Measurement of the running of the fine structure constant below 1 GeV with the KLOE detector

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on behalf of the KLOE-2 Collaboration

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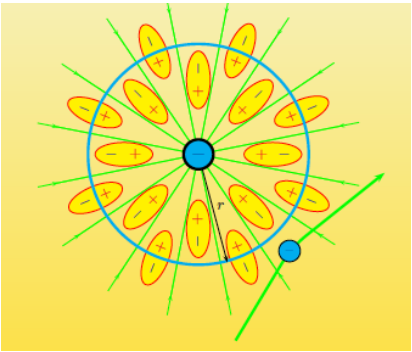
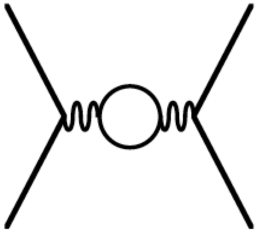
α_{em} running and the Vacuum Polarization

- Due to Vacuum Polarization effects $\alpha_{em}(q^2)$ is a running parameter from its value at vanishing momentum transfer to the effective q^2.



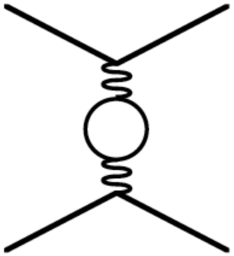
- The "Vacuum Polarization" function $\Pi(q^2)$ can be "absorbed" in a redefinition of an effective charge:



$$e^2 \to e^2(q^2) = \frac{e^2}{1+(\Pi(q^2)-\Pi(0))} \qquad \alpha(q^2) = \frac{\alpha(0)}{1-\Delta\alpha}; \quad \Delta\alpha = -\Re e\left(\Pi(q^2)-\Pi(0)\right)$$

$$\Delta\alpha = \Delta\alpha_l + \Delta\alpha^{(5)}_{had} + \Delta\alpha_{top}$$

- Δa takes a contribution by non perturbative hadronic effects ($\Delta a^{(5)}_{had}$) which exibits a different behaviour in time-like and space-like region



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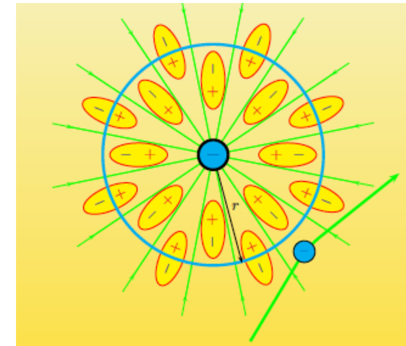

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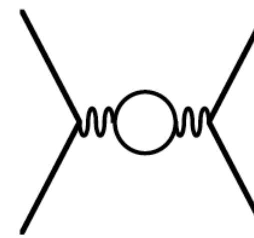

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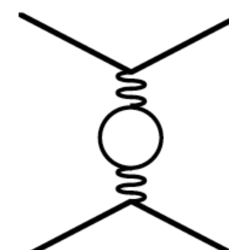
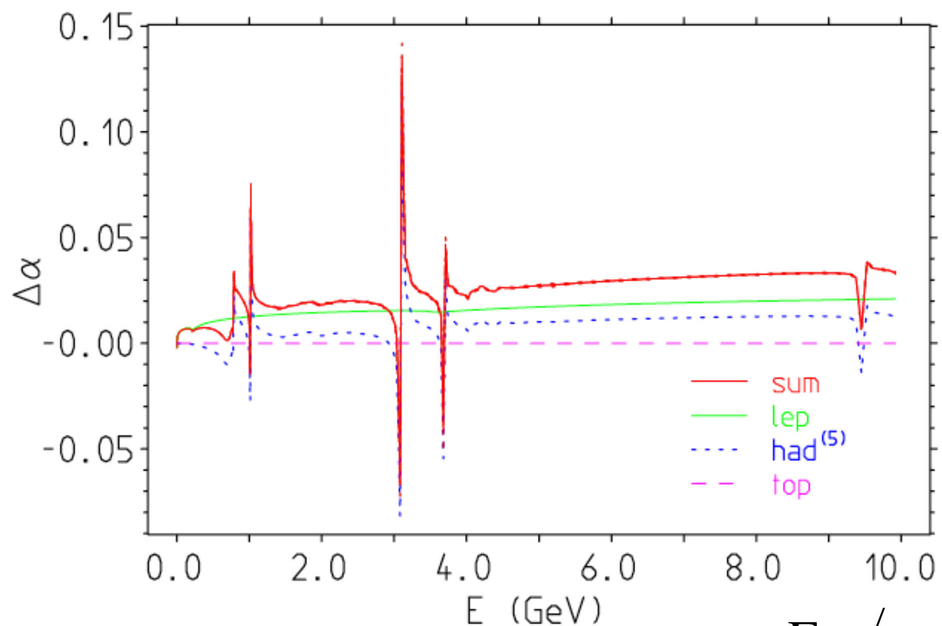
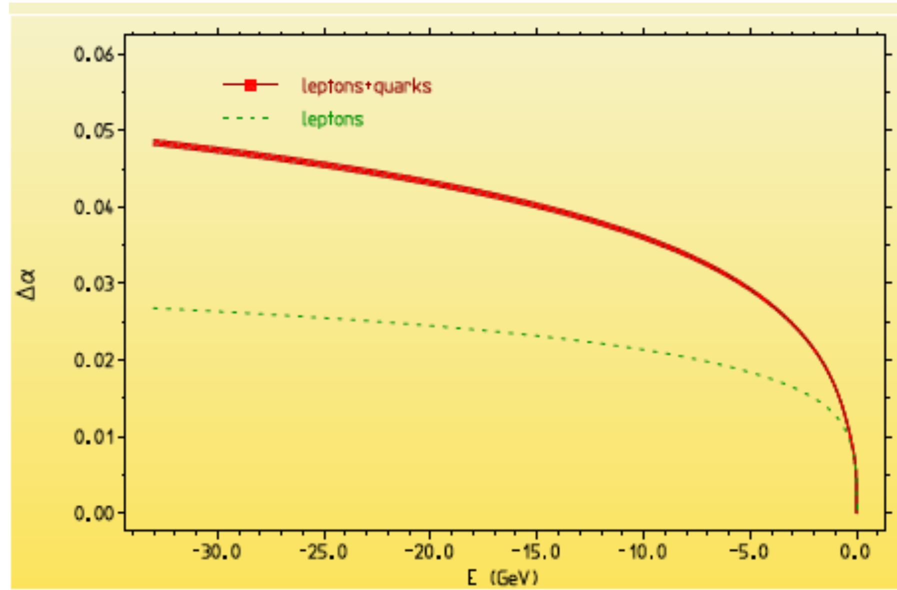


Running of α_{em}





$E=\sqrt{s}$

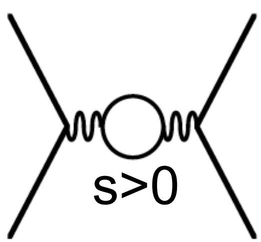
Time-like

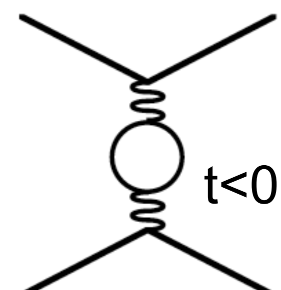
$E=-\sqrt{-t}$

Space-like

Behaviour characterized by the opening of resonances

Very smooth behaviour



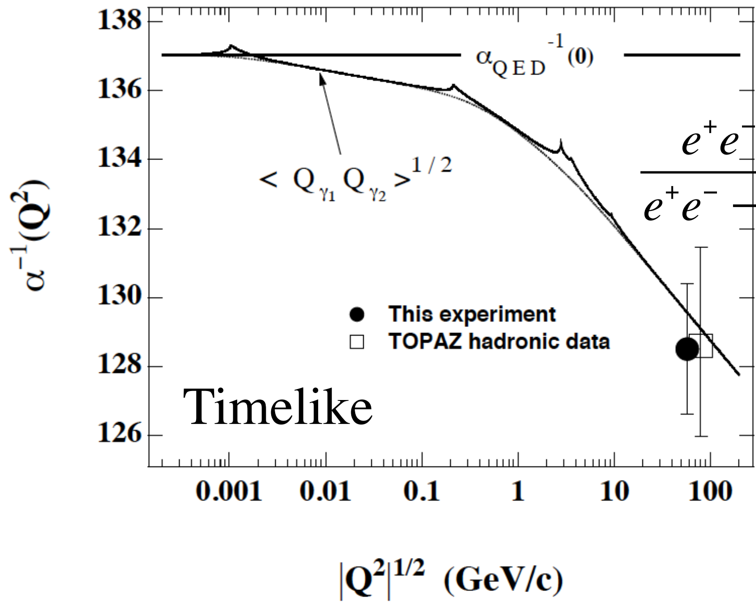


$$\Delta\alpha_{had}^{(5)}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi}\,\mathrm{Re}\int_{4m_\pi^2}^{\infty} ds\,\frac{R(s)}{s(s-M_Z^2-i\varepsilon)}$$

$$\Delta\alpha_{had}^{(5)}(-q_0^2) = -\frac{\alpha M_Z^2}{3\pi}\,\mathrm{Re}\int_{4m_\pi^2}^{\infty} ds\,\frac{R(s)}{s(s+q_0^2)}$$

