The Muon g-2 Theory Initiative

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Muon g-2 Theory Initiative

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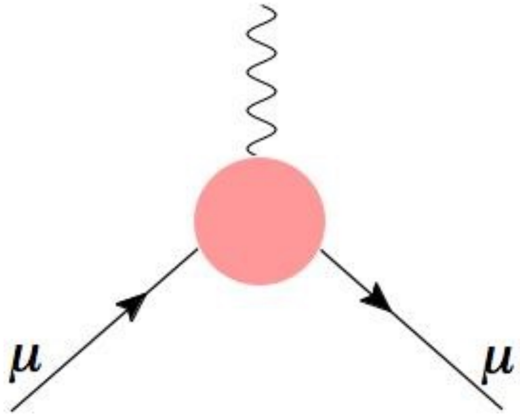
Introduction



$$= (-i\,e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p)$$

✦ muon anomalous magnetic moment: $a_\mu = F_2(0)$

- is generated by quantum effects (loops).
- receives contributions from QED, EW, and QCD effects in the SM.
- is a sensitive probe of new physics.

✦ QED + EW correction are known precisely:

$$\delta a_\mu^{\text{QED}} \times 10^{11} = 0.08 \qquad \delta a_\mu^{\text{EW}} \times 10^{11} = 1$$

✦ QCD corrections are the dominant source of error in the SM prediction:

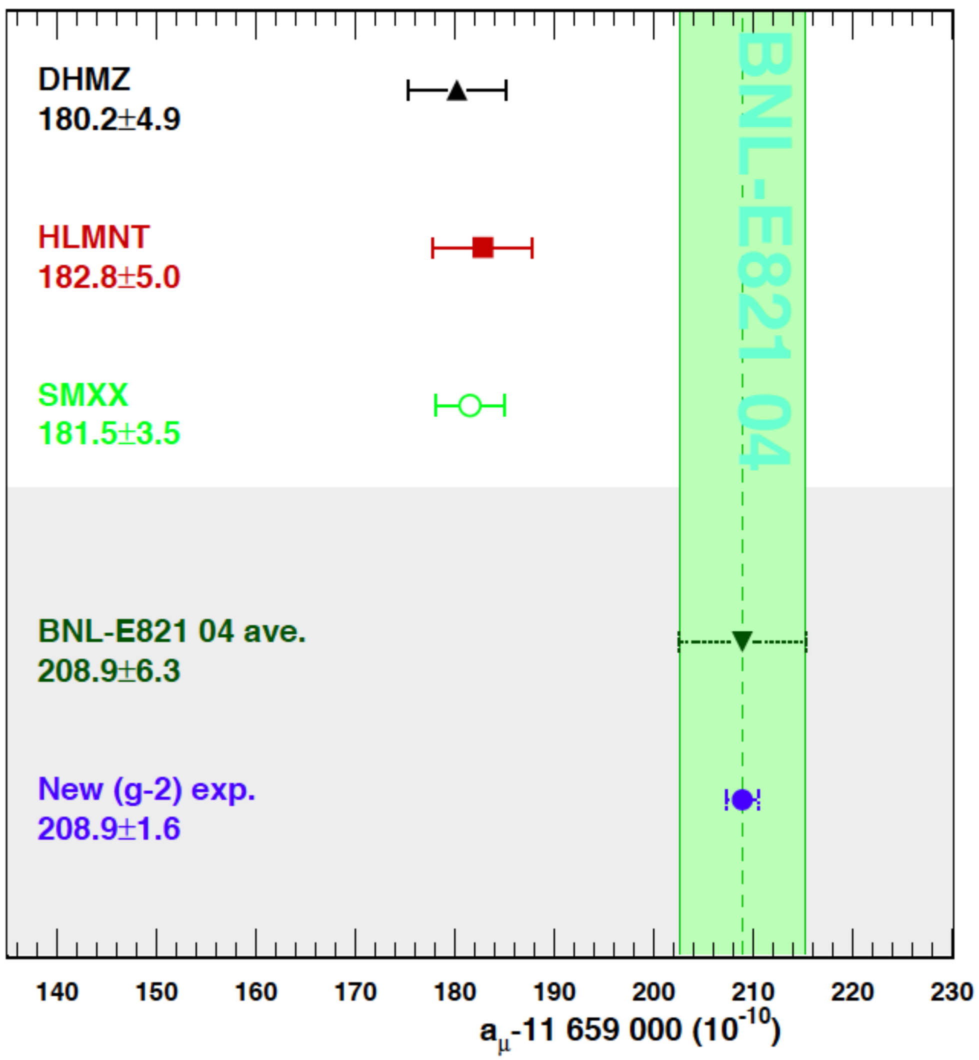
$$\delta a_\mu^{\text{had}} \times 10^{11} \sim 50$$

(Davier et al. 2011, Hagiwara et al 2011, Kurz et al 2014, Prades et al 2009, Colangelo et al 2014, Jegerlehner 2015, Benayoun et al 2015,…)

Introduction

Experiment vs SM theory

T. Blum et al. (arXiv:1311.2198)



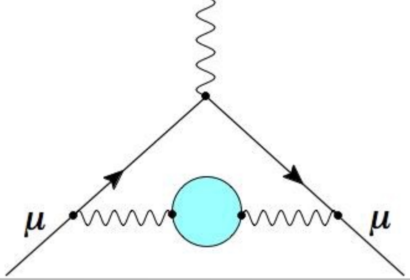
Fermilab g-2 experiment:

- ✦ reduce exp. error by a factor of 4
- ✦ first result with "Brookhaven level" statistics expected in 2018.
- ✦ Commissioning of beam, ``wiggle party".

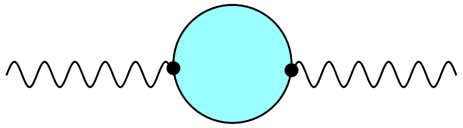
J-PARC experiment:

- ✦ complementary, different exp. method
- ✦ expect measurement at 0.3-0.4 ppm level

Need to reduce and better control theory errors for the hadronic corrections.



Hadronic vacuum polarization

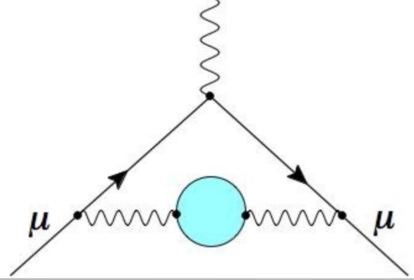


$$\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$$

$$\Pi_{\mu\nu} = \int d^4x e^{iqx} \langle j_\mu(x) j_\nu(0) \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(q^2)$$

Leading order HVP correction: $$a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2\, w(q^2)\, \hat{\Pi}(q^2)$$

- Use optical theorem and dispersion relation to rewrite the integral in terms of the hadronic cross section
  ➟ talks by Teubner, Jegerlehner, Zhang



Hadronic vacuum polarization

Leading order HVP correction: $$a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2\, w(q^2)\, \hat{\Pi}(q^2)$$

- Calculate a_μ^{HVP} in Lattice QCD:
  - ✦ Calculate $\hat{\Pi}(q^2)$ and evaluate the integral (Blum,PRL 03, Lautrup et al, 71)
  - ✦ Time-momentum representation: reorder the integrations and compute $C(t) = \frac{1}{3}\sum_{i,x}\langle j_i(x,t) j_i(0,0)\rangle$

$$a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$

(Bernecker & Meyer, EPJ 12)

  - ✦ Time-moments: (Chakraborty et al, PRD 14)

Taylor expand $\hat{\Pi}(q^2) = \sum_k q^{2k}\Pi_k$

and compute Taylor coefficients from time moments:

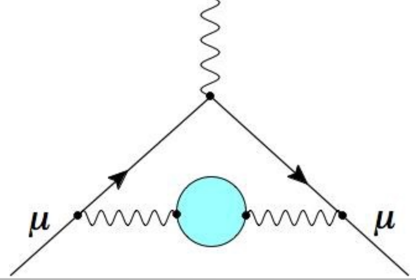
$$C_{2n} = a\sum_t t^{2n} C(t)$$

A brief ``Lattice" guide

- Need to have several (≧2) lattice spacings.
- "physical mass ensemble" means pion mass is at (or near) its physical value.
- Finite volume: should have $m_\pi L > 4$ for simple QCD quantities
  With QED included, finite volume effects are larger
- Sea quark flavors: 2+1, 2+1+1, 1+1+1+1
- Comparing lattice results obtained with different actions provides good cross checks of methods used.
- FLAG: compare/combine results from different lattice groups for specific quantities.

See appendix:

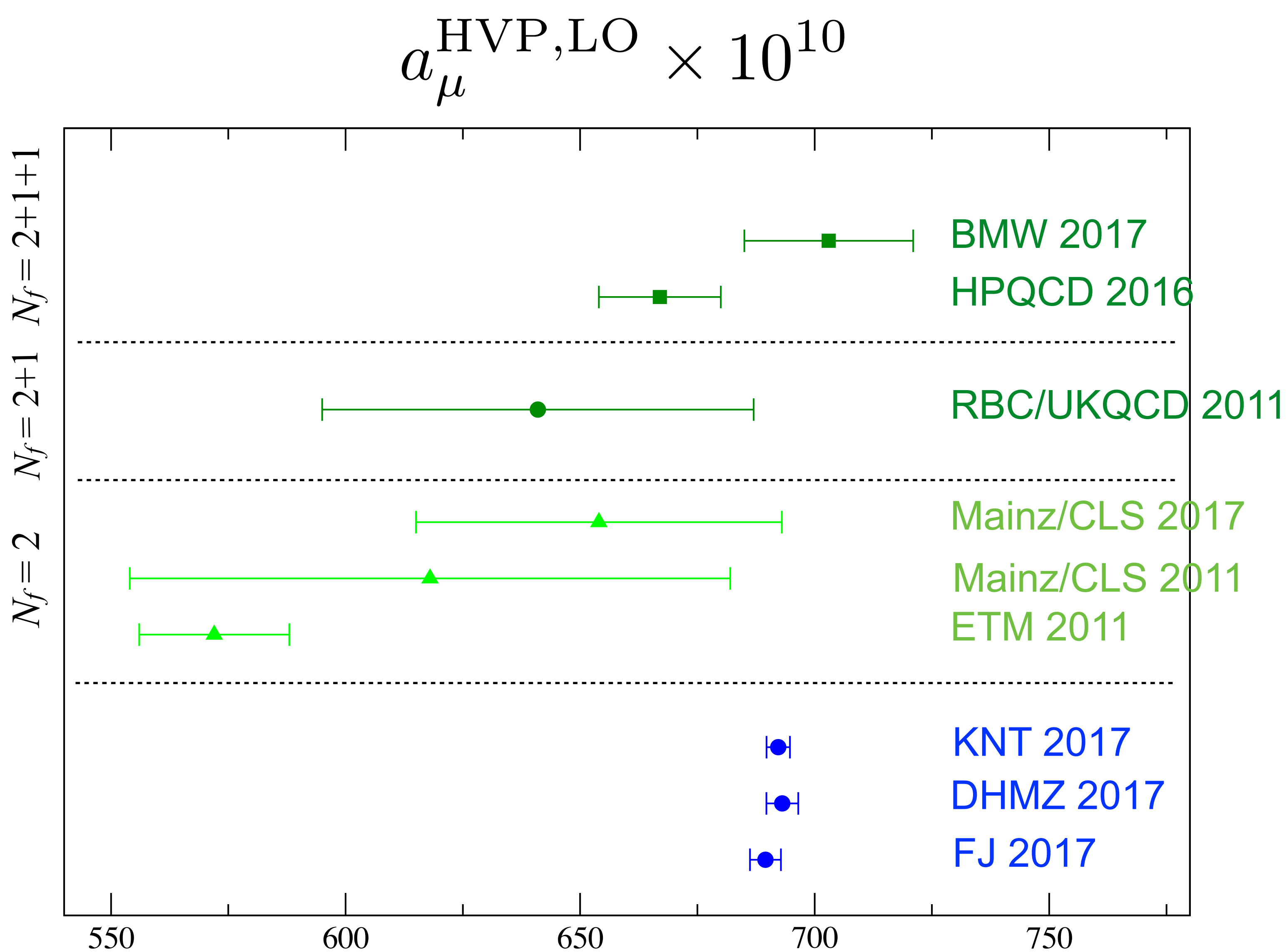
- more detailed introduction to LQCD
- LQCD success examples



