Study of the $\tau \to 3\pi\nu_\tau$ decay within extended RχT including tensor and scalar resonances

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Introduction

We analyze the contribution from intermediate spin-0 and spin-2 resonances to the $\tau \to 3\pi\nu_\tau$ decay by means of a chiral invariant Lagrangian incorporating these mesons. The advantage of this procedure with respect to previous analyses is that it incorporates chiral (and isospin) invariance and, hence, the partial conservation of the axial-vector current. This ensures the recovery of the right low-energy limit, described by chiral perturbation theory, and the transversality of the current in the chiral limit at all energies. Using the updated version of MC TAUOLA we have compared the theoretical predictions with the experimental distributions for both two- and three-pion invariant masses.

1. General description

- Lorenz invariant current for $\tau \to 3\pi\nu_\tau$

$$H^{3\pi}_\mu(p_1,p_2,p_3) = iP_T(q)^\alpha_{\ \mu}\left((p_1^\mu - p_3)^\mu \mathcal{F}_2(s_1,s_2,q^2) + (p_2^\mu - p_3)^\mu \mathcal{F}_1(s_1,s_2,q^2)\right)$$
$$+ iq^\alpha \mathcal{F}_P(s_1,s_2,q^2), \quad s_i = (p_j - p_k)^2 \quad q^2 = (p_1+p_2+p_3)^2$$

$F_i(s_1,s_2,q^2)$ are hadronic form-factors (FF): $F_2(s_1,s_2,q^2) = F_1(s_2,s_1,q^2)$

- Isospin symmetry relation [1]:

$$H^{--+}_\mu(p_1,p_2,p_3) = H^{00-}_\mu(p_3,p_1,p_2) + H^{00-}_\mu(p_3,p_2,p_1) \qquad (1)$$
$$\mathcal{F}_1^{--+}(s_1,s_2,q^2) = \mathcal{F}_1^{00-}(s_1,s_3,q^2) - \mathcal{F}_1^{00-}(s_2,s_3,q^2) - \mathcal{F}_1^{00-}(s_3,s_2,q^2)$$

2. Theoretical model

Resonance Chiral theory (RχT) with V, A resonanses [2]

$$\mathcal{L}_{R\chi T} = \mathcal{L}_{\chi PT} + \sum_R \mathcal{L}_R + \sum_{R,R'} \mathcal{L}_{RR'} \qquad \mathcal{L}^{(2)}_{\chi PT} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+ \rangle \qquad (2)$$
$$u^2 = \exp\{\pi^a\lambda^a/F\} \quad u_\mu = iu^\dagger(D_\mu U)u^\dagger$$

- reproduces NLO χPT (at least)
- shows appropriate high energy FF behaviour by imposing relation on the vertex constant
- V, A are described by *antisymmetric tensor field* $\epsilon^{\mu\nu}$

$\tau \to 3\pi\nu_\tau$: RχT FF $\quad \mathcal{F}_i = \mathcal{F}_i^{RR} + \mathcal{F}_i^{R} + \mathcal{F}_i^{\chi PT}$

Inclusion of scalar and tensor resonances

- Neglecting $s\bar{s}$ component: $S_{I=0} \sim u\bar{u} + d\bar{d} \qquad S = \begin{pmatrix} \frac{S_{I=0}}{\sqrt{2}} & 0 \\ 0 & -\frac{S_{I=0}}{\sqrt{2}} \end{pmatrix}$

S-resonance Lagrangian with min. derivative number

$$\mathcal{L}_S = c_d\langle Su_\mu u^\mu\rangle + c_m\langle S\chi_+\rangle, \quad \mathcal{L}_{AS} = \lambda_1^{AS}\langle\{\nabla_\mu S, A^{\mu\nu}\}u_\nu\rangle \qquad (3)$$

- Neglecting $s\bar{s}$ component: $T \sim u\bar{u} + d\bar{d} \qquad T^{\mu\nu} = \begin{pmatrix} \frac{f_2^{\mu\nu}}{\sqrt{2}} & 0 \\ 0 & \frac{f_2^{\mu\nu}}{\sqrt{2}} \end{pmatrix}$

