Proton Radius Review

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SFB 1044
THE LOW-ENERGY FRONTIER
OF THE STANDARD MODEL

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CHARTERED 1693

PRISMA

Beginning

A really fine talk on the experiments is coming, so here I will just give first an overview, generically telling how the proton radius is measured, and why there is a problem.

Topics,

- How the radius is defined.
- Measuring the radius in scattering.
- Measuring the radius via atomic energy level splittings.

And then discuss some specific points that may cause trouble or may be of interest

- Impact of new completed hydrogen measurement.
- Troublesome corrections: the ones from two photon exchange
- Are beyond the standard model (BSM) explanations dead?
- Obtaining the radius from scattering—disagreements.

Radius from elastic electron scattering, $e^- p \to e^- p$

- Measure differential cross section, fit results to form factors,

$$\frac{d\sigma}{d\Omega} \propto G_E^2(Q^2) + \frac{\tau}{\varepsilon} G_M^2(Q^2)$$

$$\left[\tau = Q^2/4m_p^2 \; ; \quad 1/\varepsilon = 1 + 2(1+\tau)\tan^2(\theta_e/2)\right]$$

- Low Q^2, mainly sensitive to G_E.
- Extrapolate to $Q^2 = 0$, and define,

$$\boxed{R_E^2 = -6 \left(dG_E/dQ^2 \right)_{Q^2=0}}$$

- Historical note: Nonrelativistically, form factors are Fourier transforms of charge densities, so NR the above is the RMS radius (squared). Not so good for proton. Above is the modern charge radius definition.

Scattering data

- Most extensive current data comes from Mainz, the city of Gutenberg and a city with a good electron accelerator.

- Data, Bernauer et al., PRL 2010 and later articles.
- Low Q^2 range, 0.004 to 1 GeV^2
- From their eigenanalysis,

$$R_E \text{ or } R_p = 0.879(8)\,\text{fm}$$

Atomic measurements

- Early low uncertainty proton radius quotation is from Saclay, 1962,

$$R_p = 0.86(4)\,\mathrm{fm}$$

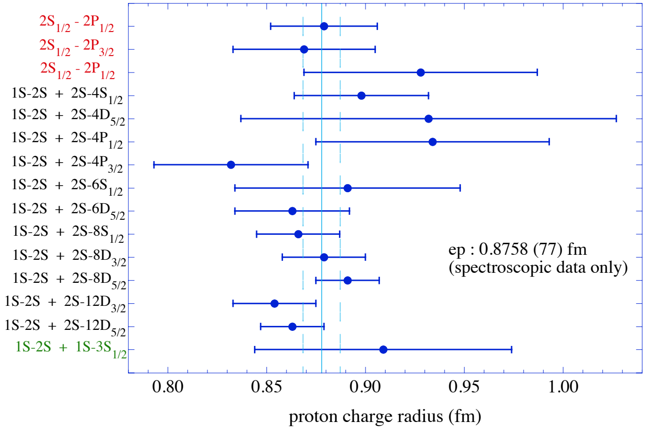
- Early CODATA proton radii come only from electron scattering.
- Atomic measurements also possible, if accuracy of energy level splittings is very high. The energy of a given state is

$$E = E_{\mathrm{QED}} + (\text{coeff.})R_E^2 + \text{other corrections}$$

- About the year 2000, the theory for the QED corrections became accurate enough to extract the small proton radius term
- Diagram (next frame) of results as of early 2016.

Proton radii from hydrogen energy level splittings

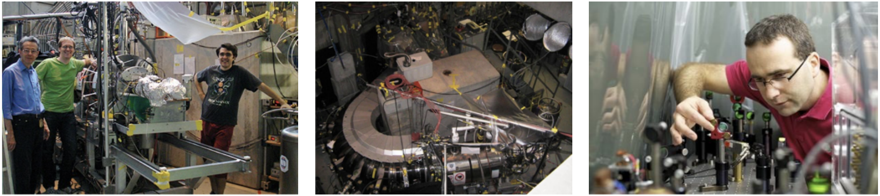
As of early 2016



- Sub 1% error obtained by dividing by $\sqrt{\text{no. of meas.}} = \sqrt{15}$
- Should we instead divide by $\sqrt{\text{no. indep. labs}} \approx \sqrt{3}$?

