Event generators for Multi Hadronic Production in e^+e^- collisions

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https:

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Outline

- Introduction
- Framework of event generator
- Optimization of LUARLW parameters
- Validation of tuned parameters
- Discussion and summary

Ref. [1] Ronggang Ping, et al., Chin. Phys. C40, 113002(2016). arXiv: 1605.09208
[2] Ronggang Ping, Chin. Phys. C38, 083001 (2014). arXiv: 1309.3932

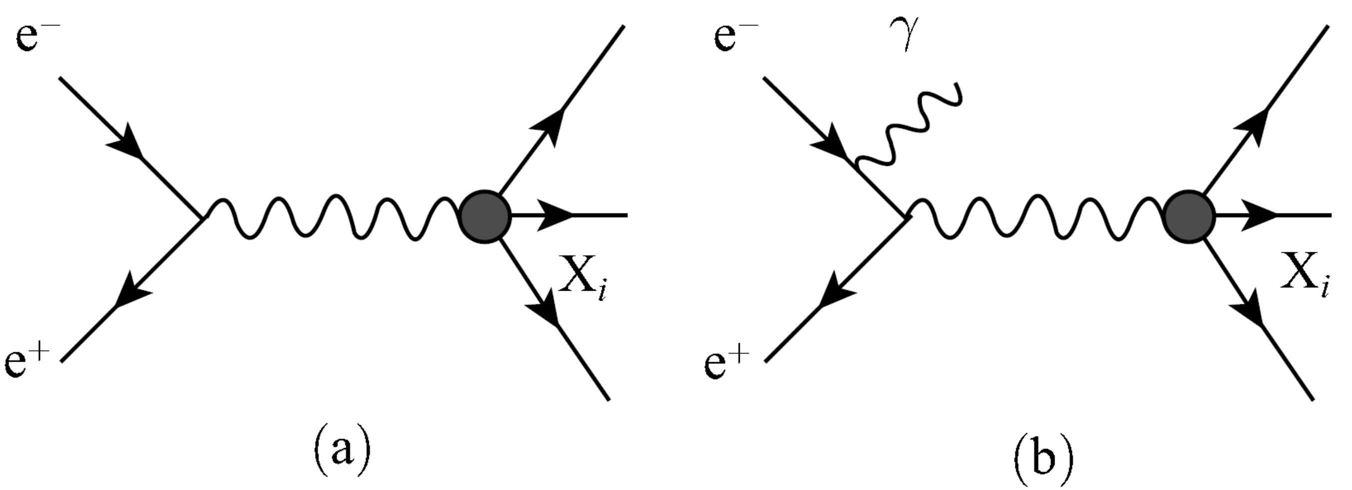
Introduction

- **Light hadron cross sections in e^+e^- collisions is one of fundamental observables in particle physics.**
- **Cross sections for most processes were exclusively measured, and R-values have been measured by many experiments from $M_{2\pi}$ to Z-boson peak, but precision needs to be improved.**
- **High precision generators are essential for cross section and R-value measurements.**
- **To well deal with ISR/FSR effects, vacuum polarization, intermediate states, angular distribution of final states, and so on.**
- **A few exclusive modes are available in PHOKHARA, and for inclusive hadron event generation, Lund Area Law model has been developed to describe the quark hadronization at low energy region.**
- **Parameter optimization of Lund Area Law model is essential but tedious for the work of R-value measurement.**

Framework of event generator

- Constructed as a model of BesEvtGen package, ConExc
- Measured processes (exclusive generator)+ unknown processes (Lund Area Law model)

■ Initial state radiation correction



Observed cross section

$$\sigma(s) = \int_{M_{\mathrm{th}}}^{\sqrt{s}} \mathrm{d}m \frac{2m}{s} W(s,x) \frac{\sigma_0(m)}{|1 - \Pi(m)|^2}, \qquad (1)$$

$\sigma_0(m)$: Born cross section
$\Pi(m)$: vacuum polarization function
$W(s,x)$: radiative function, up to order of α^2

$$W(s,x) = \Delta\beta x^{\beta-1} - \frac{\beta}{2}(2-x) + \frac{\beta^2}{8}\left\{ (2-x)[3\ln(1-x) - 4\ln x] - 4\frac{\ln(1-x)}{x} - 6 + x \right\}, \qquad (2)$$

