Meson spectroscopy from lattice calculations

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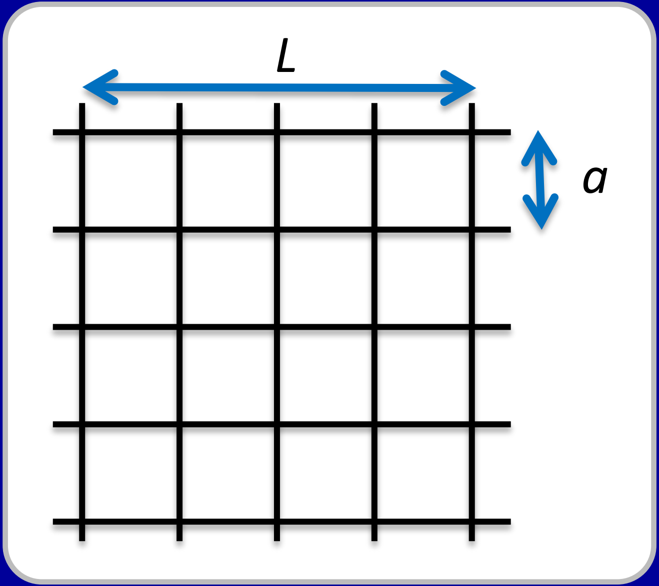
UNIVERSITY OF CAMBRIDGE

Hadron Spectrum Collaboration

Lattice QCD Spectroscopy

Systematically-improvable first-principles calculations



- **Discretise** spacetime in a **finite volume**
- Compute correlation fns. numerically (Euclidean time, $t \rightarrow i\,t$)

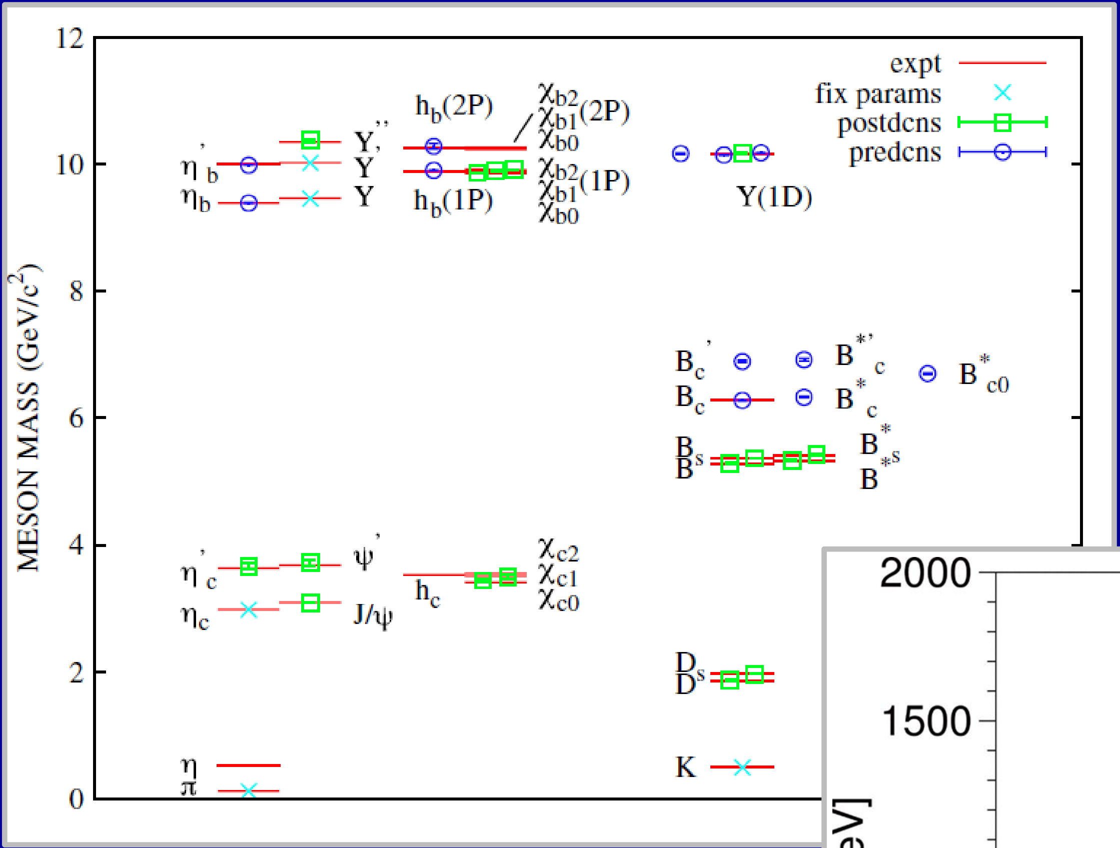
Note:

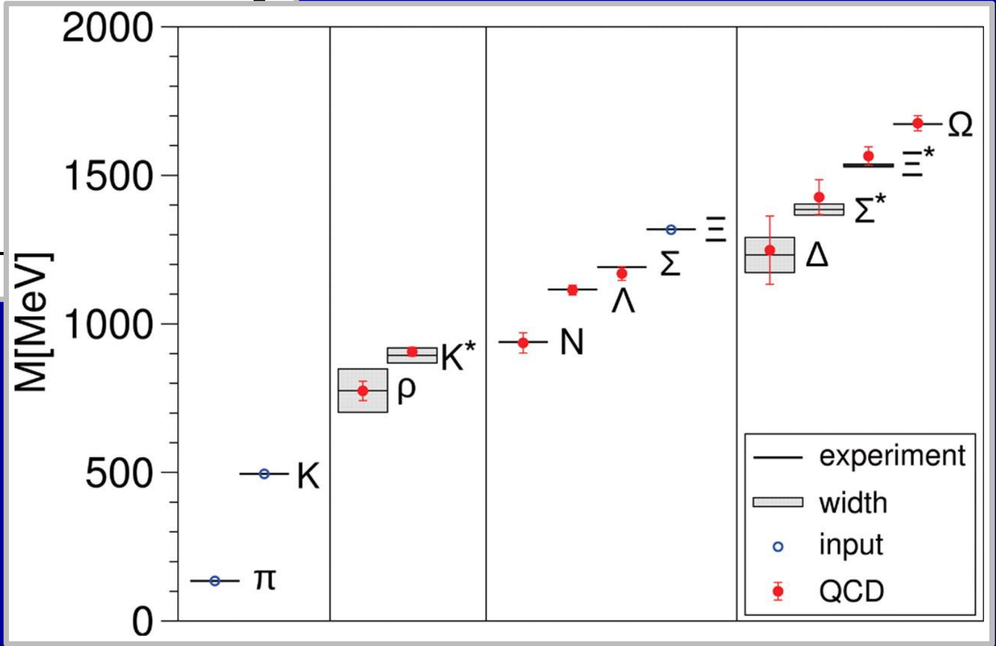
- Finite a and L
- Possibly heavy u, d ($\rightarrow$ unphysical m_π)

Finite-volume energy eigenstates from:

$$C_{ij}(t) = \left\langle 0 \left| \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0) \right| 0 \right\rangle$$

Lower-lying mesons (and baryons)

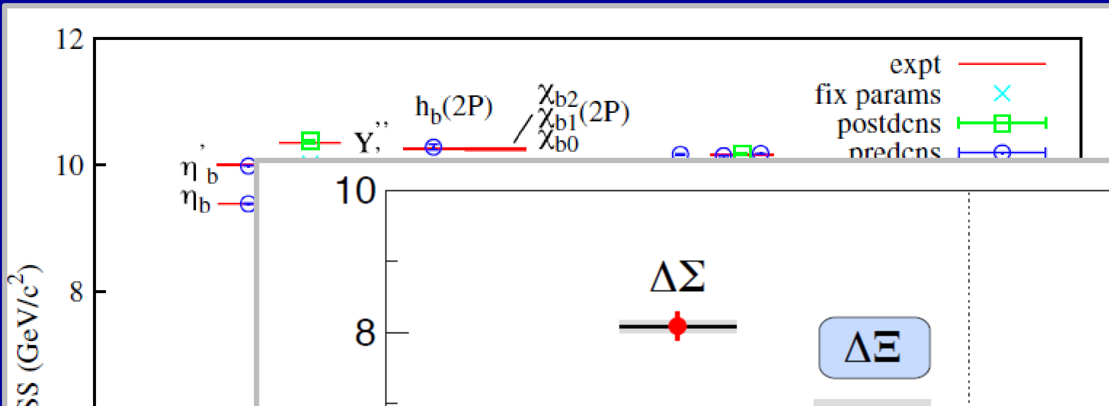




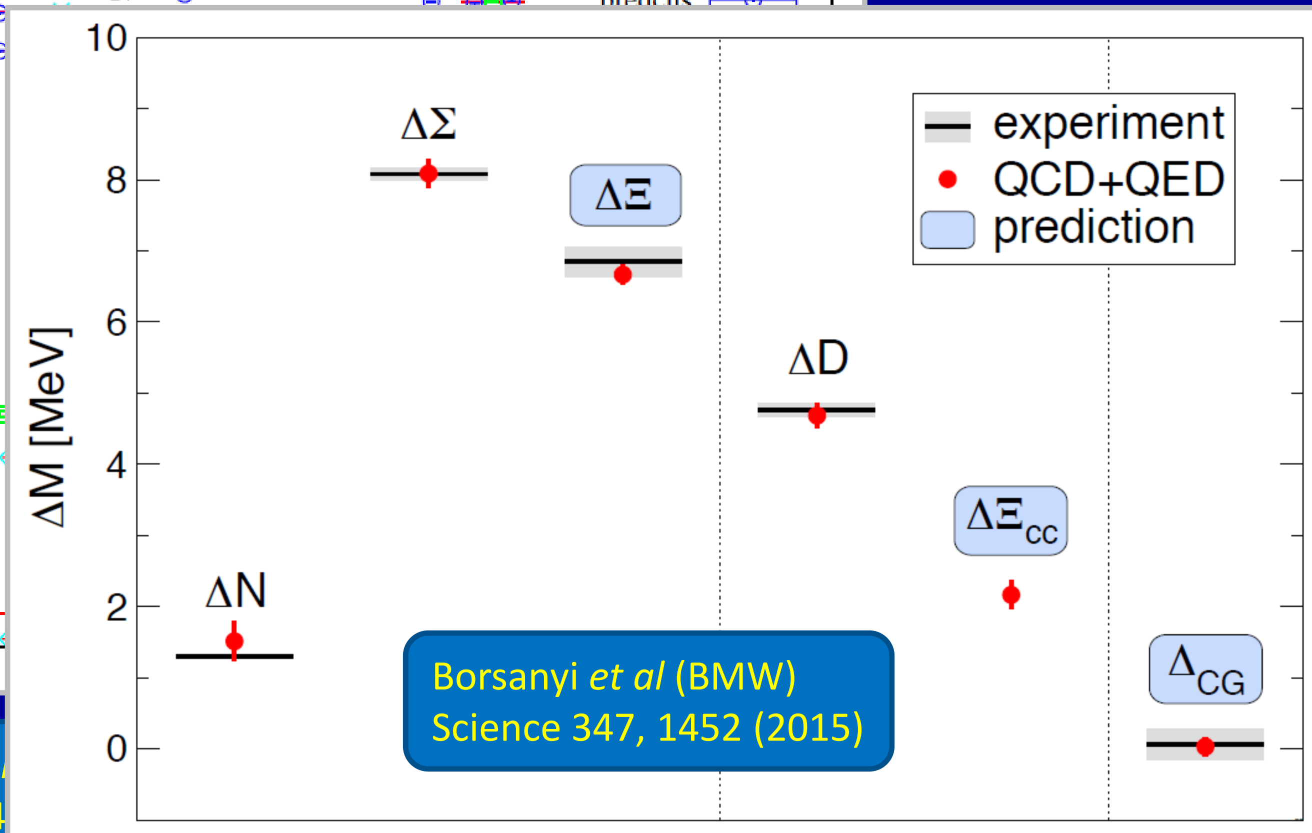
Durr *et al* (BMW Collaboration)
[Science 322, 1224 (2008)]

Dowdall *et al* (HPQCD)
[PR D86, 094510 (2012)]

Lower-lying mesons (and baryons)

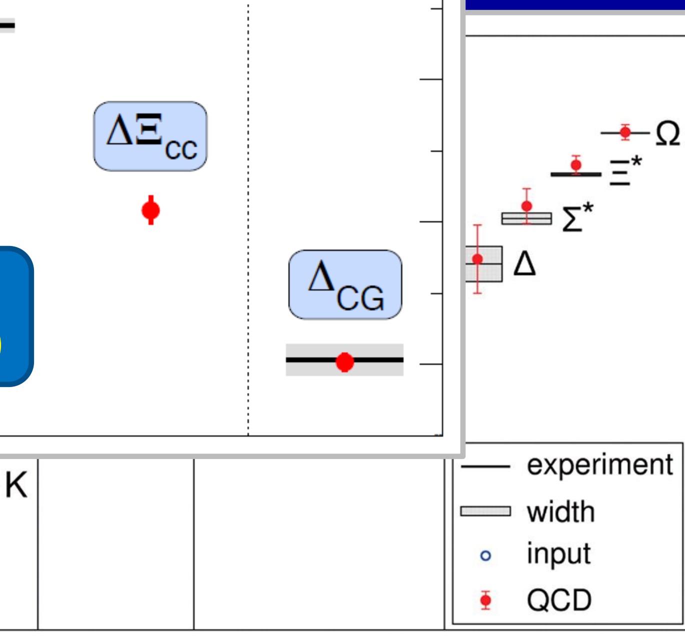


Dowdall *et al*
[PR D86, 094

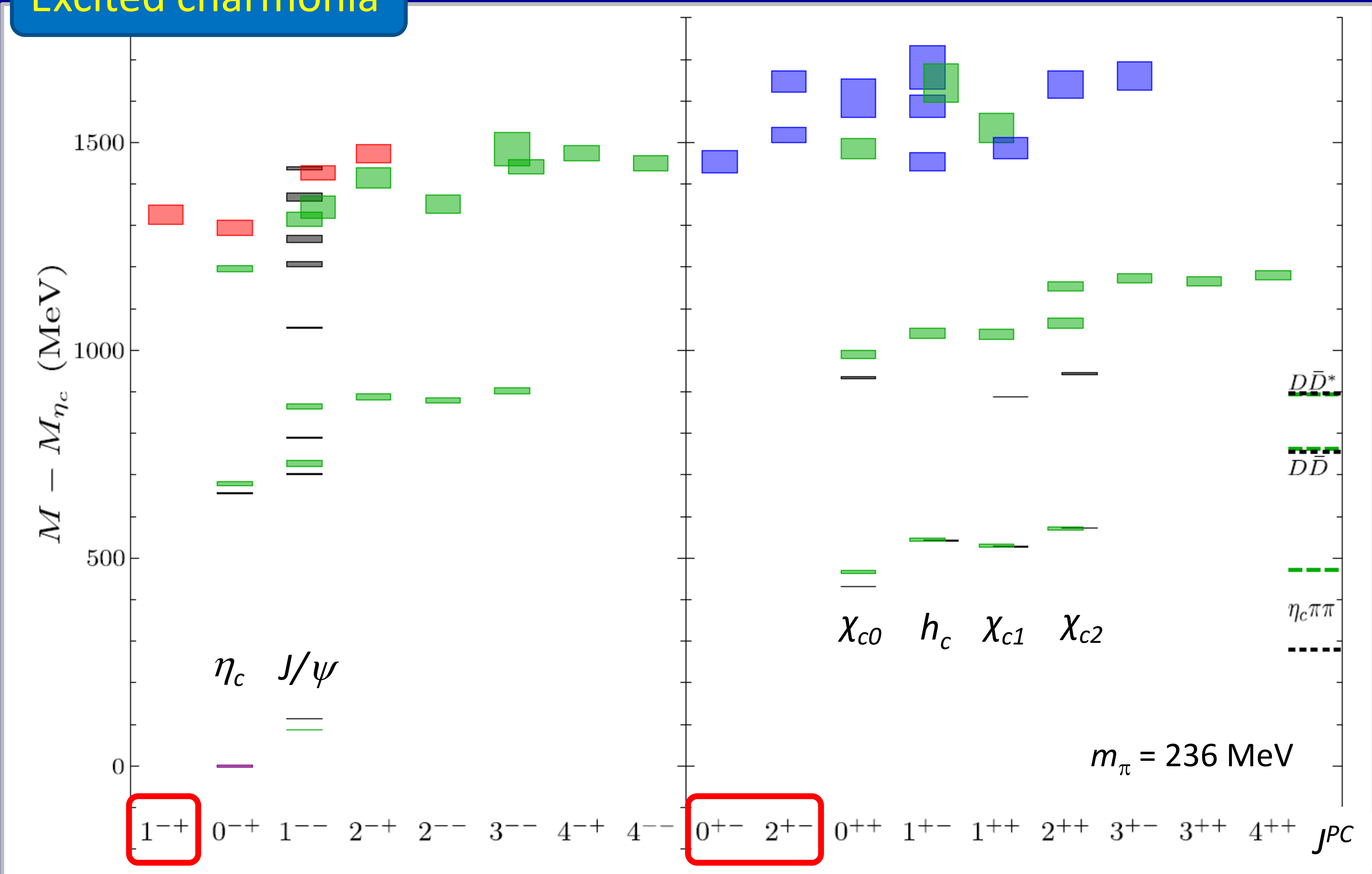


Borsanyi *et al* (BMW)
Science 347, 1452 (2015)

llaboration)
(2008)]