Then in 2010

- Can do analogous measurements with muonic atoms.
- Muons weigh 200× what electron does. Muons orbit 200× closer. Proton looks 200× bigger and proton size effects are magnified.
- Opportunity to obtain more accurate proton radius, despite short muon lifetime.



- Done by CREMA for $2S$-$2P$ splitting (Lamb shift)
- Obtained

$$\boxed{R_p = 0.84087(39)\,\mathrm{fm}}$$

Conflict!

Repeat

$$\boxed{R_p = 0.84087(39)\,\mathrm{fm}}$$

Appreciation,

- Delivered on uncertainty limit.
- But CODATA 2014 based on combining all electron numbers gave

$$R_p = 0.8751(61)\,\mathrm{fm}$$

- Muon value 4% or many σ low.

Second part

Some specific points,

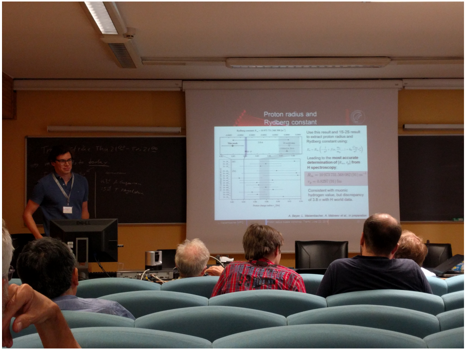
- Impact of new completed hydrogen measurement.
- Troublesome corrections: the ones from two photon exchange
- Are beyond the standard model (BSM) explanations dead?
- Obtaining the radius from scattering—disagreements.

New $2S$-$4P$ splitting measurement

- From MPI-Q (Garching), new measurement of $2S$-$4P$ splitting in hydrogen
- Accurate enough to give $\approx 1\%$ uncertainty limit by itself

New $2S$-$4P$ splitting measurement

- Announced at proton radius workshop (Trento) June 2016



Proton radius and Rydberg constant

Use this result and 1S-2S result to extract proton radius and Rydberg constant using:

Leading to the **most accurate determination of $[R_\infty, r_p]$ from H spectroscopy**:

$R_\infty = 10\,973\,731.568\,082\,(91)\,\mathrm{m}^{-1}$

$r_p = 0.8297\,(91)\,\mathrm{fm}$

Consistent with muonic hydrogen value, but discrepancy of 3.8 σ with H world data.

A. Beyer, L. Maisenbacher, A. Matveev et al., in preparation

- Data heard around the world,

$$R_p(2S\text{-}4P) = 0.8297(91)\,\mathrm{fm}$$

- Now have proton radius problem for ordinary hydrogen all by itself!

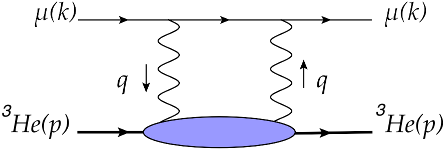
H-spectrosopy future

May also expect:

- York University (Canada): Ordinary hydrogen 2S-2P Lamb shift. (Maybe this year??)
- Laboratoire Kastler Brossel (Paris): 1S-3S transition
- More from Garching
- NIST (USA): Measure Rydberg using Rydberg states, very high n states, uncontaminated by proton size. (Very relevant: recall previous discussion.)
- $+$ National Physical Lab (U.K.), several $2S$-nS, nD transitions

Two Photon Exchange (TPE)

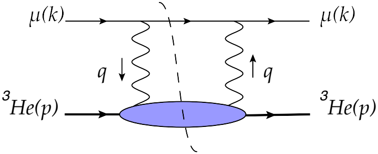
- One of the "other corrections":
  not the biggest term, but the biggest source of uncertainty.
  E.g.,



- Blob is off shell proton or any higher state. Makes calculation hard.
- How good are we?
- How good do we have to be?