Hadronic vacuum polarization

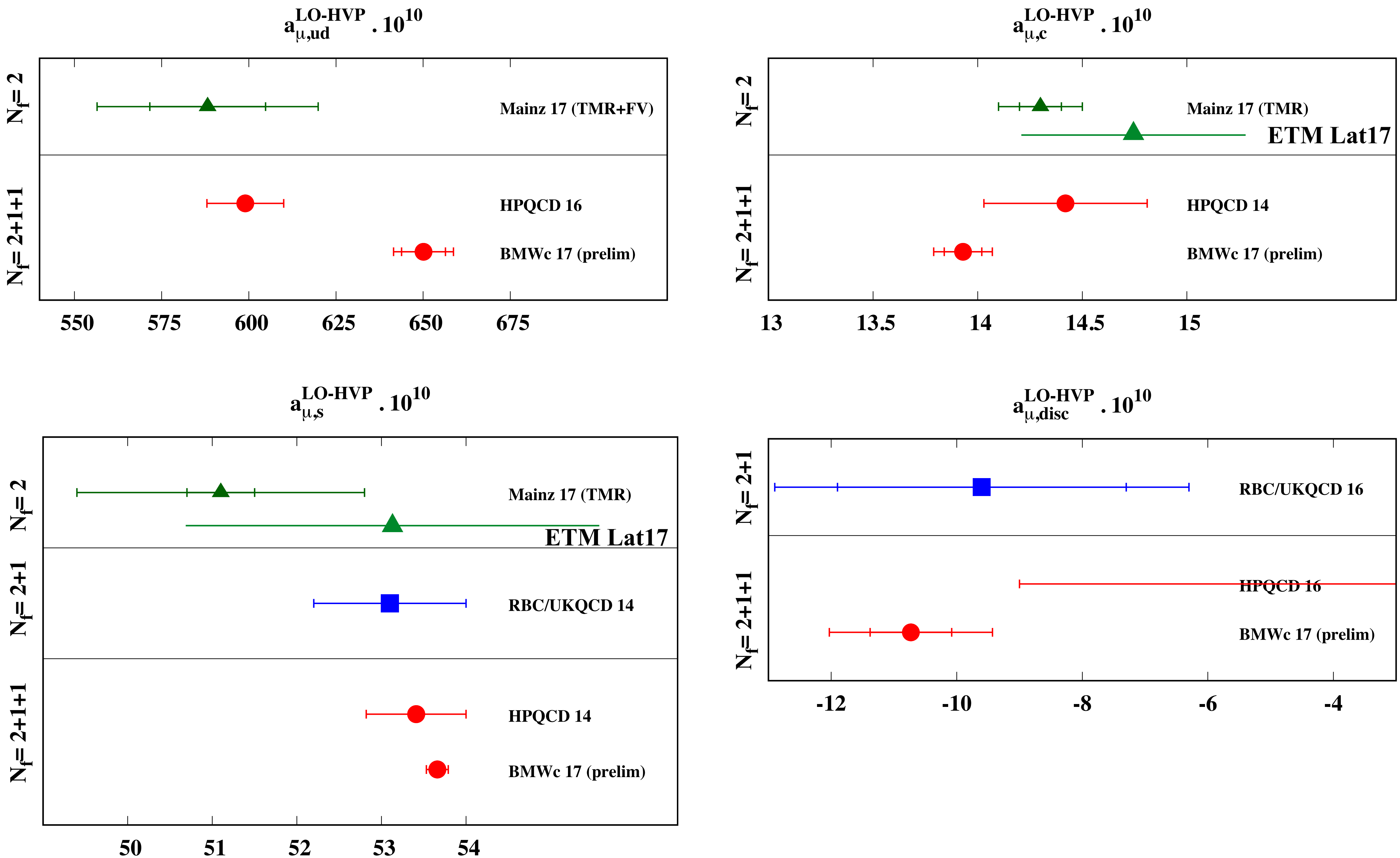
- ✦ Dispersion relation + experimental data for $e^+e^- \to \mathrm{hadrons}$ (and τ data)
  - current uncertainty ~0.4-0.5%
  - can be improved with more precise experimental data
  - new experimental measurements expected/ongoing at BaBar, BES-III, Belle/Belle-II, CMD-3, SND, KLOE,….
- ✦ Lattice QCD
  for (sub)percent precision, calculations of HVP corrections need to
  - be based on physical mass ensembles
  - include disconnected contributions
  - include QED and strong isospin breaking corrections ($m_u \neq m_d$)
  - include finite volume corrections
    ➟ talks by Witting and Lehner
- ✦ compare intermediate quantities (moments, taylor coefficients) with exp. data
- ✦ hybrid method: combine LQCD with experimental data
  (Lehner @ Lattice 2017)

Summary of recent HVP results



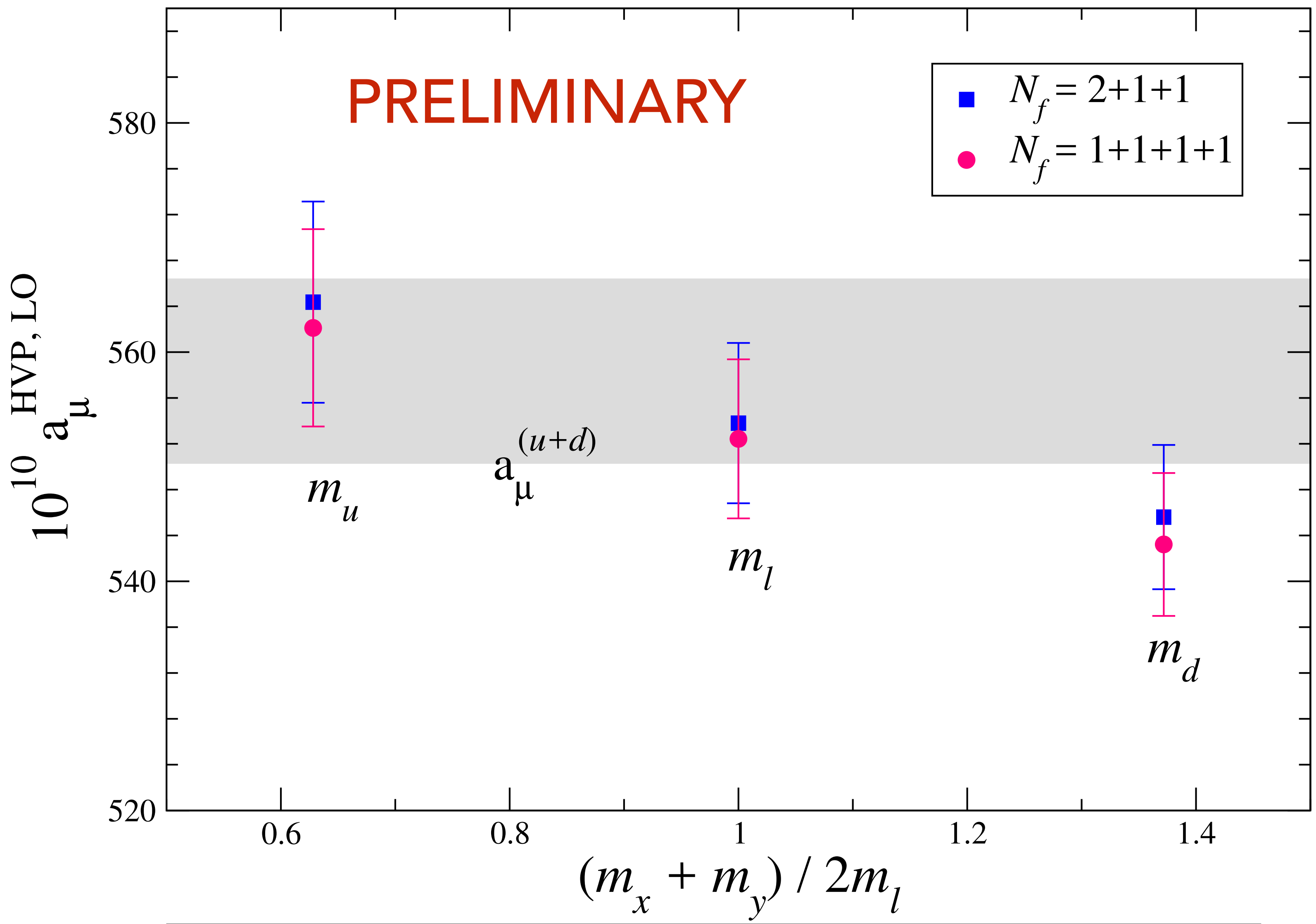
Comparison of recent lattice HVP results

L. Lellouch, K. Miura (for BMWc @ Fermilab workshop & Lattice 2017)



New: Isospin corrections

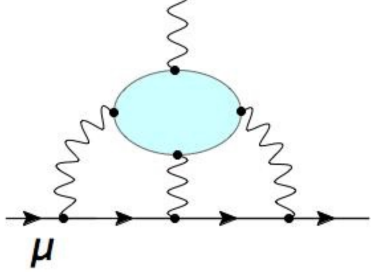
R. Van de Water @ Lattice 2017 for FNAL/HPQCD/MILC



$$\Delta a_\mu^{\mathrm{HVP}} (m_u \neq m_d) = +(1.1 \pm 0.4)\%$$

First LQCD calculation at physical pion mass.

Consistent with recent RBC calculation at heavier pion mass (P. Boyle et al, arXiv:1706.05293).



