

# Measurement of R at KEDR

PHIPSI: INTERNATIONAL WORKSHOP  
on  $e^+e^-$  collisions from Phi to Psi 2017

Phi<sup>17</sup>  
Psi

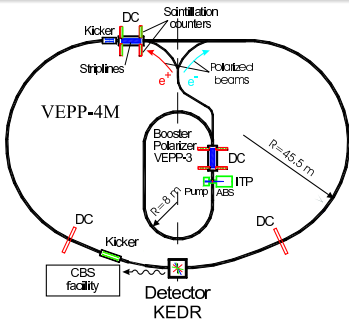
*Korneliy Todyshev*

**KEDR collaboration**

26-29 June 2017  
Mainz, Germany



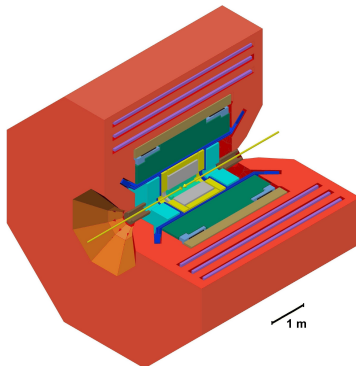
# VEPP-4M and KEDR



Beam energy	$1 \div 5\text{ GeV}$
Number of bunches	$2 \times 2$
Luminosity 1.8 GeV	$1.5 \times 10^{30}\text{ cm}^{-2}\text{ s}^{-1}$

Energy measurement:

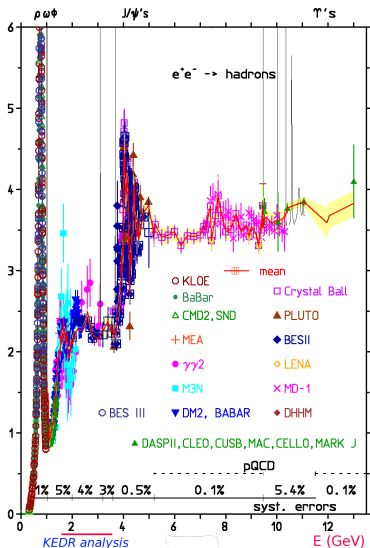
- Resonant depolarization method:
  - Instant measurement accuracy  $\sim 1\text{ keV}$
  - Energy interpolation accuracy  $10 \div 30\text{ keV}$
- Compton backscattering method  $\sim 100\text{ keV}$



- Vertex detector
- Drift chamber
- Aerogel threshold counters
- ToF counters
- Lkr calorimeter
- Superconducting coil
- Yoke
- Muon chambers
- CsI calorimeter
- Compensating solenoid

1 m

# $R(s)$ measurement



$$R = \frac{\sigma(e^-e^+ \rightarrow \text{hadrons})}{\sigma(e^-e^+ \rightarrow \mu^-\mu^+)} \approx \frac{\text{Diagram 1}}{\text{Diagram 2}}$$

In first approximation:

$$R(s) \simeq 3 \sum e_q^2$$

$R(s)$  is used to determine:

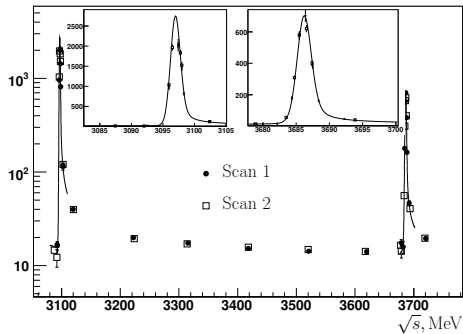
- $\alpha_s(s)$
- $(g_\mu - 2)/2$
- $\alpha(M_Z^2)$

F. Jegerlehner arXiv:1511.0447



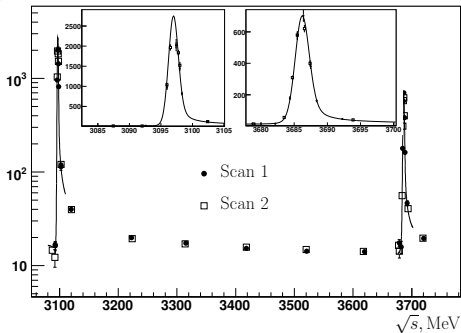
# R measurement between $J/\psi$ and $\psi(2S)$

$\sigma_{mh}^{obs}$ , nb



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$\sigma_{mh}^{obs}$ , nb



KEDR collaboration presented

poster by *Tatiana Kharlamova*

Measurement of  $\Gamma_{ee} \times \mathcal{B}_{hadrons}(J/\psi)$

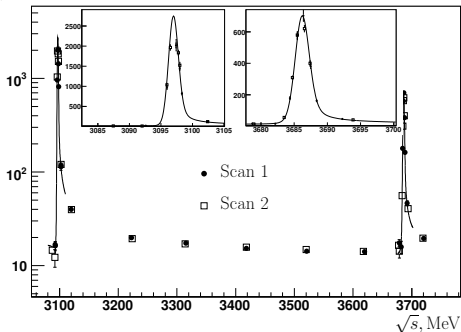
poster by *Andrey Sukharev*

Measurement of  $\Gamma_{ee} \times \mathcal{B}_{\mu\mu}(\psi(2S))$



# R measurement between $J/\psi$ and $\psi(2S)$

$\sigma_{mh}^{obs}$ , nb



KEDR collaboration presented

poster by *Tatiana Kharlamova*

Measurement of  $\Gamma_{ee} \times \mathcal{B}_{hadrons}(J/\psi)$

poster by *Andrey Sukharev*

Measurement of  $\Gamma_{ee} \times \mathcal{B}_{\mu\mu}(\psi(2S))$

- The c.m. energy range between 3.12 and 3.72 GeV studied
- An integrated luminosity of  $1.4 \text{ pb}^{-1}$  collected at 7 equidistant points with a step of  $\sim 0.1 \text{ GeV}$ : 3.12, 3.22, . . . , 3.72 GeV
- $(2 - 3) \cdot 10^3$  events per point,  $\sim 18 \cdot 10^3$  in total
- Simulation of the  $uds$  continuum based on the tuned JETSET generator, alternatively used LUARLW (H.M. Hu and A. Tai, hep-ex/0106017)



