Variations on Photon Vacuum Polarization

Fred Jegerlehner*, DESY Zeuthen/HU Berlin,
fjeger@physik.hu-berlin.de

International Workshop on e^+e^- collisions from Phi to Psi 2017
26-29 June 2017, Mainz, Germany

Outline of Talk:

- Evaluation of $\alpha(M_Z^2)$
- Reducing uncertainties via the Euclidean split trick
- My `alphaQED` and `alpha2SM` packages
- The coupling α_2, M_W and $\sin^2\Theta_f$
- HVP possible improvements
- HVP from lattice QCD
- News on VP subtraction
- HVP for the muon anomaly

Evaluation of $\alpha(M_Z^2)$

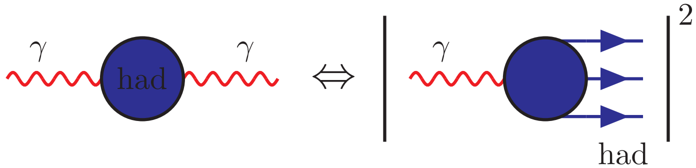
Non-perturbative hadronic contributions $\Delta\alpha^{(5)}_{\rm had}(s) = -\left(\Pi'_\gamma(s) - \Pi'_\gamma(0)\right)$ **can be evaluated in terms of** $\sigma(e^+e^- \to \text{hadrons})$ **data via dispersion integral:**

$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha\, s}{3\pi}\left(\oint_{4m_\pi^2}^{E^2_{\rm cut}} ds' \frac{R^{\rm data}_\gamma(s')}{s'(s'-s)} + \oint_{E^2_{\rm cut}}^{\infty} ds' \frac{R^{\rm pQCD}_\gamma(s')}{s'(s'-s)} \right)$$

Compilation: FJ 15

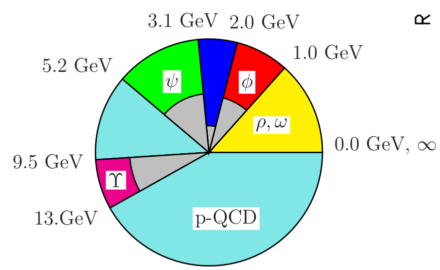
Theory = pQCD: Gorishny et al. 91, Chetyrkin et al. 97...09

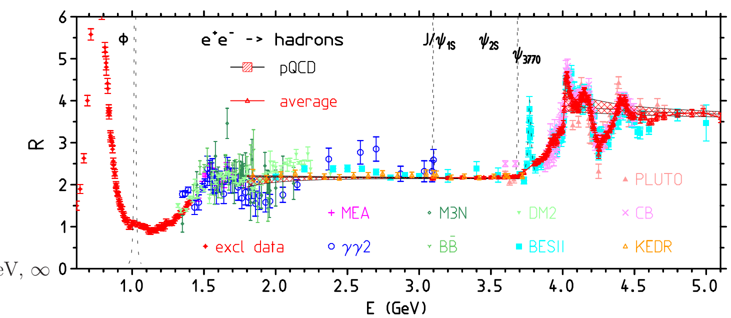
where $R_\gamma(s) \equiv \dfrac{\sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})}{\frac{4\pi\alpha^2}{3s}}$

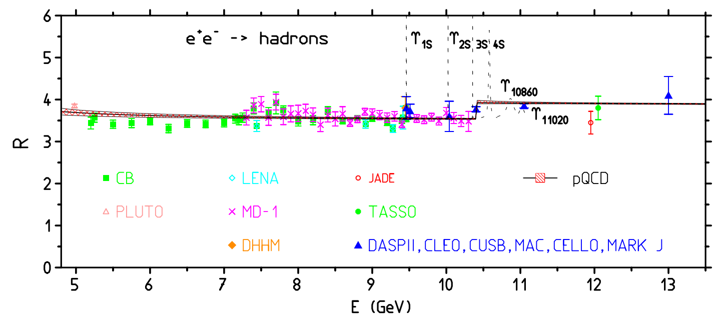


$\Pi'^{\ \rm had}_\gamma(q^2)$ $\quad\sim \sigma^{\rm had}_{\rm tot}(q^2)$

hadronic vacuum polarization



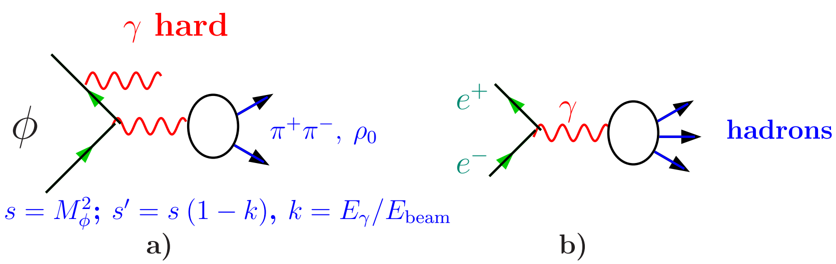




$$\boxed{\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)}\ ; \quad \Delta\alpha(s) = \Delta\alpha_{\rm lep}(s) + \Delta\alpha^{(5)}_{\rm had}(s) + \Delta\alpha_{\rm top}(s)}$$

Present situation: (after KLOE, BaBar and first BESIII results)

$\Delta\alpha^{(5)}_{\mathrm{hadrons}}(M_Z^2)$	$=$	0.027738 ± 0.000158	
		0.027523 ± 0.000119	Adler
$\alpha^{-1}(M_Z^2)$	$=$	128.919 ± 0.022	
		128.958 ± 0.016	Adler

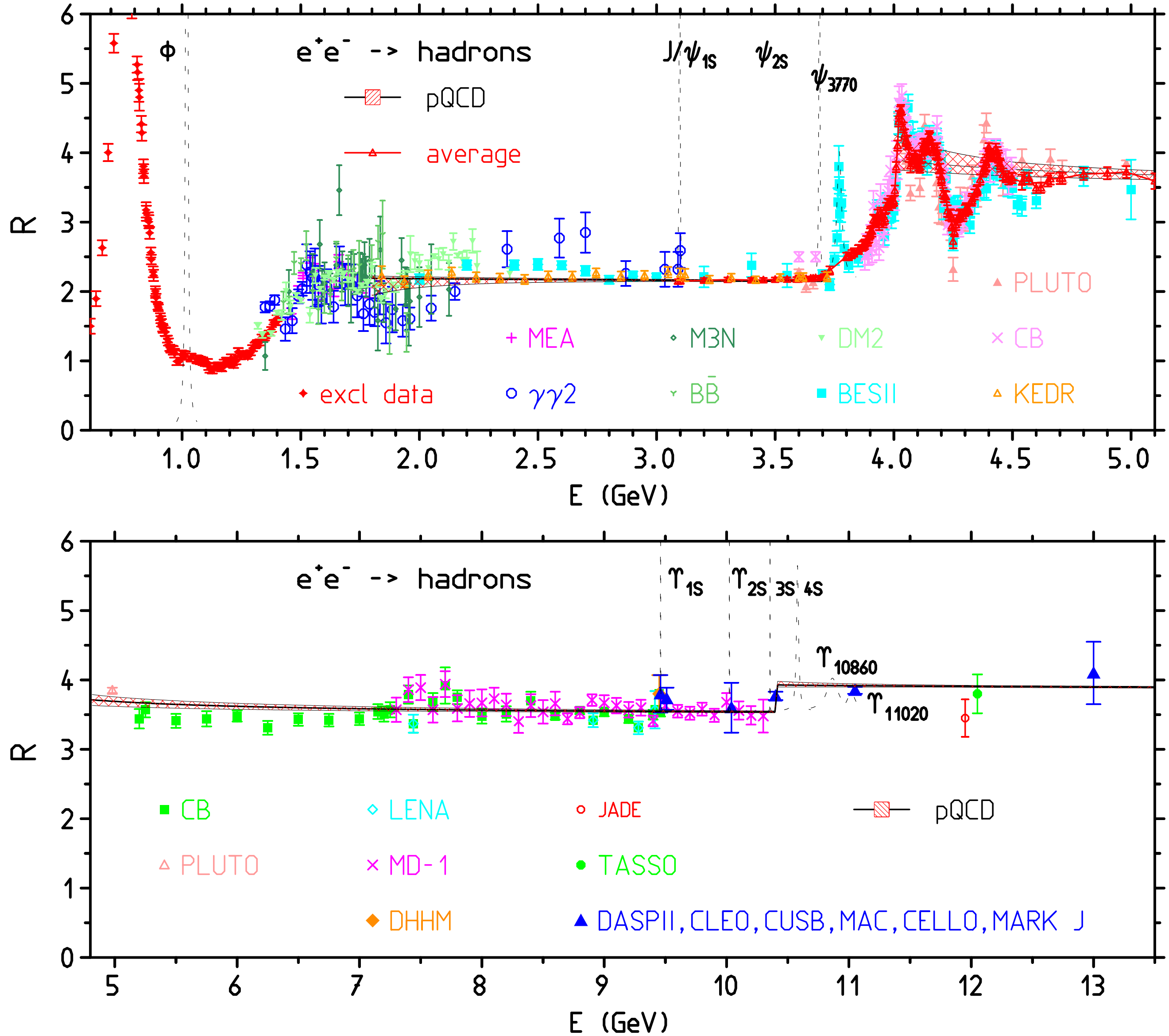


a) Initial state radiation (ISR), b) Standard energy scan.

