

Theory of Two-Photon Physics

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in coll. with Marc Vanderhaeghen

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(g-2) theory vs exp

Experiment:

$$a_{\mu}^{exp} = (11\,659\,208.9 \pm 6.3) \times 10^{-10}$$

BNL, (2006)
PRD 73 072003

Theory:

$$a_{\mu}^{SM} = (11\,659\,182.8 \pm 4.9) \times 10^{-10}$$

HLMNT, (2011)
J. Ph. G 38, 085003

$$a_{\mu}^{exp} - a_{\mu}^{SM} = \\ (26.1 \pm 4.9_{th} \pm 6.3_{exp}) \times 10^{-10}$$

3 - 4 σ
deviation !

?

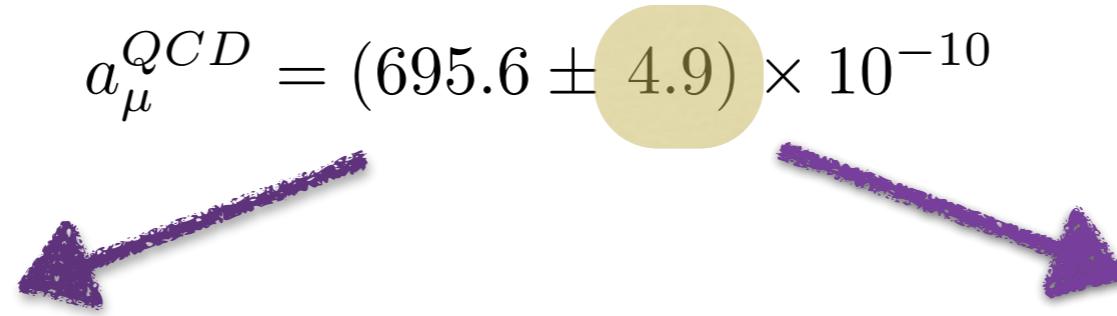
1.6_{exp}

FNAL, J-PARC
experiments

QCD contribution to ($g-2$)

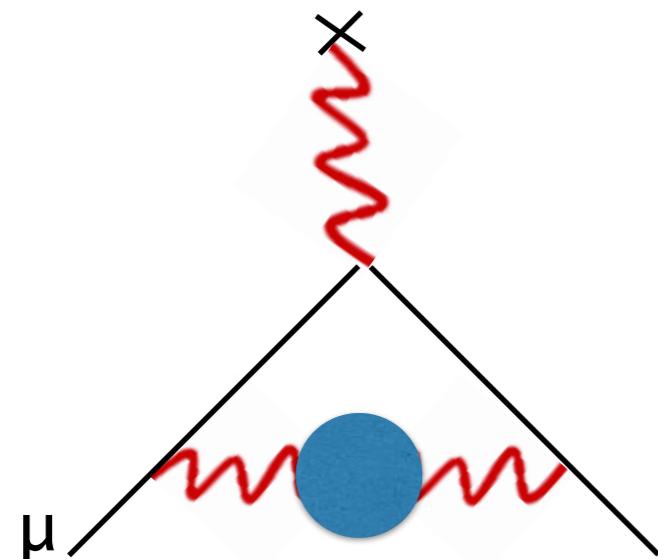
$$a_{\mu}^{QCD} = (695.6 \pm 4.9) \times 10^{-10}$$

Hagiwara (2011)
Jegerlehner (2015)



Hadronic vacuum polarization

$$a_{\mu}^{QCD, VP[LO]} = (694.9 \pm 4.3) \times 10^{-10}$$
$$a_{\mu}^{QCD, VP[HO]} = (-9.8 \pm 0.1) \times 10^{-10}$$

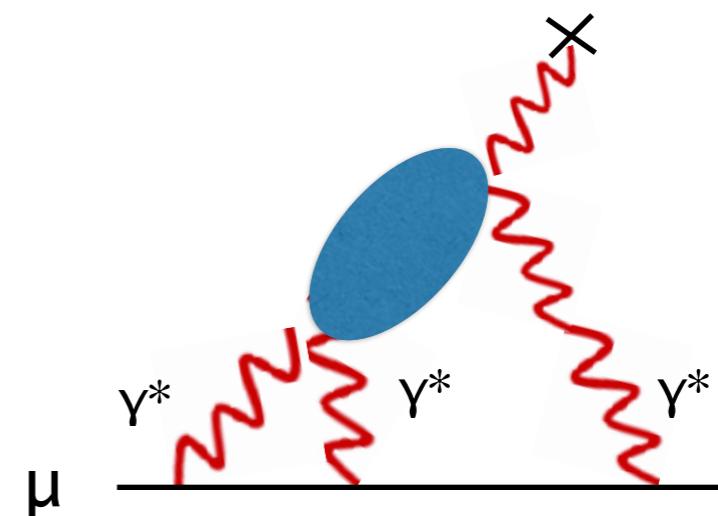


relies on experiment $e^+e^- \rightarrow \text{hadrons}$
through unitarity

$$\sigma(s)_{e^+e^- \rightarrow \text{hadrons}}$$

Hadronic light-by-light scattering

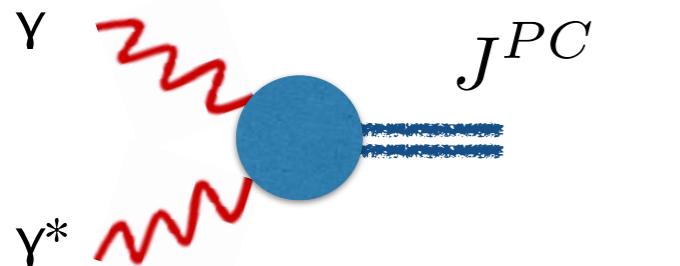
$$a_{\mu}^{QCD, LbL} = (10.5 \pm 2.6) \times 10^{-10}$$
$$= (10.2 \pm 3.9) \times 10^{-10}$$



relies on measurements of **TFF** to
reduce model dependence

$$\pi^0 \gamma^* \gamma^{(*)}, \eta \gamma^* \gamma^{(*)}, \dots$$
$$f_1 \gamma^* \gamma^{(*)}, f_2 \gamma^* \gamma^{(*)}, \dots$$

LbL sum rules: Meson production



$C=+1: J^{PC}=0^{-+}, 0^{++}, 1^{++}, 2^{++}, \dots$

$$Q^2 \neq 0$$

Landau-Yang
theorem

Narrow width approximation

$$\sigma(\gamma^* \gamma \rightarrow J^P(\Lambda)) = \delta(s - m^2) 8\pi^2 \frac{(2J+1) \Gamma_{\gamma\gamma}(J^P)}{m} \left(1 + \frac{Q^2}{m^2}\right) \left[T^{(\Lambda)}(Q^2)\right]^2$$

Sum rules will relate 2γ width or TFFs:

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s + Q^2)} [\sigma_2 - \sigma_0]$$

$$\begin{array}{l} Q^2 = 0 \\ \longrightarrow \\ Q^2 \neq 0 \end{array}$$

$$0 = - \sum_{\mathcal{P}} 16\pi^2 \frac{\Gamma_{\gamma\gamma}(\mathcal{P})}{m_{\mathcal{P}}^3} - \sum_{\mathcal{S}} \dots$$

$$0 = - \sum_{\mathcal{P}} 16\pi^2 \frac{\Gamma_{\gamma\gamma}(\mathcal{P})}{m_{\mathcal{P}}^3} [T(Q^2)]^2 - \sum_{\mathcal{S}} \dots$$

Pascalutsa, Pauk
Vanderhaeghen
(2012), (2014)

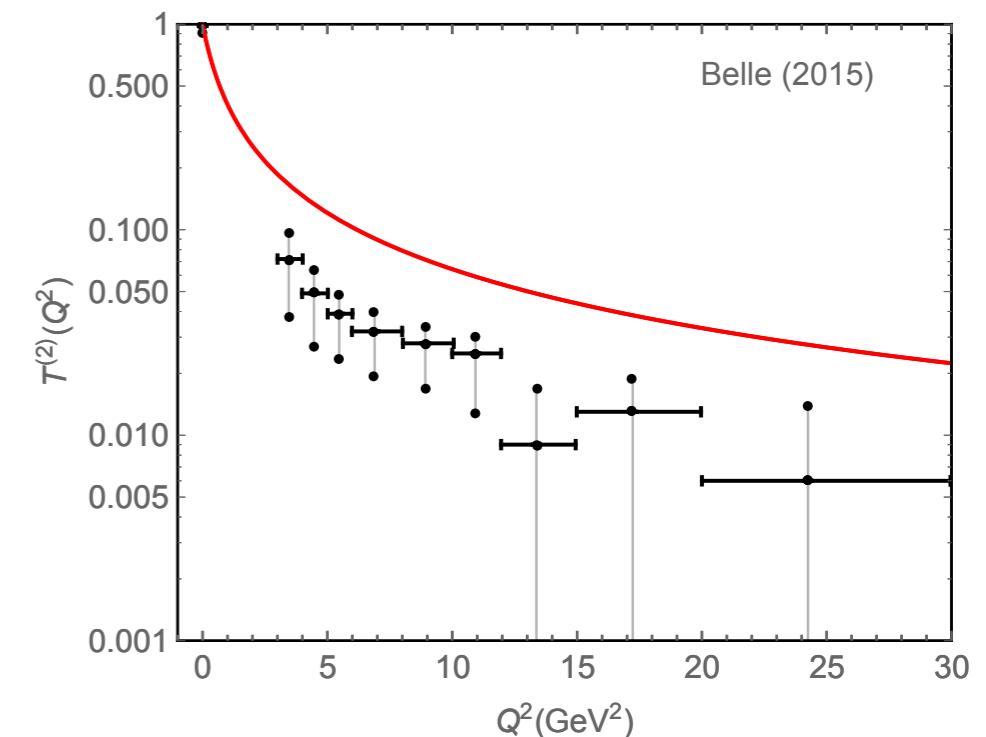
Sum rule I (*Isospin*=0)

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s + Q^2)} [\sigma_2 - \sigma_0]$$

$$0 = - \sum_{\mathcal{P}} 16\pi^2 \frac{\Gamma_{\gamma\gamma}(\mathcal{P})}{m_{\mathcal{P}}^3} \left[T_{\mathcal{P}}(Q^2) \right]^2 - \sum_{\mathcal{S}, \mathcal{A}} \dots \\ + \sum_{\mathcal{T}} 16\pi^2 \frac{\Gamma_{\gamma\gamma}(\mathcal{T})}{m_{\mathcal{T}}^3} \left(\left[T_{\mathcal{T}}^{(\Lambda=2)}(Q^2) \right]^2 - \left[T_{\mathcal{T}}^{(\Lambda=0)}(Q^2) \right]^2 \right)$$

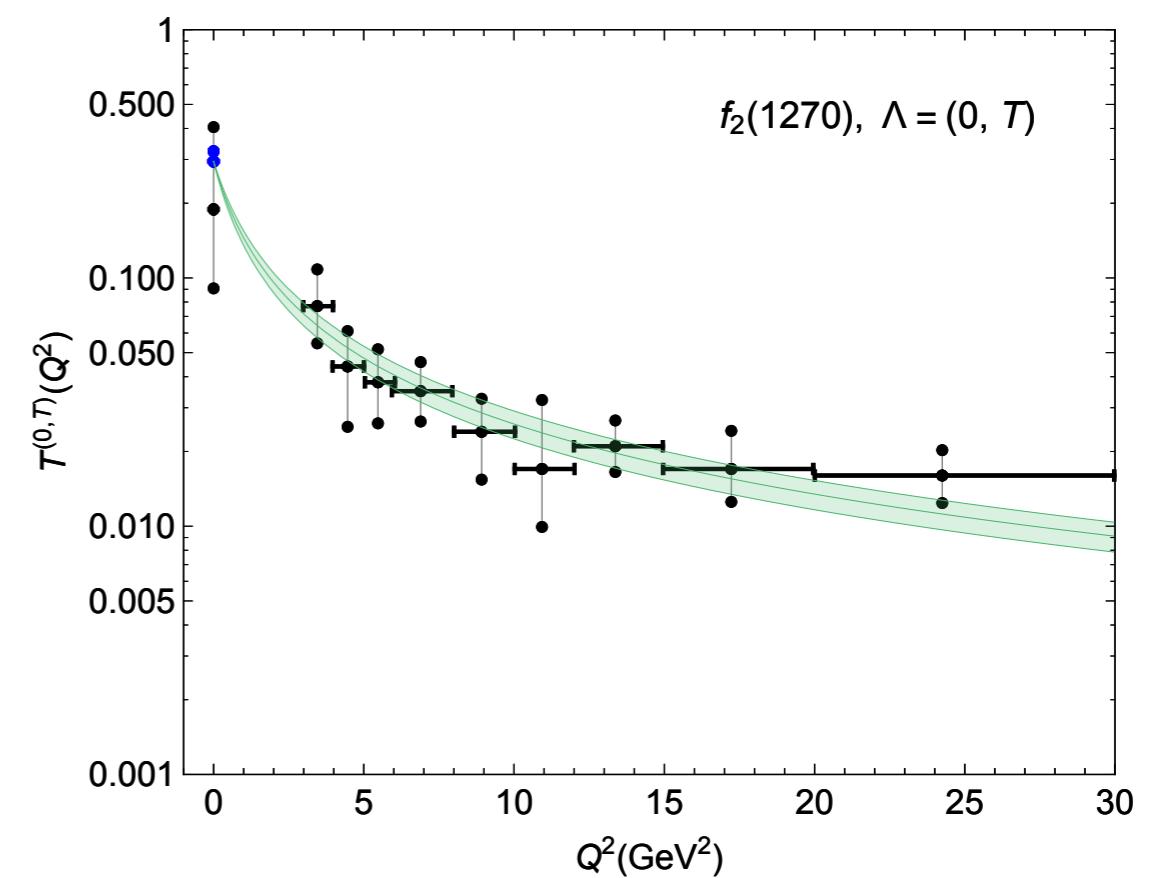
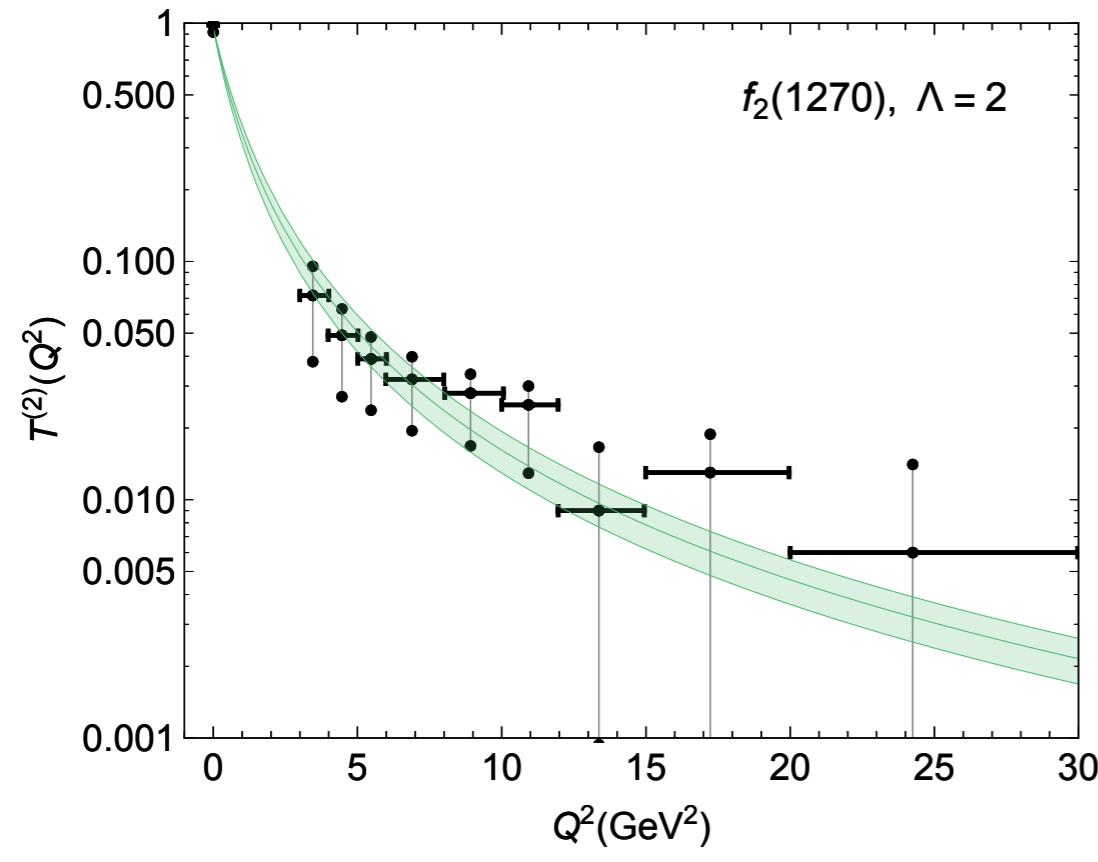
Dominant contributions