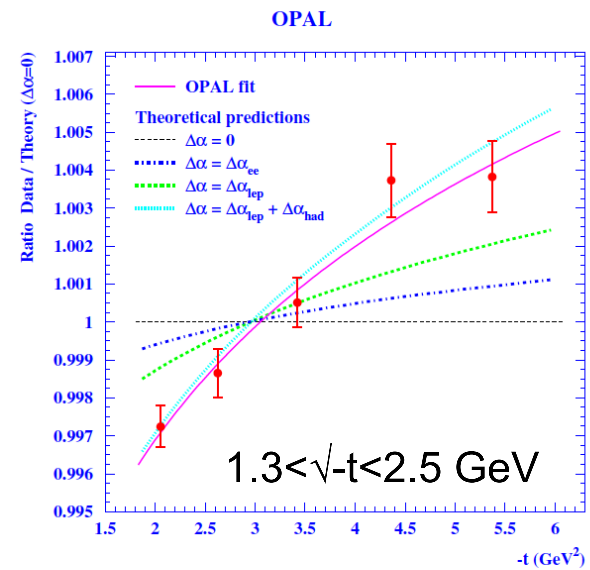
Direct measurement of α_{em} running

e+e- collider TRISTAN at √s=57.8 GeV,

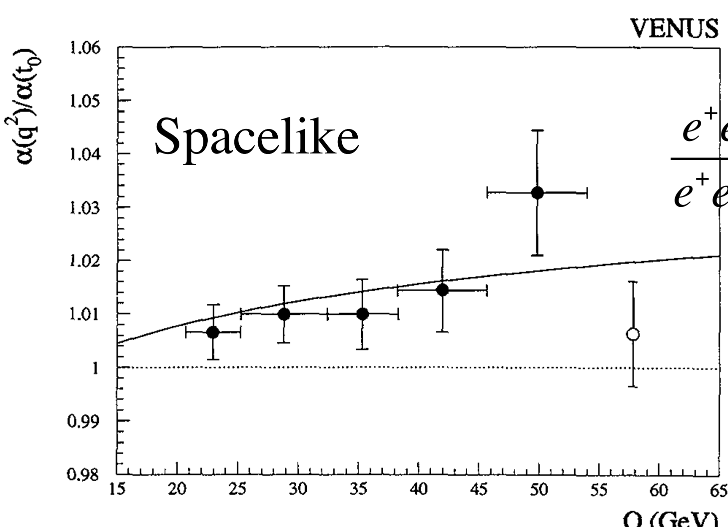


$$\frac{e^+e^- \to \mu^+\mu^-}{e^+e^- \to e^+e^-\mu^+\mu^-}$$

e+e- collider LEP at √s=189 GeV, using **Bhabha** events

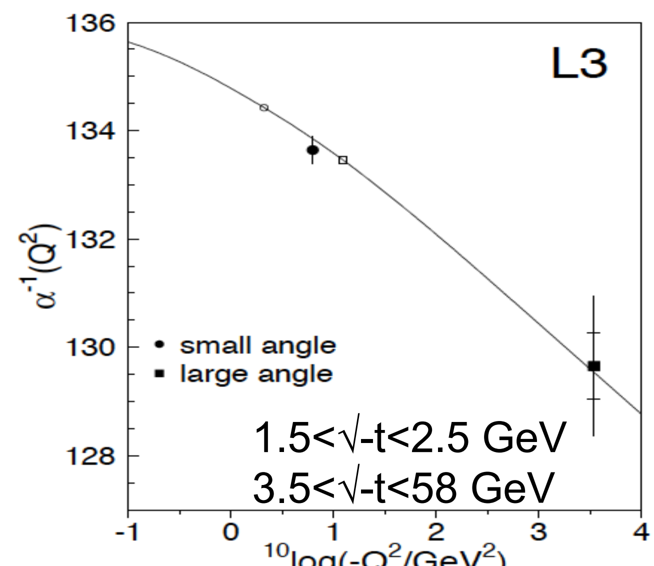


Spacelike



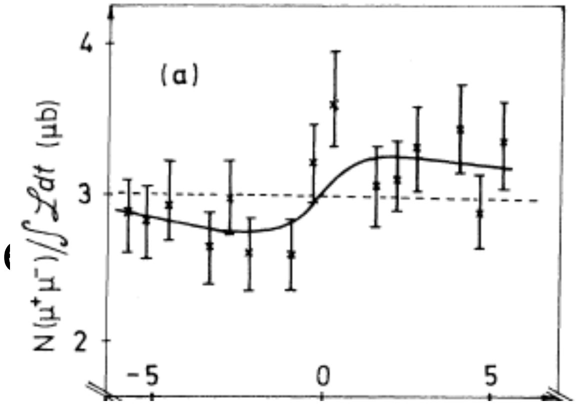
$$\frac{e^+e^- \to e^+e^-}{e^+e^- \to \mu^+\mu^-}$$

10<√-t<54 GeV



Previous tests of the hadronic contribution to VP

1) '73: $\phi(1020)$ contribution to VP at ACO (Orsay e+e-) in the $e^+e^- \rightarrow \mu+\mu-$ process: evidence at 3σ in the region ±5 MeV around the ϕ peak **[Phys.Rev.Lett. 30 (1973) 462-464)]**



-5 MeV m_ϕ 5 MeV

2) 70's: g-2 experiment at CERN: evidence for hadronic contribution to g-2 at 6σ **[Phys. Lett. 67B (1977) 225; Phys. Lett. 68B (1977) 191]**

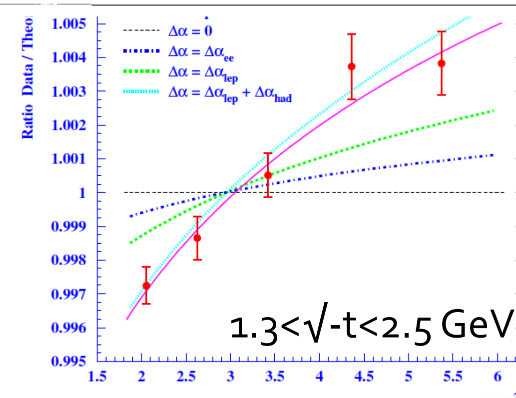
exp

$a_\mu = 1\ 165\ 924\ (8.5) \times 10^{-9}$ (7 ppm).

theo

QED terms	Muon	Numerical values ($\times 10^9$)	
2nd order: A	0.5	Total QED:	1 165 852 (1.9)
4th order: B	0.765 782 23	Strong interactions:	66.7 (8.1)
6th order: C	24.452 (26)	Weak interactions:	2.1 (0.2)
8th order: D	135 (63)	Total theory:	1 165 921 (8.3)
10th order: E	420 (30)		

3) 2006: OPAL at LEP: evidence for hadronic contributon Δa_{had} (t) (t<0) at 3σ in Bhabha scattering at small angle **[Eur.Phys.J. C45 (2006) 1-21]**



KLOE measurement of $\alpha(s)$ below 1 GeV with 1.7 fb^{-1} *Phys. Lett. B 767 (2017) 485-492*

- Measurement of the running of the fine structure constant α in the time-like region $0.6<\sqrt{s}<0.975$ GeV obtained via ISR :

$$\left|\frac{\alpha(s)}{\alpha(0)}\right|^2 = \frac{d\sigma_{data}(e^+e^- \to \mu^+\mu^-\gamma(\gamma))|_{ISR}/d\sqrt{s}}{d\sigma^0_{MC}(e^+e^- \to \mu^+\mu^-\gamma(\gamma))|_{ISR}/d\sqrt{s}} \quad : \frac{\text{data}}{\text{MC with } \alpha(s) = \alpha(0)}$$

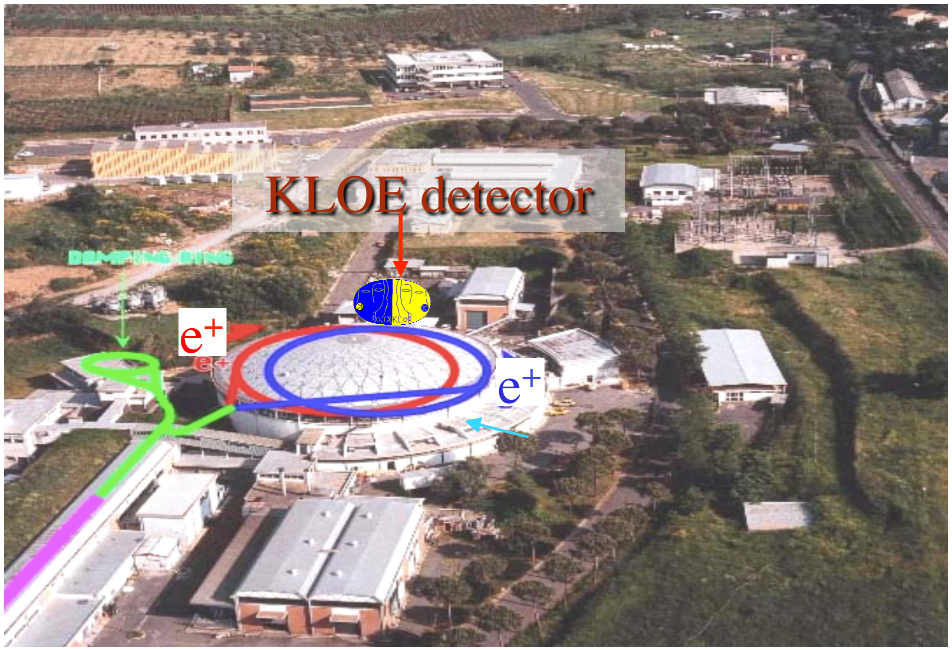
- FSR correction done using PHOKHARA MC event generator
  [H. Czyz, A. Grzelinska, J.H. Kuhn, G. Rodrigo, Eur. Phys. J. C 39 (2005) 411]

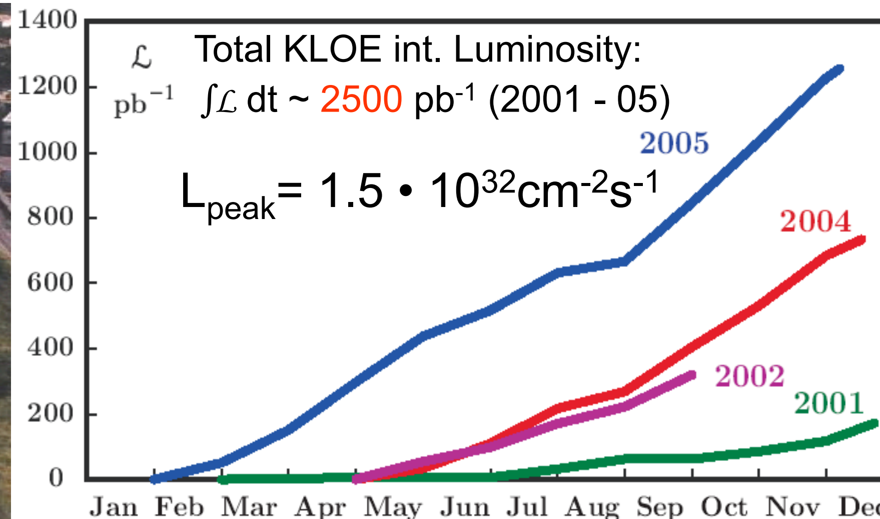
- Statistical significance of the hadron contribution to the effective $\alpha(s)$ is evaluated
- For the first time in a single experiment the real and Imaginary part of $\Delta\alpha$
- Measurement of BR($\omega \to \mu^+\mu^-$)

DAΦNE: A Φ-Factory in Frascati (near Rome)

e^+e^- - collider with $\sqrt{s}=m_\Phi \approx 1.0195$ GeV

Integrated Luminosity





2006:

- Energy scan (4 points around m_Φ-peak)
- 240 pb^{-1} at = 1000 MeV (off-peak data)