The way that we are measuring  $R$ :

$$R = \frac{\sigma_{obs}(s) - \sum \varepsilon_{\psi}^{tail}(s)\sigma_{\psi}^{tail}(s) - \sum \varepsilon_{bg}^i(s)\sigma_{bg}^i(s)}{\varepsilon(s)(1 + \delta(s))\sigma_{\mu\mu}^0} \quad (1)$$

with  $\sigma_{obs}(s) = \frac{N_{mh} - N_{res.bg.}}{\int \mathcal{L} dt}$  where  $N_{mh}$  represent all events pass hadronic selection criteria,  $N_{res.bg.}$  – residual machine background

$\sum \varepsilon_{\psi}^{tail}(s)\sigma_{\psi}^{tail}(s)$  is contribution from  $J/\psi$  and  $\psi(2S)$  resonances

$\sum \varepsilon_{bg}^i(s)\sigma_{bg}^i(s)$  is contribution from physical processes:  $e^+e^- \rightarrow l^+l^-$ ,  $\gamma\gamma$ -processes.

$\varepsilon(s)$  – multihadron efficiency.

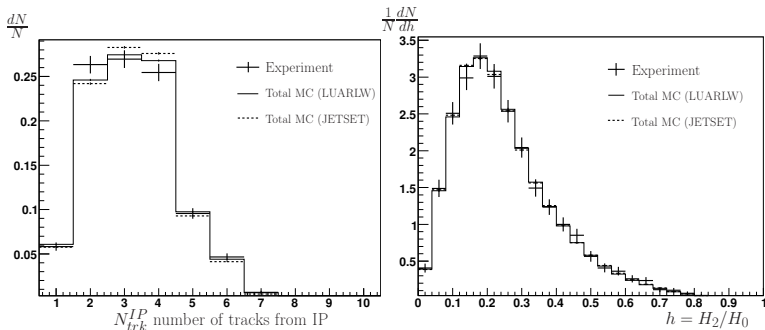
$$1 + \delta(s) = \int dx \frac{1}{1-x} \frac{\mathcal{F}(s,x)}{|1 - \tilde{\Pi}(s(1-x))|^2} \frac{\tilde{R}(s(1-x))\varepsilon(s(1-x))}{R(s)\varepsilon(s)} \quad (2)$$

$\mathcal{F}(s,x)$  – radiative correction kernel (E.A.Kuraev, V.S.Fadin

Sov. J. Nucl. Phys. 41(466-472)1985) Here  $\tilde{\Pi}$  and  $\tilde{R}$  does not includes  $J/\psi$  and  $\psi(2S)$  resonances.



# Simulation: JETSET and LUARLW



Number of tracks and ratio of Fox-Wolfram moments

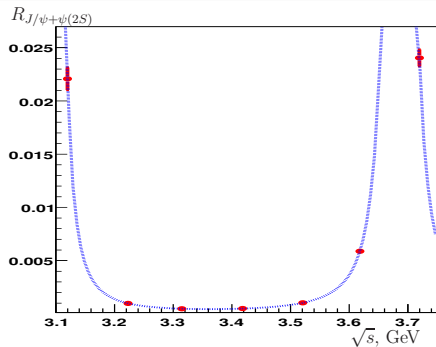
Experimental distribution and two variants of MC simulation based on LUARLW and tuned JETSET are plotted ( $\sqrt{s} = 3.12$  GeV).





# R for $\sqrt{s} = 3.12 - 3.72$ GeV

Source	Syst. uncertainty, %
Luminosity	1.1
Rad. corr.	0.4 ÷ 0.6
<i>uds</i> simulation	1.4 ÷ 2.1
$J/\psi$	0.1 ÷ 2.7
$\psi(2S)$	1.4 at 3.72 GeV
$l^+l^-$	0.1 ÷ 0.2
$e^+e^-X$	0.1 ÷ 0.2
Trigger	0.2
Nuclear interaction	0.2
Machine background	0.7 ÷ 1.1
Cuts	0.6
Total	2.1 ÷ 3.5



Using  $J/\psi$  and  $\psi(2S)$  parameters, we obtain  $R_{uds}(s) + R_{J/\psi+\psi(2S)} \Rightarrow R(s)$

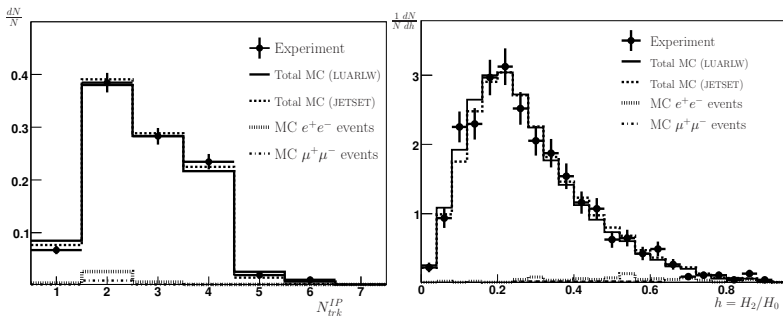
$\sqrt{s}$ , MeV	$R_{uds}(s)$	$R(s)$
$3119.9 \pm 0.2$	$2.215 \pm 0.089 \pm 0.066$	$2.237 \pm 0.089 \pm 0.066$
$3223.0 \pm 0.6$	$2.172 \pm 0.057 \pm 0.045$	$2.173 \pm 0.057 \pm 0.045$
$3314.7 \pm 0.7$	$2.200 \pm 0.056 \pm 0.043$	$2.200 \pm 0.056 \pm 0.043$
$3418.2 \pm 0.2$	$2.168 \pm 0.050 \pm 0.042$	$2.168 \pm 0.050 \pm 0.042$
$3520.8 \pm 0.4$	$2.200 \pm 0.050 \pm 0.044$	$2.201 \pm 0.050 \pm 0.044$
$3618.2 \pm 1.0$	$2.201 \pm 0.059 \pm 0.044$	$2.207 \pm 0.059 \pm 0.044$
$3719.4 \pm 0.7$	$2.187 \pm 0.068 \pm 0.060$	$2.211 \pm 0.068 \pm 0.060$

