Quark mass dependence in $\gamma\pi \to \pi\pi$

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Motivation

- lattice QCD calculations of scattering processes carried out at unphysical high pion masses (e.g. $M_\pi = 400$ MeV)
- need to extrapolate *a priori* lattice data to physical point
- ChPT and dispersion relations allow for extrapolation

process $\gamma\pi \to \pi\pi$ linked to
- $g-2$ of μ
- the Wess-Zumino-Witten anomaly: $\mathcal{F}(0,0,0) = \frac{eN_c}{12\pi^2 F_\pi^3}$

complications:
- ρ resonance $\Rightarrow$ ChPT alone not sufficient
- unitarity relation of $\gamma\pi \to \pi\pi$ does not allow for IAM
- describe also virtual photons
  - currently: $q^2 < (3M_\pi)^2$
  - extension beyond $(3M_\pi)^2$ in principle possible

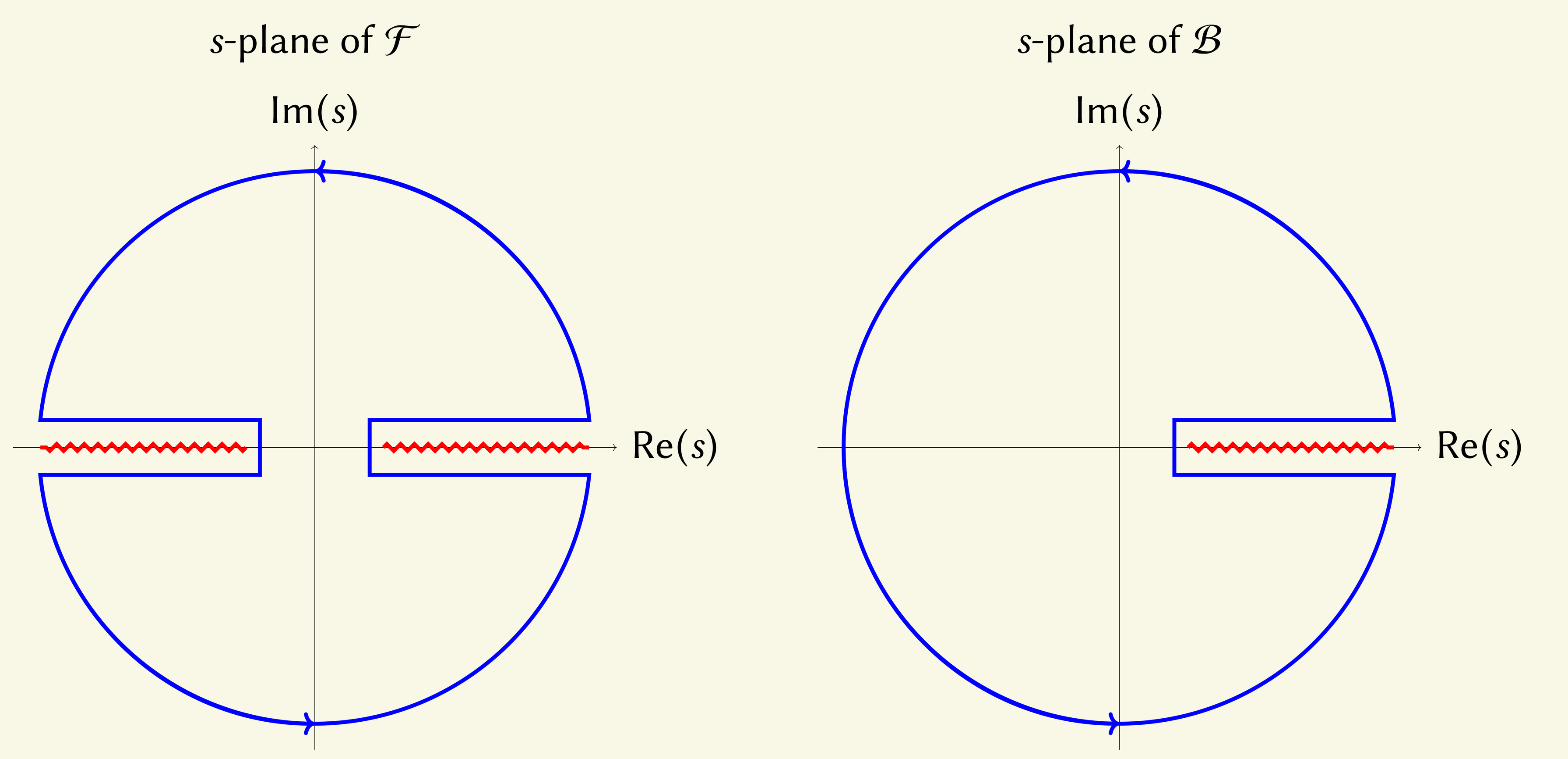
Starting point: $\pi\pi \to \pi\pi$ and $\gamma\pi \to \pi\pi$

