Radiative corrections to elastic electron proton scattering and the uncertainty in the proton charge radius

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Outline

- Motivation
- MAMI experiment
- Types of corrections to *ep* scattering
- Vacuum polarization
- Exponentiation of photonic corrections
- Light pair correction in LLA
- Complete second order NLO corrections
- Numerical results
- Open questions and Conclusions

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- Here: effects of radiative corrections in elastic *ep* scattering
- Dependence on event selection procedure is discussed

The MAMI experiment (I)

Mainz Microtron experimental set-up:

— the electron beam energy $E_e \equiv E \lesssim 855$ MeV (1.6 GeV)

— momentum transfer range: $0.003 < Q^2 < 1$ GeV^2

— the outgoing electron energy $E_e{}' \equiv E' > E_e - \Delta E$

— no any other condition: neither on enrgies nor on angles

— experimental precision (point-to-point) $\simeq 0.37\% \quad \to \quad 0.1\%$ (?)

$\Rightarrow$ all effects at least of the 10^{-4} order should be taken into account. That is not a simple task in any case

N.B. $E_e^2 \gg m_e^2, \quad Q^2 \gg m_e^2, \quad (\Delta E)^2 \gg m_e^2, \quad \Delta E \ll E_e$

Ref.: J.C. Bernauer et al. [A1 Coll.] PRC 90 (2014) 015206

The MAMI experiment (III)

The Born cross section is written via the Sachs form factors:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0 = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} \left[\frac{G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)}{1+\tau} + 2\tau G_M^2\left(Q^2\right) \tan^2\frac{\theta}{2}\right]$$
$$= \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} \frac{\varepsilon G_E^2 + \tau G_M^2}{\varepsilon\left(1+\tau\right)}, \qquad \tau = \frac{Q^2}{4m_P^2}, \quad \varepsilon = E_e$$

The proton charge radius is defined then via

$$\left\langle r^2 \right\rangle = -\frac{6}{G_E\left(0\right)} \left.\frac{\mathrm{d}G_E\left(Q^2\right)}{\mathrm{d}Q^2}\right|_{Q^2=0}$$

i.e., from the slope of the G_E form factor at $Q^2 = 0$

Types of RC to elastic *ep* scattering

— Virtual (loop) and/or real emission

— QED, QCD, and (electro)weak effects

— Perturbative and/or non-perturbative contributions

— Perturbative QED effects in $\mathcal{O}(\alpha)$, $\mathcal{O}(\alpha^2)$, ...

— Leading and next-to-leading logarithmic approximations

— Corrections to the electron line, to the proton line, and their interference

— Vacuum polarization, vertex corrections, double photon exchange etc.

First order QED RC (I)

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_1 = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0 (1+\delta)$$

The $\mathcal{O}(\alpha)$ QED RC with point-like proton are well known:
Refs.: see eg. L. C. Maximon & J. A. Tjon, PRC 2000

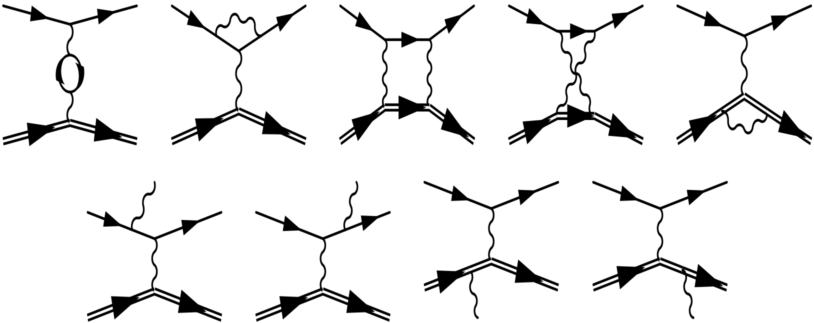
Virtual RC: Vacuum polarization, vertex, and box Feynman diagrams

Real RC: emission off the initial and final electrons and protons

N.B.1. UV divergences are regularized and renormalized

N.B.2. IR divergences cancel out in sum of virtual and real RC

First order QED RC (II)



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Size of RC

The problem has several small and large parameters to be used in expansions:

- $\alpha/(2\pi) \approx 0.001$
- $(\alpha/(2\pi))^2 \approx 10^{-6}$
- $L \equiv \ln(Q^2/m_e^2) \approx 16$ the large log for $Q^2 = 1$ GeV^2
- $\ln(\Delta) \sim 5$, where $\Delta = \Delta E_e/E_e \ll 1$

N.B. Some $\mathcal{O}(\alpha^2)$ corrections are enhanced with 2nd, 3rd or even 4th power of large logs. So, they should be treated with care.