V.V. Anashin et al., Phys.Lett. B 753, 533 (2016)



# R for $\sqrt{s} = 1.84 - 3.05$ GeV

- An integrated luminosity  $0.66 \text{ pb}^{-1}$  collected at 13 equidistant points with a step  $\sim 0.1$  GeV: 1.841, 1.937 ... 3.048 GeV
- $\sim 10^3$  hadronic events per point,  $14.8 \cdot 10^3$  events in total
- Simulation of the  $uds$  continuum based on the LUARLW generator, tuned JETSET alternatively used at 6 points for a cross-check.



Number of tracks and ratio of Fox-Wolfram moments.

Experimental distribution and two variants of MC simulation based on LUARLW and tuned JETSET are plotted ( $\sqrt{s} = 2.14$  GeV).



# R for $\sqrt{s} = 1.84 - 3.05$ GeV

$$\text{Measured value of } R = \frac{\sigma_{\text{obs}}(s) - \sum \varepsilon_{\text{bg}}^i(s) \sigma_{\text{bg}}^i(s)}{\varepsilon(s)(1 + \delta(s)) \sigma_{\mu\mu}^0}$$

The main systematic uncertainties in the  $R$ :

Point	$\sqrt{s}$ , MeV	$R(s)$
1	1841.0	$2.226 \pm 0.139 \pm 0.158$
2	1937.0	$2.141 \pm 0.081 \pm 0.073$
3	2037.3	$2.238 \pm 0.068 \pm 0.072$
4	2135.7	$2.275 \pm 0.072 \pm 0.055$
5	2239.2	$2.208 \pm 0.069 \pm 0.053$
6	2339.5	$2.194 \pm 0.064 \pm 0.048$
7	2444.1	$2.175 \pm 0.067 \pm 0.048$
8	2542.6	$2.222 \pm 0.070 \pm 0.047$
9	2644.8	$2.220 \pm 0.069 \pm 0.049$
10	2744.6	$2.269 \pm 0.065 \pm 0.050$
11	2849.7	$2.223 \pm 0.065 \pm 0.047$
12	2948.9	$2.234 \pm 0.064 \pm 0.051$
13	3048.1	$2.278 \pm 0.075 \pm 0.048$

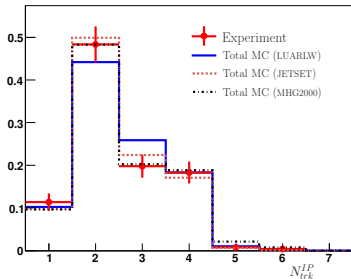
Source	Error, %
Luminosity	1.2
Rad. corr.	$0.5 \div 2.0$
$uds$ simulation	$1.2 \div 6.6$
$J^+ J^-$	$0.3 \div 0.6$
$e^+ e^- X$	0.2
Trigger	0.3
Nuclear interaction	0.4
Machine background	$0.4 \div 0.9$
Cuts	0.7
Total	$2.1 \div 7.1$

V.V. Anashin et al., Phys.Lett. B 770C, 174 (2017)



# Detection efficiency uncertainty

- Used two essentially different MC generators (LUARLW and tuned JETSET)
- We validated our estimate of the systematic uncertainty related to simulation of the  $uds$  continuum using an unfolding method (Chinise Physics C Vol. 37, No. 6 (2013) 063001).
- The estimate at the most problematic energy point 1.84 GeV was additionally verified using the exclusive generator MHG2000.

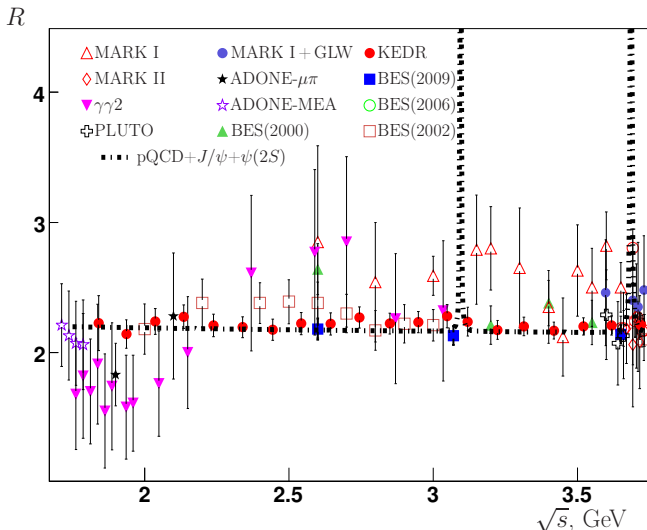
 $\frac{dN}{N}$ 

Detection efficiency uncertainties obtained by different methods

	$\delta\epsilon/\epsilon$		
	LUARLW/JETSET	Unfolding method	LUARLW/MHG2000
<b>point 1</b>	<b>6.6%</b>	<b>3.6%</b>	<b>3.8%</b>
<b>point 2-3</b>	<b>2.5%</b>	<b>1.9%</b>	–
<b>point 4-13</b>	<b>1.2%</b>	<b>0.5%</b>	–