Hadronic Light-by-light

Hadronic light-by-light:

- ✦ current estimate "Glasgow consensus" based on different QCD models
- ✦ theory error not well known

- ✦ dispersive approach: more complicated than for HVP
  (Colangelo et al, arXiv:1702.07347; Kubis et al, 2012, 2014; Hoferichter et al, 2012, 2014; Hanhardt et al, 2013; Pascalutsa et al, Pauk et al, Danilkin et al,…)
  combine with exp. data and/or LQCD calculations
  ➟ talks by Kubis, Pauk, Colangelo

Direct lattice QCD calculations:

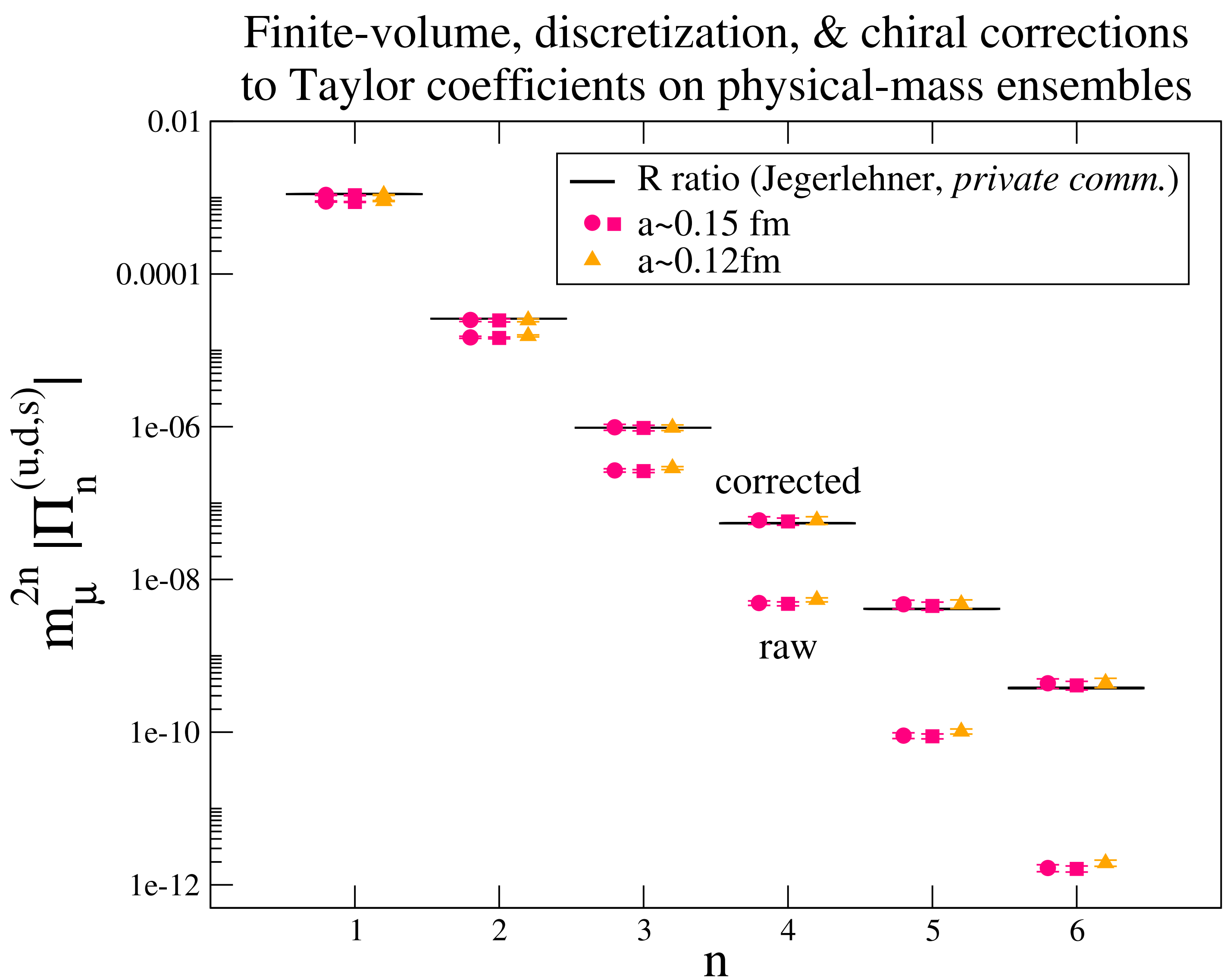
- ✦ QCD + stochastic QED ➟ talk by Lehner
  (Jin et al, arXiv:1610.04603, 2016 PRL; arXiv:1705.01067)
- ✦ QCD + exact QED kernel ➟ talk by Wittig
  (Asmussen @Lattice 2016; Green et al, arXiv:PRL 2015)
- ✦ dominant exclusive contributions (transition form factors)
  (Gerardin et al, arXiv:1607.08174, 2016 PRD, Lattice 2016)

Muon g-2 Theory Initiative: Goals

- theory support to the Fermilab and J-PARC experiments to maximize their impact
  ➟ need theoretical predictions of the hadronic corrections with reduced and reliably estimated uncertainties
- summarize the theoretical calculations of the hadronic corrections to the muon g-2
  ➟ comparisons of intermediate quantities between the different approaches. For example, lattice vs experiment
  ➟ assess reliability of uncertainty estimates

Compare HPQCD results to R-ratio data

R. Van de Water @ Lattice 2017



A good test of the corrections, because the comparison was performed **after** the first version of the HPQCD paper (1601.03071) was posted.

Lowest moments make the largest contributions to a_μ.

Muon g-2 Theory Initiative: Goals

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  ➟ need theoretical predictions of the hadronic corrections with reduced and reliably estimated uncertainties
- summarize the theoretical calculations of the hadronic corrections to the muon g-2
  ➟ comparisons of intermediate quantities between the different approaches. For example, lattice vs experiment
  ➟ assess reliability of uncertainty estimates
- combine to provide theory predictions for $a_\mu^{\rm HVP}$ and $a_\mu^{\rm HLbL}$ and write a report **before** the Fermilab and J-PARC experiments announce their first results.

Muon g-2 Theory Initiative: Plan

- Organize ``plenary” workshops to bring the different communities together
  - First workshop: held near Fermilab, June 2017: kick-off

First Workshop of the Muon g-2 Theory Initiative

3-6 June 2017 *Q Center*
US/Central timezone

Search

- Sponsors
- Committees
- Timetable
- Registration
  - Registration Form
- List of registrants
- List of confirmed speakers
- workshop photos
- Accommodations
- Wilson Hall
- Visa Information

In the coming years, experiments at Fermilab and at J-PARC plan to reduce the uncertainties on the already very precisely measured anomalous magnetic moment of the muon by a factor of four. The goal is to resolve the current tantalizing tension between theory and experiment of three to four standard deviations. On the theory side the hadronic corrections to the anomalous magnetic moment are the dominant sources of uncertainty. They must be determined with better precision in order to unambiguously discover whether or not new physics effects contribute to this quantity.

There are a number of complementary theoretical efforts underway to better understand and quantify the hadronic corrections, including dispersive methods, lattice QCD, effective field theories, and QCD models. We have formed a new theory initiative to facilitate interactions between the different groups through organizing a series of workshops. The goal of this first workshop is to bring together theorists from the different communities to discuss, assess, and compare the status of the various efforts, and to map out strategies for obtaining the best theoretical predictions for these hadronic corrections in advance of the experimental results.

All sessions in this workshop will be plenary, featuring a mix of talks and discussions.

First Workshop of the Muon g-2 Theory Initiative

3-6 June 2017 *Q Center*
US/Central timezone

Search

66 registered participants, 40 talks, 15 discussion sessions (525 minutes)

Muon g-2 Theory Initiative: Plan

- Organize ``plenary" workshops to bring the different communities together
  - First workshop: held near Fermilab, June 2017: kick-off
  - Second workshop: Mainz, 18-22 June 2018: organize first report
  - 2019 & 2020 workshops: Japan? Seattle?
- Form two working groups, one for HVP and one for HLbL:
  - invite community participation
  - organize focused workshops to advance the work: winter/spring 2018
- Finalize the first report before the Fermilab experiment announces its first result with ``Brookhaven level" statistics

  target date for first report: September 2018

Muon g-2 Theory Initiative: WGs

- sign-up for the HVP or HLbL WG in the google sheet
  or send email to one of the WG coordinators
- HVP WG coordinators:
  - Michel Davier davier@lal.in2p3.fr
  - Simon Eidelman eidelman@cern.ch
  - Aida El-Khadra axk@illinois.edu
  - Thomas Teubner thomas.teubner@liverpool.ac.uk
- HLbL WG coordinators:
  - Gilberto Colangelo gilberto@itp.unibe.ch
  - Christoph Lehner clehner@bnl.gov
  - Andreas Nyffeler nyffeler@uni-mainz.de

Summary

- ☆ The First Workshop of the Muon g-2 Theory Initiative kicked off the activities.
- ☆ Participation from all groups engaged in hadronic muon g-2 theory is essential.
- ☆ Two Working Groups (HVP and HLbL) are being formed.
  Let us know if you want to participate.
- ☆ Plenary workshop in Mainz (summer 2018) and a focused HVP workshop in Japan (Feb 2018).
  Still looking for a venue for a HLbL workshop in Winter/Spring 2018
- ☆ A web page for the initiative is currently under construction.
- ☆ Plan to coordinate with other working groups/efforts, for example Radio MonteCarLow and FLAG.

Thank you!

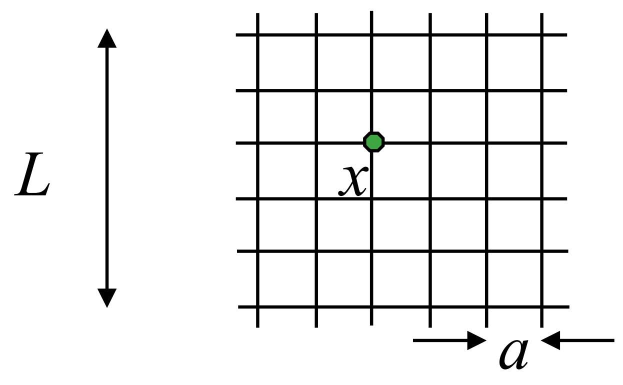
Danke!