Dispersive calculation

- Some calculate by noting putting the intermediate states on shell (a) gives the Imaginary part of the whole diagram, and (b) means each half of the diagram is an amplitude for a real scattering process, and hence can be gotten from scattering data.



- What matters is the lower vertex, so can use electron scattering data.
- Mostly need low Q^2, low energy data
- Reconstruct whole diagram using dispersion relations.

Begin with the proton

- Theory for Lamb shift splitting, with numbers for proton,

$$\Delta E_L^{\text{theo}} = \Delta E_{\text{QED}} - \frac{m_r^3 Z^4 \alpha^4}{12} R_p^2 - \Delta E_{\text{TPE}}$$
$$= 206.0336(15) - 5.2275(10) R_p^2 + 0.0332(20)$$

(units are meV and fm)

- Faith,

$$\Delta E_L^{\text{theo}} = \Delta E_L^{\text{expt}} = 202.3706(23)\,\text{meV}$$

- Solve,

$$R_p = 0.84087(39)\,\text{fm} \qquad [0.038\%]$$

- IF THE TPE WERE PERFECT,

$$R_p = 0.84087(32)\,\text{fm}$$

- Conclude: for the proton theorists have done their job.
  Uncertainty in TPE not dominant.

Deuteron

- Trouble: the deuteron is loosely bound, a little energy turns it into other states. Proton remains just a proton until there is enough energy to make a pion.
- Theory with numbers for deuteron is now,

$$\Delta E_L^{\text{theo}} = 228.7766(10) - 6.1103(3) R_d^2 + \Delta E_{\text{TPE}}$$

- and there are now two ways to obtain the TPE,

how	*who*	ΔE_{TPE} (meV)
Nuclear potentials	Hernandez *et al.*	1.6900(200)
Nuclear potentials	Pachucki-Wienczek	1.7170(200)
Dispersion theory	Carlson *et al.*	2.0100(7400)
Summary	Krauth *et al.*	1.7096(200)

- Work out, with $\Delta E_L^{\text{expt}} = 202.8785(34)\,\text{meV}$

$$R_d = 2.12562(78)\,\text{fm}$$

- If TPE be perfect,

$$R_d = 2.12562(15)\,\text{fm}$$

^{3}He

- For dispersion theorists, better case than the deuteron because the binding is stronger, the thresholds are higher, and there is data near the thresholds, which is the important region for this calculation.
- With ^{3}He numbers,

$$\Delta E_L^{\text{theo}} = 1644.4643(150) - 103.5184(98) R_T^2 + \Delta E_{\text{TPE}}$$

- and for the TPE,

how	*who*	ΔE_{TPE} (meV)
Nuclear potentials	Hernandez *et al.* (2016)	15.46(39)
Dispersion theory	CEC, Gorchtein, Vanderhaeghen	15.14(49)
Summary	Franke *et al.*	15.30(52)

^{3}He — How good do we have to be?

- comparison will be to current electron scattering data for R_T
- direct electron scattering on ^{3}He: $R_T = 1.973(14)\,\mathrm{fm}$
- can do somewhat better using ^{4}He data, $R_\alpha = 1.681(4)$ and isotope shift, except that:

group	$R_T^2 - R_\alpha^2$ (fm^2)	R_T (fm)
Cancio Pastor *et al.* (2012)	1.074(4)	1.975(4)
Shiner *et al.* (1995)	1.066(4)	1.973(4)
van Rooij *et al.* (2011)	1.028(11)	1.963(6)
subsumption		*1.968(11)*

- How well will the μ-^{3}He Lamb shift do? Use the result given for ΔE_{TPE} and work out the anticipated uncertainty:

$$R_T = 1.96xxx(13)\,\mathrm{fm}$$

- Uncertainty about 8× smaller than that from e^- scattering. (Although, (13) → (2) if TPE were perfect.)
- Still, if no BSM, will easily separate results from different isotope shift measurements.

^{3}He — what about BSM?