# Comparison with others experiments



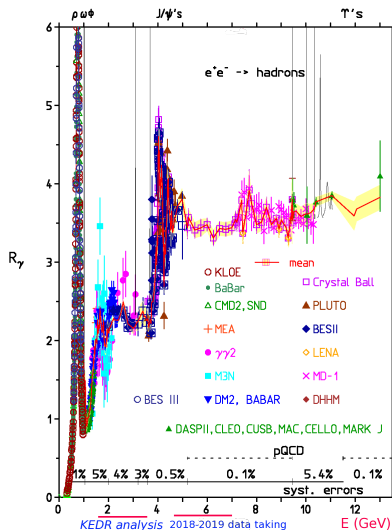
The quantity  $R$  versus the c.m. energy and the sum of the prediction of perturbative QCD and a contribution of narrow resonances.



# Conclusion

- We have determined the values of  $R$  at thirteen points of the center-of-mass energy between 1.84 and 3.05 GeV. The achieved accuracy is about or better than 3.9% at most of energy points with a systematic uncertainty less than 2.4%.
- We measured the values of  $R$  at seven points of the center-of-mass energy between 3.12 and 3.72 GeV. The total achieved accuracy is about or better than 3.3% at most of energy points with a systematic uncertainty of about 2.1%.
- We have taken data in the energy range 3.08 – 3.72 GeV after detector upgrade. An integrated luminosity of 1.4  $\text{pb}^{-1}$  collected at 8 points. Analysis in progress.





F. Jegerlehner arXiv:1511.0447

- We plan to measure  $R$  value in the energy range 5-7 GeV



Thank you for your attention



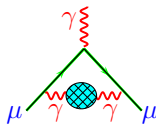
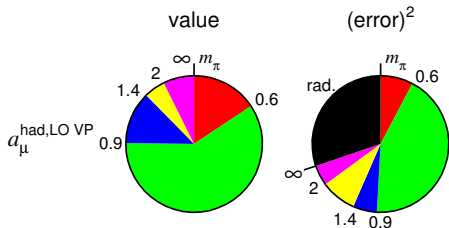


# BACKUP SLIDES

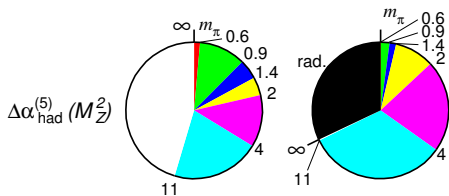


# R contribution in $a_\mu$ and $\alpha(M_Z^2)$

$$a_\mu^{\text{exp}} = (g_\mu - 2)/2$$



$$a_\mu^{\text{LO VP}} = \frac{\alpha^2}{3\pi^2} \int_{m_\pi^2}^{\infty} \frac{K(s)R(s)}{s} ds$$



$$\alpha(s) = \frac{\alpha}{1 - \Delta\alpha(s)}$$

$$\Delta\alpha = \sum_f \text{loop} = \Delta\alpha_{\text{lep}}(s) + \Delta\alpha_{\text{had}}(s)$$

$$\Delta\alpha^{(5)}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi} \text{Re} \int_{m_\pi^2}^{\infty} \frac{R(s) ds}{s(s - M_Z^2 - i\epsilon)}$$

K.Hagiwara et al. arxiv:1105.3149



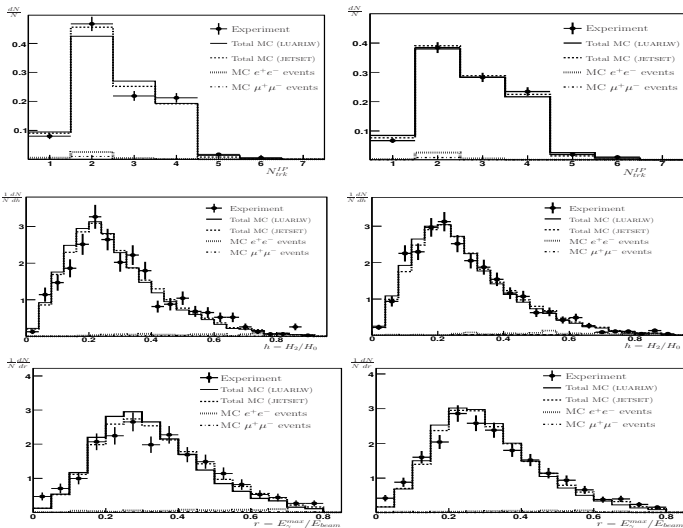
# Selection criteria

Selection criteria for hadronic events which were used by AND.

Variable	Allowed range	
	3.12-3.72 GeV	1.84 - 3.05 GeV
$N_{\text{track}}^{\text{IP}}$	$\geq 1$	$\geq 1$
$E_{\text{obs}}$	$> 1.6 \text{ GeV}$	$> 1.4 \text{ GeV}$ ( $> 1.3 \text{ GeV}$ if $E_{\text{beam}} < 1.05 \text{ GeV}$ )
$E_{\gamma}^{\text{max}}/E_{\text{beam}}$	$< 0.8$	$< 0.8$
$E_{\text{obs}} - E_{\gamma}^{\text{max}}$		$> 1.2 \text{ GeV}$ ( $> 1.1 \text{ GeV}$ if $E_{\text{beam}} < 1.05 \text{ GeV}$ )
$E_{\text{cal}}$	$> 0.75 \text{ GeV}$	$> 0.55 \text{ GeV}$
$H_2/H_0$	$< 0.85$	$< 0.9$
$ P_z^{\text{miss}}/E_{\text{obs}} $	$< 0.6$	$< 0.7$
$E_{\text{LKr}}/E_{\text{cal}}^{\text{tot}}$	$> 0.15$	$> 0.15$
$ Z_{\text{vertex}} $	$< 20.0 \text{ cm}$	$< 15.0 \text{ cm}$
	$N_{\text{particles}} \geq 4$ or $\tilde{N}_{\text{track}}^{\text{IP}} \geq 2$	$N_{\text{particles}} \geq 3$ or $\tilde{N}_{\text{track}}^{\text{IP}} \geq 2$