Excited charmonia



m_π = 236 MeV [Cheung *et al* (HadSpec) JHEP 12 (2016) 089], similar pattern to older m_π = 391 MeV

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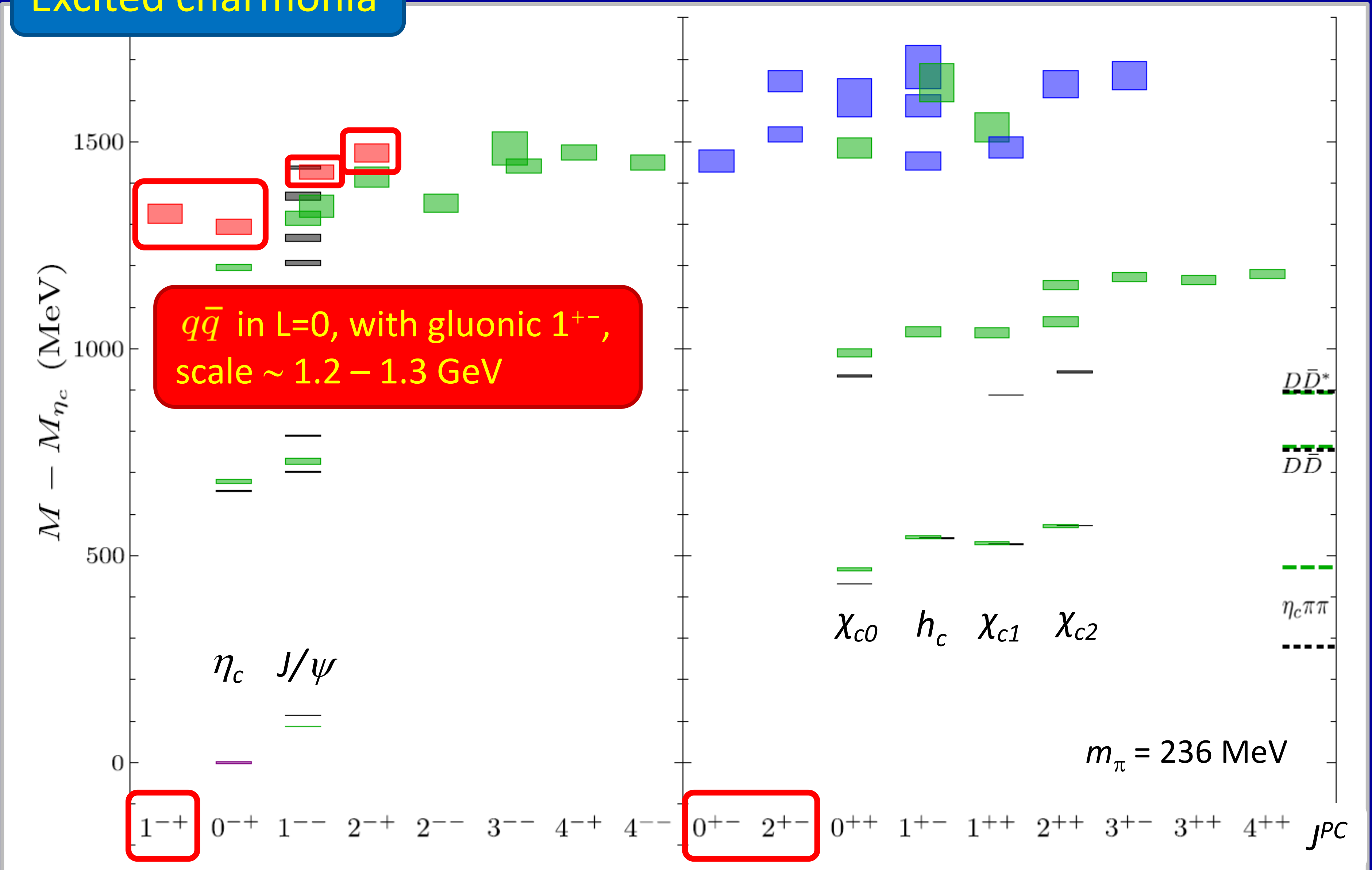

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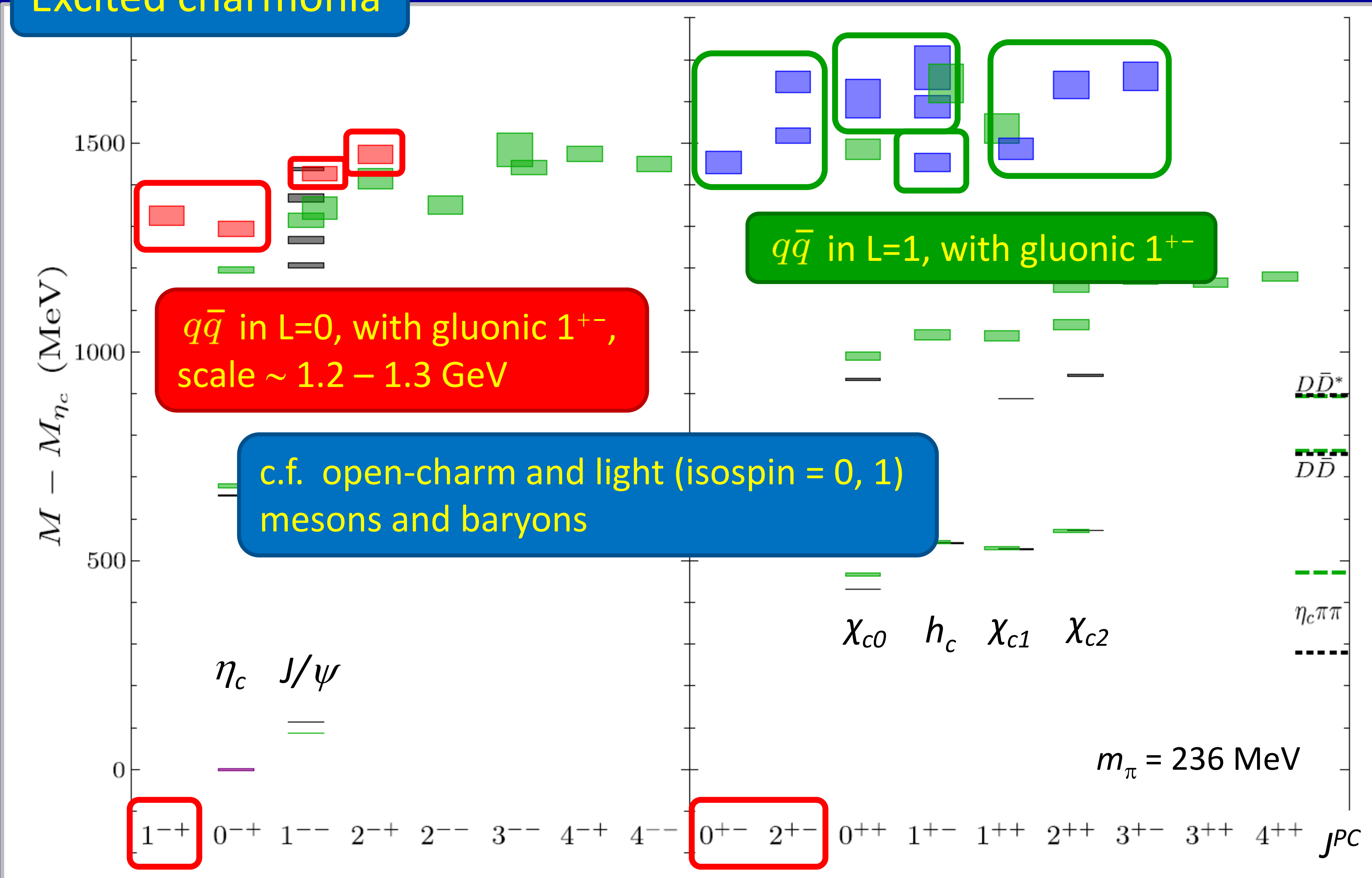

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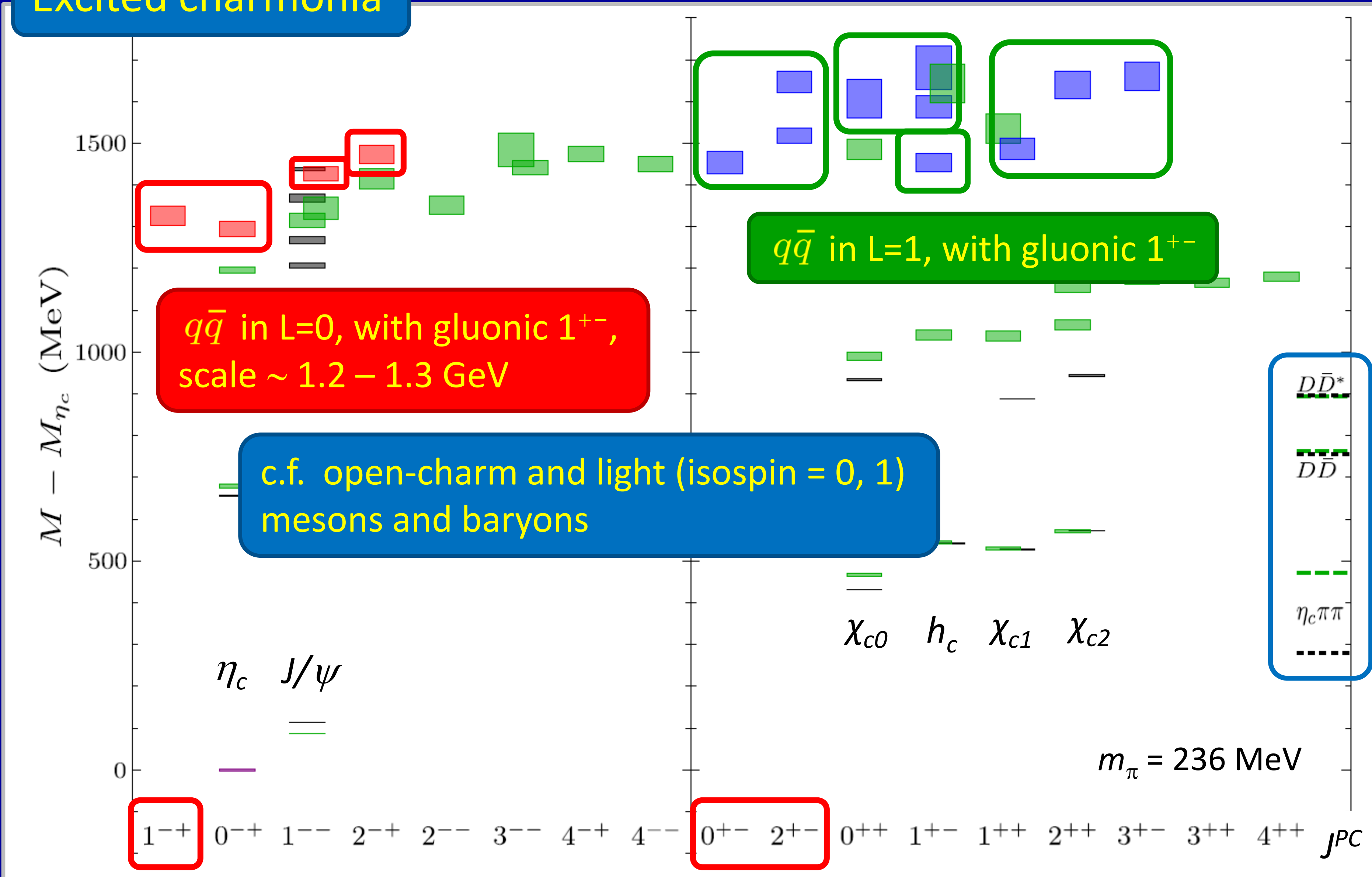


$q\bar{q}$ in L=0, with gluonic 1^{+-}, scale ~ 1.2 – 1.3 GeV

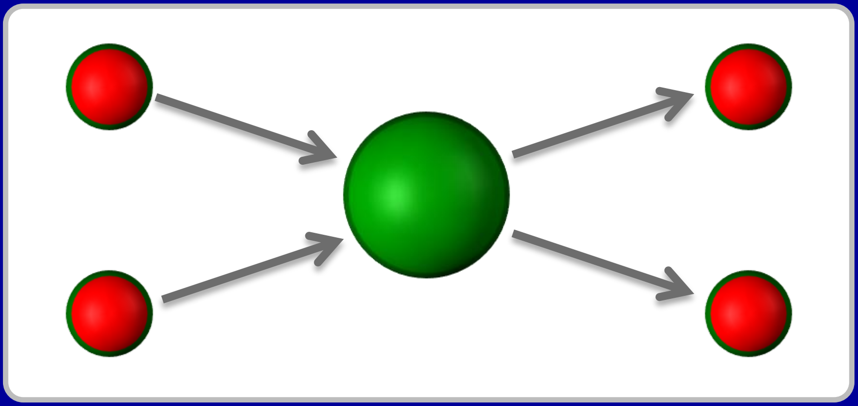
$q\bar{q}$ in L=1, with gluonic 1^{+-}

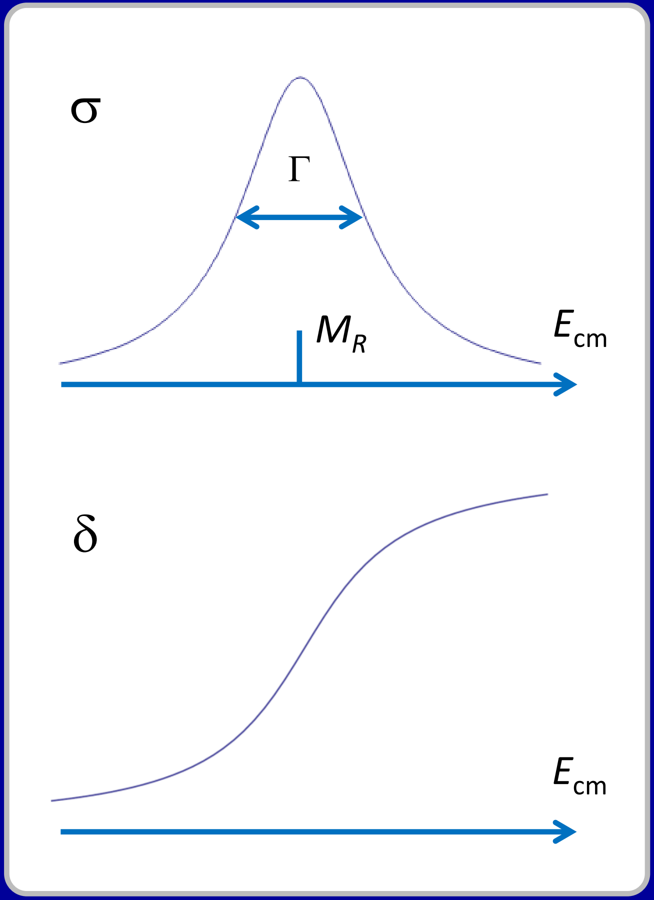
c.f. open-charm and light (isospin = 0, 1) mesons and baryons

m_π = 236 MeV [Cheung *et al* (HadSpec) JHEP 12 (2016) 089], similar pattern to older m_π = 391 MeV