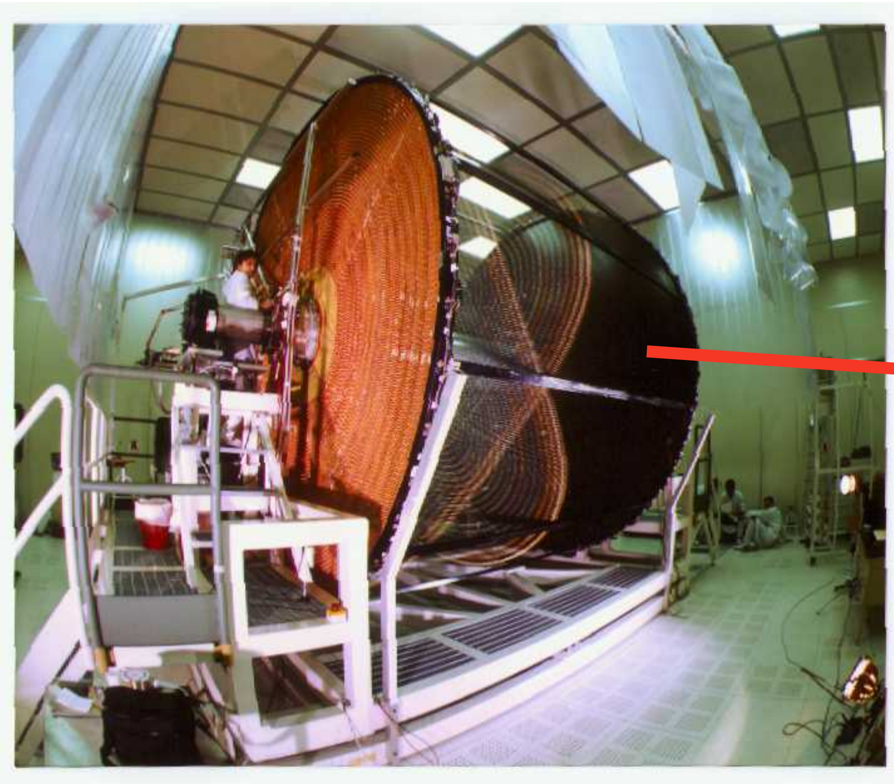
DAΦNE is running with the KLOE-2 detector with the aim to collect additional 5fb^{-1} (see Berlowski & Perez del Rio presentations) [~4fb^{-1} collected so far]

Our analysis is based on 1.7pb^{-1} of data taken in 2004/05

Phys. Lett. B 767 (2017) 485-492

KLOE Detector

Drift chamber

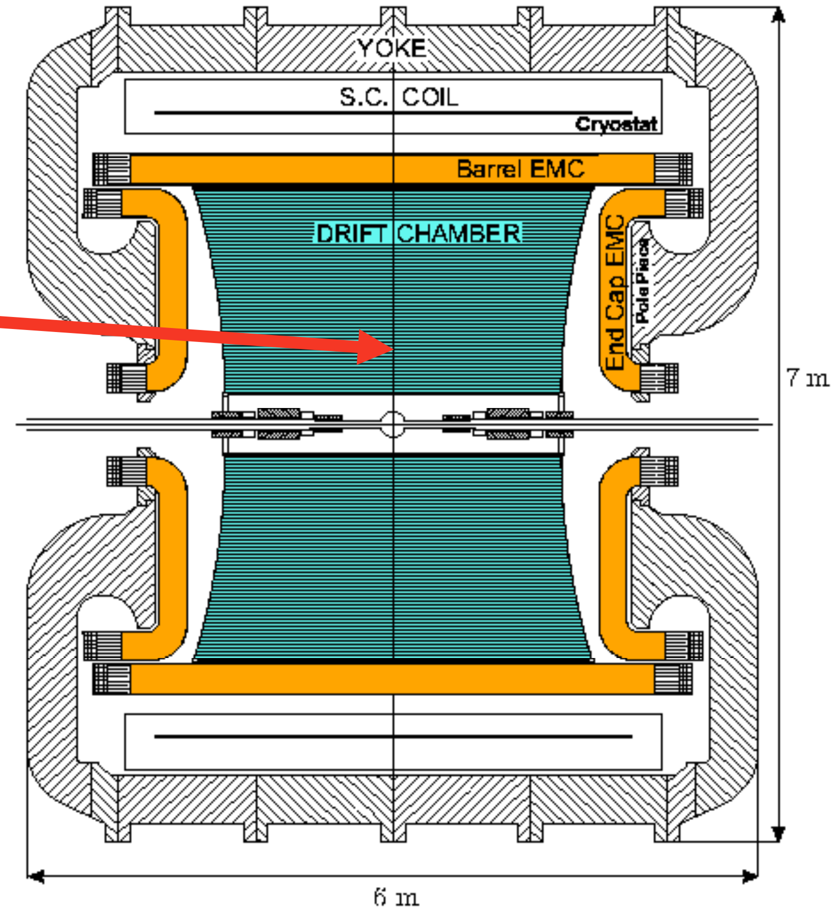


σ_p/p = 0.4% (for 90^0 tracks)

$\sigma_{xy} \approx$ 150 mm, $\sigma_z \approx$ 2 mm

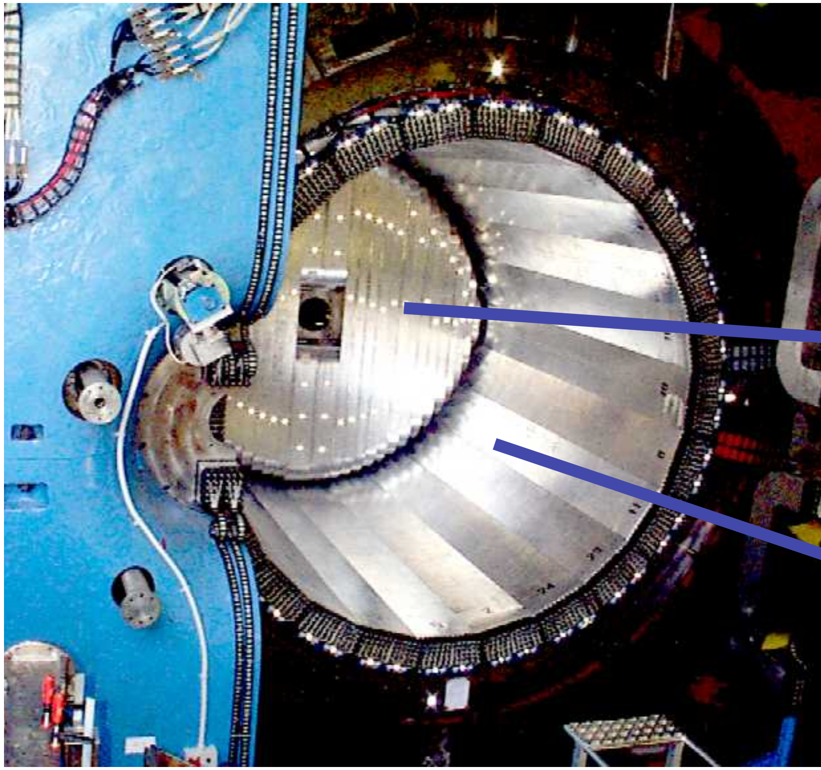
Excellent momentum resolution

Full stereo geometry, 4m diameter,
52.140 wires 90% Helium, 10% iC_4H_{10}

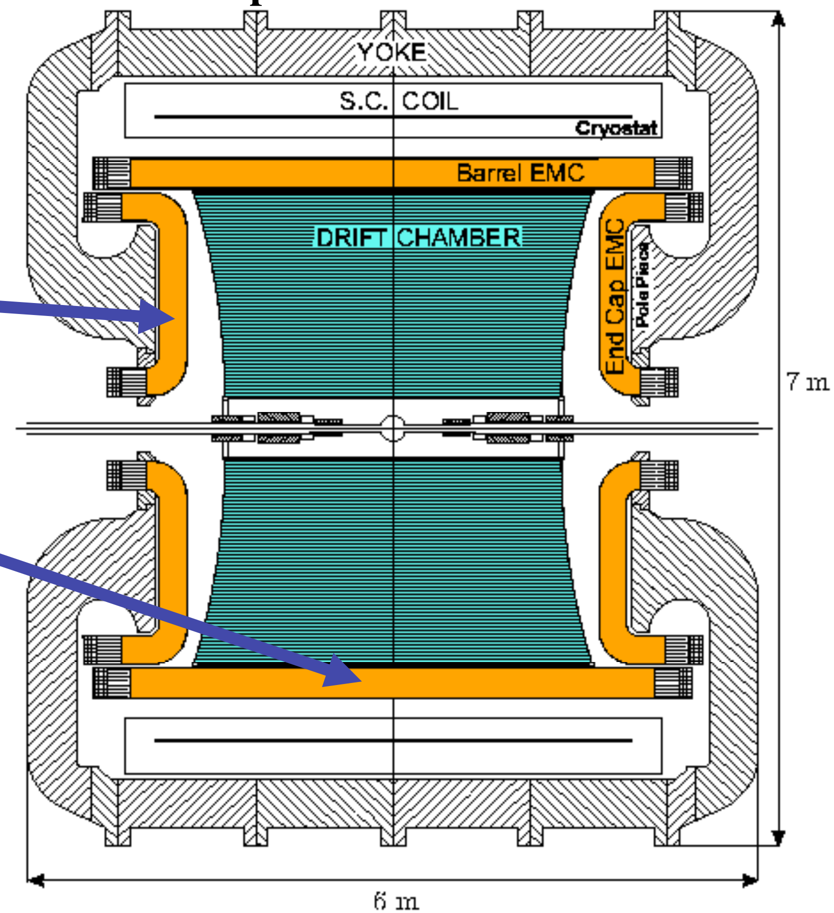


KLOE Detector

Electromagnetic Calorimeter



Pb / scintillating fibres (4880 PMT)
Endcap - Barrel - Modules



$$\sigma_E/E = 5.7\% / \sqrt{E(\text{GeV})}$$

$$\sigma_T = 54 \text{ ps} / \sqrt{E(\text{GeV})} \oplus 100 \text{ ps}$$

(Bunch length contribution subtracted from constant term)

Excellent timing resolution

Event Selection: Small Angle (SA)