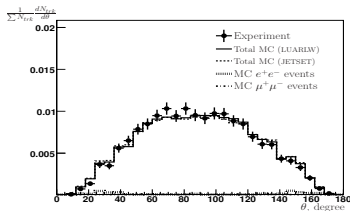
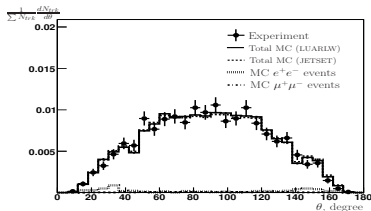
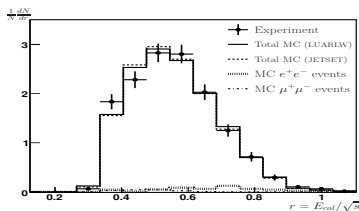
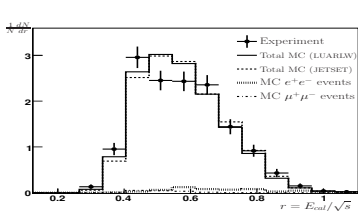
# Simulation at 1.94 and 2.14 GeV: JETSET and LUARLW



Properties of hadronic events produced in uds continuum at 1.94 GeV (left) and 2.14 GeV (right). Here,  $N$  is the number of events,  $H_2$  and  $H_0$  are Fox-Wolfram moments,  $E_\gamma^{max}$  is energy of the most energetic photon,  $N_{trk}$  is the number of tracks in event.



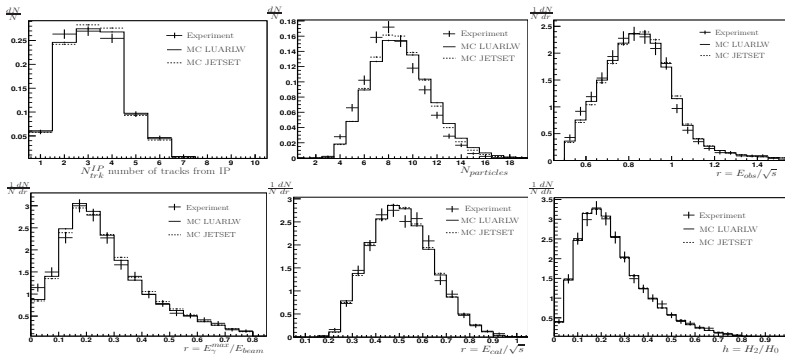
# Simulation at 1.94 and 2.14 GeV: JETSET and LUARLW



Properties of hadronic events produced in uds continuum at 1.94 GeV (left) and 2.14 GeV (right). Here,  $N$  is the number of events,  $E_{cal}$  is energy deposited in the calorimeter,  $\theta$  is polar angle,  $N_{trk}$  is the number of tracks in event. Integrals of all distributions are normalized to unity.

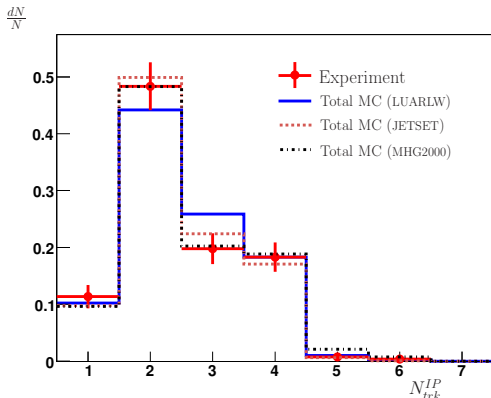


# Simulation at 3.12 GeV: JETSET and LUARLW



Properties of hadronic events produced in  $uds$  continuum at 3.12 GeV. Here  $N$  is the number of events,  $H_2$  and  $H_0$  are Fox-Wolfram moments. Integrals of all distributions are normalized to unity.





To obtain the detection efficiency required for calculation of the radiative correction, we performed simulation of the hadronic events using LUARLW and the event generator MHG2000 developed by the CMD-3 collaboration. MGH2000 generates about 30 exclusive channels accounting for the resonance production below 1.9 GeV.

# Detection efficiency: JETSET and LUARLW

Detection efficiency for the uds continuum in % (statistical errors only).

$\sqrt{s}$ , MeV	$\epsilon_{LUARLW}$	$\epsilon_{JETSET}$	$\delta\epsilon/\epsilon$
1841.0	$42.2 \pm 0.1$	$45.0 \pm 0.1$	$-6.6 \pm 0.3$
1937.0	$47.2 \pm 0.1$	$46.0 \pm 0.1$	$-2.5 \pm 0.3$
2037.3	$53.4 \pm 0.1$		
2135.7	$52.5 \pm 0.1$	$51.3 \pm 0.1$	$-1.2 \pm 0.3$
2239.2	$57.0 \pm 0.1$		
2339.5	$61.6 \pm 0.1$		
2444.1	$64.3 \pm 0.1$		
2542.6	$66.7 \pm 0.1$		
2644.8	$68.2 \pm 0.1$	$68.0 \pm 0.1$	$-0.2 \pm 0.2$
2744.6	$70.3 \pm 0.1$	$70.6 \pm 0.1$	$+0.4 \pm 0.2$
2849.7	$71.6 \pm 0.1$		
2948.9	$73.0 \pm 0.1$		
3048.1	$72.4 \pm 0.1$	$73.2 \pm 0.1$	$+1.1 \pm 0.2$