SCAN: CMD-2, SND (NSK); ISR: KLOE (pioneered the method), BaBar, BESIII

New experimental input for HVP: BESIII-ISR,VEPP-2000,KEDR,SNC,CMD-3,BaBar excl.
New data: see various talks this meeting

Data vs pQCD



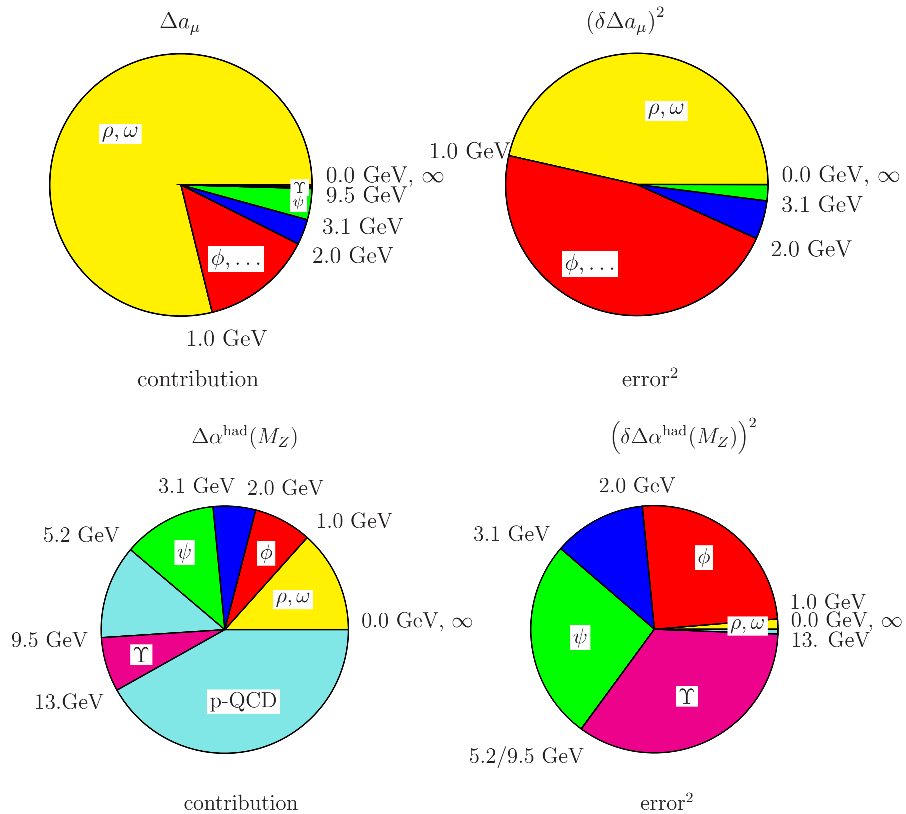
e⁺e⁻ -> hadrons
pQCD
average
φ
J/ψ1S
ψ2S
ψ3770
excl data
γγ2
MEA
M3N
DM2
CB
BB̄
BESII
KEDR
PLUTO
R
E (GeV)
e⁺e⁻ -> hadrons
Υ1S
Υ2S 3S 4S
Υ10860
Υ11020
CB
LENA
JADE
pQCD
PLUTO
MD-1
TASSO
DHHM
DASPII, CLEO, CUSB, MAC, CELLO, MARK J
R
E (GeV)

$\Delta\alpha_{\rm had}(M_Z^2)$ results from ranges:

for $M_Z = 91.1876$ GeV in units 10^{-4}. 2017 update in terms of e^+e^--data and pQCD. 43% data, 57% perturbative QCD. pQCD is used between 5.2 GeV and 9.5 GeV and above 11.5 GeV.

final state	range (GeV)	$\Delta\alpha^{(5)}_{\rm had} \times 10^4$ (stat) (syst) [tot]	rel	abs
ρ	(0.28, 1.05)	33.91 (0.05) (0.18)[0.19]	0.6%	1.4%
ω	(0.42, 0.81)	3.10 (0.04) (0.08)[0.09]	3.0%	0.3%
ϕ	(1.00, 1.04)	4.76 (0.07) (0.11)[0.13]	2.7%	0.7%
J/ψ		12.38 (0.60) (0.67)[0.90]	7.2%	32.1%
Υ		1.30 (0.05) (0.07)[0.09]	6.9%	0.3%
had	(1.05, 2.00)	16.53 (0.06) (0.83)[0.83]	5.0%	27.4%
had	(2.00, 3.20)	15.34 (0.08) (0.61)[0.62]	4.0%	15.2%
had	(3.10, 3.60)	4.98 (0.03) (0.09)[0.10]	1.9%	0.4%
had	(5.20, 5.20)	16.84 (0.12) (0.21)[0.25]	0.0%	2.4%
pQCD	(5.20, 9.46)	33.84 (0.12) (0.25)[0.03]	0.1%	0.0%
had	(9.46,11.50)	11.12 (0.07) (0.69)[0.69]	6.2%	19.2%
pQCD	(11.50,∞)	123.29 (0.00) (0.05)[0.05]	0.0%	0.1%
data	(0.28,11.50)	120.25 (0.63) (1.45)[1.58]	1.0%	0.0%
total		277.38 (0.63) (1.45)[1.58]	0.6%	100.0%

Correlation between different contributions to $a_\mu^{\rm had}$ and $\Delta\alpha^{{\rm had}\,(5)}$



Contributions from e^+e^- data ranges and form pQCD to $a_\mu^{\rm had}$ and $\Delta\alpha^{{\rm had}\,(5)}$.

4. Reducing uncertainties via the Euclidean split trick: Adler function controlled pQCD

- experiment side: new more precise measurements of $R(s)$
- future direct measurements Patrick Janot, Luca Trentadue et al
- theory side: $\alpha_{\rm em}(M_Z^2)$ by the "Adler function controlled" approach

$$\alpha(M_Z^2) \quad = \quad \alpha^{\rm data}(-s_0) + \left[\alpha(-M_Z^2) - \alpha(-s_0)\right]^{\rm pQCD} + \left[\alpha(M_Z^2) - \alpha(-M_Z^2)\right]^{\rm pQCD}$$

where the space-like $-s_0$ is chosen such that pQCD is well under control for $-s < -s_0$. The monitor to control the applicability of pQCD is the Adler function

$$D(Q^2 = -s) = \frac{3\pi}{\alpha} s \frac{d}{{\rm d}s} \Delta\alpha_{\rm had}(s) = -(12\pi^2)\, s\, \frac{{\rm d}\Pi'_\gamma(s)}{{\rm d}s} = Q^2 \int_{4m_\pi^2}^{\infty} \frac{R(s)}{(s+Q^2)^2}$$

which also is determined by $R(s)$ and can be evaluated in terms of experimental e^+e^-–data. Perturbative QCD tail: $D(Q^2) \to N_c \sum_f Q_f^2\ (1 + O(\alpha_s))$ as $Q^2 \to \infty$.

S. Eidelman, F. J., A. Kataev, O. Veretin, Phys. Lett. B **454** (1999) 369

$\Delta\alpha^{\rm had}$ Adler function controlled

✓ use old idea: Adler function: Monitor for comparing theory and data

$$D(-s) \doteq \frac{3\pi}{\alpha}\, s\, \frac{d}{ds}\Delta\alpha_{\rm had}(s) = -\left(12\pi^2\right)\, s\, \frac{d\Pi'_\gamma(s)}{ds}$$

$$\Rightarrow \qquad D(Q^2) = Q^2 \left(\int_{4m_\pi^2}^{E^2_{\rm cut}} {\rm d}s\, \frac{R(s)^{\rm data}}{(s+Q^2)^2} + \int_{E^2_{\rm cut}}^{\infty} \frac{R^{\rm pQCD}(s)}{(s+Q^2)^2}\, {\rm d}s \right).$$

pQCD $\leftrightarrow R(s)$	pQCD $\leftrightarrow D(Q^2)$
very difficult to obtain in theory	smooth simple function in Euclidean region

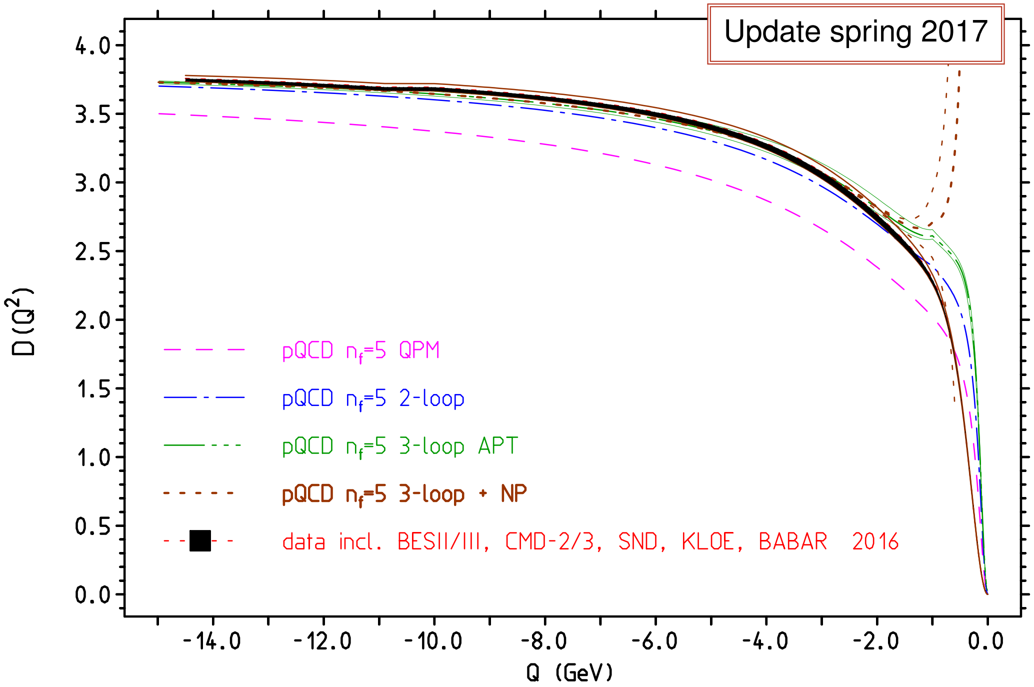
Conclusion:

❖time-like approach: pQCD works well in "perturbative windows"
3.00 - 3.73 GeV, 5.00 - 10.52 GeV and 11.50 - ∞ Kühn,Steinhauser

❖space-like approach: pQCD works well for $\sqrt{Q^2 = -q^2} > 2.0$ GeV (see plot)

“Experimental” Adler–function versus theory (pQCD + NP)

Error includes statistical + systematic here (in contrast to most R-plots showing statistical errors only)!