Muon tracks at large angles

$50^o < \theta_\mu < 130^o$

Photons at small angles

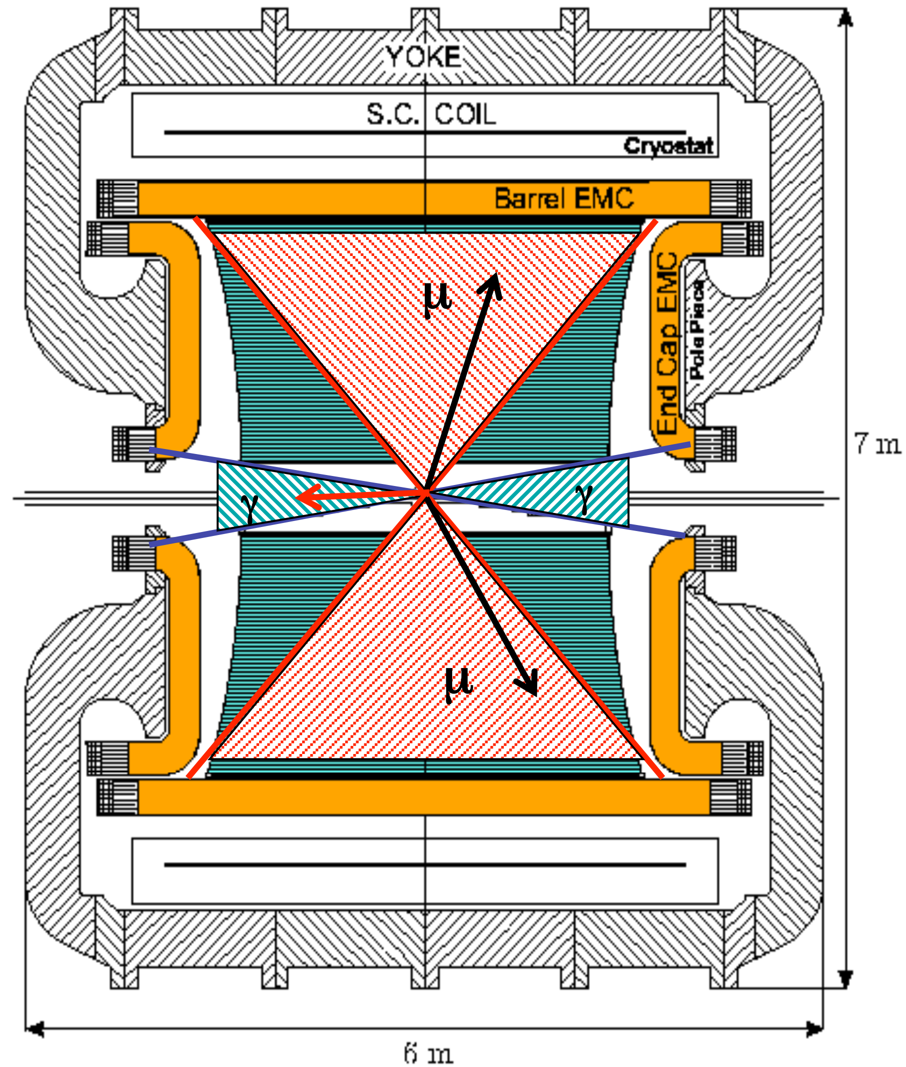
$\theta_\gamma < 15^o$ or $\theta_\gamma > 165^o$

➔ **Photon momentum from kinematics:**

$$\vec{p}_\gamma = \vec{p}_{miss} = -(\vec{p}_+ + \vec{p}_-)$$

- High statistics for ISR photons
- Very small contribution from FSR
- Reduced background contamination

KLOE



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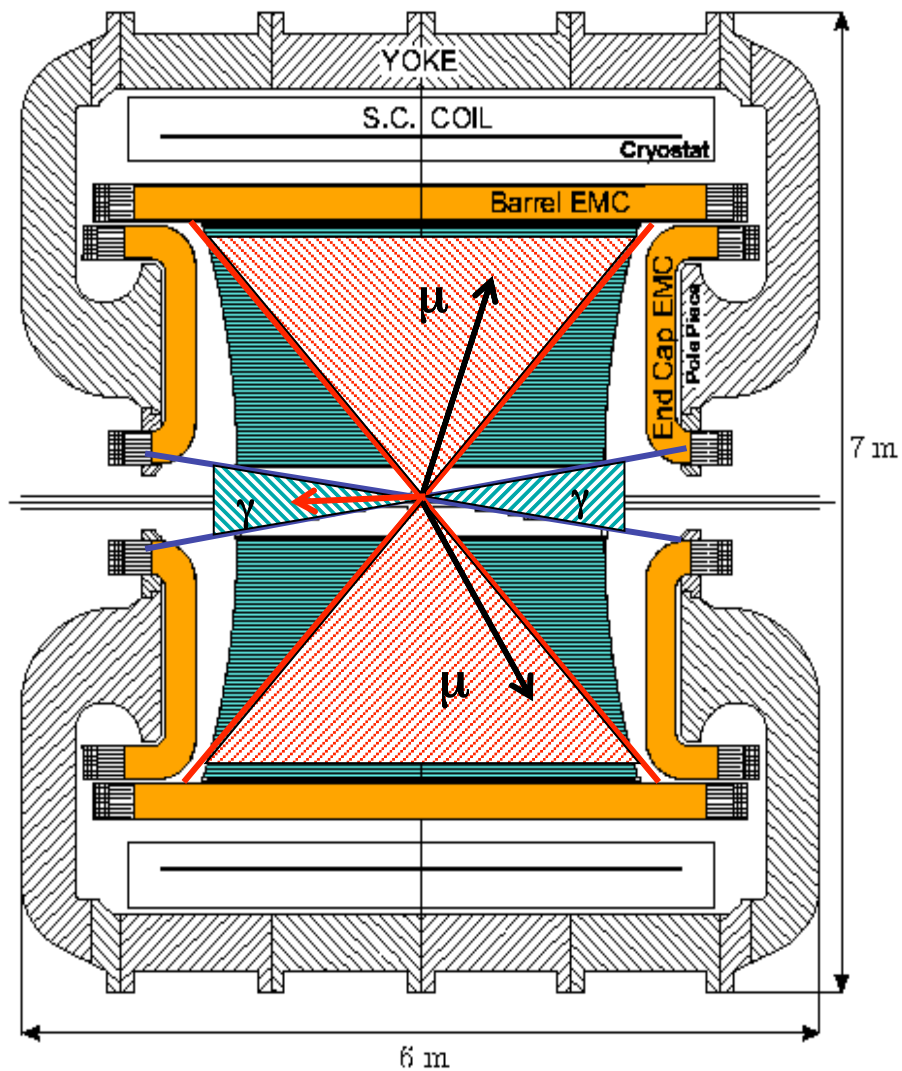
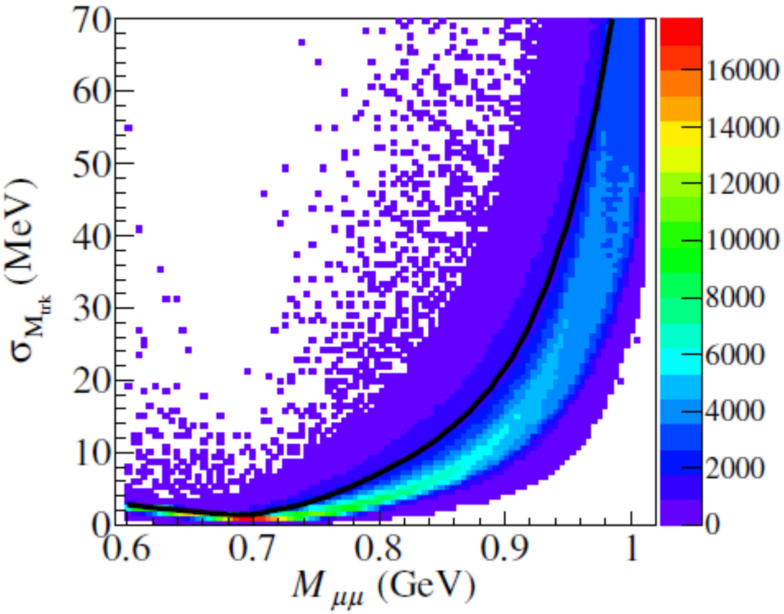
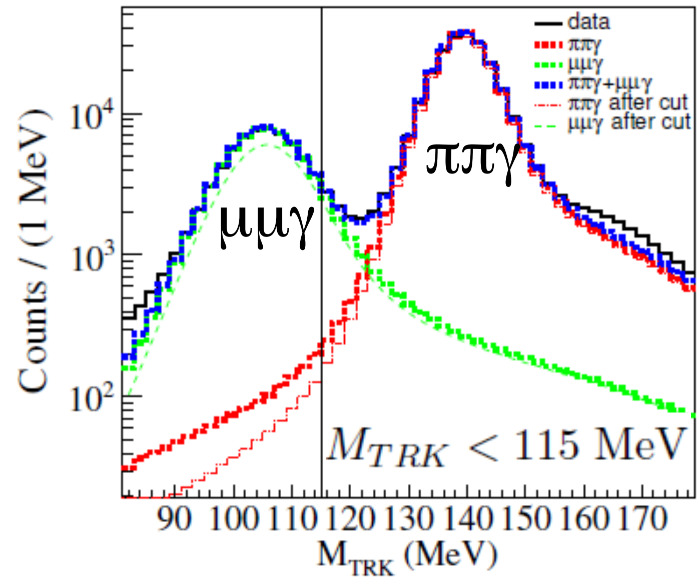


- Careful work to achieve a control of ~1% in the muon selection and background subtraction
- Efficiencies done on data

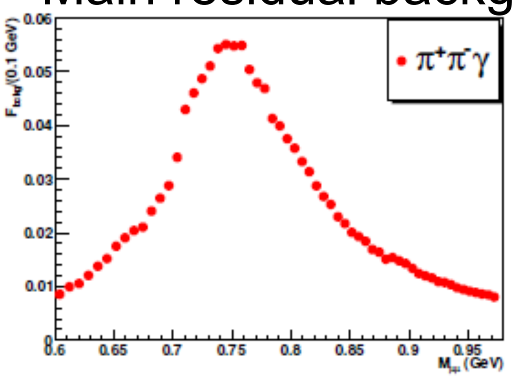
Main cuts

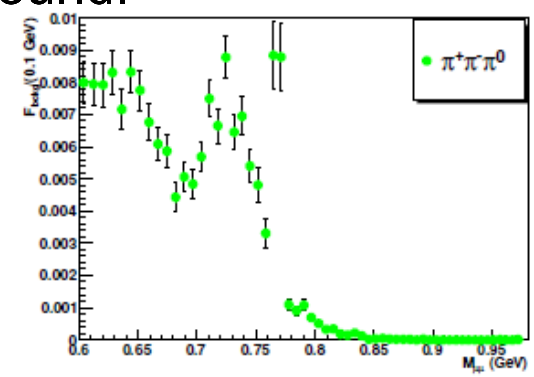


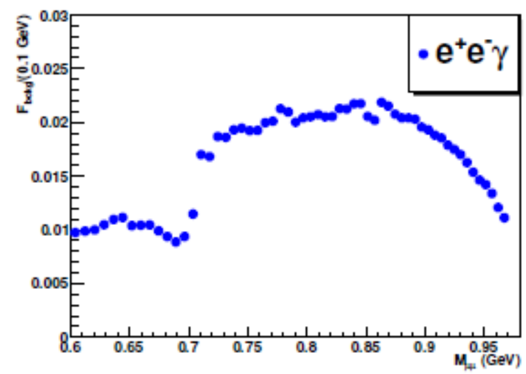
$$(\sqrt{s} - \sqrt{|p_+|^2 + M_{TRK}^2} - \sqrt{|p_-|^2 + M_{TRK}^2})^2 - (p_+ + p_-)^2 = 0$$



Main residual background:





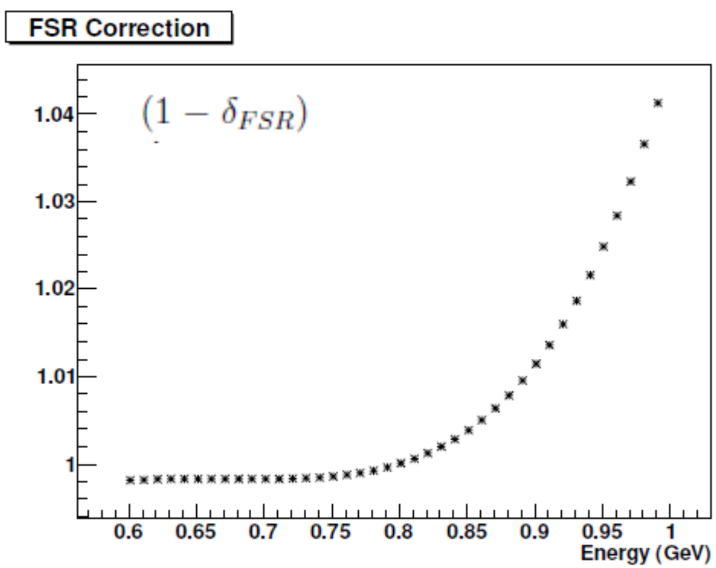


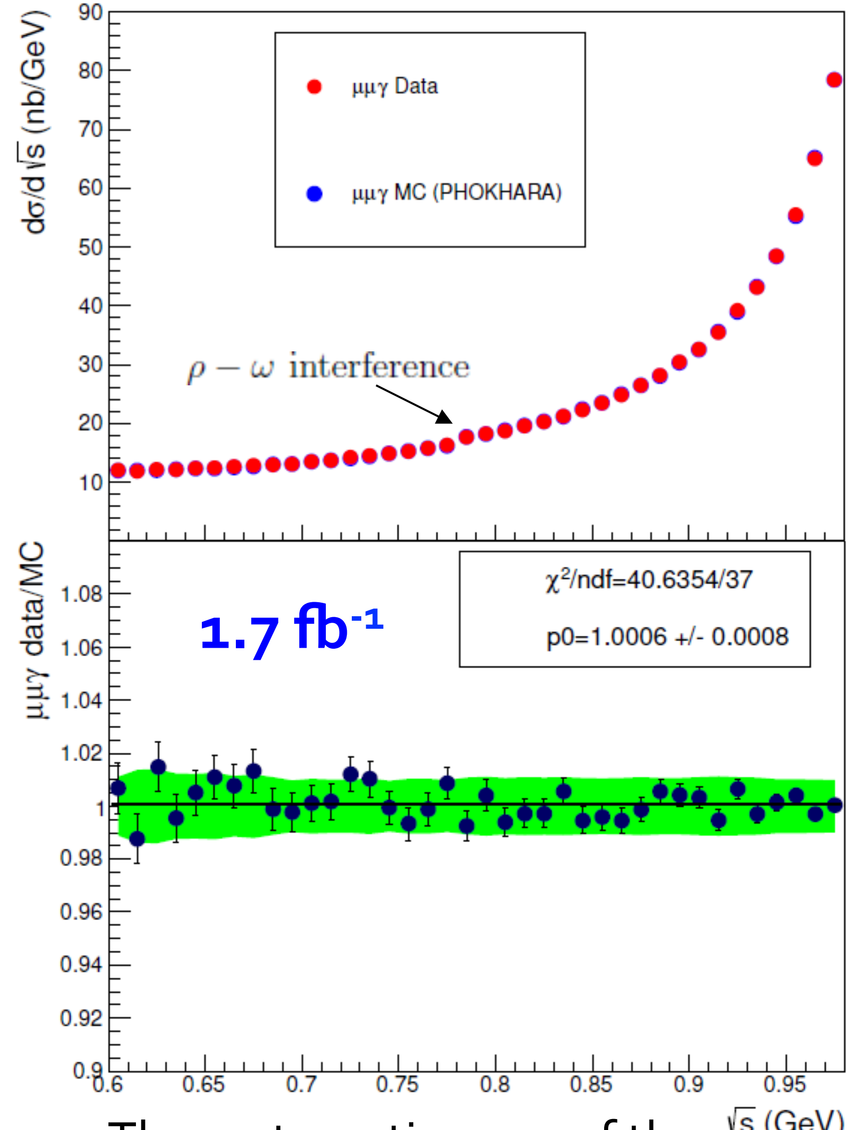
About 4.5 x 10^6 μμγ events pass these selection criteria

μμγ cross section measurement

Integrated Luminosity 1.7 fb^{-1}

$$\left.\frac{d\sigma(e^+e^- \to \mu^+\mu^-\gamma(\gamma))}{d\sqrt{s}}\right|_{ISR} = \frac{N_{obs} - N_{bkg}}{\Delta\sqrt{s}} \cdot \frac{(1-\delta_{FSR})}{\epsilon(\sqrt{s}) \cdot L}$$





Excellent agreement with NLO μμγ(γ) cross section (PHOKHARA NLO)

The systematic error of the order of 1% - (green band)

Measurement of the running of $\alpha(s)$

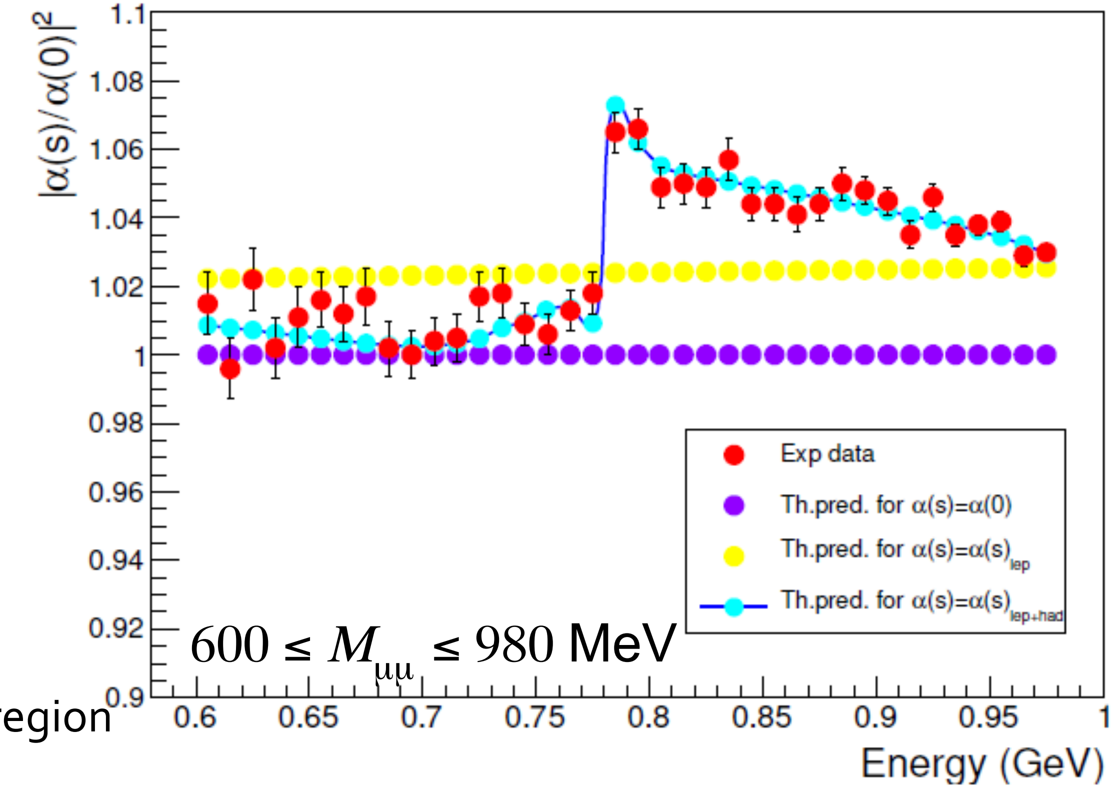
$$\left|\frac{\alpha(s)}{\alpha(0)}\right|^2 = \frac{\frac{d\sigma^{ISR}}{dM_{\mu\mu}}}{\frac{d\sigma^{MC}}{dM_{\mu\mu}}}$$

MC with $\alpha=\alpha(0)$

$$\left|\frac{\alpha(s)}{\alpha(0)}\right|^2 = \frac{1}{|1-\Delta\alpha(s)|^2}$$

$$\Delta\alpha(s) = \Delta\alpha_{lep} + \Delta\alpha_{had}$$

(we neglect the top contribution)



– Δa_{had} obtained by dispersive approach using data in time-like region provided by F. Jegerlehner

- Excellent agreement with other R compilation (Teubner / Ignatov)

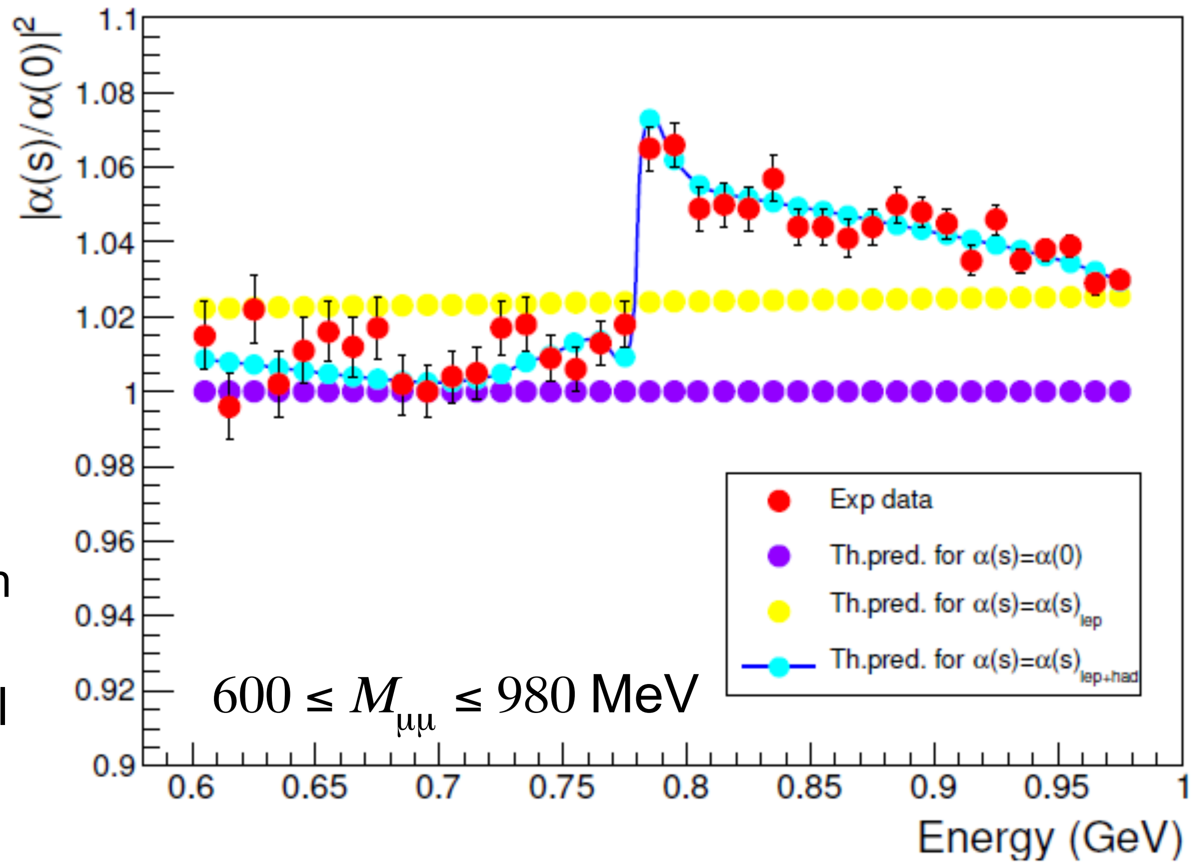
Systematic error at ~1% level

$$\Delta\alpha_{had}(s) = -\left(\frac{\alpha(0)s}{3\pi}\right) Re \int_{m_\pi^2}^{\infty} ds' \frac{R(s')}{s'(s'-s-i\epsilon)} \qquad R(s) = \frac{\sigma_{tot}(e^+e^-\to\gamma*\to hadrons)}{\sigma_{tot}(e^+e^-\to\gamma*\to\mu^+\mu^-)}$$

Test of leptonic and hadronic contribution to $\alpha(s)$

χ^2 -test for two hypotheses:
no running and running due to lepton pairs only is performed.