# Detection efficiency: JETSET and LUARLW

$\sqrt{s}$ , MeV	$\epsilon_{JETSET}$	$\epsilon_{LUARLW}$	$\delta\epsilon/\epsilon$
Scan 1			
3119.9	$75.5 \pm 0.1$	$75.0 \pm 0.1$	$-0.7 \pm 0.2$
3222.4	$76.9 \pm 0.1$	$76.2 \pm 0.1$	$-0.9 \pm 0.2$
3315.2	$77.0 \pm 0.1$	$77.0 \pm 0.1$	$0.0 \pm 0.2$
3418.1	$78.1 \pm 0.1$	$77.4 \pm 0.1$	$-0.9 \pm 0.2$
3521.0	$78.3 \pm 0.1$	$78.2 \pm 0.1$	$-0.1 \pm 0.2$
3619.7	$79.6 \pm 0.1$	$78.6 \pm 0.1$	$-1.3 \pm 0.2$
3720.4	$80.8 \pm 0.1$	$79.2 \pm 0.1$	$-2.0 \pm 0.2$
Scan 2			
3120.1	$75.3 \pm 0.1$	$74.9 \pm 0.1$	$-0.5 \pm 0.2$
3223.6	$75.9 \pm 0.1$	$75.1 \pm 0.1$	$-1.1 \pm 0.2$
3313.9	$77.5 \pm 0.1$	$77.3 \pm 0.1$	$-0.3 \pm 0.2$
3418.4	$78.7 \pm 0.1$	$78.0 \pm 0.1$	$-0.9 \pm 0.2$
3520.3	$78.8 \pm 0.1$	$78.7 \pm 0.1$	$-0.1 \pm 0.2$
3617.6	$80.0 \pm 0.1$	$79.0 \pm 0.1$	$-1.3 \pm 0.2$
3718.9	$80.9 \pm 0.1$	$79.4 \pm 0.1$	$-1.9 \pm 0.2$



# Luminosity determination: 3.12-3.72 GeV

$e^+e^- \rightarrow e^+e^-(\gamma)$  events detected by the LKr calorimeter  $41^\circ < \theta < 159^\circ$  and Csl calorimeter  $20^\circ < \theta < 32^\circ$  and  $148^\circ < \theta < 160^\circ$

Systematic uncertainties of the luminosity determination in %.

Source	Uncertainty, %
Calorimeter response	0.7
Calorimeter alignment	0.2
Polar angle resolution	0.2
Cross section calculation	0.5
Background	0.1
MC statistics	0.1
Variation of cuts	0.6
Sum in quadrature	1.1

Differences of an integrated luminosities obtained using the LKr and Csl calorimeters in two scans are  $0.5 \pm 0.5\%$  and  $0.0 \pm 0.5\%$ , respectively.



- The contribution of residual machine background was estimated using runs with separated  $e^+$  and  $e^-$  bunches.
- The residual background was evaluated and subtracted using the number of events which passed selection criteria in the background runs in the assumption that the background rate is proportional to the beam current and the measured vacuum pressure.
- As alternative we assumed that background rate is proportional to the current only. The difference between the numbers of background events obtained with the two assumption was considered as the uncertainty estimate for given energy point.

The residual machine background in % of observed cross section

Point	Scan 1	Scan 2
1	$1.3 \pm 0.2 \pm 0.4$	$1.3 \pm 0.2 \pm 0.4$
2	$2.4 \pm 0.4 \pm 0.5$	$2.7 \pm 0.4 \pm 0.5$
3	$2.7 \pm 0.5 \pm 0.4$	$3.0 \pm 0.5 \pm 0.4$
4	$2.9 \pm 0.5 \pm 0.4$	$3.6 \pm 0.6 \pm 0.4$
5	$3.1 \pm 0.6 \pm 0.5$	$3.3 \pm 0.5 \pm 0.5$
6	$2.7 \pm 0.5 \pm 0.4$	$3.7 \pm 0.6 \pm 0.4$
7	$2.1 \pm 0.4 \pm 0.2$	$2.2 \pm 0.3 \pm 0.2$



- An efficiency matrix  $\epsilon_{ij}$  describes the efficiency of an event generated with  $j$  charged tracks to be reconstructed with  $i$  charged tracks.
- The distribution of the number of observed charged track events in data,  $N_i^{obs}$ , is known. The true multiplicity distribution in data can be estimated from the observed multiplicity distribution in data and the efficiency matrix by minimizing the  $\chi^2$ .

■

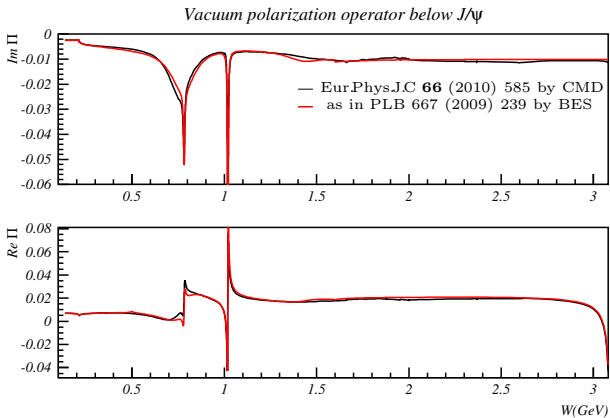
$$\chi^2 = \sum_{i=1}^{i=8} \frac{N_i^{obs} - \sum_{j=1}^{i=8} \epsilon_{ij} \times N_j}{N_i^{obs}}$$

where the  $N_j$  ( $j = 0, 2, 4, 6, 8$ ) describe the true multiplicity distribution in data and are taken as floating parameters in the fit.

- The total «true» number of events in data can be obtained by summing all fitted  $N_j$ .