(Eidelman, F. J., Kataev, Veretin 98, FJ 08/17 updates)
theory based on results by Chetyrkin, Kühn et al.

⇒ **pQCD works well controlled to predict $D(Q^2)$ down to $s_0 = (2.0\,\textbf{GeV})^2$; use this to calculate**

$$\Delta\alpha_{\rm had}(-Q^2) \sim \frac{\alpha}{3\pi}\int \mathrm{d}Q'^2 \frac{D(Q'^2)}{Q'^2}$$

$$\boxed{\begin{aligned}\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = & \left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-s_0)\right]^{\rm pQCD} \\ & + \Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data}\end{aligned}}$$

and obtain, for $s_0 = (2.0\,\textbf{GeV})^2$: **(FJ 98/17)**

$$\begin{aligned}
\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} &= 0.006409 \pm 0.000063 \\
\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) &= 0.027483 \pm 0.000118 \\
\Delta\alpha^{(5)}_{\rm had}(M_Z^2) &= 0.027523 \pm 0.000119
\end{aligned}$$

❖**shift** $+0.000008$ **from the 5-loop contribution**

❖**error** ± 0.000100 **added in quadrature form perturbative part**

QCD parameters: ● $\alpha_s(M_Z) = 0.1189(20)$**,**

● $m_c(m_c) = 1.286(13)$ $[M_c = 1.666(17)]$ **GeV** , ● $m_b(m_c) = 4.164(25)$ $[M_b = 4.800(29)]$ **GeV**

based on a complete 3–loop massive QCD analysis Kühn et al 2007

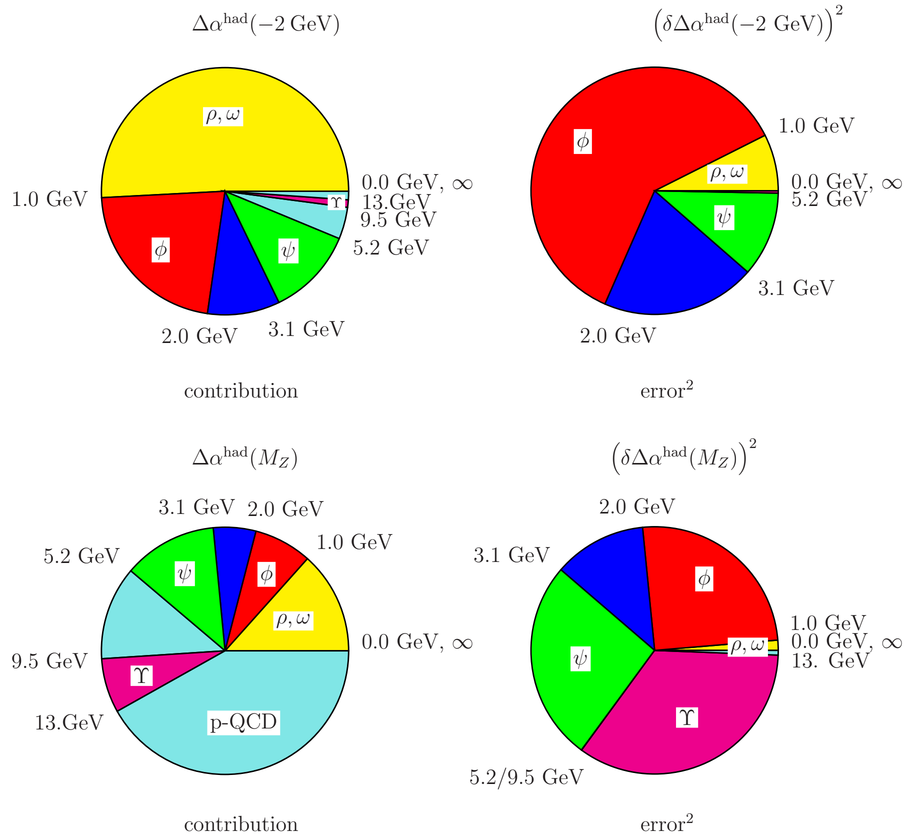
F. J., Nucl. Phys. Proc. Suppl. **181-182** (2008) 135

$\Delta\alpha_{\rm had}(-M_0^2)$ results from ranges:

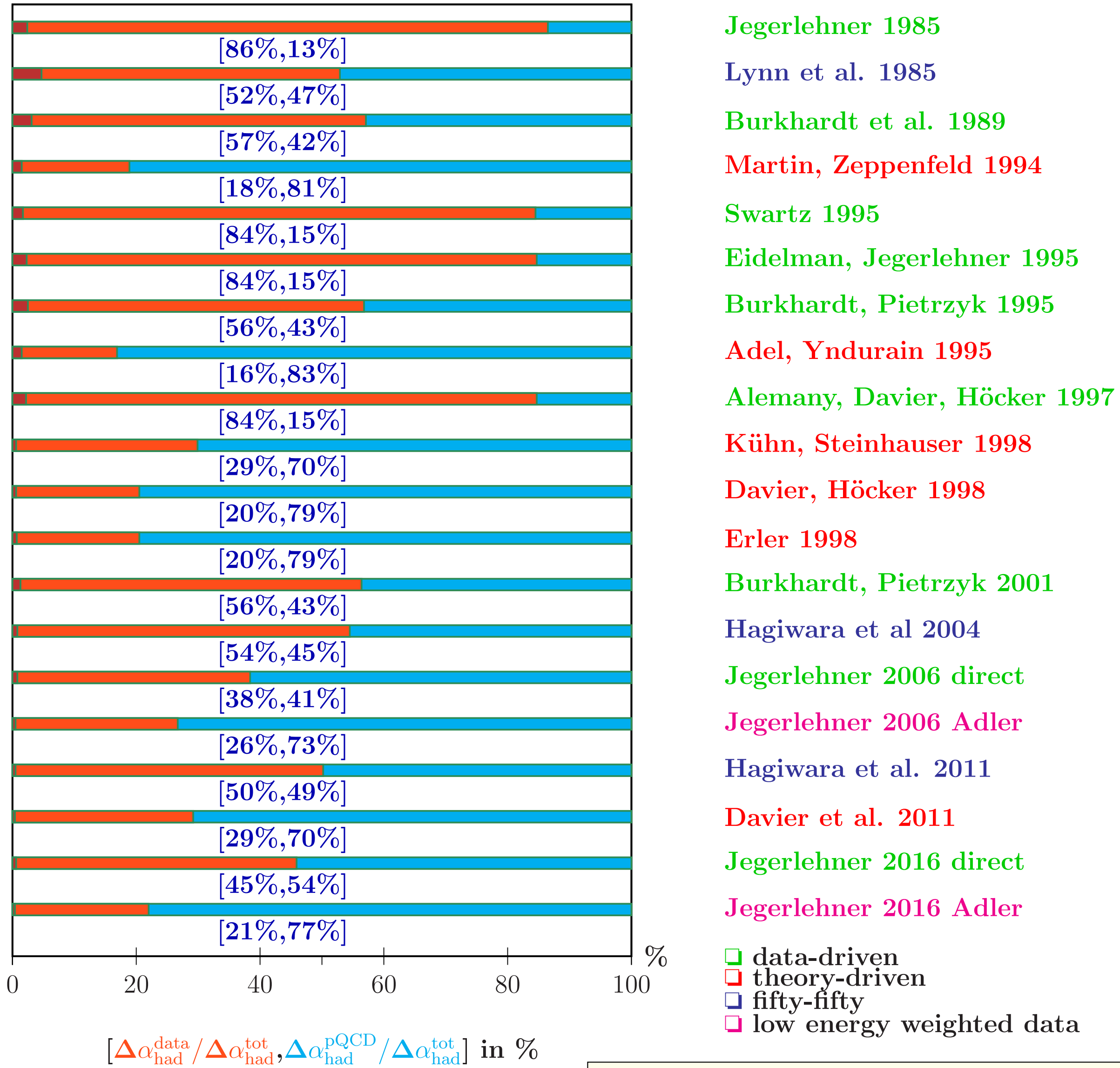
for $M_0 = 2$ GeV in units 10^{-4}. 2015 update in terms of e^+e^--data and pQCD. 94% data, 6% perturbative QCD. pQCD is used between 5.2 GeV and 9.5 GeV and above 11.5 GeV.

final state	range (GeV)	$\Delta\alpha^{(5)}_{\rm had}(-M_0^2) \times 10^4$ (stat) (syst) [tot]	rel	abs
ρ	(0.28, 1.05)	29.78 (0.04) (0.16)[0.16]	0.5%	6.6%
ω	(0.42, 0.81)	2.69 (0.03) (0.07)[0.08]	3.0%	1.6%
ϕ	(1.00, 1.04)	3.78 (0.05) (0.09)[0.10]	2.7%	2.6%
J/ψ		3.21 (0.15) (0.15)[0.21]	6.7%	11.4%
Υ		0.05 (0.00) (0.00)[0.00]	6.8%	0.0%
had	(1.05, 2.00)	10.36 (0.04) (0.49)[0.49]	4.8%	61.2%
had	(2.00, 3.20)	6.06 (0.03) (0.25)[0.25]	4.2%	16.1%
had	(3.10, 3.60)	1.31 (0.01) (0.02)[0.03]	1.9%	0.2%
had	(5.20, 5.20)	2.90 (0.02) (0.02)[0.03]	0.0%	0.2%
pQCD	(5.20, 9.46)	2.66 (0.02) (0.02)[0.00]	0.1%	0.0%
had	(9.46,11.50)	0.39 (0.00) (0.02)[0.02]	5.7%	0.1%
pQCD	(11.50,∞)	0.90 (0.00) (0.00)[0.00]	0.0%	0.0%
data	(0.28,11.50)	60.53 (0.18) (0.61)[0.63]	1.0%	0.0%
total		64.09 (0.18) (0.61)[0.63]	1.0%	100.0%

Of $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$ 22% data, 78% pQCD!



