Dispersive analysis of light-meson transition form factors

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Hadronic light-by-light scattering using dispersion relations

- Pion–pion intermediate states — Bern $\longrightarrow$ talk by G. Colangelo
- $\pi^0 \to \gamma^*\gamma^{(*)}$ transition form factor — Bonn
- $\eta \to \gamma^*\gamma^{(*)}$ transition form factor — Jülich

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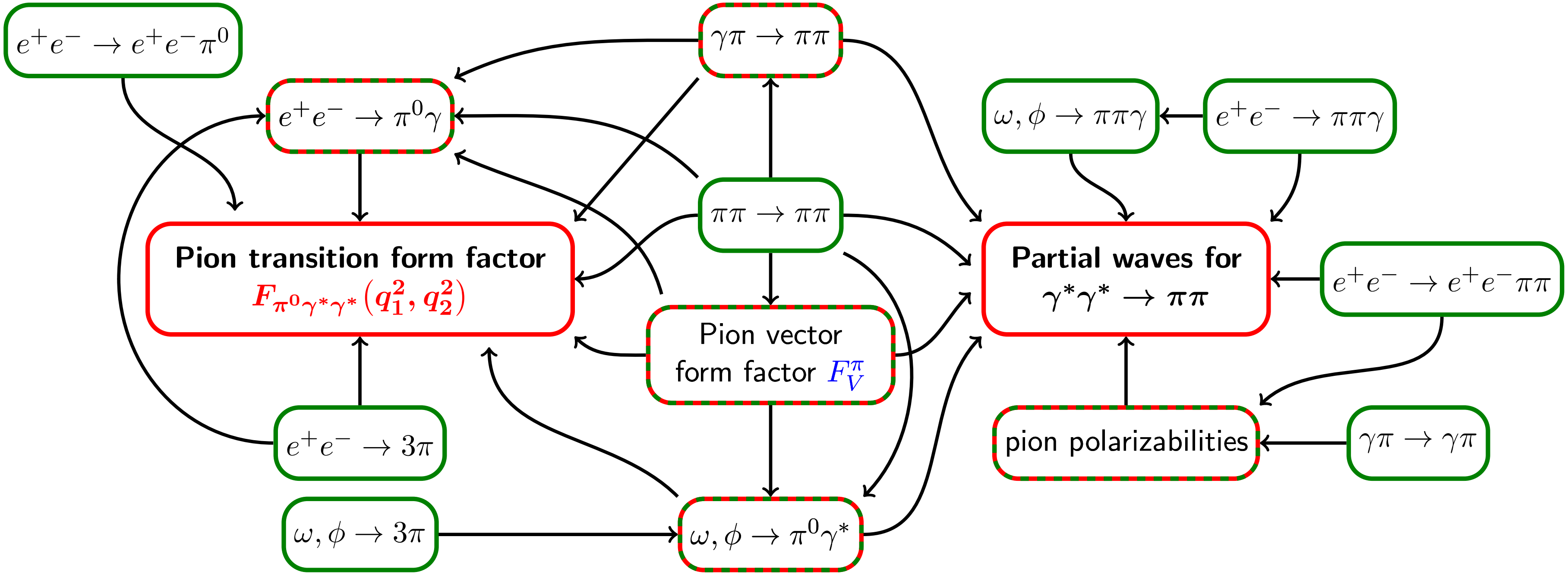
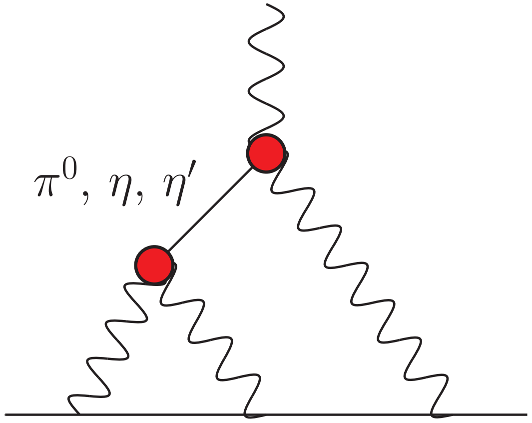


Colangelo, Hoferichter, BK, Procura, Stoffer 2014

Pseudoscalar transition form factors and $(g-2)_\mu$

- largest individual HLbL contribution:
  pseudoscalar pole terms
  singly / doubly virtual form factors
  $F_{P\gamma\gamma^*}(q^2,0)$ and $F_{P\gamma^*\gamma^*}(q_1^2,q_2^2)$



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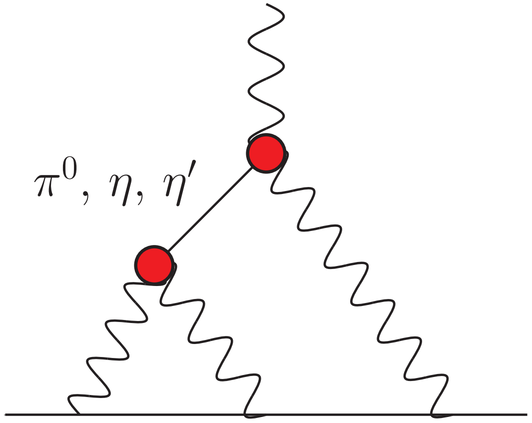

- normalisation fixed by Wess–Zumino–Witten anomaly, e.g.:

$$F_{\pi^0\gamma\gamma}(0,0) = \frac{e^2}{4\pi^2 F_\pi}$$

F_π: pion decay constant $\longrightarrow$ measured at 1.5% level   PrimEx 2011

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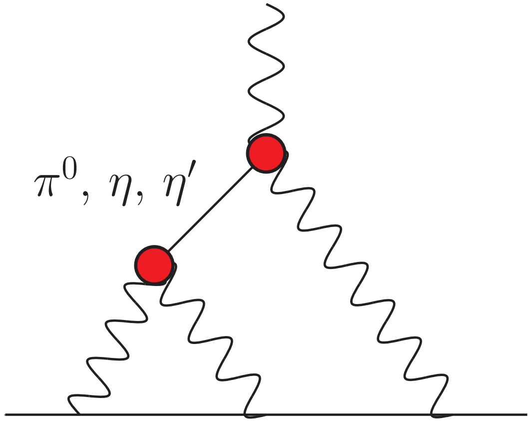

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- q_i^2-dependence: often modelled by vector-meson dominance
  $\longrightarrow$ what can we learn from analyticity and unitarity constraints?
  $\longrightarrow$ what experimental input sharpens these constraints?

Dispersive analysis of $\pi^0/\eta \to \gamma^*\gamma^*$

- isospin decomposition:

$$F_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = F_{vs}(q_1^2,q_2^2) + F_{vs}(q_2^2,q_1^2)$$
$$F_{\eta\gamma^*\gamma^*}(q_1^2,q_2^2) = F_{vv}(q_1^2,q_2^2) + F_{ss}(q_2^2,q_1^2)$$

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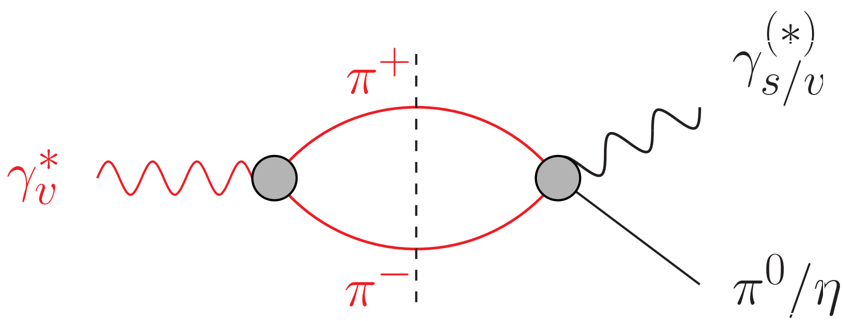
- analyse the leading hadronic intermediate states:

Hanhart et al. 2013, Hoferichter et al. 2014



▷ isovector photon: 2 pions

$\propto$ pion vector form factor $\times$ $\gamma\pi \to \pi\pi$ / $\eta \to \pi\pi\gamma$

all determined in terms of pion–pion P-wave phase shift

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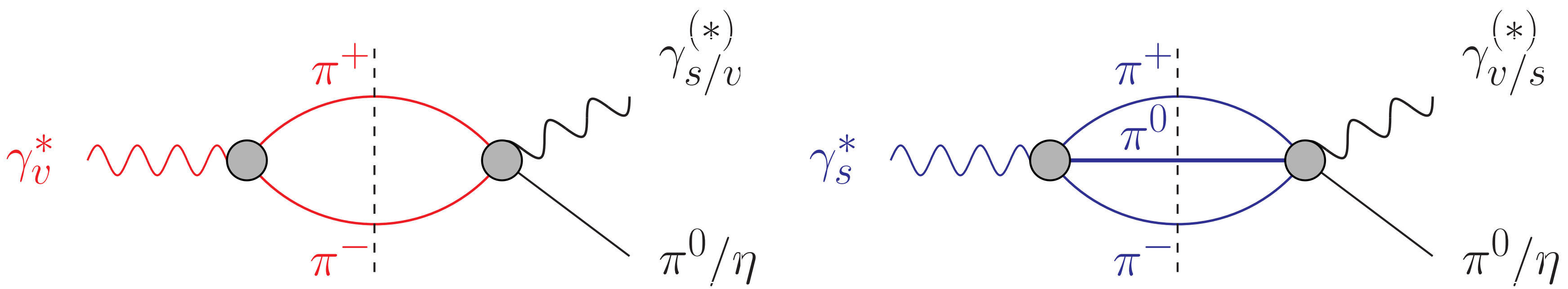

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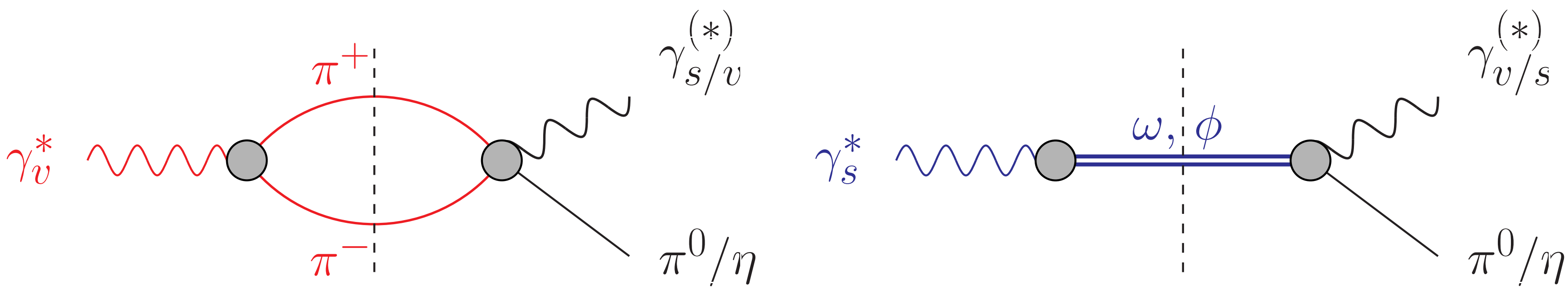

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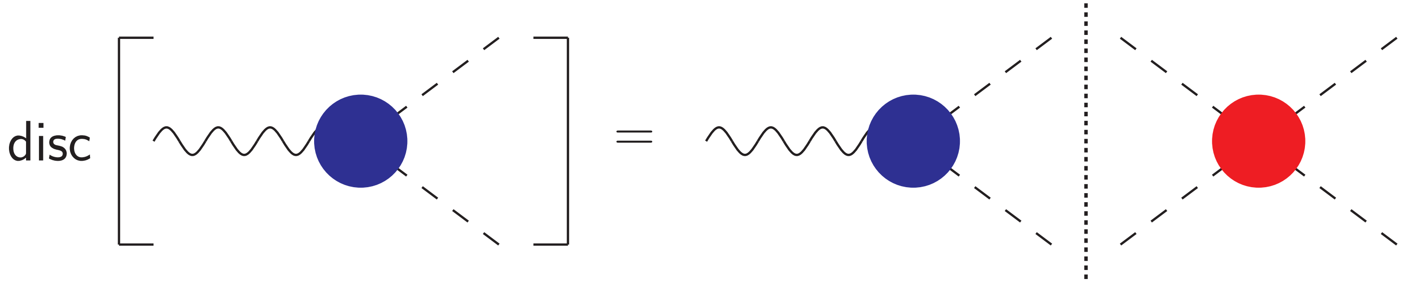
▷ isoscalar photon: 3 pions $\longrightarrow$ dominated by narrow ω, ϕ

$\leftrightarrow \omega/\phi$ transition form factors; very small for the η

Warm-up: charged pion form factor



$$\frac{1}{2i}\,\text{disc}\,F_\pi^V(s) = \text{Im}\,F_\pi^V(s) \;=\; F_\pi^V(s)\times\theta(s-4\,M_\pi^2)\times\sin\delta_1^1(s)\,e^{-i\delta_1^1(s)}$$

$\longrightarrow$ final-state theorem: phase of $F_\pi^V(s)$ is just $\delta_1^1(s)$ Watson 1954

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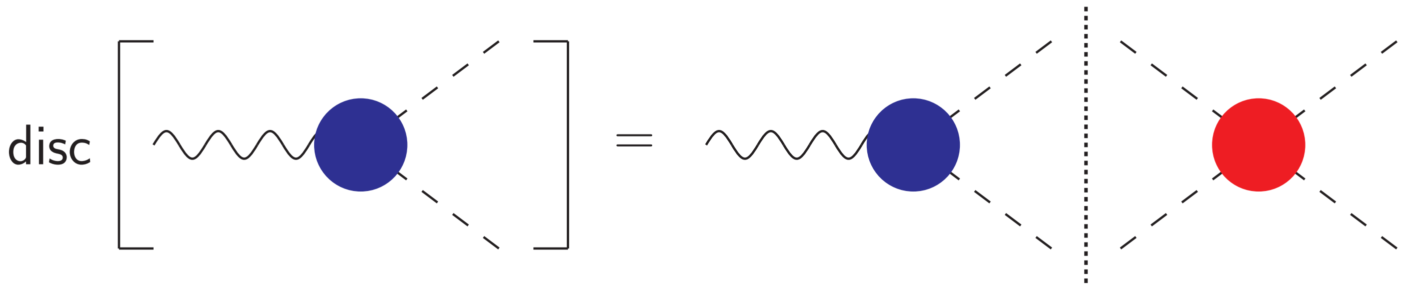

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- solution:

$$F_\pi^V(s) = P(s)\Omega(s)\,, \quad \Omega(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^\infty ds'\,\frac{\delta_1^1(s')}{s'(s'-s)}\right\}$$

$P(s)$ polynomial, $\Omega(s)$ Omnès function Omnès 1958

  - $\triangleright$ $\pi\pi$ phase shifts from Roy equations
    Ananthanarayan et al. 2001, García-Martín et al. 2011
  - $\triangleright$ $P(0) = 1$ from symmetries (gauge invariance)

- below 1 GeV: $F_\pi^V(s) \approx (1 + 0.1\,\text{GeV}^{-2}s)\Omega(s)$
  slope due to inelastic resonances ρ', ρ''... Hanhart 2012

Final-state universality: $\eta,\ \eta' \to \pi^+\pi^-\gamma$

- $\eta^{(\prime)} \to \pi^+\pi^-\gamma$ driven by the chiral anomaly, $\pi^+\pi^-$ in P-wave $\longrightarrow$ final-state interactions the same as for vector form factor
- ansatz: $\mathcal{F}^{\eta^{(\prime)}}_{\pi\pi\gamma} = A \times P(t) \times \Omega(t)$, $P(t) = 1 + \alpha^{(\prime)} t$, $t = M^2_{\pi\pi}$