State	m (MeV)	$\Gamma_{\gamma\gamma}$ (keV)	$SR_1 (Q^2 = 0)$ (nb)
η	547.862 ± 0.017	0.516 ± 0.020	-193 \pm 7
η'	957 ± 0.06	4.35 ± 0.25	-304 \pm 17
$f_2(1270)$	1275.5 ± 0.8	2.93 ± 0.40	($\Lambda=2$) 434 \pm 60 ($\Lambda=0$) ≈ 0
$f_2(1565)$	1562 ± 13	0.70 ± 0.14	56 \pm 11
.....			
sum			-7 \pm 64



Pascalutsa, Pauk
Vanderhaeghen
(2012)

Belle (2015)

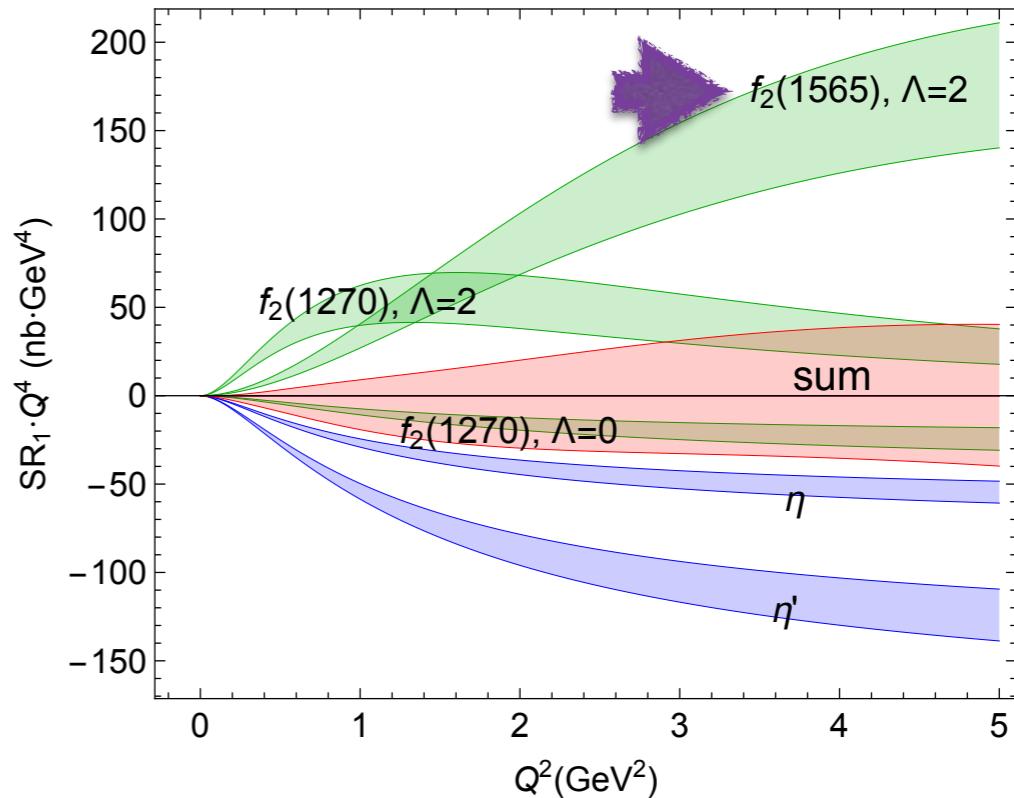
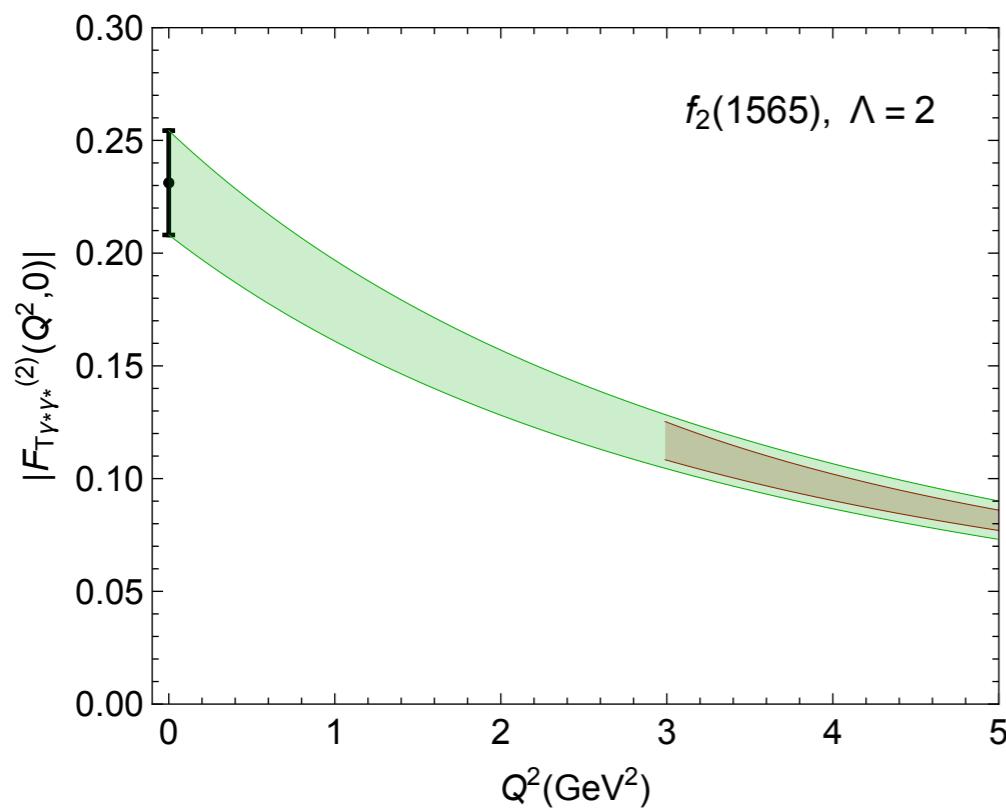
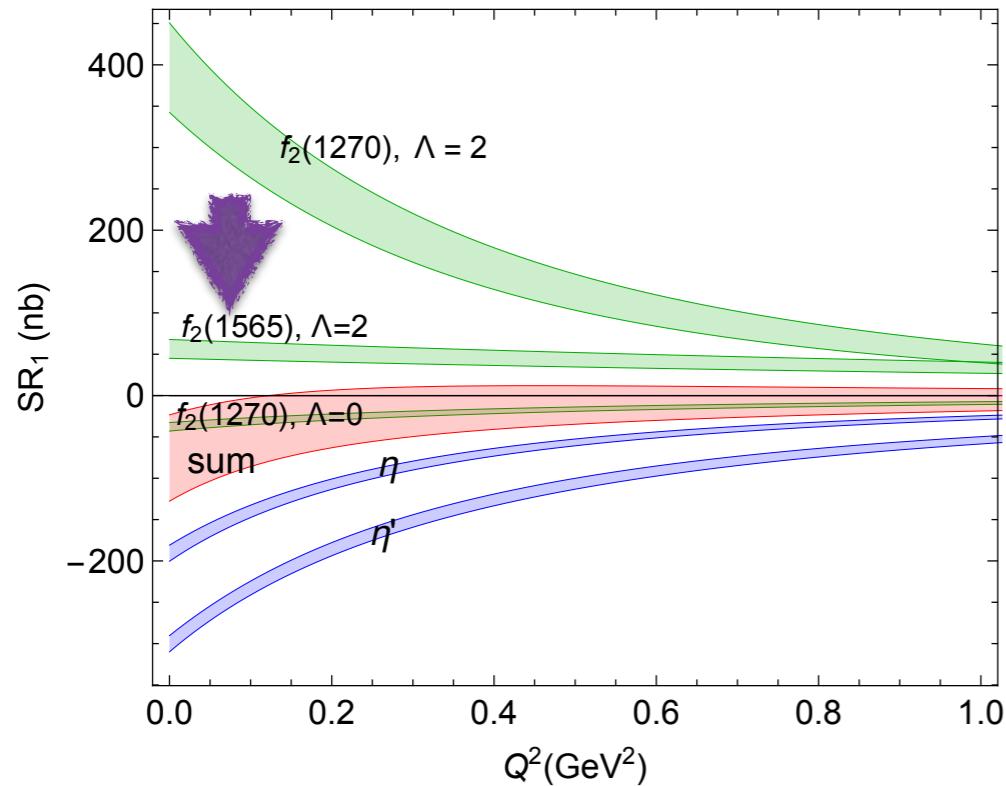


$$T^{(\Lambda)}(Q^2) = \text{Factor}(Q^2) \times \frac{1}{\left(1 + Q^2/\lambda_{(\Lambda)}^2\right)^2}$$

$$\lambda_{\Lambda=2} = 1222 \pm 66 \text{ MeV}$$

$$\lambda_{\Lambda=(0,T)} = 1051 \pm 36 \text{ MeV}$$

Sum rule I (*Isospin=0*)



$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s + Q^2)} [\sigma_2 - \sigma_0]$$

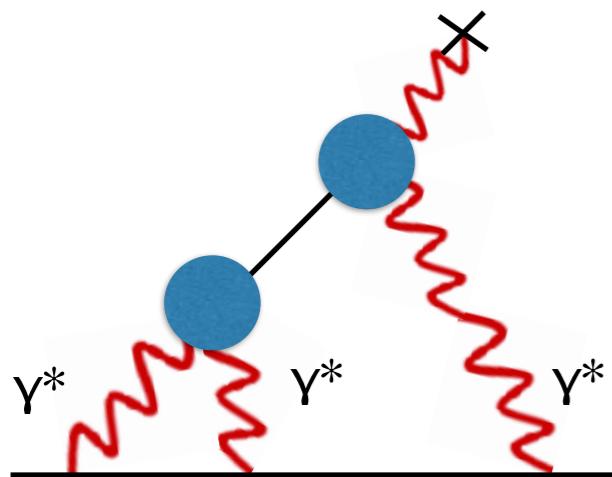
Prediction:

$f_2(1565)$
 $\lambda_{\Lambda=2} = 2719 \pm 53 \text{ MeV}$

I.D., Vanderhaeghen
 (2016)

future Belle data

Meson contributions to $(g-2)$



Lepton tensor: well known

Hadron tensor: requires input from **TFFs**

$$a_\mu^{LbL} = \lim_{k \rightarrow 0} ie^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} T^{\mu\nu\lambda\sigma}(q_1, k - q_1 - q_2, q_2) \Pi_{\mu\nu\lambda\sigma}(q_1, k - q_1 - q_2, q_2)$$

$$\frac{1}{q_1^2} \frac{1}{q_2^2} \frac{1}{(k - q_1 - q_2)^2} \frac{1}{(p + q_1)^2 - m^2} \frac{1}{(p' - q_2)^2 - m^2}$$

Results (excluding low energy region):

$$a_\mu[f_2(1270), f_2(1565)] = (0.1 \pm 0.01) \times 10^{-10}$$

New evaluation of axial vector contributions (satisfying Landau-Yang theorem)

$$a_\mu[f_1(1285), f_1(1420)] = (0.64 \pm 0.20) \times 10^{-10}$$

$$= (0.75 \pm 0.27) \times 10^{-10}$$

Pauk, Vdh (2013)
Jegerlehner (2015)

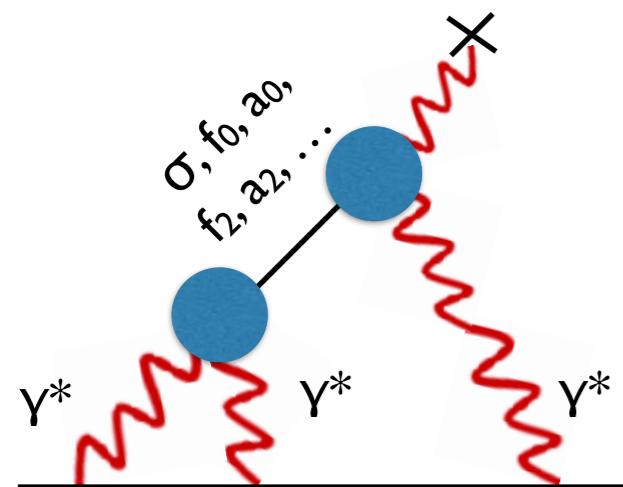
Compared to $(1.5 \pm 1.0) \times 10^{-10}$
(which enters the Glasgow consensus)

$$\delta a_\mu^{exp} = 1.6 \times 10^{-10}$$

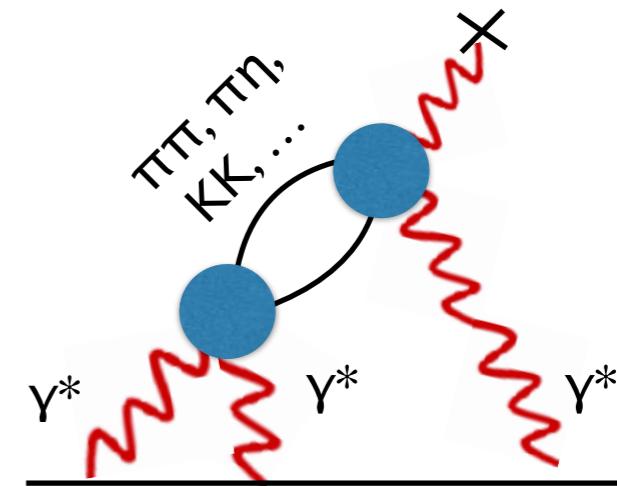
FNAL, J-PARC
experiments

Improvements: Multi-meson production

Important contributions beyond **pseudo-scalar** poles



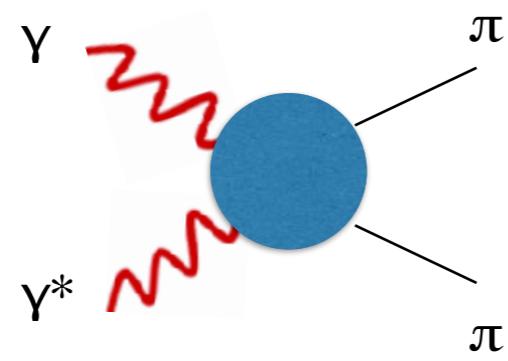
dispersive analysis for
 $\pi\pi, \pi\eta, \dots$ loops