Contributions from e^+e^- data ranges and form pQCD to $\Delta\alpha^{(5)}_{\mathrm{had}}(-M_0^2)$ vs. $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$.



How much pQCD?

Note: the Adler function monitored Euclidean data vs pQCD split approach is only moderately more pQCD-driven, than the time-like approach adopted by Davier et al. and others.

My `alphaQED` and `alpha2SM` packages

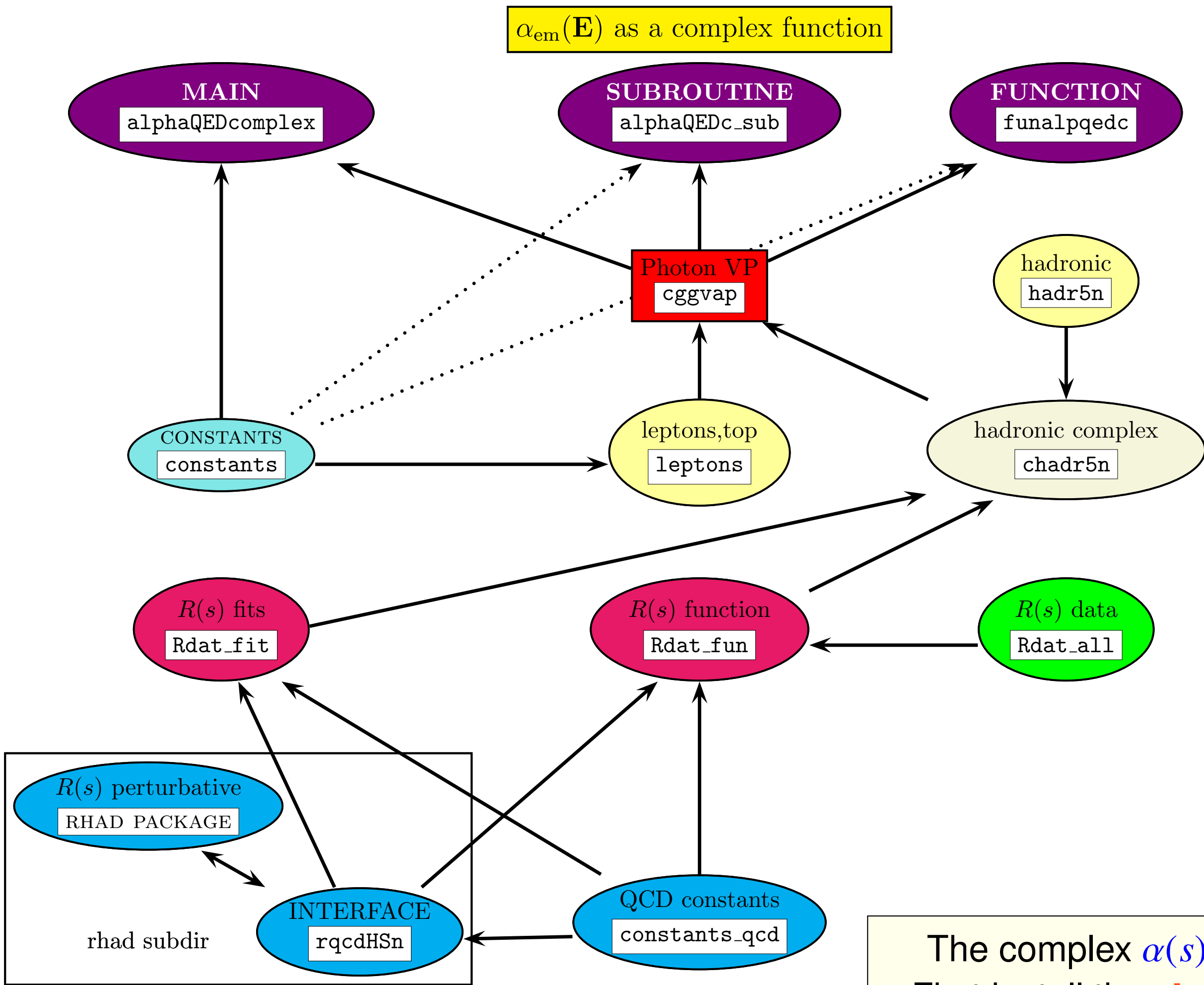
Download link: ✲⋙ [alphaQED.tar.gz]

The package for calculating the effective electromagnetic fine structure constant is available in two versions:

➠alphaQEDreal [FUNCTION funalpqed] providing the real part of the subtracted photon vacuum polarization including hadronic, leptonic and top quark contributions as well as the weak part (relevant at ILC energies)

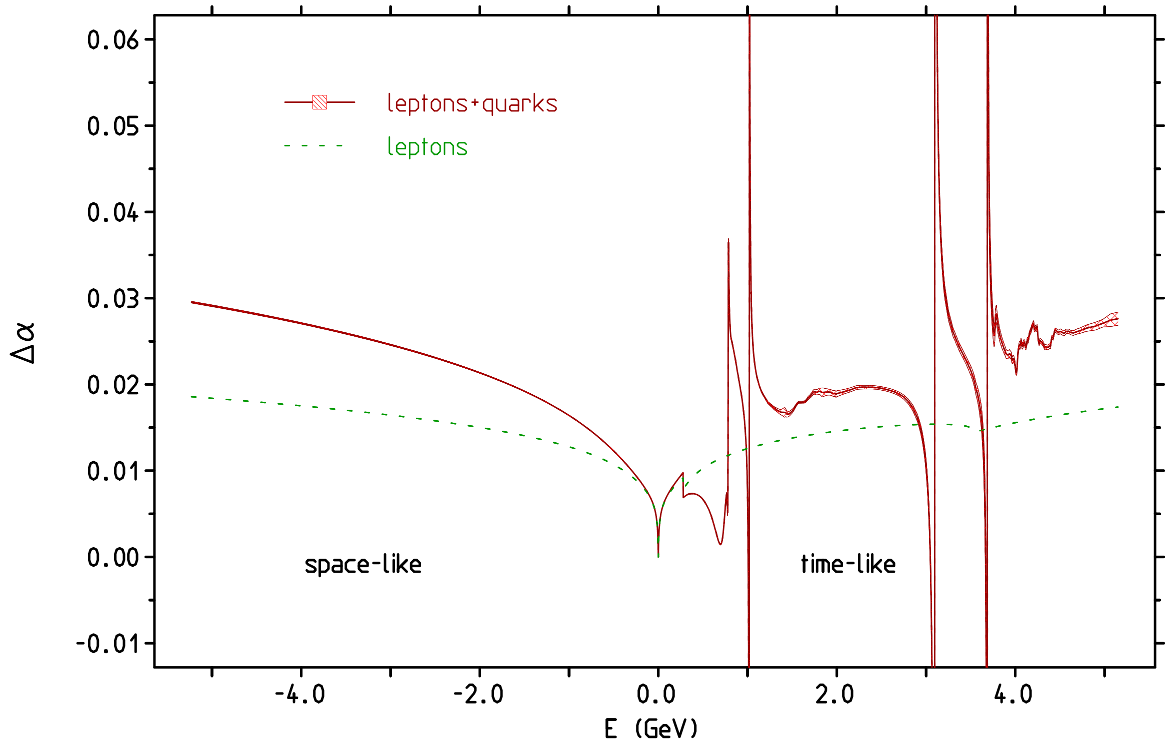
➠alphaQEDcomplex [FUNCTION funalpqedc] provides in addition the corresponding imaginary parts.

➠corresponding options for $SU(2)_L$ coupling $\alpha_2 = g^2/4\pi$ alpha2SMreal and alpha2SMcomplex



