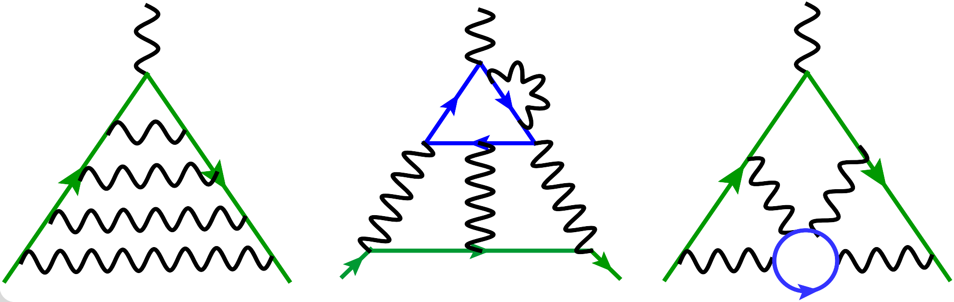
4 loop QED corrections to $(g-2)_\mu$

PhiPsi17, Mainz, June 26-29, 2017

Matthias Steinhauser | in collaboration with A. Kurz, T. Liu, P. Marquard, A.V. Smirnov, V.A. Smirnov, D. Wellmann

TTP KARLSRUHE



Highlight of the year: $(g-2)_e$

Semi-analytic calculation of 4-loop coefficient

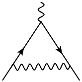
$$\frac{(g-2)_e}{2} = a_e = \frac{\alpha}{2\pi} + \cdots + c_4 \left(\frac{\alpha}{\pi}\right)^4$$

[Laporta'17]

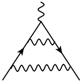
$$\begin{aligned} c_4 = {} & T_0 + T_2 + T_3 + T_4 + T_5 + T_6 + T_7 \\ & + \sqrt{3}\,(V_{4a} + V_{6a}) + V_{6b} + V_{7b} + W_{6b} + W_{7b} \\ & + \sqrt{3}\,(E_{4a} + E_{5a} + E_{6a} + E_{7a}) + E_{6b} + E_{7b} \\ & + U \end{aligned}$$

I. Introduction
II. $(g-2)_\mu$: *e* and τ loops
III. $(g-2)_\mu$: photonic contribution and μ loops
IV. Conclusions

Muon $g - 2$

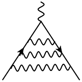


[Schwinger'48]



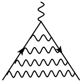
[Petermann'57;

Sommerfeld'58]



[Laporta,Remiddi'96;

. . . ; Passera'06]



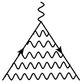
[Laporta'93;Kinoshita,Nio'06;

Aoyama,Hayakawa,Kinoshita,Nio'07'08;

Kurz,Marquard,Liu,Steinhauser'14;

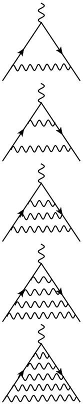
Kurz,Marquard,Liu,Smirnov,Smirnov,Steinhauser'15'16;

Volkov'17; Laporta'17]



[Aoyama,Hayakawa,Kinoshita,Nio'12]

Muon $g - 2$



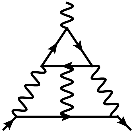
$0.5\,\frac{\alpha}{\pi}$

$\left(-0.328\ldots + 1.094\ldots|_e\right)\frac{\alpha^2}{\pi^2}$

$\left(1.181\ldots + 22.868\ldots|_e\right)\frac{\alpha^3}{\pi^3}$

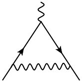
$\left(-1.910\ldots + 132.682\ldots|_e\right)\frac{\alpha^4}{\pi^4}$

$\left(9.168(571) + 742.18(87)|_e\right)\frac{\alpha^5}{\pi^5}$

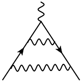


Muon $g-2$

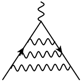
- $a_\mu(\mathrm{exp}) - a_\mu(\mathrm{SM}) \sim 250(90) \times 10^{-11}$
- $\Delta a_\mu^{\mathrm{hadr}} \sim 40 \times 10^{-11}$
- $\Delta a_\mu^{\mathrm{hadr,lbl}} \sim 40 \times 10^{-11}$