Farah Willenbrock

Backup slides

Lattice QCD Introduction

$$\mathcal{L}_{\rm QCD} = \sum_f \bar\psi_f (\not{D} + m_f)\psi_f + \frac{1}{4}{\rm tr} F_{\mu\nu}F^{\mu\nu}$$

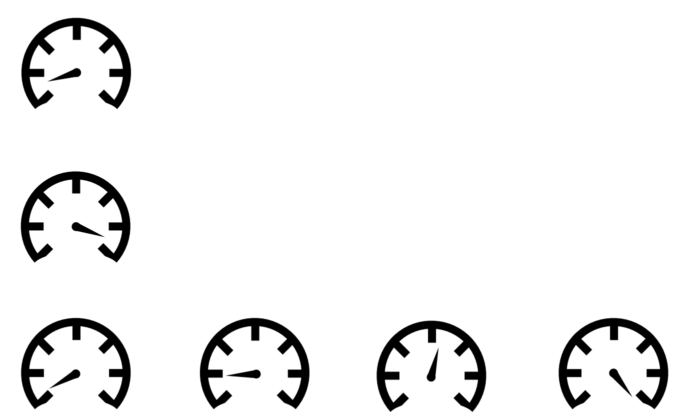


- ✦ discrete Euclidean space-time (spacing a)
  derivatives ➙ difference operators, etc…
- ✦ finite spatial volume (L)
- ✦ finite time extent (T)

adjustable parameters

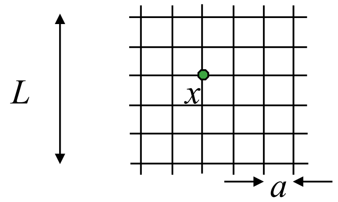
- ❖ lattice spacing: $a \to 0$
- ❖ finite volume, time: $L \to \infty,\ T > L$
- ❖ quark masses (m_f): $M_{H,\rm lat} = M_{H,\rm exp}$, $m_f \to m_{f,\rm phys}$

  tune using hadron masses
  extrapolations/interpolations



m_{ud} m_s m_c m_b

- ❖ also: n_f = number of sea quarks: 3 (2+1), 4 (2+1+1)



Lattice QCD Introduction

systematic error analysis

...of lattice spacing, chiral, heavy quark, and finite volume effects is based on EFT (Effective Field Theory) descriptions of QCD

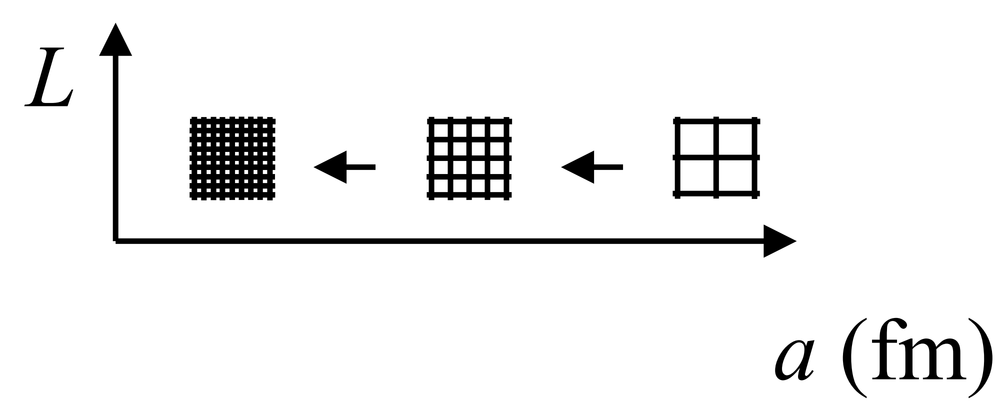
➙ ab initio

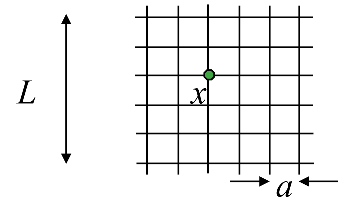
The EFT description:

- provides functional form for extrapolation (or interpolation)
- can be used to build improved lattice actions/methods
- can be used to anticipate the size of systematic effects

To control and reliably estimate the systematic errors

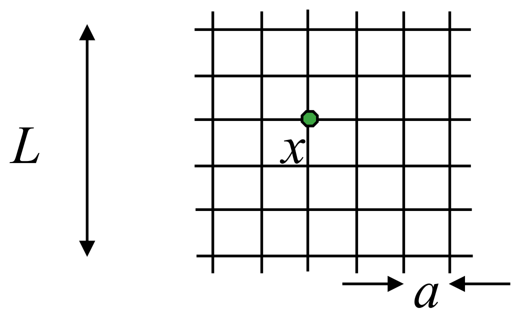
- repeat the calculation on several lattice spacings, light quark masses, spatial volumes, ...





Lattice QCD Introduction

discretization effects



discrete space-time ➙ discrete QCD action

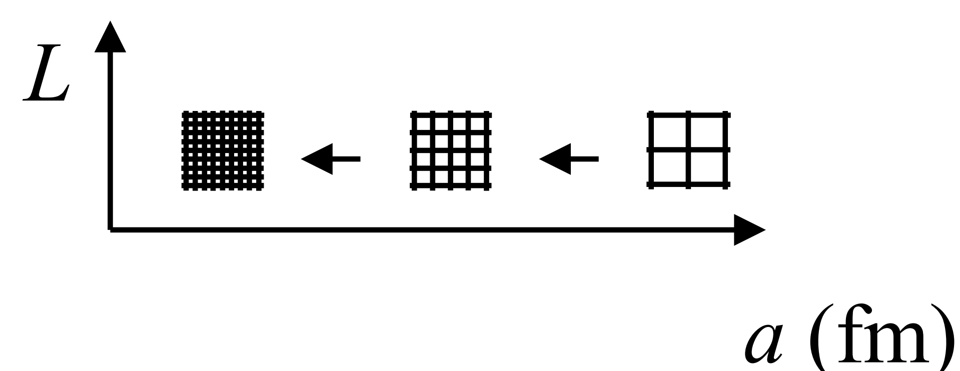
Symanzik EFT: $\langle \mathcal{O} \rangle^{\mathrm{lat}} = \langle \mathcal{O} \rangle^{\mathrm{cont}} + O(ap)^n$

p is the typical momentum scale associated with $\langle \mathcal{O} \rangle$

for light quark systems, $p \sim \Lambda_{\mathrm{QCD}}$

The form of $O(ap)^n$ depends on the details of the lattice action.

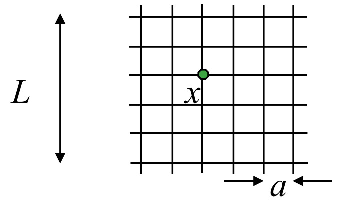
All modern light-quark actions start at $n = 2$ (improved Wilson, twisted-mass Wilson, asqtad, HISQ, Domain Wall, Overlap, ...).



We can now use improved light-quark actions also for charm.

for b quarks: $am_b > 1$ at currently available lattice spacings

➙ use HQET, NRQCD, gen. Symanzik EFT to avoid terms of $O(am_b)^n$



Lattice QCD Introduction

$$\langle \mathcal{O} \rangle \sim \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \, \mathcal{O}(\psi, \bar{\psi}, A) \, e^{-S} \qquad S = \int d^4x \left[\bar{\psi}(\not{D} + m)\psi + \frac{1}{4}(F^a_{\mu\nu})^2 \right]$$

use monte carlo methods (importance sampling) to evaluate the integral.

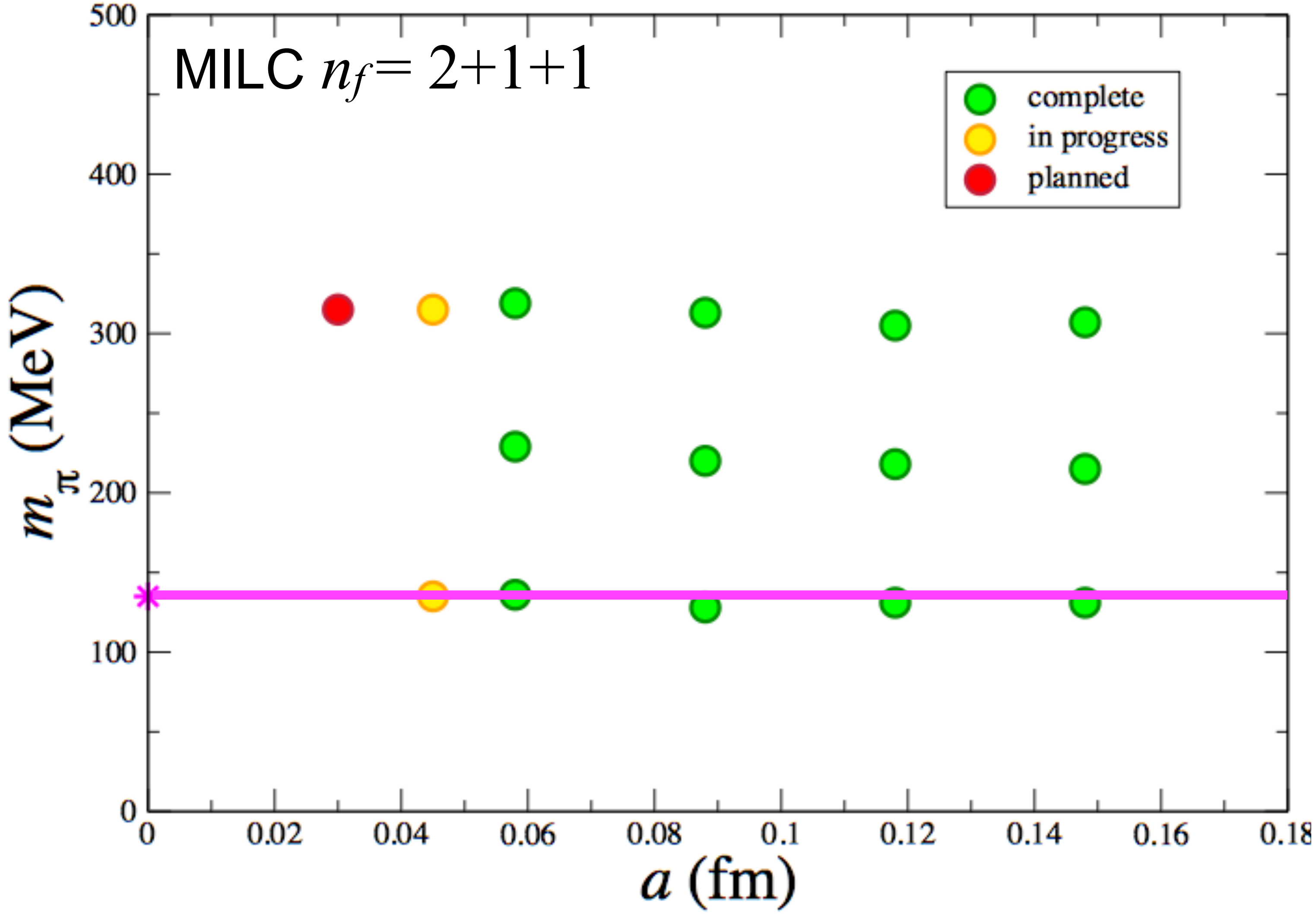
Note: Integrating over the fermion fields leaves $\det(\not{D} + m)$ in the integrand. The correlation functions, $\mathcal{O}$, are then written in terms of $(\not{D}+m)^{-1}$ and gluon fields.

steps of a lattice QCD calculation:

1. generate gluon field configurations according to $\det(\not{D}+m)\, e^{-S}$
2. calculate quark propagators, $(\not{D}+m_q)^{-1}$, for each valence quark flavor and source point
3. tie together quark propagators into hadronic correlation functions (usually 2 or 3-pt functions)
4. statistical analysis to extract hadron masses, energies, hadronic matrix elements, …. from correlation functions
5. systematic error analysis

Lattice set-up

Example: Set of ensembles by MILC collaboration



Five collaborations have now generated sets of ensembles that include sea quarks with physical light-quark masses: PACS-CS, BMW, MILC, RBC/UKQCD, ETM

FNAL/HPQCD/MILC g-2 group

Participants

Fermilab Lattice Collaboration

- Aida El Khadra (Illinois)
- Andreas Kronfeld (Fermilab)
- Ethan Neil (Colorado)
- Ruth Van de Water (Fermilab)