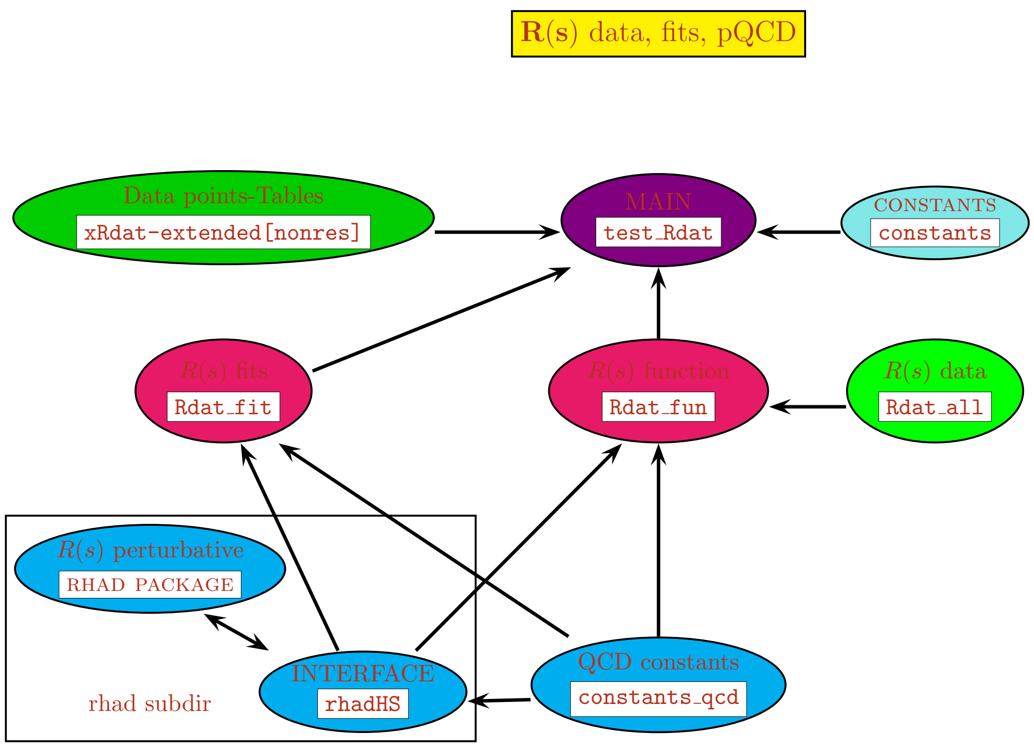
The complex $\alpha(s)$ requires $R(s)$ in addition to the real $\alpha(s)$.
First install the rhad package by Harlander and Steinhauser
(FORTRAN package version rhad-1.01 (March 2009 issue))

Sample Plots:

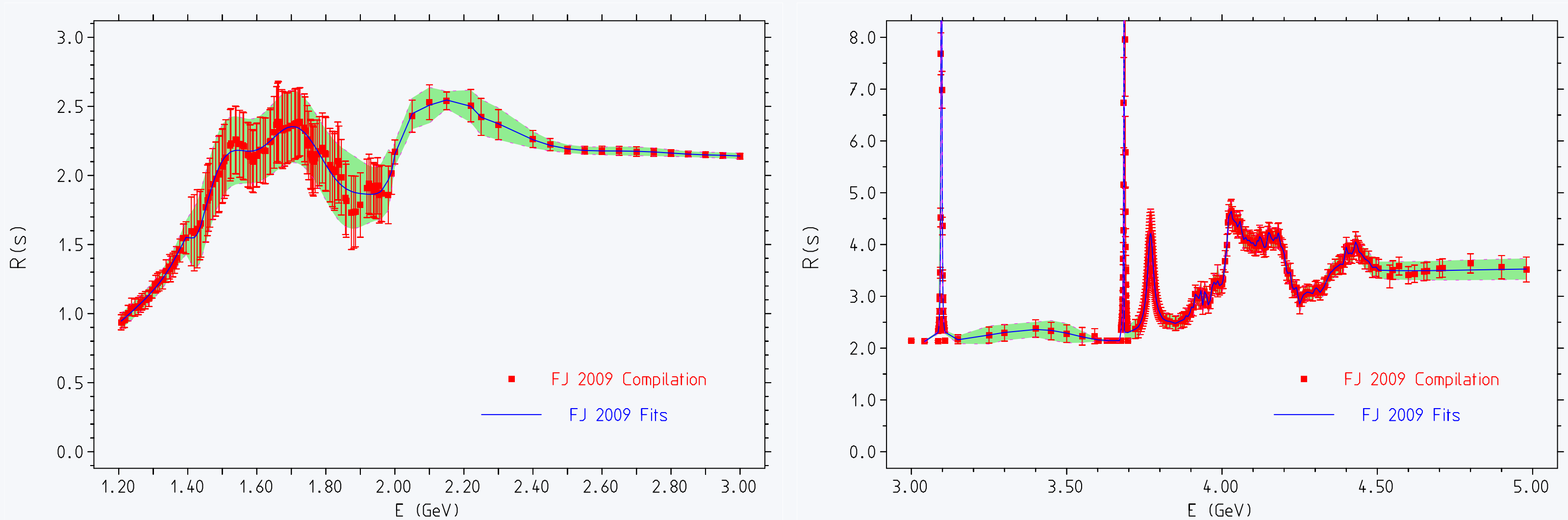


Shift of the effective fine structure constant $\Delta\alpha$ as a function of the energy scale in the time–like region $s > 0$ ($E = \sqrt{s}$) vs the space–like region $-s > 0$ ($E = -\sqrt{-s}$). The band indicates the uncertainties

Sample program `test_Rdat.f` for extracting $R(s)$ data, fits and pQCD calculation.



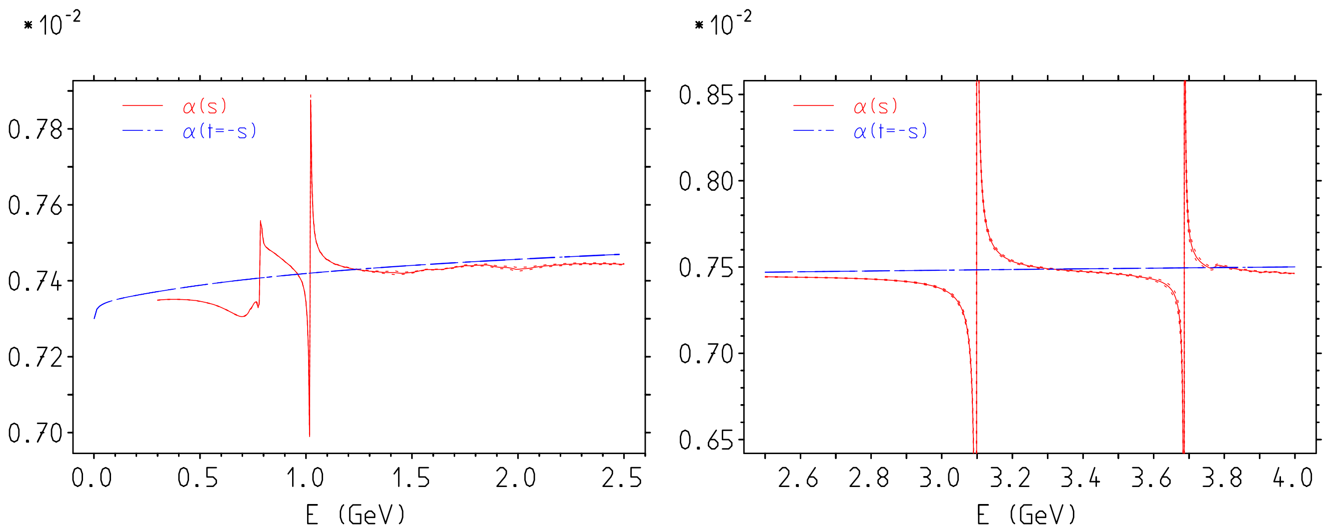
Sample results:



$R(s)\ e^+e^- \to \mathrm{hadrons}$ data vs. Chebyshev polynomial fits

[no fit for $\psi_3 \ldots \psi_6$ region yet]

The time-like vs space-like effective charge

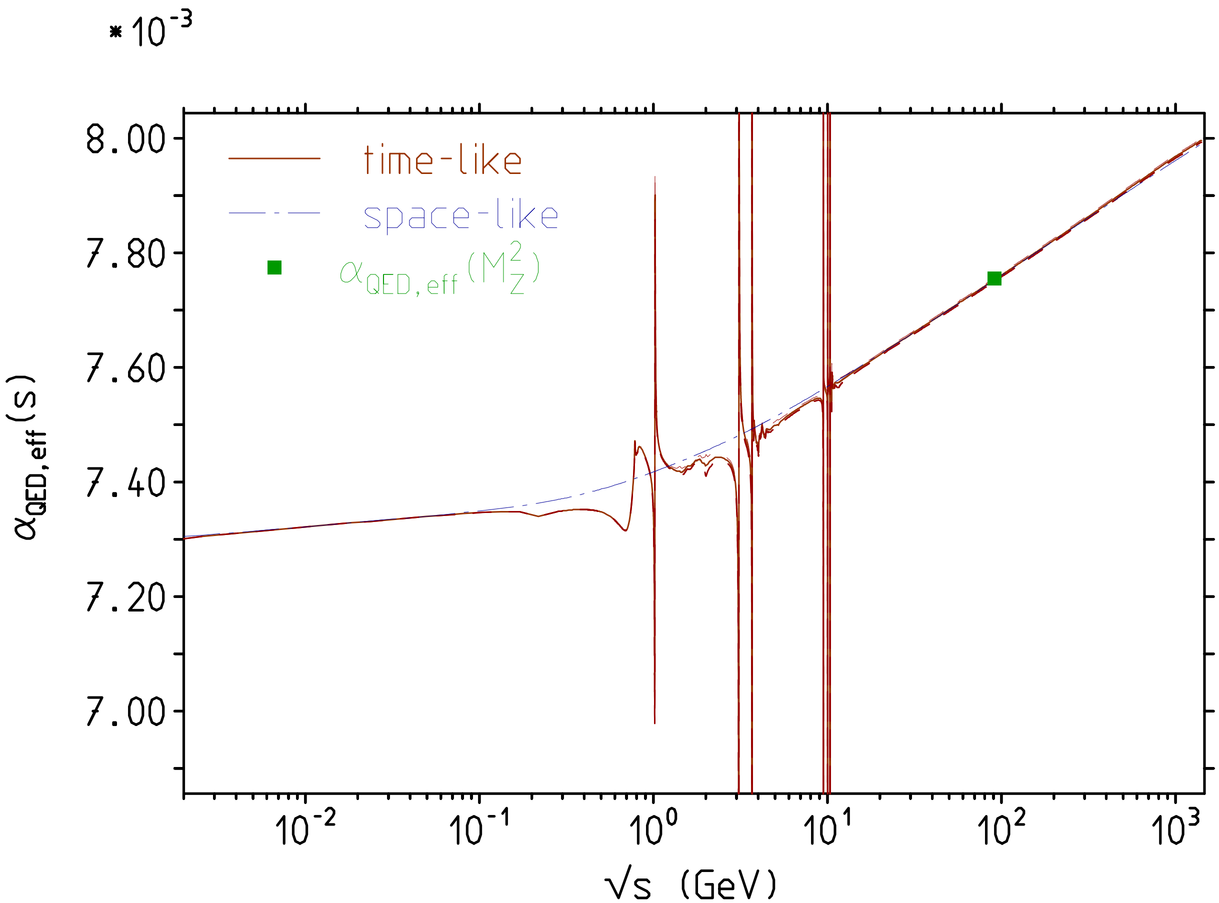


Note that the smooth space-like effective charge agrees rather well with the non-resonant "background" above the Φ (kind of duality)

No proof that this cannot produce non-negligible shifts!