# $\Pi(s)$ calculation



# Luminosity determination

$e^+e^- \rightarrow e^+e^-(\gamma)$  events detected by the LKr calorimeter  $41^\circ < \theta < 159^\circ$  and Csl calorimeter  $20^\circ < \theta < 32^\circ$  and  $148^\circ < \theta < 160^\circ$

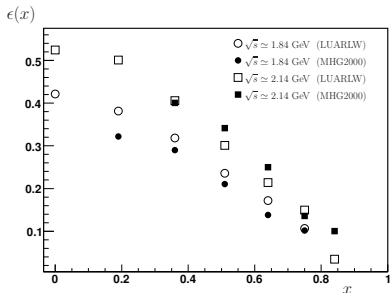
Systematic uncertainties of the luminosity determination in %.

Source	Uncertainty, %
Calorimeter response	0.7
Calorimeter alignment	0.2
Polar angle resolution	0.2
Cross section calculation	0.5
Background	0.1
MC statistics	0.1
Variation of cuts	0.6
Sum in quadrature	1.1

Differences of an integrated luminosities obtained using the LKr and Csl calorimeters in two scans are  $0.5 \pm 0.5\%$  and  $0.0 \pm 0.5\%$ , respectively.



# Radiation correction calculation in the energy range 1.84 – 3.05 GeV



Detection efficiency vs variable  $x$  at 1.84 and 2.14 GeV.

$$1 + \delta(s) = \int \frac{dx}{1-x} \frac{\mathcal{F}(s, x)}{|1 - \Pi((1-x)s)|^2} \frac{R((1-x)s)\epsilon((1-x)s)}{R(s)\epsilon(s)}$$

$$R(s) = -\frac{3}{\alpha} \text{Im} \Pi_{\text{hadr}}(s)$$

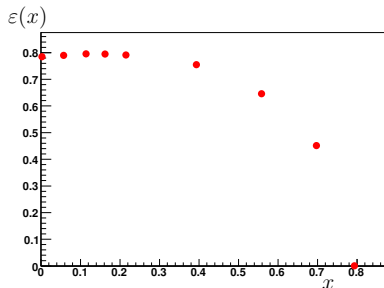
Vacuum polarization according to  
CMD-2 data compilation:  
Eur. Phys. J. C66 (2010) 585

Radiative correction factor  $1 + \delta$

$\sqrt{s}$ , MeV	$1 + \delta$	$\sqrt{s}$ , MeV	$1 + \delta$
1841.0	$1.0423 \pm 0.0208$	2542.6	$1.0739 \pm 0.0054$
1937.0	$1.0429 \pm 0.0156$	2644.8	$1.0796 \pm 0.0054$
2037.3	$1.0515 \pm 0.0126$	2744.6	$1.0809 \pm 0.0054$
2135.7	$1.0634 \pm 0.0106$	2849.7	$1.0823 \pm 0.0054$
2239.2	$1.0645 \pm 0.0096$	2948.9	$1.0774 \pm 0.0054$
2339.5	$1.0664 \pm 0.0075$	3048.1	$1.0584 \pm 0.0053$
2444.1	$1.0684 \pm 0.0064$		



# Radiation correction calculation in the energy range 3.12 – 3.72 GeV



$$1 + \delta(s) = \int \frac{dx}{1-x} \frac{\mathcal{F}(s, x)}{|1 - \bar{\Pi}((1-x)s)|^2} \frac{\bar{R}((1-x)s)\varepsilon((1-x)s)}{R(s)\varepsilon(s)}$$

$$R(s) = -\frac{3}{\alpha} \text{Im} \Pi_{\text{hadr}}(s)$$

Vacuum polarization according to  
CMD-2 data compilation:  
Eur. Phys. J. C66 (2010) 585

Detection efficiency vs variable  $x$  (scan 1,  $\sqrt{s} = 3.52$  GeV).

$\sqrt{s}$ , MeV	$1 + \delta$		Uncertainty, %				Total
	Scan 1	Scan 2	$\Pi(s)$	$\delta R$	$\delta \varepsilon$	$\delta_{\text{calc.}}$	
3119.9	$1.0941 \pm 0.0066$	$1.1074 \pm 0.0066$	0.3	0.5	0.2	0.2	0.6
3223.0	$1.0949 \pm 0.0055$	$1.1049 \pm 0.0055$	0.1	0.4	0.2	0.2	0.5
3314.7	$1.0959 \pm 0.0055$	$1.1100 \pm 0.0056$	0.1	0.4	0.2	0.2	0.5
3418.2	$1.0982 \pm 0.0044$	$1.1094 \pm 0.0044$	0.1	0.3	0.2	0.2	0.4
3520.8	$1.1032 \pm 0.0044$	$1.1102 \pm 0.0044$	0.1	0.3	0.2	0.2	0.4
3618.2	$1.1021 \pm 0.0044$	$1.1098 \pm 0.0044$	0.1	0.3	0.2	0.2	0.4
3719.4	$1.1049 \pm 0.0055$	$1.1067 \pm 0.0055$	0.4	0.3	0.2	0.2	0.5





# List of systematic uncertainties in the energy range 1.84-3.05 GeV

R systematic uncertainties (in %) assigned to each energy point.