HPQCD

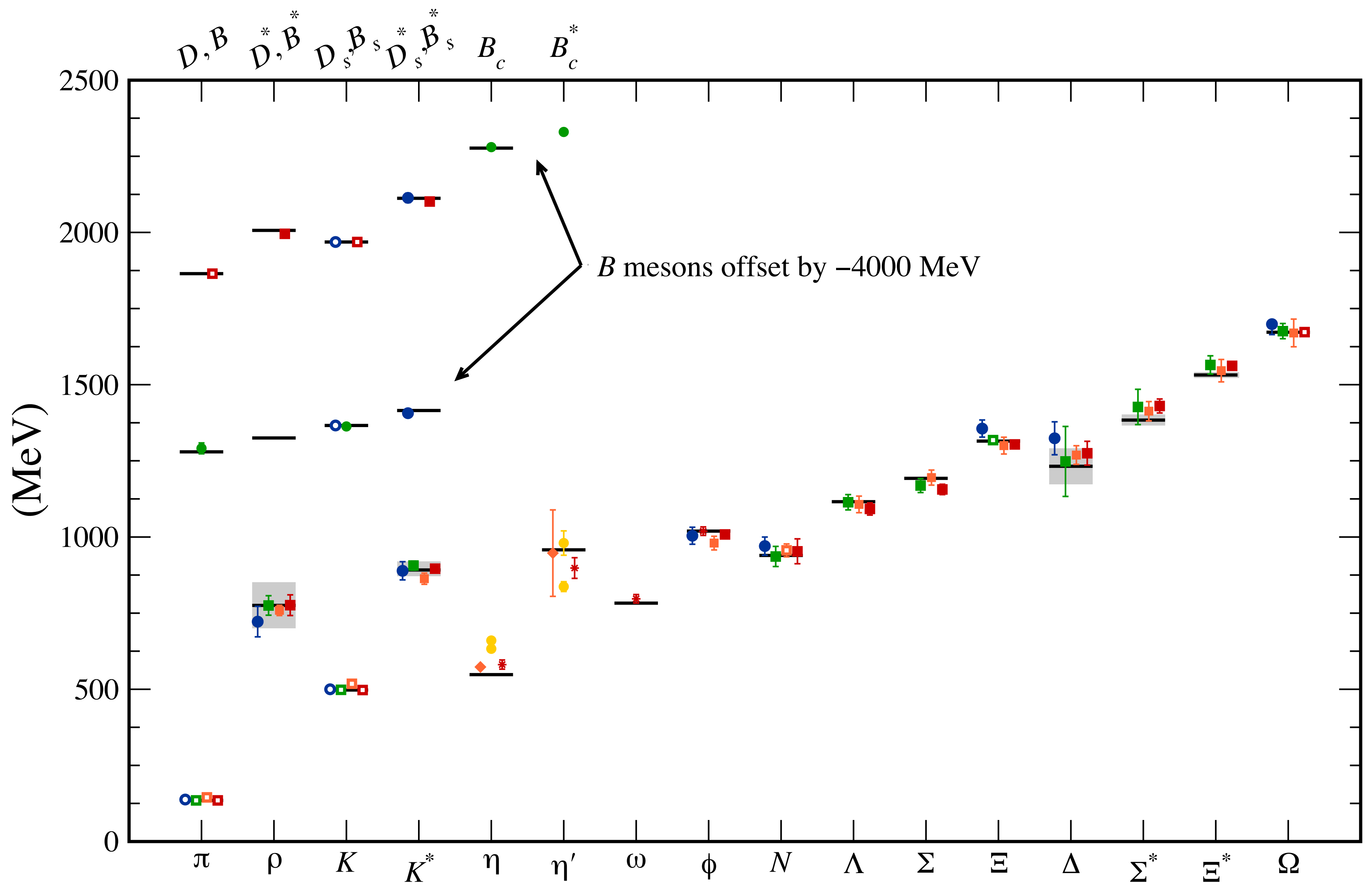
- Bipasha Chakraborty (JLAB)
- Daniel Hatton (Glasgow)
- Christine Davies (Glasgow)
- Jonna Koponen (INFN, Rome)
- Peter Lepage (Cornell)
- Andrew Lytle (Glasgow)
- Craig McNeile (Plymouth)

MILC Collaboration

- Carleton DeTar (Utah)
- Steve Gottlieb (Indiana)
- Jack Laiho (Syracuse)
- Yuzhi Liu (Indiana)
- Doug Toussaint (Arizona)
- Alejandro Vaquero (Utah)

LQCD success: spectrum

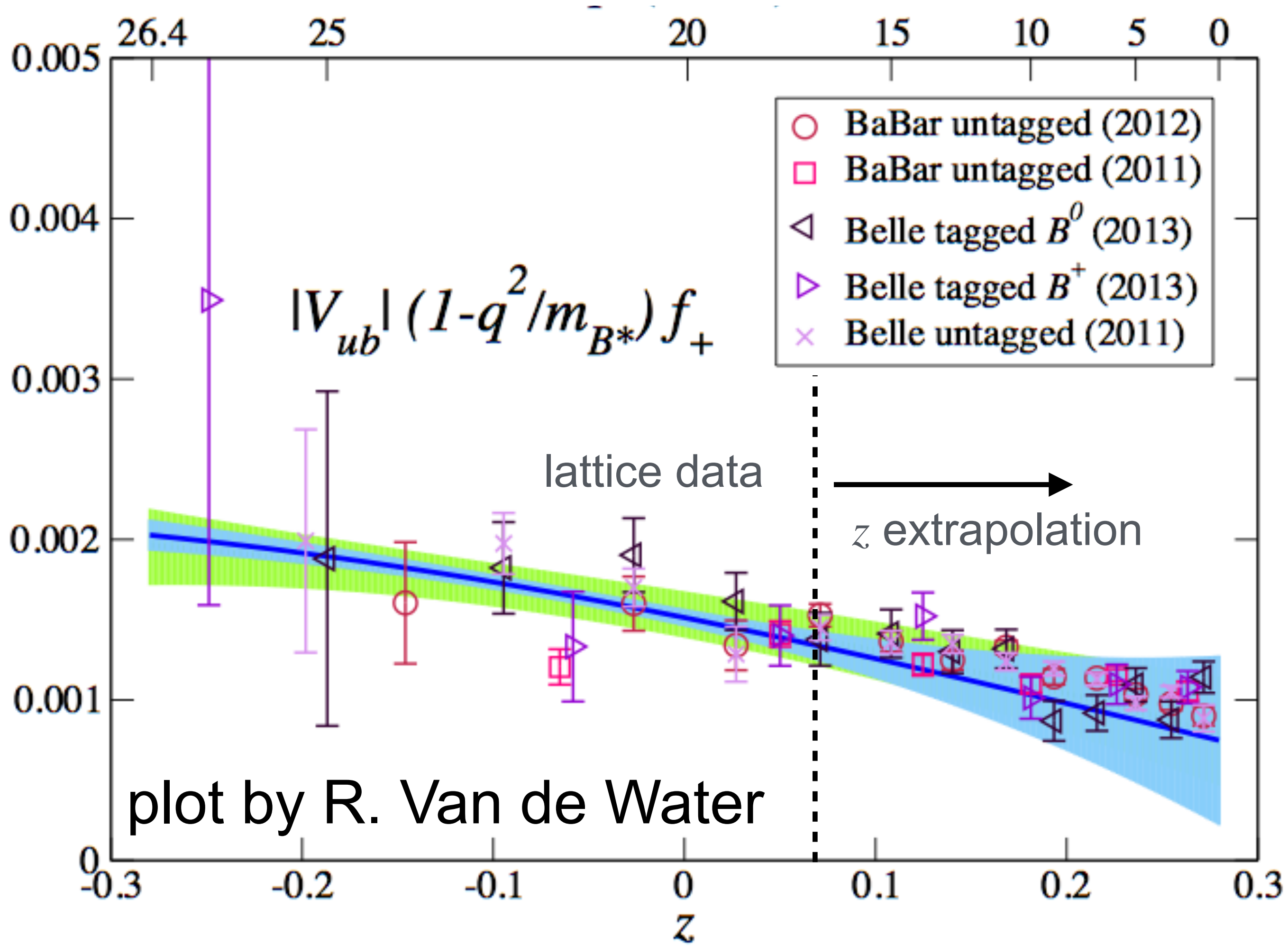
A. Kronfeld (Annu. Rev. Part. & Nucl. Sci, arXiv:1203.1204, updated)



π…Ω: BMW, MILC, PACS-CS, QCDSF; η-η′: RBC, UKQCD, Hadron Spectrum (ω);

D, *B*: Fermilab, HPQCD, Mohler-Woloshyn

LQCD success: form factors



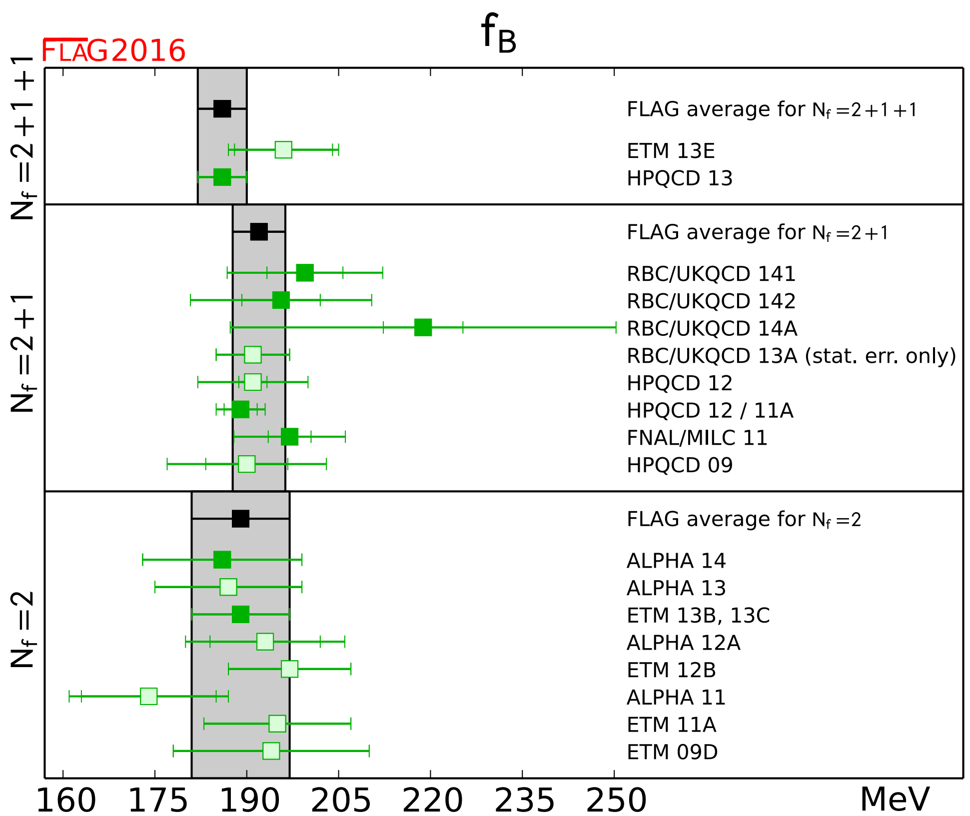
RBC (arXiv:1501.05373, PRD 2015)