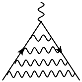
$0.5\,\frac{\alpha}{\pi}$



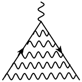
$(-0.328\ldots + 1.094\ldots|_e)\,\frac{\alpha^2}{\pi^2}$



$(1.181\ldots + 22.868\ldots|_e)\,\frac{\alpha^3}{\pi^3}$



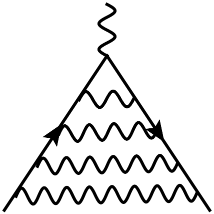
$(-1.910\ldots + 132.682\ldots|_e)\,\frac{\alpha^4}{\pi^4} \sim 380 \times 10^{-11}$

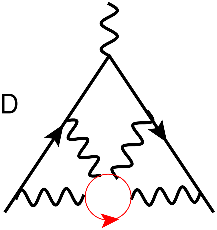


$(9.168(571) + 742.18(87)|_e)\,\frac{\alpha^5}{\pi^5}$

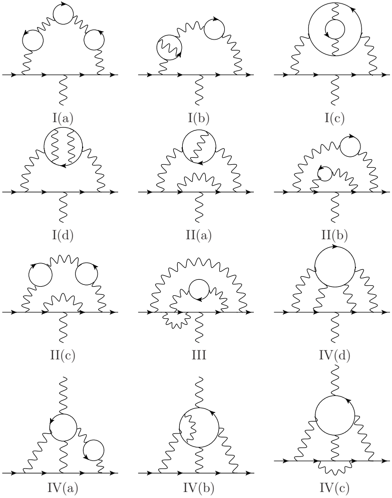
$(g-2)_\mu$

- 4-loop massive form factor
- $\Gamma^\mu(q_1, q_2) = \left[F_1(q^2)\gamma^\mu - \frac{i}{2m} F_2(q^2)\sigma^{\mu\nu}q_\nu\right]$
- $F_2(0) = \frac{(g-2)}{2} = a = \frac{\alpha}{2\pi} + \mathcal{O}(\alpha^2)$
- $q \to 0$: Taylor expansion ➪ 4-loop OS integrals
- subset of $\overline{\mathrm{MS}}$-OS heavy quark mass relation in QCD
  358 out 386 MIs [Marquard,Smirnov,Smirnov,Steinhauser,Wellmann'16]
  \+ many improvements in numerical results
- e and τ loops ➪ in addition asymptotic expansion





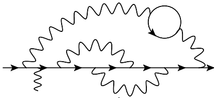
e loops



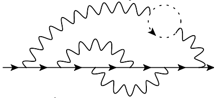
- expand in $x = m_e/m_\mu$ up to $\mathcal{O}(x^3)$
- apply methods of region [Beneke,Smirnov'98; Smirnov]
- lbl diagrams: $m_e = 0$ not possible

Example

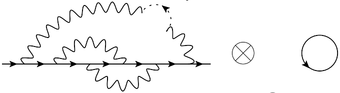
full diagram:



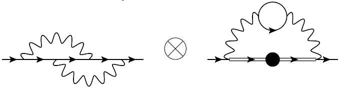
1. all-hard region:



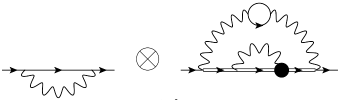
2. three hard momenta:



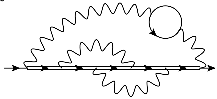
3. two hard momenta:



4. one hard momentum:



5. all-soft region:

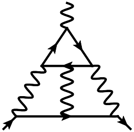


Structure

- asymptotic expansion for $x \equiv m_e/m_\mu \ll 1$

3 loops

$$
\begin{aligned}
a_\mu = \; & a_\mu^{(0,0)} + a_\mu^{(0,1)} \ln \frac{m_\mu}{m_e} \\
& + \frac{m_e}{m_\mu} \left(a_\mu^{(1,0)} + a_\mu^{(1,1)} \ln \frac{m_\mu}{m_e} \right) \\
& + \frac{m_e^2}{m_\mu^2} \left(a_\mu^{(2,0)} + a_\mu^{(2,1)} \ln \frac{m_\mu}{m_e} + a_\mu^{(2,2)} \ln^2 \frac{m_\mu}{m_e} \right. \\
& \left. \qquad + a_\mu^{(2,3)} \ln^3 \frac{m_\mu}{m_e} \right) \\
& + \ldots
\end{aligned}
$$



$$\ln \frac{m_\mu}{m_e} \approx 5.332$$

$$\frac{m_e}{m_\mu} \approx 0.005 \qquad \frac{m_e}{m_\mu} \ln \frac{m_\mu}{m_e} \approx 0.026 \qquad \frac{m_e}{m_\mu} \ln^2 \frac{m_\mu}{m_e} \approx 0.137$$

$$\frac{m_e^2}{m_\mu^2} \ln^3 \frac{m_\mu}{m_e} \approx 0.004 \qquad \frac{m_e^2}{m_\mu^2} \ln^4 \frac{m_\mu}{m_e} \approx 0.019$$

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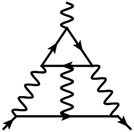
& \qquad \left. + a_\mu^{(2,3)} \ln^3 \frac{m_\mu}{m_e} \right) \\
& + \ldots \\
= \; & (20.52801865 + 0.42645778 + 0.00655695 + \ldots) \left(\frac{\alpha}{\pi}\right)^3
\end{aligned}
$$



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Structure

Karlsruhe Institute of Technology

- asymptotic expansion for $x \equiv m_e/m_\mu \ll 1$

4 loops

$$
\begin{aligned}
a_\mu \;=\; & a_\mu^{(0,0)} + a_\mu^{(0,1)} \ln\frac{m_\mu}{m_e} + a_\mu^{(0,2)} \ln^2\frac{m_\mu}{m_e} + a_\mu^{(0,3)} \ln^3\frac{m_\mu}{m_e} \\
& + \frac{m_e}{m_\mu}\left(a_\mu^{(1,0)} + a_\mu^{(1,1)} \ln\frac{m_\mu}{m_e}\right) \\
& + \frac{m_e^2}{m_\mu^2}\left(a_\mu^{(2,0)} + a_\mu^{(2,1)} \ln\frac{m_\mu}{m_e} + a_\mu^{(2,2)} \ln^2\frac{m_\mu}{m_e} \right. \\
& \qquad \left. + a_\mu^{(2,3)} \ln^3\frac{m_\mu}{m_e} + a_\mu^{(2,4)} \ln^4\frac{m_\mu}{m_e}\right) \\
& + \ldots
\end{aligned}
$$

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Structure

- asymptotic expansion for $x \equiv m_e/m_\mu \ll 1$

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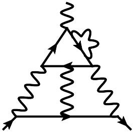
& \qquad \left. + a_\mu^{(2,3)} \ln^3 \frac{m_\mu}{m_e} + a_\mu^{(2,4)} \ln^4 \frac{m_\mu}{m_e} \right) \\
& + \ldots \\
= \; & (1.08 - 1.52 + 0.06 - 0.0002 + \ldots) \left(\frac{\alpha}{\pi}\right)^4
\end{aligned}
$$



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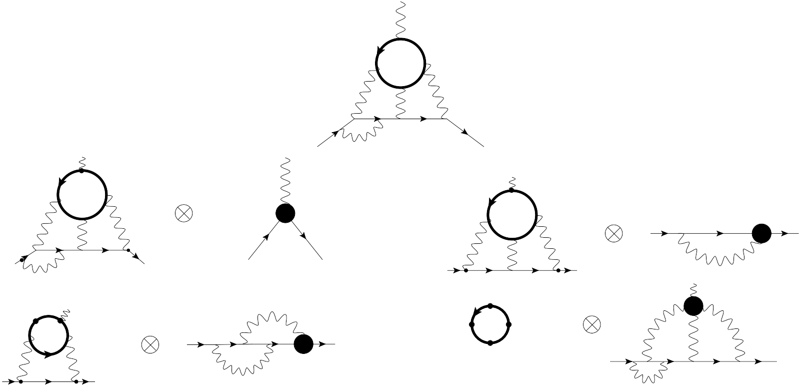
Techniques