- include intermediate $\pi\pi$ states/interactions
- unitarity ($SS^\dagger = \mathbb{1}$) links $\gamma\pi \to \pi\pi$ to $\pi\pi \to \pi\pi$ via *Watson's theorem*:

$$\underbrace{\arg\left[f_1(s)\right]}_{\text{phase of lowest partial wave of } \gamma\pi\to\pi\pi} = \underbrace{\delta(s)}_{\pi\pi\to\pi\pi\ i=j=1 \text{ phase shift}}$$

- $\Rightarrow \pi\pi$ scattering serves as input for $\gamma\pi$ scattering

Dispersive treatment of $\gamma(q)\pi^-(p_1) \to \pi^-(p_2)\pi^0(p_0)$



scalar part $\mathcal{F}$ of scattering amplitude $\mathcal{M}(s,t,u) = i\epsilon_{\mu\nu\alpha\beta}\epsilon^\mu p_1^\nu p_2^\alpha p_0^\beta \mathcal{F}(s,t,u)$:
- has right- and left-hand cut
- fulfils fixed-t dispersion relation

simplify problem:
- assume isospin symmetry
- ignore Im(f_j) for partial waves with $j \geq 3$
- $\Rightarrow$ *reconstruction theorem*

$$\mathcal{F}(s,t,u) = \mathcal{B}(s) + \mathcal{B}(t) + \mathcal{B}(u)$$

investigate $\mathcal{B}$:
- has only right-hand cut
- fulfils simpler dispersion relation (Ω = Omnès function)

$$\mathcal{B}(s) = \Omega(s)\left\{c_1 \frac{1-\dot{\Omega}(0)\,s}{3} + c_2\frac{s}{3} + \frac{s^2}{\pi}\int_{4M_\pi^2}^{\infty} \frac{\hat{\mathcal{B}}(x)\sin\left[\delta(x)\right]}{x^2(x-s)\left|\Omega(x)\right|}\mathrm{d}x\right\}$$

$$\hat{\mathcal{B}}(s) = \frac{3}{2}\int_{-1}^{1}\left(1-z^2\right)\mathcal{B}(t(z))\,\mathrm{d}z$$

numerical treatment:
- use linearity of dispersion relation to write

$$\mathcal{B}(s) = c_1\mathcal{B}_1(s) + c_2\mathcal{B}_2(s)$$

- basis functions $\mathcal{B}_i$ independent of subtraction constants c_i
- solve dispersion relation for basis functions e.g. iteratively

M_π dependence of $\gamma\pi \to \pi\pi$

- subtraction constants c_i: matching to ChPT
- $\pi\pi \to \pi\pi$ phase δ: IAM
- dispersion relation (integration limit and t)

References

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Inverse amplitude method (IAM) for $\pi\pi \to \pi\pi$

$i = j = 1$ partial wave t with phase δ:
- only identical particles involved $\Rightarrow$ simple unitarity relation:

$$\mathrm{Im}\left[t(s)\right] = \sigma(s)\left|t(s)\right|^2 \qquad \Rightarrow \qquad \mathrm{Im}\left[\frac{1}{t(s)}\right] = -\sigma(s) = -\sqrt{1-\frac{4M_\pi^2}{s}}$$