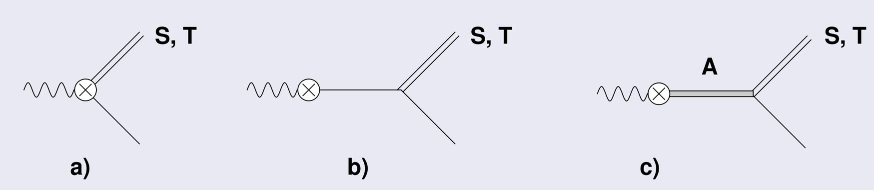
T-resonance Lagrangian with min. derivative number

$$[3]: \ \mathcal{L}_T = g_T\langle T_{\mu\nu}\{u^\mu, u^\nu\}\rangle \qquad (4)$$
$$\mathcal{L}^{(4)}_{\rm non-R} = L_1^{SD}\langle u^\mu u_\mu\rangle^2 + L_2^{SD}\langle u^\mu u^\nu\rangle\langle u_\mu u_\nu\rangle + L_3^{SD}\langle (u^\mu u_\mu)^2\rangle$$
$$\text{new:} \ \mathcal{L}_{AT} = \lambda_1^{AT}\langle\{T_{\mu\nu}, A^{\nu\alpha}\}h^\mu_\alpha\rangle + \lambda_2^{AT}\langle\{A_{\alpha\beta}, \Delta^\alpha T^{\mu\beta}\}u_\mu\rangle$$

2.1 Scalar resonance contribution

General Lorentz structure

$$\langle S_{I=0}(k)\pi^-(p)|\bar{d}\gamma^\alpha\gamma_5 u|0\rangle = -2iP_T(q)^{\alpha\nu} p_\nu \mathcal{F}^a_{S\pi}(q^2;k^2) + i\,q^\alpha \mathcal{H}^a_{S\pi}(q^2;k^2)$$



$$\mathcal{F}^a_{S\pi}(q^2;k^2) = \frac{2c_d}{F_\pi} + \frac{\sqrt{2}F_A\lambda_1^{AS}}{F_\pi}\frac{q^2}{M_A^2 - q^2}$$
$$\mathcal{H}^a_{S\pi}(q^2;k^2) = \frac{4}{F_\pi}\frac{m_\pi^2}{q^2(q^2-m_\pi^2)}\left[c_d(qp) + c_m q^2\right]$$

Three pion FF $\quad \mathcal{F}_1^{\pi^0\pi^0\pi^-}(s_1,s_2,q^2)\Big|_S = \frac{2}{3}\mathcal{F}^a_{S\pi}(q^2)\mathcal{G}_{S\pi\pi}(s_3)$

$\mathcal{F}_1^{\pi^-\pi^-\pi^+}(s_1,s_2,q^2)\Big|_S$: (1) $\to \mathcal{F}_1^{\pi^-\pi^-\pi^+}(s_1,s_2,q^2)\Big|_S$ with the $S \to \pi\pi$ propagation factor $\mathcal{G}_{S\pi\pi}(s) = \frac{\sqrt{2}}{F^2}\frac{c_d(s-2m_\pi^2)+2c_m m_\pi^2}{M_s^2-s}$

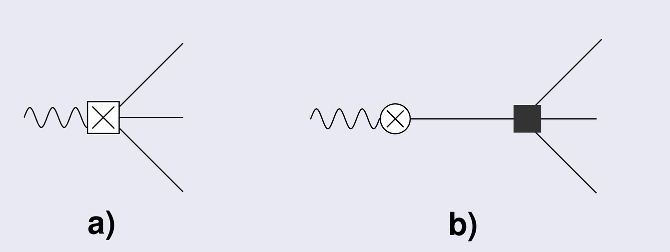
- high energy constraint $\mathcal{F}^a_{S\pi}(q^2) \longrightarrow 0$ for $q^2 \to \infty$: $F_A\lambda_1^{AS} = \sqrt{2}c_d$
- σ-f_0 splitting $\frac{1}{M_S^2 - s} \to \frac{\sin^2\phi_S}{M_{f_0}^2 - s - iM_{f_0}\Gamma_{f_0}(s)} + \frac{\cos^2\phi_S}{M_\sigma^2 - s - c_\sigma s^k \overline{B}_0(s,m_\pi^2,m_\pi^2)}$
- widths of the S-resonances: $f_0(980)$ the Flatte form $\Gamma(f_0)(s) = \frac{c_{f_0}}{16\pi M_{f_0}}\sqrt{1 - \frac{4m_K^2}{s}}$

$\sigma(f_0(500))$ the two-point subtracted Feyman integral ($\overline{B}_0(0,m_\pi^2,m_\pi^2) = 0$)

$$\overline{B}_0(s,m_\pi^2,m_\pi^2) = \frac{1}{16\pi^2}\left[2 - \rho(s)\ln\left|\frac{\rho(s)+1}{1-\rho(s)}\right| + i\pi\rho(s)\theta(s-4m_\pi^2)\right]$$