- generation of amplitudes with `qgraf` [Nogueira'91]
- traces, projectors, ... `FORM` [Vermaseren,...]
- asymptotic expansion: `asy` [Pak,Smirnov'10] and in-house program
  basic idea: alpha parametrization
- 4-loop on-shell integrals:
  $\approx 70 = 40_{\text{ana/high prec.}} + 30_{\text{num}}$ [Marquard,Smirnov,Smirnov,Steinhauser,Wellmann'15]
- 4-loop "linear" integrals:
  $\approx 70 = 20_{\text{ana/high prec.}} + 50_{\text{num}}$ [Kurz,Liu,Marquard,Smirnov,Smirnov,Steinhauser'15'16]
- numerical integration with `FIESTA` [Smirnov'13]

τ loops:

- asymptotic expansion for $m_\tau \gg m_e$
- analytic calculation
- numerically small

[Kurz,Marquard,Liu,Steinhauser'14]



$(g-2)_\mu$: results

$A_2^{(8)}(e)$	[Kurz,Marquard,Liu,Smirnov, Smirnov,Steinhauser'15'16]		
I(a0)	7.223076	7.223077 ± 0.000029	[Kinoshita et al.'04]
		7.223076	[Laporta'93]
I(a1)	0.494072	0.494075 ± 0.000006	[Kinoshita et al.'04]
		0.494072	[Laporta'93]
I(a2)	0.027988	0.027988 ± 0.000001	[Kinoshita et al.'04]
		0.027988	[Laporta'93]
I(a)	7.745136	7.74547 ± 0.00042	[Aoyama et al.'12]
I(bc0)	8.56876 ± 0.00001	8.56874 ± 0.00005	[Kinoshita et al.'04]
I(bc1)	0.1411 ± 0.0060	0.141184 ± 0.000003	[Kinoshita et al.'04]
I(bc2)	0.4956 ± 0.0004	0.49565 ± 0.00001	[Kinoshita et al.'04]
I(bc)	9.2054 ± 0.0060	9.20632 ± 0.00071	[Aoyama et al.'12]
I(d)	− 0.2303 ± 0.0024	− 0.22982 ± 0.00037	[Aoyama et al.'12]
		− 0.230362 ± 0.000005	[Baikov et al.'95]
II(a)	− 2.77885	− 2.77888 ± 0.00038	[Aoyama et al.'12]
		− 2.77885	[Laporta'93]
II(bc0)	−12.212631	−12.21247 ± 0.00045	[Kinoshita et al.'04]
II(bc1)	− 1.683165 ± 0.000013	− 1.68319 ± 0.00014	[Kinoshita et al.'04]
II(bc)	−13.895796 ± 0.000013	−13.89457 ± 0.00088	[Aoyama et al.'12]
III	10.800 ± 0.022	10.7934 ± 0.0027	[Aoyama et al.'12]
IV(a0)	116.76 ± 0.02	116.759183 ± 0.000292	[Kinoshita et al.'04]
		111.1 ± 8.1	[Calmet et al.'75]
		117.4 ± 0.5	[Chlouber et al'77]
IV(a1)	2.69 ± 0.14	2.697443 ± 0.000142	[Kinoshita et al.'04]
IV(a2)	4.33 ± 0.17	4.328885 ± 0.000293	[Kinoshita et al.'04]
IV(a)	123.78 ± 0.22	123.78551 ± 0.00044	[Aoyama et al.'12]
IV(b)	− 0.38 ± 0.08	− 0.4170 ± 0.0037	[Aoyama et al.'12]
IV(c)	2.94 ± 0.30	2.9072 ± 0.0044	[Aoyama et al.'12]
IV(d)	− 4.32 ± 0.30	− 4.43243 ± 0.00058	[Aoyama et al.'12]