- ChPT expansion $t(s) = t_2(s) + t_4(s) + \ldots$ satisfies unitarity only perturbatively, but:

$$\mathrm{Im}\left[\frac{t_4(s)}{t_2^2(s)}\right] = \sigma(s) = -\mathrm{Im}\left[\frac{1}{t(s)}\right]$$

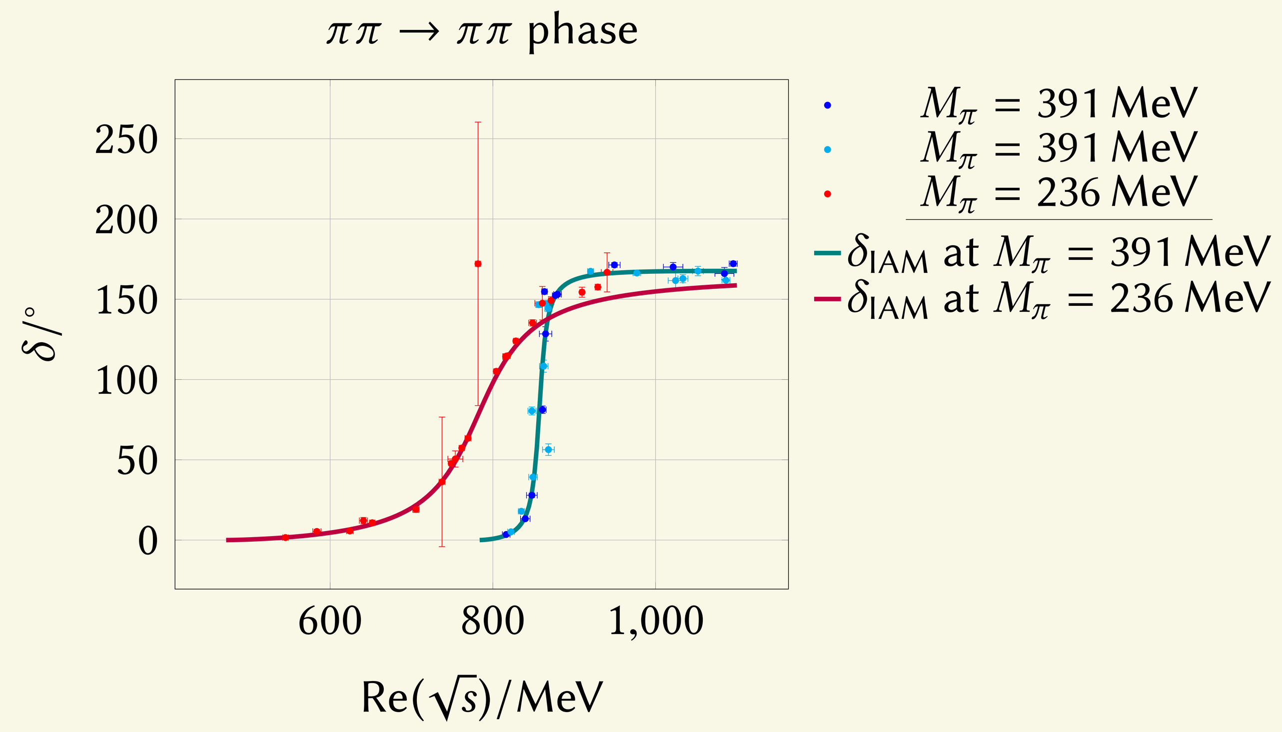
- use fixed-t dispersion relation to obtain