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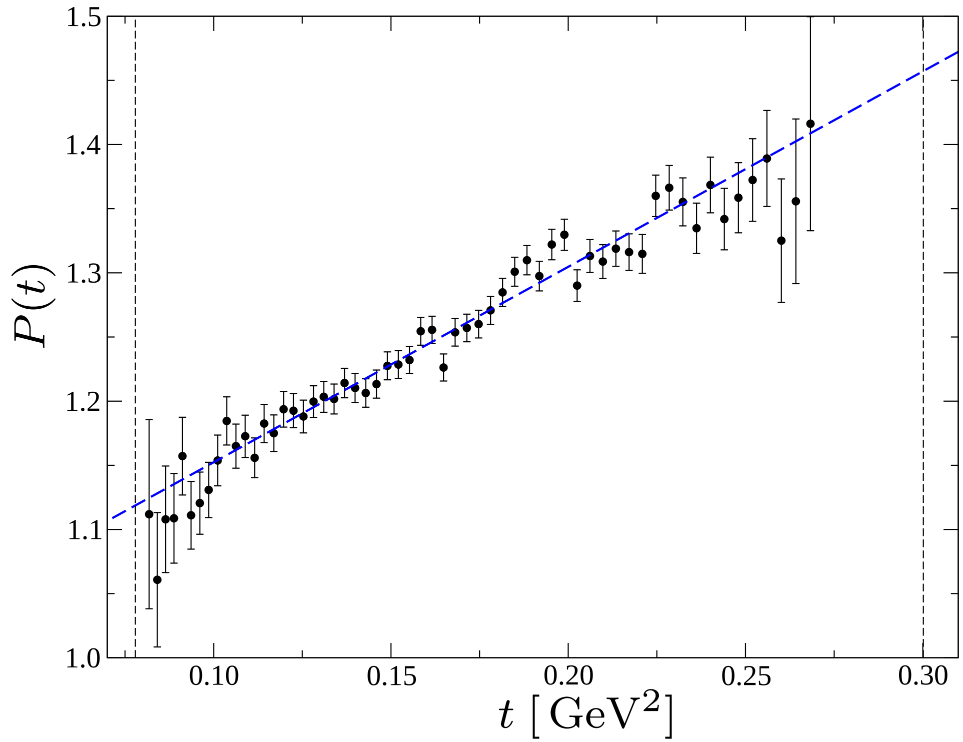
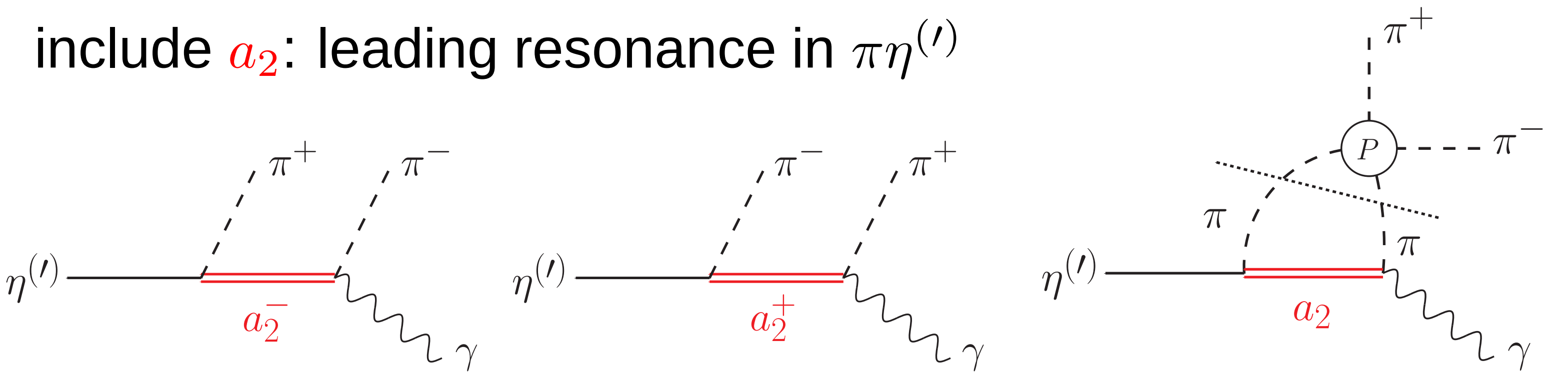
- divide data by pion form factor $\longrightarrow P(t)$ Stollenwerk et al. 2012



$\longrightarrow$ exp.: $\alpha_{\mathrm{KLOE}} = (1.52 \pm 0.06)$ GeV^{-2} cf. KLOE 2013

$\eta,\ \eta' \to \pi^+\pi^-\gamma$ with left-hand cuts

- include a_2: leading resonance in $\pi\eta^{(\prime)}$



BK, Plenter 2015

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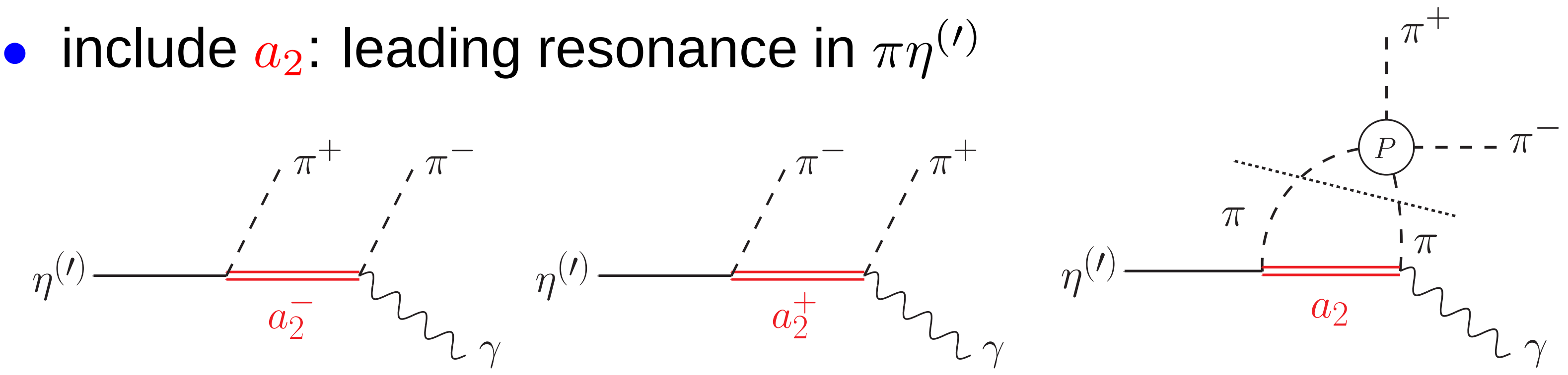

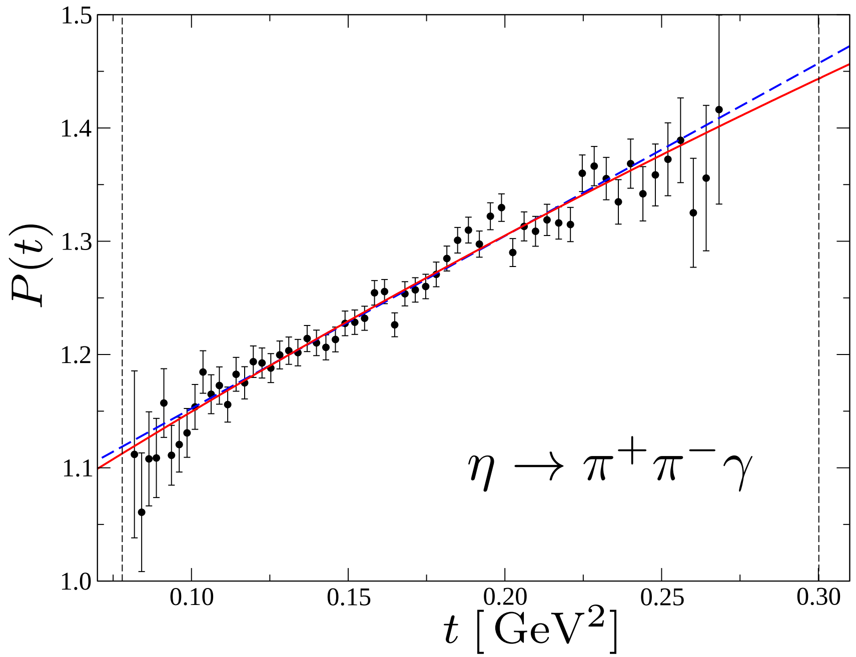

KLOE 2013; BK, Plenter 2015

- induces curvature in $P(t)$

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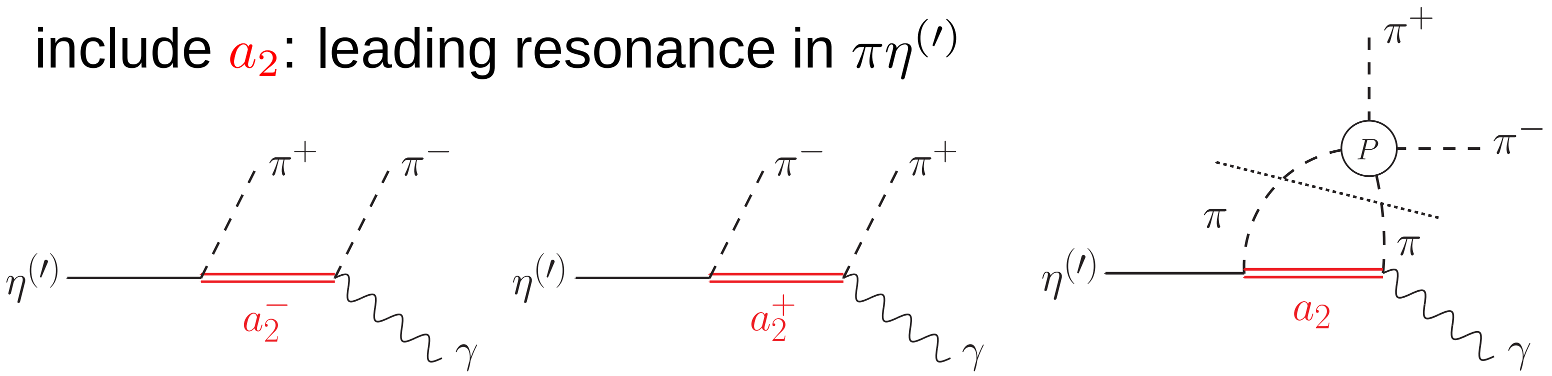

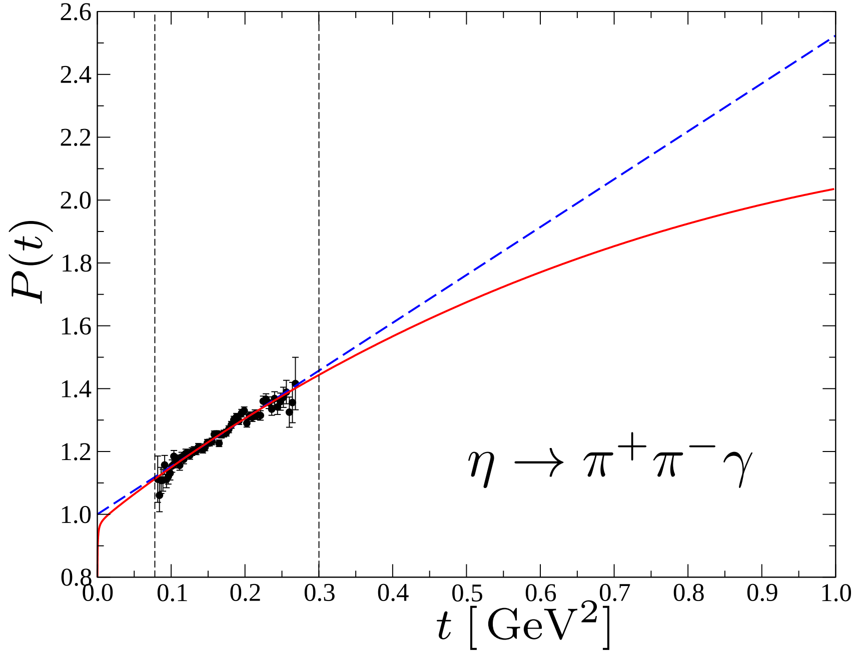

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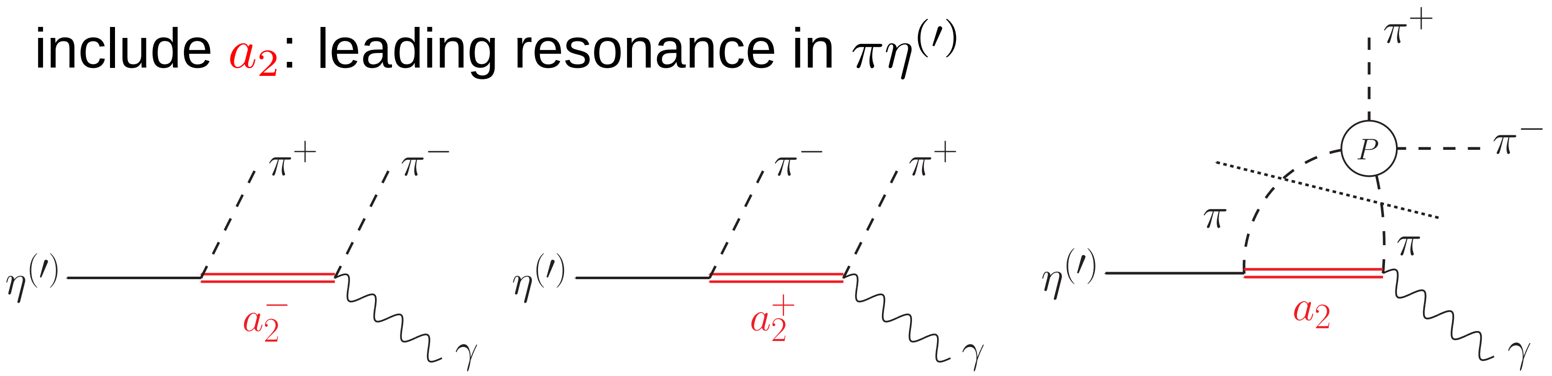

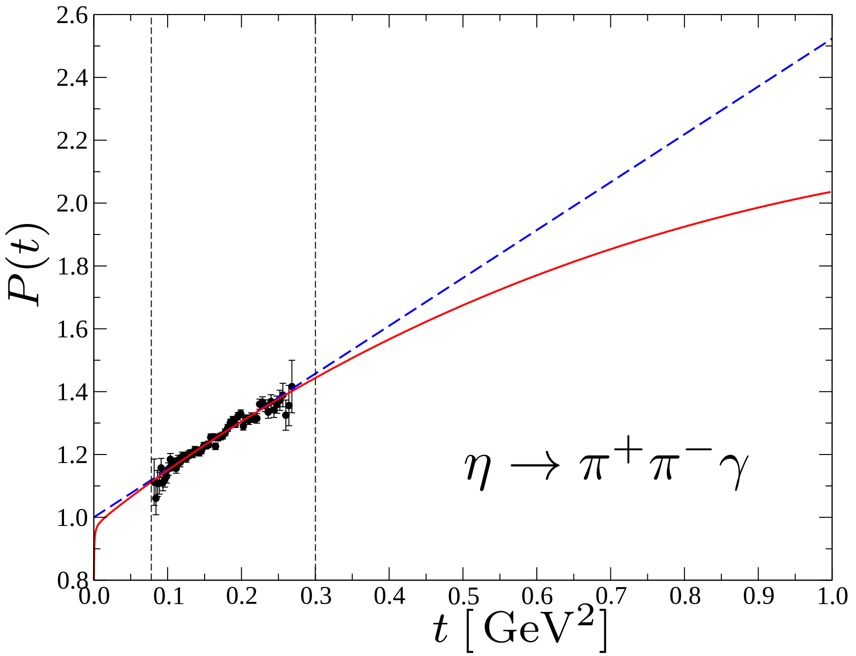

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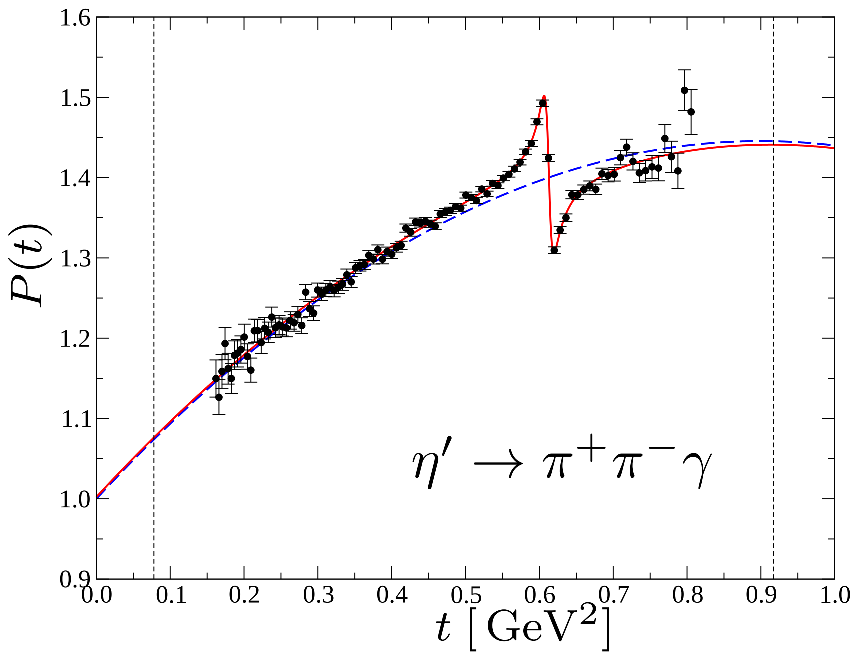


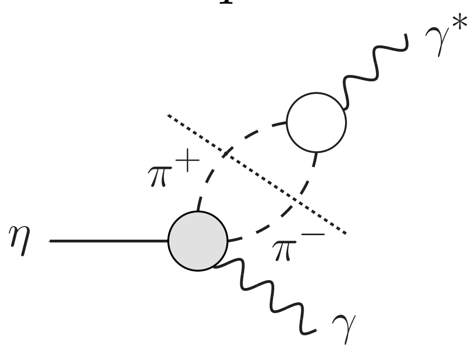
BESIII prel.; Hanhart et al. 2017

- induces curvature in $P(t)$
- curvature, plus ρ–ω mixing

Transition form factor $\eta \to \gamma^*\gamma$

Hanhart et al. 2013

$$\bar{F}_{\eta\gamma^*\gamma}(q^2,0) = 1 + \frac{\kappa_\eta q^2}{96\pi^2 F_\pi^2}\int_{4M_\pi^2}^{\infty} ds \sigma(s)^3 P_V(s) P_\eta(s) \frac{|\Omega(s)|^2}{s-q^2}$$
$$+ \Delta F^{I=0}_{\eta\gamma^*\gamma}(q^2,0) \; [\longrightarrow \text{VMD}]$$