Scattering and resonances

Most hadrons appear as resonances in scattering of lighter hadrons





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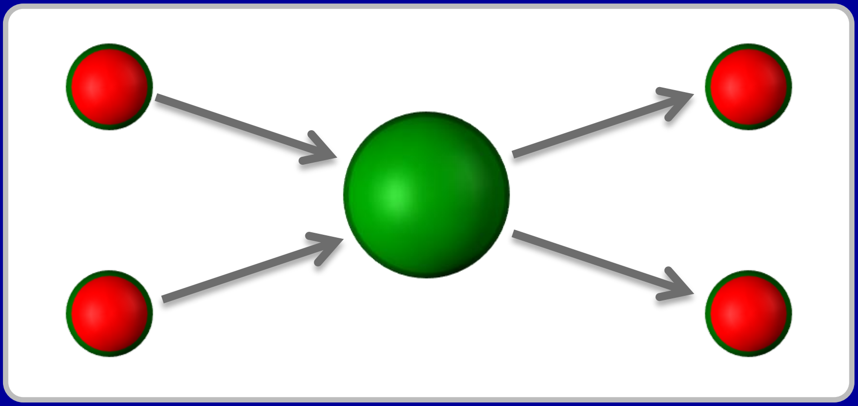
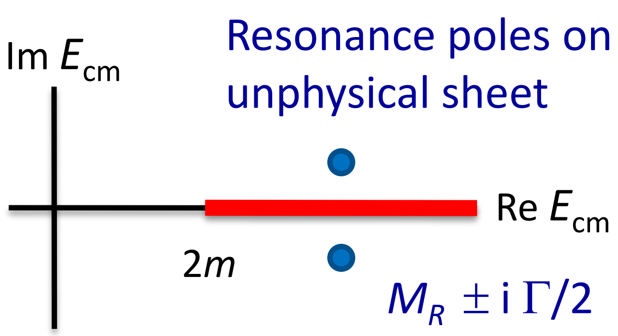
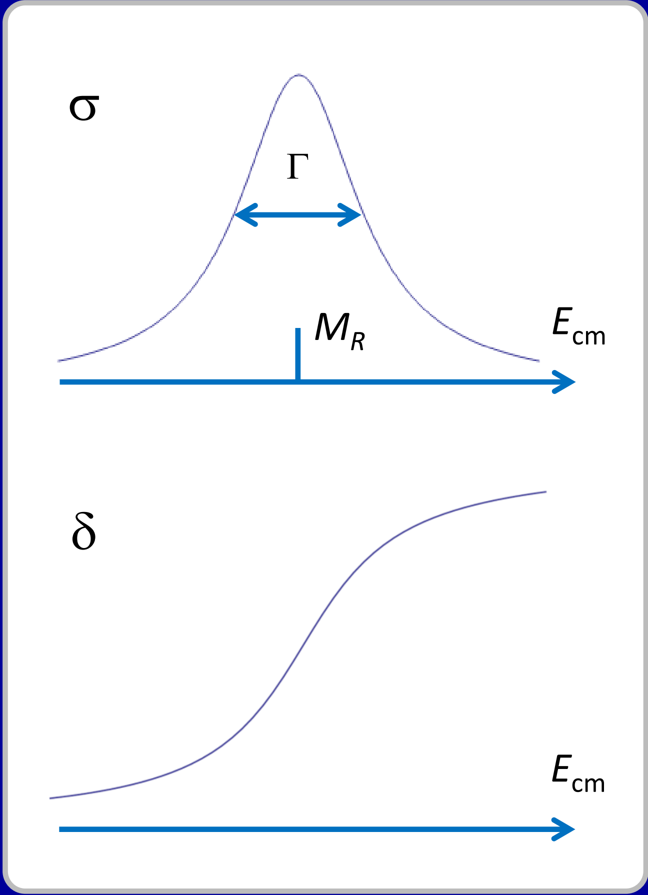


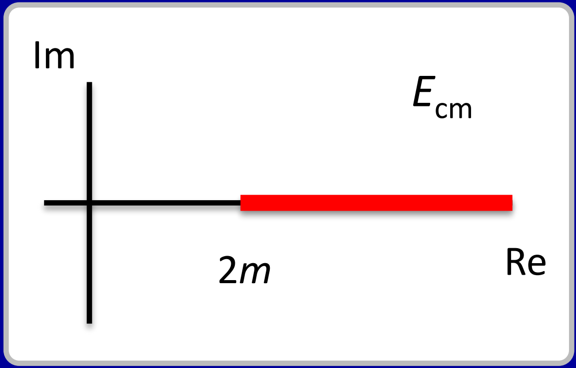
Singularity structure of scattering matrix





Scattering in Lattice QCD

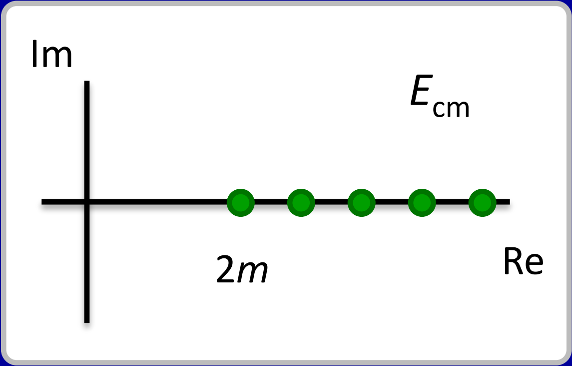
Infinite volume – contin. spectrum above thresh.

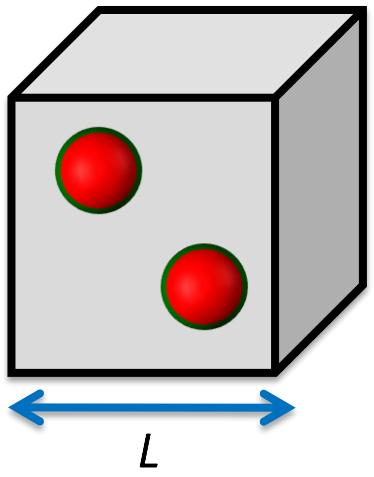


Scattering in Lattice QCD

Infinite volume – contin. spectrum above thresh.

Finite volume – discrete spectrum





Non-interacting: $\vec{k}_{A,B} = \frac{2\pi}{L}(n_x, n_y, n_z)$

Interacting: $\vec{k}_{A,B} \neq \frac{2\pi}{L}(n_x, n_y, n_z)$

[periodic b.c.s]

Scattering in Lattice QCD

Infinite volume – contin. spectrum above thresh.

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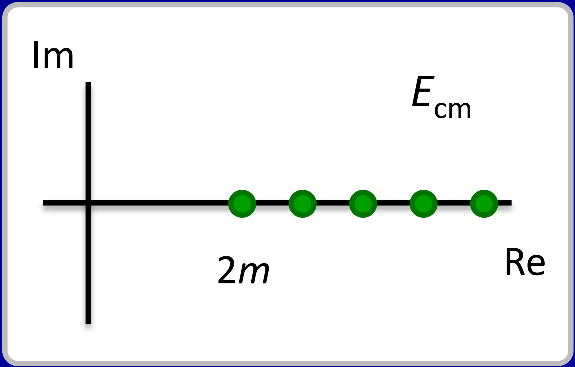

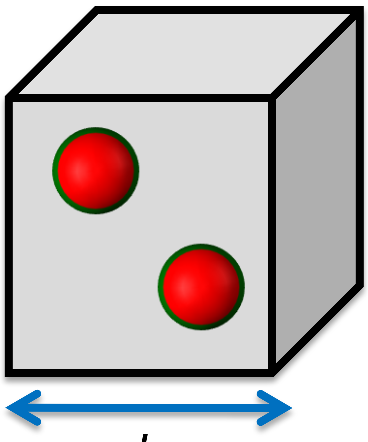

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Interacting: $\vec{k}_{A,B} \neq \frac{2\pi}{L}(n_x, n_y, n_z)$

$$\mathrm{t}(E_{\mathrm{cm}}) = \begin{pmatrix} t_{\pi\pi\to\pi\pi}(E_{\mathrm{cm}}) & t_{\pi\pi\to K\bar{K}}(E_{\mathrm{cm}}) \\ t_{K\bar{K}\to\pi\pi}(E_{\mathrm{cm}}) & t_{K\bar{K}\to K\bar{K}}(E_{\mathrm{cm}}) \end{pmatrix}$$

Lüscher method (and extensions): relate **finite-volume energy levels** to **infinite-volume scattering *t*-matrix**

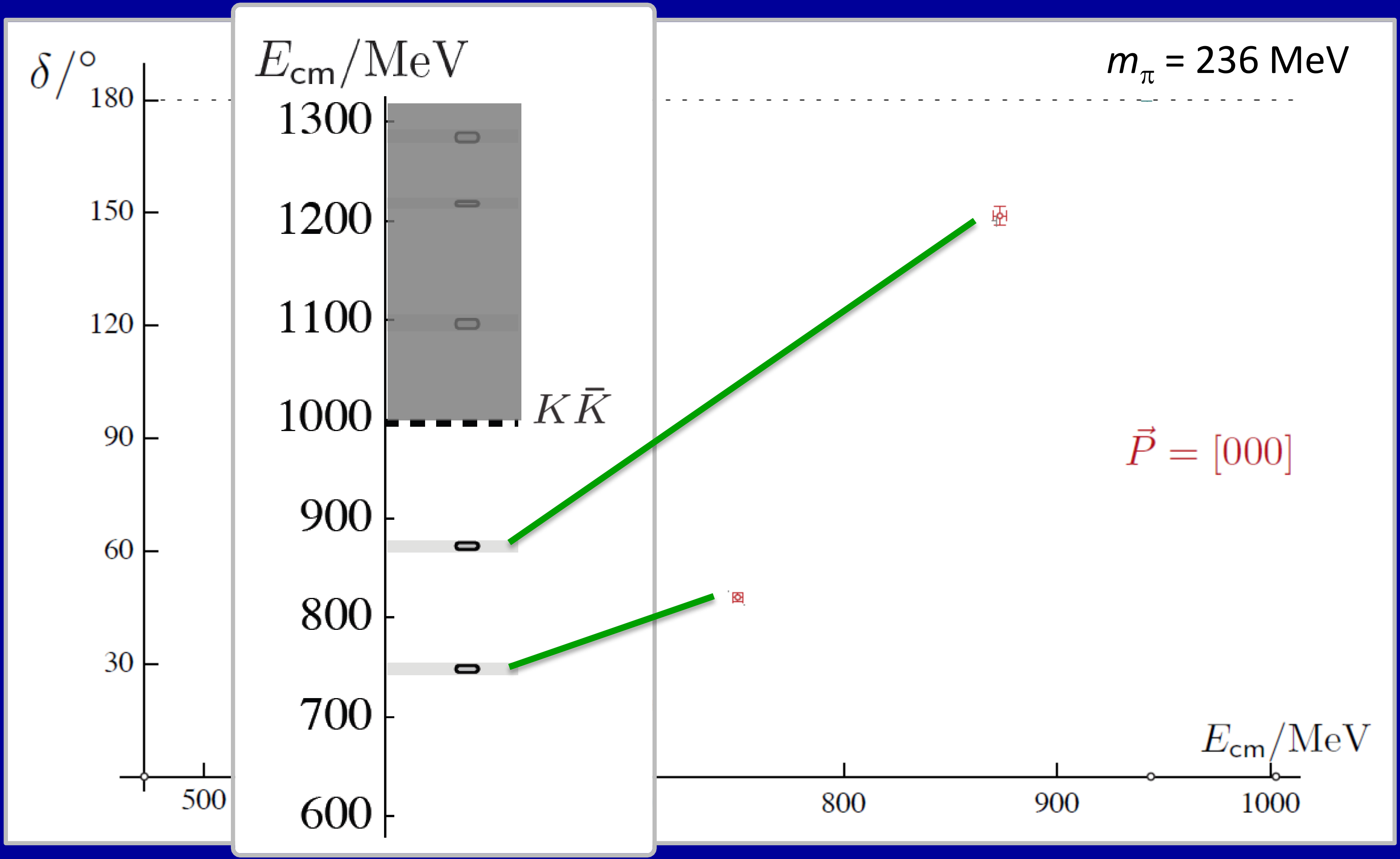
In general under-constrained problem (determinant equ. at each E_{cm})

→ parameterize E_{cm} dependence of *t*-matrix and fit $\{E_{\mathrm{lat}}\}$ to $\{E_{\mathrm{param}}\}$

Consider many different parameterizations (e.g. *K*-matrix, eff. range, B.W.)

The ρ resonance: **elastic** ππ scattering

$(J^{PC} = 1^{--}, I = 1)$



(HadSpec) [PR D87, 034505 (2013); PR D92, 094502 (2015)]

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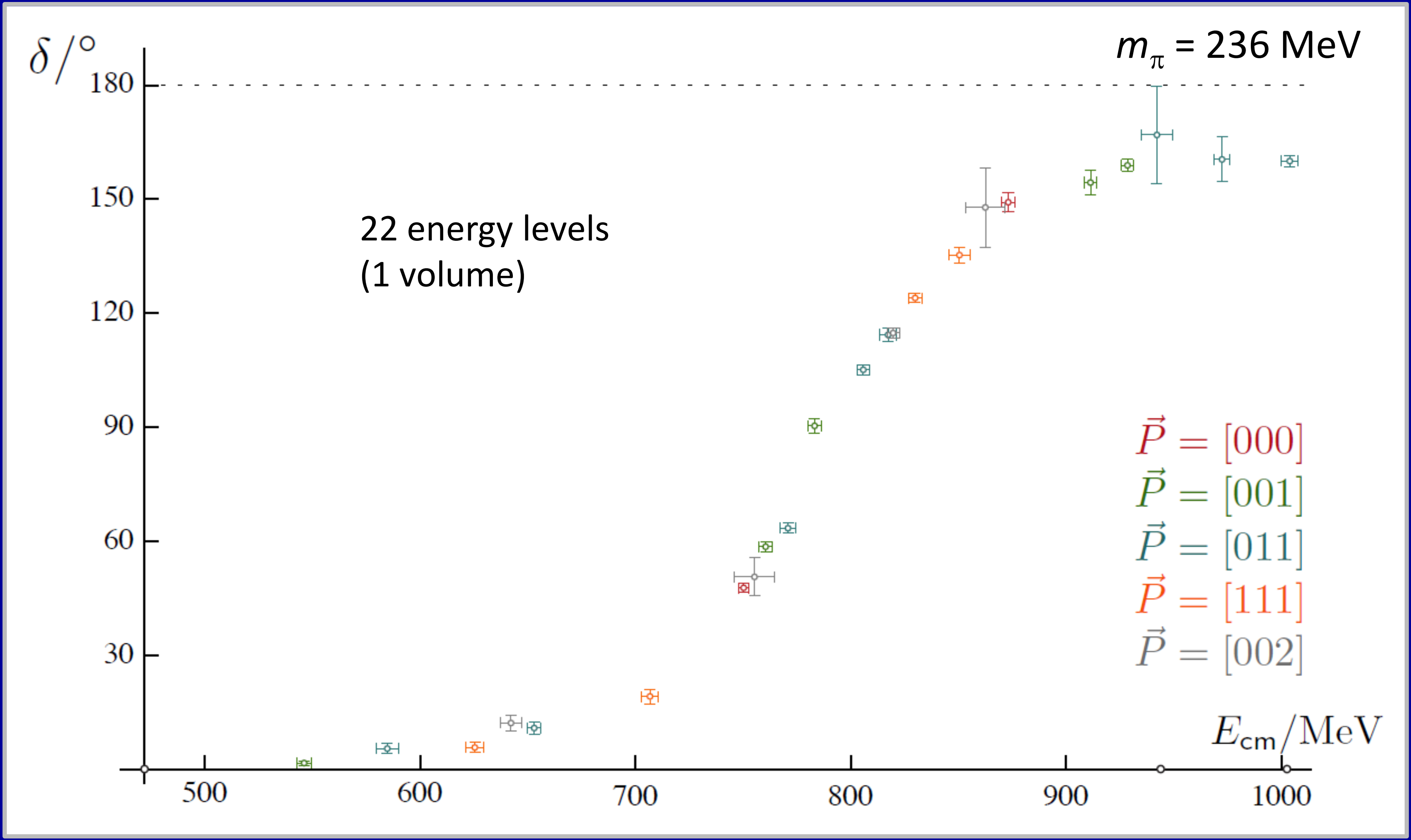