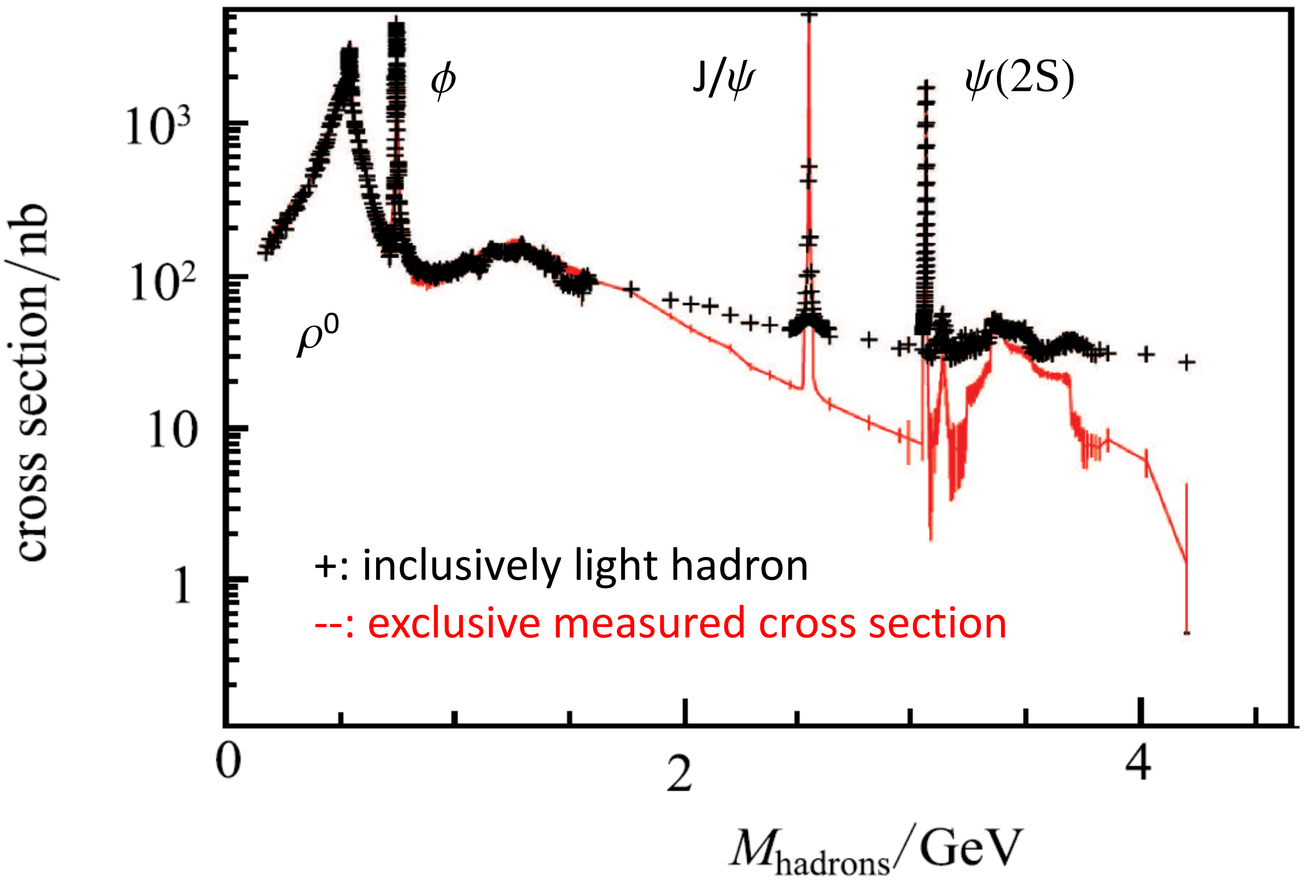
Multi-soft photon

$$L = 2\ln\frac{\sqrt{s}}{m_{\mathrm{e}}},$$

$$\Delta = 1 + \frac{\alpha}{\pi}\left(\frac{3}{2}L + \frac{1}{3}\pi^2 - 2\right) + \left(\frac{\alpha}{\pi}\right)^2 \delta_2,$$

$$\delta_2 = \left(\frac{9}{8} - 2\xi_2\right)L^2 - \left(\frac{45}{16} - \frac{11}{2}\xi_2 - 3\xi_3\right)L - \frac{6}{5}\xi_2^2 - \frac{9}{2}\xi_3 - 6\xi_2\ln 2 + \frac{3}{8}\xi_2 + \frac{57}{12},$$

Born cross section, σ_0(m), for light hadron production in tau-charm energy region



ISR photon energy and angular distributions

Nucl.Phys.B27, 381 (1971)

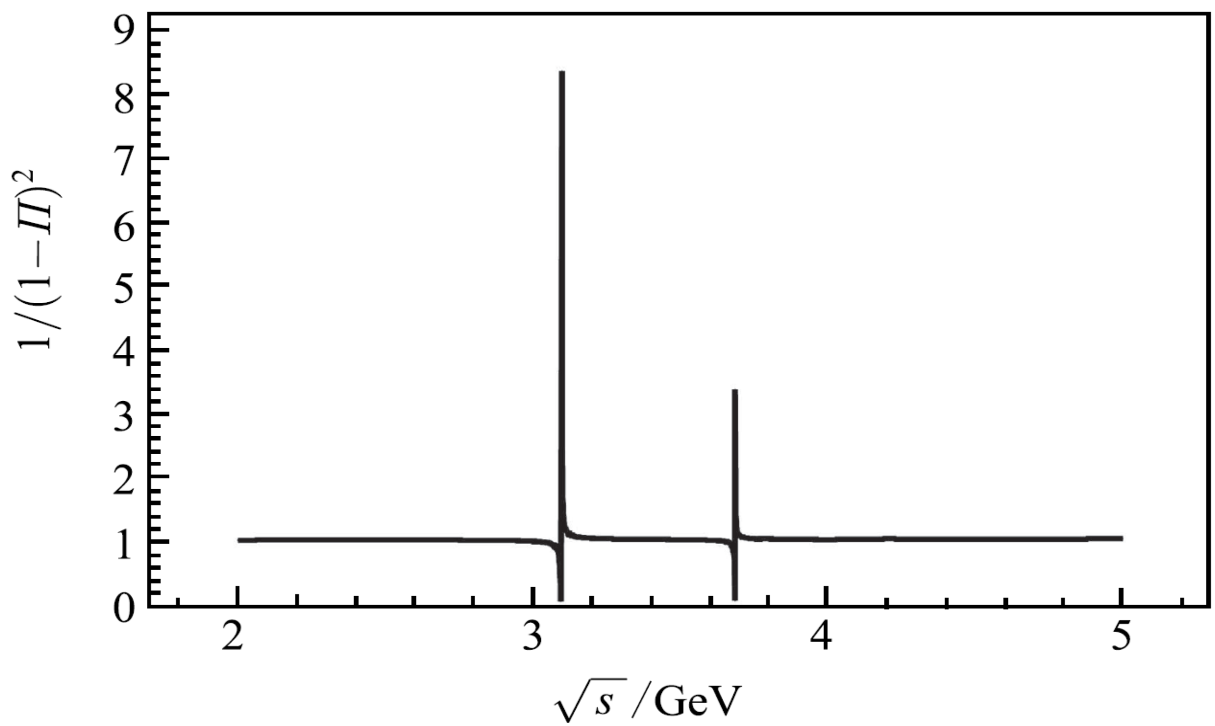
$$\frac{\mathrm{d}\sigma(s,x)}{\mathrm{d}x\mathrm{d}\cos\theta} = \frac{2\alpha}{\pi x}\left(1 - x + \frac{x^2}{2}\right)\sigma_0(s(1-x))P(\theta),$$

$$P(\theta) = \frac{\sin^2\theta - \dfrac{x^2\sin^4\theta}{2(x^2-2x+2)} - \dfrac{m_\mathrm{e}^2}{E^2}\dfrac{(1-2x)\sin^2\theta - x^2\cos^4\theta}{x^2-2x+2}}{\left(\sin^2\theta + \dfrac{m_\mathrm{e}^2}{E^2}\cos^2\theta\right)^2}$$

ISR photon : soft energy and beam collinear

■ Vacuum polarization $\Pi(s)$

- **Essential for R-value measurements in tau-charm energy region**
- **Only consider time-like case**
- **Calculated by many groups, we use HADR5N subroutine, by Fred Jegerlehner group**