Important ingredient: $\gamma\gamma^* \rightarrow \pi\pi, \pi\eta, \dots$

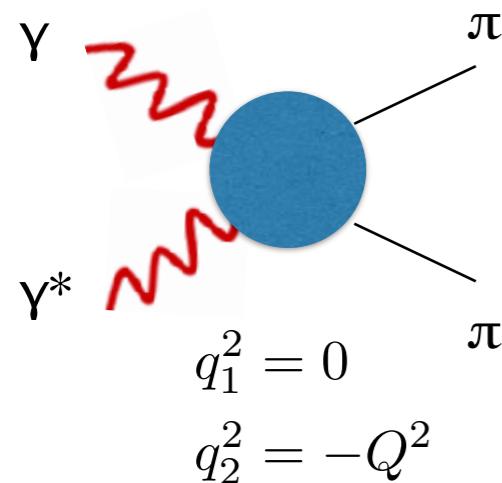
Pauk,
Vanderhaeghen,
(2014)

Colangelo,
Hoferichter, Procura,
Stoffer, (2014, 2015)



$\gamma\gamma \rightarrow \pi\pi, KK, \eta\eta, \pi\eta$ (Belle: 07,08,09,10,..)
 $\gamma\gamma^* \rightarrow \pi\pi, \pi\eta$ (BESIII in progress)

Cross section



$C=+1: J^{PC}=0^{++}, 2^{++}, 1^{-+}, \dots$

$$Q^2 \neq 0$$

Landau-Yang
theorem

Helicity amplitudes

$$H_{\lambda_1 \lambda_2} = H^{\mu\nu} \epsilon_\mu(\lambda_1) \epsilon_\nu(\lambda_2), \quad \lambda_1 = \pm 1, \lambda_2 = \pm 1, 0$$

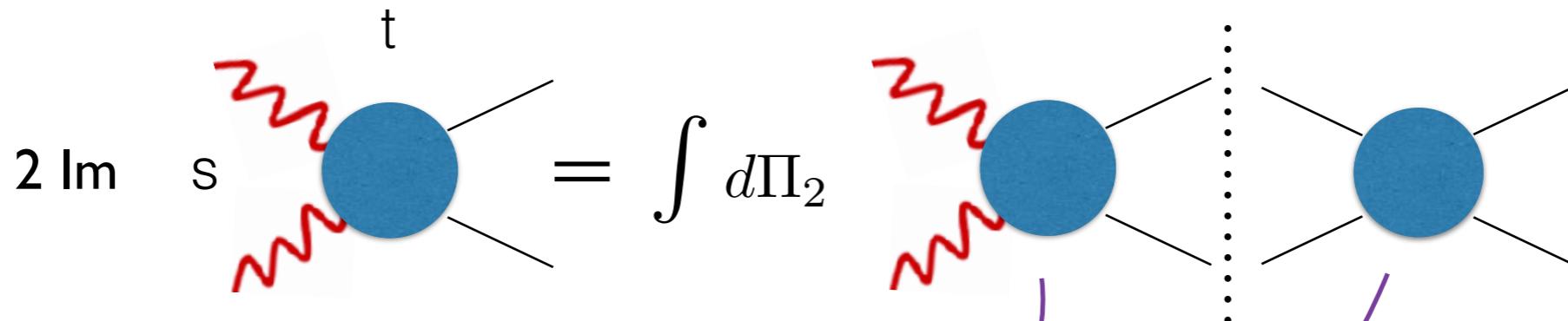
P symmetry: **6** **3** independent amplitudes

$$H_{++}, H_{+-}, H_{+0}$$

Differential cross section

$$\frac{d\sigma}{d \cos \theta} = \pi \alpha^2 \frac{\rho(s)}{4(s + Q^2)} (|H_{++}|^2 + |H_{+-}|^2 + |H_{+0}|^2)$$

Unitarity



Partial wave expansion

$$H_{\lambda_1 \lambda_2}(s, t) = \sum_{J=0}^{\cancel{\infty}} (2J+1) h_{J, \lambda_1 \lambda_2}(s) d_{\lambda_1 - \lambda_2, 0}^J(\theta)$$

J_{max} = 2

$$T(s, t) = \sum_{J=0}^{\cancel{\infty}} (2J+1) t_J(s) P_J(\theta)$$

J_{max} = 2

These “diagonalise unitarity” and contain resonance information

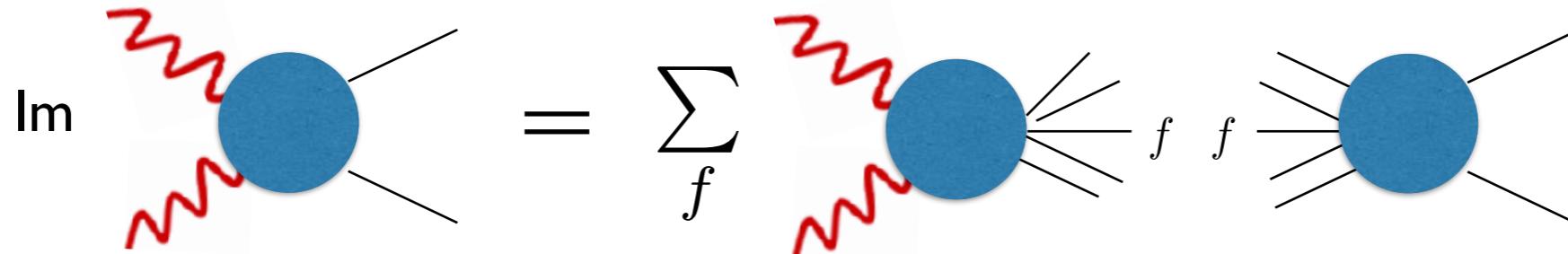
Definite: J, λ_1, λ_2

$$\text{Im } h_{\gamma\gamma^* \rightarrow \pi\pi}(s) = h_{\gamma\gamma^* \rightarrow \pi\pi}(s) \rho_{\pi\pi}(s) t_{\pi\pi \rightarrow \pi\pi}^*(s)$$

Coupled channel Unitarity

Coupled-channel unitarity

Definite: J, λ_1, λ_2



$$\text{Im } h_{\gamma\gamma^*,b}(s) = \sum_f h_{\gamma\gamma^*,f}(s) \rho_f(s) t_{fb}^*(s)$$

$$\text{Im } h_{\gamma\gamma^*,1}(s) = \rho_1 h_{\gamma\gamma^*,1} t_{11}^* + \rho_2 h_{\gamma\gamma^*,2} t_{21}^*$$

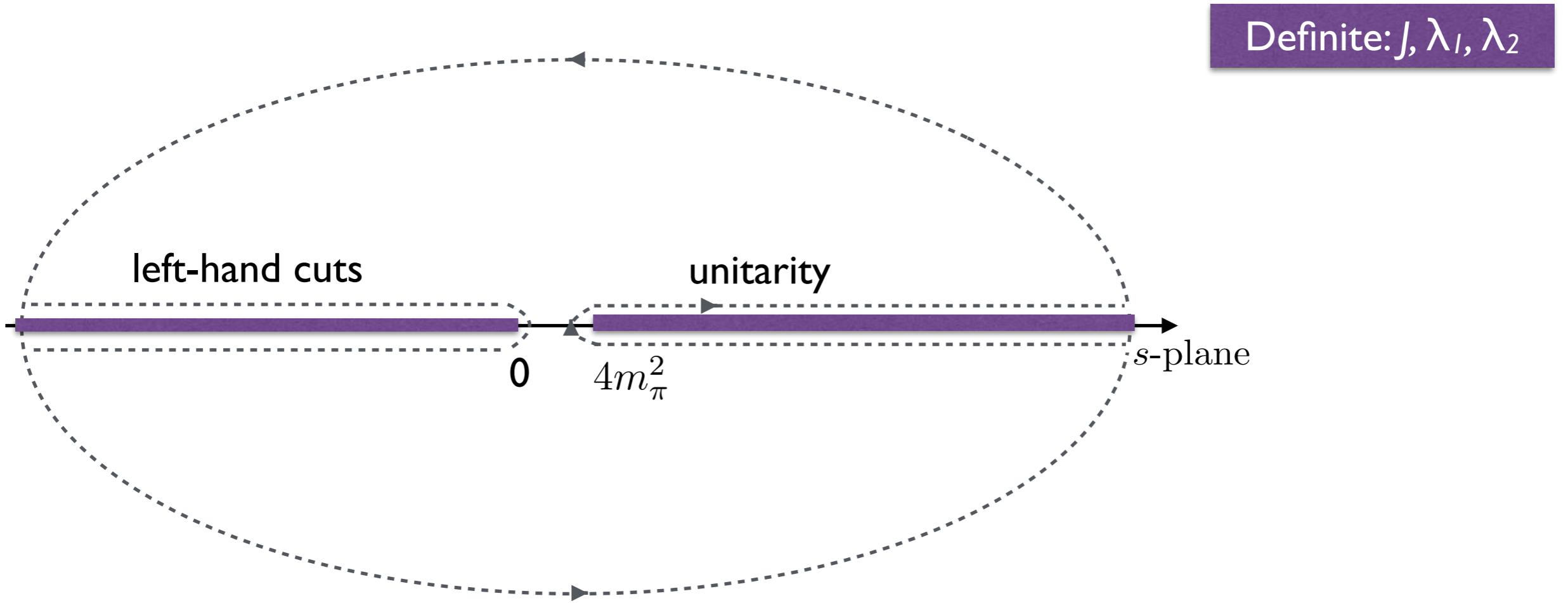
$$\text{Im } h_{\gamma\gamma^*,2}(s) = \rho_1 h_{\gamma\gamma^*,1} t_{12}^* + \rho_2 h_{\gamma\gamma^*,2} t_{22}^*$$

1 = $\pi\pi$

2 = KK

Entire dynamical information that does not depend on the underlying theory (e.g. QCD) comes from **unitarity**

Dispersion relation

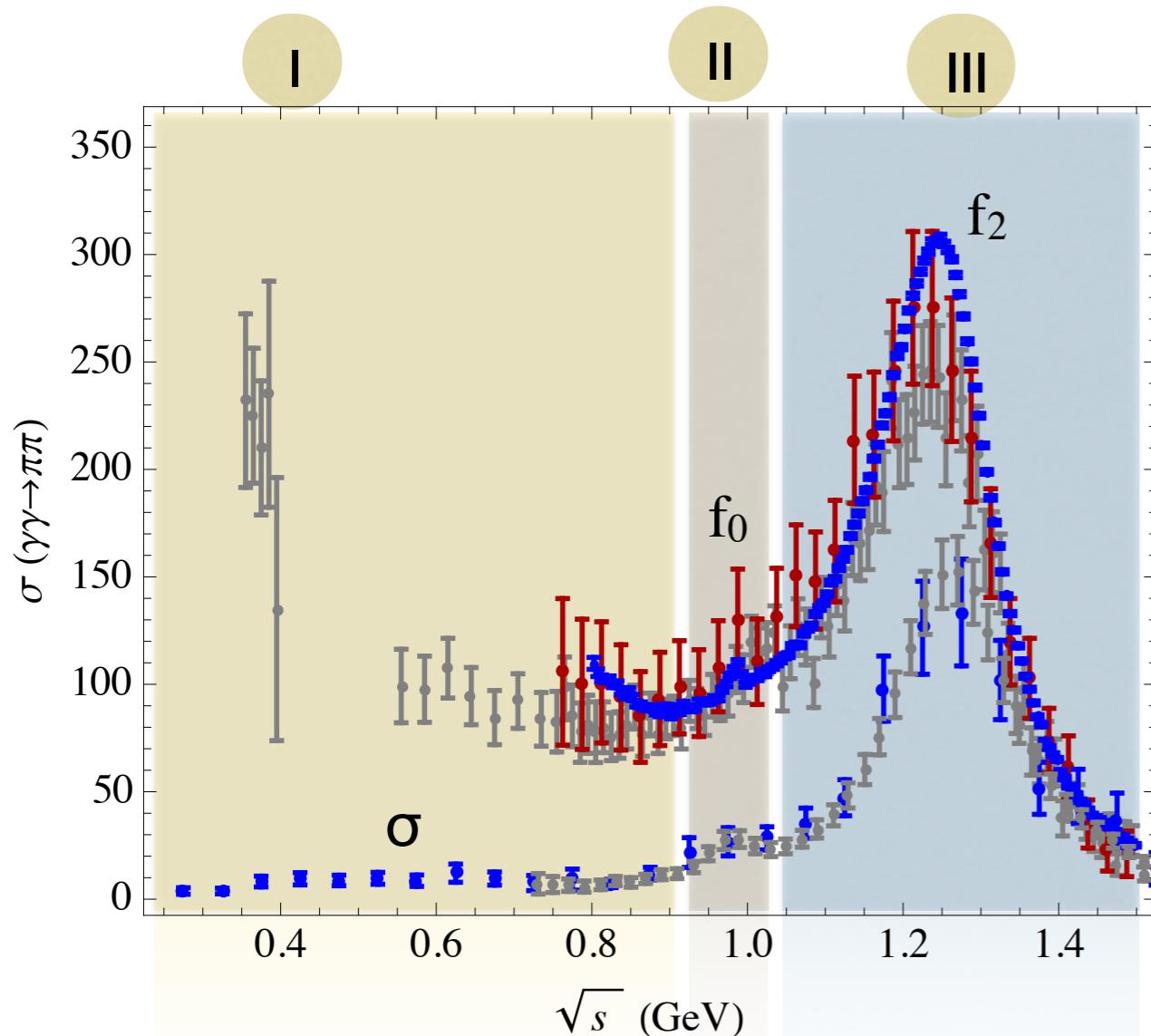


$$h(s) = \frac{1}{2\pi i} \int_C ds' \frac{h(s')}{s' - s} = \int_{-\infty}^0 \frac{ds'}{\pi} \frac{\text{Im } h(s')}{s' - s} + \int_{4m_\pi^2}^\infty \frac{ds'}{\pi} \frac{\text{Im } h(s')}{s' - s}$$

analyticity relates scattering amplitude at different energies

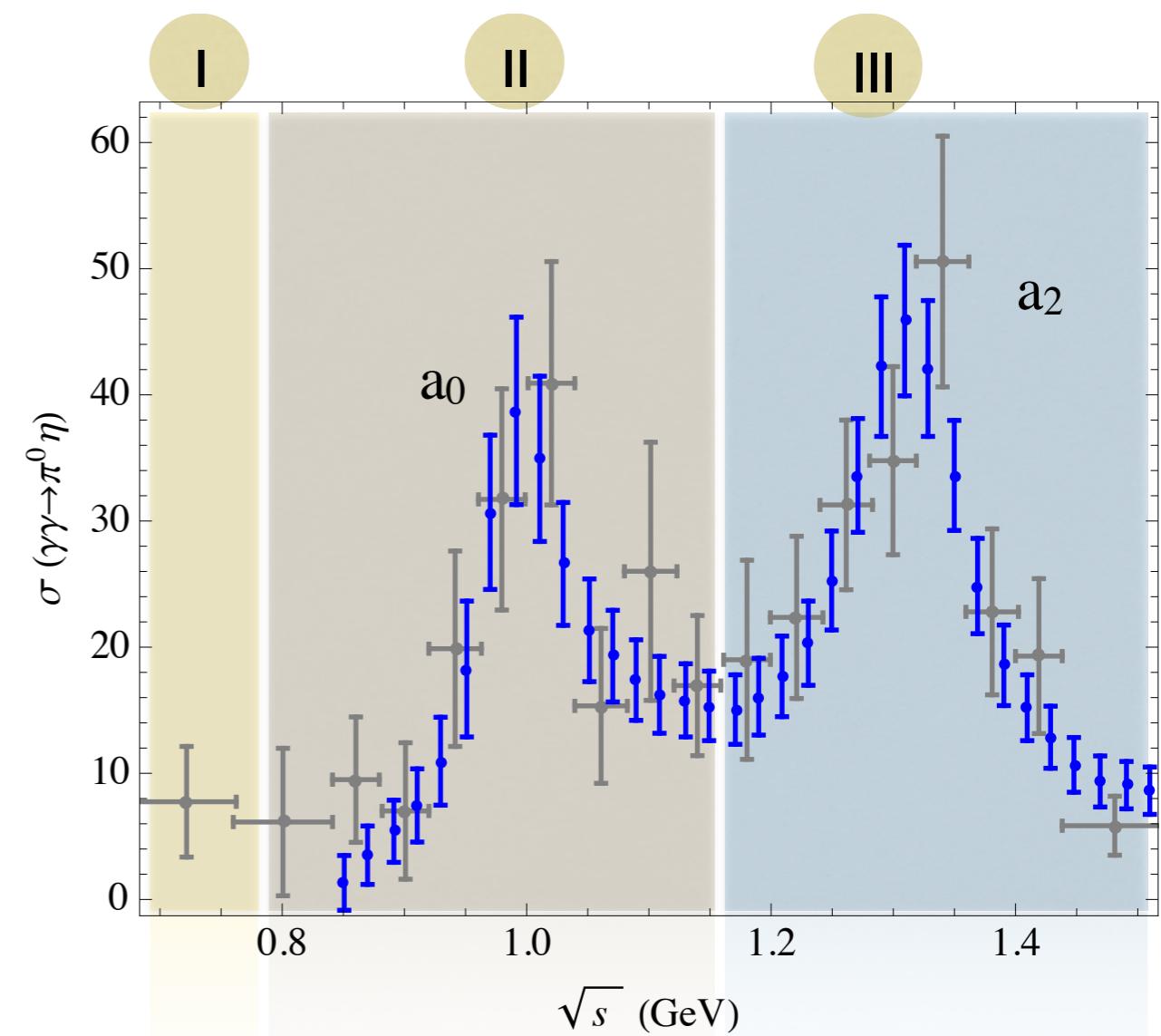
Experimental data

$\gamma\gamma \rightarrow \pi^+\pi^-, \pi^0\pi^0$



$\gamma\gamma \rightarrow \pi^+\pi^-$: Mark II ('90), CELLO ('92), Belle ('07)
 $\gamma\gamma \rightarrow \pi^0\pi^0$: Crystal Ball ('90), Belle ('09)
 $\gamma\gamma \rightarrow \pi^0\eta$: Crystal Ball ('86), Belle ('09)