	1841.0	1937.0	2037.3	2135.7	2239.2	2339.5	2444.1
Luminosity	1.2	1.2	1.2	1.2	1.2	1.2	1.2
Radiative correction	2.0	1.5	1.2	1.0	0.9	0.7	0.6
Continuum simulation	6.6	2.5	2.5	1.2	1.2	1.2	1.2
Track reconstruction	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$I^+I^-$ contribution	0.6	0.5	0.4	0.4	0.4	0.4	0.3
$e^+e^-X$ contribution	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Trigger efficiency	0.3	0.3	0.3	0.3	0.3	0.3	0.3
Nuclear interaction	0.4	0.4	0.4	0.4	0.4	0.4	0.4
Neutral events	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Cuts variation	0.7	0.7	0.7	0.7	0.7	0.7	0.7
Machine background	0.6	0.5	0.4	0.7	0.8	0.6	0.8
Energy determination	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Sum in quadrature	7.1	3.4	3.2	2.4	2.4	2.2	2.2
	2542.6	2644.8	2744.6	2849.7	2948.9	3048.1	
Luminosity	1.2	1.2	1.2	1.2	1.2	1.2	
Radiative correction	0.5	0.5	0.5	0.5	0.5	0.5	
Continuum simulation	1.2	1.2	1.2	1.2	1.2	1.2	
Track reconstruction	0.5	0.5	0.5	0.5	0.5	0.5	
$I^+I^-$ contribution	0.4	0.4	0.4	0.4	0.4	0.4	
$e^+e^-X$ contribution	0.2	0.2	0.2	0.2	0.2	0.2	
Trigger efficiency	0.3	0.3	0.3	0.3	0.3	0.3	
Nuclear interaction	0.4	0.4	0.4	0.4	0.4	0.4	
Neutral events	0.2	0.2	0.2	0.2	0.2	0.2	
Cuts variation	0.7	0.7	0.7	0.7	0.7	0.7	
Machine background	0.4	0.6	0.8	0.4	0.9	0.5	
Energy determination	0.1	0.1	0.1	0.1	0.1	0.1	
Sum in quadrature	2.1	2.2	2.2	2.1	2.3	2.1	



# List of systematic uncertainties in the energy range 3.12-3.72 GeV

$R_{uds}$  systematic uncertainties (in %) assigned to each energy point.

	3119.9	3223.0	3314.7	3418.2	3520.8	3618.2	3719.4
<i>Scan 1</i>							
Luminosity	1.1	1.1	1.1	1.1	1.1	1.1	1.1
Radiative correction	0.6	0.5	0.5	0.4	0.4	0.4	0.5
Continuum simulation	1.4	1.4	1.4	1.4	1.4	1.4	2.1
$J/\psi$ contribution	2.7	0.5	0.3	0.2	0.2	0.1	0.1
$\psi(2S)$ contribution							1.4
$e^+e^-X$ contribution	0.1	0.1	0.1	0.2	0.2	0.2	0.2
$I^+I^-$ contribution	0.1	0.1	0.1	0.1	0.1	0.2	0.2
Trigger efficiency	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Nuclear interaction	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Cuts variation	0.6	0.6	0.6	0.6	0.6	0.6	0.6
Machine background	1.1	0.8	0.7	0.7	0.9	0.7	0.7
Sum in quadrature	3.5	2.2	2.1	2.1	2.2	2.1	3.0
<i>Scan 2</i>							
Luminosity	1.1	1.1	1.1	1.1	1.1	1.1	1.1
Radiative correction	0.6	0.5	0.5	0.4	0.4	0.4	0.5
Continuum simulation	1.4	1.4	1.4	1.4	1.4	1.4	2.1
$J/\psi$ contribution	2.8	0.6	0.3	0.2	0.2	0.1	0.1
$\psi(2S)$ contribution							1.3
$e^+e^-X$ contribution	0.1	0.1	0.1	0.2	0.2	0.2	0.2
$I^+I^-$ contribution	0.1	0.1	0.1	0.1	0.1	0.2	0.2
Trigger efficiency	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Nuclear interaction	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Cuts variation	0.6	0.6	0.6	0.6	0.6	0.6	0.6
Machine background	1.1	0.8	0.7	0.8	0.8	0.7	0.5
Sum in quadrature	3.6	2.2	2.1	2.1	2.1	2.1	2.9



$R(s)$ , obtained in:

**P.A.Baikov et al. Nucl. and Part. Phys. Proceed. 261-262(2015):**

$$R^{n_f=3}(s) = 2 \left[ 1 + \frac{\alpha_s}{\pi} + 1.6398 \left( \frac{\alpha_s}{\pi} \right)^2 - 10.2839 \left( \frac{\alpha_s}{\pi} \right)^3 - 106.8798 \left( \frac{\alpha_s}{\pi} \right)^4 \right].$$

$\alpha_s$  obtained in **K.G.Chetyrkin, B.A.Kniehl, M.Steinhauser PRL 79 (1997)**

$$\alpha_s = \frac{1}{\beta_0 L} - \frac{1}{(\beta_0 L)^2} \frac{\beta_1}{\beta_0} \ln L + \frac{1}{(\beta_0 L)^3} \left[ \left( \frac{\beta_1}{\beta_0} \right)^2 (\ln^2 L - \ln L - 1) + \frac{\beta_2}{\beta_0} \right] \\ + \frac{1}{(\beta_0 L)^4} \left[ \left( \frac{\beta_1}{\beta_0} \right)^3 \left( -\ln^3 L + \frac{5}{2} \ln^2 L + 2 \ln L - \frac{1}{2} \right) - 3 \frac{\beta_1 \beta_2}{\beta_0^2} \ln L + \frac{\beta_3}{2\beta_0} \right]$$

for  $n_f = 3$   $\beta_0 = \frac{9}{4}$ ,  $\beta_1 = 4$ ,  $\beta_2 = \frac{3863}{384}$ ,  $\beta_3 = \frac{445}{32} \zeta(3) + \frac{140599}{4608}$ ,  $L = \ln^2 \frac{Q^2}{\Lambda_{MS}^2}$

$\alpha_s(m_\tau^2) = 0.331 \pm 0.013$  (**A.Pich Nucl. and Part. Phys. Proceed. 260 (2015) 61-69**) allow to get  $R_{uds}^{pQCD} = 2.16 \pm 0.01$  in energy range  $3.1 \div 3.7$  GeV.