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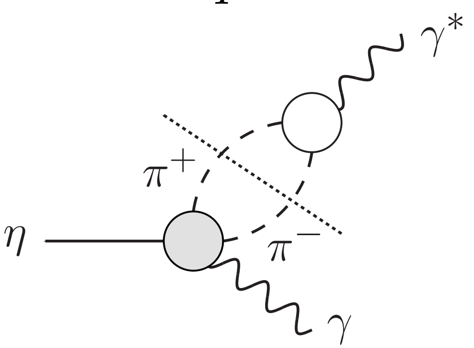
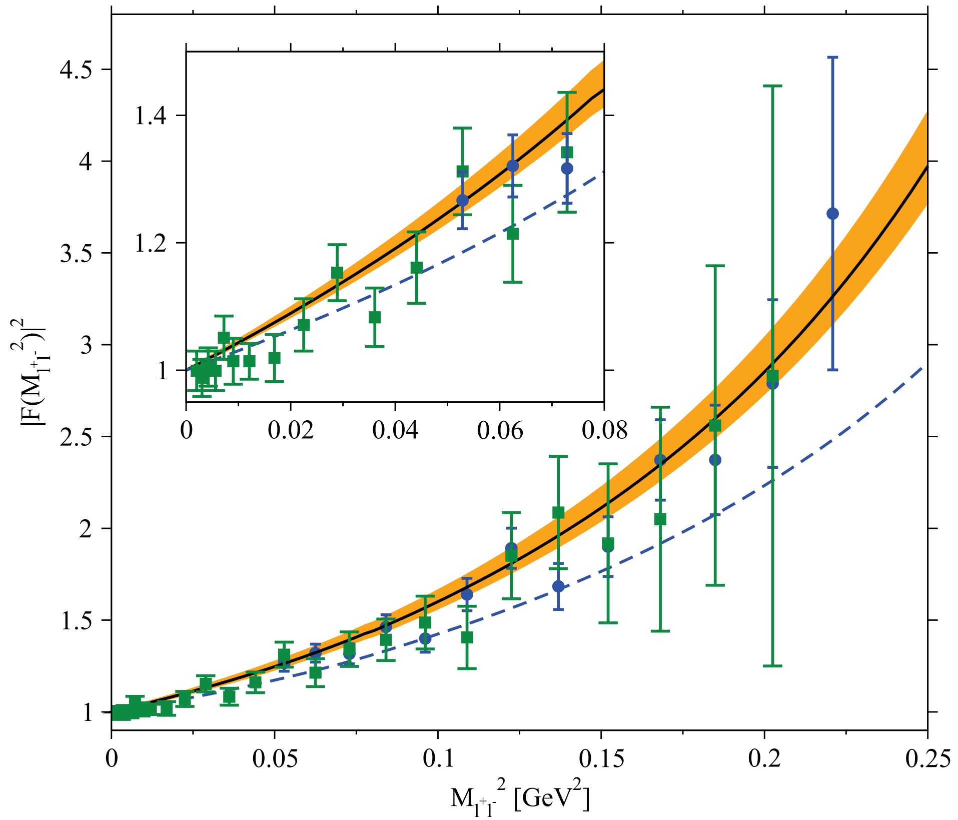




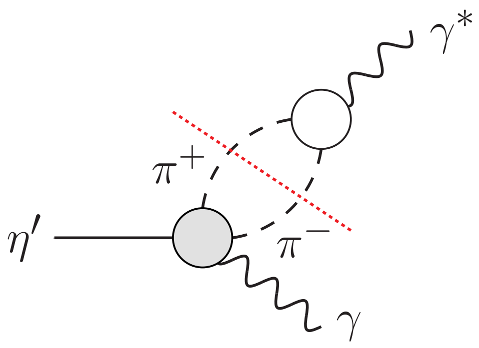
$\longrightarrow$ huge statistical advantage of using hadronic input for $\eta \to \pi^+\pi^-\gamma$ over direct measurement of $\eta \to e^+e^-\gamma$ (rate suppressed by α^2_{QED})

figure courtesy of C. Hanhart

data: NA60 2011, A2 2014

Prediction for η' transition form factor

- isovector: combine high-precision data on $\eta' \to \pi^+\pi^-\gamma$ and $e^+e^- \to \pi^+\pi^-$
- isoscalar: VMD, couplings fixed from $\eta' \to \omega\gamma$ and $\phi \to \eta'\gamma$



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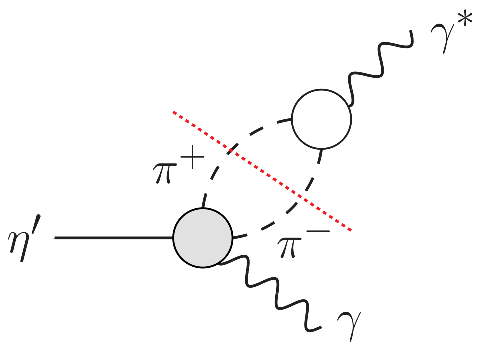

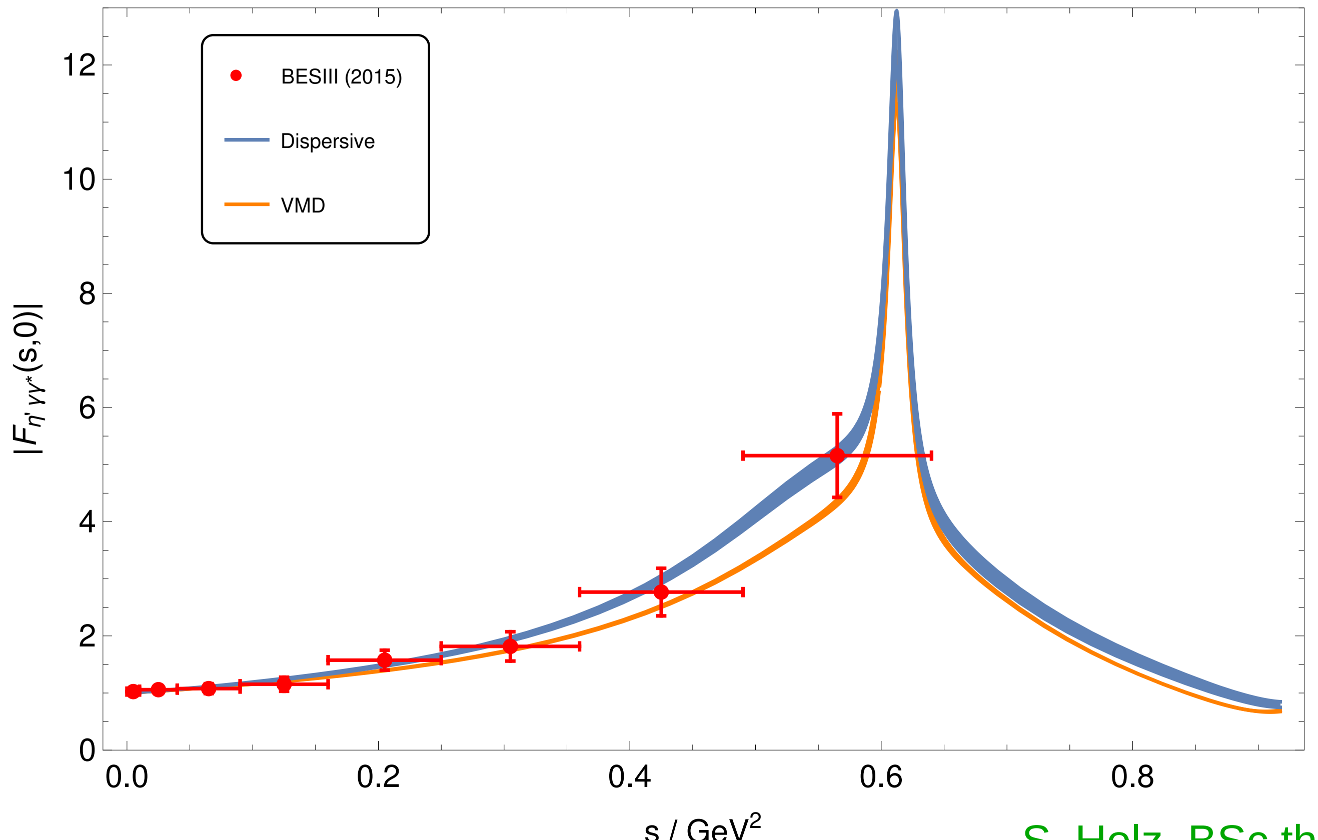

S. Holz, BSc thesis 2016

Anomalous process $\gamma\pi \to \pi\pi$

- $\gamma\pi \to \pi\pi$: crossing symmetry, WZW low-energy theorem

$$\mathcal{F}(s,t,u) = \mathcal{F}(s) + \mathcal{F}(t) + \mathcal{F}(u)\,, \qquad \mathcal{F}(0,0,0) = F_{3\pi} = \frac{e}{4\pi^2 F_\pi^3}$$

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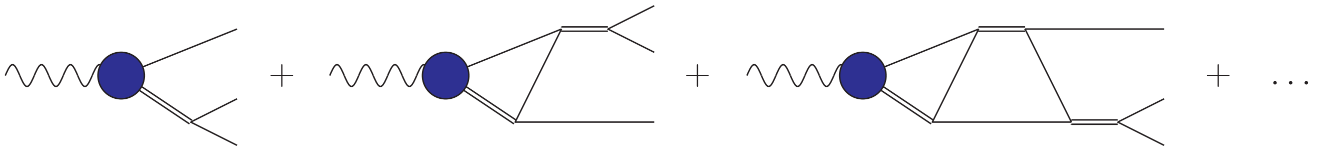
$$\mathcal{F}(s) = \Omega(s)\left\{ \frac{C_1}{3} + \frac{s}{\pi}\int_{4M_\pi^2}^{\infty} \frac{ds'}{s'} \frac{\sin\delta_1^1(s')\hat{\mathcal{F}}(s')}{|\Omega(s')|(s'-s)} \right\}$$

$=$ 

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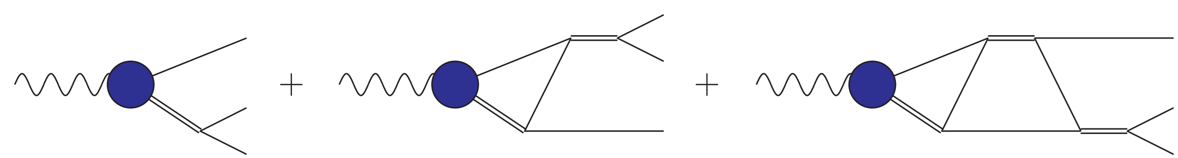
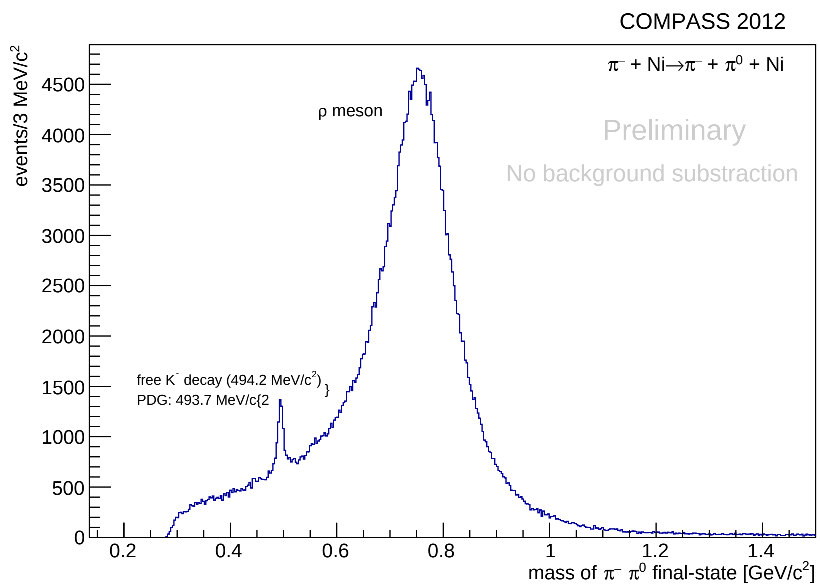
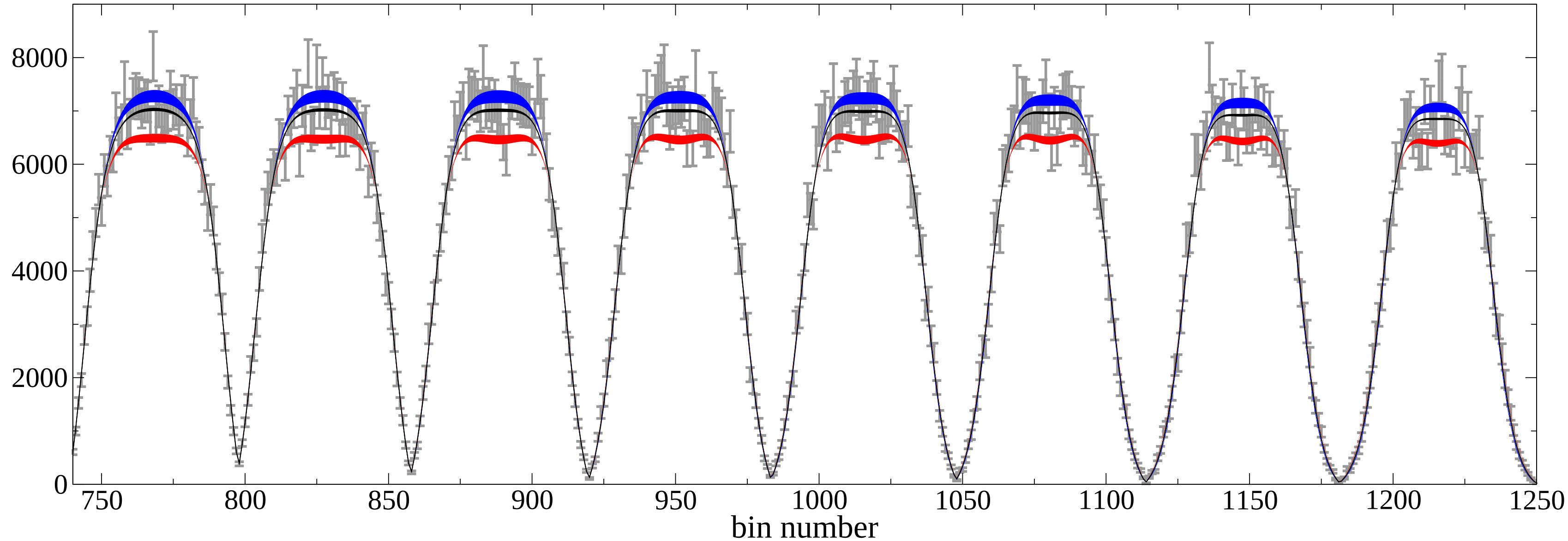
$=$  $+ \;\dots$

- high-accuracy data from Primakoff spectrum COMPASS
  Seyfried, MSc thesis 2017
- fit dispersive representation to data, extract $F_{3\pi} \simeq C_1$
  Hoferichter, BK, Sakkas 2012
- lattice $\longrightarrow$ M. Niehus, poster sess.