Vacuum polarization factor $1/|1-\Pi(s)|^2$

Cross section for exclusive processes

ID	$e^+e^- \to$	$\sqrt{s}$/GeV	Ref.	ID	$e^+e^- \to$	$\sqrt{s}$/GeV	Ref.
1	$p\bar{p}$	1.877—4.500	[15]	39	$\omega\pi^+\pi^-$	1.150—2.525	[17]
2	$n\bar{n}$	1.90—2.44	[16]	40	$\omega f_0(980)$	1.700—2.475	[17]
3	$\Lambda\bar{\Lambda}$	2.23—5.00	[17]	41	$\eta'\pi^+\pi^-$	1.58—3.42	[17]
4	$\Sigma\bar{\Sigma}^0$	2.385—5.000	[17]	42	$f_1(1285)\pi^+\pi^-$	1.66—3.50	[17]
5	$\Lambda\bar{\Sigma}^0$	2.308—5.000	[17]	43	ωK^+K^-	1.57—3.45	[17]
6	$\Sigma^0\bar{\Lambda}$	2.308—5.000	[17]	44	$\omega\pi^+\pi^-\pi^0$	1.500—4.423	[21]
7	$\pi^+\pi^-$	0.305—2.950	[18]	45	$\Sigma^-\bar{\Sigma}^+$	2.308—5.000	[17]
8	$\pi^+\pi^-\pi^0$	1.063—2.989	[19]	46	K^+K^-	1.009—4.170	[27, 29]
9	$K^+K^-\pi^0$	1.34—4.68	[20]	47	$K_S K_L$	1.004—2.140	[27]
10	$K_S K^+\pi^-$	1.26—4.66	[20]	48	$\omega\eta$	1.371—3.178	[26]
11	$K_S K^-\pi^+$	1.26—4.66	[20]	49	$p\bar{p}\pi^0$	4.009—4.200	[28]
12	$K^+K^-\eta$	1.69—3.13	[20]	50	$p\bar{p}\eta$	4.009—4.200	[28]
13	$2(\pi^+\pi^-)$	0.615—4.45	[21]	51	$D^-D^{*0}\pi^+$	4.020 -5.171	[33]
14	$\pi^+\pi^-2\pi^0$	0.185—2.98	[22]	52	$D^+D^{*0}\pi^-$	4.020—5.171	[33]
15	$K^+K^-\pi^+\pi^-$	1.425—4.988	[23]	53	$D^{*0}\text{-}D^{*0}$	4.033—4.991	[31]
16	$K^+K^-2\pi^0$	1.50— 4.02	[23]	54	$D^0\bar{D}^{*0}$	4.033—4.991	[30]
17	$2(K^+K^-)$	2.02—4.54	[24]	55	$\bar{D}^0 D^{*0}$	3.814—4.990	[30]
18	$2(\pi^+\pi^-)\pi^0$	1.013—4.488	[25]	56	$D^0\bar{D}^0$	3.814—4.990	[30]
19	$2(\pi^+\pi^-)\eta$	1.313—4.488	[25]	57	D^+D^-	3.814—4.990	[30]
20	$K^+K^-\pi^+\pi^-\pi^0$	1.613—4.488	[25]	58	D^+D^{*-}	3.890—4.994	[31]
21	$K^+K^-\pi^+\pi^-\eta$	2.113—4.488	[25]	59	D^-D^{*+}	3.890—4.994	[31]