$$t_{\mathrm{IAM}}(s) = \frac{t_2^2(s)}{t_2(s) - t_4(s)}$$

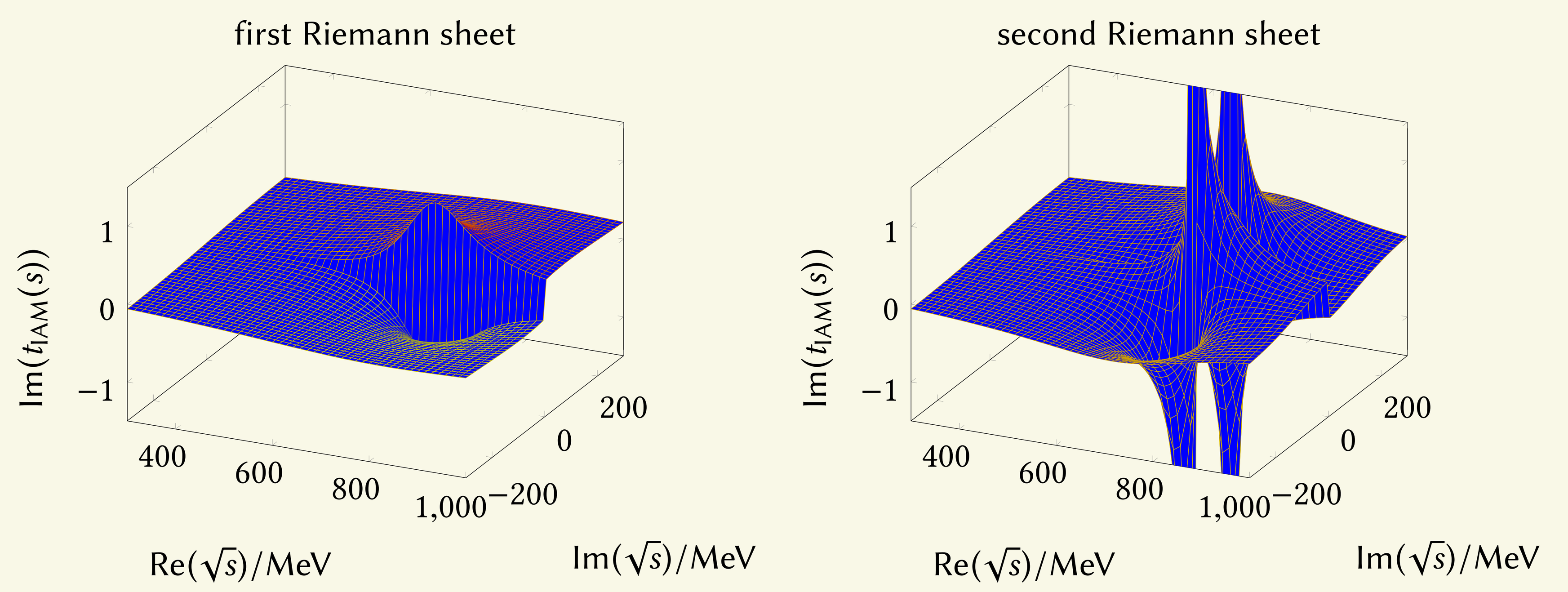
t_{IAM} depends on
- low energy constant (LEC) $\bar{l}_2 - \bar{l}_1$
- M_π