Time-like VP-subtraction cannot be implemented locally near OZI suppressed resonances: $J/\psi, \psi'$ and $\Upsilon_1, \Upsilon_2, \Upsilon_3$

$\alpha_{\rm QED,eff}$: time-like vs. space-like



$\alpha_{\rm QED,eff}$ duality: $\alpha_{\rm QED,eff}(s)$ is varying dramatically near resonances, but agrees quite well in average with space-like version

The coupling α_2, M_W and $\sin^2\Theta_f$

How to measure α_2:

❖charged current channel M_W ($g \equiv g_2$):

$$M_W^2 = \frac{g^2\,v^2}{4} = \frac{\pi\,\alpha_2}{\sqrt{2}\,G_\mu}$$

❖neutral current channel $\sin^2\Theta_f$

In fact here running $\sin^2\Theta_f(E)$: LEP scale $\Longleftrightarrow$ low energy $\nu_e e$ scattering

$$\sin^2\Theta_e = \left\{\frac{1-\Delta\alpha_2}{1-\Delta\alpha} + \Delta_{\nu_\mu e,\mathrm{vertex+box}} + \Delta\kappa_{e,\mathrm{vertex}}\right\}\sin^2\Theta_{\nu_\mu e}$$

The first correction from the running coupling ratio is largely compensated by the ν_μ charge radius which dominates the second term. The ratio $\sin^2\Theta_{\nu_\mu e}/\sin^2\Theta_e$ is close to 1.002, independent of top and Higgs mass. Note that errors in the ratio $\frac{1-\Delta\alpha_2}{1-\Delta\alpha}$ can be taken to be 100% correlated and thus largely cancel.

Above result allow us to calculate non-perturbative hadronic correction in $\gamma\gamma$, γZ, ZZ and WW self energies, as

$$
\begin{aligned}
\Pi^{\gamma\gamma} &= e^2 \quad \hat{\Pi}^{\gamma\gamma} \\
\Pi^{Z\gamma} &= \frac{eg}{c_\Theta} \quad \hat{\Pi}_V^{3\gamma} \quad - \quad \frac{e^2 s_\Theta}{c_\Theta} \quad \hat{\Pi}_V^{\gamma\gamma} \\
\Pi^{ZZ} &= \frac{g^2}{c_\Theta^2} \quad \hat{\Pi}_{V-A}^{33} \quad - \quad 2\,\frac{e^2}{c_\Theta^2} \quad \hat{\Pi}_V^{3\gamma} \quad + \quad \frac{e^2 s_\Theta^2}{c_\Theta^2} \quad \hat{\Pi}_V^{\gamma\gamma} \\
\Pi^{WW} &= g^2 \quad \hat{\Pi}_{V-A}^{+-}
\end{aligned}
$$

with $\hat{\Pi}(s) = \hat{\Pi}(0) + s\hat{\pi}(s)$. Leading hadronic contributions:

$$
\begin{aligned}
\Delta\alpha_{\rm had}^{(5)}(s) &= -e^2 \left[\mathrm{Re}\,\hat{\pi}^{\gamma\gamma}(s) - \hat{\pi}^{\gamma\gamma}(0)\right] \\
\Delta\alpha_{2\,\rm had}^{(5)}(s) &= -\frac{e^2}{s_\Theta^2} \left[\mathrm{Re}\,\hat{\pi}^{3\gamma}(s) - \hat{\pi}^{3\gamma}(0)\right]
\end{aligned}
$$

which exhibit the leading hadronic non-perturbative parts, i.e. the ones involving the photon field via mixing. $\Delta\alpha_{\rm had}^{(5)}(s)$ and $\Delta\alpha_{2\,\rm had}^{(5)}(s)$ via e^+e^--data and isospin arguments [(u,d), s flavor separation]:

$$\Pi^{3\gamma}_{ud} = \frac{1}{2}\,\Pi^{\gamma\gamma}_{ud} \quad ; \quad \Pi^{3\gamma}_{s} = \frac{3}{4}\,\Pi^{\gamma\gamma}_{s}$$

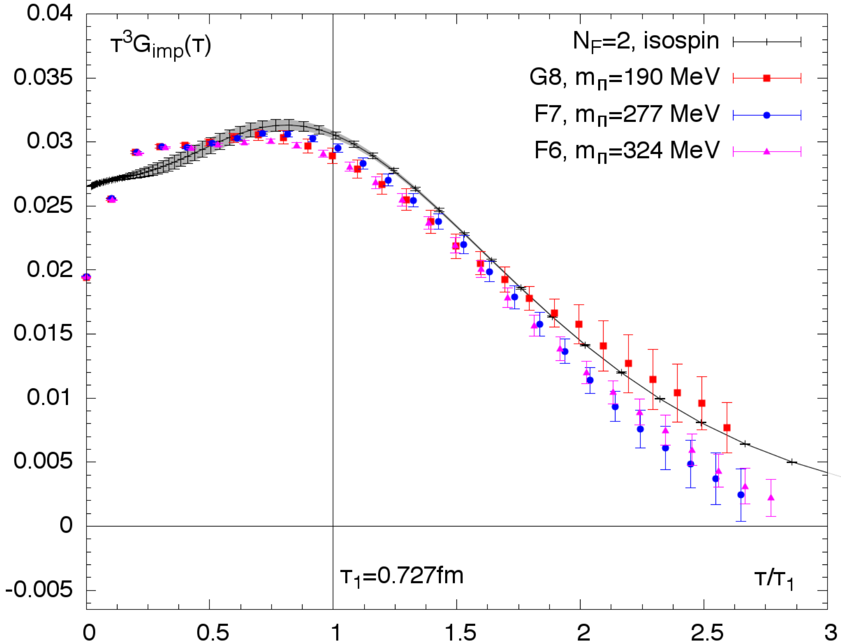
$$\Pi^{\gamma\gamma} = \Pi^{(\rho)} + \Pi^{(\omega)} + \Pi^{(\phi)} + \cdots \quad \Rightarrow \quad \Pi^{3\gamma} = \frac{1}{2}\,\Pi^{(\rho)} + \frac{3}{4}\,\Pi^{(\phi)} + \cdots$$

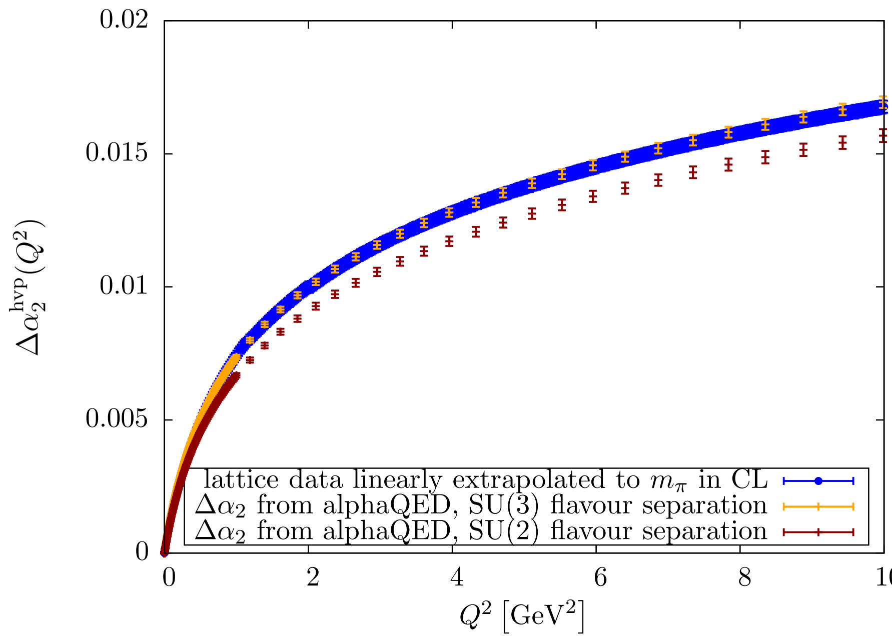
Flavor separation assuming OZI violating terms to be small $\Rightarrow$ perturbative rewighting $\Rightarrow$ disagrees with lattice QCD results!!!

Note that the “wrong” perturbative weighting

$$\Pi^{3\gamma}_{ud} = \frac{9}{20}\,\Pi^{\gamma\gamma}_{ud}\ ;\ \ \Pi^{3\gamma}_{s} = \frac{3}{4}\,\Pi^{\gamma\gamma}_{s}$$

has been proven to clearly mismatch lattice results, while the correction $\frac{9}{20} \Rightarrow \frac{10}{20}$ is in good agreement. This also means the OZI suppressed contributions should be at the 5% level and not negligibly small.