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		0.230362 ± 0.000005	[Baikov et al.'05]
II(a)	− 2.7788		
II(bc0)	−12.2126		
II(bc1)	− 1.6831		
II(bc)	−13.8957		
III	10.800 ±		
IV(a0)	116.76 ±		
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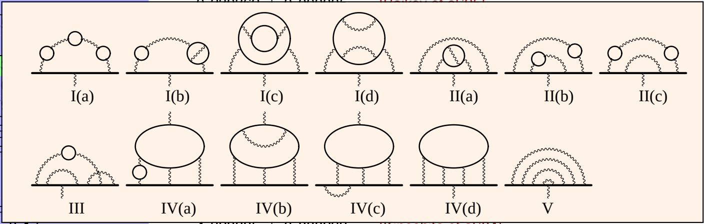

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I(a)	7.745136	7.74547 ± 0.00042	[Aoyama et al.'12]
I(bc0)	8.56876 ± 0.00001	8.56874 ± 0.00005	[Kinoshita et al.'04]
I(bc1)	0.1411 ± 0.0060	0.141184 ± 0.000003	[Kinoshita et al.'04]
I(bc2)	0.4956 ± 0.0004	0.49565 ± 0.00001	[Kinoshita et al.'04]
I(bc)	9.2054 ± 0.0060	9.20632 ± 0.00071	[Aoyama et al.'12]
I(d)	− 0.2303 ± 0.0024	− 0.22982 ± 0.00037	[Aoyama et al.'12]
IV(d)	− 4.32 ± 0.30	− 4.43243 ± 0.00058	[Aoyama et al.'12]

$$a_\mu^{(8)} = a_\mu^{(8)}|_{\rm univ.} \; \overset{e^-}{+132.86(48)} \; \overset{\tau}{+0.0424941(53)} \; \overset{e^-+\tau}{+0.062722(10)}$$ [Kurz et al.'14'15'16]

$$a_\mu^{(8)} = -1.9106(20) + 132.6852(60) + 0.04234(12) + 0.06272(4)$$ [Aoyama et al.12]

final uncertainty

$0.48 \times (\alpha/\pi)^4 \approx 1.4 \times 10^{-11}$

much smaller than uncertainty of

$a_\mu(\mathrm{exp}) - a_\mu(\mathrm{SM}) \sim 250(90) \times 10^{-11}$

even after a projected improvement by a factor 4

Highlight of the year: $(g-2)_e$

Semi-analytic calculation of 4-loop coefficient

$$a_e = \frac{\alpha}{2\pi} + \cdots + c_4 \left(\frac{\alpha}{\pi}\right)^4$$

[Laporta'17]

$$\begin{aligned} c_4 = & \; T_0 + T_2 + T_3 + T_4 + T_5 + T_6 + T_7 \\ & + \sqrt{3}\,(V_{4a} + V_{6a}) + V_{6b} + V_{7b} + W_{6b} + W_{7b} \\ & + \sqrt{3}\,(E_{4a} + E_{5a} + E_{6a} + E_{7a}) + E_{6b} + E_{7b} \\ & + U \end{aligned}$$

- "finalizing a 20-year effort"
- high-precision (few 1000 digits) numerical result for master integrals
- fit to analytic expressions (PSLQ)
  - "usual" transcendental constants
  - HPLs with arguments $1, 1/2, e^{i\pi/2}, e^{i\pi/3}, e^{i2\pi/3}$
  - one-dimensional integrals of products of elliptic integrals
  - six finite parts of master integrals;

$(g-2)_\mu$: photonic contr. and μ loops

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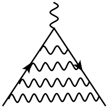
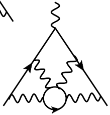
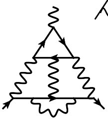
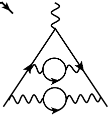
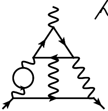
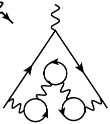
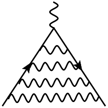
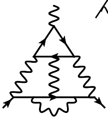
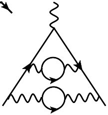
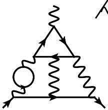
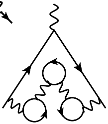
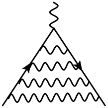
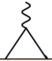
	[Marquard,Smirnov,Smirnov, Steinhauser,Wellmann]	[Aoyama,Hayakawa, Kinoshita,Nio'12]
	-2.161 ± 0.065	-2.1755 ± 0.0020
	0.077 ± 0.031	0.05596 ± 0.0001
	-0.3048 ± 0.021	-0.3162 ± 0.0002
	-0.07461 ± 0.00008	-0.074665 ± 0.000005
	0.597204 ± 0.0012	0.598838 ± 0.000019
	$0.000876865\ldots$	$0.000876865\ldots$