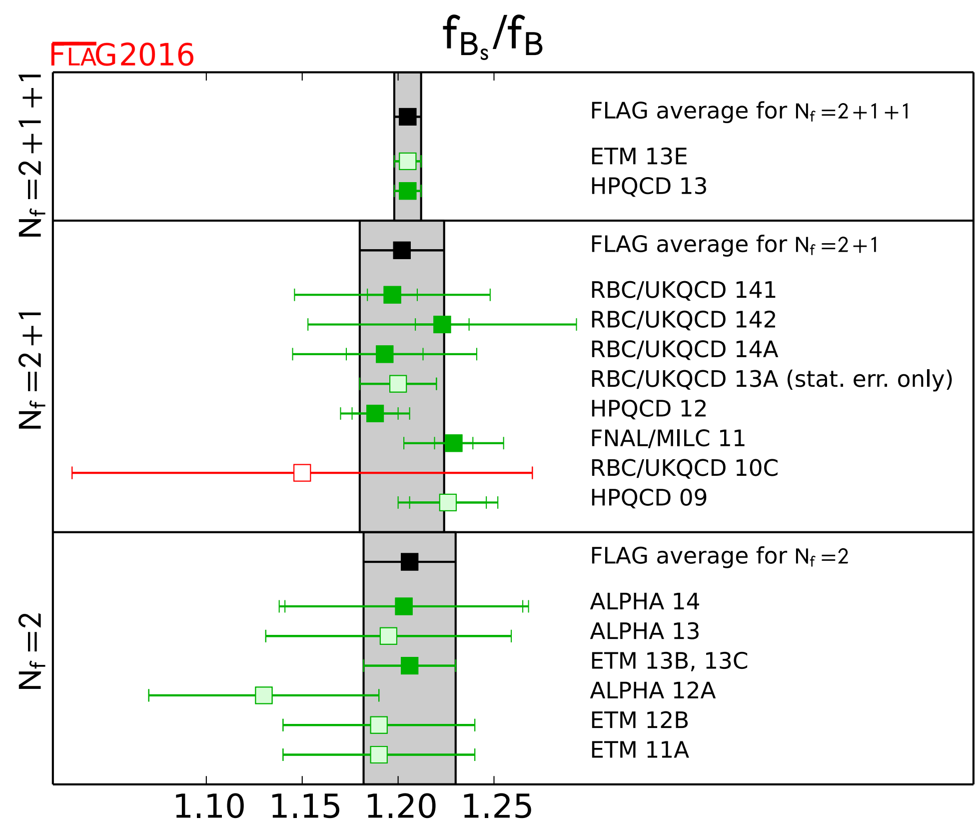
FNAL/MILC (arXiv:1503.07839, PRD 2015)

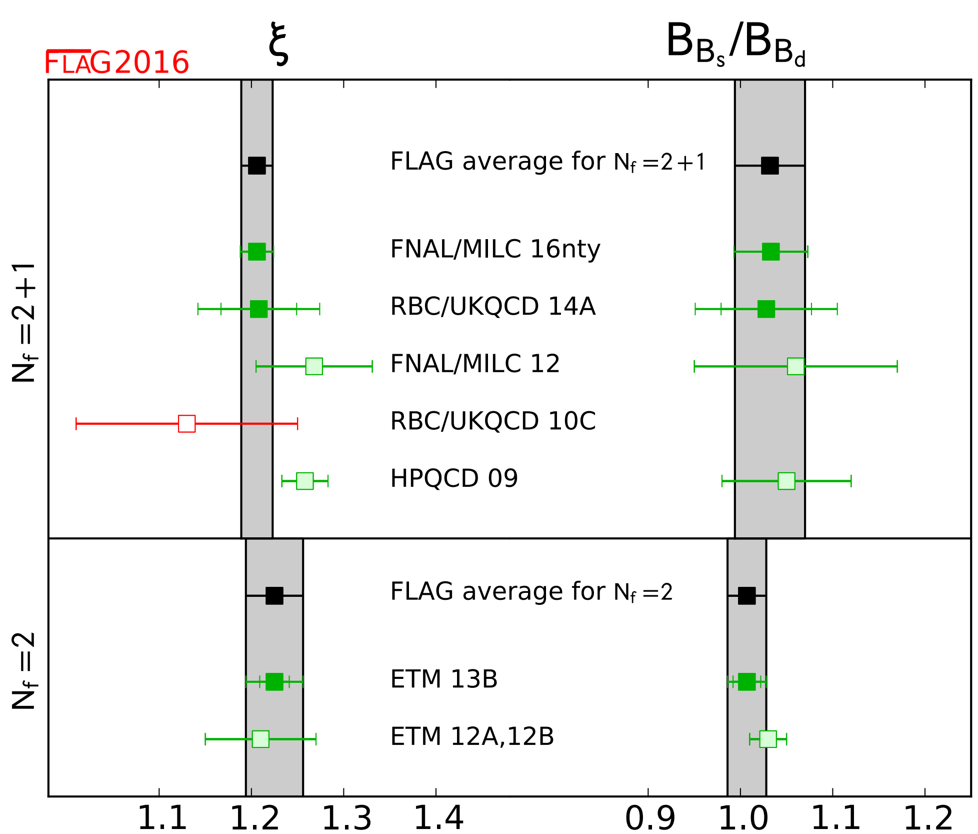
$|V_{ub}| = 3.72\ (16)\ 10^{-3}$

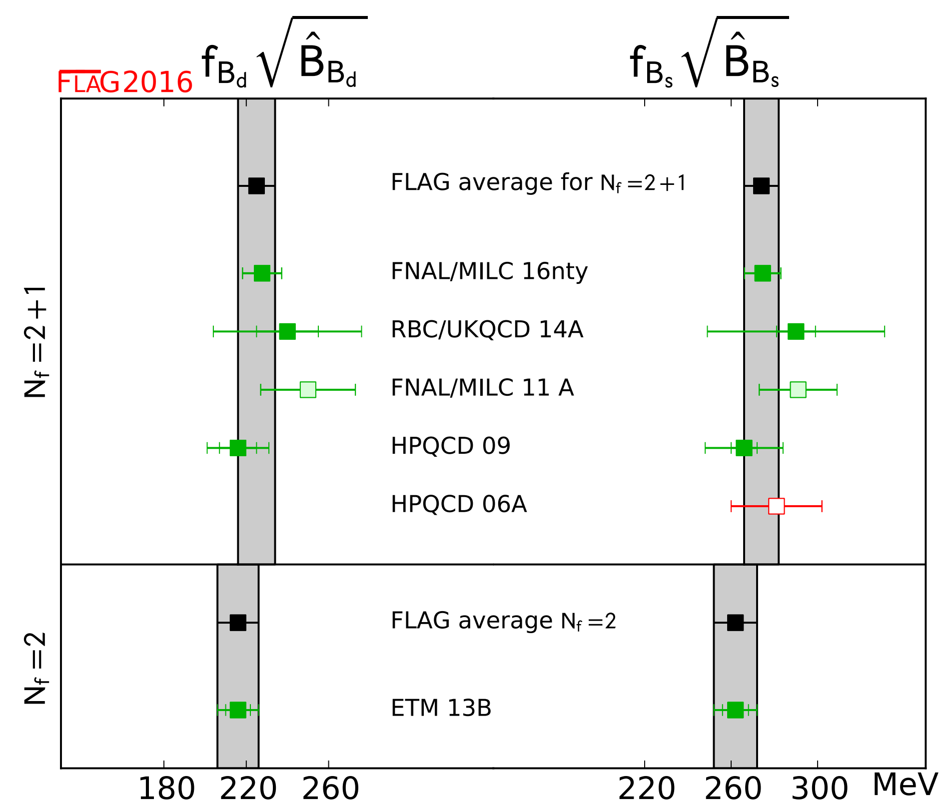
- shape of f_+ agrees with experiment and uncertainties are **commensurate**
- fit lattice form factors together with experimental data to determine $|V_{ub}|$ **and** obtain form factors (f_+, f_0) with improved precision…