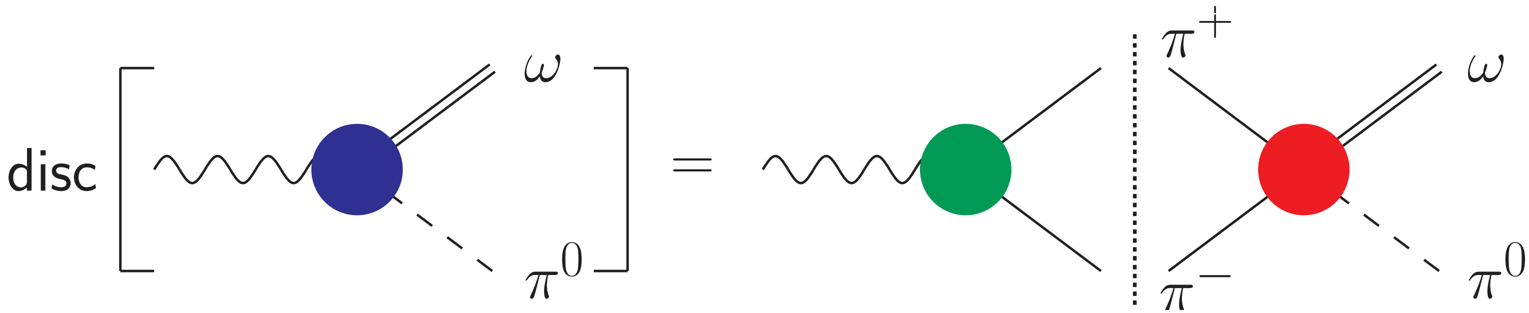
Extension to decays: $\omega/\phi \to 3\pi$

- same quantum numbers as $\gamma\pi \to \pi\pi$ — Niecknig, BK, Schneider 2012
- first $\omega \to 3\pi$ Dalitz plot measurement — WASA-at-COSY 2017
- test accuracy on $\phi \to 3\pi$ Dalitz plot: $2 \cdot 10^6$ events — KLOE 2005



	$\hat{\mathcal{F}} = 0$	once-subtracted	twice-subtracted
χ^2/ndof	$1.71 \ldots 2.06$	$1.17 \ldots 1.50$	$1.02 \ldots 1.03$

Transition form factor $\omega(\phi) \to \pi^0\ell^+\ell^-$



- ω transition form factor related to

  pion vector form factor $\times$ $\omega \to 3\pi$ decay amplitude

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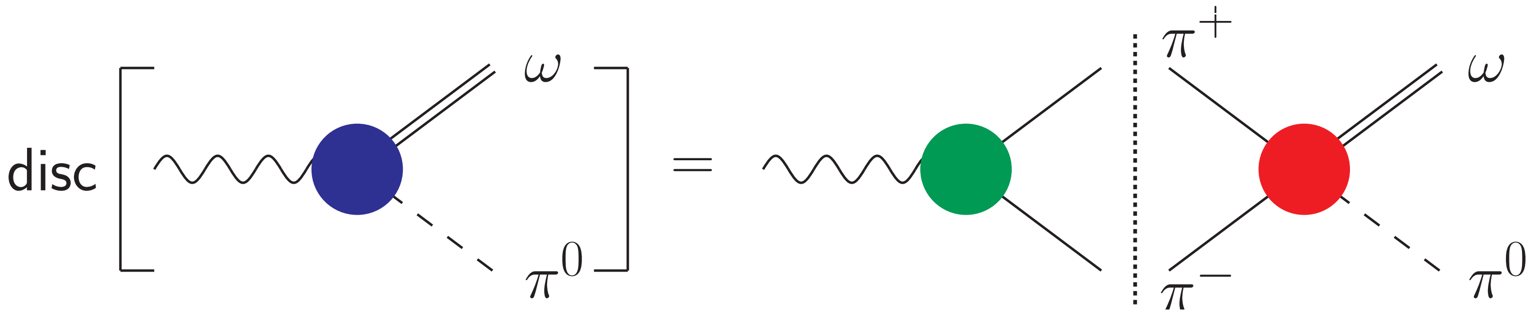


- ω transition form factor related to

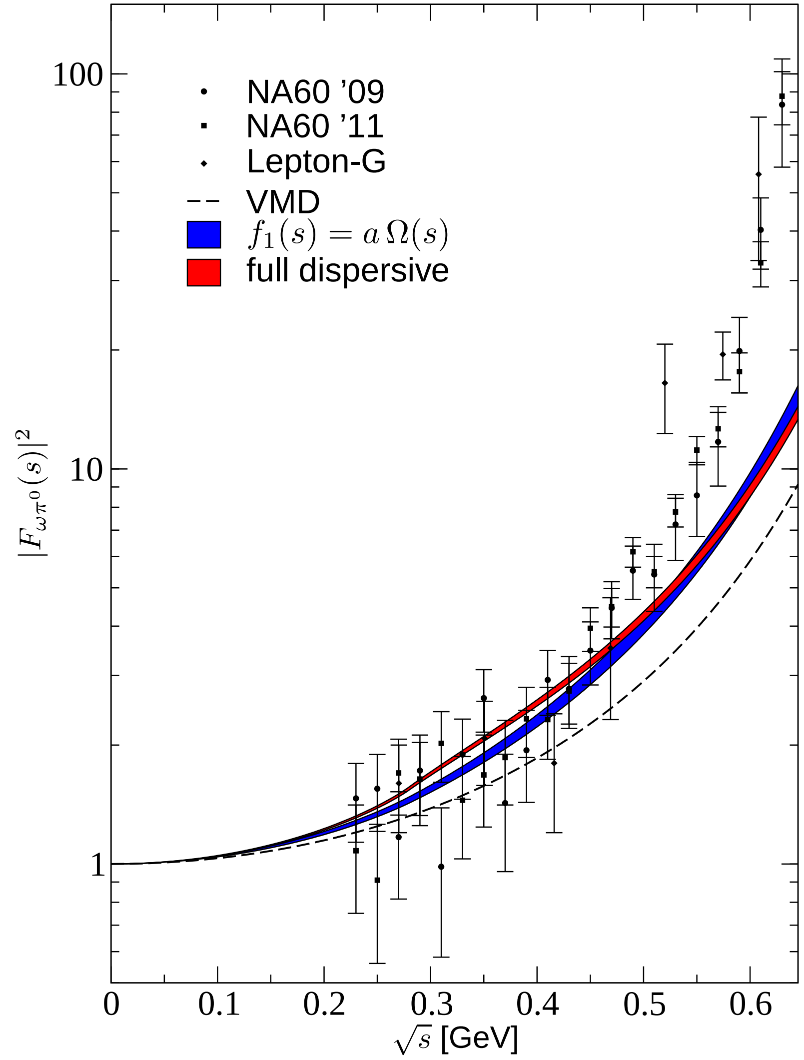
  pion vector form factor $\times$ $\omega \to 3\pi$ decay amplitude

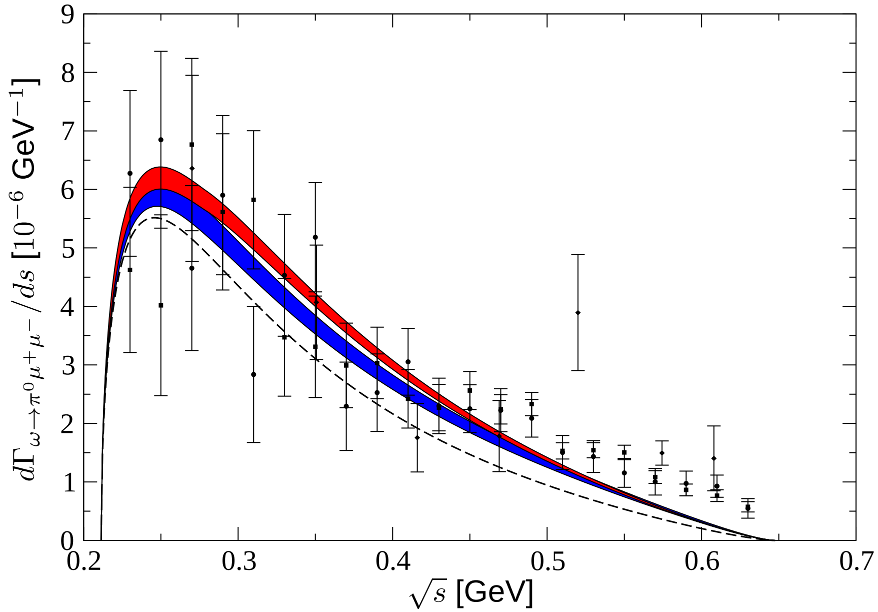
- form factor normalization yields rate $\Gamma(\omega \to \pi^0\gamma)$

  (2nd most important ω decay channel)

  $\longrightarrow$ works at 95% accuracy Schneider, BK, Niecknig 2012

Numerical results: $\omega \to \pi^0\mu^+\mu^-$





- clear enhancement vs. VMD

  cf. also Danilkin et al. 2015
- incompatible with data from heavy-ion coll. NA60 2009, 2011
- $\omega \to \pi^0 e^+e^-$ data: no tension (but less precise) A2 2016

- NA60 data potentially in conflict with unitarity bounds

  Ananthanarayan, Caprini, BK 2014, Caprini 2015

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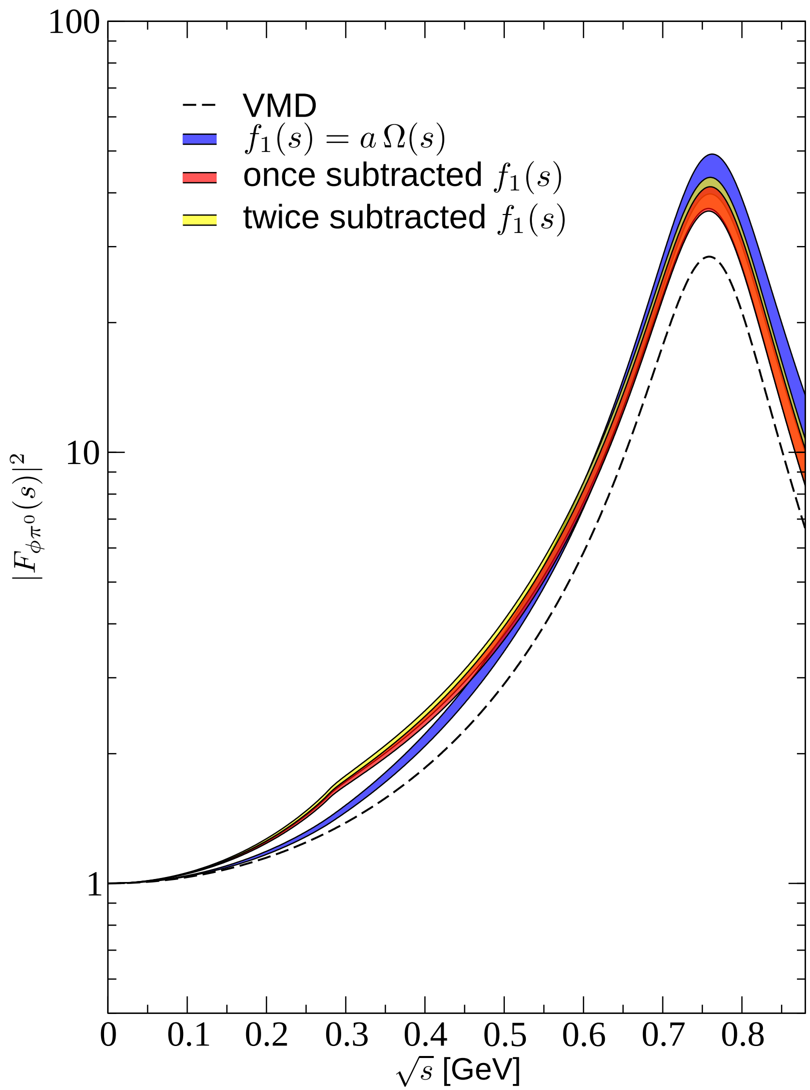
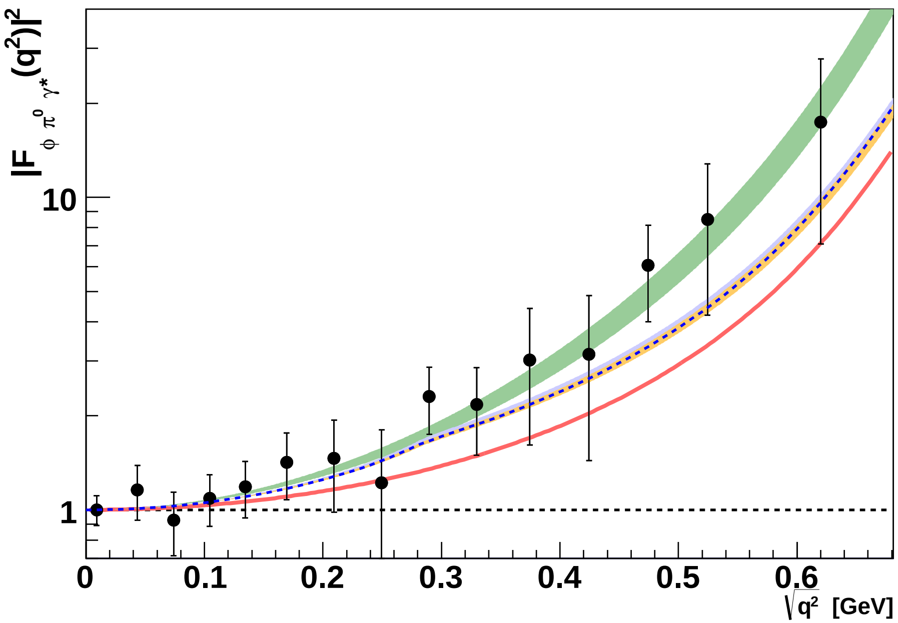


Schneider, BK, Niecknig 2012

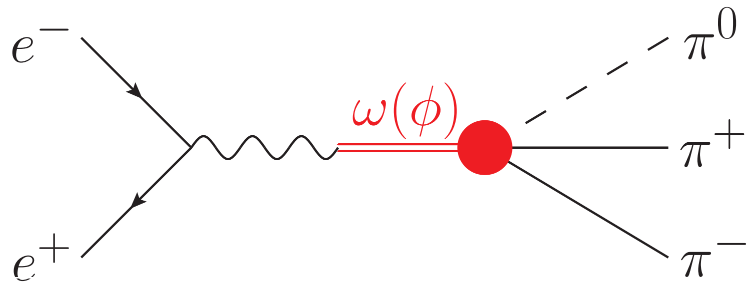


KLOE 2016

- measurement in ρ peak region would be extremely helpful
- $\phi \to 3\pi$ partial-wave amplitude backed up by experiment

Niecknig, BK, Schneider 2012

One step further: $e^+e^- \to 3\pi$, $e^+e^- \to \pi^0\gamma^*$

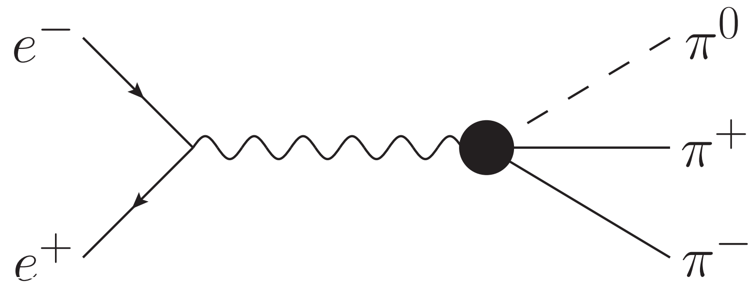


- decay amplitude for $\omega/\phi \to 3\pi$: $\mathcal{M}_{\omega/\phi} \propto \mathcal{F}(s) + \mathcal{F}(t) + \mathcal{F}(u)$

$$\mathcal{F}(s) = a_{\omega/\phi}\,\Omega(s)\left\{1 + \frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\sin\delta_1^1(s')\hat{\mathcal{F}}(s')}{|\Omega(s')|(s'-s)}\right\}$$

$a_{\omega/\phi}$ adjusted to reproduce total width $\omega/\phi \to 3\pi$

One step further: $e^+e^- \to 3\pi$, $e^+e^- \to \pi^0\gamma^*$



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$a_{e^+e^-}(q^2)$ adjusted to reproduce spectrum $e^+e^- \to 3\pi$

- parameterisation:

$$a_{e^+e^-}(q^2) = \frac{F_{3\pi}}{3} + \beta\, q^2 + \frac{q^4}{\pi}\int_{\text{thr}}^{\infty} ds'\,\frac{\mathrm{Im}BW(s')}{s'^2(s'-q^2)}$$

$$BW(q^2) = \sum_{V=\omega,\phi}\frac{c_V}{M_V^2 - q^2 - i\sqrt{q^2}\,\Gamma_V(q^2)}$$

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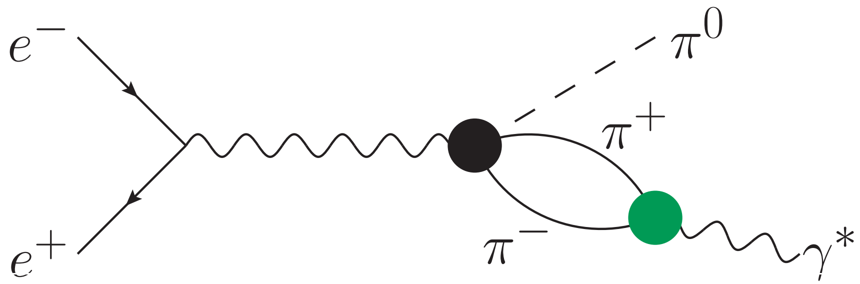