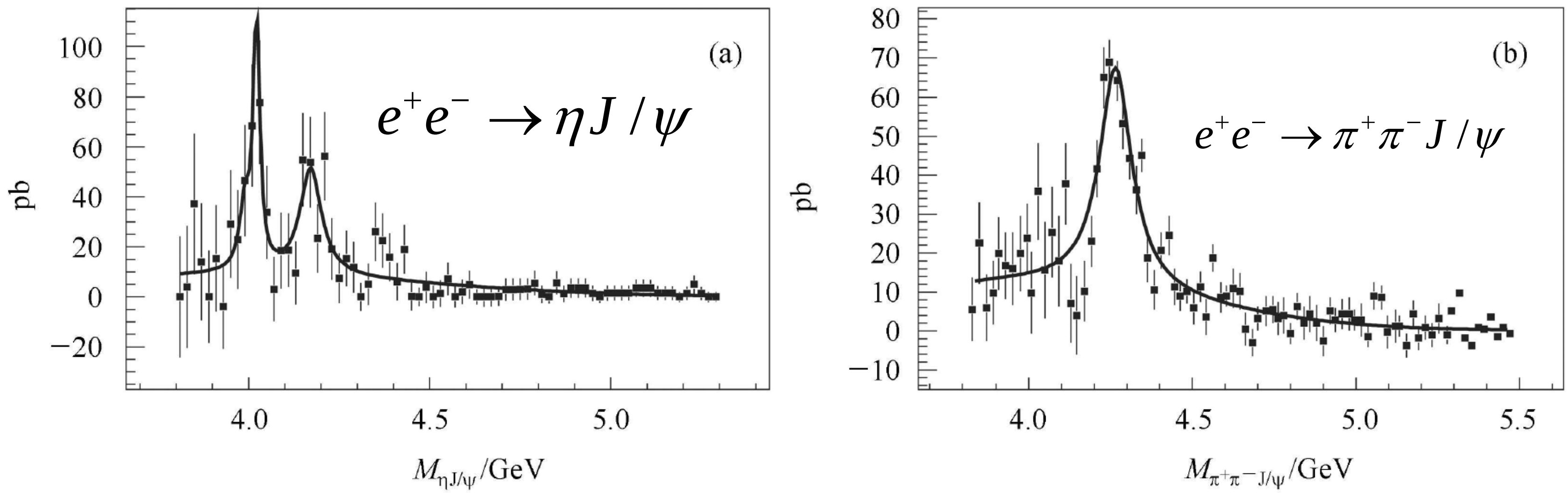
Table continued

22	$3(\pi^+\pi^-)$	1.313—4.488	[26]	60	$D^{*+}D^{*-}$	4.033—4.991	[31]
23	$2(\pi^+\pi^-\pi^0)$	1.313—4.488	[26]	61	$D^0D^-\pi^+$	4.015—4.974	[32]
24	$\phi\eta$	1.57—3.45	[20]	62	$\bar{D}^0D^+\pi^-$	4.015—4.974	[32]
25	$\phi\pi^0$	1.25—1.45	[20]	63	$D^0D^{*-}\pi^+$	4.020—5.171	[33]
26	K^+K^{*-}	1.37—1.99	[20]	64	$\bar{D}^0D^{*+}\pi^-$	4.020—5.171	[33]
27	K^-K^{*+}	1.37—1.99	[20]	65	$\psi(2S)\pi^0\pi^0$	4.127—5.480	[37]
28	$K_S\bar{K}^{*0}(892)$	1.37— 1.99	[20]	66	$\eta J/\psi$	3.81—4.68	[35]
29	$K^*(892)^0K^+\pi^-$	1.588—3.963	[23]	67	$\pi^+\pi^-h_c$	4.009—4.420	[38]
30	$K^*(892)^0K^-\pi^+$	1.588—3.963	[23]	68	$\pi^0\pi^0h_c$	4.009—4.420	[39]
31	$K^*(892)^-K^+\pi^0$	1.588—3.963	[23]	69	K^+K^-J/ψ	4.179—4.970	[41]
32	$K^*(892)^+K^-\pi^0$	1.588—3.963	[23]	70	$K_S^0K_S^0J/\psi$	4.179—4.970	[40]
33	$K_2^*(1430)^0K^+\pi^-$	2.348—3.965	[23]	71	$J/\psi\pi^+\pi^-$	3.829—5.471	[36]
34	$K_2^*(1430)^0K^-\pi^+$	2.348—3.965	[23]	72	$\psi(2S)\pi^+\pi^-$	4.127—5.480	[37]
35	$K^+K^-\rho$	1.777—3.830	[23]	73	$D_s^+D_s^-$	3.97—4.26	[42]
36	$\phi\pi^+\pi^-$	1.488—2.863	[23]	74	$D_s^{*+}D_s^-$	4.12—4.26	[42]
37	$\phi f_0(980)$	1.888—2.963	[23]	75	$D_s^{*-}D_s^+$	4.12—4.26	[42]
38	$\eta\pi^+\pi^-$	1.025—2.975	[17]	76	$\Lambda_c^+\Lambda_c^-$	4.57—4.64	[34]

For vector resonances, $\psi(3770)$, $\psi(2S)$, J/ψ, $\rho(1770)^0$, and $\omega(1420)$, cross section is taken

$$\sigma_{\rm BW}(s) = 12\pi\frac{\gamma_{\rm ee}\gamma}{(s-M^2)+M^2\gamma^2},$$

- Measured cross sections are parametrized with multi-Gaussian functions, eg.

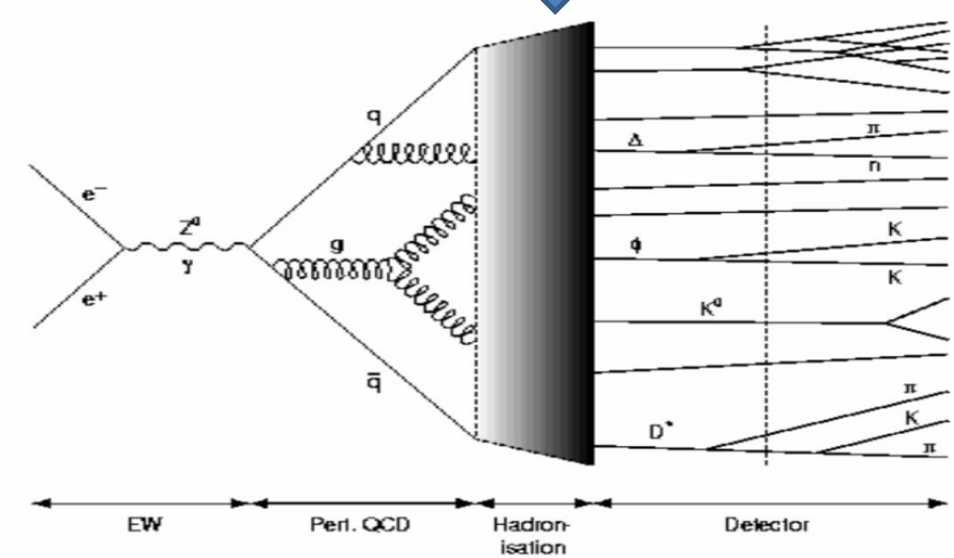


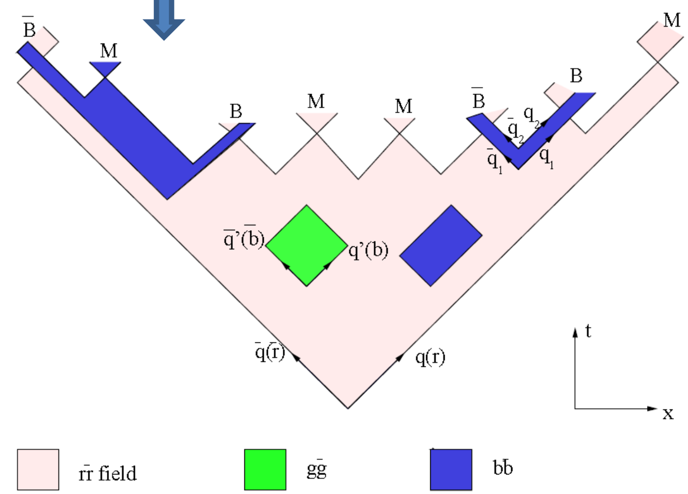
- Multi-body decays are generated with PHSP model
- Two-body decays of angular distributions are generated according to $1+\alpha\cos^2\theta$ in the helicity system of two-body CMS with

$$a = \begin{cases} 1 \quad \text{for VP mode} \\ \text{-1 for PP mode} \\ 0 \sim 1 \text{ for } B\bar{B} \text{ mode, quark model prediction} \end{cases}$$

- FSR effects are considered with PHOTOS package.