We exclude the only-leptonic hypothesis at 6σ
Our result is consistent with the lepton and hadron hypothesis with a statistical significance of 0.3
(χ^2/ndf = 41.2/37).



Hypothesis	$\chi^2/d.o.f$ stat.	$\chi^2/d.o.f.$ stat.+ syst.
$\Delta\alpha(s) = 0$	2189/37	382/37
$\Delta\alpha(s)_{lep}$	475/37	119/37

Systematics

Syst. errors	$\sigma_{\mu\mu\gamma}$	$\lvert\alpha(s)/\alpha(0)\rvert^2$
Trigger	$< 0.1\%$	
Tracking	s dep. (0.5% at ρ-peak)	
Particle ID	$< 0.1\%$	
Background subtraction	s dep. (0.1% at ρ-peak)	
M_{TRK}	0.4%	
σ_{MTRK}	s dep. (0.05% at ρ-peak)	
Acceptance	s dep. (0.3% at ρ-peak)	
Software Trigger	0.1%	
Luminosity	0.3%	
$\Delta\alpha_{had}$ dep. (Normalization)	-	0.2%
FSR treatment	0.2%	
Rad. function H	-	0.5%
Total systematic error	s dep. (0.7% at $\rho-$peak)	(0.9% at $\rho-$peak)

Real and Imaginary part of $\Delta\alpha(s)$

In the contribution to the running of α the imaginary part is usually neglected. This approximation is not sufficient in the presence of resonances like the ρ meson, where the accuracy of the cross section measurements reaches the order of (or even less than) 1%.

$$\alpha(s) = \frac{\alpha(0)}{1-\Delta\alpha}; \quad \Delta\alpha = -\left(\Pi(s) - \Pi(0)\right)$$

$$Re\,\Delta\alpha = 1 - \sqrt{|\alpha(0)/\alpha(s)|^2 - (Im\,\Delta\alpha)^2}.$$

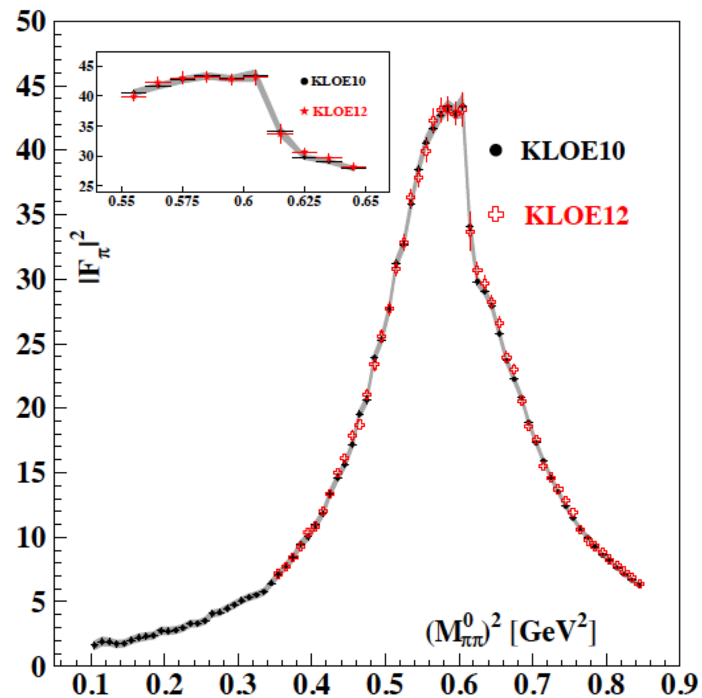
$$Im\Delta\alpha = -\tfrac{\alpha}{3} R(s) \qquad R(s) = \sigma_{tot} / \frac{4\pi\alpha(s)^2}{3s}$$

$$R(s) = R_{lep}(s) + R_{had};$$

$$R_{lep} = \sqrt{1 - \frac{4m_l^2}{s}}\left(1 + \frac{2m_l^2}{s}\right) \qquad (l = e, \mu, \tau)$$

$$R_{had}(s) = \frac{1}{4}\left(1 - \frac{4m_\pi^2}{s}\right)^{\frac{3}{2}} |F_\pi^0(s)|^2$$

$$|F_\pi^0(s)|^2 = |F_\pi(s)|^2 \left|\frac{\alpha(0)}{\alpha(s)}\right|^2$$



Physics Letters B 720 (2013) 336-343

In collaboration with F. Jegerlehner

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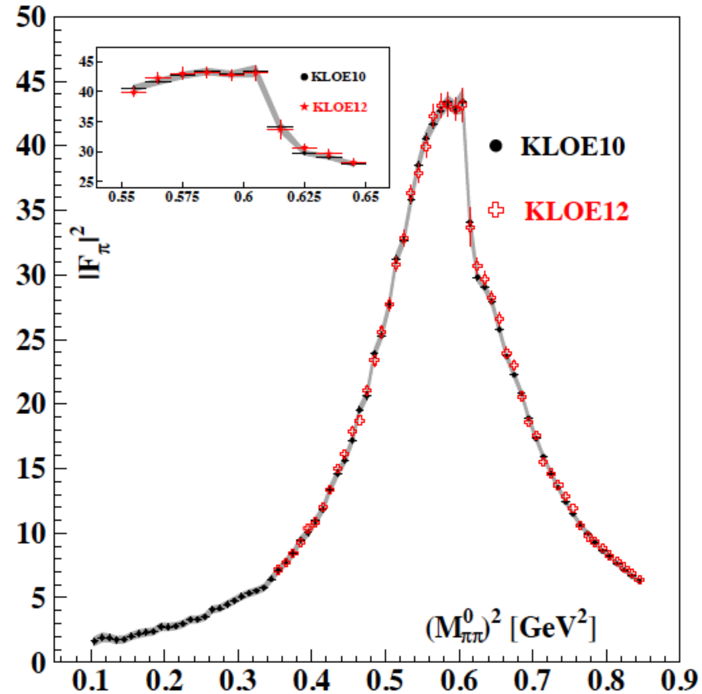

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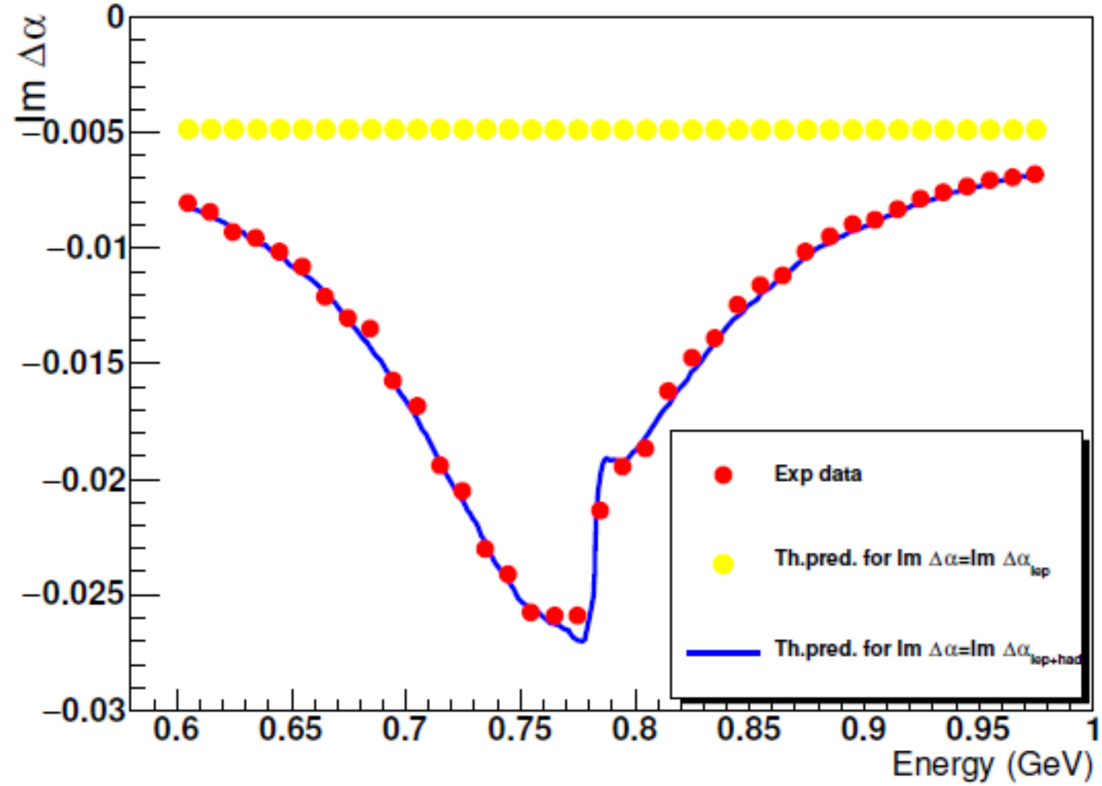
Imaginary part of $\Delta\alpha(s)$