- BSM here means (Tucker-Smith & Yavin; Battel, McKeen & Pospelov; CC & Rislow)
  - proton radius is fixed number
  - observed energy discrepancy is real
  - and due to BSM μ-philic interaction
- Model somehow:
  - vector interaction, new exchange boson ϕ of some mass
  - coupling to $\mu \gg$ coupling to e
  - coupling to hadron like dark photon, *i.e.*, $\propto Z$
- Get result from energy deficit in hydrogen upon scaling to T,

$$\Delta E^T_{\mathrm{L,BSM}} = Z^4 \left(\frac{m_r^T}{m_r^p}\right)^3 \frac{f(x_T)}{f(x_p)} \Delta E^p_{\mathrm{L,BSM}} \overset{m_\phi \gg \text{few MeV}}{=} 6.59\,\mathrm{meV}$$

$$\text{for } f(x) = x^4/(1+x)^4 = m_\phi^4/(Z m_r \alpha + m_\phi)^4$$

- The 0.52 meV uncertainty in the TPE is good enough to kill/confirm BSM idea (for many m_ϕ).

BSM possibilities generally

- Reports have it that CREMA finds ^{3}He radius compatible with electron scattering number, with small error limit.
- Incompatible with 6.59 meV shift expected from BSM explanation of original puzzle, for m_ϕ (mass of BSM force carrier) not small.
- Does this kill BSM idea?
  - maybe
  - maybe not
- One difference between ^{3}He and hydrogen is size of atomic state. ^{3}He is factor 2 smaller.
- Recall zero mass exchange particle (photon) gives no $2P$-$2S$ splitting. Something long range to ^{3}He can look like short range to hydrogen. Light boson exchange can give $\approx$ no splitting in ^{3}He but notable splitting for H.
- Works numerically—for present uncertainty limits—for $m_\phi \approx 1$ MeV.

Reanalyses of electron scattering data

- Point: Measurements at finite Q^2. Need to extrapolate to $Q^2 = 0$ to obtain charge radius. (Mainz group itself: $R_p = 0.879(8)$ fm.)
- Because of importance, others have tried, using different ways of fitting data. Three recent fits found big values:
- Graczyk & Juszczak (2014), using Bayesian ideas and pre-Mainz world data, obtained
$$R_p = 0.899(3)\,\text{fm}.$$
- Lee, Arrington, & Hill (2015) using Mainz data and neat mapping ideas to ensure convergence of expansions, obtained
$$R_p = 0.895(20)\,\text{fm}.$$
- Arrington & Sick (2015) found
$$R_p = 0.879(11)\,\text{fm}.$$

Alternative reanalyses

- There are also low results, using ostensibly the same data sets
- Lorenz, Meißner, Hammer, & Dong (2015 and earlier), dispersive ideas, also using timelike data, obtained

$$R_p = 0.840(15)\,\text{fm}.$$

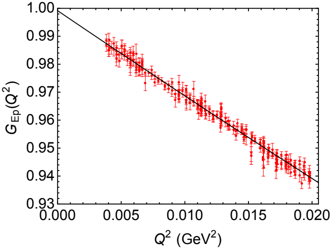
- Horbatsch and Hessels, PRC (2016), got both high and low values.
- Griffioen, Maddox, Carlson, PRC 2016, quote

$$R_p = 0.840(16)\,\text{fm}.$$

- Higinbotham, Kabir, Lin, Meekins, Norum, Sawatzky, PRC (2016) Consistent with low value of R_p.

One plot

- Viewpoint: Charge radius is a $Q^2 = 0$ concept, should be able to obtain just from low Q^2 data.