■ LUND Area Law (LUARLW) model





- In high energy region, available models, e.g. cluster model in HERWIG, LUND model in JETSET/PYTHIA
- In tau-charm energy region, LUARLW is designed to describe multiplicity distribution of primary hadrons, [arXiv: hep-ph/9910285]

$$P_n = \frac{\mu^n}{n!}\exp[c_0 + c_1(n-\mu) + c_2(n-\mu)^2] \quad \text{with } \mu = \alpha + \beta\exp(\gamma\sqrt{s})$$

Parameters to be tuned: $c_0, c_1, c_2, \alpha, \beta, \gamma$

Monte-Carlo algorithm

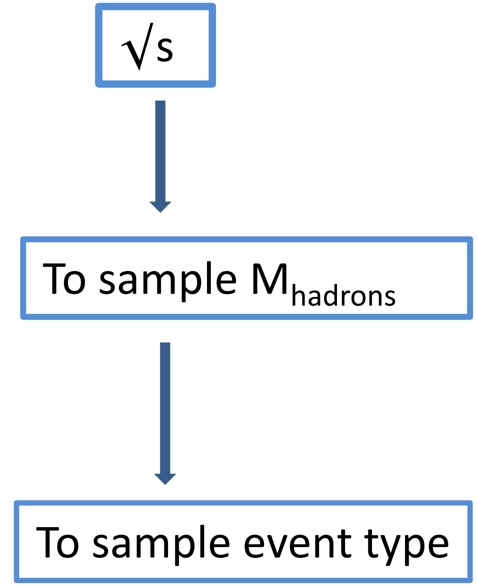
- Sampling of $M_{hadrons}$

$$\sigma(s) \equiv \sigma^{I}(s) + \sigma^{II}(s)$$

$$\sigma^{I}(s) = \int_{M_{\rm th}}^{M_0} \mathrm{d}m \frac{2m}{s} W(s,x) \frac{\sigma_0(m)}{|1-\Pi(m)|^2}$$

$$\sigma^{II}(s) = \int_{M_0}^{\sqrt{s}} \mathrm{d}m \frac{2m}{s} W(s,x) \frac{\sigma_0(m)}{|1-\Pi(m)|^2},$$

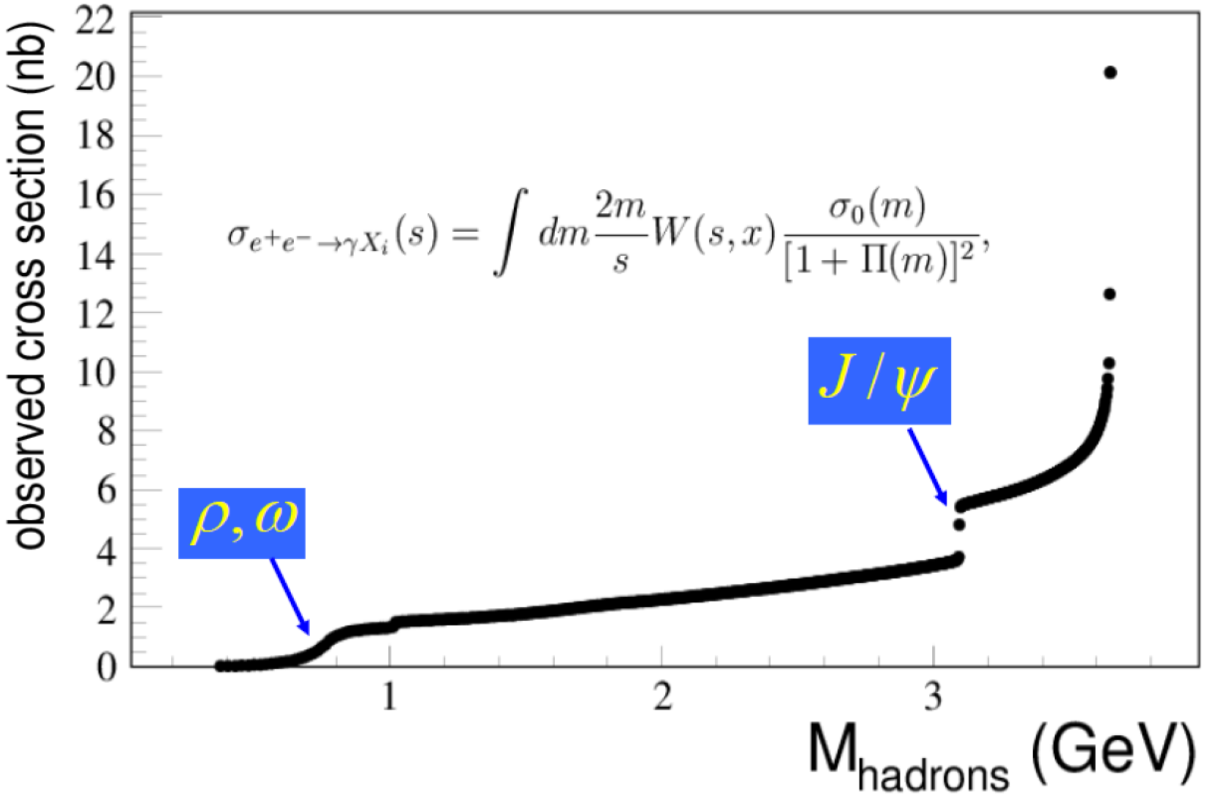
with $M_0 = \sqrt{s - 2\sqrt{s}E_\gamma^{\rm cut}}$ $\quad E_\gamma^{\rm cut} = 1\ \mathrm{MeV}$



The ISR returned energy for the σ^{I}(s) events is imposed by one ISR photon
The ISR photons for the σ^{II}(s) events are not produced, since below detector sensitivity

MC technique: to discretize the $\sigma(s)$

$$\hat{\sigma}(m_i) = \frac{1}{\sigma(s)} \int_{M_{\rm th}}^{m_i} \mathrm{d}m \frac{2m}{s} W(s,x) \frac{\sigma_0(m)}{|1-\Pi(m)|^2}$$



For example

To sample M_{hadrons} at $\sqrt{s}$= 3.65 GeV

- Sampling of event type

Exclusive process *m,* is sampled according to the probability

$$c_m = \sigma_m(M_{\text{hadrons}})/\sigma^{\text{tot}}(M_{\text{hadrons}})$$

Remainder probability, $1-\Sigma\, c_m$, is set for LUNARLW model.