$\gamma\gamma \rightarrow \pi^0\eta$



$\gamma\gamma \rightarrow \eta\eta$: Belle ('10)
 $\gamma\gamma \rightarrow KK$: ARGUS ('90), TASSO ('85),
 CELLO ('89), Belle ('13)

Ongoing experiment:
BES III

What has been done so far?

$Q^2 = 0$	Approach	Inelasticity	Number of fitted parameters to $\sigma_{\gamma\gamma \rightarrow \text{MM}}$	Range of applicability
[Hoferichter et. al. 2011]	Roy-Steiner	$\pi\pi$	0	$\sqrt{s} < 0.98 \text{ GeV}$
[Morgan et. al. 1998]	Disp, Omnes	$\pi\pi$	0	$\sqrt{s} \lesssim 0.6 \text{ GeV}$
[Dai et. al. 2014]	Amplitude anal.	$\pi\pi, KK$	>20	$\sqrt{s} < 1.5 \text{ GeV}$
[Garcia-Martin et.al. 2010]	Disp, Omnes	$\pi\pi, KK$	6	$\sqrt{s} < 1.3 \text{ GeV}$
[Current work]	Disp, Omnes	$\pi\pi, KK$ $\pi\eta, KK$	0	$\sqrt{s} < 1.4 \text{ GeV}$

$Q^2 \neq 0$	Approach	Inelasticity	Number of fitted parameters to $\sigma_{\gamma\gamma \rightarrow \text{MM}}$	Range of applicability
[Moussallam 2013]	Disp, Omnes	$\pi\pi, J=0$	0	$\sqrt{s} \lesssim 0.8 \text{ GeV}$
[Colangelo et.al. 2017]	Roy-Steiner	$\pi\pi, J=0$	0	$\sqrt{s} \lesssim 0.8 \text{ GeV}$
[Current work]	Disp, Omnes	$\pi\pi, KK, J=0,2$ $\pi\eta, KK$	0	$\sqrt{s} < 1.4 \text{ GeV}$

Only dispersive analyses are shown

Dispersion relation

Left and right-hand cuts

Definite: J, λ_1, λ_2

$$h(s) = \int_{-\infty}^0 \frac{ds'}{\pi} \frac{\text{Im } h(s')}{s' - s} + \int_{4m_\pi^2}^\infty \frac{ds'}{\pi} \frac{\text{Im } h(s')}{s' - s}$$

Looking for a solution in the form (N/D technique)

$$h(s) = h^{Born}(s) + \Omega(s) N(s)$$

$$s > 4m_\pi^2$$

$$\text{Im } \Omega(s) = \Omega(s) \rho(s) t^*(s)$$

$$\text{Im } h(s) = h(s) \rho(s) t^*(s)$$

Omnes (1958)
Morgan et. al. (1998)

Dispersive integral (twice subtracted) for $J=0$

$$h(s) = h^{Born}(s) + \Omega(s) \left(a + b s + \frac{s^2}{\pi} \int_{-\infty}^{-s_L} \frac{ds'}{s'^2} \frac{\text{Im}(h(s')) \Omega^{-1}(s)}{s' - s} - \frac{s^2}{\pi} \int_{4m_\pi^2}^\infty \frac{ds'}{s'^2} \frac{h^{Born}(s') \text{Im}(\Omega^{-1}(s'))}{s' - s} \right)$$

Q² - dependent

see also Moussallam
(2013)

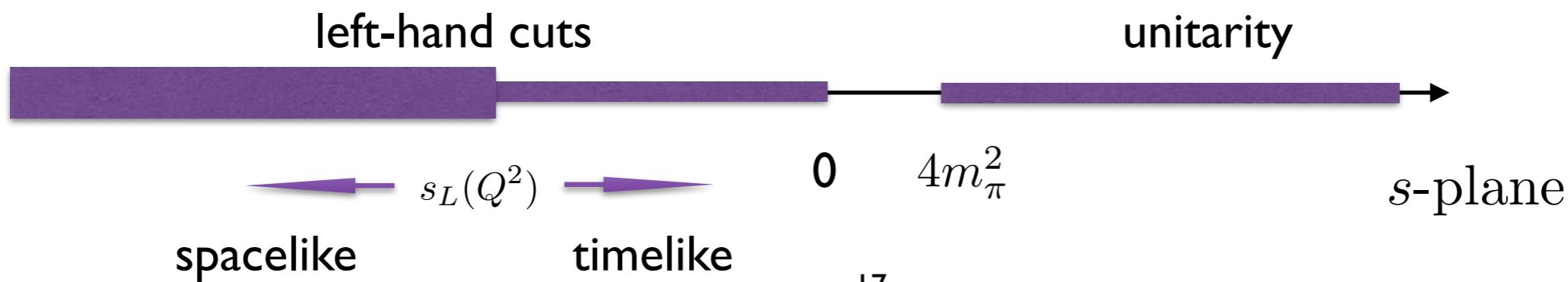
similar eq. for coupled-channel ($\pi\pi, KK$)

Left-hand cuts

Dispersive integral for $J=0$

$$h(s) = h^{Born}(s) + \Omega(s) \left(a + b s + \frac{s^2}{\pi} \int_{-\infty}^{-s_L} \frac{ds'}{s'^2} \frac{\text{Im}(h(s')) \Omega^{-1}(s)}{s' - s} \right. \\ \left. - \frac{s^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^2} \frac{h^{Born}(s') \text{Im}(\Omega^{-1}(s'))}{s' - s} \right)$$

Singularities Born: Left-hand cut $s = [-\infty, 0]$ and pole $s = -Q^2$
 V-exch: Left-hand cut $s = [-\infty, s_L(Q^2)]$



Omnes functions

Dispersive integral for J=0

$$h(s) = h^{Born}(s) + \Omega(s) \left(a + b s + \frac{s^2}{\pi} \int_{-\infty}^{-s_L} \frac{ds'}{s'^2} \frac{\text{Im}(h^V(s')) \Omega^{-1}(s)}{s' - s} \right. \\ \left. - \frac{s^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^2} \frac{h^{Born}(s') \text{Im}(\Omega^{-1}(s'))}{s' - s} \right)$$

Coupled channel Omnes

$$\Omega(s) = \begin{pmatrix} \Omega_{\pi\pi \rightarrow \pi\pi} & \Omega_{\pi\pi \rightarrow K\bar{K}} \\ \Omega_{K\bar{K} \rightarrow \pi\pi} & \Omega_{K\bar{K} \rightarrow K\bar{K}} \end{pmatrix}$$

Solve p.w. dispersion relation using N/D technique using model-independent form for the left-hand cuts

$$T(s) = U(s) + \frac{s}{\pi} \int_R \frac{ds'}{s'} \frac{\rho(s') |T(s')|^2}{s' - s}$$

$$\sum_k C_k \xi(s)^k$$

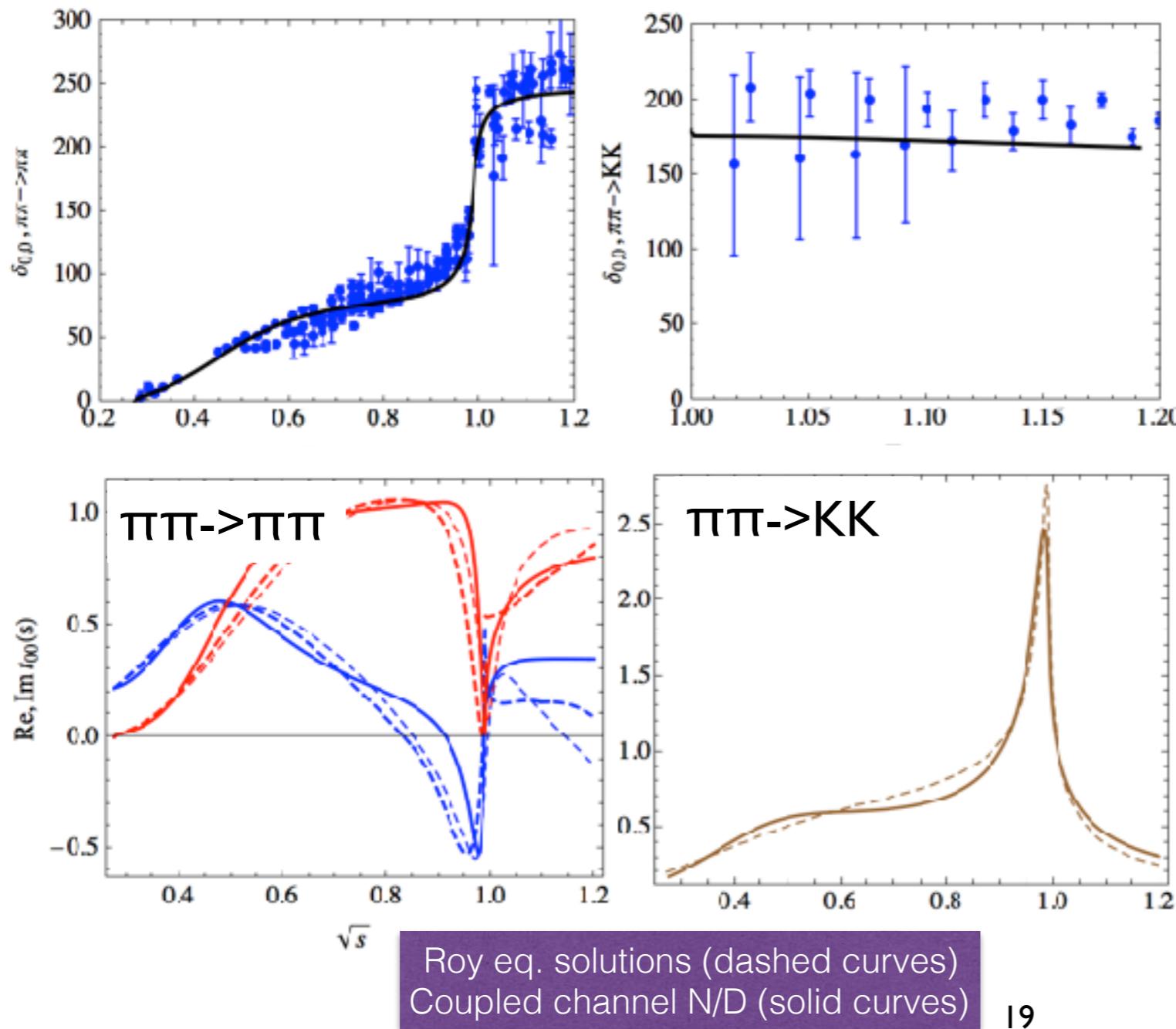
conformal mapping expansion

C_k fitted to exp data and Roy eq. solutions

Omnes functions

Coupled channel Omnes

$$\Omega(s) = \begin{pmatrix} \Omega_{\pi\pi \rightarrow \pi\pi} & \Omega_{\pi\pi \rightarrow K\bar{K}} \\ \Omega_{K\bar{K} \rightarrow \pi\pi} & \Omega_{K\bar{K} \rightarrow K\bar{K}} \end{pmatrix}$$



Bounded p.w. amplitudes and
Omnes at large energies

$$T(s) = \frac{N(s)}{D(s)} = \Omega(s) N(s)$$

$$N(s) = U(s) + \frac{s}{\pi} \int_R \frac{ds'}{s'} \frac{\rho(s') N(s') (U(s) - U(s'))}{s' - s}$$

$$D(s) = 1 - \frac{s}{\pi} \int_R \frac{ds'}{s'} \frac{\rho(s') N(s')}{s' - s}$$

$$U(s) = \sum_{k=0}^2 C_k \xi(s)^k$$

$f_2(1270)$ contribution

Watson theorem (for elastic unitarity) J=2:

$$\phi(\gamma\gamma \rightarrow \pi\pi) = \phi(\pi\pi \rightarrow \pi\pi) = \delta(\pi\pi \rightarrow \pi\pi)$$

$$\Omega(s) = \exp \left(\frac{s}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'} \frac{\phi_{\gamma\gamma \rightarrow \pi\pi}(s')}{s' - s} \right)$$

Roy analysis (2011)
R. Garcia-Martin
at.al.