2.2 Tensor resonance contribution

General Lorentz structure $\langle f_2(k,\epsilon)\,\pi^-(p_3)|\,\bar{d}\gamma^\alpha\gamma_5 u\,|0\rangle = \epsilon^*_{\mu\nu}H^{\alpha,\,\mu\nu}_{T\pi} =$
$i\,\epsilon^*_{\mu\nu}\left[P_T(q)^{\alpha\rho}\,p_3^\nu\,\left(g^\mu_\rho\,\mathcal{F}^a_{T\pi}(q^2;k^2) + p_{3\,\rho}p_3^\mu\,\mathcal{G}^a_{T\pi}(q^2;k^2)\right) + p_3^\mu p_3^\nu q^\alpha\,\mathcal{H}^a_{T\pi}(q^2;k^2)\right]$,



$$\mathcal{F}^a_{T\pi}(q^2;k^2) = -\frac{8g_T}{F_\pi} + \frac{4\sqrt{2}F_A\lambda_1^{AT}}{F_\pi}\frac{(qp_3)}{M_A^2 - q^2} - \frac{2\sqrt{2}F_A\lambda_2^{AT}}{F_\pi}\frac{(qk)}{M_A^2 - q^2}$$
$$\mathcal{G}^a_{T\pi}(q^2;k^2) = -\frac{4\sqrt{2}F_A\lambda_1^{AT}}{F_\pi}\frac{1}{M_A^2 - q^2} - \frac{2\sqrt{2}F_A\lambda_2^{AT}}{F_\pi}\frac{1}{M_A^2 - q^2}$$
$$\mathcal{H}^a_{T\pi}(q^2;k^2) = 0$$

Three pion FF

$\mathcal{F}_1^{\pi^0\pi^0\pi^-}(s_1,s_2,q^2)\Big|_T = \mathcal{F}_{1,\,(0)}^{\pi^0\pi^0\pi^-}(s_1,s_2,q^2) + \mathcal{F}_{1,({\rm RSD})}^{\pi^0\pi^0\pi^-}(s_1,s_2,q^2) +$ (equal to 0 with (5))

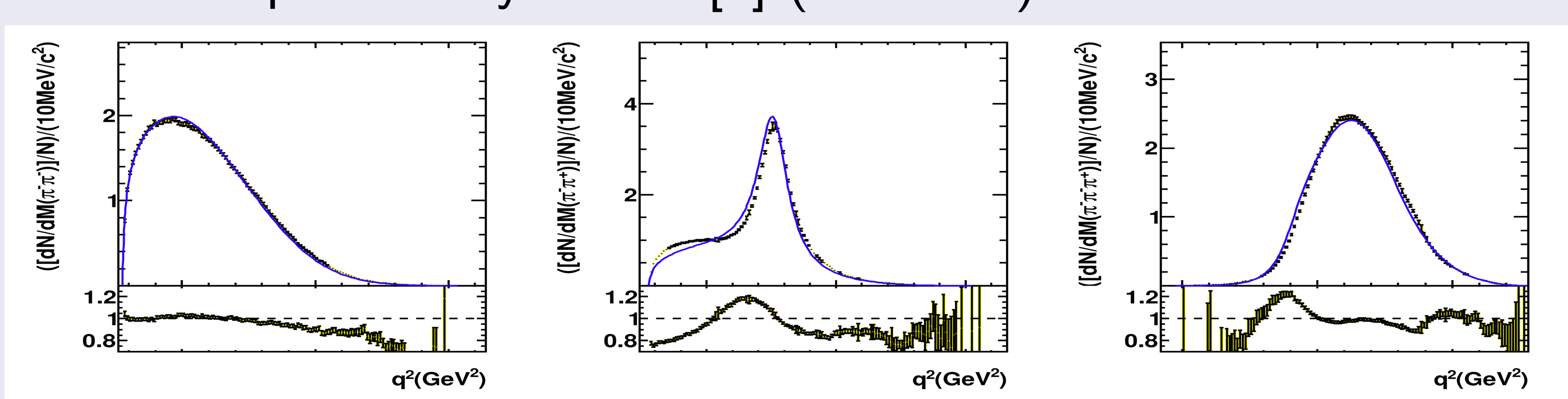
- high energy constraints $\mathcal{F}(\mathcal{G})^a_{T\pi}(q^2, M_T^2) \longrightarrow 0$ for $q^2 \to \infty$:

$$F_A\lambda_2^{AT} = -2F_A\lambda_1^{AT} = 2\sqrt{2}g_T \qquad (5)$$

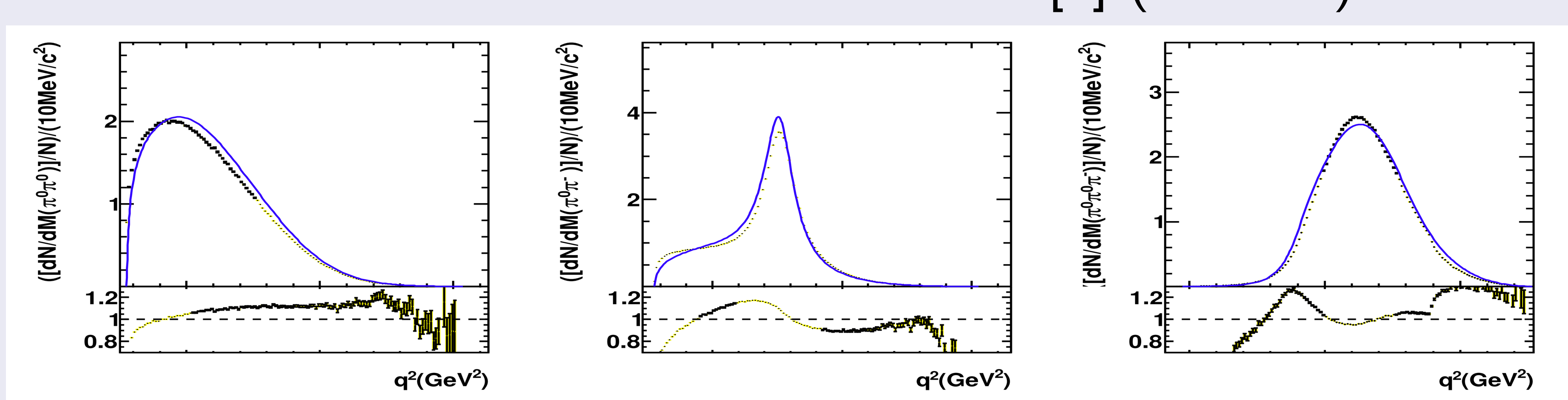
- width of $f_2(1270)$: $\frac{1}{M_T^2 - s} \to \frac{1}{M_{f_2}^2 - s - iM_{f_2}\Gamma_{f_2}(s)}$, $\Gamma_{f_2}(s) = \Gamma_0^{f_2}\frac{s^2}{M_{f_2}^4}\frac{\rho_\pi(s)^5}{\rho_\pi(M_{f_2}^2)^5}$

3. Comparison with experimental data

preliminary BaBar [4] ($\pi^-\pi^-\pi^+$)



'emulated' on the basis of Cleo current [5] ($\pi^0\pi^0\pi^-$)



V, A parameters from fit [6], $f_2(1270)$ from PDG

f_0 and σ estimation on the basis of $s^{pole}_{f_0} = (900 - i70/2)^2$MeV^2, $s^{pole}_\sigma = (441 - i544/2)^2$MeV^2

M_ρ	$M_{\rho'}$	$\Gamma_{\rho'}$	M_{a_1}	F_V	F_A	β_ρ	M_σ	M_{f_0}	c_{f_0}	M_{f_2}	Γ_{f_2}	F_π	g_T	c_d	c_σ
0.772	1.35	0.448	1.10	0.168	0.131	−0.32	0.8064	1.024	$17.7M_{f_0}^2$	1.275	0.185	0.0922	0.028	0.026	76.12

4. Comparison with other results. Summary and plans

$\tau \to 3\pi\nu$ including S- and T-resonance contributions. Hadronic FFs reproduce high energy and low energy χPT behaviour, satisfy isospin symetry relations.

- [5] S, T contributions coincide with our in the resonance regions; T-contribution has a substhreshold singularity at $s_3 = 0$ and does not reproduce χPT
- [6] S included phenomenologically: isospin symmetry relation broken, no T
- [7] $\tau \to T\pi\nu$: fulfills the transversality condition only for a_1 on-shell; coincides with our only in chiral limit and a_1 on-shell

Plans:

- Fit to the BaBar/Cleo data
- $\sigma(500)$ parametrization study
- Study of T-resonances in $e^+e^- \to \pi\pi\eta$ and $\gamma^*\gamma^* \to T$

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