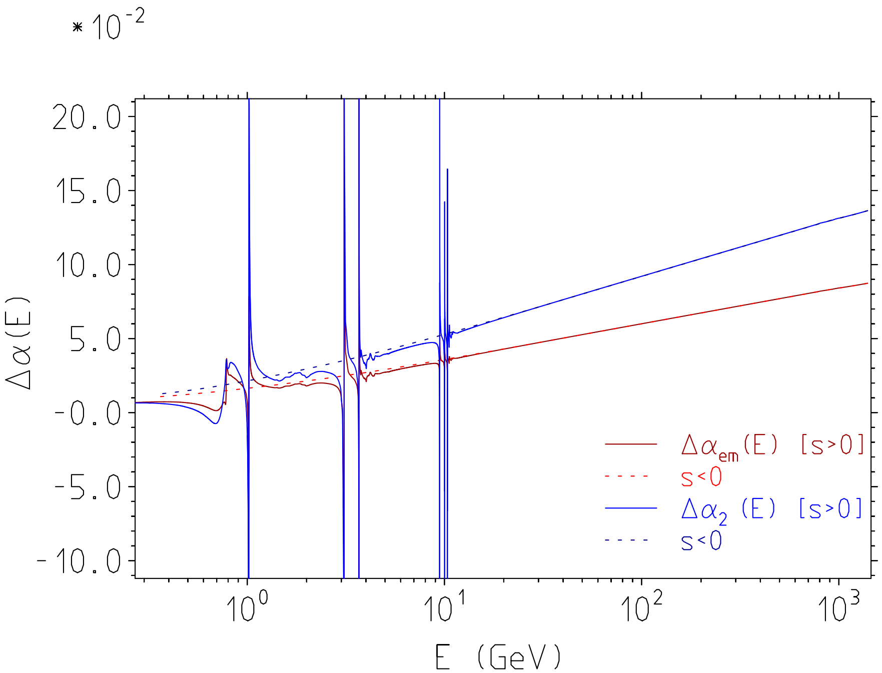




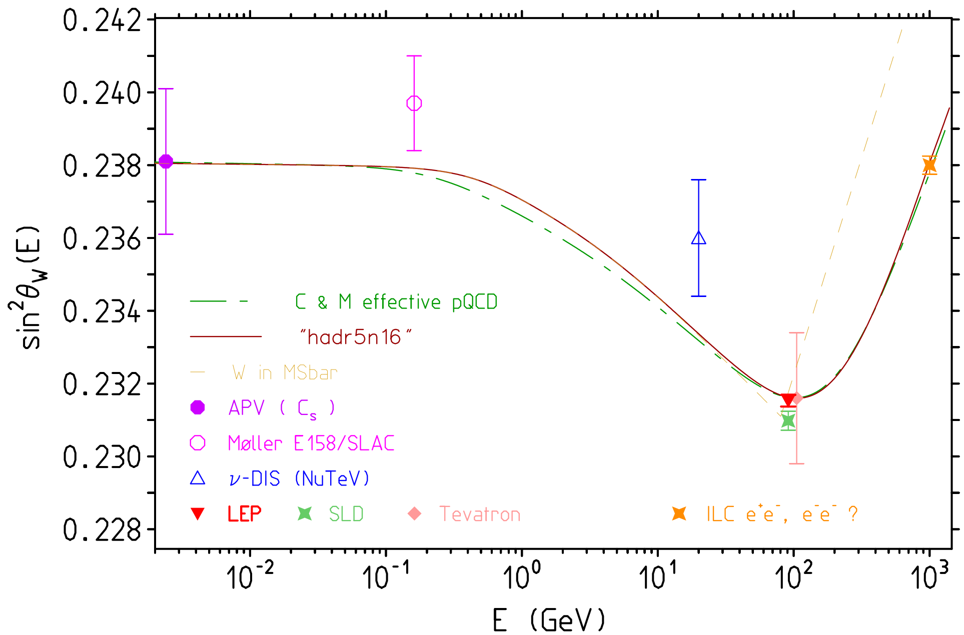
Testing flavor separation H. Meyer et al. [l], arXiv:1312.0035, K. Jansen et al. arXiv:1505.03283[r]

Note: gauge boson SE potentially very sensitive to New Physics (oblique corrections)

➠new physics may be obscured by non-perturbative hadronic effects; need to fix this!

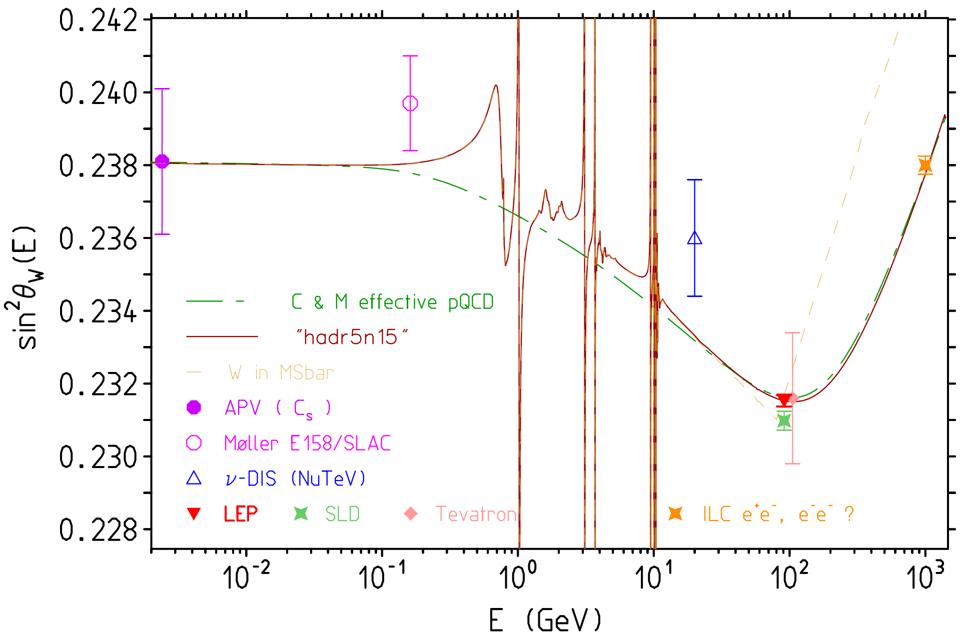


$\Delta\alpha_{\rm em}(E)$ and $\Delta\alpha_2(E)$ as functions of energy E in the time-like and space-like domain. The smooth space-like correction (dashed line) agrees rather well with the non-resonant "background" above the ϕ-resonance (kind of duality). In resonance regions as expected "agreement" is observed in the mean, with huge local deviations.



$\sin^2\Theta_W(Q)$ as a function of Q in the space-like region. Hadronic uncertainties are included but barely visible. Uncertainties from the input parameter $\sin^2\theta_W(0) = 0.23822(100)$ or $\sin^2\theta_W(M_Z^2) = 0.23153(16)$ are not shown. Future ILC/FCC measurements at 1 TeV would be sensitive to Z', H^{--} etc.

Except from the LEP and SLD points (which deviate by 1.8 σ), all existing measurements are of rather limited accuracy unfortunately!



$\sin^2\Theta_W(E)$ as a function of E in the time-like region. Note that $\sin^2\theta_W(0)/\sin^2\theta_W(M_Z^2) = 1.02876$ a 3% correction established at $6.5\ \sigma$.

$\sin^2\Theta_{\rm eff}$

exhibiting a specific dependence on the gauge boson SEs
is an excellent monitor for New Physics

HVP possible improvements

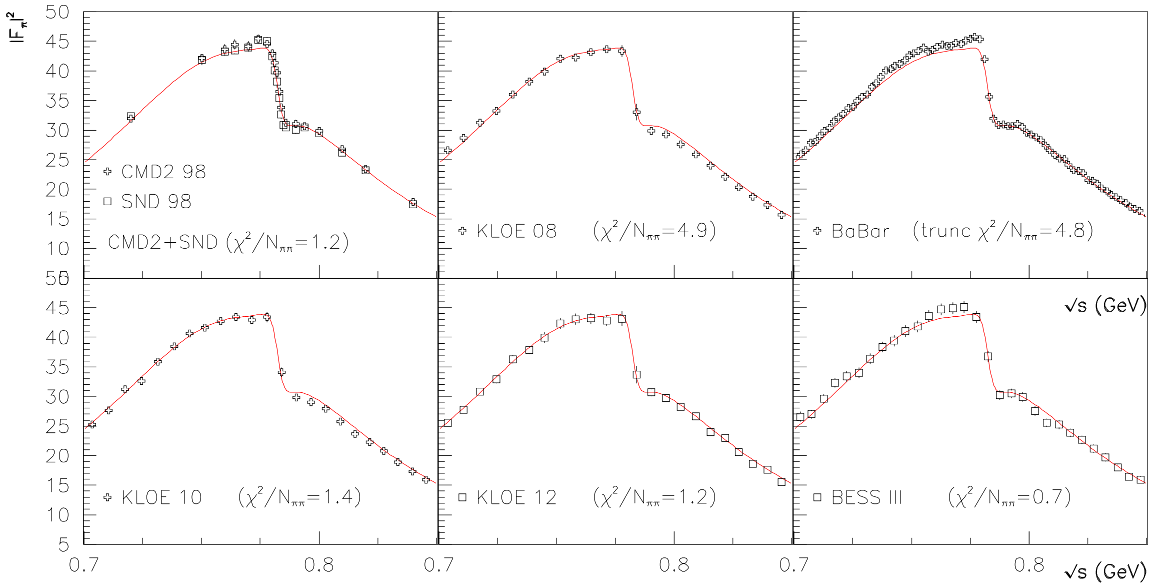
Additional data besides e^+e^- ones providing improvements:

Requires modeling: CHPT + spin 1 resonances (VMD) $\Rightarrow$ Resonance Lagrangian Approach e.g. HLS (massive Yang-Mills) $\Rightarrow$ dynamical widths, dynamical mixing of $\gamma, \rho^0, \omega, \phi$