When there are **no VM**, it is not possible to describe J=2 partial wave using Omnes functions and we parametrize it with the Breit Wigner + Background

$$h_{J=2}^{f_2} = \frac{C_{f_2 \rightarrow \gamma\gamma} C_{f_2 \rightarrow \pi\pi}}{10\sqrt{6}} \frac{s(s+Q^2)\beta(s)}{s - M^2 + iM\Gamma(s)} T_{f_2}^{(\Lambda=2)}(Q^2)$$

$$h_{J=2} = B_{D2} + h_{J=2}^{f_2} e^{i\phi_0} = |h_{J=2}| e^{i\delta(\pi\pi \rightarrow \pi\pi)}$$



Background: Born

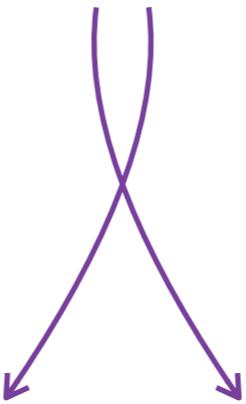


Relative phase: unitarization

Subtraction constants

Dispersive integral for J=0

$$h(s) = h^{Born}(s) + \Omega(s) \left(a + b s + \frac{s^2}{\pi} \int_{-\infty}^{-s_L} \frac{ds'}{s'^2} \frac{\text{Im}(h^V(s')) \Omega^{-1}(s)}{s' - s} \right. \\ \left. - \frac{s^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^2} \frac{h^{Born}(s') \text{Im}(\Omega^{-1}(s'))}{s' - s} \right)$$



NO Vector mesons
fix: **a**

With Vector mesons
fix: **a, b**

Soft photon limit ($q_1=0$)

$$H_{\lambda_1 \lambda_2} \rightarrow H_{\lambda_1 \lambda_2}^{Born}$$

$$s = -Q^2, t = u = m_\pi^2$$

For space like photons: generalized polarizabilities

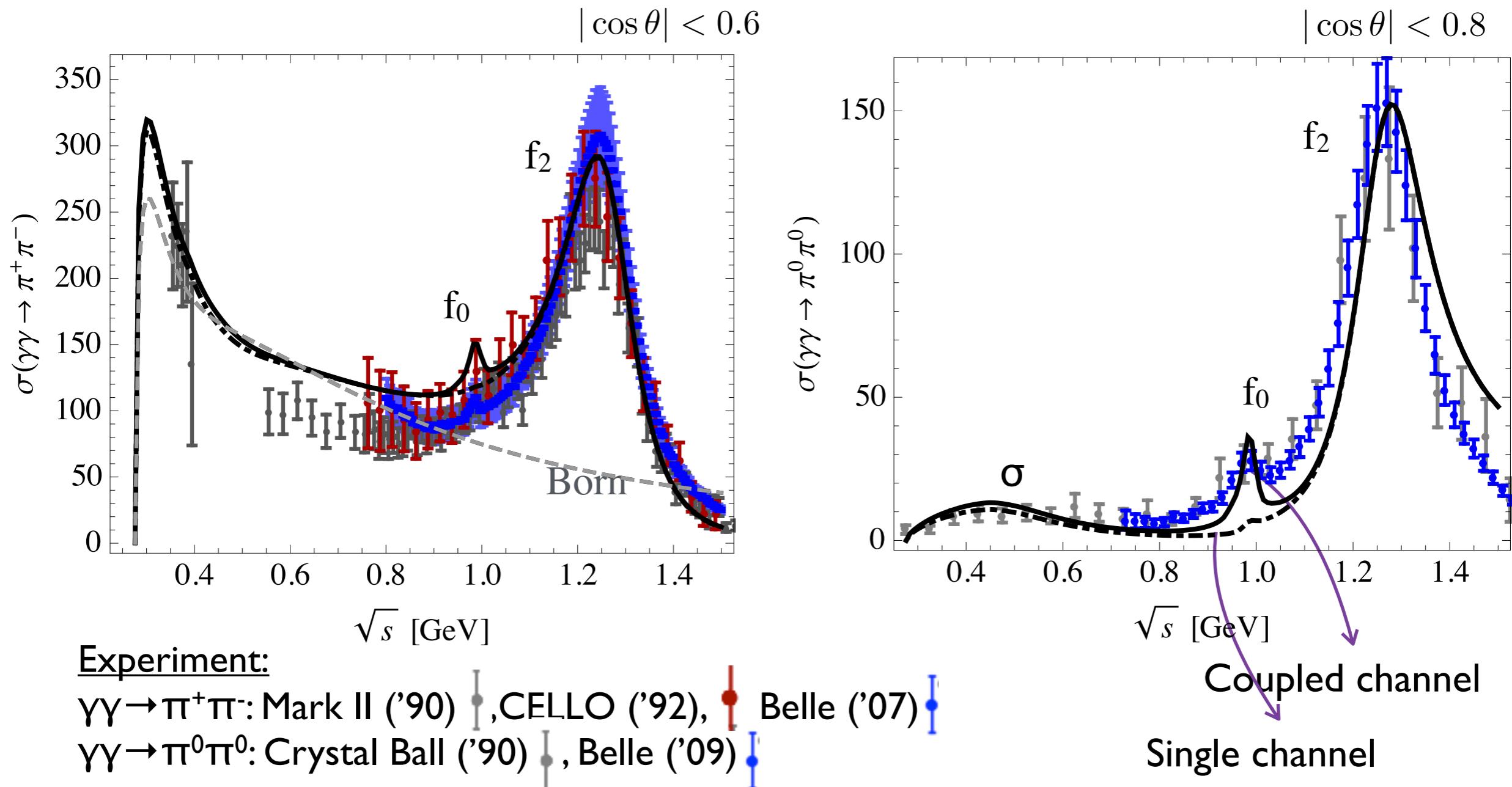
$$\pm \frac{2\alpha}{m_\pi} \frac{H_{+\pm}^n}{s + Q^2} = (\alpha_1 \mp \beta_1)_{\pi^0} + \dots$$

$$\pm \frac{2\alpha}{m_\pi} \frac{(H_{+\pm}^c - H_{+\pm}^{Born})}{s + Q^2} = (\alpha_1 \mp \beta_1)_{\pi^+} + \dots$$

prediction for **b**: generalised polarizabilities

more realistic l.h.cut: fix **b** from **ChPT** and **COMPASS**

no VM ($Q^2=0$)



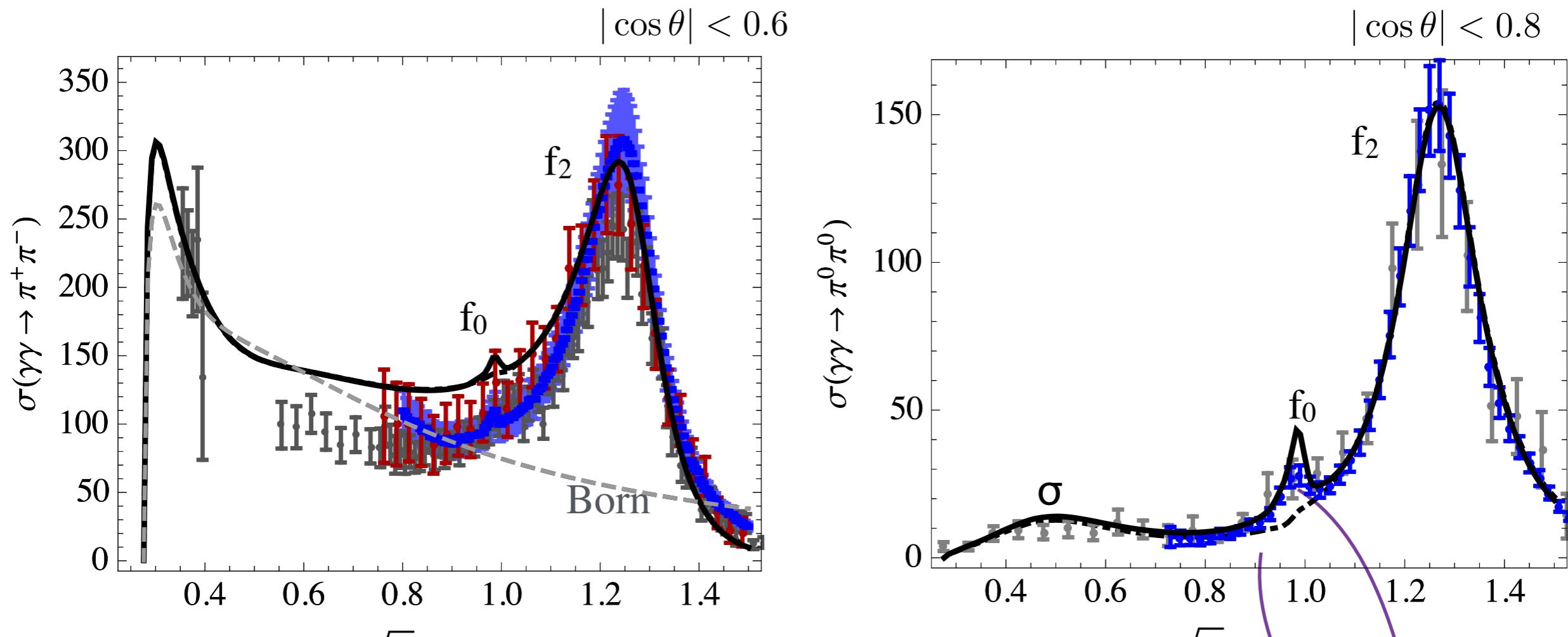
$$C_{f_2 \rightarrow \gamma\gamma}^{exp} = 0.20 \pm 0.02$$

$$C_{f_2 \rightarrow \gamma\gamma}^{eff} = 0.22$$

I.D., Vanderhaeghen
(work in progress)

see also Dai ('14),
Hoferichter ('11),
Garcia-Martin et. al
('10)

with VM ($Q^2=0$)



Experiment:

$\gamma\gamma \rightarrow \pi^+\pi^-$: Mark II ('90), CELLO ('92), Belle ('07)
 $\gamma\gamma \rightarrow \pi^0\pi^0$: Crystal Ball ('90), Belle ('09)

Coupled channel
Single channel

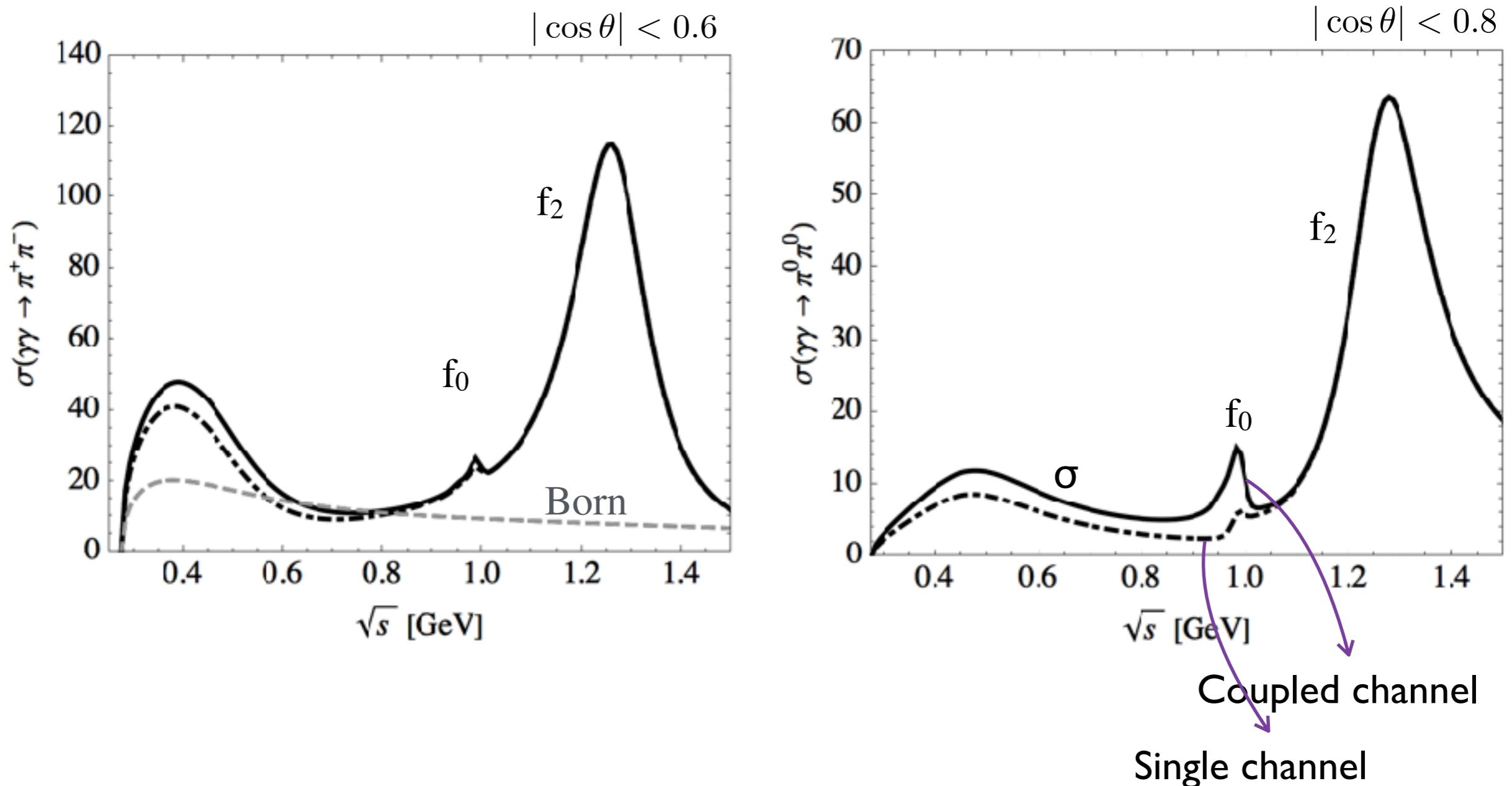
$$C_{\rho^\pm \rightarrow \pi^\pm \gamma}^{exp} = 0.365 \pm 0.020$$

$$C_{V \rightarrow P\gamma}^{eff} = 0.325$$

I.D., Vanderhaeghen
(work in progress)

see also Dai ('14),
Hoferichter ('11),
Garcia-Martin et. al ('10)

no VM ($Q^2=0.5$)

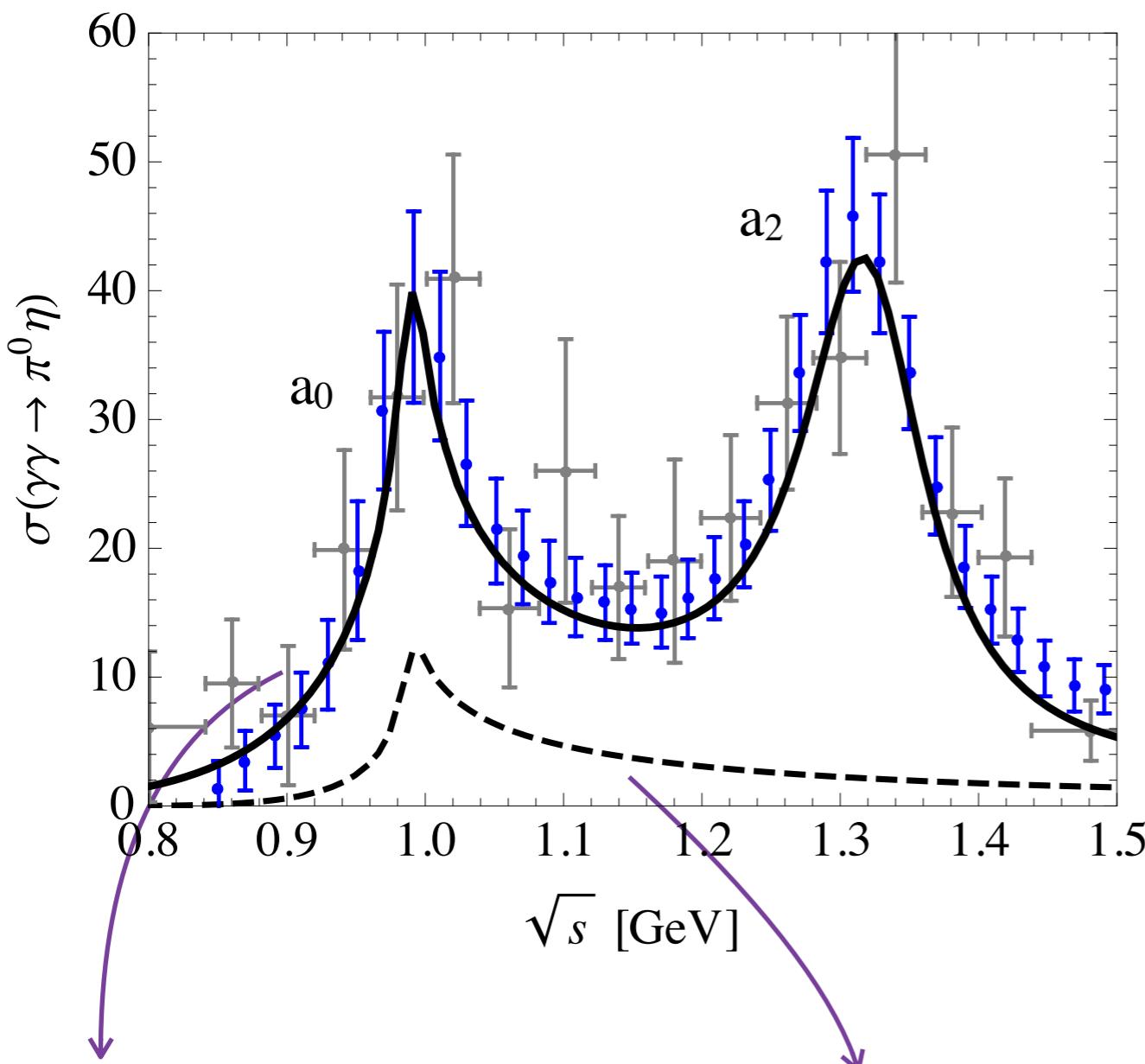


Results for $Q^2=0.5$ **with VM** and fully **dispersive $f_2(1270)$** contribution are on their way