Vacuum polarization in one-loop

$$\delta^{(1)}_{\rm vac} = \frac{\alpha}{\pi}\frac{2}{3}\left\{\left(v^2 - \frac{8}{3}\right) + v\frac{3-v^2}{2}\ln\left(\frac{v+1}{v-1}\right)\right\}$$

$$\stackrel{Q^2 \gg m_l^2}{\longrightarrow} \frac{\alpha}{\pi}\frac{2}{3}\left\{-\frac{5}{3} + \ln\left(\frac{Q^2}{m_l^2}\right)\right\}, \quad v = \sqrt{1+\frac{4m_l^2}{Q^2}}, \quad l = e, \mu, \tau$$

Two ways of re-summation:

1) geometric progression

$$\Rightarrow \alpha(Q^2) = \frac{\alpha(0)}{1-\Pi(Q^2)}, \quad \Pi(Q^2) = \frac{1}{2}\delta^{(1)}_{\rm vac} + \ldots$$

2) exponentiation

$$\alpha(Q^2) = \alpha(0)e^{\delta^{(1)}_{\rm vac}/2}$$

the latter option was used by A1 Coll.

Other $\mathcal{O}(\alpha)$ effects

$$\delta^{(1)}_{\mathrm{vertex}} = \frac{\alpha}{\pi}\left\{\frac{3}{2}\ln\left(\frac{Q^2}{m^2}\right) - 2 - \frac{1}{2}\ln^2\left(\frac{Q^2}{m^2}\right) + \frac{\pi^2}{6}\right\}$$

$$\begin{aligned}\delta^{(1)}_{\mathrm{real}} &= \frac{\alpha}{\pi}\left\{\ln\left(\frac{(\Delta E_s)^2}{E\cdot E'}\right)\left[\ln\left(\frac{Q^2}{m^2}\right) - 1\right] - \frac{1}{2}\ln^2\eta + \frac{1}{2}\ln^2\left(\frac{Q^2}{m^2}\right)\right.\\ &- \left.\frac{\pi^2}{3} + \mathrm{Li}_2\left(\cos^2\frac{\theta_e}{2}\right)\right\}, \quad \eta = \frac{E}{E'}, \quad \Delta E_s = \eta\cdot\Delta E'\end{aligned}$$

Interference δ_1 and radiation off proton δ_2 do not contain the large log.
A1 Coll. applied RC in the exponentiated form:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{exp}}(\Delta E') = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0 e^{\delta_{\mathrm{vac}}+\delta_{\mathrm{vertex}}+[\delta_R+\delta_1+\delta_2](\Delta E')}$$

Higher order effects are partially taken into account by exponentiation.
Remind the Yennie-Frautschi-Suura theorem

Multiple soft photon radiation (I)

Exponentiation corresponds to independent emission of soft photons, while the cut on the total lost energy leads to sizable shifts.

For two photons:

$$\mathrm{e}^{\delta_{\rm soft}} \to \mathrm{e}^{\delta_{\rm soft}} - \left(\frac{\alpha}{\pi}\right)^2 \frac{\pi^2}{3} (L-1)^2$$

at $Q^2 = 1$ GeV^2 this gives $-3.5 \cdot 10^{-3}$

In the leading log approximation

$$\delta^{(3)}_{\rm LLA} = (L-1)^3 \left(\frac{\alpha}{\pi}\right)^3 \frac{1}{6} \left(P^{(0)} \otimes P^{(0)} \otimes P^{(0)}\right)_\Delta ,$$

$$\left(P^{(0)} \otimes P^{(0)} \otimes P^{(0)}\right)_\Delta = 8 \left(P^{(0)}_\Delta\right)^3 - 24\zeta(2) P^{(0)}_\Delta + 16\zeta(3)$$

$$\Rightarrow \quad \delta^{(3)}_{\rm cut} = (L-1)^3 \left(\frac{\alpha}{\pi}\right)^3 \left[-4\zeta(2) P^{(0)}_\Delta + \frac{8}{3}\zeta(3)\right]$$

which is not small and reaches $2 \cdot 10^{-3}$

Multiple soft photon radiation (II)