$$Im\Delta\alpha = -\frac{\alpha}{3} R(s)$$

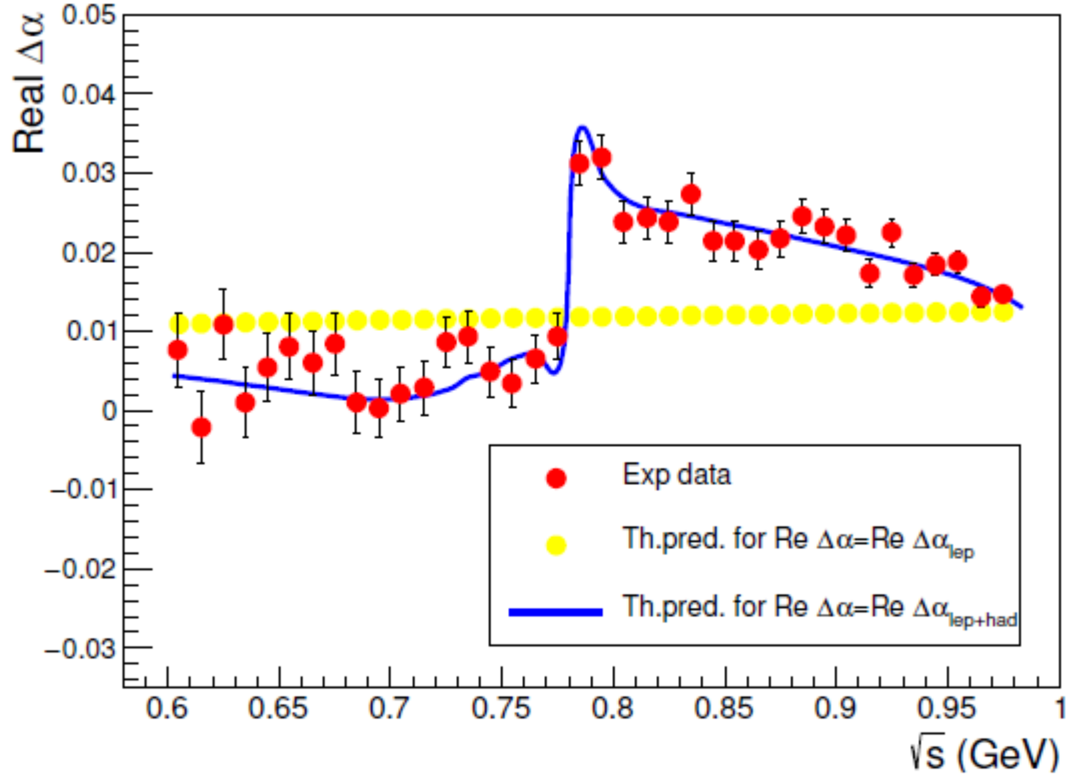
Results obtained for the 2π contribution to $\Delta\alpha$ by using KLOE12 measurement of the pion form factor (red full circles) and the ones obtained by using the $R_{had}(s)$ compilation with the 2π channel only and removing KLOE data (blue solid line).



Real part of $\Delta\alpha(s)$

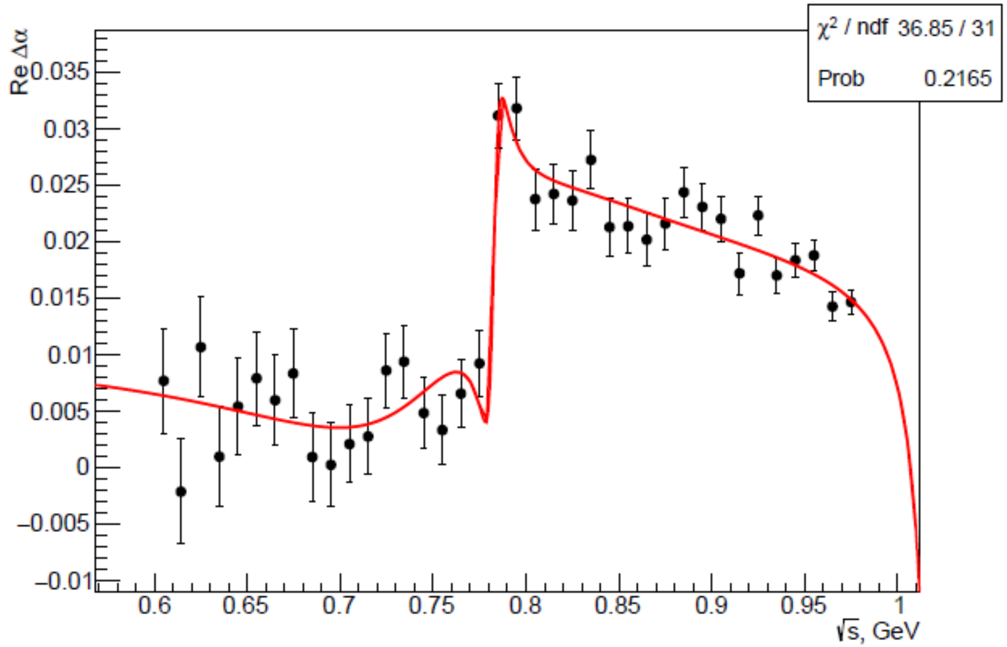
$$Re\,\Delta\alpha = 1 - \sqrt{|\alpha(0)/\alpha(s)|^2 - (Im\,\Delta\alpha)^2}.$$

Re$\Delta\alpha$ obtained by KLOE $\mu\mu\gamma$ data compared with theoretical prediction with leptonic contribution only and with leptonic and hadronic contributions. Excellent agreement for Re $\Delta\alpha(s)$ has been obtained with the data-based compilation



Fit of Re Δα(s)

We fit ReΔα by a sum of the leptonic and hadronic contributions, where the hadronic contribution is parametrized as a sum of ρ(770), ω(782) and ϕ(1020) resonances components and a non resonant term (param. with a pol1)



$$Re\,\Delta\alpha_{V=\omega,\phi} = \frac{3\sqrt{BR(V\to e^+e^-)\cdot BR(V\to\mu^+\mu^-)}}{\alpha M_V}\frac{s(s-M_V^2)}{(s-M_V^2)^2+M^2\Gamma_V^2}$$

For ω, ϕ

$$F_\pi(s) = BW^{GS}_{\rho(s)} = \frac{M_\rho^2(1+d\Gamma_\rho/M_\rho)}{M_\rho^2 - s + f(s) - iM_\rho\Gamma_\rho(s)}$$

For ρ, neglecting interference with ω and high exc. stat. of ρ

Γ_ω , M_ϕ, Γ_ϕ, and $BR(\phi\to e^+e^-)BR(\phi\to\mu^+\mu^-)$ fixed to PDG values [pdg]

[pdg] K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014) and 2015 update.

Fit of Re Δα(s)

$$BR(\omega \to \mu^+\mu^-)BR(\omega \to e^+e^-) = (4.3 \pm 1.8 \pm 2.2) \cdot 10^{-9}$$

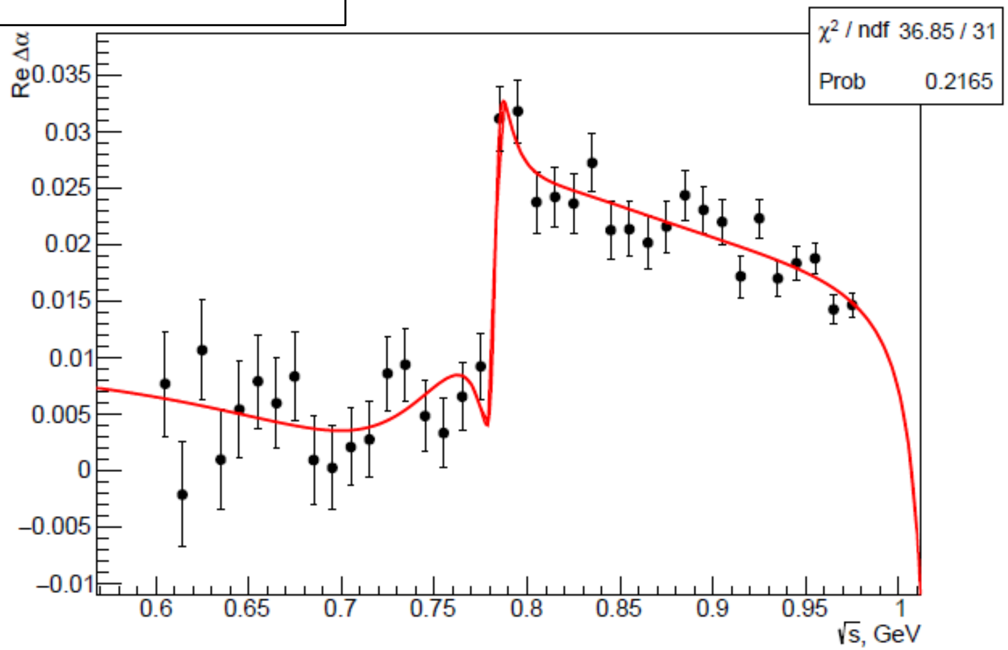
Assuming lepton universality and multiplying for the phase space correction

$$\xi = \left(1 + 2\frac{m_\mu^2}{m_\omega^2}\right)\left(1 - 4\frac{m_\mu^2}{m_\omega^2}\right)^{1/2}$$

$$BR(\omega \to \mu^+\mu^-) =$$

$$= (6.6 \pm 1.4_{stat} \pm 1.7_{syst}) \cdot 10^{-5}$$

$$(9.0 \pm 3.1) \cdot 10^{-5} \text{ from PDG}$$

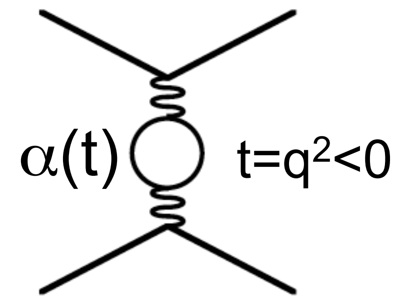


Parameter	Result from the fit	PDG
M_ρ, MeV	775 ± 6	775.26 ± 0.25
Γ_ρ, MeV	146 ± 9	147 ± 0.9
M_ω, MeV	782.7 ± 1.0	782.65 ± 0.12
$BR(\omega \to \mu^+\mu^-)BR(\omega \to e^+e^-)$	$(4.3 \pm 1.8) \cdot 10^{-9}$	$(6.5 \pm 2.3) \cdot 10^{-9}$
χ^2/ndf	1.19	-

Inclusion of ω−ρ interference doesn't change the results (within the error)