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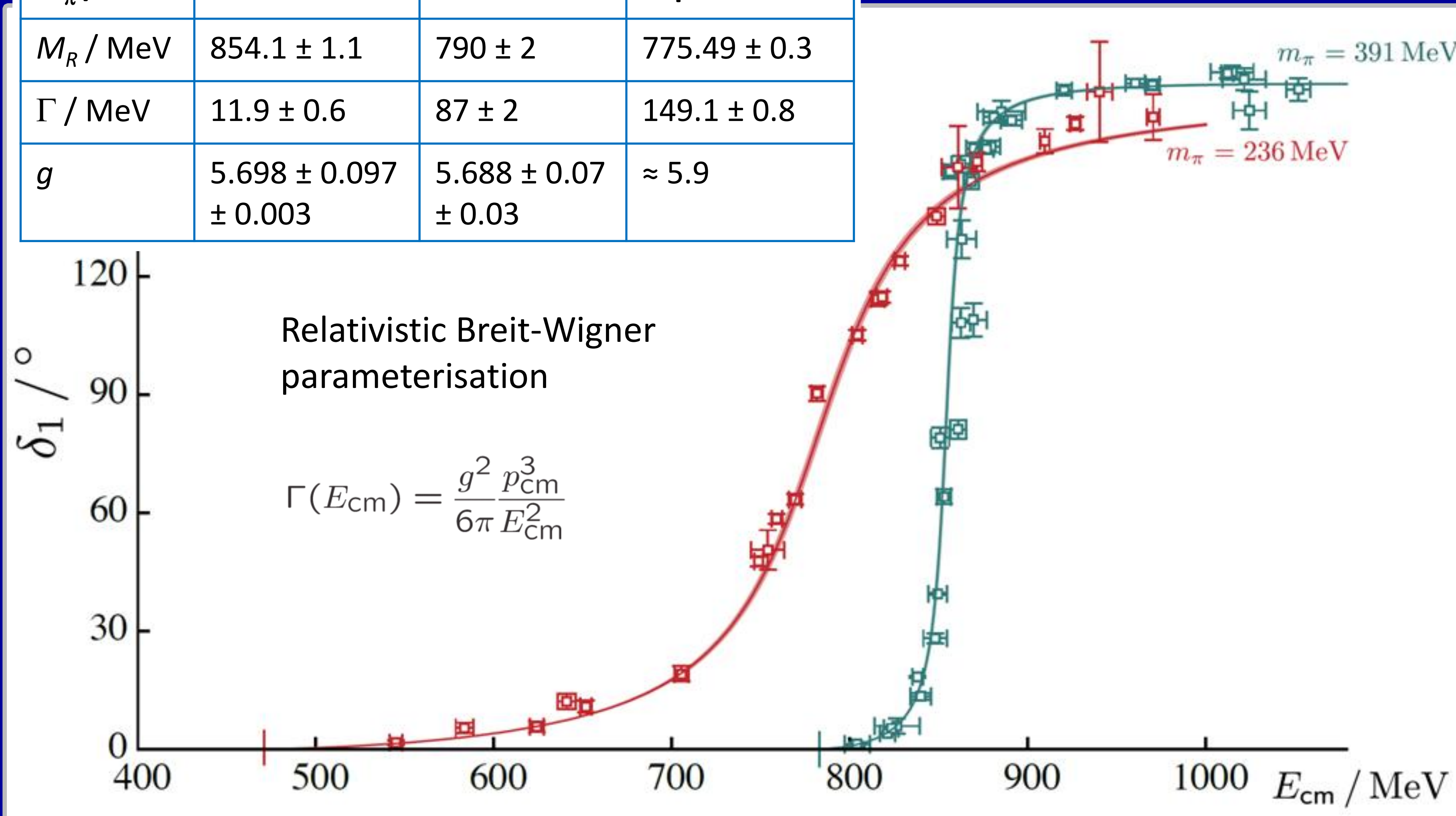
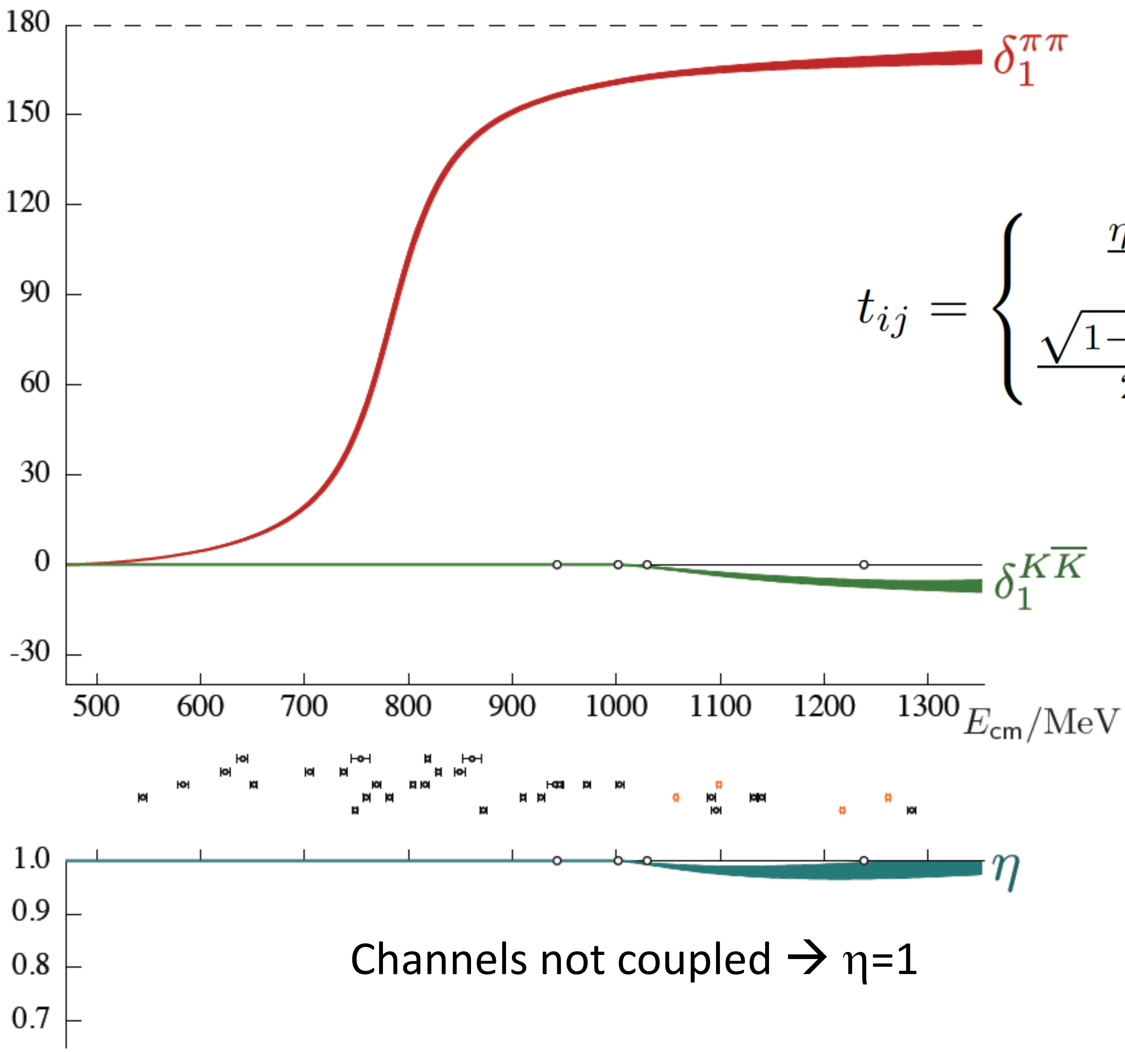
The ρ resonance: **elastic** ππ scattering

m_π / MeV	391	236	Experimental
M_R / MeV	854.1 ± 1.1	790 ± 2	775.49 ± 0.3
Γ / MeV	11.9 ± 0.6	87 ± 2	149.1 ± 0.8
g	5.698 ± 0.097 ± 0.003	5.688 ± 0.07 ± 0.03	≈ 5.9



(HadSpec) [PR D87, 034505 (2013); PR D92, 094502 (2015)]

The ρ resonance: **coupled-channel** $\pi\pi, K\bar{K}$



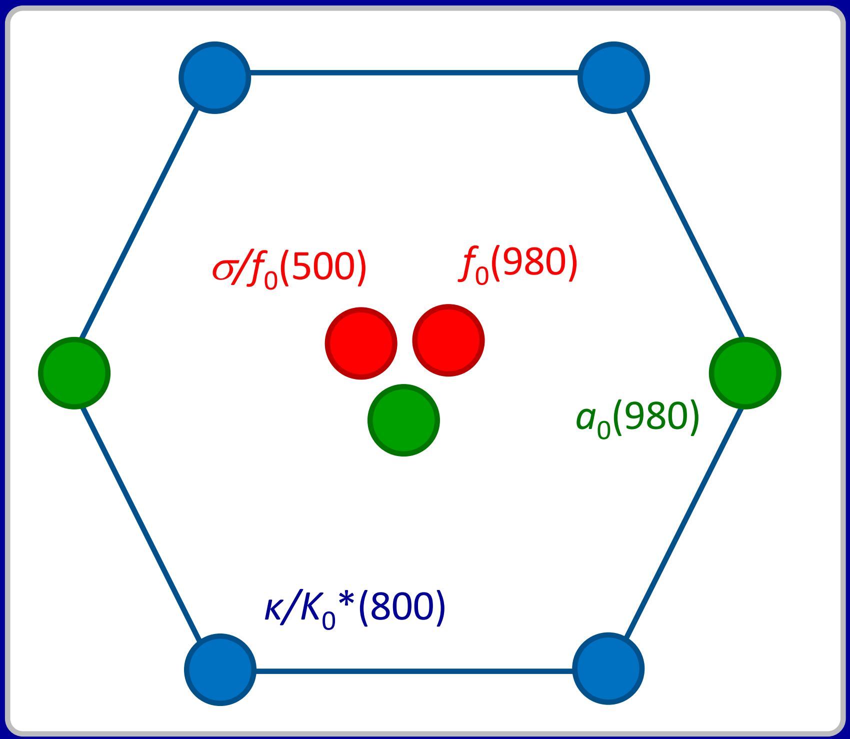
m_π = 236 MeV

$$t_{ij} = \begin{cases} \dfrac{\eta\, e^{2i\delta_i} - 1}{2i\,\rho_i} & (i = j) \\[2ex] \dfrac{\sqrt{1-\eta^2}\; e^{i(\delta_i+\delta_j)}}{2\,\sqrt{\rho_i\,\rho_j}} & (i \neq j) \end{cases}$$

34 energy levels
(1 volume)

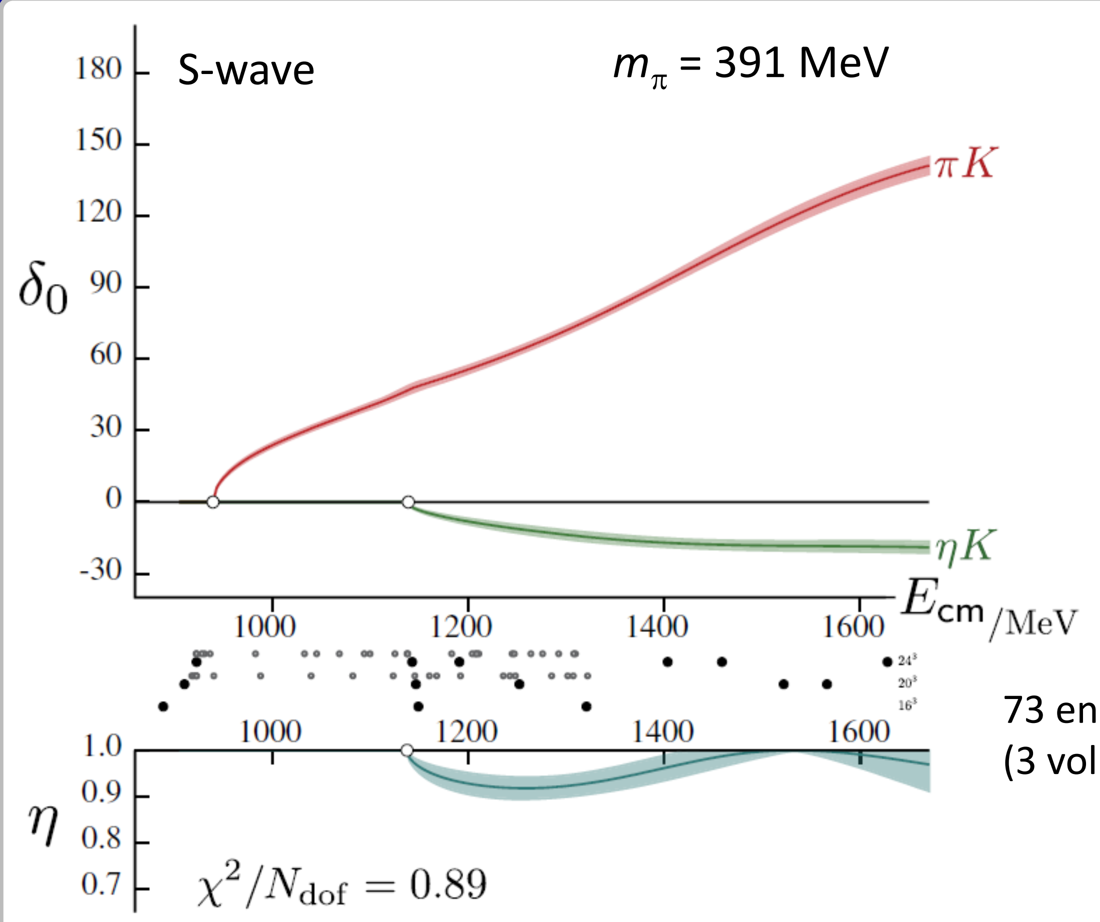
Wilson *et al* (HadSpec) [PR D92, 094502 (2015)]

Light scalar mesons (< 1 GeV)



κ in πK, ηK

$J^P = 0^+$, Isospin = ½, Strangeness = 1



73 energy levels
(3 volumes)

Wilson, Dudek, Edwards, CT
(HadSpec) [PRL 113, 182001 (2014);
PR D91, 054008 (2015)]

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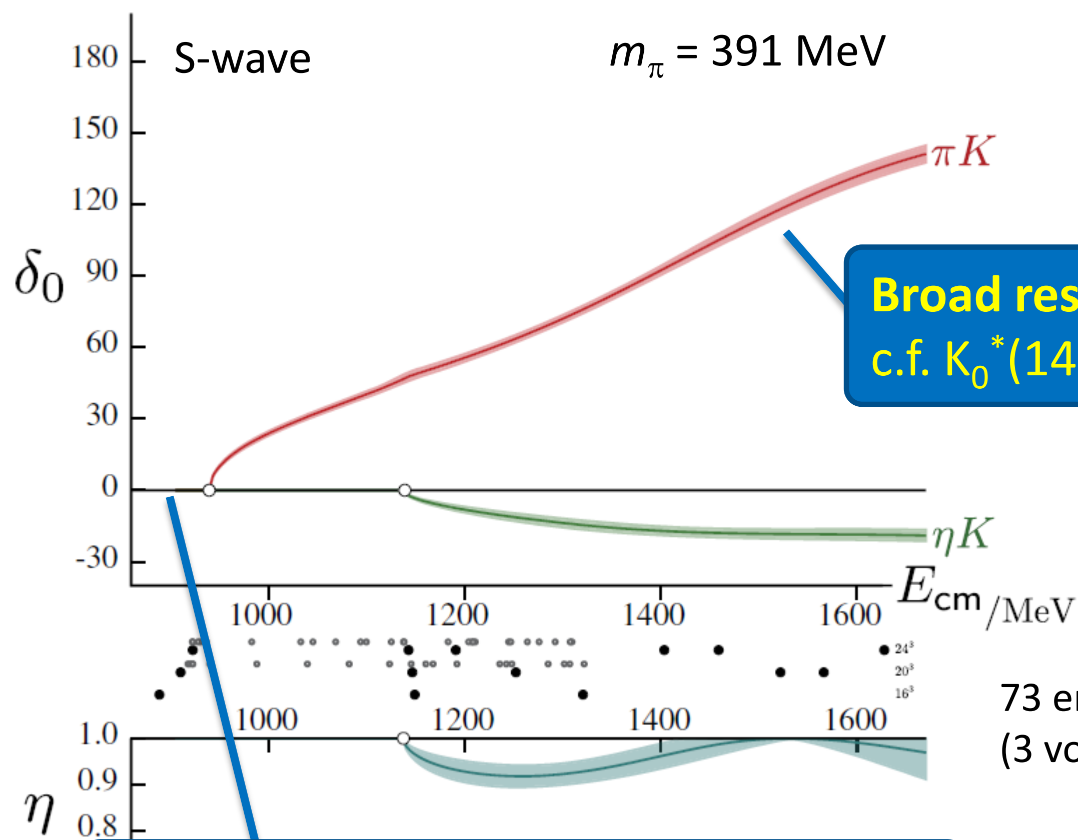

Broad resonance
c.f. $K_0^*(1430)$

73 energy levels
(3 volumes)

Virtual bound state [pole on real axis below threshold on unphysical sheet]

c.f. κ in unitarised χpt [Nebreda & Pelaez, PR D81, 054035 (2010)]

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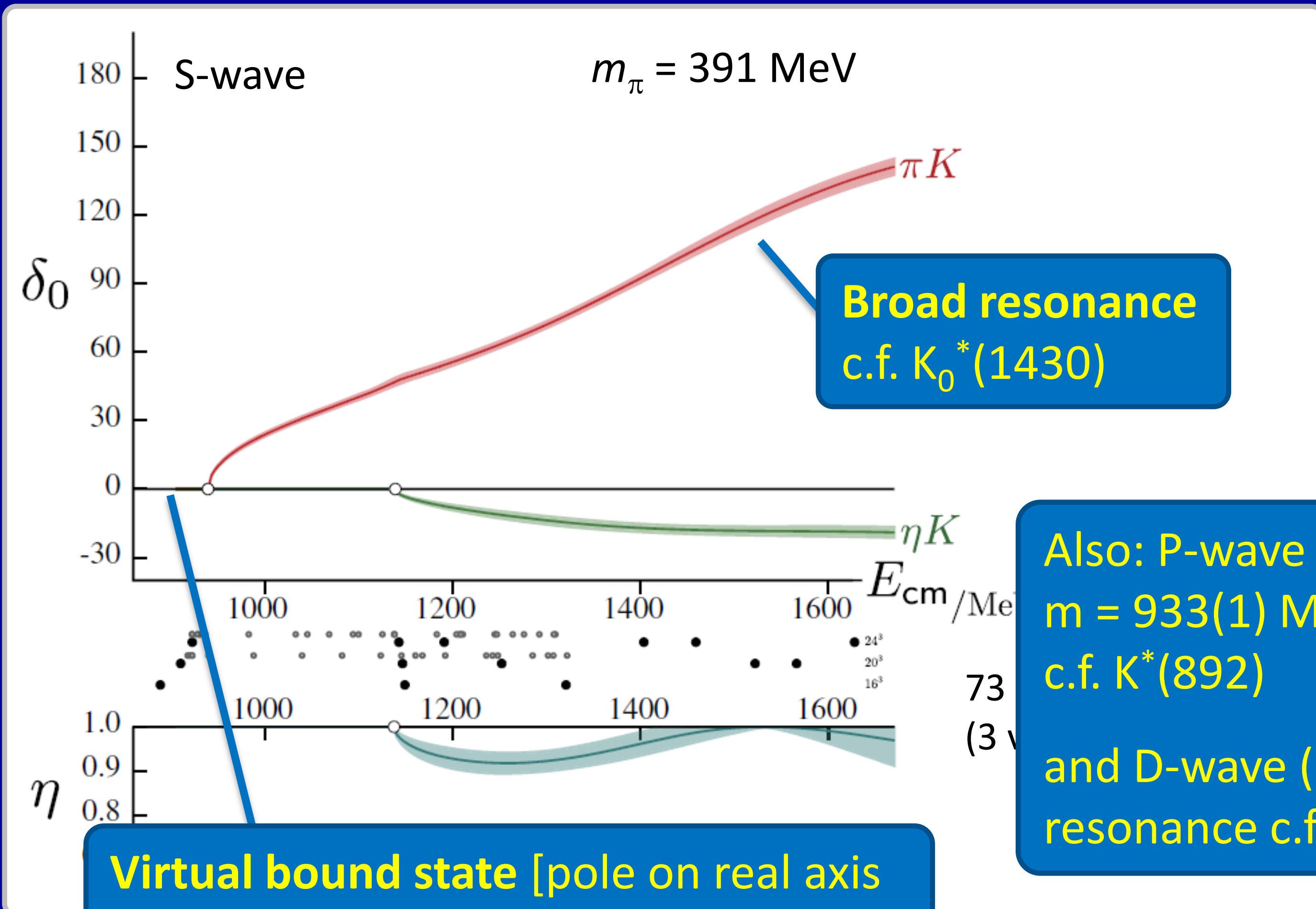