The exact LLA solution of the evolution equation for the photonic part of the non-singlet structure function in the soft limit is known

$$\mathcal{D}_\gamma^{\mathrm{N}S}(z,Q^2)\Big|_{z\to 1} = \frac{\beta}{2}\,\frac{(1-z)^{\beta/2-1}}{\Gamma(1+\beta/2)}\exp\left\{\frac{\beta}{2}\left(\frac{3}{4}-C\right)\right\}$$

where C is the Euler constant, $\beta = \frac{2\alpha}{\pi}(\ln\frac{Q^2}{m^2}-1)$

$$\int\limits_{1-\Delta}^{1} \mathrm{d}z\, \mathcal{D}_\gamma^{\mathrm{N}S}(z,Q^2) = \exp\left\{\frac{\beta}{2}\ln\Delta + \frac{3\beta}{8}\right\}\frac{\exp(-C\beta/2)}{\Gamma(1+\beta/2)},$$

$$\frac{\exp(-C\beta/2)}{\Gamma(1+\beta/2)} = 1 - \frac{1}{2}\left(\frac{\beta}{2}\right)^2\zeta(2) + \frac{1}{3}\left(\frac{\beta}{2}\right)^3\zeta(3) + \frac{1}{16}\left(\frac{\beta}{2}\right)^4\zeta(4) + \frac{1}{5}\left(\frac{\beta}{2}\right)^5\zeta(5) - \frac{1}{6}\left(\frac{\beta}{2}\right)^5\zeta(2)\zeta(3) + \mathcal{O}(\beta^6)$$

[V. Gribov, L. Lipatov, Sov. J. Nucl. Phys. **15** (1972) 451; 675]

Light pair corrections

A quick estimate can be done within LLA:

$$\delta^{LLA}_{\rm pair} = \frac{2}{3}\left(\frac{\alpha}{2\pi}L\right)^2 P^{(0)}_{\Delta} + \frac{4}{3}\left(\frac{\alpha}{2\pi}L\right)^3 \left\{\left(P^{(0)} \otimes P^{(0)}\right)_{\Delta} + \frac{2}{9}P^{(0)}_{\Delta}\right\} + \mathcal{O}\left(\alpha^2 L, \alpha^4 L^4\right)$$

$$P^{(0)}_{\Delta} = 2\ln\Delta + \frac{3}{2}, \qquad \left(P^{(0)} \otimes P^{(0)}\right)_{\Delta} = \left(P^{(0)}_{\Delta}\right)^2 - \frac{\pi^2}{3}$$

The energy of the emitted pair is limited by the same parameter: $E_{\rm pair} \leq \Delta E$. Both virtual and real e^+e^- pair corrections are taken into account.

Typically, $\mathcal{O}\left(\alpha^2\right)$ pair RC are a few times less than $\mathcal{O}\left(\alpha^2\right)$ photonic ones, see e.g. A.A. JHEP'2001

Complete NLLA corrections (I)

The NLO structure function approach for QED was first introduced in F.A. Berends et al. NPB'1987, and then developed in A.A. & K.Melnikov PRD'2002; A.A. JHEP'2003

The master formula for *ep* scattering reads

$$\mathrm{d}\sigma = \int_{\bar{z}}^{1} \mathrm{d}z \mathcal{D}_{ee}^{\mathrm{str}}(z) \left(\mathrm{d}\sigma^{(0)}(z) + \mathrm{d}\bar{\sigma}^{(1)}(z) + \mathcal{O}\left(\alpha^2 L^0\right) \right) \int_{\bar{y}}^{1} \frac{\mathrm{d}y}{Y} \mathcal{D}_{ee}^{\mathrm{frg}} \left(\frac{y}{Y} \right)$$

where $\mathrm{d}\bar{\sigma}^{(1)}$ is the $\mathcal{O}(\alpha)$ correction to the *ep* scattering with a "massless electron" in the $\overline{\mathrm{MS}}$ scheme