Ongoing experiment:
BES III

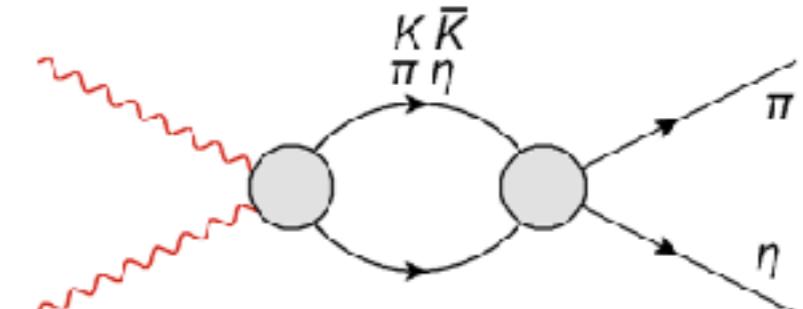
I.D., Vanderhaeghen
(work in progress)

$\gamma\gamma \rightarrow \pi\eta$ ($Q^2=0$)



Coupled channel:
with VM

Coupled channel:
no VM



$$\Omega(s) = \begin{pmatrix} \Omega_{\pi\eta \rightarrow \pi\eta} & \Omega_{\pi\eta \rightarrow K\bar{K}} \\ \Omega_{K\bar{K} \rightarrow \pi\eta} & \Omega_{K\bar{K} \rightarrow K\bar{K}} \end{pmatrix}$$

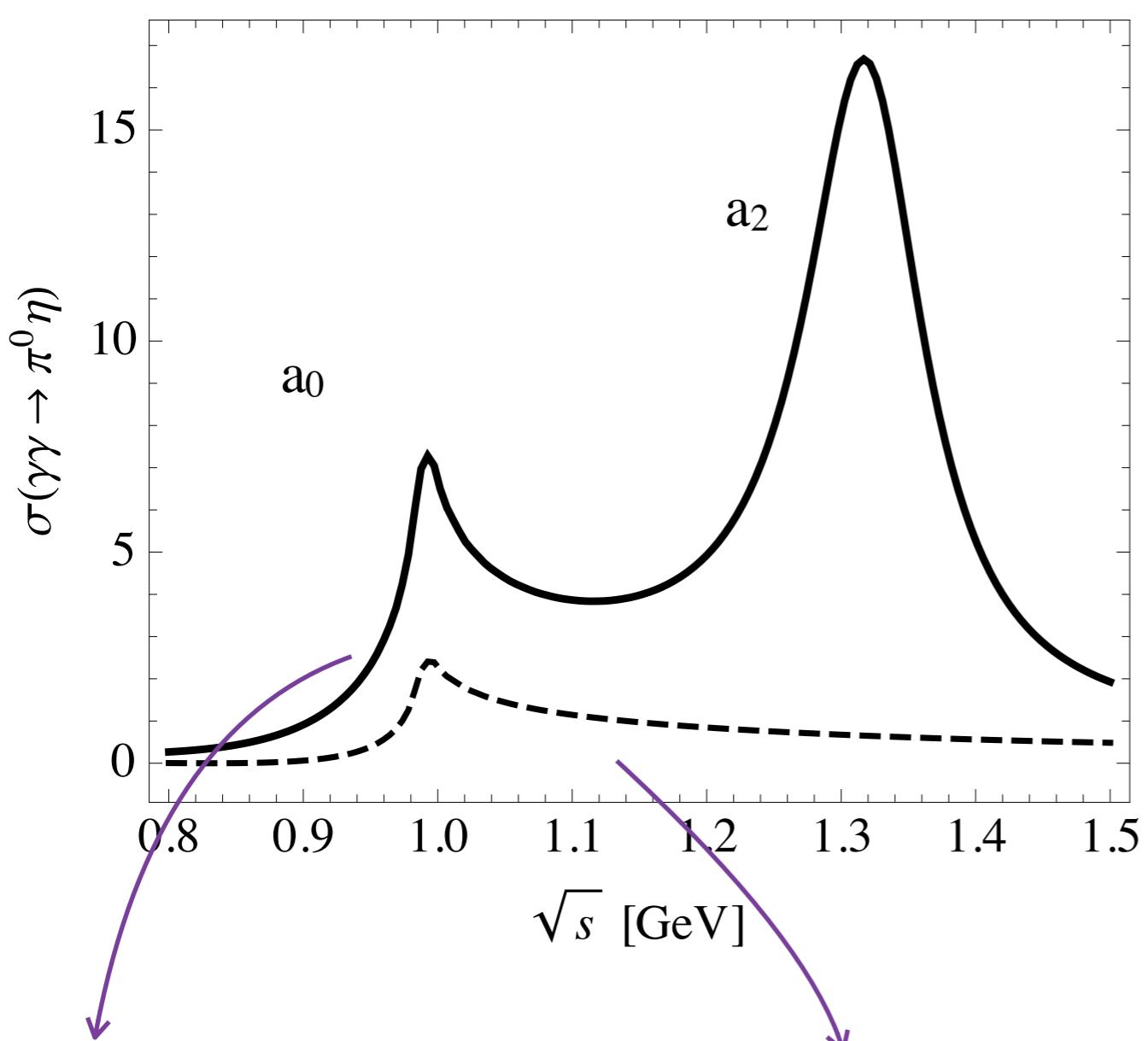
I.D., Gil, Lutz
(2011), (2013)

Coupled-channel dispersive treatment for $J=0$ is **crucial**

$a_2(1230)$ described as a Breit Wigner resonance

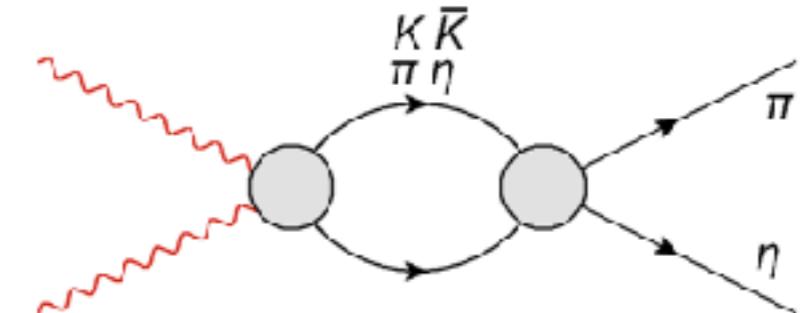
I.D., Deineka,
Vanderhaeghen
(work in progress)

$\gamma\gamma \rightarrow \pi\eta$ ($Q^2=0.5$)



Coupled channel:
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I.D., Gil, Lutz
(2011), (2013)

Coupled-channel dispersive treatment for J=0 is **crucial**

a₂(1230) described as a Breit Wigner resonance

I.D., Deineka,
Vanderhaeghen
(work in progress)

Summary and Outlook

- ▶ In light of the **new Belle data (2015)** for $f_2(1270)$ TFFs and using LbL sum rules we **predicted** ($\Lambda=2$) TFF for $f_2(1565)$
- ▶ **Update for meson contributions to (g-2) LbL**
Tensor mesons contributions found to be small compared to anticipated exp. uncertainty $1.6 \cdot 10^{-10}$
- ▶ Axial vector mesons contributions (satisfying Landau-Yang theorem constraint) evaluated by 2 groups and found to be between $(0.64 - 0.75 \pm 0.27) \cdot 10^{-10}$
- ▶ **Next steps?**
Need to take into account $f_0(500)$ and non resonant contributions in a dispersive approach
- ▶ Main ingredients: $\gamma\gamma^*\rightarrow\pi\pi$, $\pi\eta, \dots$ (work in progress). Can be used in **different** (g-2) dispersive approaches.

It is important to **validate** dispersive treatment of $\gamma\gamma^*\rightarrow\pi\pi$, $\pi\eta, \dots$ with upcoming BES III data

Extra slides

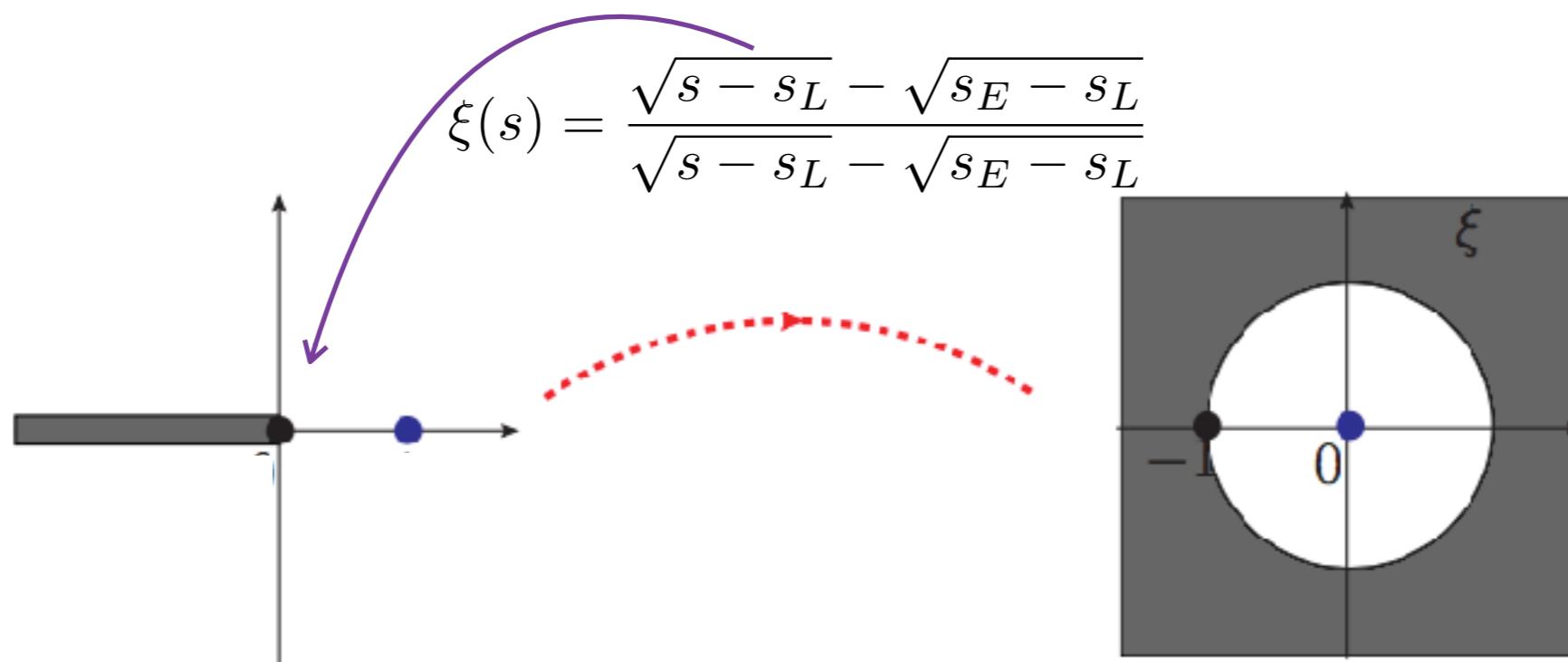
Conformal mapping

Solve p.w. dispersion relation using N/D technique using model-independent form for the left-hand cuts

$$T(s) = U(s) + \frac{s}{\pi} \int_R \frac{ds'}{s'} \frac{\rho(s')|T(s')|^2}{s' - s}$$

$\sum_k C_k \xi(s)^k$ conformal mapping expansion
 C_k fitted to exp data and Roy solutions

Chew, Mandelstam
Lutz, Gasparyan



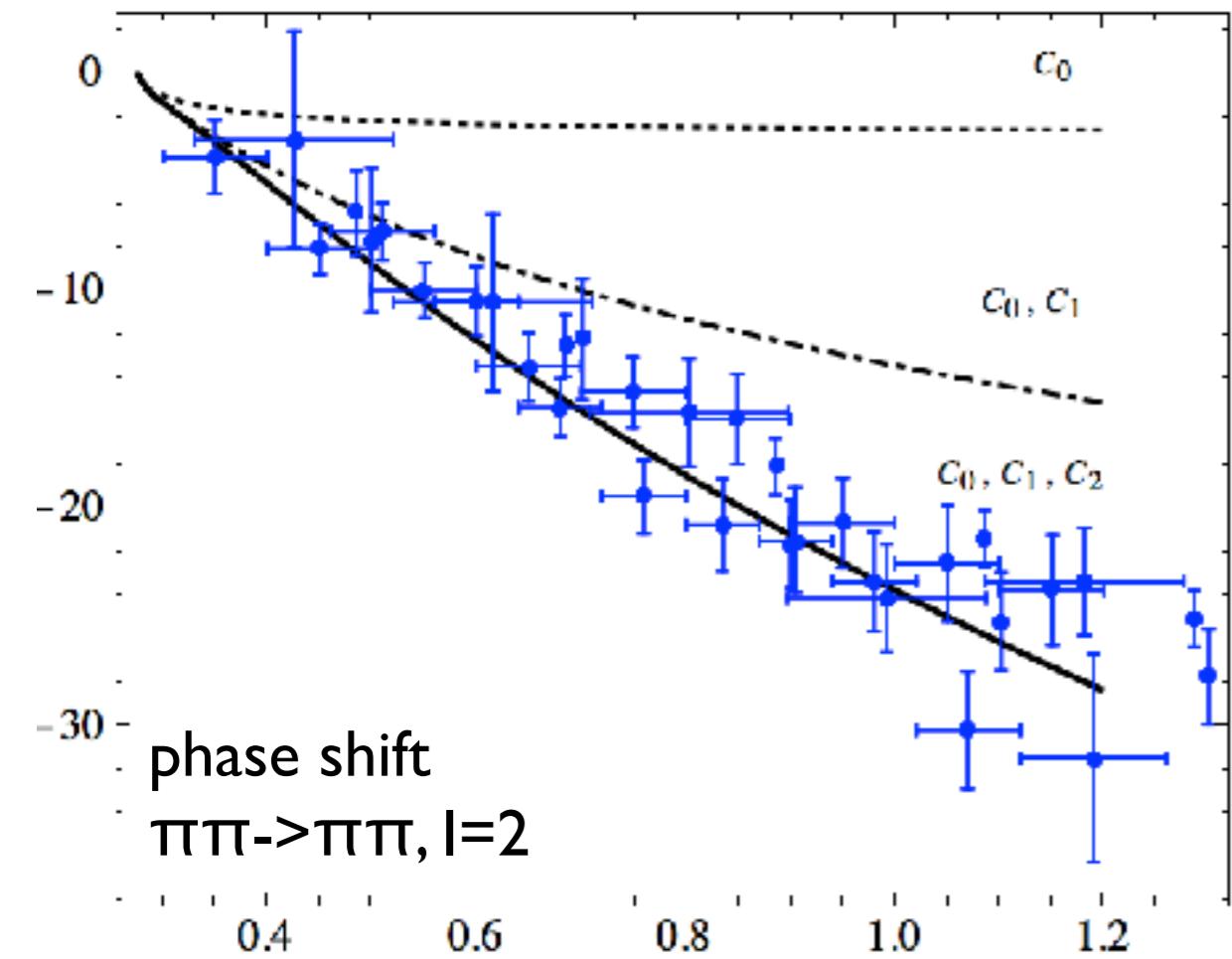
Example: $I=2$

Solve p.w. dispersion relation using N/D technique using model-independent form for the left-hand cuts

$$T(s) = U(s) + \frac{s}{\pi} \int_R \frac{ds'}{s'} \frac{\rho(s') |T(s')|^2}{s' - s}$$

$$\downarrow$$

$$\sum_k C_k \xi(s)^k = C_0 + C_1 \xi(s) + C_2 \xi(s)^2 + \dots$$



← fixed from the threshold parameter \mathbf{a}_2 (scattering length)