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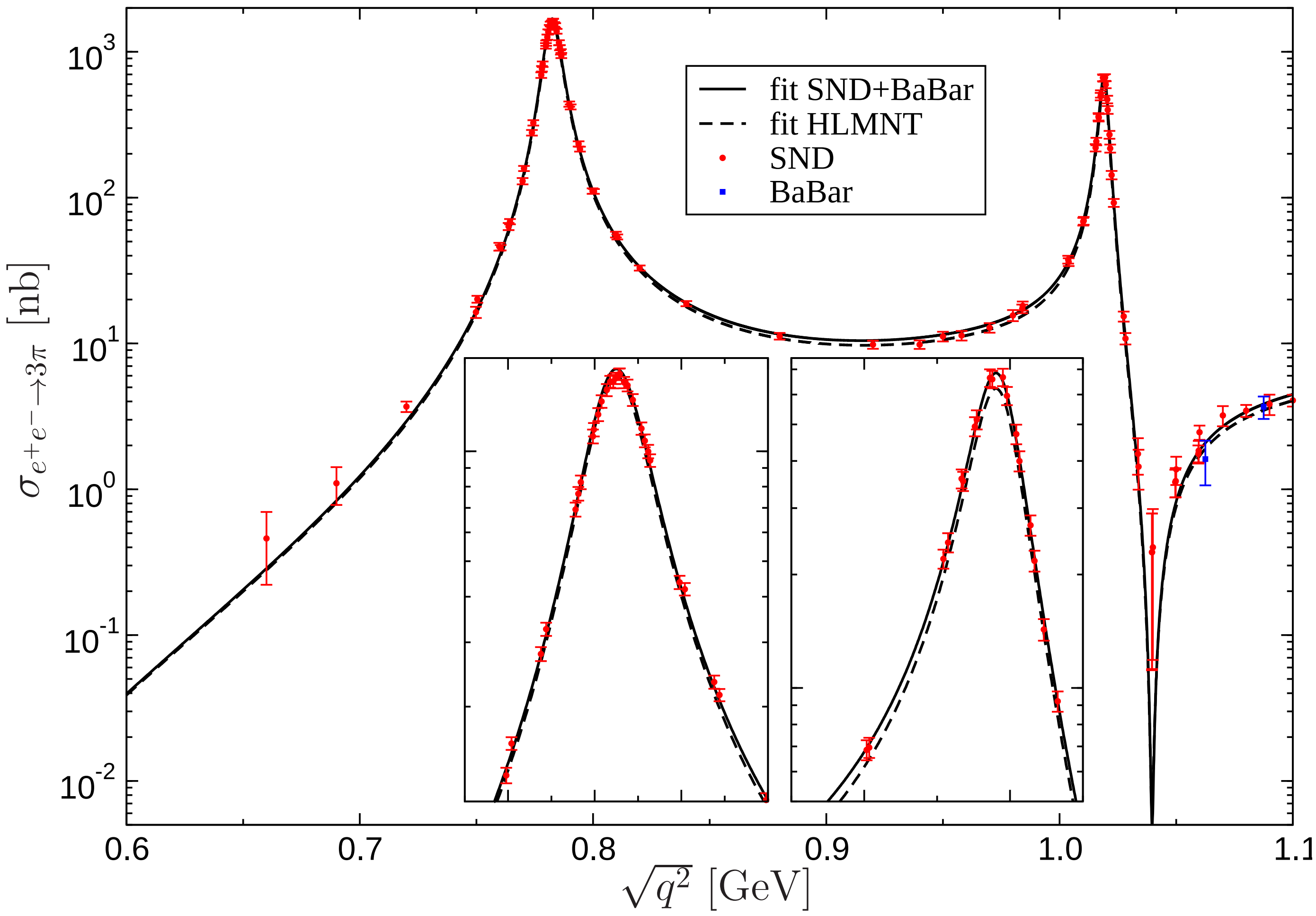
- fit to $e^+e^- \to 3\pi$ data $\longrightarrow$ prediction for $e^+e^- \to \pi^0\gamma^*$

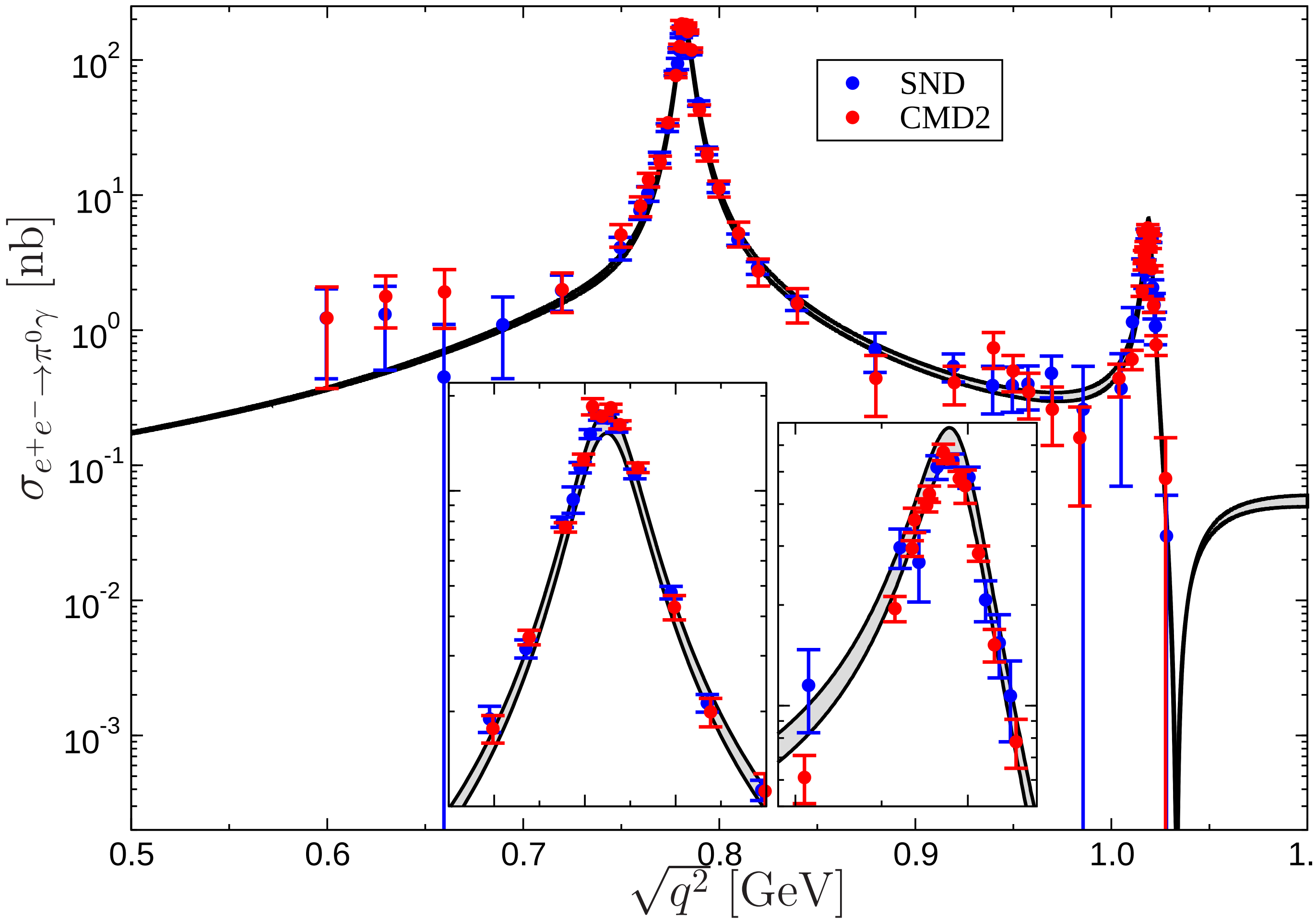
Fit to $e^+e^- \to 3\pi$ data



Hoferichter, BK, Leupold, Niecknig, Schneider 2014

- one subtraction/normalisation at $q^2 = 0$ fixed by $\gamma \to 3\pi$
- fitted: ω, ϕ residues, linear subtraction β

Comparison to $e^+e^- \to \pi^0\gamma$ data



Hoferichter, BK, Leupold, Niecknig, Schneider 2014 $\longrightarrow$ L. Bai, poster session

- "prediction"—no further parameters adjusted
- data very well reproduced

Extension to spacelike region; slope

- continuation to spacelike region: use another dispersion relation

$$F_{\pi^0\gamma^*\gamma}(q^2,0) = F_{\pi\gamma\gamma} + \frac{q^2}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\mathrm{Im}\,F_{\pi^0\gamma^*\gamma}(s',0)}{s'(s'-q^2)}$$

- sum rule for slope $\quad F_{\pi^0\gamma^*\gamma}(q^2,0) = F_{\pi\gamma\gamma}\left\{1 + a_\pi \frac{q^2}{M_{\pi^0}^2} + \mathcal{O}(q^4)\right\}$

$$a_\pi = \frac{M_{\pi^0}^2}{F_{\pi\gamma\gamma}} \times \frac{1}{\pi}\int_{4M_\pi^2}^{\infty} \frac{ds'}{s'^2}\,\mathrm{Im}\,F_{\pi^0\gamma^*\gamma}(s',0)$$

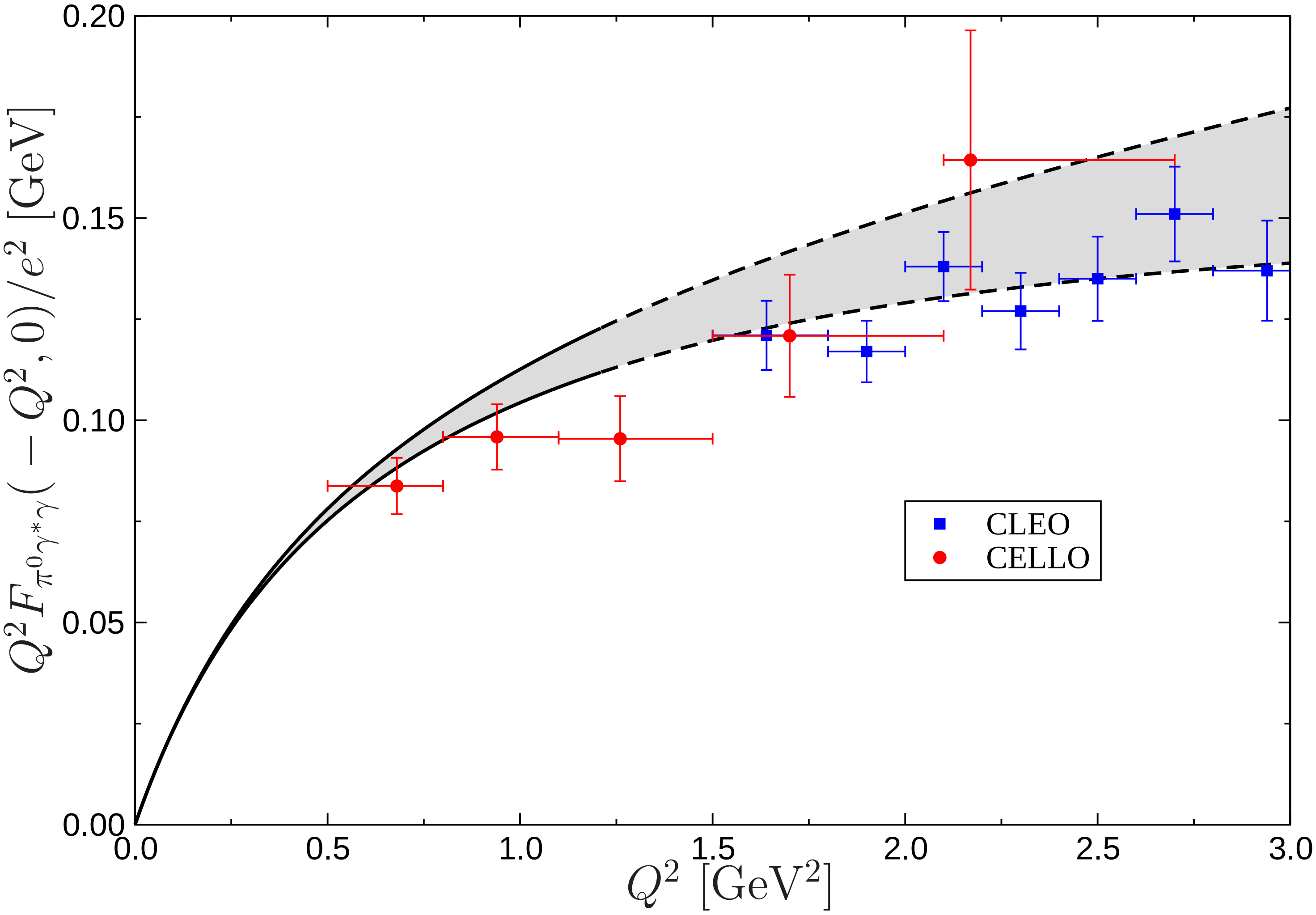
$$= (30.7 \pm 0.6) \times 10^{-3}$$

Hoferichter et al. 2014

compare: $\quad a_\pi = (32 \pm 4) \times 10^{-3}$ PDG 2014

- theory error estimate:
  - ▷ $\pi\pi$ phases
  - ▷ cutoff effects in $\gamma^* \to 3\pi$ partial waves and $[\gamma^* \to 3\pi] \longrightarrow [\gamma^* \to \pi^0\gamma]$

Prediction spacelike form factor



Hoferichter, BK, Leupold, Niecknig, Schneider 2014 $\longrightarrow$ L. Bai, poster session

$\longrightarrow$ more precise low-energy spacelike data to come

BESIII

Summary / Outlook

Dispersive analyses of $\boldsymbol{\pi^0}$, $\boldsymbol{\eta^{(\prime)}}$ transition form factors:

- high-precision data on

  $\eta \to \pi^+\pi^-\gamma$ KLOE / $\eta' \to \pi^+\pi^-\gamma$ BESIII / $e^+e^- \to \pi^+\pi^-\pi^0$ var.

  allow for high-precision dispersive predictions of π^0, $\eta^{(\prime)} \to \gamma^*\gamma$

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Further experimental input:

- Primakoff reactions $\gamma\pi \to \pi\pi$, $\gamma\pi \to \pi\eta$ COMPASS
- $\omega \to 3\pi$ precision Dalitz plot CLAS, BESIII?
- $\omega/\phi \to \pi^0\gamma^*$ test doubly virtual $F_{\pi^0\gamma^*\gamma^*}$ with precision
- $e^+e^- \to \eta\pi^+\pi^-$ differential data Xiao et al., in progress

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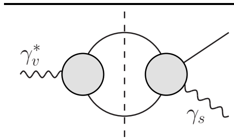
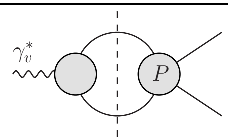
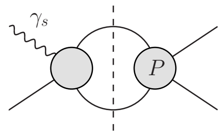
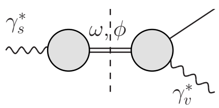
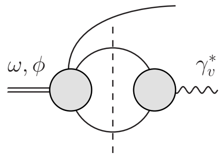
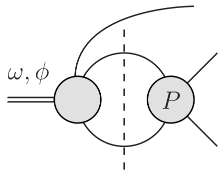
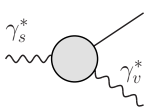
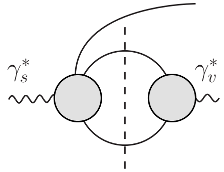
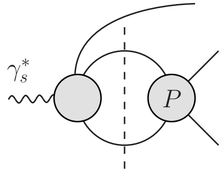
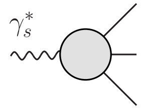
Theoretical work in progress:

- include high-energy constraints $\longrightarrow$ L. Bai, poster session
- improved doubly-virtual $\eta^{(\prime)}$ ($\leftrightarrow \eta^{(\prime)} \to 4\pi$) $\longrightarrow$ J. Plenter, poster sess.
- link to lattice $\longrightarrow$ M. Niehus, A. Gerardin, poster session

Spares

Summary: processes and unitarity relations for $\pi^0 \to \gamma^*\gamma^*$

process	unitarity relations	SC 1	SC 2
			$F_{\pi^0\gamma\gamma}$
		$F_{3\pi}$	$\sigma(\gamma\pi \to \pi\pi)$
			$\Gamma_{\pi^0\gamma}$
		$\Gamma_{3\pi}$	$\frac{d^2\Gamma}{ds\,dt}(\omega,\phi \to 3\pi)$
			$\sigma(e^+e^- \to \pi^0\gamma)$
		$\sigma(e^+e^- \to 3\pi)$	$\sigma(\gamma\pi \to \pi\pi)$ $\frac{d^2\Gamma}{ds\,dt}(\omega,\phi \to 3\pi)$
		$F_{3\pi}$	$\sigma(e^+e^- \to 3\pi)$