Broad resonance
c.f. $K_0^*(1430)$

Also: P-wave (1^-) bound state,
m = 933(1) MeV, g = 5.93(26)
c.f. $K^*(892)$

and D-wave (2^+) narrow resonance c.f. $K_2^*(1430)$

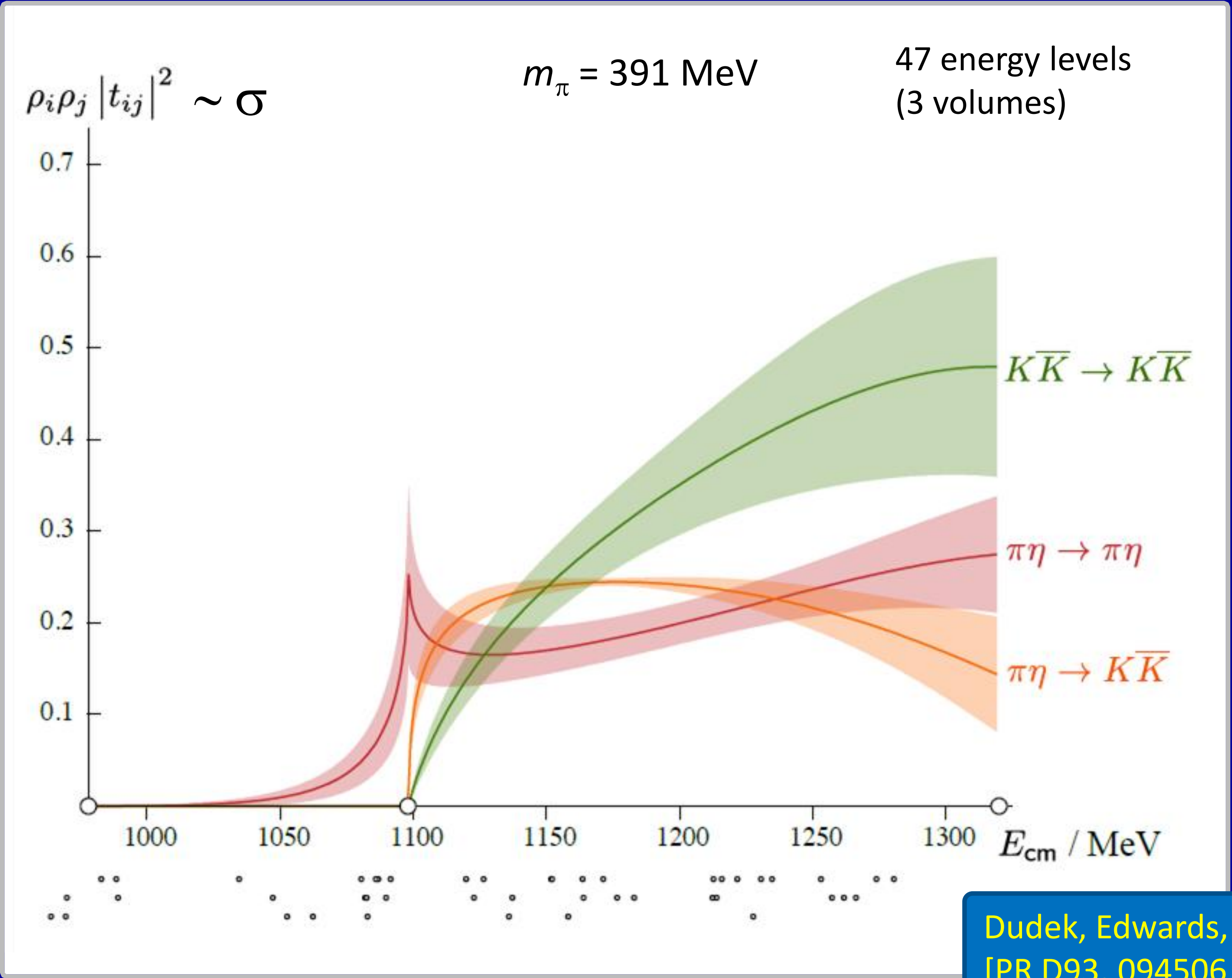
Virtual bound state [pole on real axis below threshold on unphysical sheet]

c.f. κ in unitarised χpt [Nebreda & Pelaez, PR D81, 054035 (2010)]

Wilson, Dudek, Edwards, CT (HadSpec) [PRL 113, 182001 (2014); PR D91, 054008 (2015)]

a_0 resonance in $\pi\eta$, $K\bar{K}$

$J^P = 0^+$, $I = 1$

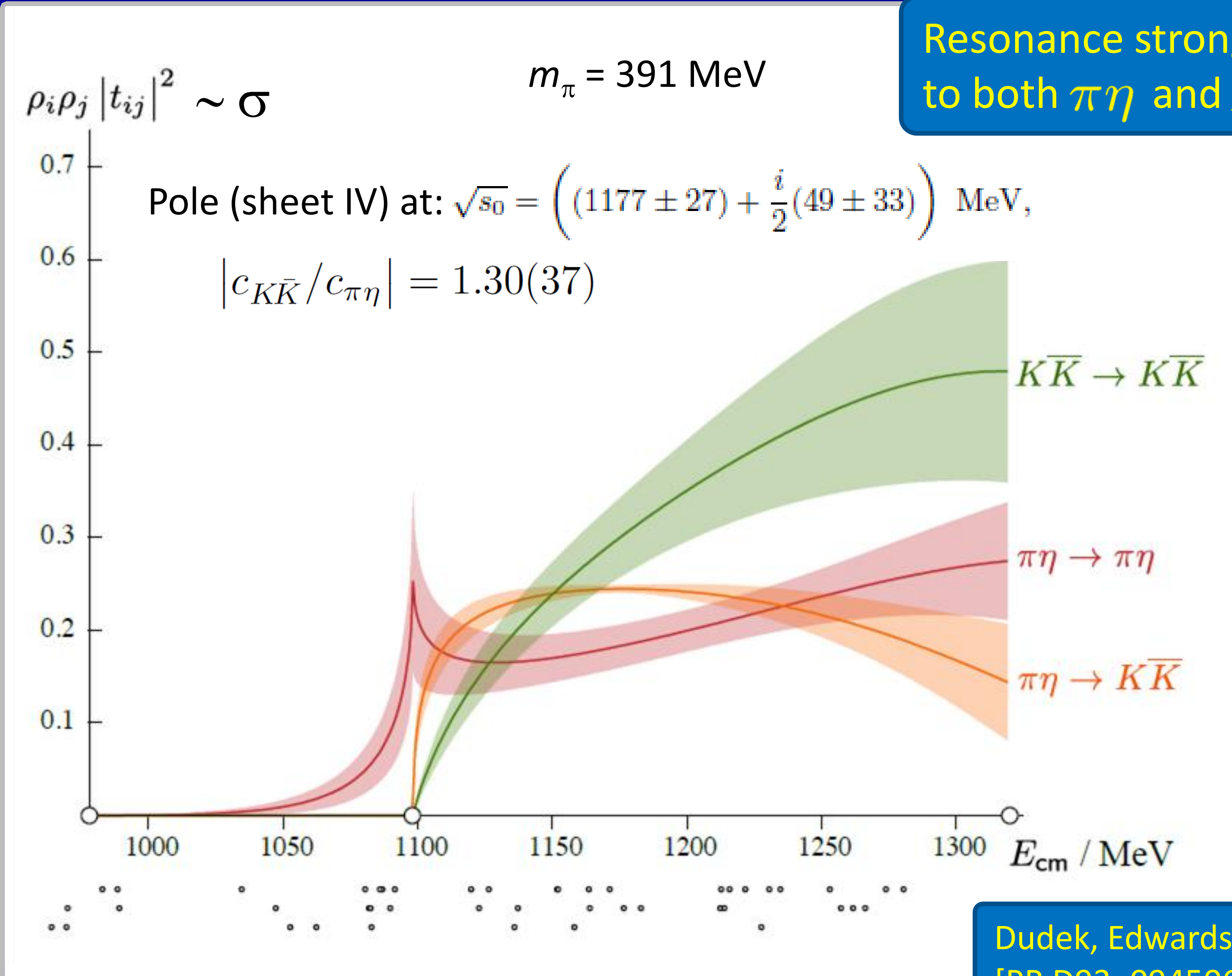


Dudek, Edwards, Wilson (HadSpec)
[PR D93, 094506 (2016)]

a_0 resonance in $\pi\eta,\ K\bar{K}$

$J^P = 0^+, I = 1$

Resonance strongly coupled to both $\pi\eta$ and $K\bar{K}$



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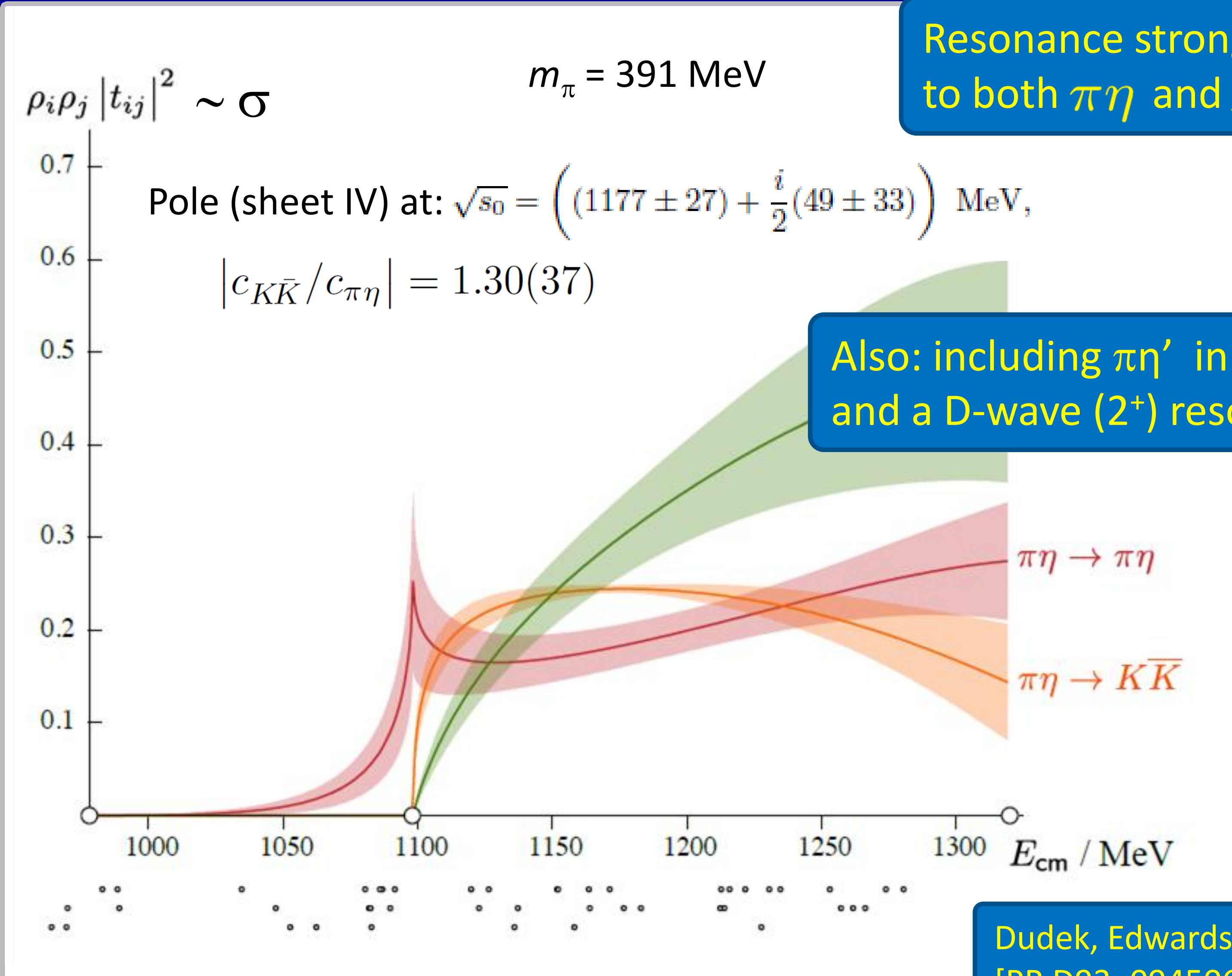


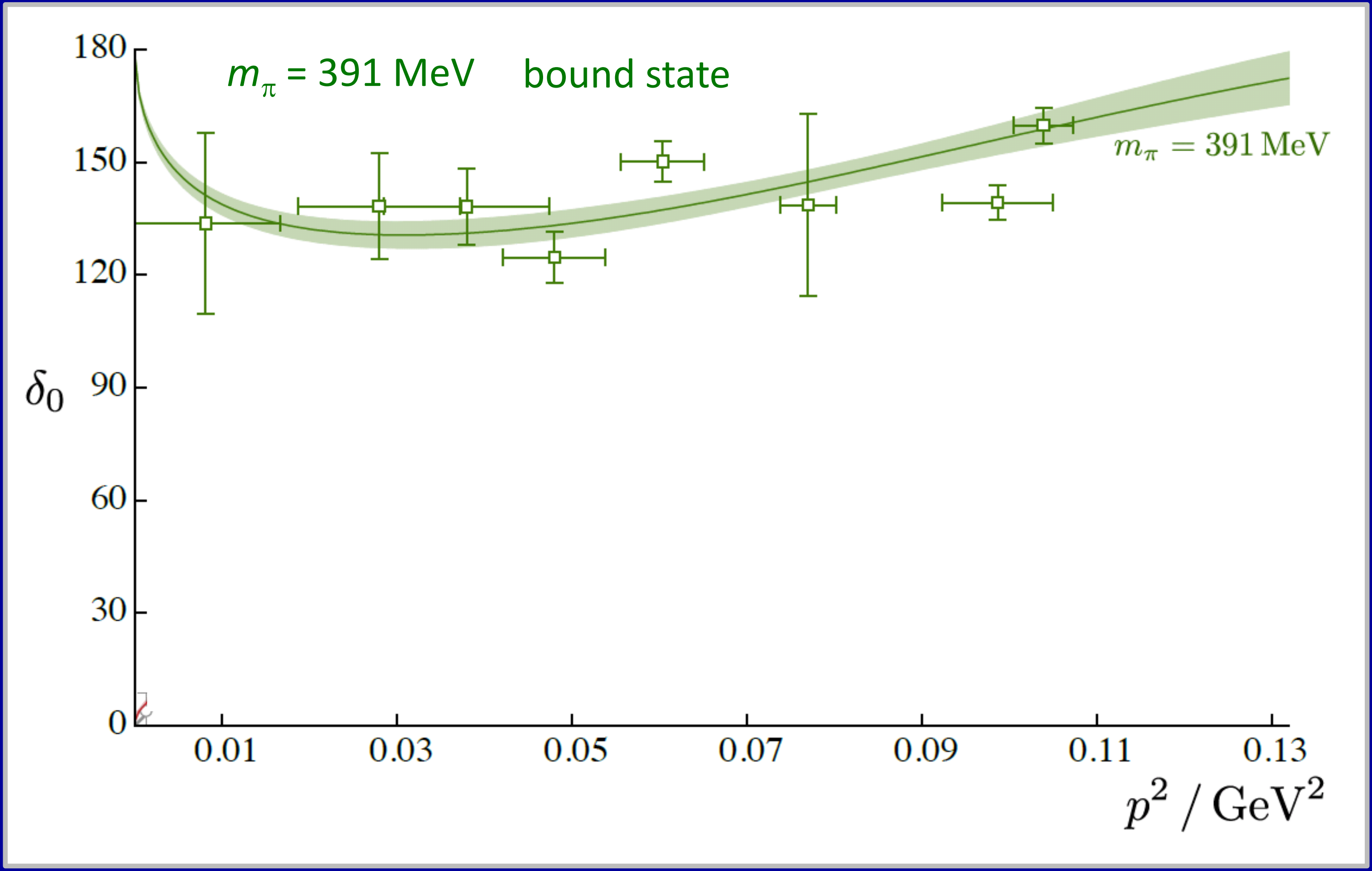
Resonance strongly coupled to both $\pi\eta$ and $K\bar{K}$