Optimization of LUARLW parameters

- Response function method
  - **Borrowed the idea from generator tuning tool: Professor and River, introduced by TASSO, later used by ALEPH, DELPHI, and recently by LHC.**
  - **To simultaneously fit data with the response function:**

$$
\begin{aligned}
f(\boldsymbol{p}_0+\delta\boldsymbol{p},x) &= a_0^{(0)}(x)+\sum_{i=1}^{n} a_i^{(1)}(x)\delta p_i \\
&\quad + \sum_{i=1}^{n}\sum_{j=i}^{n} a_{ij}^{(2)}(x)\delta p_i \delta p_j \\
&\approx MC(\boldsymbol{p}_0+\delta\boldsymbol{p},x),
\end{aligned}
$$

  - **The parameters, $\boldsymbol{a}_{\mathbf{0}}^{(\mathbf{0})}(\boldsymbol{x})$, $\boldsymbol{a}_{\boldsymbol{i}}^{(\mathbf{1})}(\boldsymbol{x})$, $\boldsymbol{a}_{\boldsymbol{ij}}^{(\mathbf{2})}(\boldsymbol{x})$, are determined with MC sample ensemble *L***

$$L=1+n+n(n+1)/2,$$

- Parameters, p_0, the error δp_0, and the correlation coefficients ρ_{ij} are determined with package MINUIT
- Distributions sensitive to parameter tuning

$$S_i(x) = \frac{\delta MC(x)}{MC(x)}\Big|_{p_i} \Big/ \frac{\delta p_i}{p_i} \approx \frac{\partial \ln MC(x)}{\partial \ln |p_i|}\Big|_{p_i}$$

Azimuthal distributions of charged tracks are insensitive to parameter tuning

12 distributions are included in the simultaneous fit: multiplicity of photons and charged tracks, momentum of charged tracks, x_f=2P_z/W, $x_\perp$= 2$P_\perp$/W, shericity, aplarinarity, and Fox-Wolfam moments (H_{20},H_{30},H_{40}) .

- To consider some special requirements on the fit quality of charged multiplicity, it is weighted with factor of 10 in the χ^2 objective function.

■ Event selection and optimization results

- Data set taken at 3.65 GeV used to optimize the LUARLW parameters

- Event selection criteria are the same as applied to the R-value measurements

- QED backgrounds are subtracted with MC simulation, e.g. $e^+e^- \to l^+l^-, \gamma\gamma, \gamma^*\gamma^*$.

- To check possible correlations among the selected distributions , different combinations are tried.

- Strong correlation among the tuned parameters are seen in the covariant matrix.

Optimization I : Exclusive processes + LUNARLW

Optimized parameters at $\sqrt{s}$ =3.65 GeV. The statistical errors are negligible. $^{(2S+1)}P_J$ denotes a meson has spin S, orbital angular momentum (L) and total spin J.

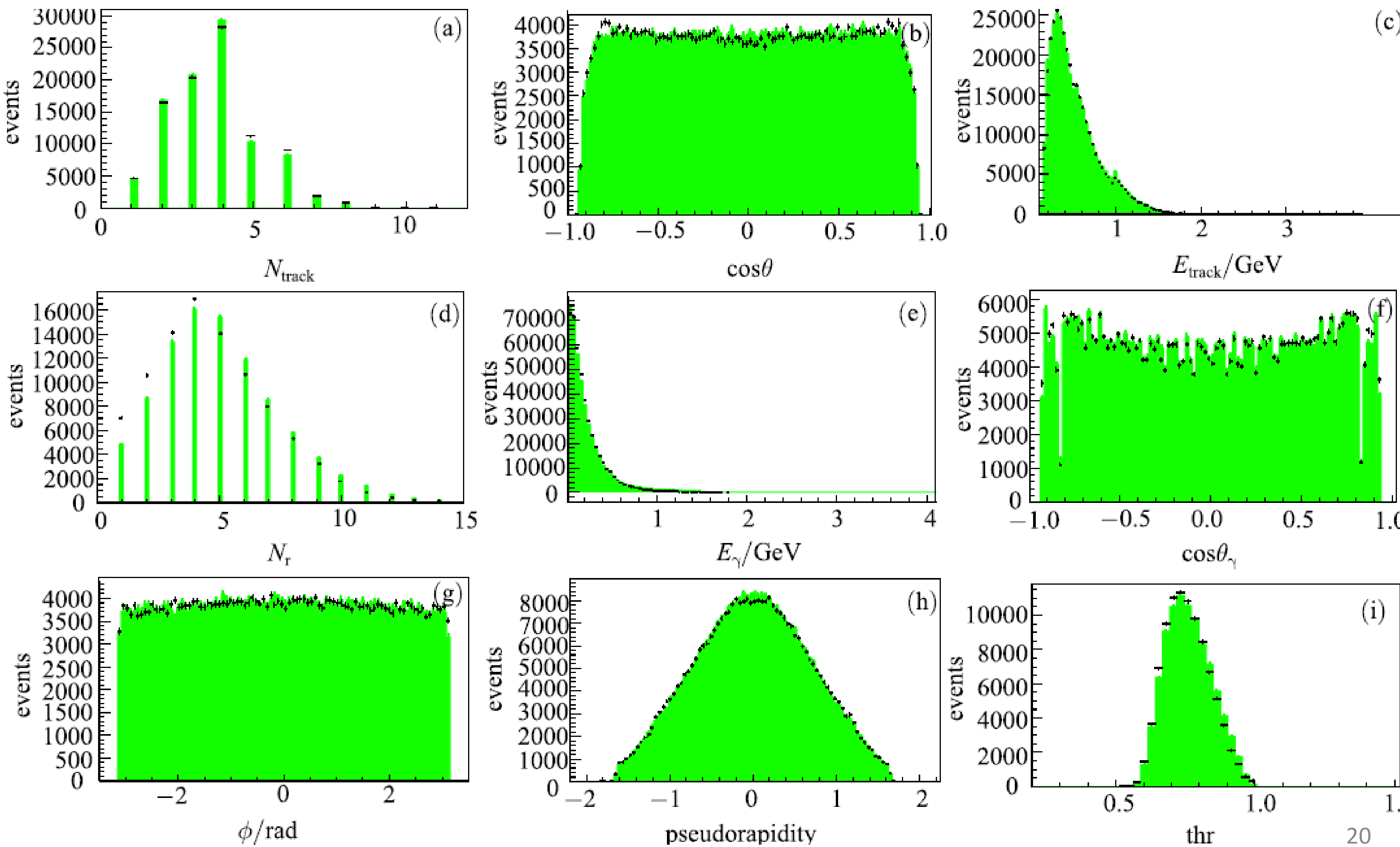
parameters	tuned	description
PARJ(1)	0.065	suppression of diquark-antidiquark pair production
PARJ(2)	0.260	suppression of s quark pair production
PARJ(11)	0.612	probability that a light meson has spin 1
PARJ(12)	0.000	probability that a strange meson has spin 1
PARJ(14)	0.244	probability for a $^1P_1$ meson production
PARJ(15)	0.000	probability for a $^3P_0$ meson production
PARJ(16)	0.437	probability for a $^3P_1$ meson production
PARJ(17)	0.531	probability for a $^3P_2$ meson production
PARJ(21)	0.066	width of Gaussian for transverse momentum
RALPA(15)	0.577	LUARLW model parameter
RALPA(16)	0.364	LUARLW model parameter
RALPA(17)	0.000	LUARLW model parameter