Colangelo, Hoferichter, BK, Procura, Stoffer 2014

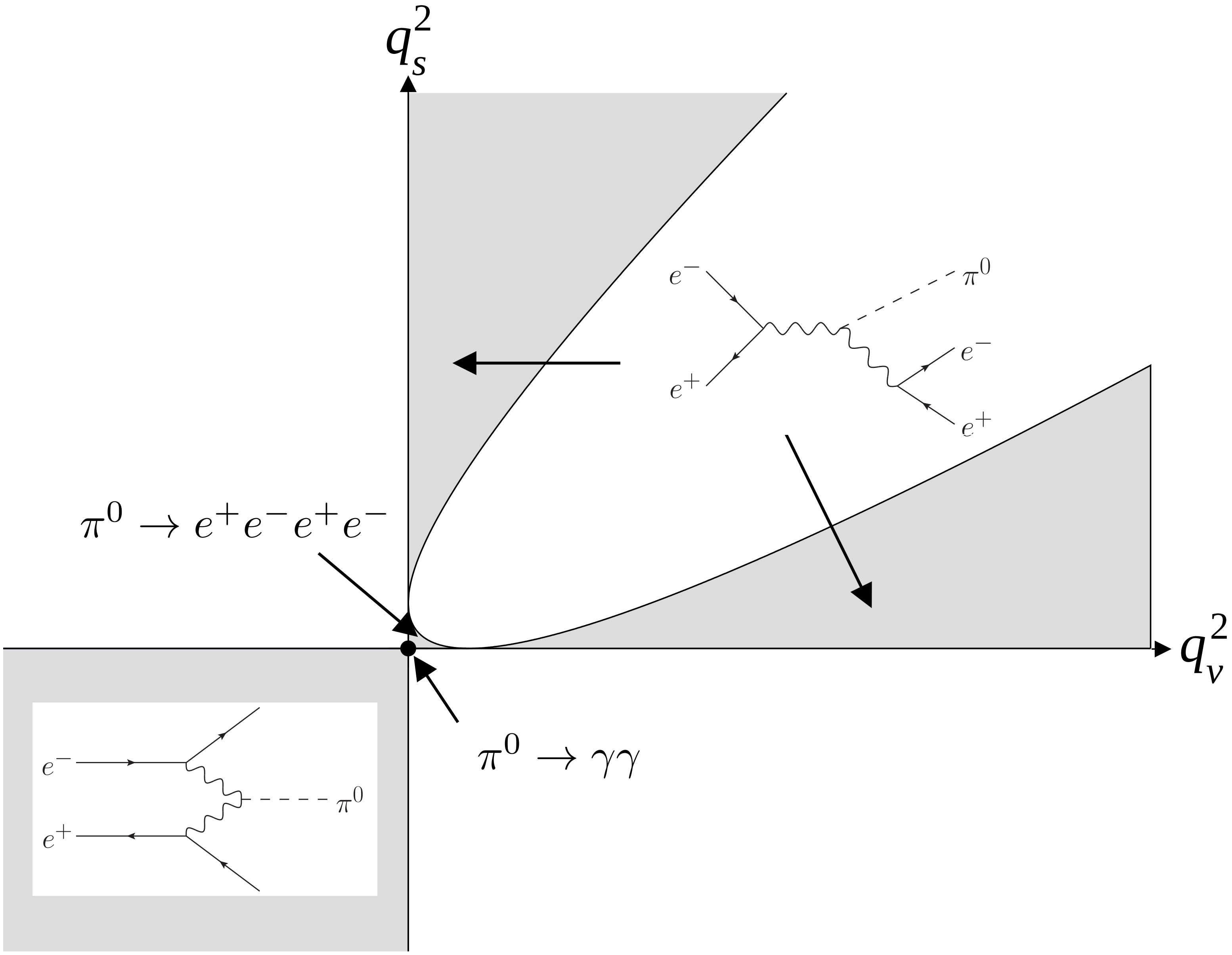
$\gamma\pi \to \pi\pi$

$\omega \to 3\pi$, $\phi \to 3\pi$

$\gamma^* \to 3\pi$

common theme:
resum $\pi\pi$ rescattering

$\pi^0 \to \gamma^*(q_v^2)\gamma^*(q_s^2)$ transition form factor



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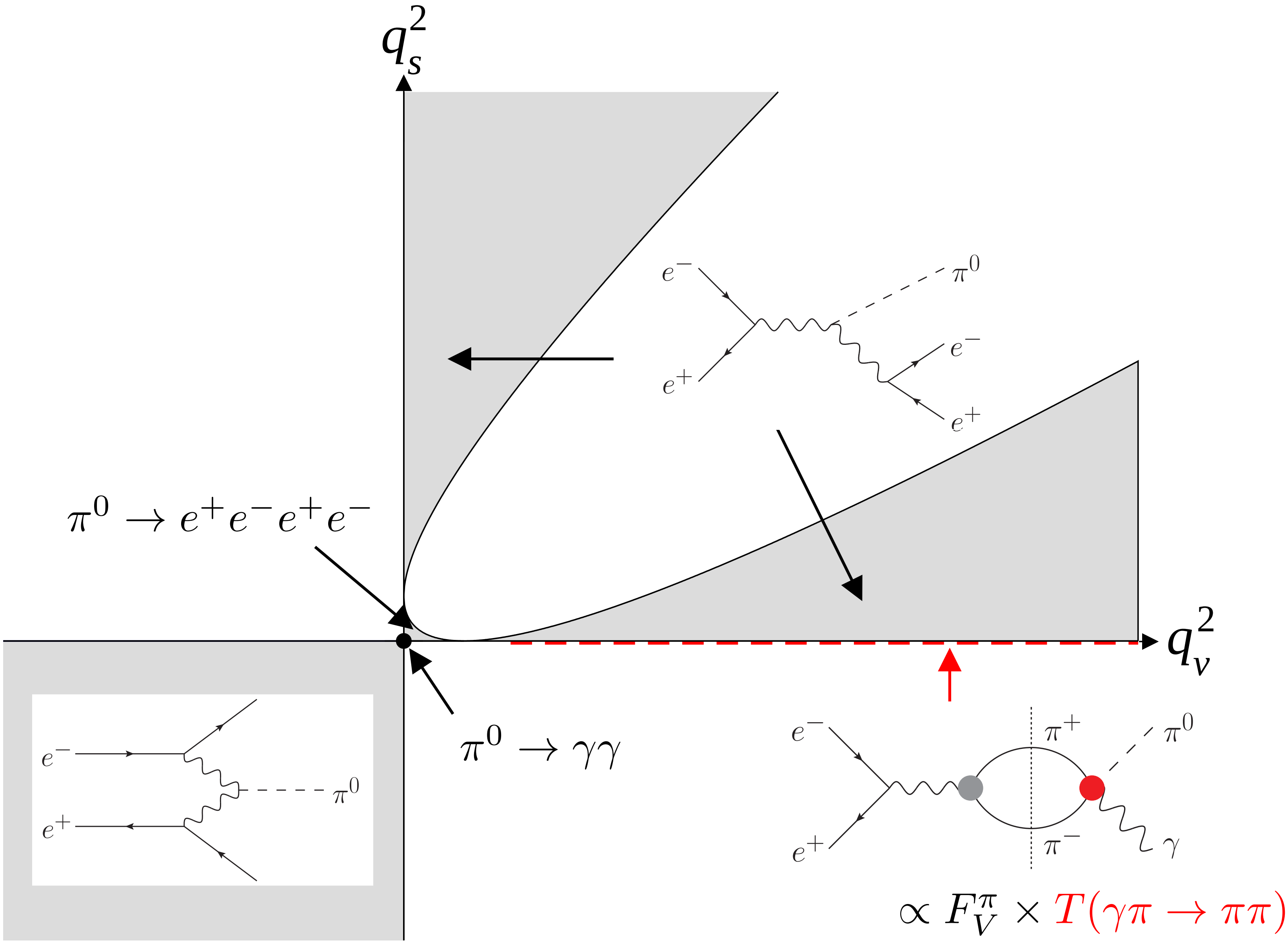

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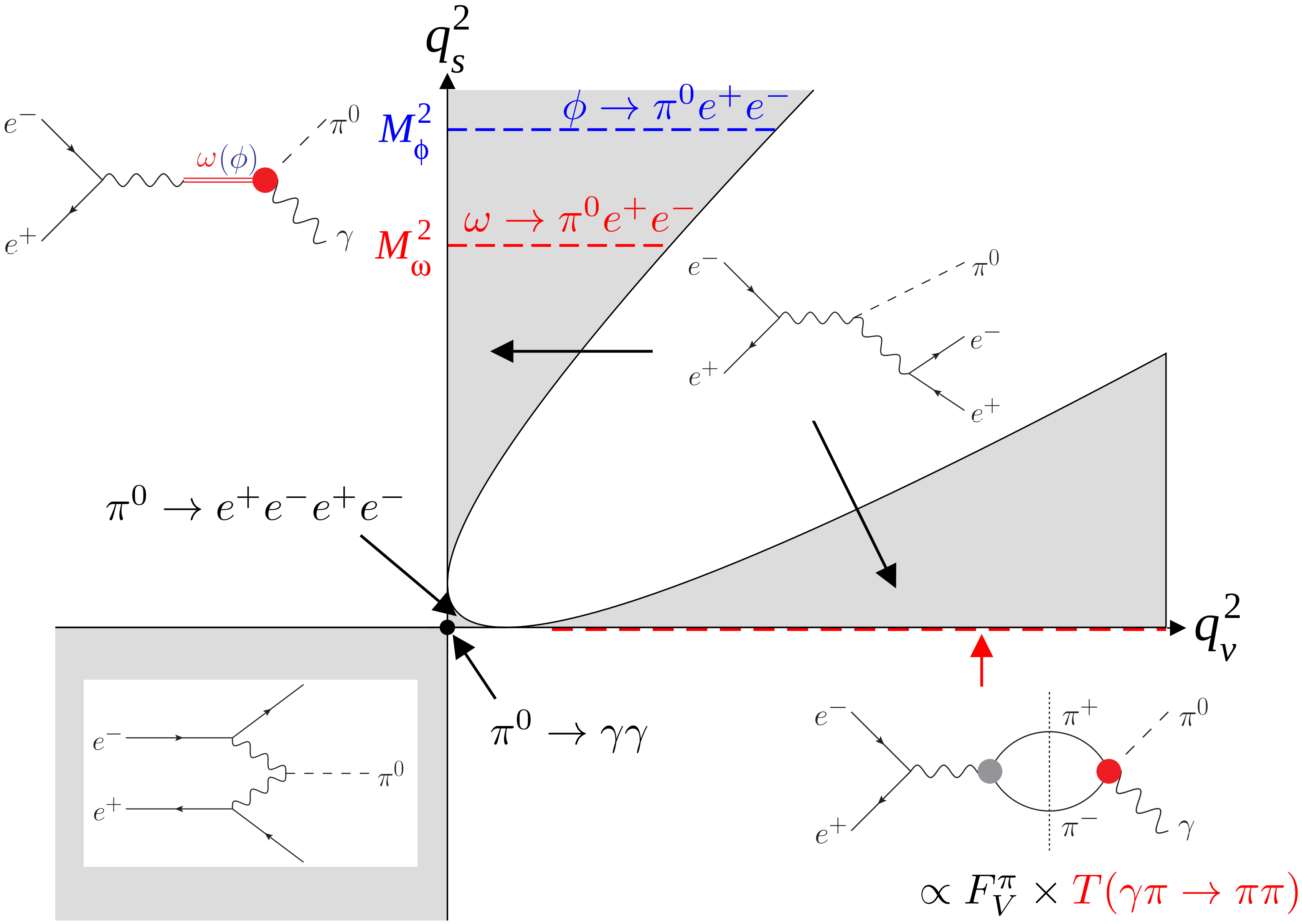
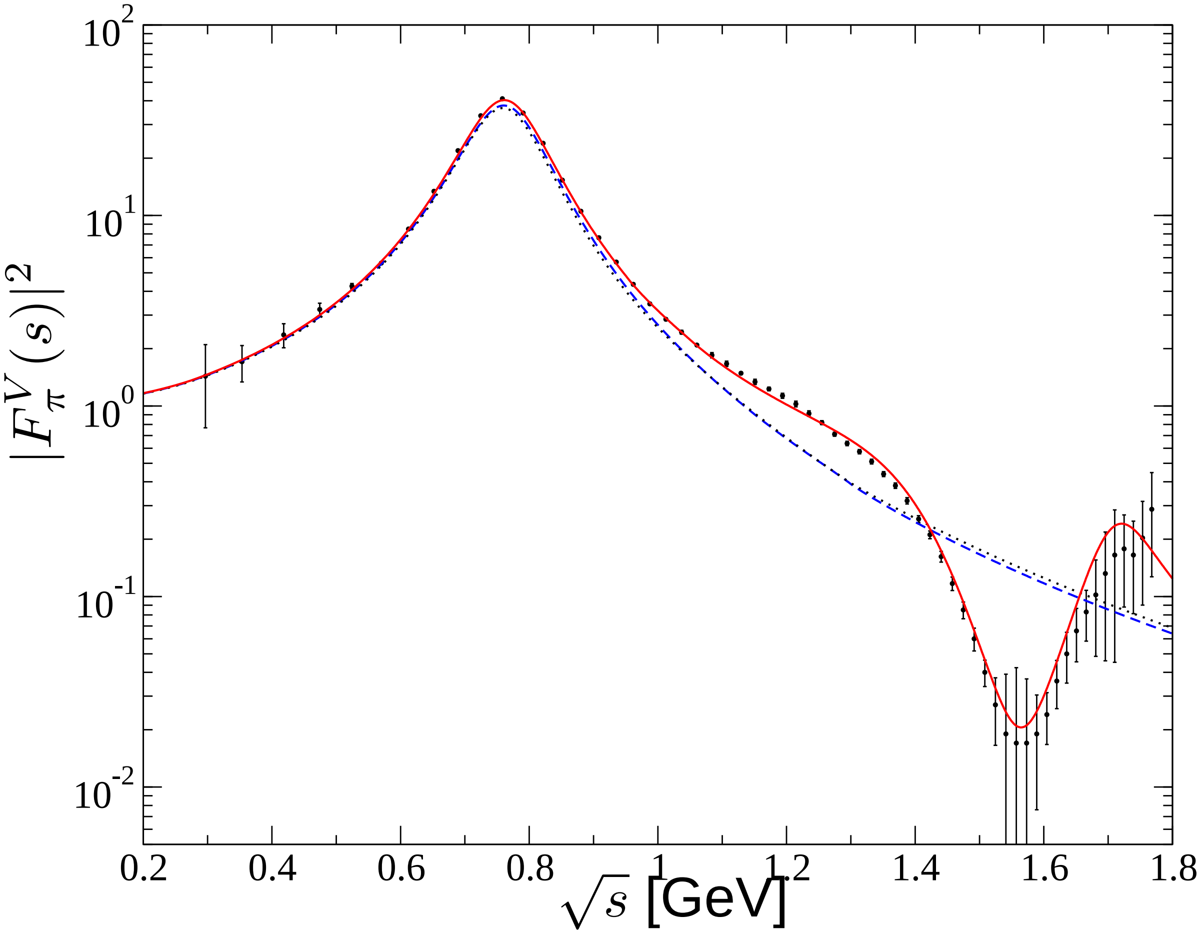


Pion vector form factor vs. Omnès representation

Data on pion form factor in $\tau^- \to \pi^-\pi^0\nu_\tau$ Belle 2008

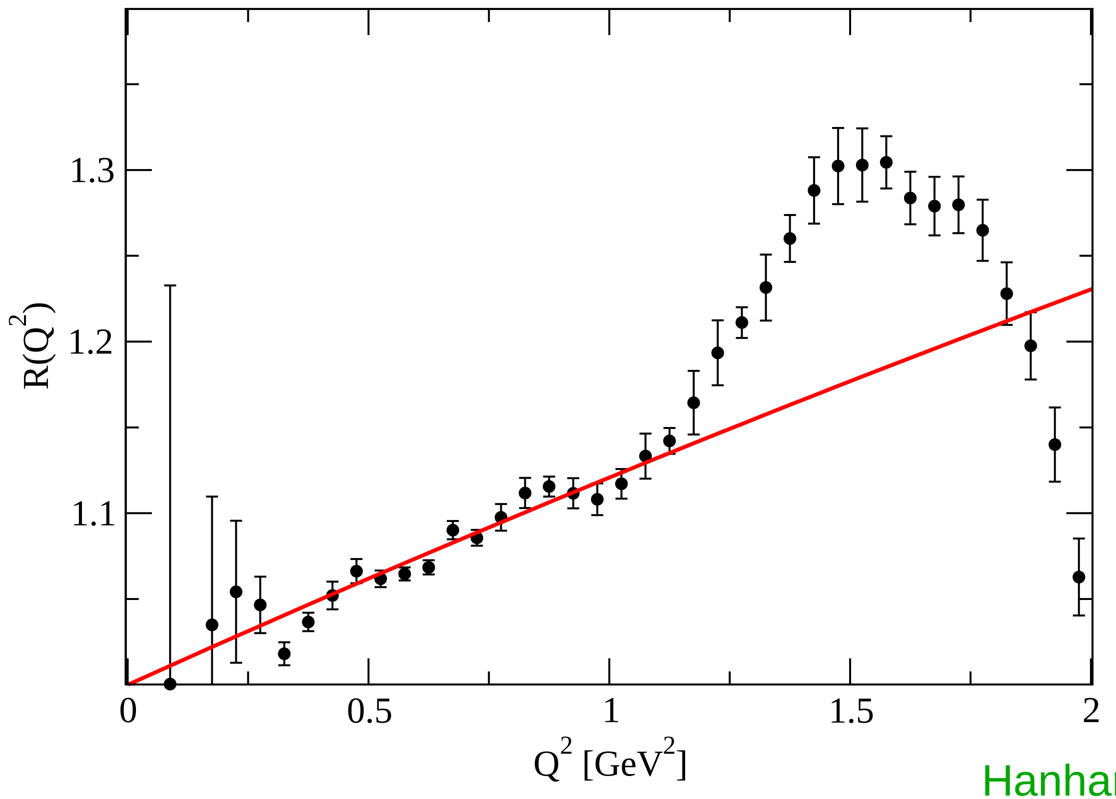


Schneider et al. 2012

Pion vector form factor vs. Omnès representation

Data on pion form factor in $\boldsymbol{\tau^- \to \pi^-\pi^0\nu_\tau}$ Belle 2008

- divide $\tau^- \to \pi^-\pi^0\nu_\tau$ form factor by Omnès function:



Hanhart et al. 2013

$\longrightarrow$ linear below 1 GeV: $F_\pi^V(s) \approx (1 + 0.1\,\text{GeV}^{-2}s)\Omega(s)$

$\longrightarrow$ above: inelastic resonances ρ', ρ''...