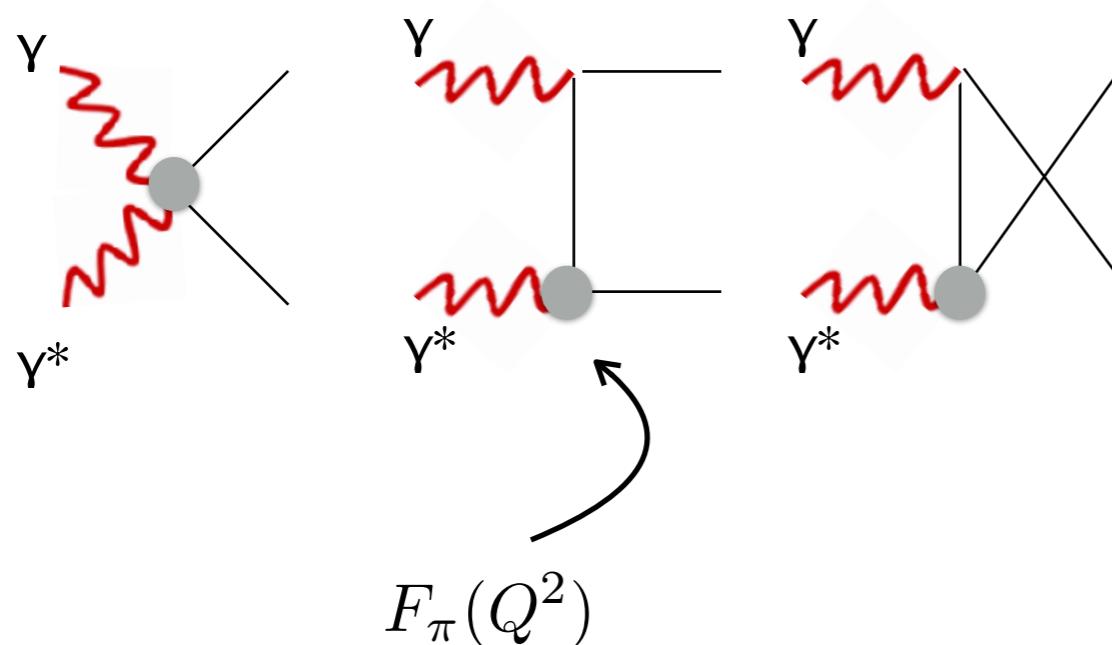
← fixed from the threshold parameters $\mathbf{a}_2, \mathbf{b}_2$ $\frac{1}{m_\pi} \text{Re}(T_{\pi\pi \rightarrow \pi\pi}/16\pi) = a + b p_{\text{cm}}^2 + \dots$

← one parameter fit

$$N(s) = U(s) + \frac{s}{\pi} \int_R \frac{ds'}{s'} \frac{\rho(s') N(s') (U(s) - U(s'))}{s' - s}$$

$$D(s) = 1 - \frac{s}{\pi} \int_R \frac{ds'}{s'} \frac{\rho(s') N(s')}{s' - s}$$

Born amplitudes ($Q^2 \neq 0$)



Vertex $\pi\pi\gamma^*$

$$\langle \pi^+ | j_\mu(0) | \pi^+(p') \rangle = (p + p')_\mu F_\pi(Q^2)$$

Space-like region

$$F_\pi(Q^2) = \frac{1}{1 + Q^2/M_\rho^2}$$

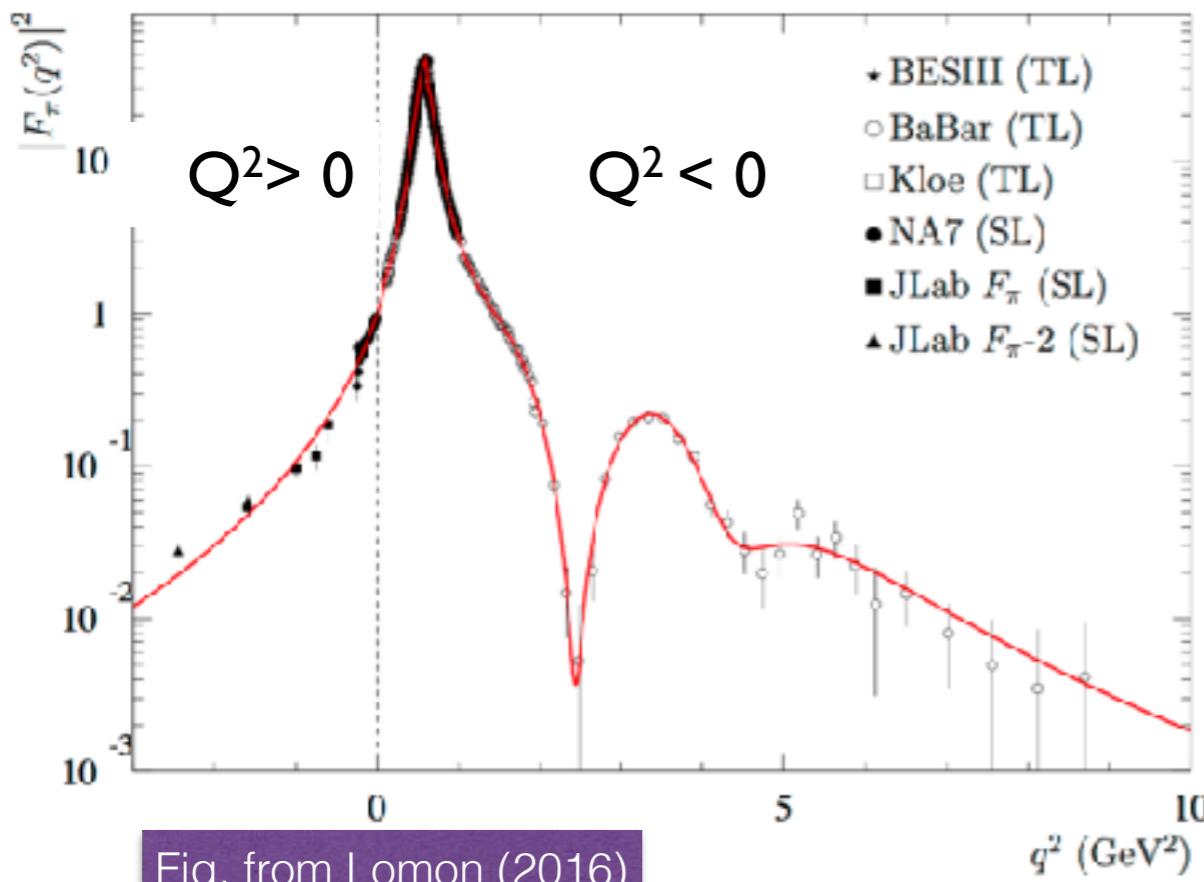
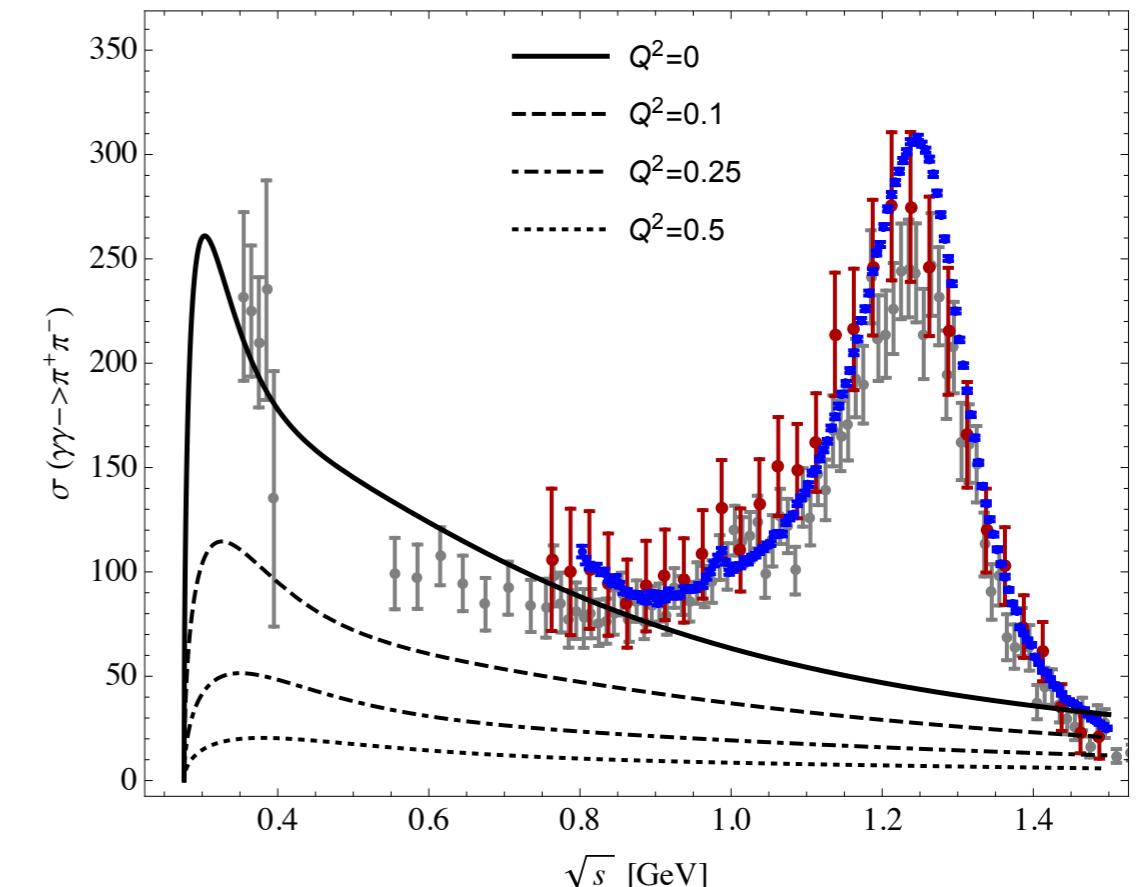


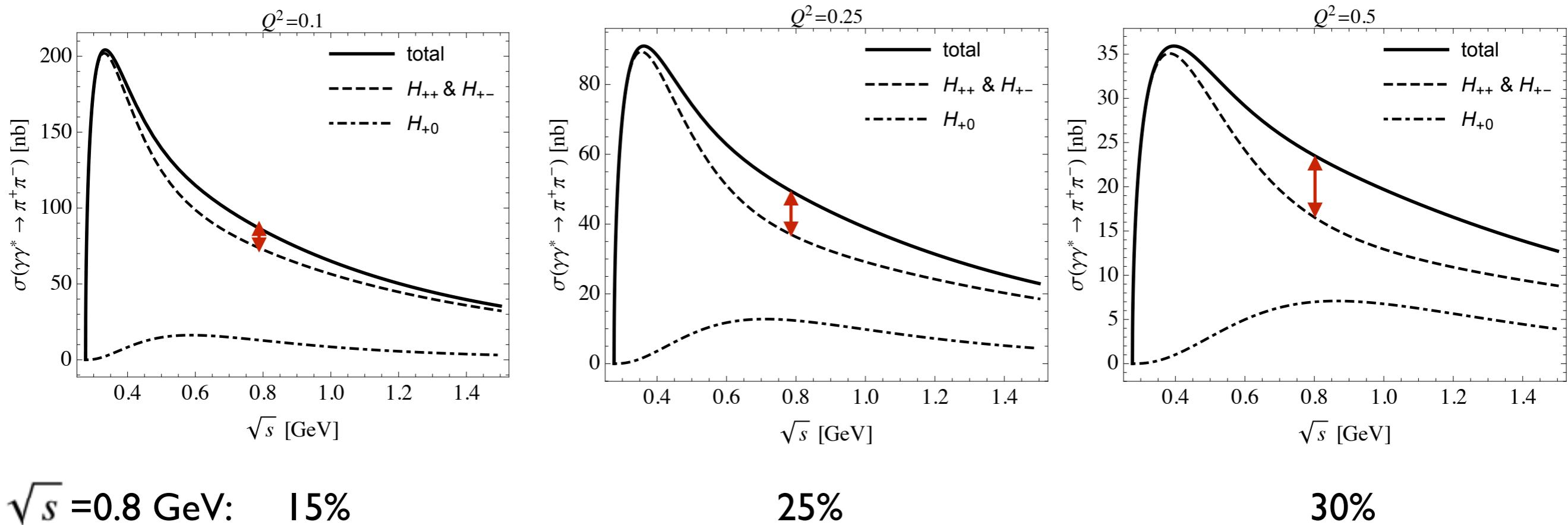
Fig. from Lomon (2016)



Born amplitudes ($Q^2 \neq 0$)