FLAG review of *B*-meson quantities



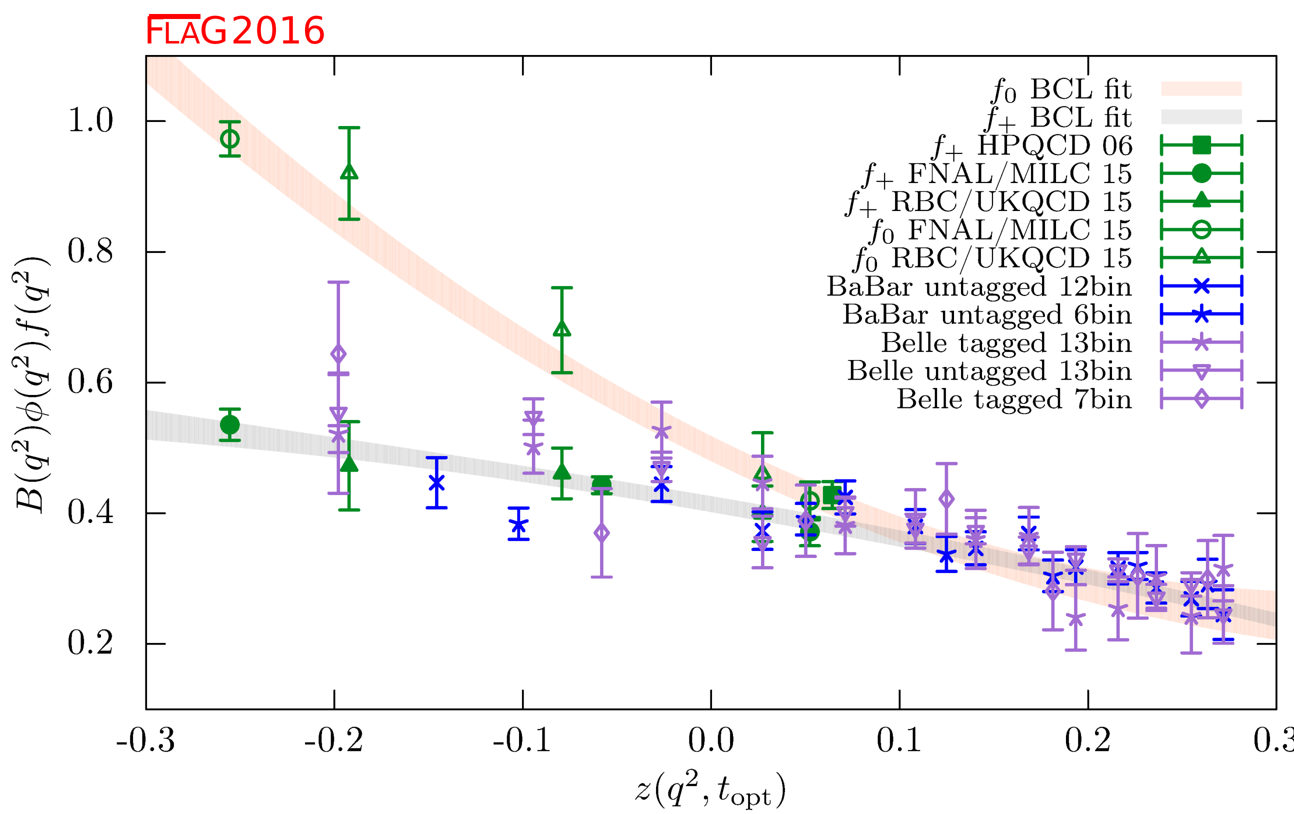






S. Aoki et al (FLAG-3 review, arXiv: 1607.00299, EPJC 17, web update)

FLAG review of *B*-meson quantities



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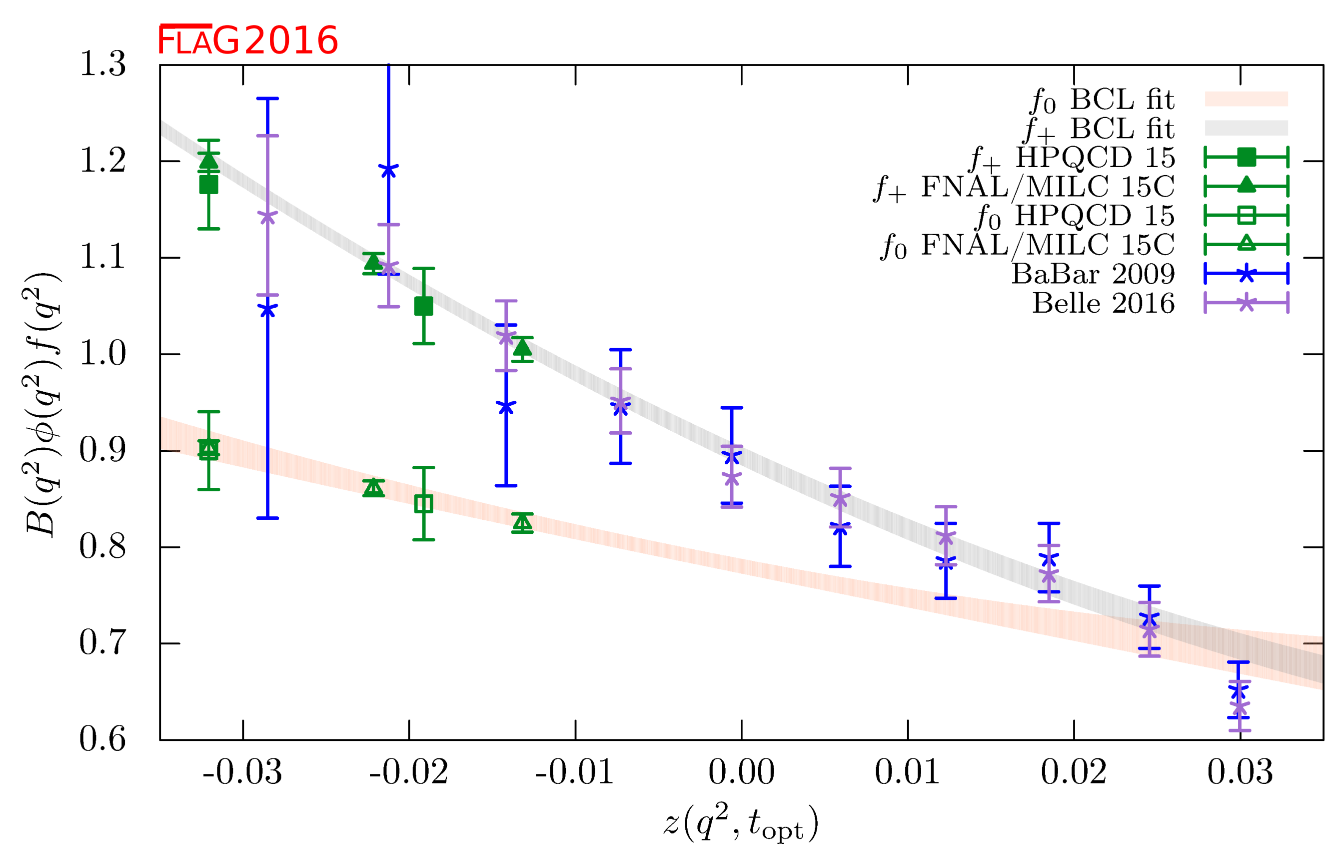
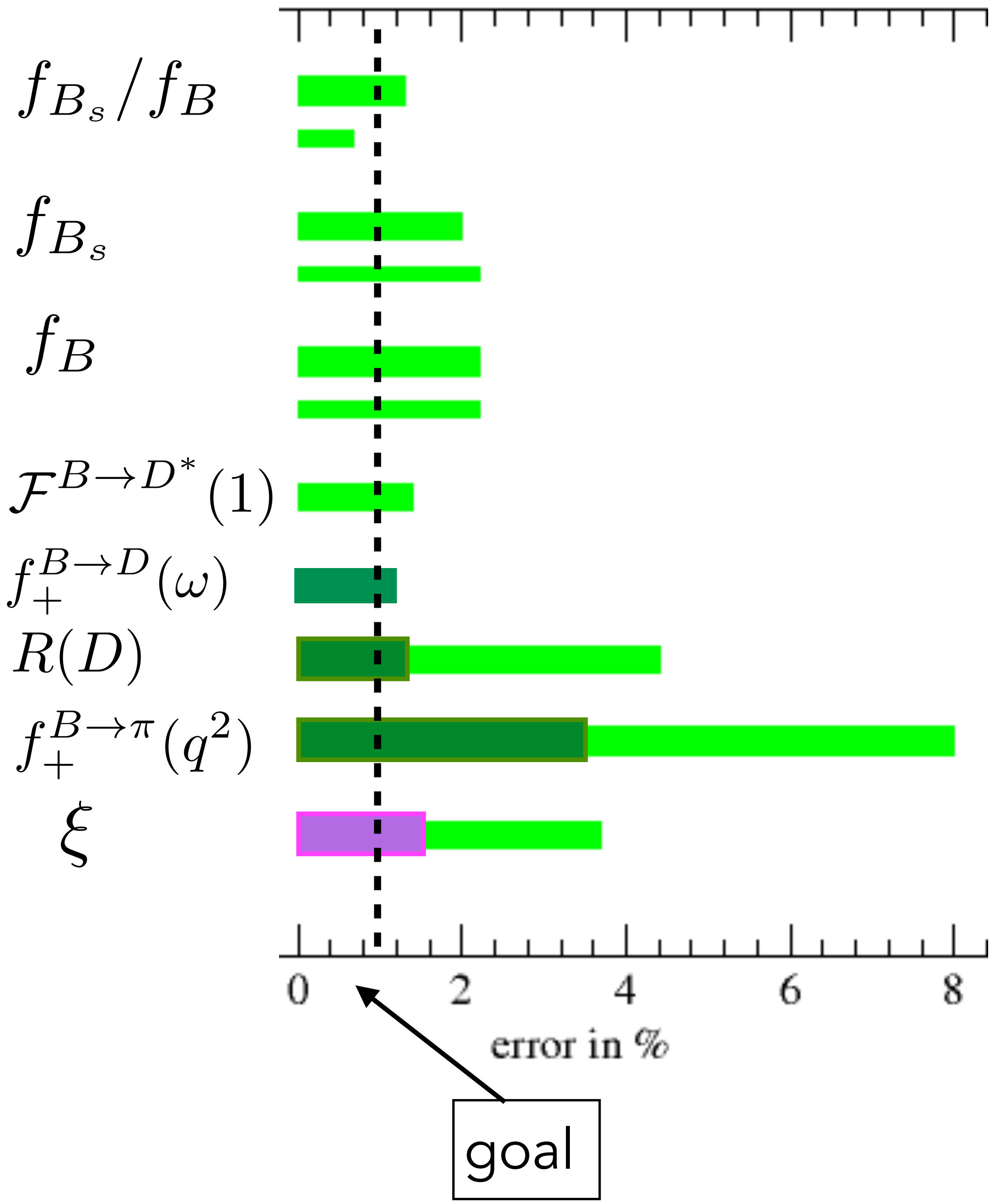


B-meson summary

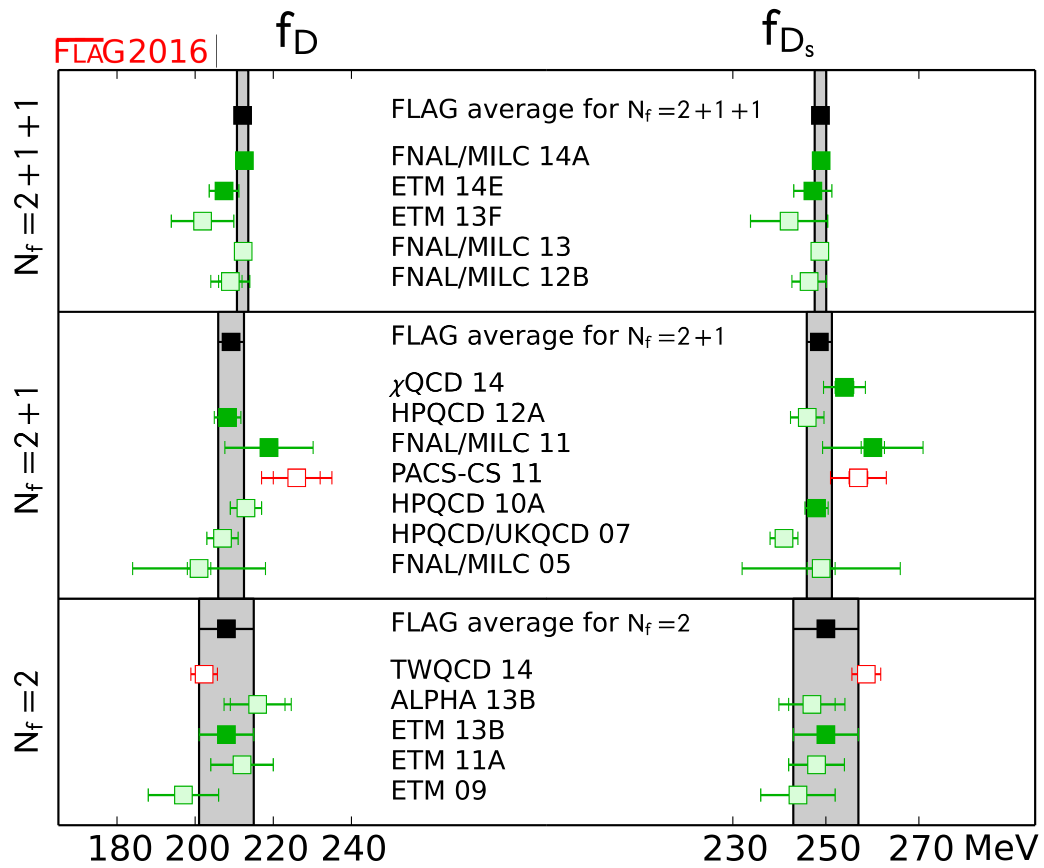
errors (in %) FLAG-2/3 averages + new results