Extension to vector-meson decays: $\omega/\phi \to 3\pi$

- identical quantum numbers to $\gamma\pi \to \pi\pi$
- beyond ChPT: copious efforts to develop EFT for vector mesons

  Bijnens et al.; Bruns, Meißner; Lutz, Leupold; Gegelia et al.; Kampf et al. . . .
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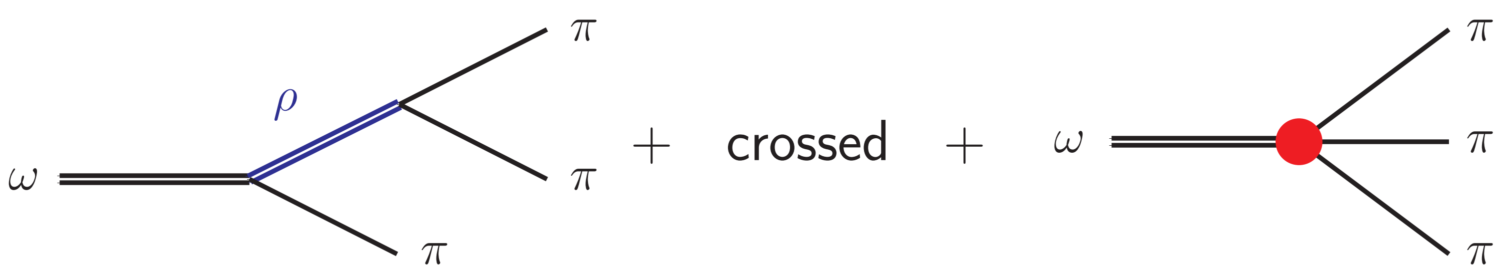
  \+ constant background term



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Problem:

$\longrightarrow$ unitarity fixes Im/Re parts

$\longrightarrow$ adding a contact term destroys this relation

$\longrightarrow$ reconcile data with dispersion relations?

$\omega/\phi \to 3\pi$: dispersive solution

- identical quantum numbers to $\gamma\pi \to \pi\pi$

$$\mathcal{F}(s) = a\,\Omega(s)\bigg\{1 + \frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\sin\delta_1^1(s')\hat{\mathcal{F}}(s')}{|\Omega(s')|(s'-s-i\epsilon)}\bigg\}$$

$$\hat{\mathcal{F}}(s) = \frac{3}{2}\int_{-1}^{1}dz\,(1-z^2)\mathcal{F}\big(t(s,z)\big)$$

$\longrightarrow$ fix subtraction constant a to partial width(s) $\omega/\phi \to 3\pi$

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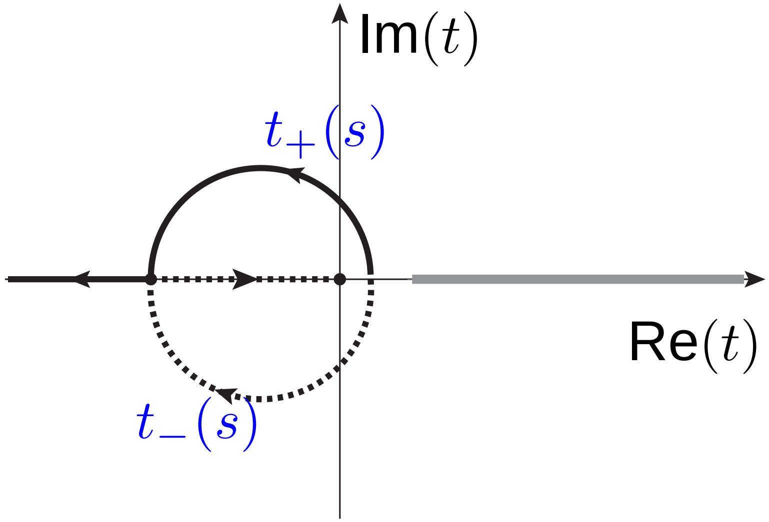
$\longrightarrow$ fix subtraction constant a to partial width(s) $\omega/\phi \to 3\pi$

- complication:
  analytic continuation in
  decay mass M_V required
- $M_V < 3M_\pi$:
  okay



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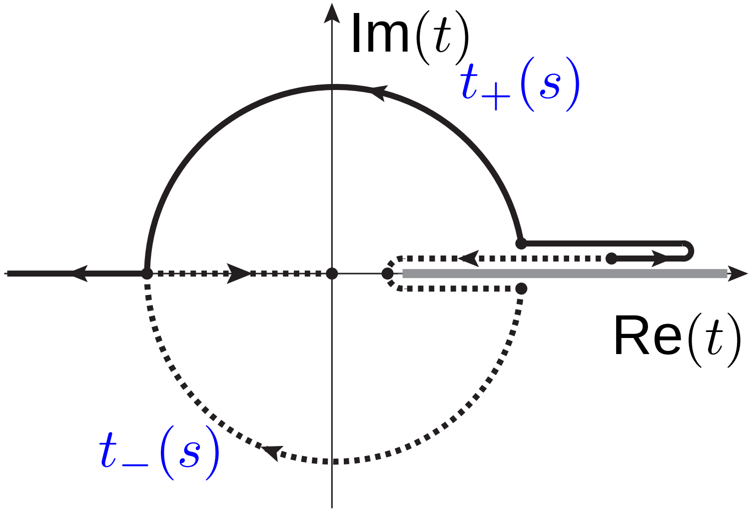

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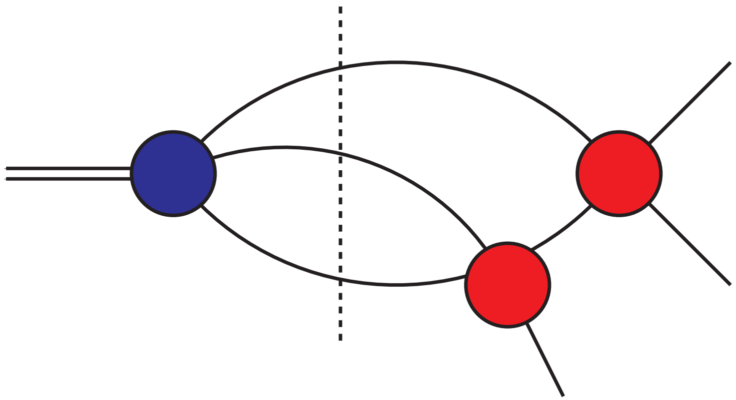
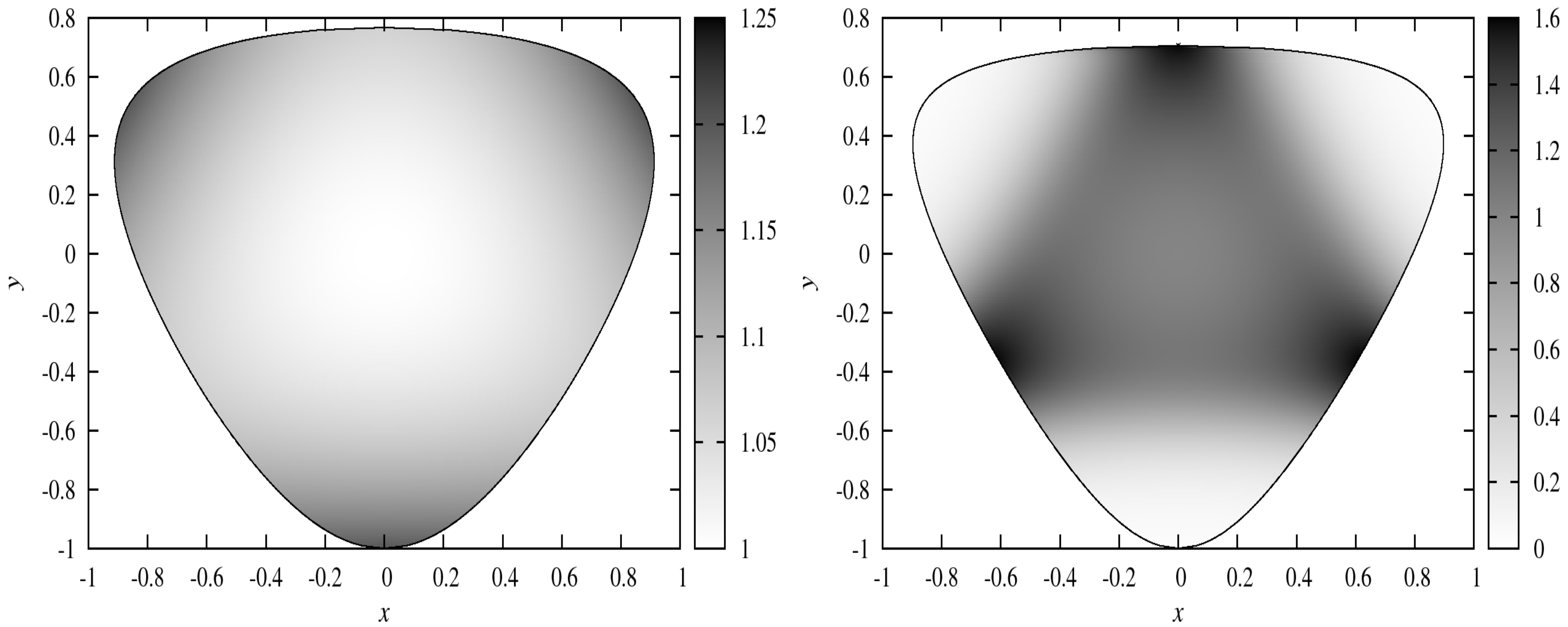
- $M_V > 3M_\pi$:
  path deformation required
  $\longrightarrow$ generates 3-particle cuts



$\omega/\phi \to 3\pi$ Dalitz plots

- subtraction constant a fixed to partial width
  $\longrightarrow$ normalised Dalitz plot a prediction

$\omega \to 3\pi$: $\phi \to 3\pi$:



- ω Dalitz plot is relatively smooth
- ϕ Dalitz plot clearly shows ρ resonance bands

Niecknig, BK, Schneider 2012

$\omega/\phi \to 3\pi$ Dalitz plots

- subtraction constant a fixed to partial width
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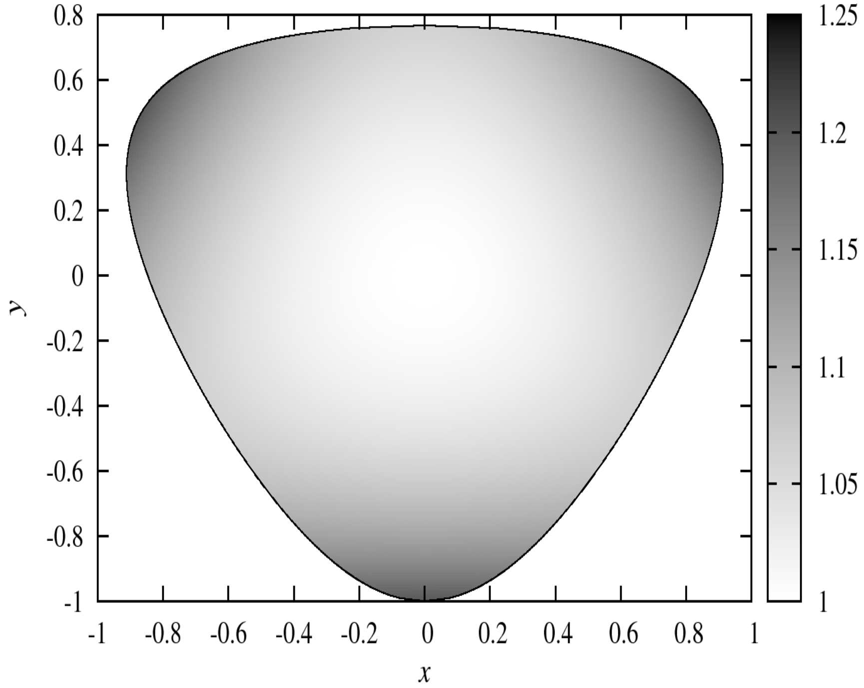
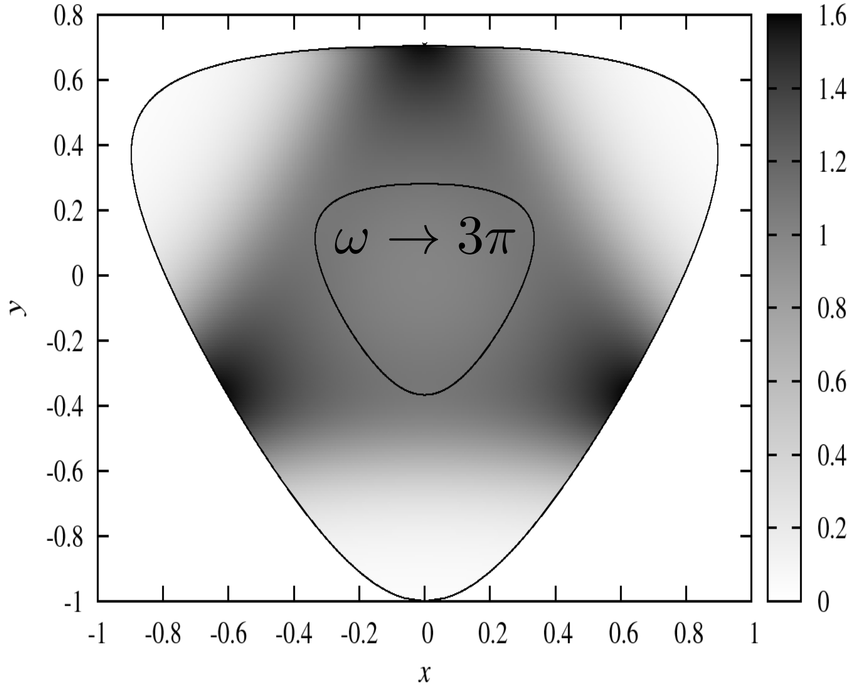
$\omega \to 3\pi$:



$\phi \to 3\pi$:



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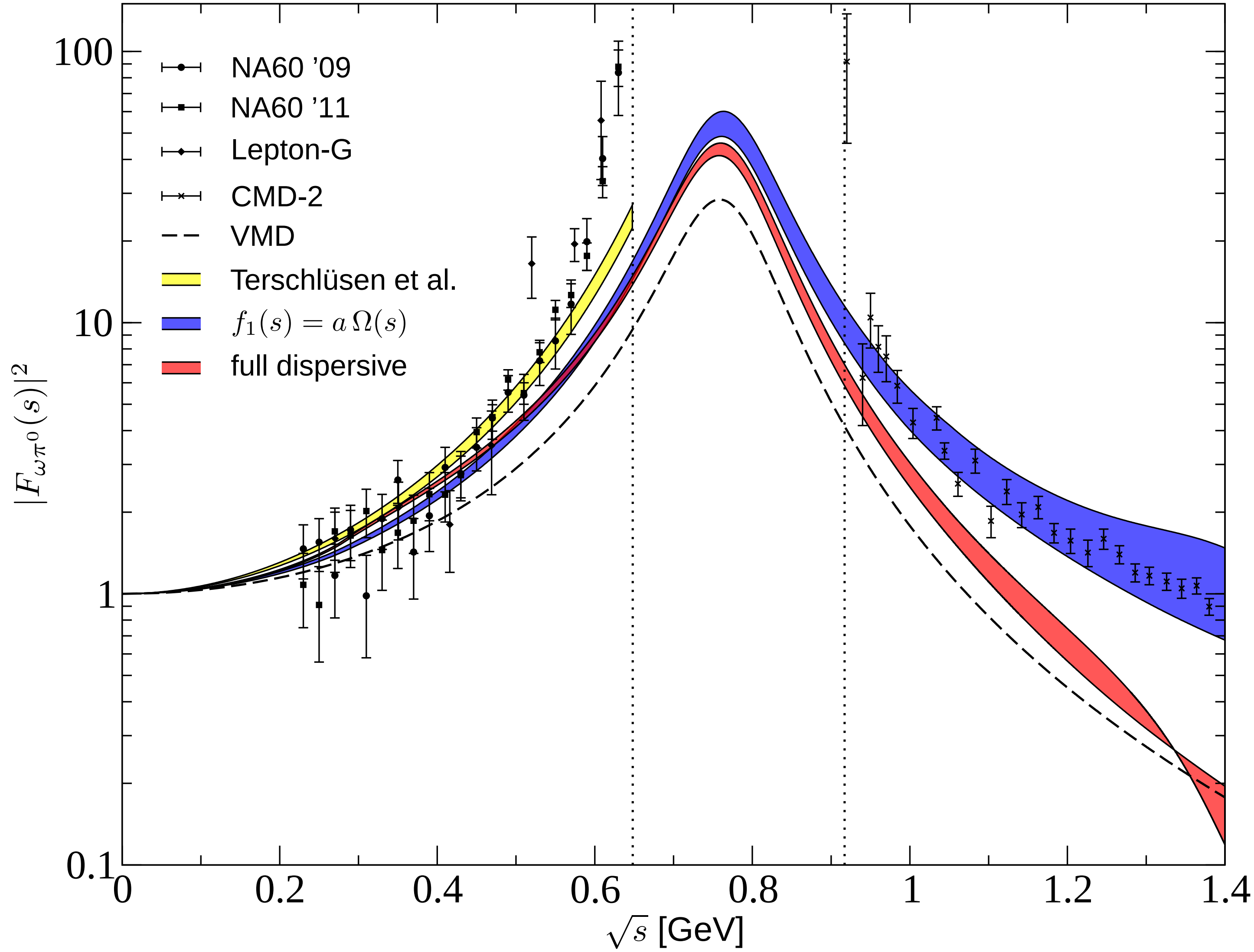
Niecknig, BK, Schneider 2012

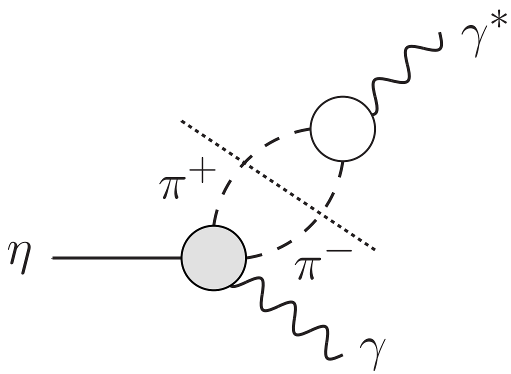
Naive extension to $e^+e^- \to \pi^0\omega$



- full solution above naive VMD, but still too low
- higher intermediate states (4π / $\pi\omega$) more important?