Differential cross section

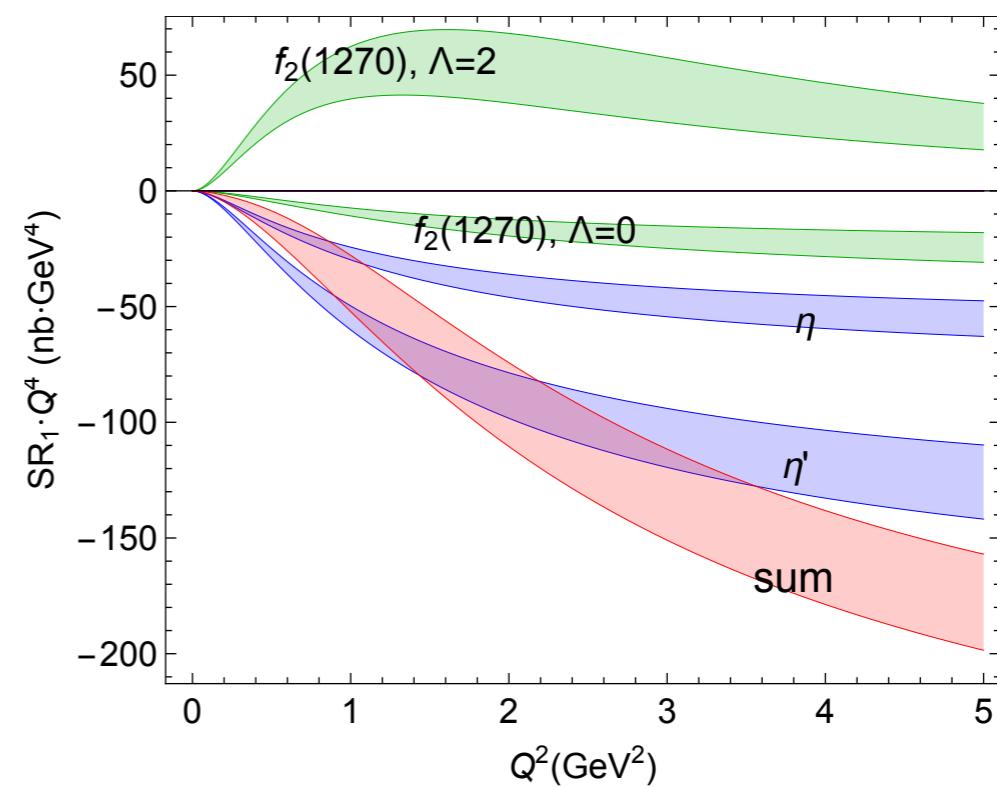
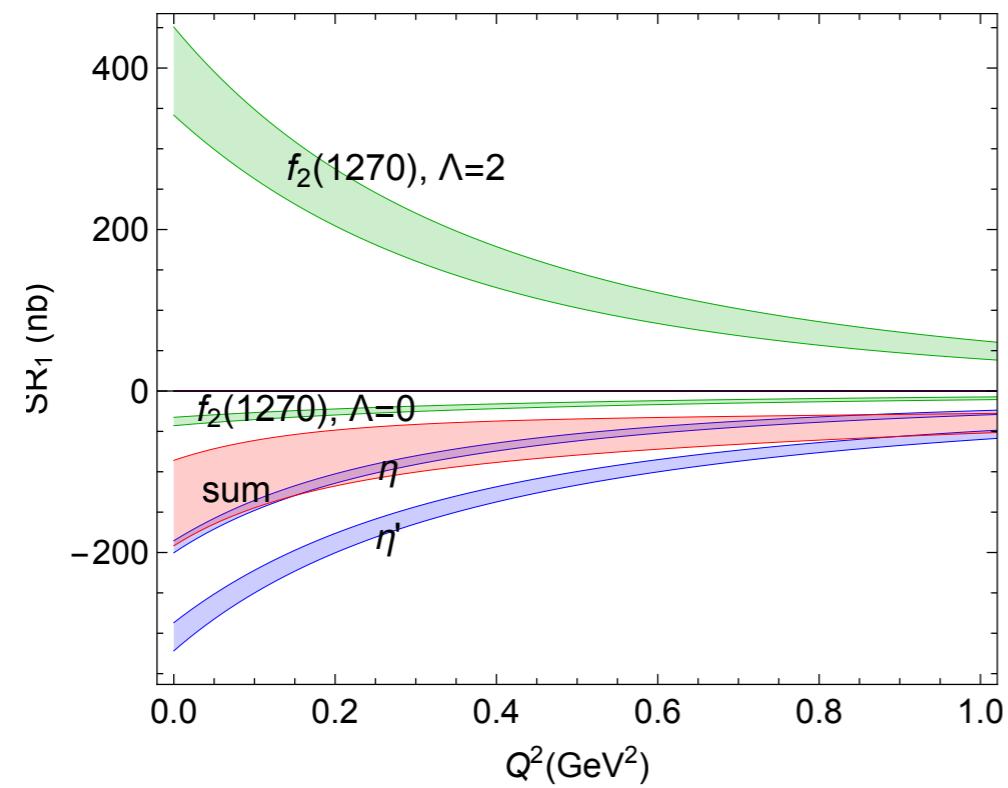
$$\frac{d\sigma}{d \cos \theta} = \pi \alpha^2 \frac{\rho(s)}{4(s + Q^2)} (|H_{++}|^2 + |H_{+-}|^2 + |H_{+0}|^2)$$



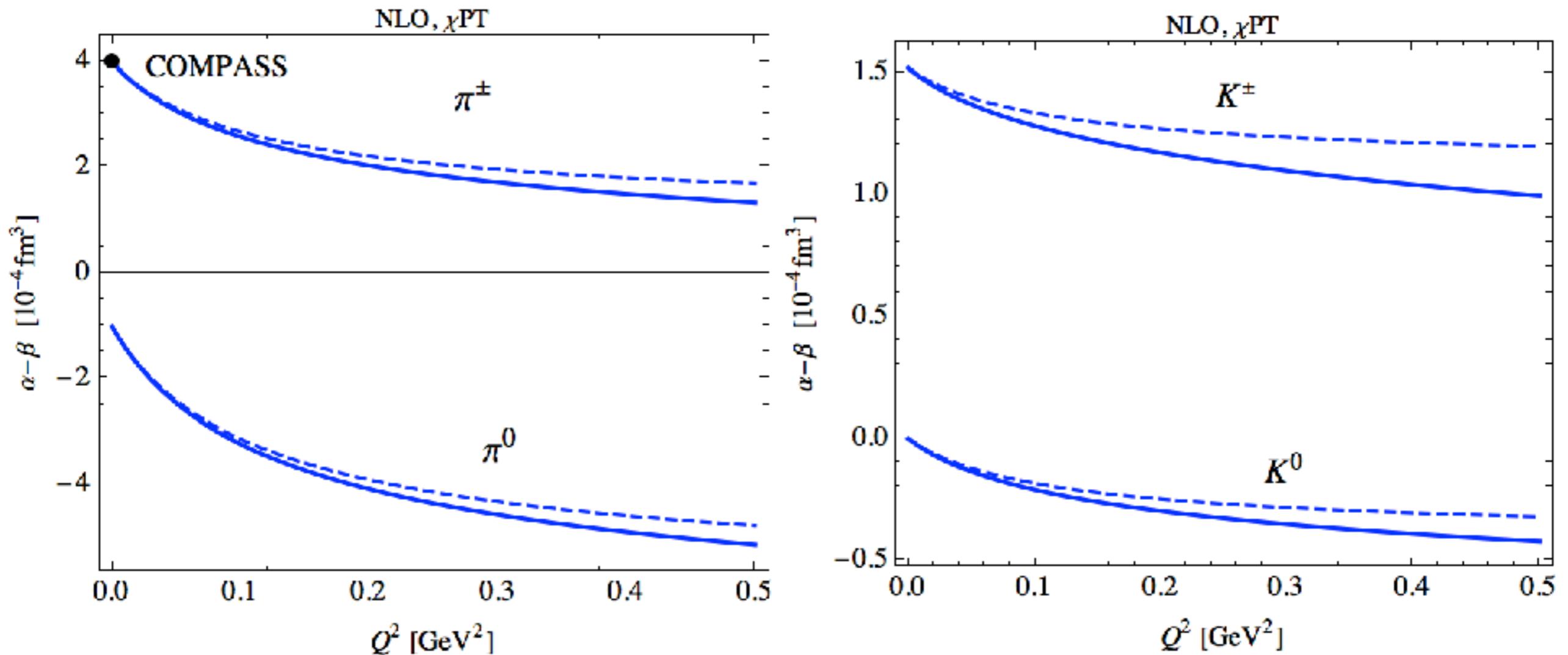
Sum rule I (*Isospin=0*)

State	m (MeV)	Γ_γ (keV)	SR ₁ ($Q^2 = 0$) (nb)
η	547.862 ± 0.017	0.516 ± 0.020	-193±7
η'	957 ± 0.06	4.35 ± 0.25	-304±17
$f_2(1270)$	1275.5 ± 0.8	2.93 ± 0.40	($\Lambda=2$) 396±54 ($\Lambda=0$) -38±5
$f_2(1565)$	1562 ± 13	0.70 ± 0.14	56±11
.....			
sum			-82±54

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s + Q^2)} [\sigma_2 - \sigma_0]$$



Generalized polarizabilities

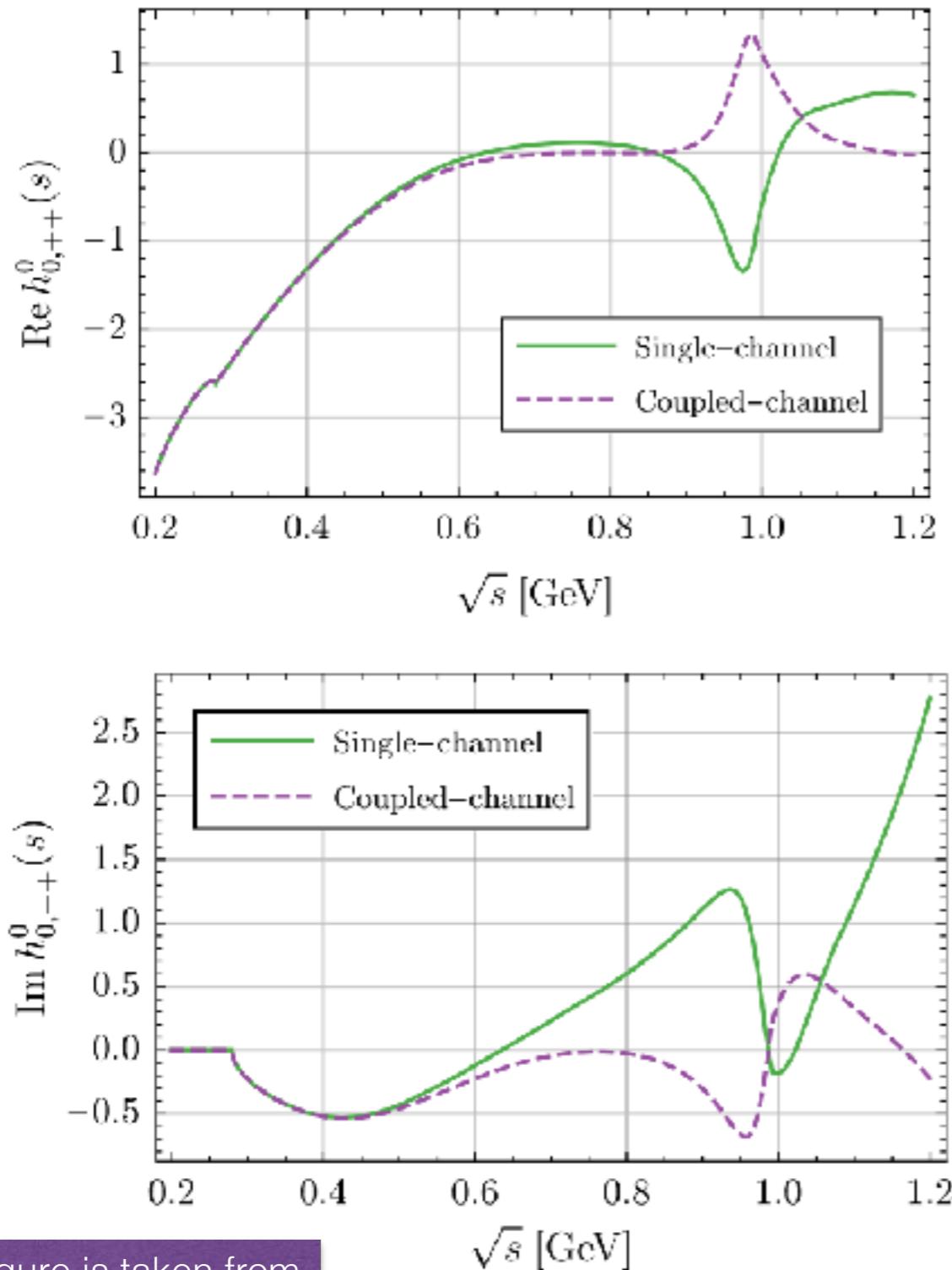


$$\pm \frac{2\alpha}{m_\pi} \frac{H_{+\pm}^n}{s + Q^2} = (\alpha_1 \mp \beta_1)_{\pi^0} + \dots$$

$$\pm \frac{2\alpha}{m_\pi} \frac{(H_{+\pm}^c - H_{+\pm}^{Born})}{s + Q^2} = (\alpha_1 \mp \beta_1)_{\pi^+} + \dots$$

dotted - only pion loops
solid - pion and kaon loops

Fuchs, Pasquini,
 Unkmeir, Scherer,
 L'vov, Drechsel
 (1999,2000,2001)



Single channel:
 $Q^2 \rightarrow 0$ limit is taken

Range of
 applicability
 $s^{1/2} < 0.8$ GeV

Coupled channel: 6 par fit
 to the cross sections

$(g-2)$

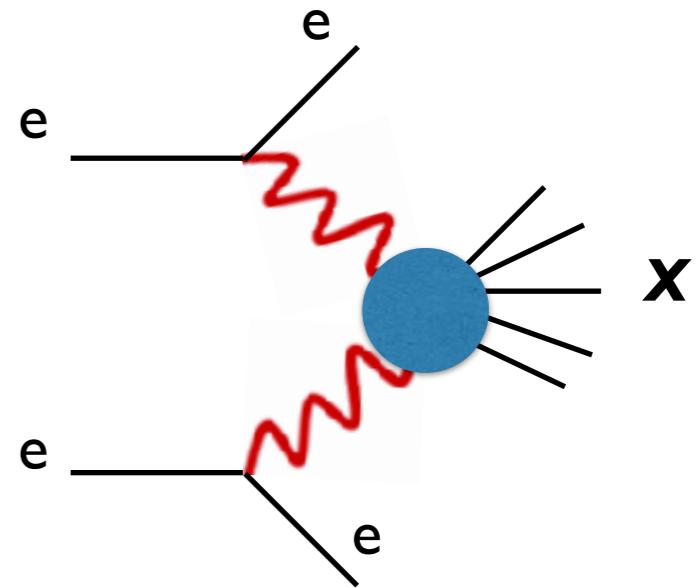
- Anomalous magnetic moment of the muon

→ Total HLbL [a_μ in units 10^{-11}]

Contribution	HKS	BPP	KN	MV	PdRV	N/JN	Jegerlehner (2015)
π^0, η, η'	82.7 ± 6.4	85 ± 13	83 ± 12	114 ± 10	114 ± 13	99 ± 16	
π, K loops	-4.5 ± 8.1	-19 ± 13	–	0 ± 10	-19 ± 19	-19 ± 13	
axial vectors	1.7 ± 1.7	2.5 ± 1.0	–	22 ± 5	15 ± 10	22 ± 5	→ 7.5 ± 2.7
scalars	–	-6.8 ± 2.0	–	–	-7 ± 7	-7 ± 2	
quark loops	9.7 ± 11.1	21 ± 3	–	–	2.3	21 ± 3	
total	89.6 ± 15.4	83 ± 32	80 ± 40	136 ± 25	105 ± 26	116 ± 39	→ 102 ± 39

Light by light scattering

Observables in experiment $e^+e^- \rightarrow e^-e^+X$



$$\begin{aligned}
 d\sigma = & \frac{\alpha^2}{16\pi^4 Q_1^2 Q_2^2} \frac{2\sqrt{X}}{s(1 - 4m^2/s)} \cdot \frac{d^3 \vec{p}'_1}{E'_1} \cdot \frac{d^3 \vec{p}'_2}{E'_2} \\
 & \times \left\{ 4 \rho_1^{++} \rho_2^{++} \sigma_{TT} + \rho_1^{00} \rho_2^{00} \sigma_{LL} + 2 \rho_1^{++} \rho_2^{00} \sigma_{TL} + 2 \rho_1^{00} \rho_2^{++} \sigma_{LT} \right. \\
 & + 2 (\rho_1^{++} - 1) (\rho_2^{++} - 1) (\cos 2\tilde{\phi}) \tau_{TT} + 8 \left[\frac{(\rho_1^{00} + 1) (\rho_2^{00} + 1)}{(\rho_1^{++} - 1) (\rho_2^{++} - 1)} \right]^{1/2} (\cos \tilde{\phi}) \tau_{TL} \\
 & \left. + h_1 h_2 4 [(\rho_1^{00} + 1) (\rho_2^{00} + 1)]^{1/2} \tau_{TT}^a + h_1 h_2 8 [(\rho_1^{++} - 1) (\rho_2^{++} - 1)]^{1/2} (\cos \tilde{\phi}) \tau_{TL}^a \right\},
 \end{aligned}$$