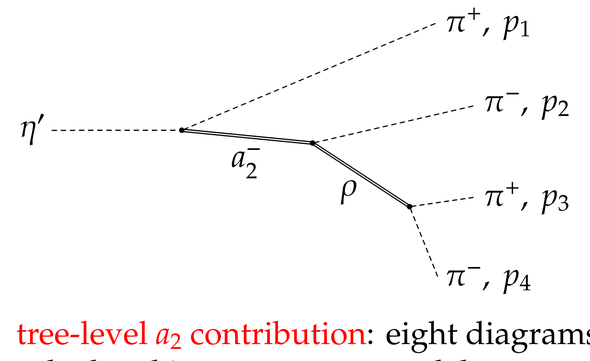
$\eta' \to \pi^+\pi^-\pi^+\pi^-$ WITH EFFECTS OF THE a_2 TENSOR-MESON

- The a_2 tensor meson corrects the amplitude of $\eta' \to \pi^+\pi^-\gamma$ noticeably [5] $\Longrightarrow$ a_2 effect on $\eta' \to \pi^+\pi^-\pi^+\pi^-$?
extend the previous (vector-)meson dominance description
- Just the tree-level correction severely violates Watson's theorem: correct phases only with full rescattering between all four pions.
- Since a correct description of the final-state interaction in terms of an extension of the Khuri-Treiman equations would be a formidable task, we limit our analysis such that the pions have rescattered at least pairwise:



$$A_{\eta'\to 4\pi} = c\varepsilon_{\mu\nu\alpha\beta}\, p_1^\mu p_2^\nu p_3^\alpha p_4^\beta \Big[F(s_{12},\ldots)\Omega(s_{34}) + F(s_{34},\ldots)\Omega(s_{12}) - F(s_{23},\ldots)\Omega(s_{14}) - F(s_{14},\ldots)\Omega(s_{23})\Big]$$

$$F(t,\ldots) = \left[P(t) + \frac{t^2}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\text{d}t'}{t'^2} \frac{\hat{G}(t',k^2)\sin\delta(t')}{(t'-t)|\Omega(t')|}\right]\Omega(t) + G(s,t,u;k^2) + G(u,t,s;k^2)$$



tree-level a_2 contribution: eight diagrams calculated in a resonance model

$G(s,t,u;k^2)\Omega(k^2)$: reduced amplitude stemming from the tree-level diagram

$\hat{G}(t,k^2)$: its projection on the P-wave of the pion pair originating at the a_2 vertices

$\Omega(k^2)$: Omnès function

As input for the dispersive determination of $A_{\eta'\pi\pi\gamma*}$ the projection of $A_{\eta'\to 4\pi}$ onto the P wave of one $\pi^+\pi^-$ pair is needed. Neglecting F waves this is (in the s_{12} channel) given by

$$\left[F(s_{12},\ldots)\right]_P \Omega(s_{34}) + \left[F(s_{34},\ldots)\right]_P \Omega(s_{12})$$

NEXT STEPS:

- reproduction of $\eta' \to \pi^+\pi^-\gamma$ data
- generalisation to $\eta' \to \pi^+\pi^-\gamma^*$ and $\eta' \to \gamma^*\gamma^*$
- quantification of factorisation-breaking contributions

[1] J. P. Miller, E. de Rafael, B. L. Roberts and D. Stöckinger, Ann. Rev. Nucl. Part. Sci. **62**, 237 (2012)

[2] C. Hanhart, A. Kupść, U.-G. Meißner, F. Stollenwerk and A. Wirzba, "Dispersive analysis for $\eta \to \gamma\gamma^*$," Eur. Phys. J. C **73**, 2668 (2013)

[3] F.-K. Guo, B. Kubis and A. Wirzba, "Anomalous decays of eta' and eta into four pions," Phys. Rev. D **85**, 014014 (2012)

[4] M. Ablikim *et al.* [BESIII Collaboration], "Observation of $\eta' \to \pi^+\pi^-\pi^+\pi^-$ and $\eta' \to \pi^+\pi^-\pi^0\pi^0$," Phys. Rev. Lett. **112**, 251801 (2014) [Addendum-ibid. **113**, 039903 (2014)]

[5] B. Kubis and J. Plenter, "Anomalous decay and scattering processes of the η meson," Eur. Phys. J. C **75**, no. 6, 283 (2015)

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