Anomalous decay $\eta \to \pi^+\pi^-\gamma$

- $\alpha_{\text{KLOE}} = (1.52 \pm 0.06)\,\text{GeV}^{-2}$ large
  $\longrightarrow$ implausible to explain through ρ', ρ''...
- for large t, expect $P(t) \to$ const. rather
- $\eta \to \gamma^*\gamma$ transition form factor:
  $\longrightarrow$ dispersion integral covers
  larger energy range



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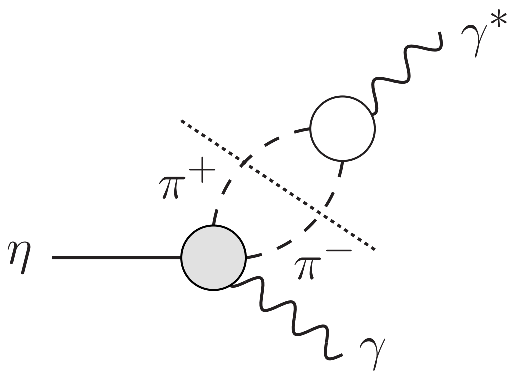


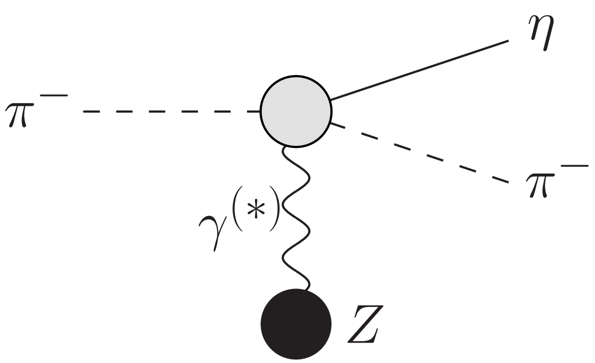
Intriguing observation:

- naive continuation of $\mathcal{F}^{\eta}_{\pi\pi\gamma} = A(1+\alpha t)\Omega(t)$ has zero at $t = -1/\alpha \approx -0.66\,\mathrm{GeV}^2$
  $\longrightarrow$ test this in crossed process $\gamma\pi^- \to \pi^-\eta$
  $\longrightarrow$ "left-hand cuts" in $\pi\eta$ system?

BK, Plenter 2015

Primakoff reaction $\gamma\pi \to \pi\eta$

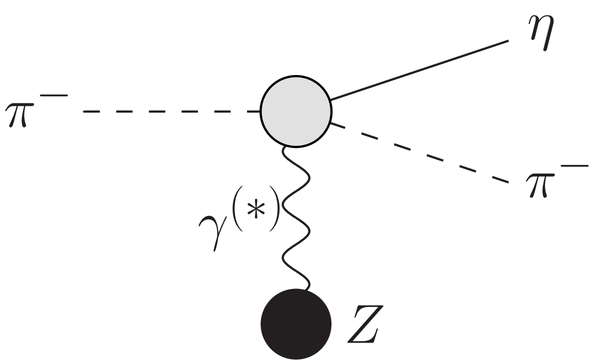
- can be measured in Primakoff reaction COMPASS
- S-wave forbidden
  P-wave exotic: $J^{PC} = 1^{-+}$
  D-wave $a_2(1320)$ first resonance



Primakoff reaction $\gamma\pi \to \pi\eta$

- can be measured in Primakoff reaction

COMPASS

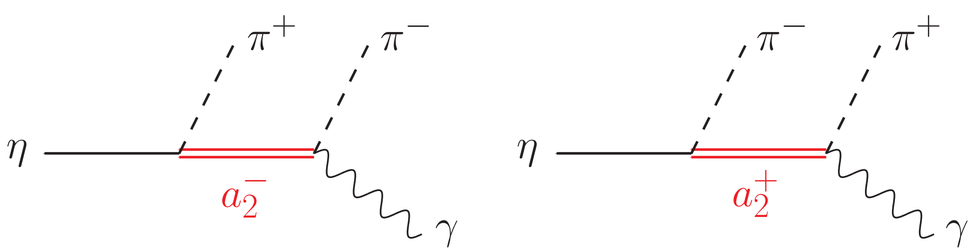


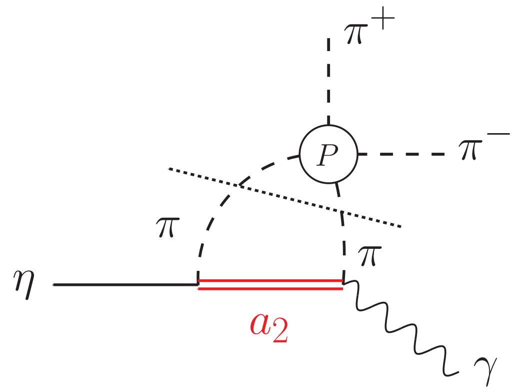
- S-wave forbidden

  P-wave exotic: $J^{PC} = 1^{-+}$

  D-wave $a_2(1320)$ first resonance

- include a_2 as left-hand cut in decay
  couplings fixed from $a_2 \to \pi\eta$, $\pi\gamma$



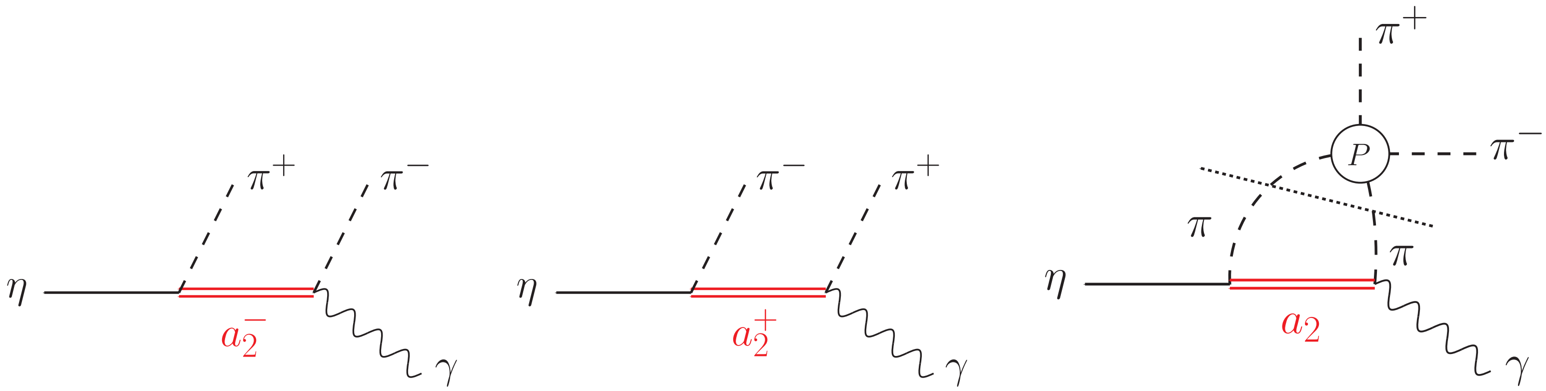


▷ compatible with decay data?

▷ predictions for $\gamma\pi \to \pi\eta$ cross sections and asymmetries
[$\longrightarrow$ spares]

BK, Plenter 2015

Formalism including left-hand cuts



- a_2 + rescattering essential to preserve Watson's theorem
- formally:

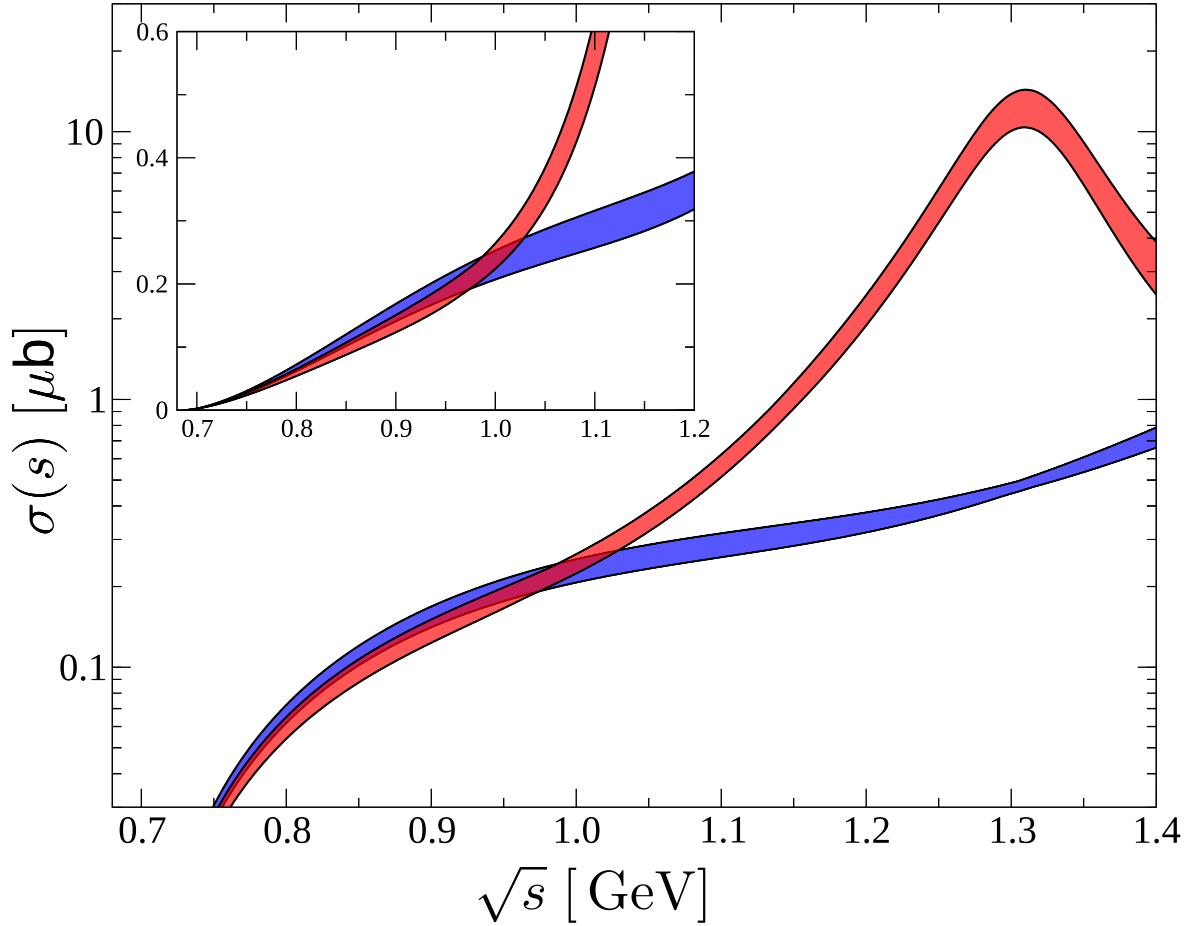
$$\mathcal{F}^{\eta}_{\pi\pi\gamma}(s,t,u) = \mathcal{F}(t) + \mathcal{G}_{a_2}(s,t,u) + \mathcal{G}_{a_2}(u,t,s)$$

$$\mathcal{F}(t) = \Omega(t)\left\{ A(1+\alpha t) + \frac{t^2}{\pi}\int_{4M_\pi^2}^{\infty} \frac{dx}{x^2}\frac{\sin\delta(x)\hat{\mathcal{G}}(x)}{|\Omega(x)|(x-t)}\right\}$$

$\hat{\mathcal{G}}$: t-channel P-wave projection of a_2 exchange graphs

- re-fit subtraction constants A, α

Total cross section $\gamma\pi \to \pi\eta$

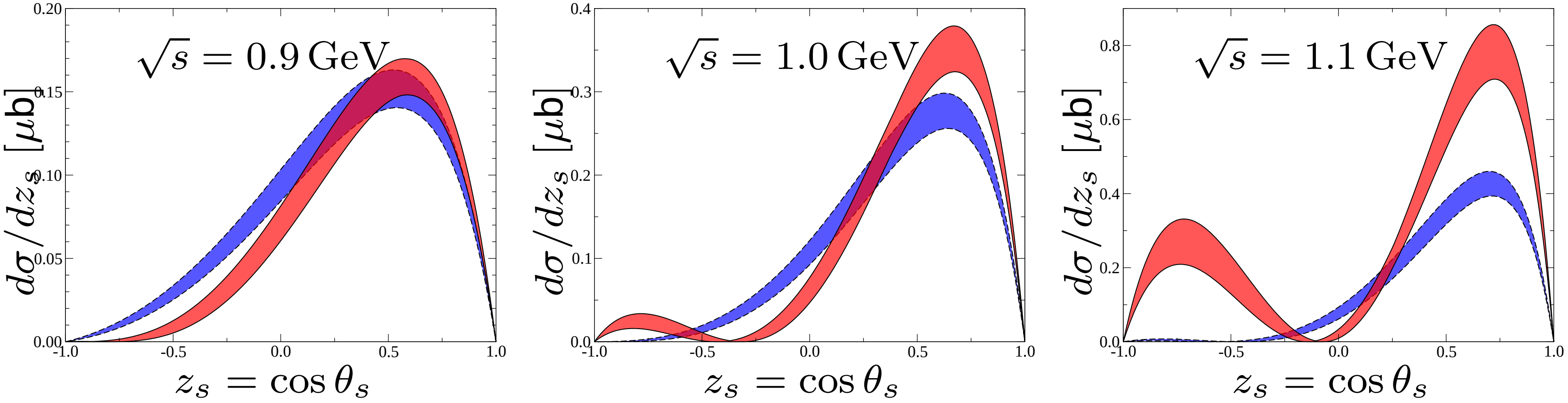


blue: t-channel dynamics / "ρ" only red: full amplitude

- t-channel dynamics dominate below $\sqrt{s} \approx 1\,\mathrm{GeV}$
- uncertainty bands: $\Gamma(\eta \to \pi^+\pi^-\gamma)$, α, a_2 couplings BK, Plenter 2015

Differential cross sections $\gamma\pi \to \pi\eta$

- amplitude zero visible in differential cross sections:



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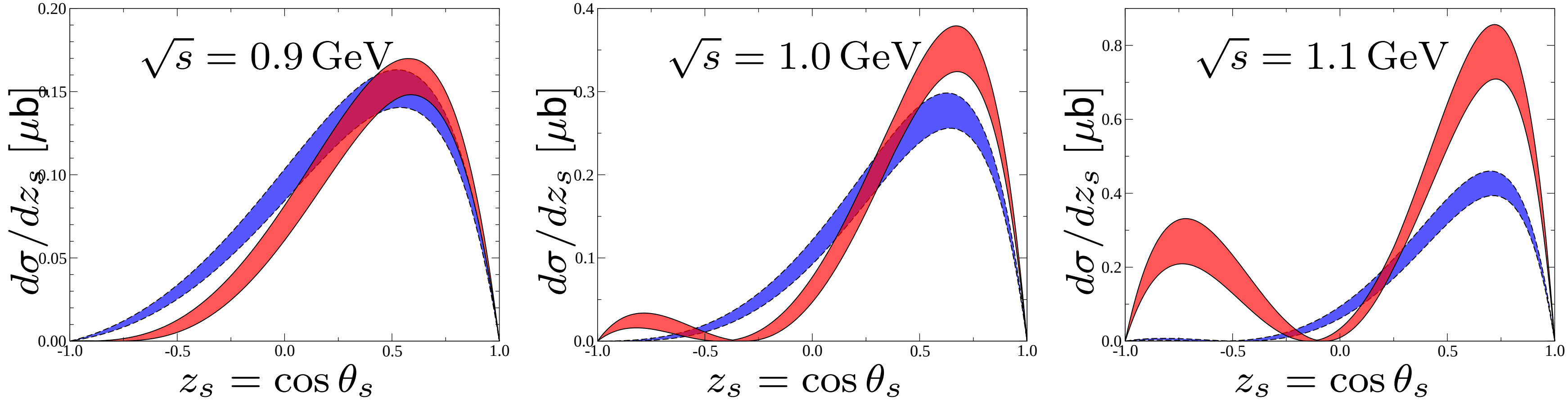
Differential cross sections $\gamma\pi \to \pi\eta$

- amplitude zero visible in differential cross sections:



blue: t-channel dynamics / "ρ" only red: full amplitude

- strong P-D-wave interference
- can be expressed as forward-backward asymmetry

$$A_{\text{FB}} = \frac{\sigma(\cos\theta > 0) - \sigma(\cos\theta < 0)}{\sigma_{\text{total}}}$$