Optimization II : optimization using data at 3.08GeV Phokhara (10 modes)+ other exclusive processes + LUNARLW

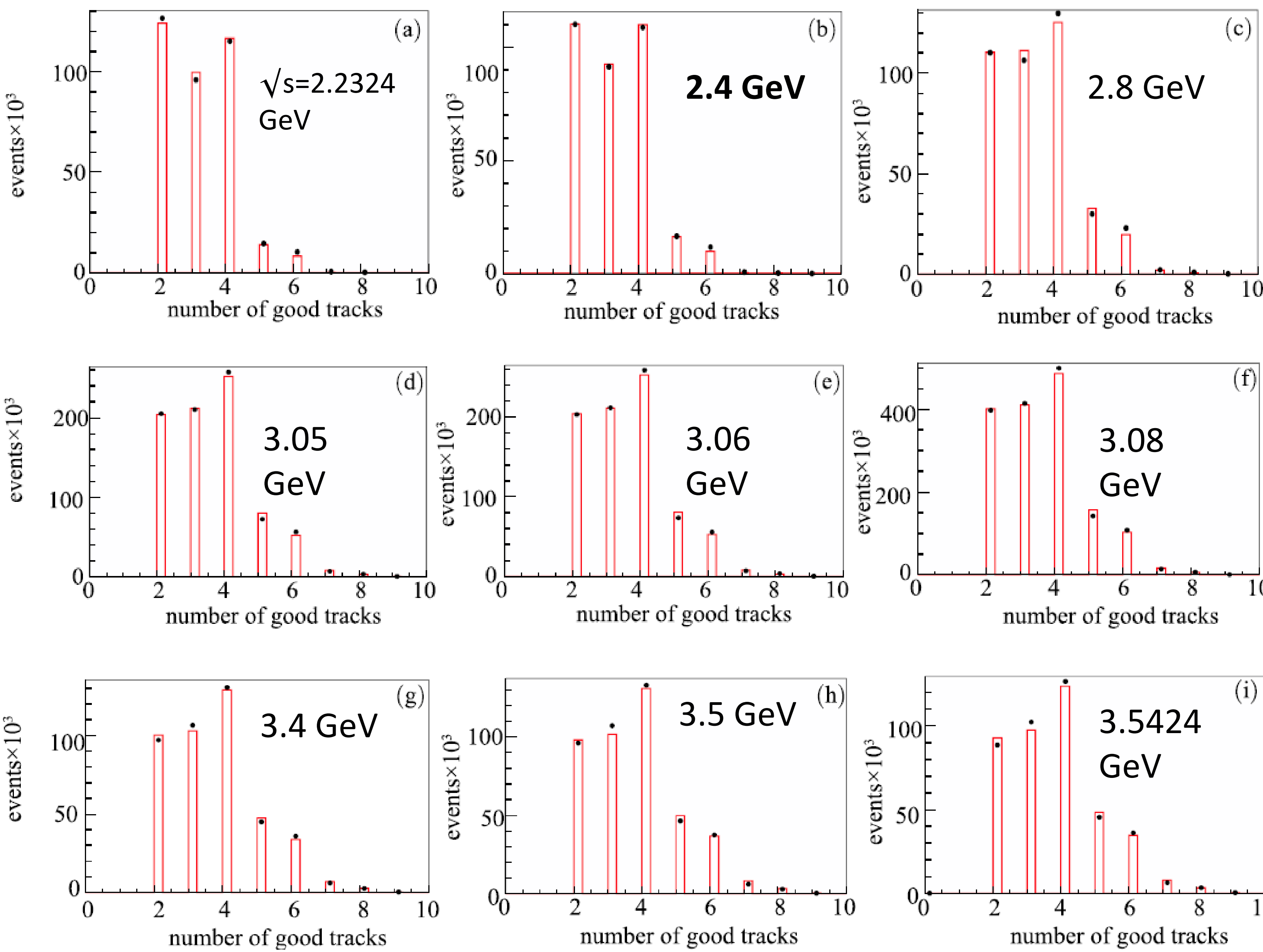
Parameters	optimized	Description
PARJ(1)	0.118	P(qq)/P(q)
PARJ(2)	0.670	P(ss)/P(uu)
PARJ(11)	0.868	V/P ratio of u- and d-quarks
PARJ(12)	0.644	V/P ratio of s-qaruk
PARJ(14)	0.188	axial vector meson ratio
PARJ(15)	0.232	scalar meson ratio
PARJ(16)	0.518	another axial vector meson
PARJ(17)	0.320	tensor meson
PARJ(21)	0.201	σ, width of Gaussian
RALPA(67)	0.191	LUNDA model parameter
RALPA(16)	1.000	LUNDA model parameter
RALPA(17)	-0.537	LUNDA model parameter

Validation of optimized parameters

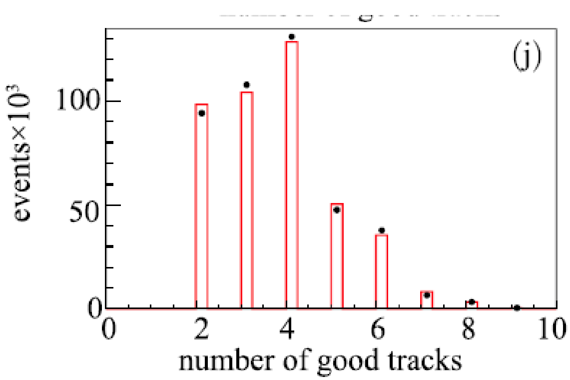
- optimization I: Comparisons between MC and data taken at 3.65 GeV