- Fit with function linear and quadratic is Q^2, with floating norm.
- Gives low R_p
- Studies seen to show little bias.
- Consult "Avoiding common pitfalls and misconceptions in extractions of the proton radius," 1606.02159

Scattering radii

- New scattering experiments coming
- PRad (JLab) does electron scattering down to $Q^2 = 0.0002$ GeV^2. Have data already.
- MUSE (PSI) will do both muon and electron scattering, down to 0.002 GeV^2

Ending

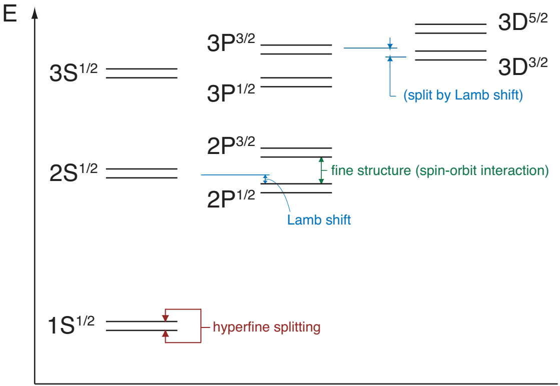
- Remarkable: 7 years after the first announcement, the problem persists.
- Interestingly little discussion of the correctness of the μ-H Lamb shift data.
- Serious and good new data coming, in spectroscopy and scattering.
- Opinion: Either
  - The puzzle isnt a puzzle: The electron based radius measurements will reduce to the muonic value.
    - The scattering analysis is under discussion, and more data coming
    - The spectroscopy measurements by themselves have a puzzle.
  - All radii correct, and a BSM muonic specific force is explanation despite problems
- Comment: the theory for $(g-2)_\mu$ cannot be considered settled until the proton radius problem is settled. Further, there may be striking corrections to other processes that involve muons.

Extras

Beyond the end

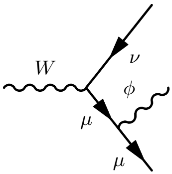
Hydrogen energy levels

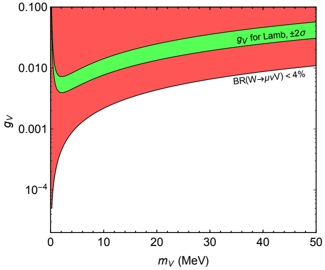
Not to scale



Possible W decay constraints

- Remark of Karshenboim, McKeen, and Pospelov: there is fast growth with energy of amplitudes involving massive vector particles
- If light new particle ϕ or V coupling to muon, it gives large radiative correction to W decay via $W \to \mu\nu V$, larger than measured error in W decay rate.

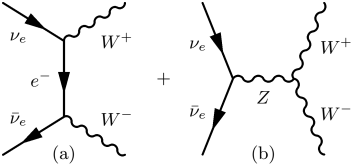




Red: forbidden
Fig. based on Karshenboim et al. (2014)

Relevant to this

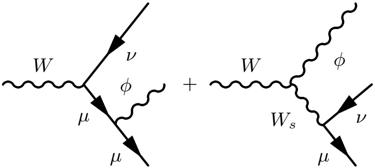
- Reminiscent of (from early days of W.S. model),



- Left diagram grew unpleasantly at high energy, right diagram cancelled it at high energy, was small at lower energy

Here

- Should have interaction also with W to make theory renormalizable.



- Problem ameliorated (see Freid and me, PRD (2015))

Why use G_E in defining the proton radius?

- Proton e.m. current matrix element is

$$\langle p'|J(0)|p\rangle = \bar{u}(p')\left(\gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_p}F_2(Q^2)\right)u(p)$$

- May reorganize Dirac and Pauli FF into electric and magnetic FF

$$G_M = F_1 + F_2 \ ; \quad G_E = F_1 - \frac{Q^2}{4m_p^2}F_2$$

- Can define DIrac radius using derivative of F_1. Why use G_E?
- Answer from considering what atomic spectroscopists see.
- Atomic state energies calculated by first solving Schroedinger or DIrac equation for pointlike proton, and then adding proton structure effect using perturbation theory.

Atomic proton radius effects

- Calculate perturbative term using extra part of proton current,

$$\text{proton current} \to$$
$$\bar{u}(p')\left(\gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_p}F_2(Q^2)\right)u(p) - \bar{u}(p')\gamma_\mu u(p)$$

- Work through and find result $\propto G_E'(Q^2)\big|_{Q^2=0}$
- So the rest of us also quote (or should quote) $R_p = R_E$ to match.