Also: including $\pi\eta'$ in S-wave, and a D-wave (2^+) resonance c.f. a_2

Dudek, Edwards, Wilson (HadSpec) [PR D93, 094506 (2016)]

$f_0(500)/\sigma$ in $\pi\pi$ scattering

$J^P = 0^+, I = 0$



Briceño, Dudek, Edwards, Wilson (HadSpec) [PRL 118, 022002 (2017)]

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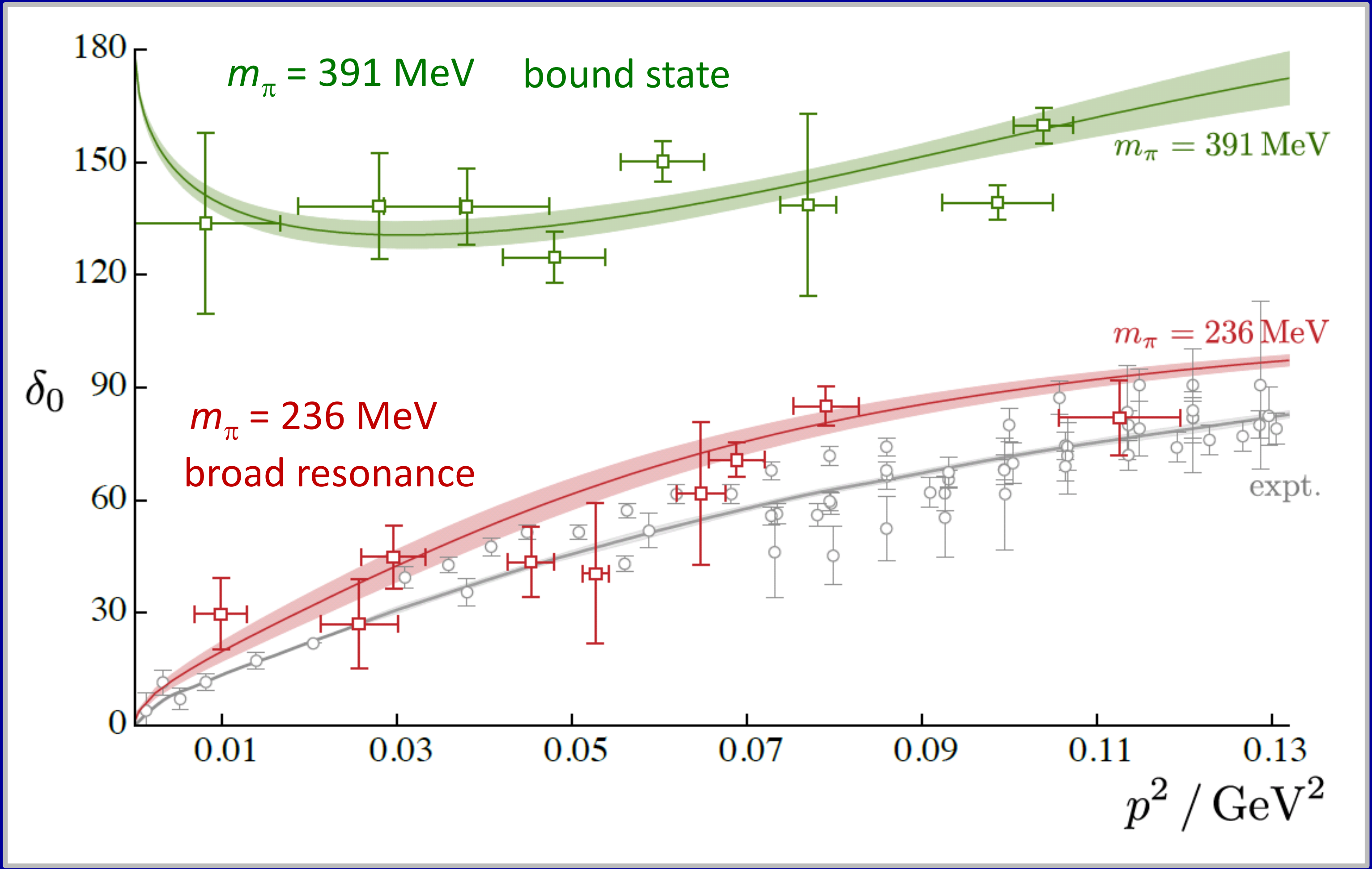

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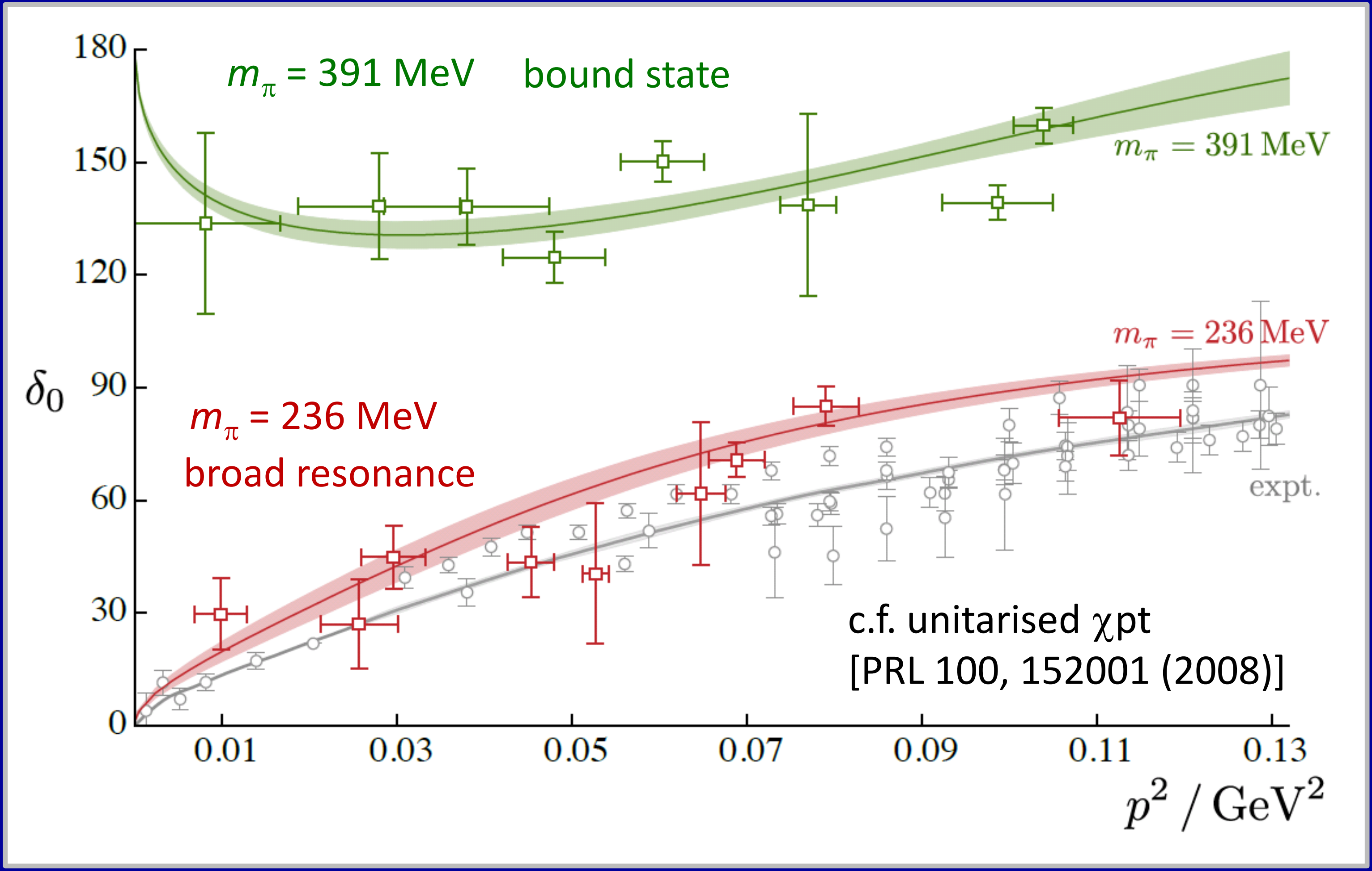

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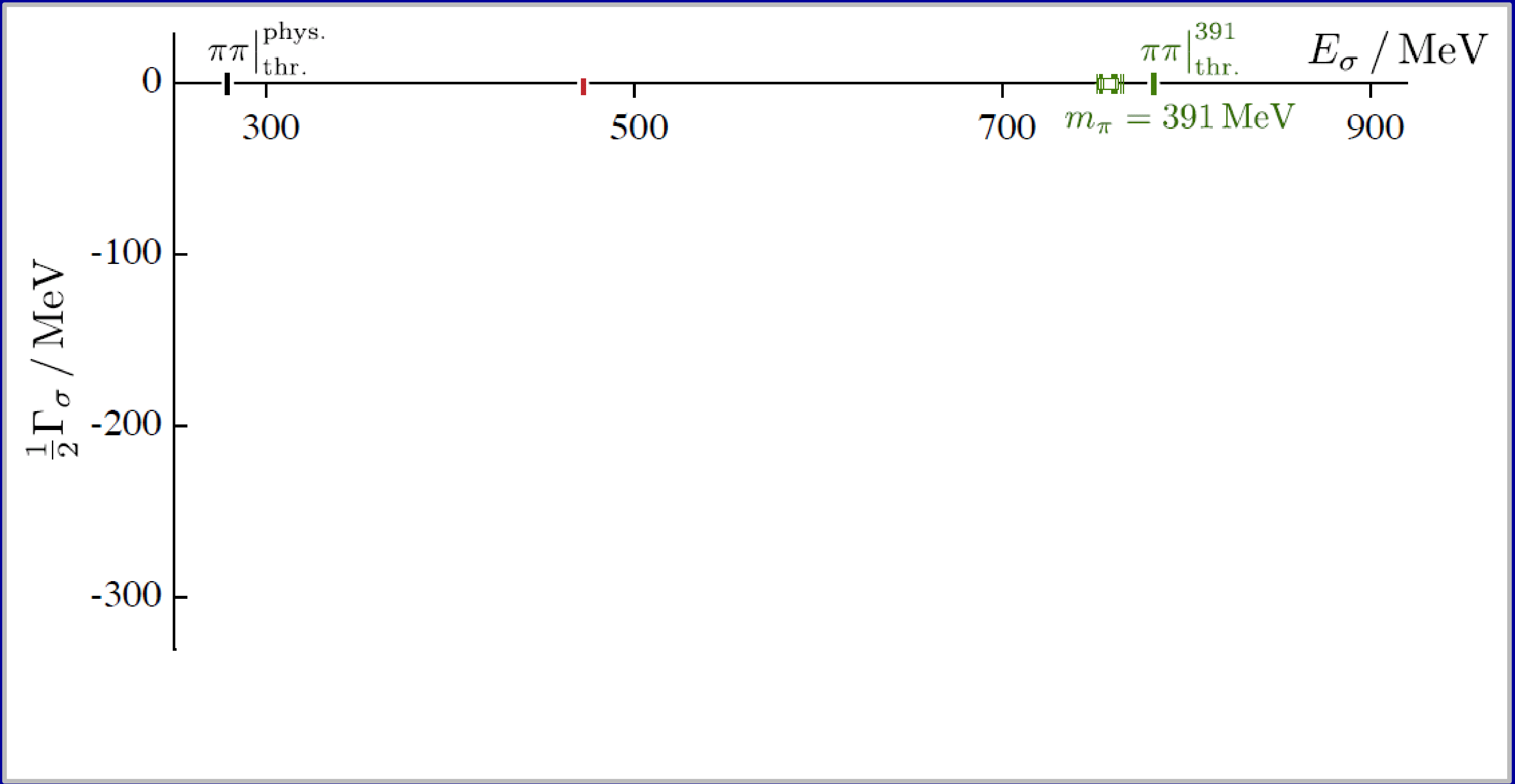

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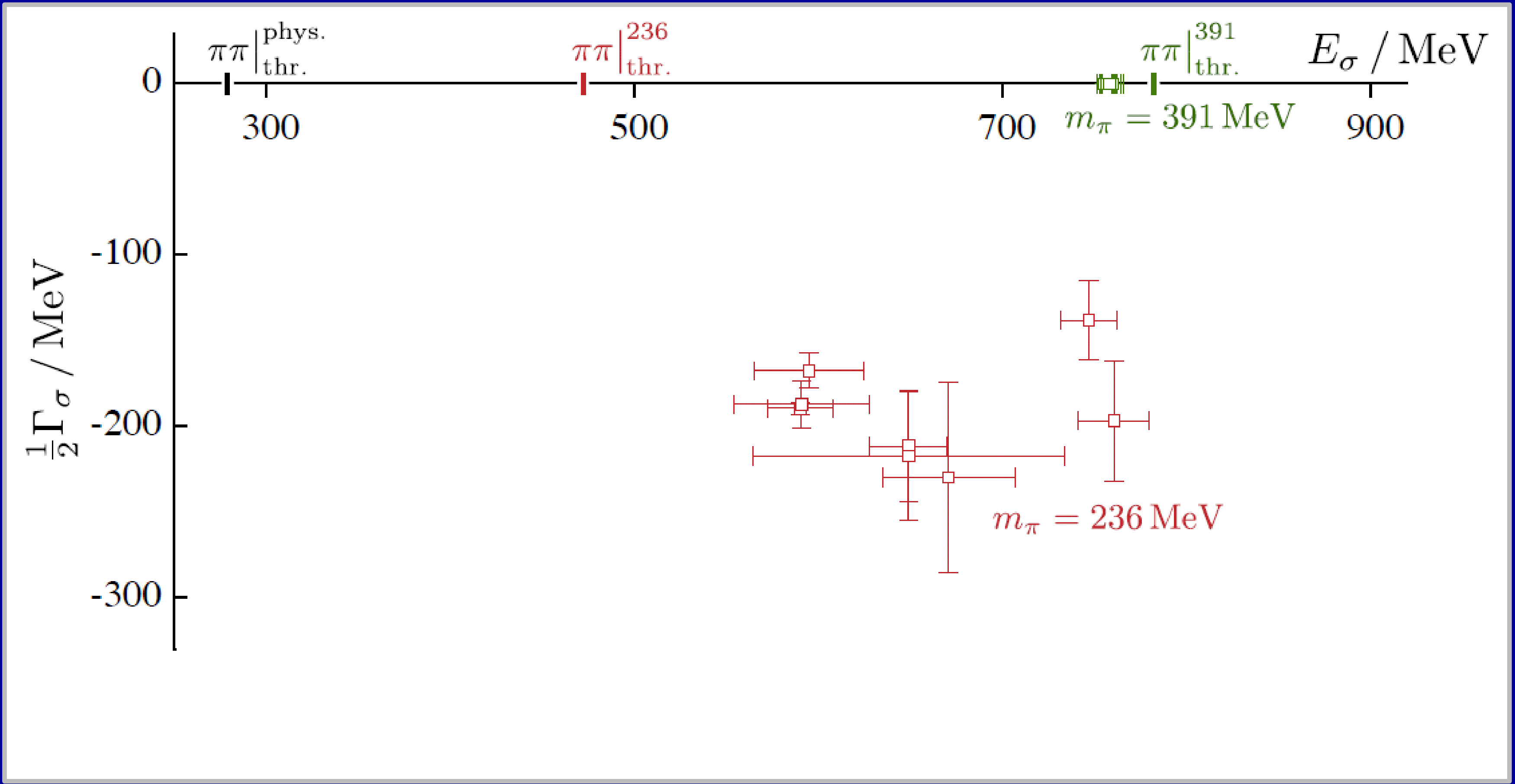