Complete NLLA corrections (II)

$$\mathrm{d}\sigma^{\mathrm{NLO}} = \int_{1-\Delta}^{1} \mathcal{D}^{\mathrm{str}}_{ee} \otimes \mathcal{D}^{\mathrm{frg}}_{ee}(z) \Big[\mathrm{d}\sigma^{(0)}(z) + \mathrm{d}\bar{\sigma}^{(1)}(z) \Big] \mathrm{d}z$$

$$= \mathrm{d}\sigma^{(0)}(1) \bigg\{ 1 + 2\frac{\alpha}{2\pi} \Big[L P^{(0)}_{\Delta} + (d_1)_{\Delta} \Big] + 2\left(\frac{\alpha}{2\pi}\right)^2 \Big[L^2 \left(P^{(0)} \otimes P^{(0)} \right)_{\Delta}$$

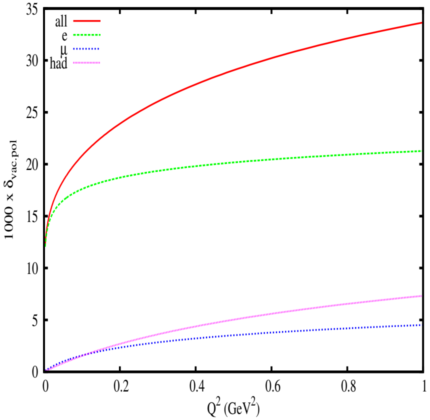
$$+ \frac{1}{3} L^2 P^{(0)}_{\Delta} + 2L(P^{(0)} \otimes d_1)_{\Delta} + L(P^{(1,\gamma)}_{ee})_{\Delta} + L(P^{(1,\mathrm{pair})}_{ee})_{\Delta} \Big] \bigg\}$$

$$+ \mathrm{d}\bar{\sigma}^{(1)}(1)\, 2\frac{\alpha}{2\pi} L P^{(0)}_{\Delta} + \mathcal{O}\left(\alpha^3 L^3\right)$$

$$(d_1)_{\Delta} = -2\ln^2\Delta - 2\ln\Delta + 2, \quad \ldots$$

N.B. Method gives complete $\mathcal{O}\left(\alpha^2 L\right)$ results for sufficiently inclusive observables.

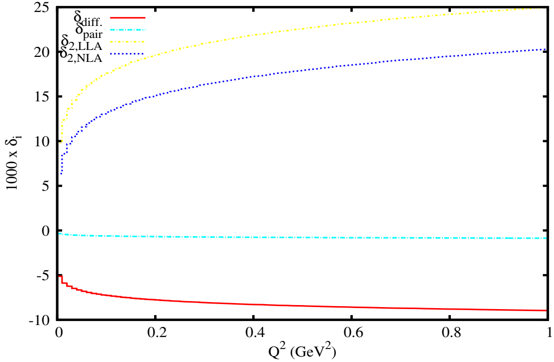
Numerical results: vacuum polarization



Vacuum polarization corrections due to electrons (e), muons (μ), hadrons (had), and the combined effect (all).
Program `AlphaQED` by F. Jegerlehner was used.

Higher-order corrections (1)

$E_{\mathrm{beam}} = 800$ MeV, $E_{\mathrm{lost}} \leq$ 10 MeV

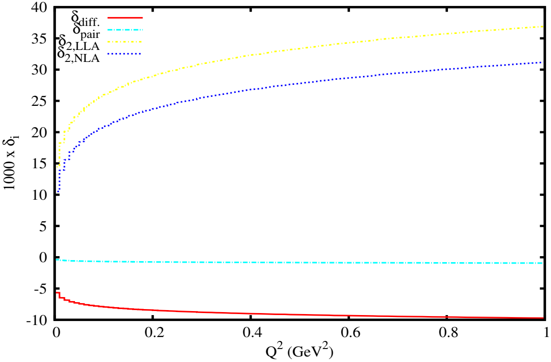


$$\delta_i = \mathrm{d}\sigma^{(i)} / \mathrm{d}\sigma^{(0)}$$

$$\delta_{\mathrm{diff.}} = \frac{\mathrm{d}\sigma^{\mathrm{NLO}}}{\mathrm{d}\sigma^{(0)}(1)} + \delta^{(3)}_{\mathrm{LLA}} + \delta^{(3)}_{\mathrm{LLA,pair}} + \delta^{(4)}_{\mathrm{LLA}} - \exp\{\delta^{(1)}\}$$