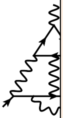
[no "security factor"]

$(g-2)_\mu$: photonic contr. and μ loops

Diagram	[Marquard,Smirnov,Smirnov, Steinhauser,Wellmann]	[Laporta'17]	[Aoyama,Hayakawa, [Kinoshita,Nio'12]
	-2.161 ± 0.065	$-2.176866027739540077443259355895893938670$	-2.1755 ± 0.0020
	0.077 ± 0.031	$0.05611089989782836483146927441890884223$	0.05596 ± 0.0001
	-0.3048 ± 0.021	$-0.31653839064894015884326038238151328482$	-0.3162 ± 0.0002
	-0.07461 ± 0.00008	$-0.0746711843261055138601599657227931268$	-0.074665 ± 0.000005
	0.597204 ± 0.0012	$0.598842072031421820464649513201747727836$	0.598838 ± 0.000019
	$0.000876865\ldots$	$0.000876865858889990697913748939713726165$	$0.000876865\ldots$

$(g-2)_\mu$: photonic contr. and μ loops

	[Marquard,Smirnov,Smirnov, Steinhauser,Wellmann]	[Laporta'17]	[Aoyama,Hayakawa, Kinoshita,Nio'12]
	-2.161 ± 0.065		-2.1755 ± 0.0020
	$-2.176866027739540077443259355895893938670$		
	0.077 ± 0.031		0.05596 ± 0.0001

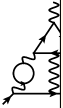


$-1.91298 \pm 0.00084 \longrightarrow -5.56893 + 0.00245 \times 10^{-11}$

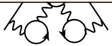
$-1.87 \pm 0.12 \longrightarrow -5.44 + 0.35 \times 10^{-11}$

$-1.9122457649264\ldots \longrightarrow -5.5667960136503 2\ldots \times 10^{-11}$

5



- μ: $a_\mu(\mathrm{exp}) - a_\mu(\mathrm{SM}) \sim 250(90) \times 10^{-11}$
- e: $a_e(\mathrm{SM}) = 1\ 159\ 652\ 18.1664(23)(16)(763) \times 10^{-11}$
  (5l)(hadr+ew)(α)



$0.00087686585888990697913748939713726165$

Conclusions

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- $(g-2)_e$ cross checked: semi-analytic calculation
- $(g-2)_\mu$

photonic, μ loops cross checked: 3 calculations

e loops
τ loops

} cross checked: { purely numerical calc.; asymptotic expansion + numerical/analytic calc.

[Aoyama,Hayakawa,Kinoshita,Nio'12]

group	$A_2^{(8)}(e)$	$A_2^{(8)}(\tau)$	$A_3^{(8)}(e,\tau)$
I(a)	7.74547 (42)	0.000032 (0)	0.003209 (0)
I(b)	7.58201 (71)	0.000252 (0)	0.002611 (0)
I(c)	1.624307 (40)	0.000737 (0)	0.001807 (0)
I(d)	-0.22982 (37)	0.000368 (0)	0
II(a)	-2.77888 (38)	-0.007329 (1)	0
II(b)	-4.55277 (30)	-0.002036 (0)	-0.009008 (1)
II(c)	-9.34180 (83)	-0.005246 (1)	-0.019642 (2)
III	10.7934 (27)	0.04504 (14)	0
IV(a)	123.78551 (44)	0.038513 (11)	0.083739 (36)
IV(b)	-0.4170 (37)	0.006106 (31)	0
IV(c)	2.9072 (44)	-0.01823 (11)	0
IV(d)	-4.43243 (58)	-0.015868 (37)	0