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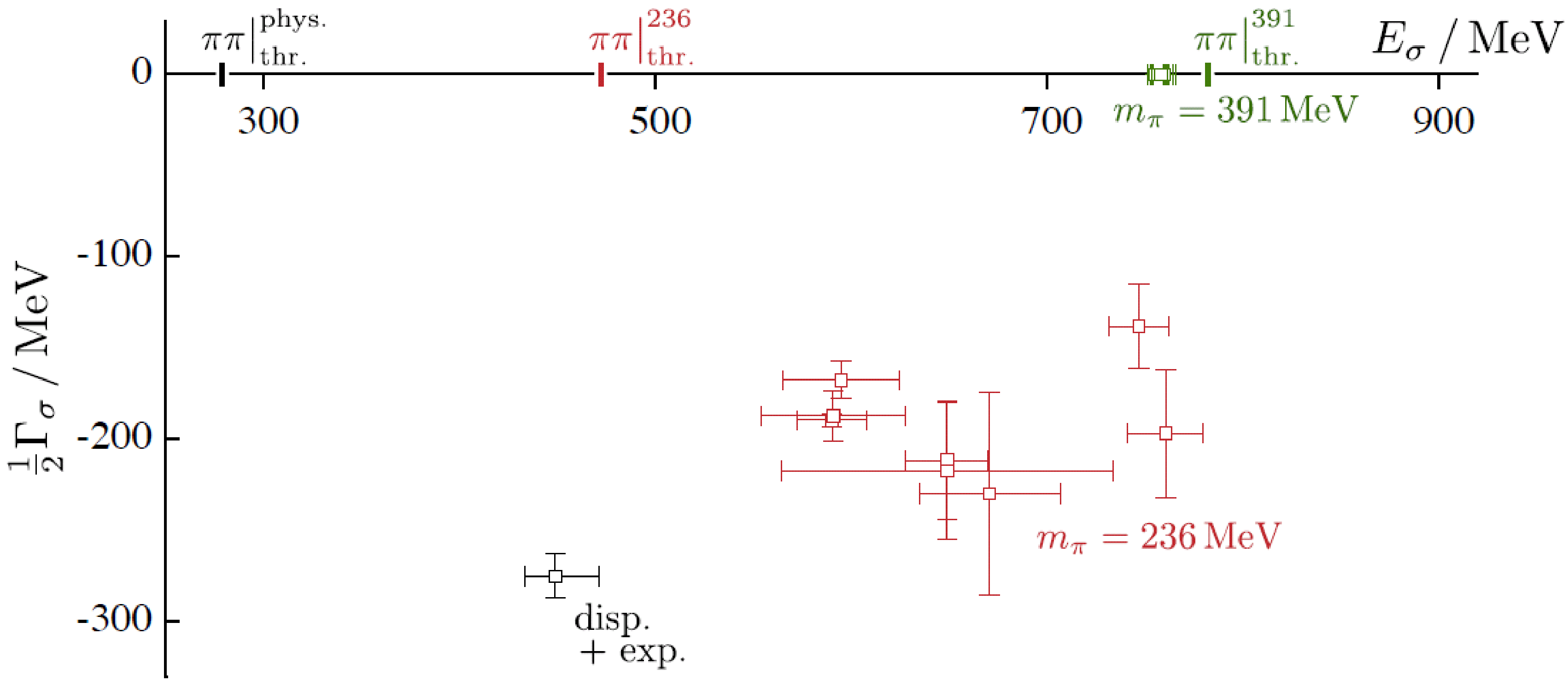

analysis of exp. data, Pelaez [Phys. Rept. 658, 1 (2016)]

Briceño, Dudek, Edwards, Wilson (HadSpec) [PRL 118, 022002 (2017)]

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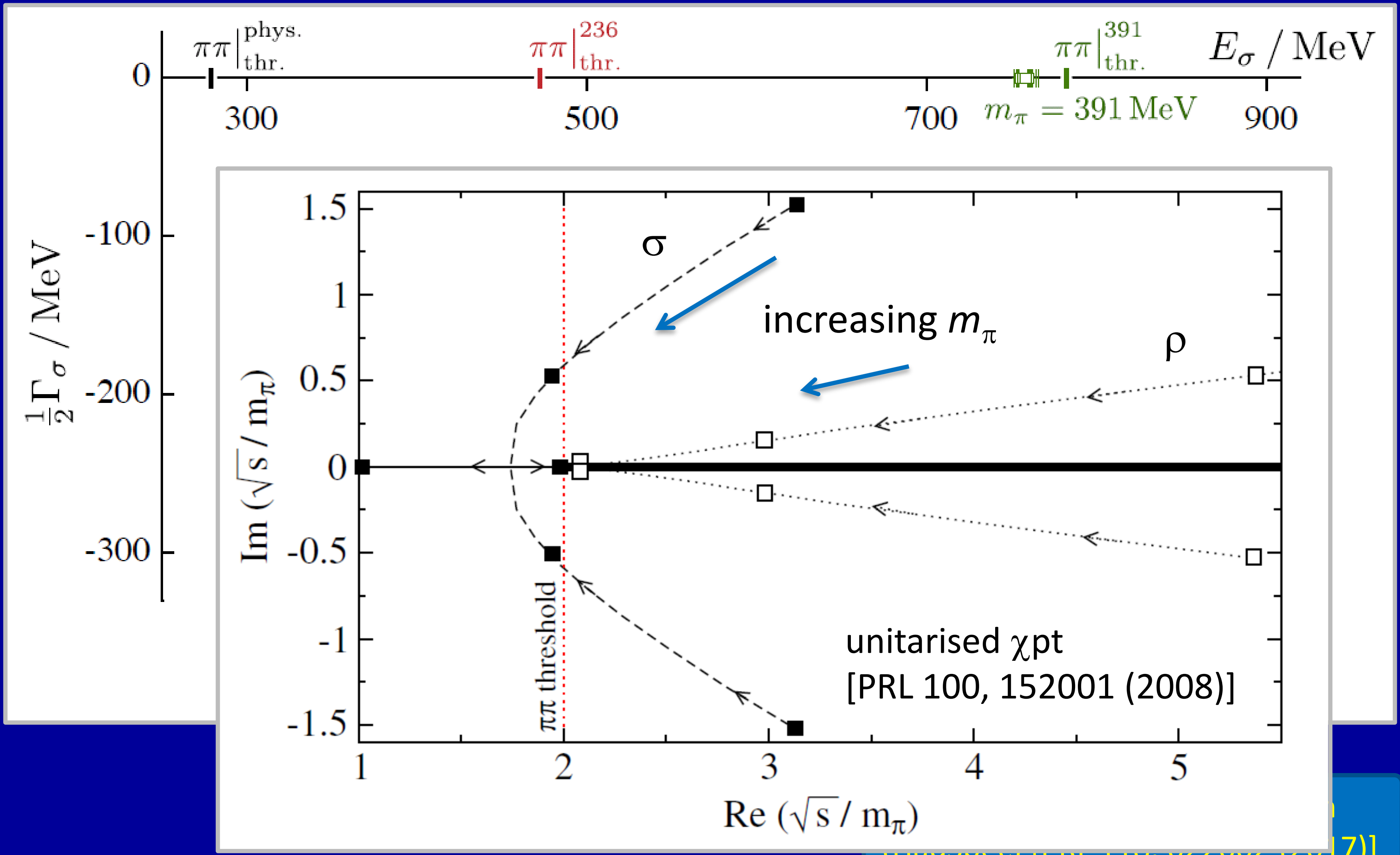
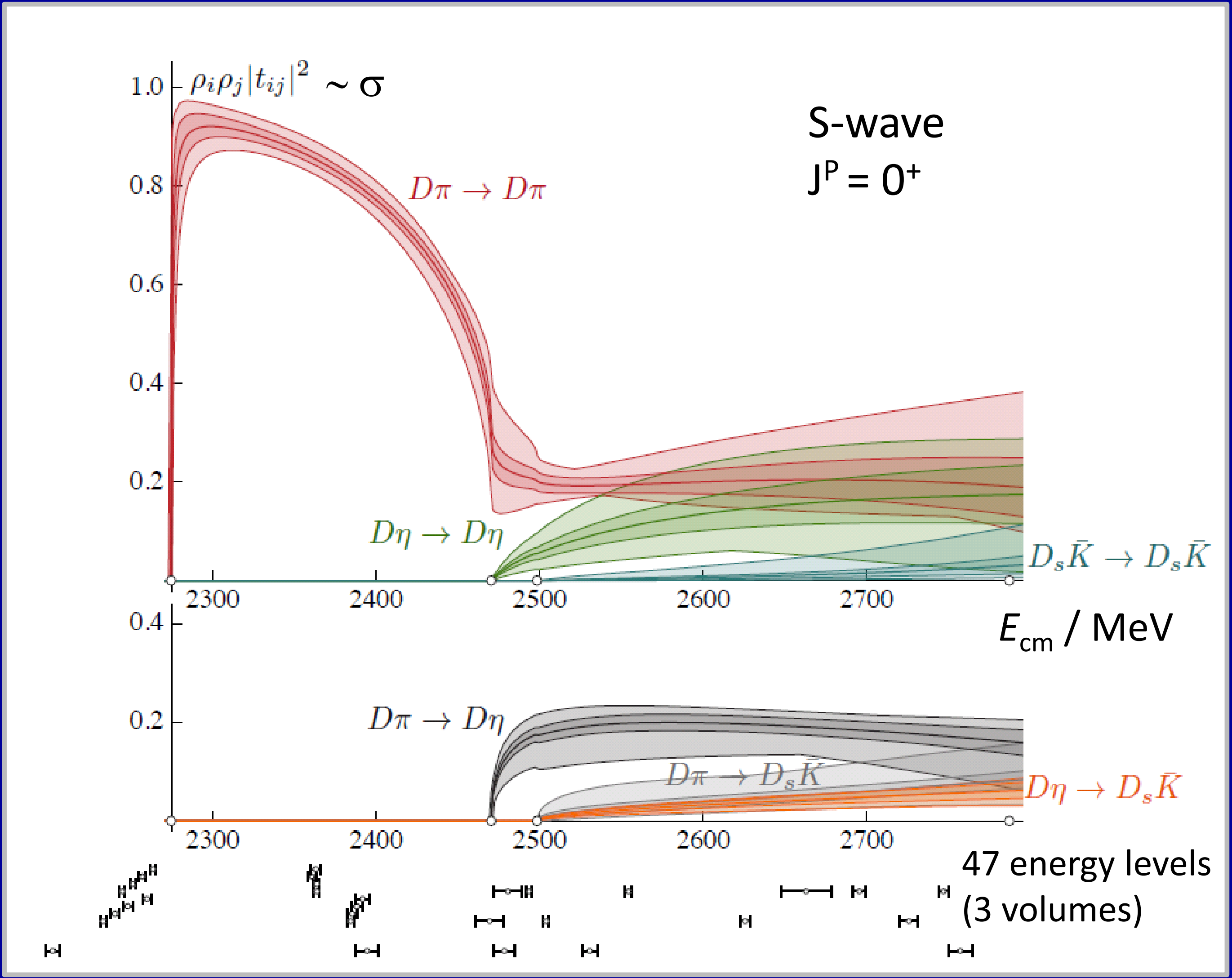


Charm-light: $D\pi$, $D\eta$, $D_s\bar{K}$ (I=½)

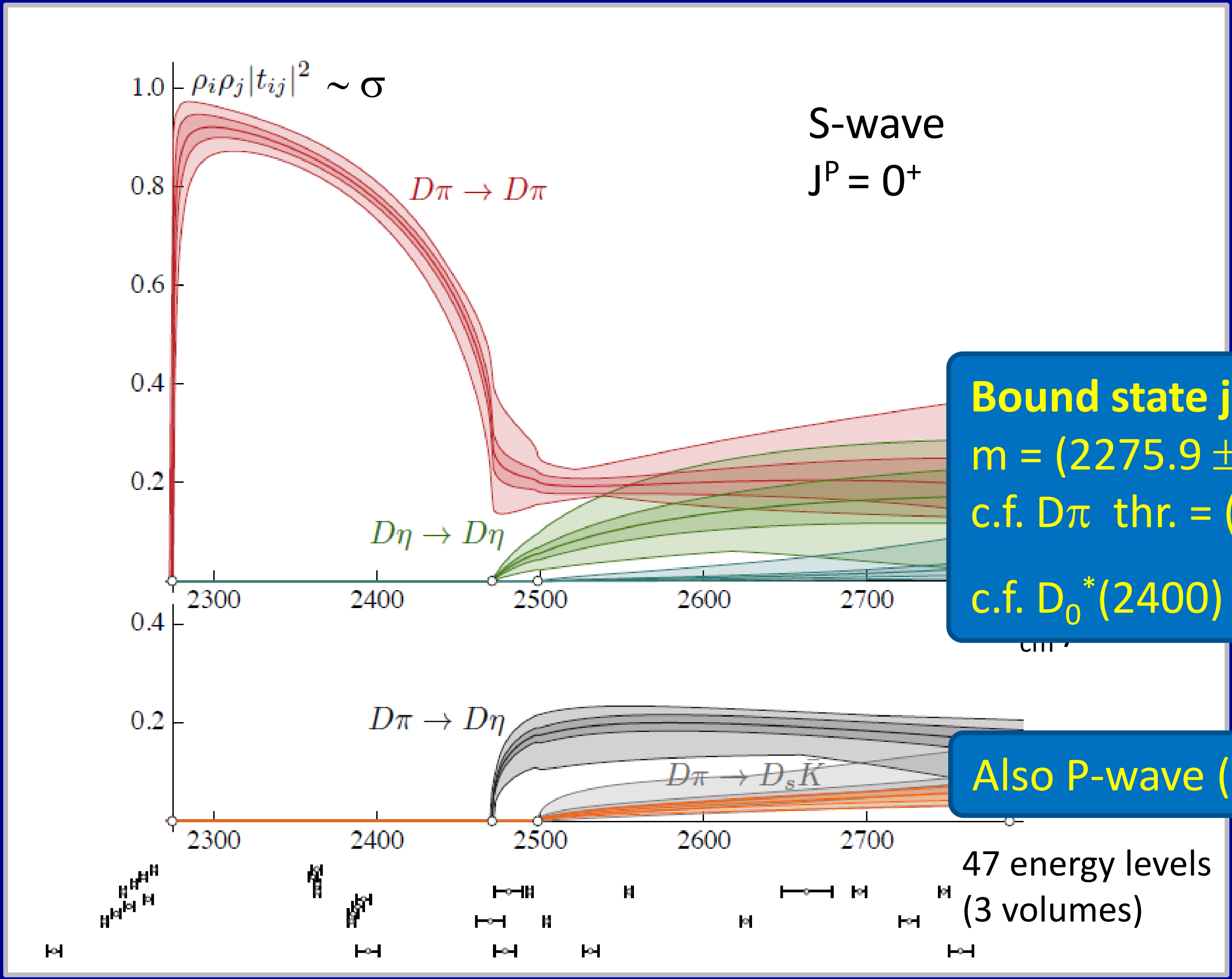
m_π = 391 MeV



Moir, Peardon, Ryan, CT, Wilson (HadSpec) [JHEP 1610, 011 (2016)]

Charm-light: $D\pi$, $D\eta$, $D_s\bar{K}$ (I=½)

m_π = 391 MeV



Bound state just below thresh.

m = (2275.9 ± 0.9) MeV

c.f. $D\pi$ thr. = (2276.4 ± 0.9) MeV

c.f. $D_0^*(2400)$

Also P-wave (1^-) and D-wave (2^+)

Moir, Peardon, Ryan, CT, Wilson (HadSpec) [JHEP 1610, 011 (2016)]

Summary

- Excited spectra of charmonia including **exotic** J^{PC}
  – supermultiplets of **hybrid mesons**
- **Significant progress** in LQCD calculations of **resonances**, near-threshold states, etc – **map out scattering amps**.
- Some examples of recent work:
  - ρ resonance, light scalars (κ, $a_0(980)$, σ)
  - Charm-light mesons
- [Also transitions, e.g. ρ resonance ($\pi\pi$) → $\pi\,\gamma$]
- Use m_π dependence as a tool
- Ongoing work on formalism (e.g. 3-hadron scattering)
- Connections with analysis of experimental data

CT acknowledges support from the U.K. Science and Technology Facilities Council

Hadron Spectrum Collaboration

Jefferson Lab, USA:

Jozef Dudek, Robert Edwards, David Richards,
Raul Briceño

Trinity College Dublin, Ireland:

Mike Peardon, Sinéad Ryan, **David Wilson**,
Cian O'Hara, David Tims

University of Cambridge, UK:

CT, **Graham Moir**, *Gavin Cheung, Antoni Woss*

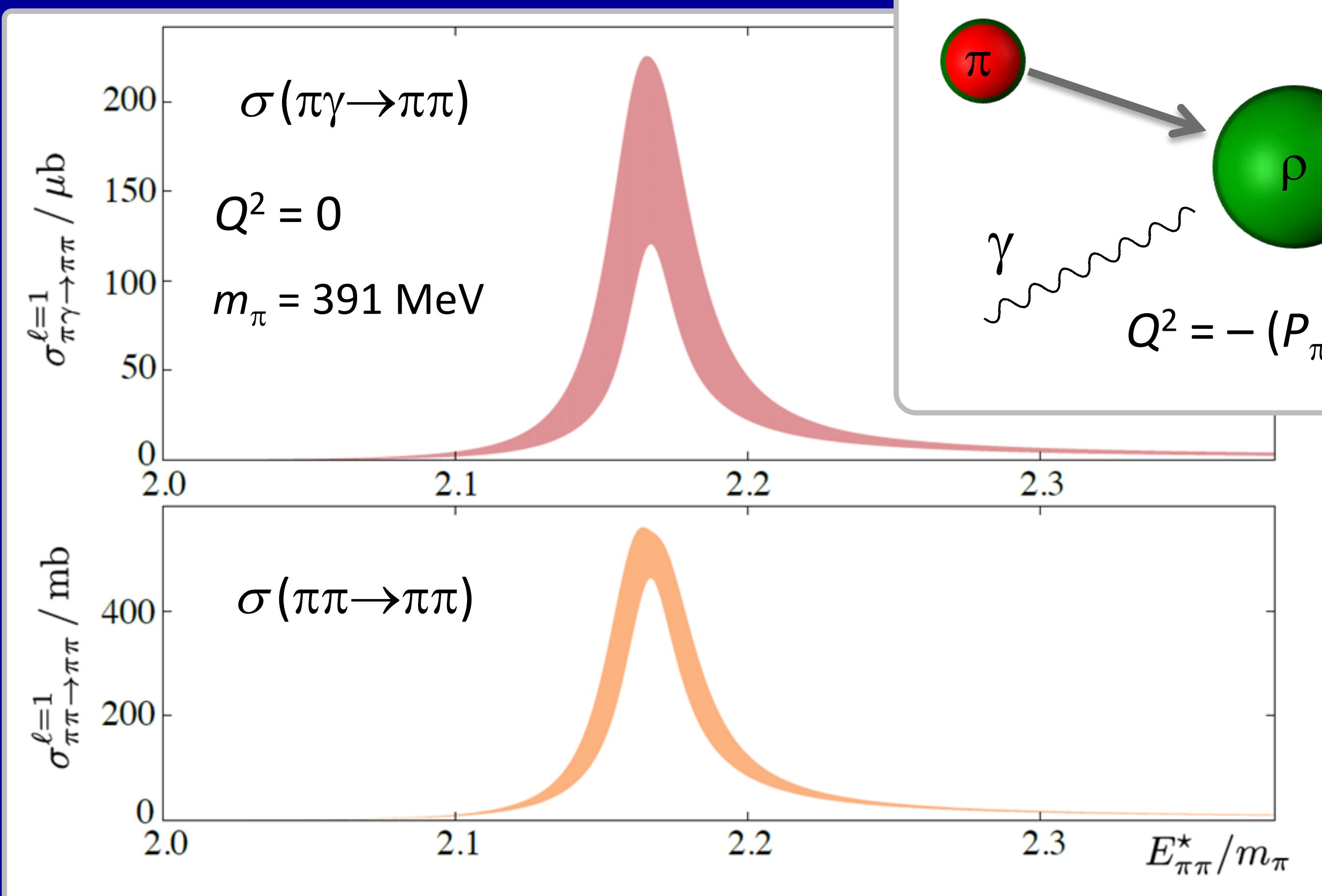
Tata Institute, India:

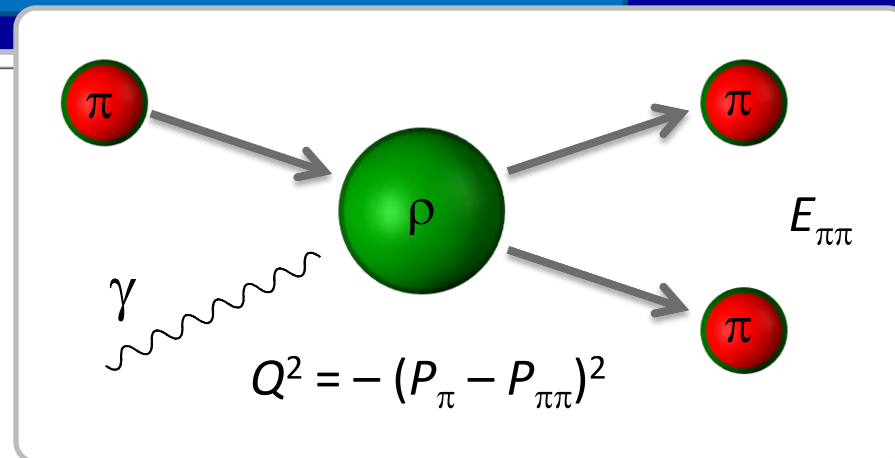
Nilmani Mathur

Extra slides

Resonant $\pi^+ \gamma \to \rho \to \pi^+ \pi^0$ amplitude

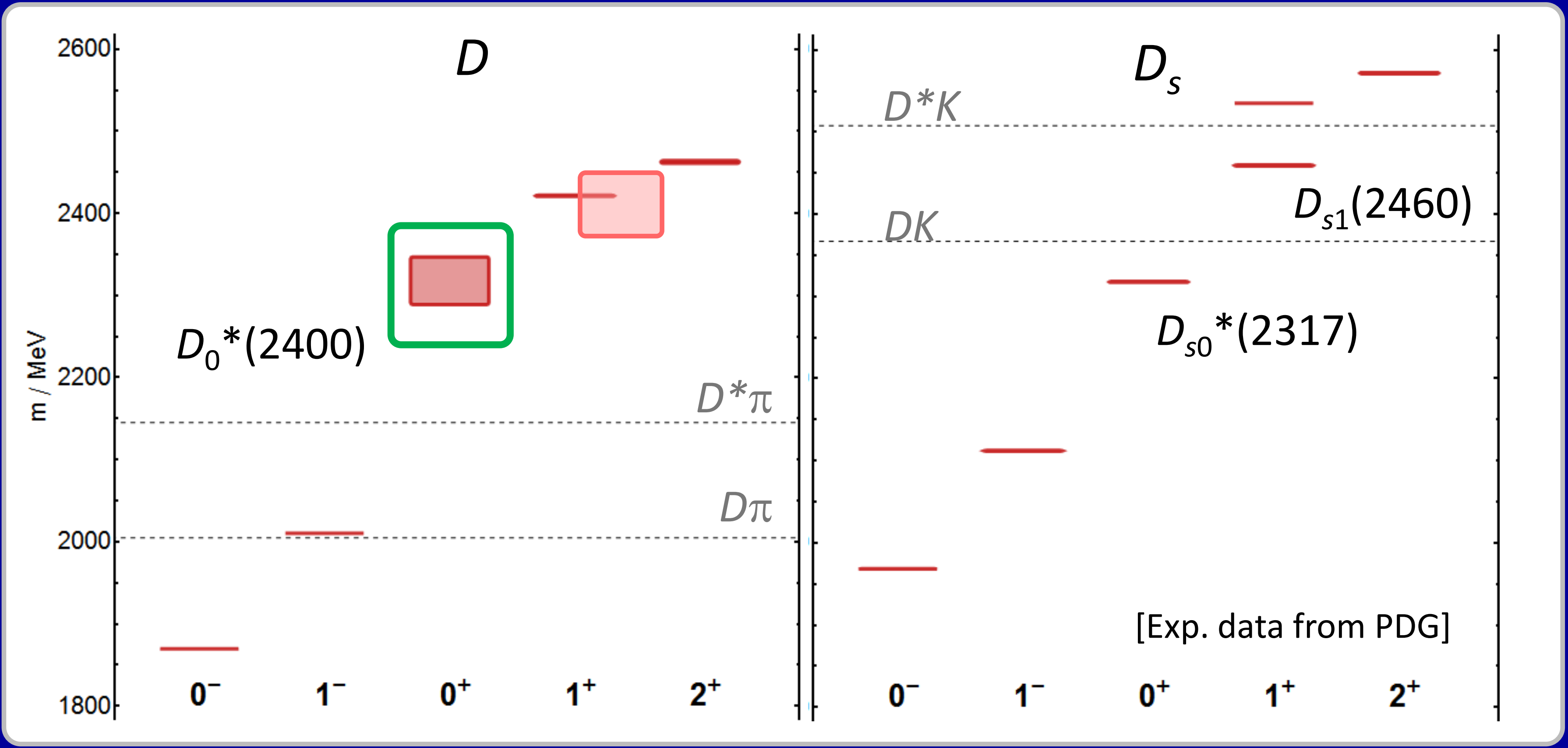
Need: $C_{ij}(t_f, t, t_i) =< 0|O_i(t_f)\ \bar{\psi}(t)\gamma^\mu\psi(t)\ O_j(t_i)|0 >$





Briceño *et al* (HadSpec) [PRL 115, 242001 (2015); PRD 93, 114508 (2016)]

Charm-light (D) and charm-strange (D_s) mesons



Some earlier LQCD studies:

- Mohler *et al* [PR D87, 034501 (2012)] – 0^+ $D\,\pi$ and 1^+ $D^*\,\pi$ resonances
- Mohler *et al* [PRL 111, 222001 (2013)] – 0^+ $D_s(2317)$ below $D\,K$ threshold
- Lang *et al* [PRD 90, 034510 (2014)] – 0^+ $D_s(2317)$ and 1^+ $D_{s1}(2460)$, $D_{s1}(2536)$

Some other recent work on charmonium(-like) mesons:

- Ozaki, Sasaki [PR D87, 014506 (2013)] – no sign of *Y*(4140) in $J/\psi\ \varphi$
- Prelovsek & Leskovec [PRL 111, 192001 (2013)] – 1^{++} I=0 near $D\bar{D}^*$ – *X*(3872)?
- Prelovsek *et al* [PL B727, 172; PR D91, 014504 (2015)] – no sign of $Z^+(3900)$ in 1^{+-}
- Chen *et al* (CLQCD) [PR D89, 094506 (2014)] – 1^{++} I=1 $D\bar{D}^*$ weakly repulsive
- Padmanath *et al* [PR D92, 034501 (2015)] – 1^{++} I=0 [X(3872)?]; no I=1 or *Y*(4140)
- Lang *et al* [JHEP 1509, 089 (2015)] – I=0 $D\bar{D}$: 1^{--} $\psi(3770)$ and 0^{++}
- Chen *et al* (CLQCD) [PR D92, 054507 (2015)] – 1^{+-} I=1 $D^*\bar{D}^*$ weakly repulsive?
- Chen *et al* (CLQCD) [PR D93, 114501 (2016)] – 0^{--}, 1^{+-} I=1 $D^*\bar{D}_1$ some attraction?
- Ikeda *et al* (HAL QCD) [PRL 117, 242001 (2016)] – $\pi\ J/\psi$, $\rho\ \eta_c$, $D\bar{D}^*$ using HAL QCD method – suggest $Z^+(3900)$ is a threshold cusp
- Albaladejo *et al* [EPJ C76, 573 (2016)] – different scenarios for PR D91, 014504

Spectroscopy on the lattice

Energy eigenstates from: $C_{ij}(t) = < 0|\boxed{\mathcal{O}_i(t)}\boxed{\mathcal{O}_j^\dagger(0)}|0 >$

Interpolating operators $\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\ \bar{\psi}(x)\left[\Gamma \overleftrightarrow{D}\overleftrightarrow{D}\dots\right]\psi(x)$

$$C_{ij}(t) = \sum_n \frac{e^{-\boxed{E_n}t}}{2\boxed{E_n}} \boxed{< 0|\mathcal{O}_i(0)|n >< n|\mathcal{O}_j^\dagger(0)|0 >}$$

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$$C_{ij}(t) = \sum_n \frac{e^{-E_n t}}{2E_n} \langle 0|\mathcal{O}_i(0)|n\rangle\langle n|\mathcal{O}_j^\dagger(0)|0\rangle$$

Large basis of ops → matrix of corrs. – generalised eigenvalue problem

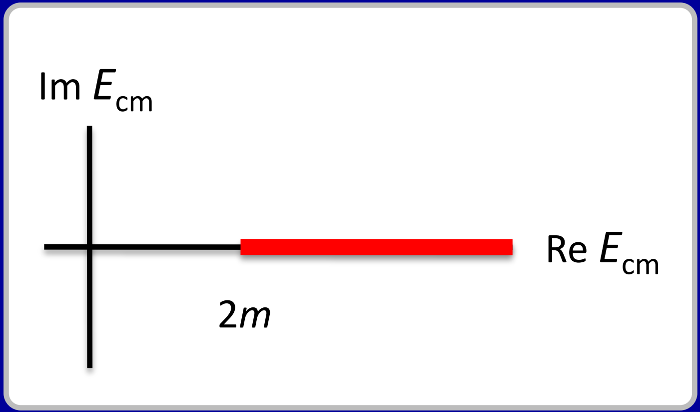
$$C_{ij}(t)v_j^{(n)} = \lambda^{(n)}(t)C_{ij}(t_0)v_j^{(n)}$$

$\lambda^{(n)}(t) \rightarrow e^{-E_n(t-t_0)}$ $\quad v_i^{(n)} \rightarrow Z_i^{(n)} \equiv \langle 0|\mathcal{O}_i|n\rangle$ $\quad$ ($t >> t_0$)

Scattering in Lattice QCD

Scattering in Lattice QCD

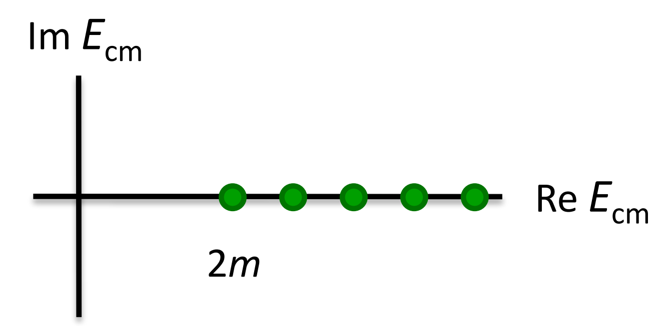
Infinite volume – continuous spectrum above threshold

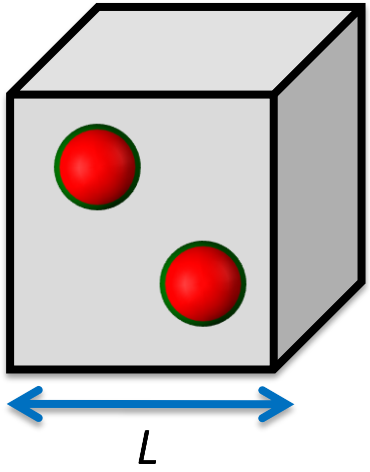


Scattering in Lattice QCD

Infinite volume – continuous spectrum above threshold

Finite volume – discrete spectrum





[periodic b.c.s]

Non-interacting: $\vec{k}_{A,B} = \frac{2\pi}{L}(n_x, n_y, n_z)$

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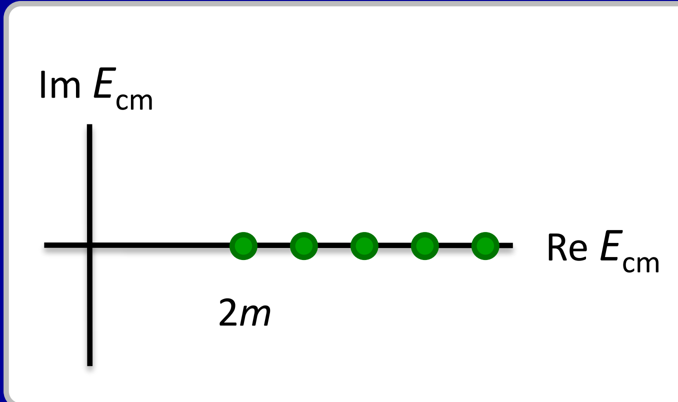

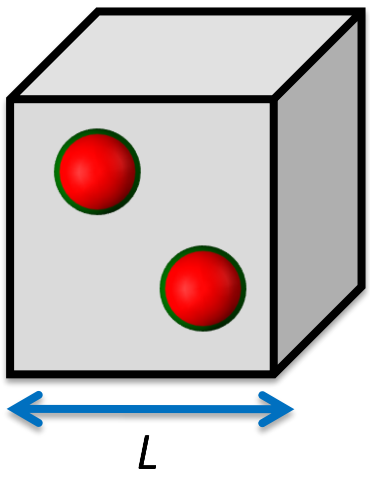

Non-interacting: $\vec{k}_{A,B} = \frac{2\pi}{L}(n_x, n_y, n_z)$

Interacting: $\vec{k}_{A,B} \neq \frac{2\pi}{L}(n_x, n_y, n_z)$

c.f. 1-dim: $k = \frac{2\pi}{L} n + \frac{2}{L}\delta(k)$

[periodic b.c.s]

scattering phase shift

Scattering in Lattice QCD

Lüscher method [NP B354, 531 (1991)] extended by many others:
relate **finite-volume energy levels** $\{E_{\mathrm{cm}}\}$ to **infinite-volume scattering *t*-matrix**

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Elastic scattering: from $E_{\rm cm}$ get $t(E_{\rm cm})$ or equivalently $\delta(E_{\rm cm})$

Scattering *t*-matrix: $S = I + 2i\rho\ t$

$$\rho = \frac{2k_{\rm cm}}{E_{\rm cm}}$$

$$t^{(\ell)} = \frac{1}{\rho} e^{i\delta_\ell} \sin \delta_\ell$$

Larger set of $E_{\rm cm}$ by e.g. overall non-zero mom., twisted b.c.s, different vols.

[Complication: reduced symmetry of lattice volume → mixing of partial waves]

Scattering in Lattice QCD

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Coupled-channel scattering:

$$\text{E.g.} \quad \mathbf{t}(E_{\rm cm}) = \begin{pmatrix} t_{\pi\pi\to\pi\pi}(E_{\rm cm}) & t_{\pi\pi\to K\bar{K}}(E_{\rm cm}) \\ t_{K\bar{K}\to\pi\pi}(E_{\rm cm}) & t_{K\bar{K}\to K\bar{K}}(E_{\rm cm}) \end{pmatrix}$$

Determinant equation for $\mathbf{t}(E_{\rm cm})$ at each $E_{\rm cm}$

- Given $\mathbf{t}(E_{\rm cm})$: solns. of equ. → finite-vol. spec. $\{E_{\rm cm}\}$
  But we need: spectrum → $\mathbf{t}(E_{\rm cm})$
- **Under-constrained problem** (e.g. 2 channels: 3 unknowns but 1 equ.)

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  But we need: spectrum → $\mathbf{t}(E_{cm})$
- **Under-constrained problem** (e.g. 2 channels: 3 unknowns but 1 equ.)
  → Parameterize E_{cm} dependence of *t*-matrix and fit $\{E_{lattice}\}$ to $\{E_{param}\}$

Try different parameterizations, e.g. various *K*-matrix forms
(for elastic scattering also Breit Wigner, effective range expansion).