Higher-order corrections (2)

$E_{\rm beam} = 1600$ MeV, $E_{\rm lost} \leq$ 10 MeV

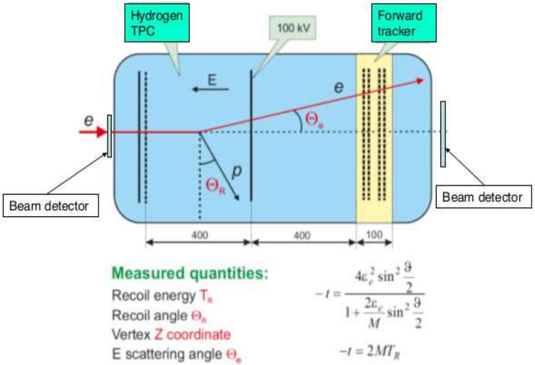


$$\delta_i = \mathrm{d}\sigma^{(i)}/\mathrm{d}\sigma^{(0)}$$

$$\delta_{\rm diff.} = \frac{\mathrm{d}\sigma^{\rm NLO}}{\mathrm{d}\sigma^{(0)}(1)} + \delta^{(3)}_{\rm LLA} + \delta^{(3)}_{\rm LLA,pair} + \delta^{(4)}_{\rm LLA} - \exp\{\delta^{(1)}\}$$

New experiment is proposed at MAMI

Proposal to perform an experiment at the A2 hall, MAMI:
High Precision Measurement of the ep elastic cross section at small Q^2
Contact persons for the Experiment:
Alexey Vorobyev, Petersburg Nuclear Physics Institute
Achim Denig, Institute for Nuclear Physics, JGU Mainz



Leading logs in the new set-up

FSR large log corrections are cancelled out (KLN theorem)

ISR provides an effective reduction of the beam energy.
It affects the the proton Q^2 distribution rather weakly

Some PRELIMINARY results in the collinear leading log approximation were obtained for

$$E_{\mathrm{beam}} = 500\ \mathrm{MeV}, \qquad 0.001 < Q^2 < 0.02\ \mathrm{GeV}^2$$

Q^2 [GeV]	0.001	0.01	0.02
δ_1^{LLA}	$-6.0 \cdot 10^{-4}$	$-2.9 \cdot 10^{-3}$	$-4.6 \cdot 10^{-3}$
δ_2^{LLA}	$-1.1 \cdot 10^{-5}$	$-4.0 \cdot 10^{-5}$	$-5.4 \cdot 10^{-5}$
$\delta_{2+3+\mathrm{pairs}}^{\mathrm{LLA}}$	$-1.3 \cdot 10^{-5}$	$-5.1 \cdot 10^{-5}$	$-7.4 \cdot 10^{-5}$

$$\delta_n(Q^2) = \sigma^{\mathrm{LLA}}(Q^2)/\sigma^{\mathrm{Born}}(Q^2) - 1$$

Further steps

In 2014 "A new event generator for the elastic scattering of charged leptons on protons" was presented [A.V. Gramolin, V.S. Fadin, A.L. Feldman, R.E. Gerasimov, D.M. Nikolenko, I.A. Rachek, D.K. Toporkov, J.Phys.G 41 (2014) 115001]

The code already contains:
— a library of proton form factors
— vacuum polarization
— complete one-loop QED
— the dependence on m_e^2/Q^2 (not complete)
— double photon exchange treatment
— etc.

Next: update higher-order effects:
— higher order leading and next-to-leading RC
— complete $\mathcal{O}(\alpha^2)$ to electron line
— tests and tuned comparisons

Conclusions

1. Application of RC in the analysis of MAMI data was discussed

2. An advanced treatment of higher order QED RC to the electron line is given

3. In particular, effects due to multiple radiation and pair emission in the LLA and NLLA are calculated

4. Vacuum polarization by hadrons should be taken into account

5. The size of the higher order effects make them relevant for the high-precision experiment

6. Higher order RC to the electron line should be combined with an advanced treatment of two-photon exchange and other relevant effects

7. Radiative corrections for the new proposed experimental set-up have to be re-considered