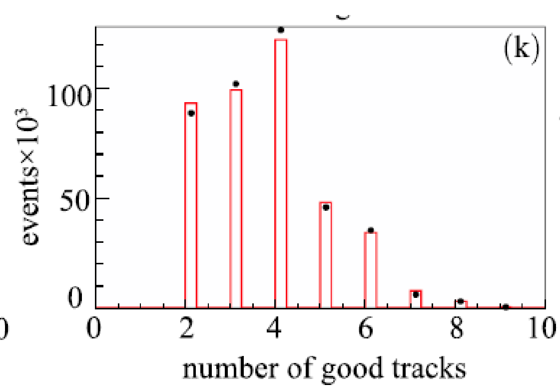
Comparisons with data below DDbar threshold. Dots: data, red bar: MC



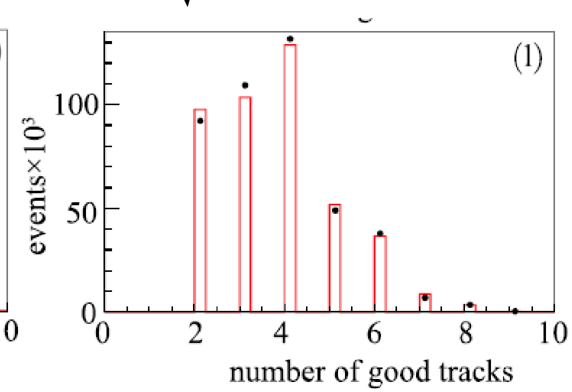
√s=3.5538 GeV



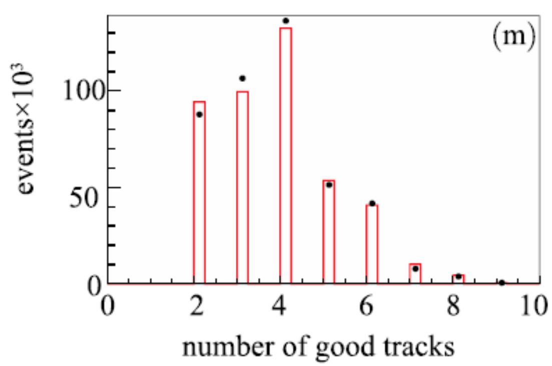
√s=3.5611 GeV



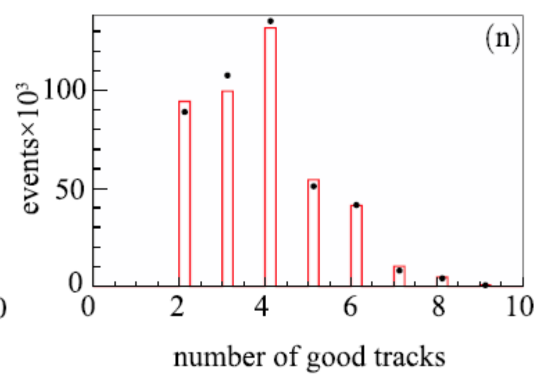
√s=3.6002 GeV



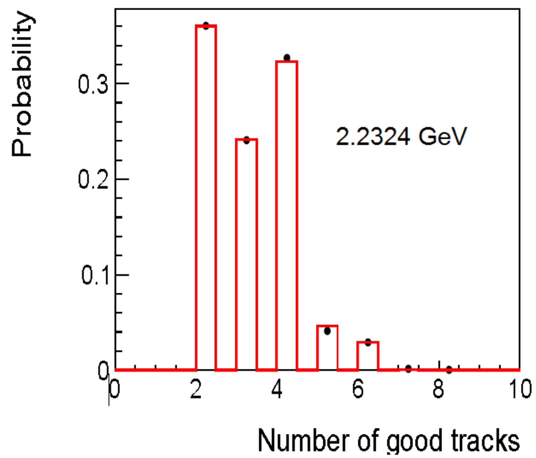
√s=3.65 GeV

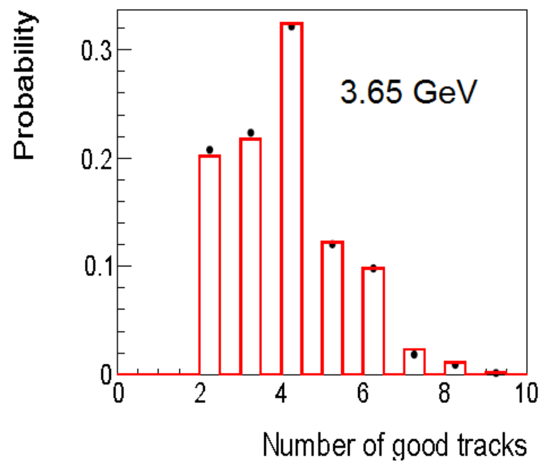


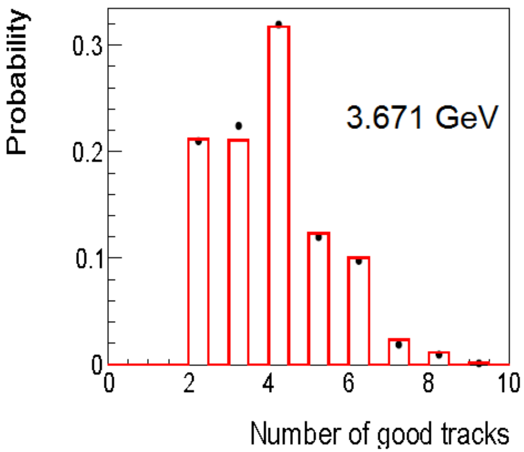
√s=3.671 GeV



optimization II







Comparison of optimizations, where $\Delta^2 = \sum_J (p^J_{MC} - p^J_{data})^2$ sums over data sets taken at 2.2324, 2.4, 2.8, 3.05, 3.06, 3.08, 3.4, 3.5, 3.5424, 3.5538, 3.5611, 3.6, 3.65 and 3.671 GeV

Distributions	N_{trk}	$\cos\theta$	P_{trk}
Opti. I	Δ^2=15.5	Δ^2=1.7	Δ^2=14.0
Opti. II	Δ^2=6.0	Δ^2=1.1	Δ^2=6.4

Discussion and summary

- We developed an event generator for multi-hadron production for R-value measurements
- Events are generated with the scheme of "exclusive + inclusive" processes.
- Parameters for inclusive generation are optimized below DDbar threshold.
- Inclusion of PHOKHARA modes is helpful to describe data