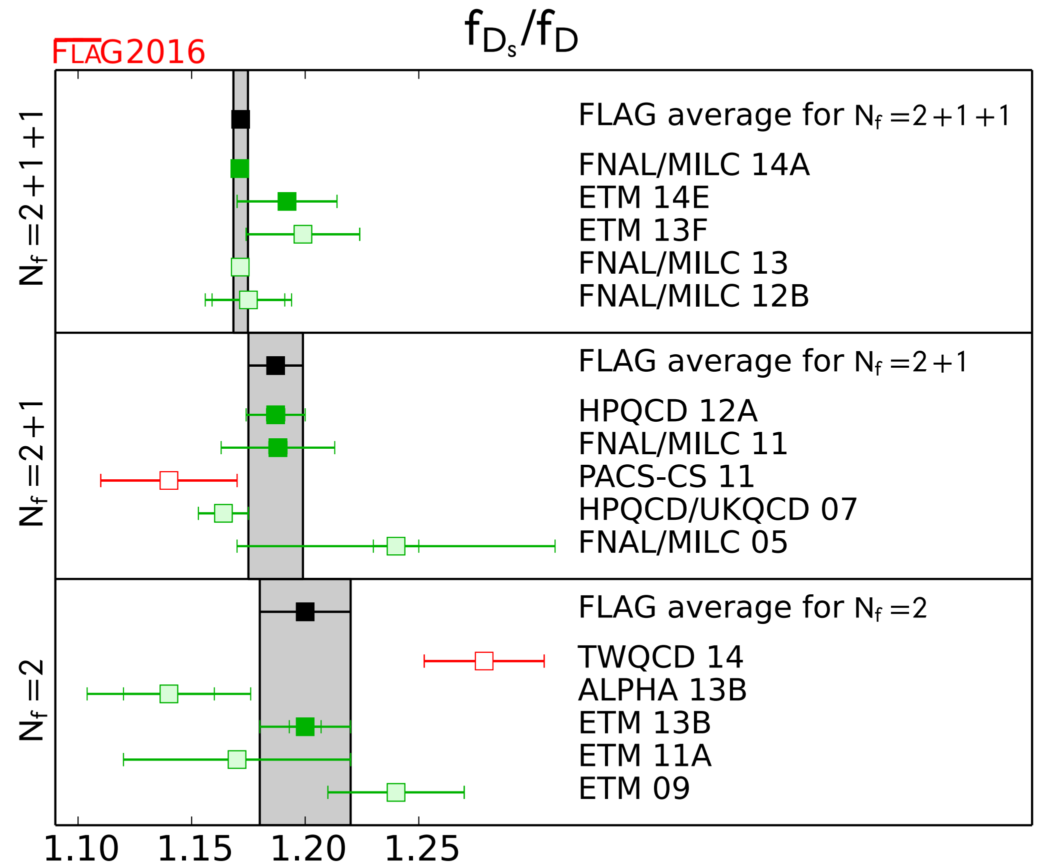


Several quantities where lattice errors are commensurate with experimental uncertainties

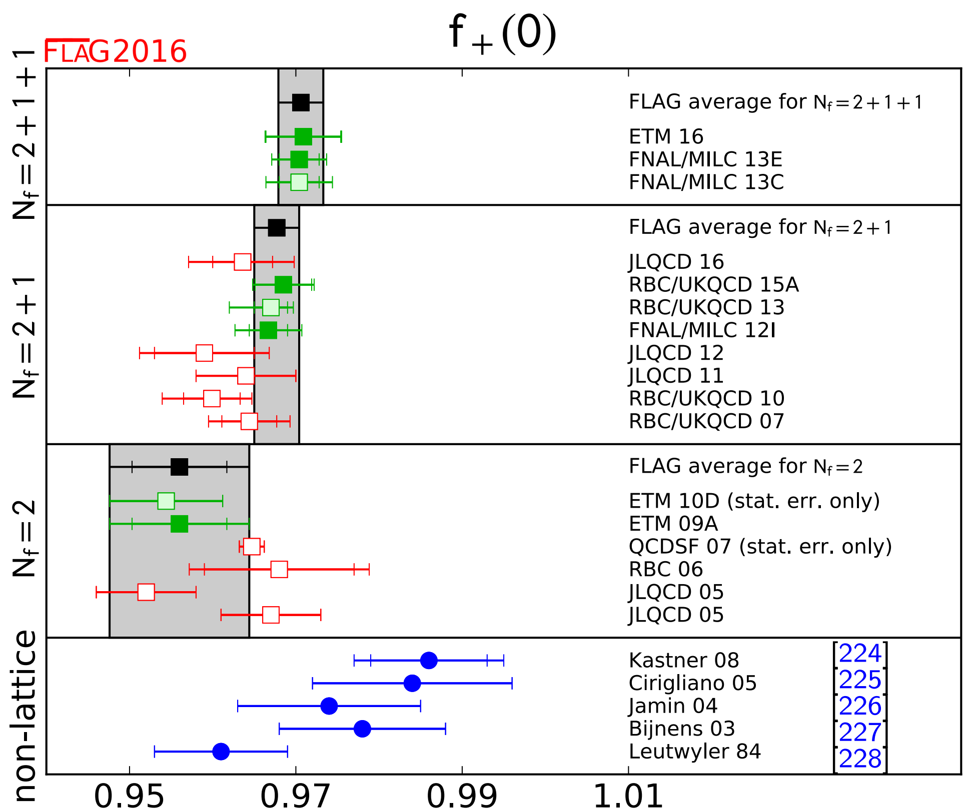
FLAG review of *D*-meson quantities

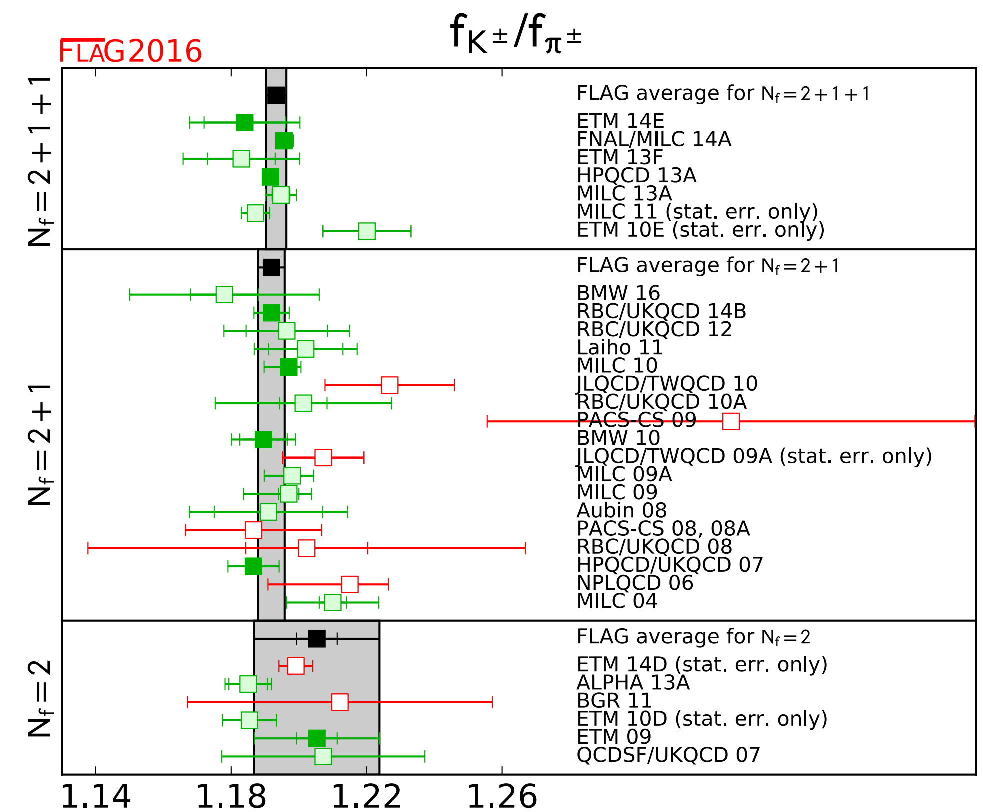
S. Aoki et al (FLAG-3 review, arXiv:1607.00299, EPJC 17, web update)

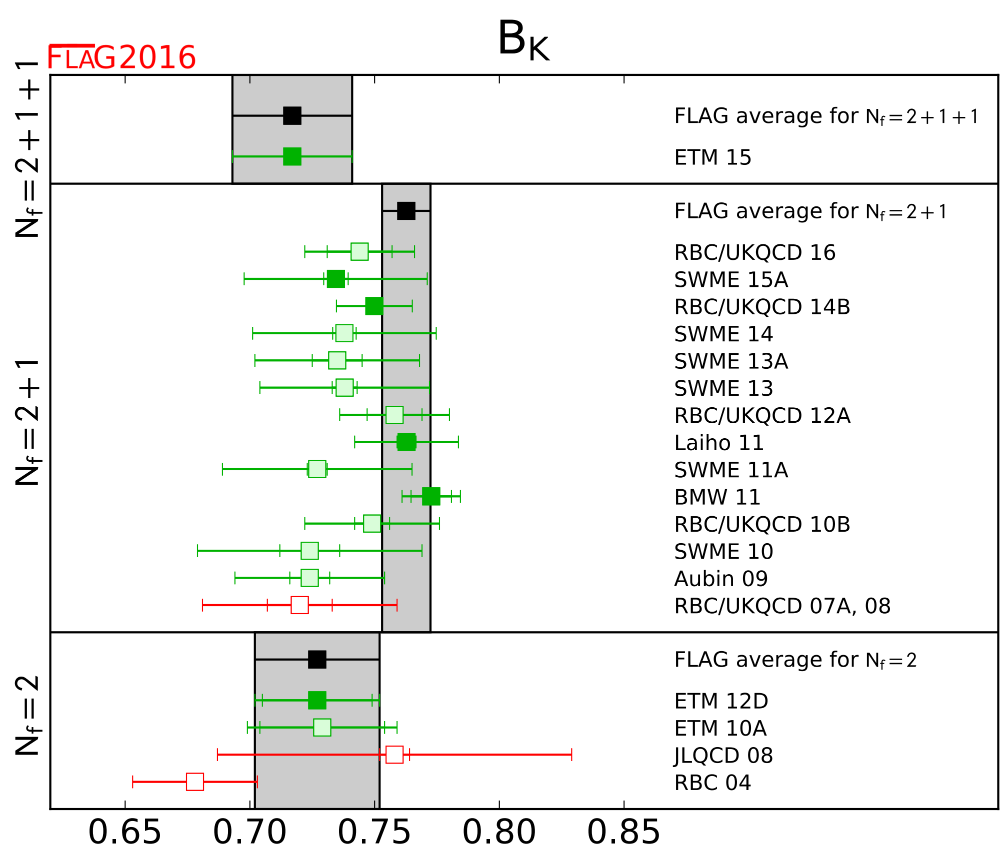




FLAG review of Kaon quantities







S. Aoki et al (FLAG-3 review, arXiv:1607.00299, EPJC 17, web update)

Kaon summary

For all quantities there are results that use **physical mass** ensembles

errors (in %) **FLAG-3 averages**