FUTURE: a_μ^{HLO} from $\Delta\alpha(t)$ in the space-like region

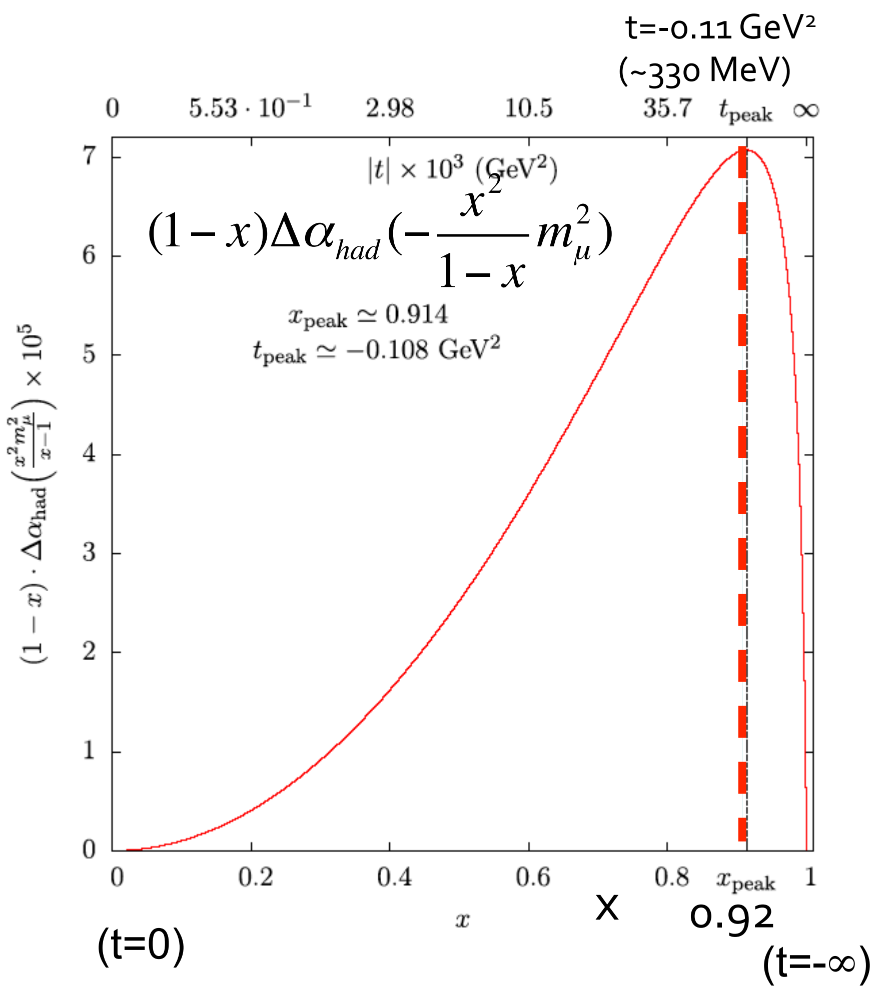
$$a_\mu^{HLO} = -\frac{\alpha}{\pi}\int_0^1 (1-x)\Delta\alpha_{had}\left(-\frac{x^2}{1-x}m_\mu^2\right)dx$$



$$t = \frac{x^2 m_\mu^2}{x-1} \qquad 0 \le -t < +\infty$$

$$x = \frac{t}{2m_\mu^2}\left(1-\sqrt{1-\frac{4m_\mu^2}{t}}\right); \qquad 0 \le x < 1;$$

- **a_μ^{HLO} is given by the integral of the curve** (smooth behaviour)
- **It requires a measurement of** the hadronic contribution to the effective electromagnetic coupling in the space-like region **$\Delta\alpha_{had}(t)$ ($t=q^2<0$)**
- **It enhances** the contribution from **low** q^2 region (below 0.11 GeV²)
- **Its precision** is determined by the **uncertainty** on $\Delta\alpha_{had}(t)$ in this region



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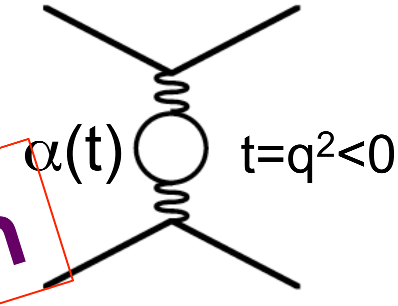

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See Trentadue's presentation

- **a_μ^{HLO} is given by the integral of the curve** (smooth behaviour)
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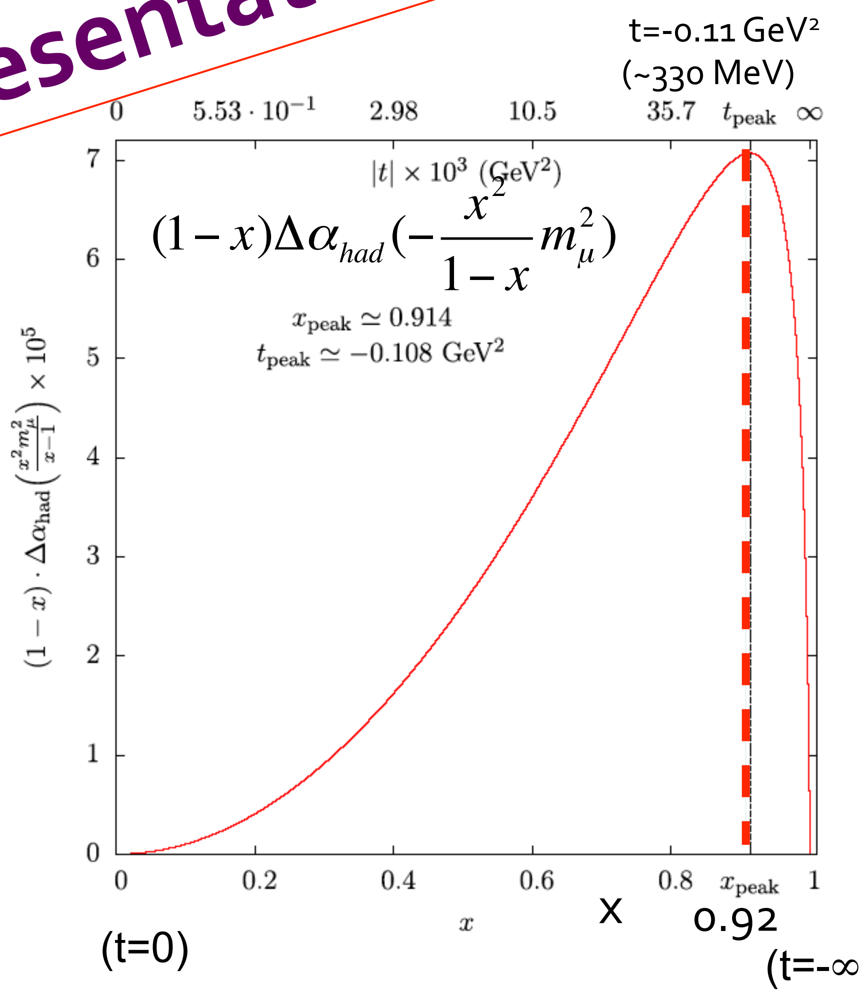

Conclusions

- The running of the e.m. coupling constant α has been measured in the ISR process $e^+e^- \rightarrow \mu^+\mu^-\gamma$ in the 0.6 - 0.98 GeV $M_{\mu\mu}$ invariant mass range with 1.7 fb^{-1}.
- The $\mu\mu\gamma$ cross section has been measured at <1% accuracy and shows excellent agreement with NLO Phokhara MC generator.
- Clear contribution of the $\rho-\omega$ interference to the photon propagator with 6σ statistical significance.
- Imaginary and Real part of $\Delta a(s)$ extracted.
- By a fit of the real part of $\Delta a(s)$ and assuming lepton universality the branching ratio of $\omega\rightarrow\mu^+\mu^-$ has been extracted.
- New proposal to measure $\Delta a(t)$ at high precision in the space-like region for a_μ^{HLO} (see L. Trentadue's presentation).

SPARES

Outlook: measurement of $\Delta\alpha(q^2)$ in the space-like region for a_μ^{HLO}

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A new approach to evaluate the leading hadronic corrections to the muon *g*-2

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Regular Article - Experimental Physics

Measuring the leading hadronic contribution to the muon *g*-2 via μe scattering

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Eur. Phys. J C **77** (2017) 139

60’-70’(PRE)History of ISR (Apologize for missing articles)

- Photon emission in muon pair production in electron-positron collisions ($e^+ e^- \rightarrow \mu^+ \mu^- \gamma$, σtot, ISR, FSR, interference)V. N. Baier, V. A. Khoze, ZhETF 48 (1965) 946, Yad. Fiz. 2 (1965) 287
- Radiation accompanying two particle annihilation of an electron - positron pair (scalar final states $e^+ e^- \rightarrow \pi^+ \pi^- \gamma$, σtot, ISR, FSR, interference, formfactors) V. N. Baier, V. A. Khoze, Sov. Phys. JETP 21 (1965) 629, 1145 [Zh. Eksp. Theor. Fiz. 48 (1965) 946]
- Infra-red radiative corrections for resonant processes (ϱ, ω, ϕ intermediate states) G. Pancheri, Nuovo Cim. A 60 (1969) 321
- Radiative Corrections for Colliding Beam Resonances (application to ψ(3.1), ψ′ (3.7) intermediate states) M. Greco, G. Pancheri, Y. N. Srivastava, Nucl. Phys. B 101 (1975) 234
- **Secondary Reactions in electron - positron (electron) Collisions (pion form factor) M. S. Chen, P. M. Zerwas, Phys. Rev. D 11 (1975) 58**

End of the 1990s: ISR at DAFNE and KLOE

- Hadronic Cross Sections in Electron-Positron Annihilation with Tagged Photon A. B. Arbuzov, E. A. Kuraev, N. P. Merenkov, L. Trentadue, JHEP 9812:009, 1998
- The hadronic contribution to the muon g-2 from hadronic production in ISR events at e+e- collider DAFNE, S. Spagnolo, Eur. Phys. J. C6 (1999) 637
- Measuring σ (**e+ e– →** hadrons) using tagged photon S. Binner, H. Kühn, K. Melnikov, Phys. Lett. B 459 (1999) 279
- **Measurement of σ (e+ e– → $\pi^+\pi^-$) from ISR with KLOE (first realistic study) G. Cataldi, A. Denig, W. K., G. Venanzoni, KLOE memo #195, August 1999,**
- Bottomonium ϒ(ns) spectroscopy at **B**- Factories via hard photon emission M. Benayoun, S. I. Eidelman, V. N. Ivanchenko, Z. K. Silagadze Mod. Phys. Lett. A 14 (1999) 2605

Years 2000s : ISR at the per mill precision: PHOKHARA enters the game (selected entries)

- NLO QED corrections to ISR in e+ e- annihilation and the measurement of sigma(e+ e- → hadrons) using tagged photons, G. Rodrigo, A. Gehrmann-De Ridder, M. Guilleaume and J. H. Kühn, Eur. Phys. J. C 22 (2001) 81
- The radiative return at small angles: Virtual corrections, J. H. Kühn and G. Rodrigo, Eur. Phys. J. C 25 (2002) 215
- Radiative return at NLO and the measurement of the hadronic cross-section in electron positron annihilation, G. Rodrigo, H. Czyż, , J. H. Kühn and M. Szopa, Eur. Phys. J. C 24 (2002) 71
- The radiative return at Phi- and B-factories: Small-angle photon emission at next to leading order, H. Czyż , A. Grzelińska, J. H. Kühn and G. Rodrigo, Eur. Phys. J. C 27 (2003) 563
- The radiative return at Phi and B-factories: FSR at next-to-leading order, H. Czyż , A. Grzelińska, J. H. Kühn and G. Rodrigo, Eur. Phys. J. C 33 (2004) 333
- The radiative return at Phi- and B-factories: FSR for muon % pair production at next-to-leading order, H. Czyż , A. Grzelińska, J. H. Kühn and G. Rodrigo, Eur. Phys. J. C 39 (2005) 411
- **Complete QED NLO contributions to the reaction e+e-→$\mu^+\mu^-\gamma$ and their implementation in the event generator PHOKHARA, F. Campanario, H. Czyż, J. Gluza, M. Gunia, T. Riemann, G. Rodrigo and V. Yundin, JHEP 1402 (2014) 114**