IAM and lattice QCD

fit IAM at unphysical pion mass to lattice data to fix LEC



ρ resonance

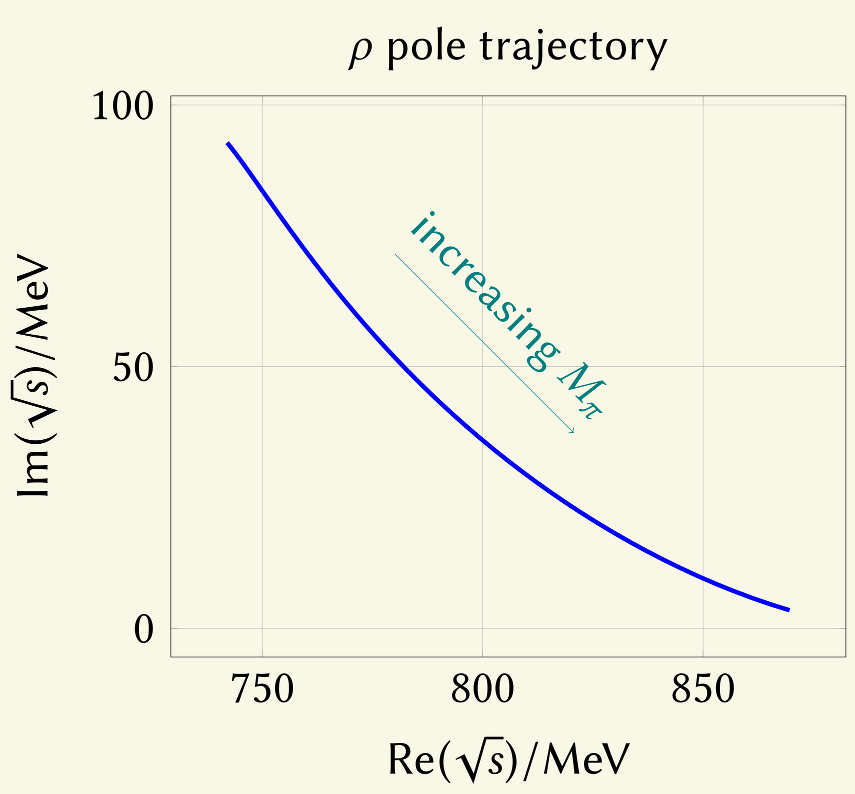


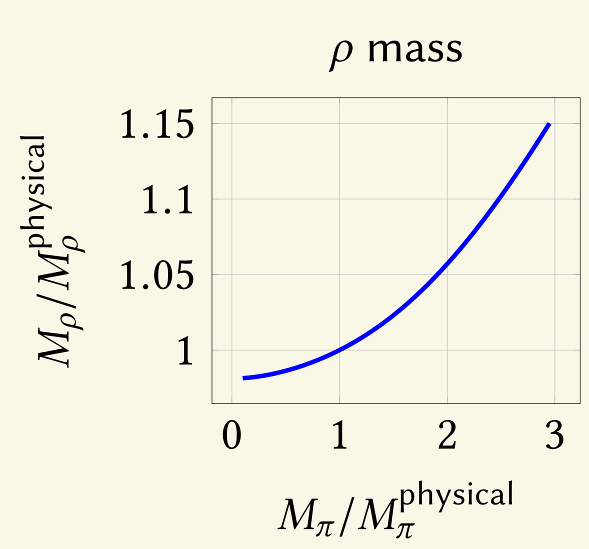
- t_{IAM} shows correct analytic properties, e.g. ρ pole on second Riemann sheet
- pole position $\mathcal{P}_\pm$ associated with mass M_ρ and width Γ_ρ of resonance:

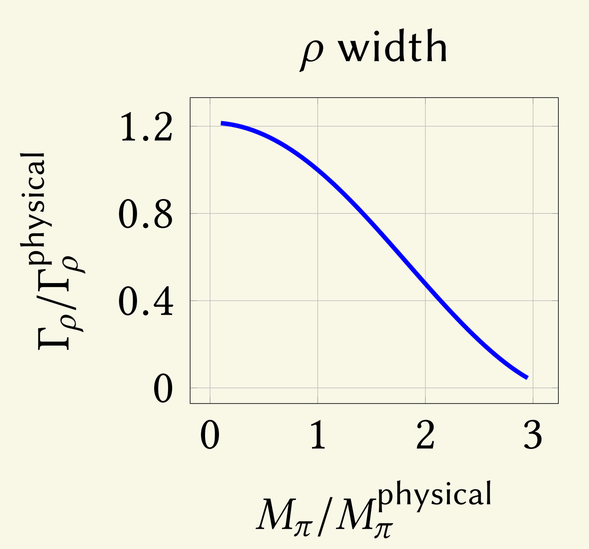
$$\mathcal{P}_\pm = M_\rho \pm i\frac{\Gamma_\rho}{2}$$

M_π dependence of $\pi\pi \to \pi\pi$ with IAM

- with LEC fixed, vary M_π to investigate mass dependence
- observe movement of ρ pole on second sheet







Summary of workflow

1. fit IAM to lattice data for $\pi\pi \to \pi\pi$ to fix δ [in progress]
2. calculate basis functions with δ [in progress]
3. fix subtraction constants by fit to lattice data for $\gamma\pi \to \pi\pi$ [to be done]
4. use i.a. matching to ChPT to extrapolate to physical point [to be done]