❒ Global Fit strategy: Benayoun et al.
Data below $E_0 = 1.05$ GeV (just above the ϕ) constrain effective Lagrangian couplings, using 45 different data sets (6 annihilation channels and 10 partial width decays).
❒ Effective theory predicts cross sections:
$\pi^+\pi^-,\ \pi^0\gamma,\ \eta\gamma,\ \eta'\gamma,\ \pi^0\pi^+\pi^-,\ K^+K^-,\ K^0\bar{K}^0$ (83.4%),

- Missing part: $4\pi, 5\pi, 6\pi, \eta\pi\pi, \omega\pi$ and regime $E > E_0$ evaluated using data directly and pQCD for perturbative region and tail
- Including $\boxed{\text{self-energy effects}}$ is mandatory ($\gamma\rho$-mixing, $\rho\omega$-mixing ..., decays with proper phase space, energy dependent width etc)

- Method works in reducing uncertainties by using indirect constraints
- Able to reveal inconsistencies in data, e.g. KLOE vs BaBar



Comparing the τ+PDG prediction (red curve) of the pion form factor in e^+e^- annihilation in the $\rho - \omega$ interference region. Benayoun et al. 2015

- τ data missing $\gamma - \rho^0$ mixing correction Szafron et al.

$$v_-(s) \to v_0(s) = r_{\rho\gamma}(s)\, R_{\rm IB}(s)\, v_-(s)$$

for the I=1 part of $a_\mu^{\rm had}[\pi\pi]$ results in

$$\delta a_\mu^{\rm had}[\rho\gamma] \simeq (-5.1 \pm 0.5) \times 10^{-10}\;.$$

as a correction applied for the range [0.63,0.96] GeV. The correction is not too large, but at the level of 1 σ and thus non-negligible.

Including IB corrected τ data:

$$a_\mu^{\rm had(1)} = (688.07 \pm 4.14)[688.77 \pm 3.38] \times 10^{-10}$$

based on e^+e^-–data [incl. τ-decay spectra].

● using $\pi\pi$ scattering phase shifts Caprini et al. 2013 contribution to a_μ from below 0.63 GeV yields

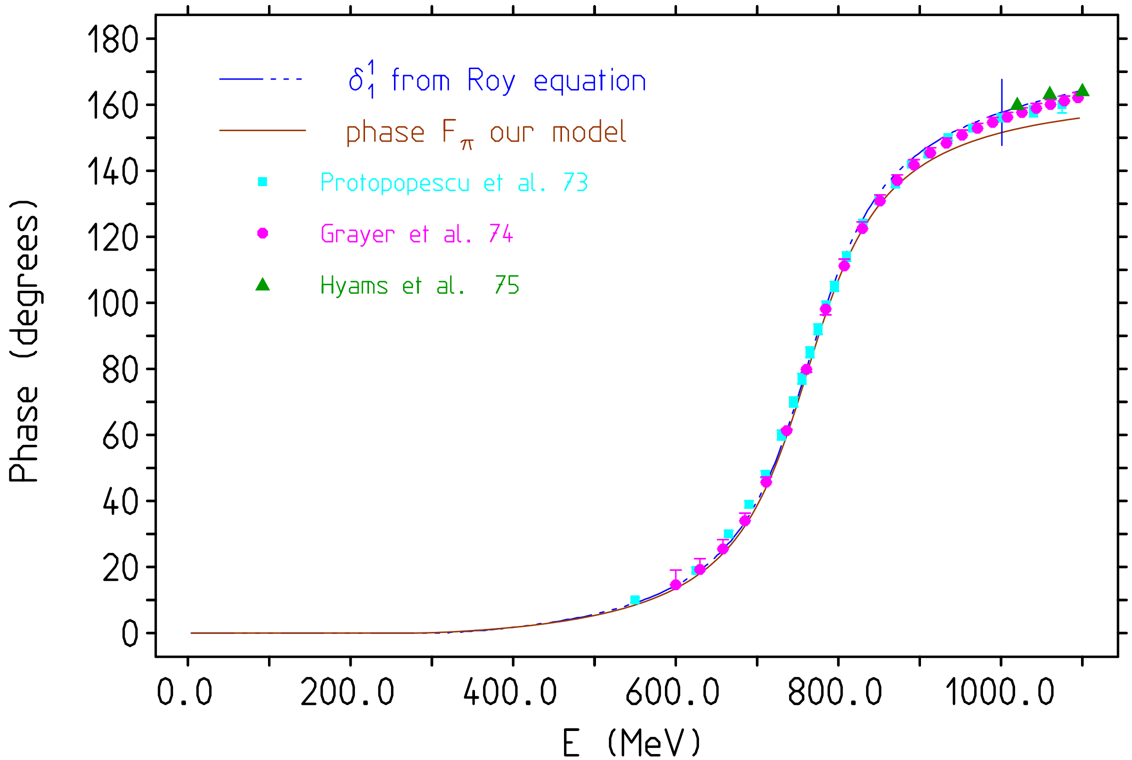
$$a_\mu^{\pi\pi(\gamma),\mathrm{LO}}[2m_\pi,\ 0.63\ \mathrm{GeV}] = (133.258 \pm 0.723) \times 10^{-10}\,,$$

a 40% reduction of the error estimated in a standard calculation in terms of e^+e^- data which yields $132.57(055)(0.93)[1.19] \times 10^{-10}$.

One obtains

$$a_\mu^{\mathrm{had}(1)} = (689.46 \pm 3.25) \times 10^{-10}\,,$$

as a best estimate.



The phase of $F_\pi(E)$ as a function of the c.m. energy E. We compare the result of the elaborate Roy equation analysis of Leutwyler 02, Colangelo 03, Caprini 16 with the one due to the sQED pion-loop and data Hyams 73, Grayer 74, Protopopescu 73. By analyticity the phase determines the modulus of F_π

HVP from lattice QCD

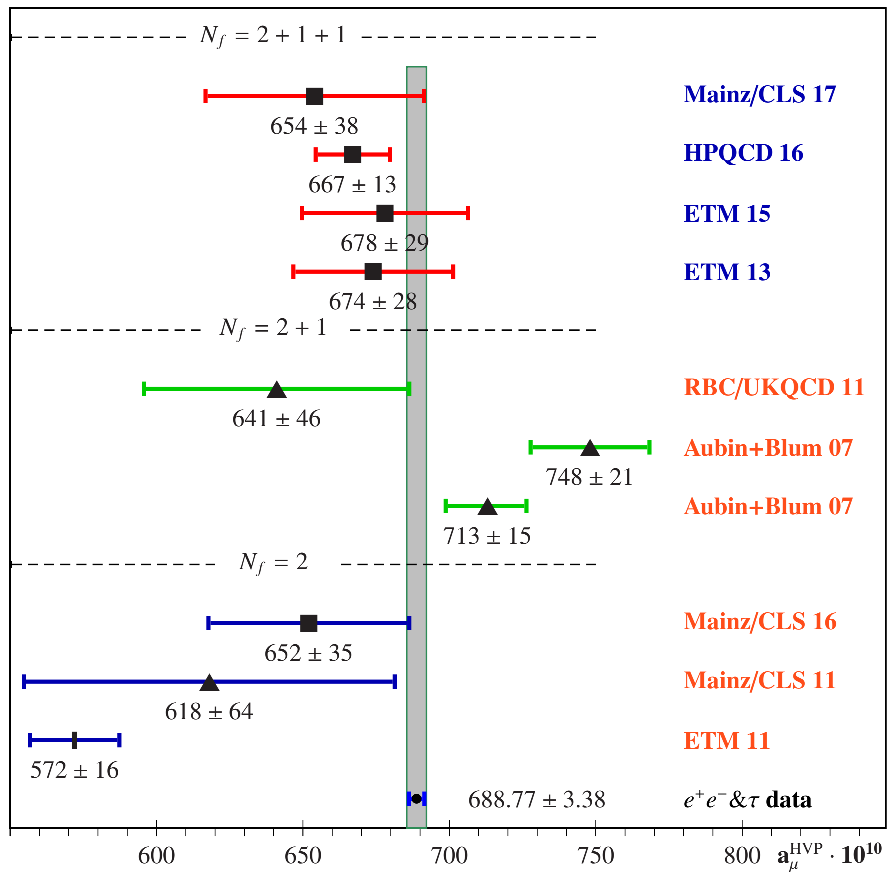
Hartmut Wittig's Talk

Lattice QCD can provide important input on HVP and HLbL to $a_\mu^{\rm had}$

The hope is that LQCD can deliver estimates of accuracy

$$\delta a_\mu^{\rm HVP}/a_\mu^{\rm HVP} < 0.5\% \;, \;\; \delta a_\mu^{\rm HLbL}/a_\mu^{\rm HLbL} \lesssim 10\%$$

in the coming years.



Summary of recent LQCD results for the leading order a_μ^{HVP}, in units 10^{-10}. Labels: ■ marks u, d, s, c, ▲ u, d, s and ▮ u, d contributions. Individual flavor contributions from light (u, d) amount to about 90%, strange about 8% and charm about 2%.

Brookhaven, Zeuthen, Mainz, Edinburgh, ...

Details of possible missing effects in lattice QCD calculations of LO HVP $a_\ell^{\rm had}$ in Tables below (backup slides):

$m_{\pi^0} \to m_{\pi^\pm}$ shifts should be performed on lattice data directly! $e^+e^- \to \rm hadrons$ measures all-inclusive, precise m_π-dependence model dependent

So corrections left

Total shift a_μ

type of correction	$\delta a_\mu \times 10^{-10}$	$\delta a_e \times 10^{12}$	$\delta a_\tau \times 10^{8}$
iso+em from $\pi\pi$ channel :	+4.16	+ 1.42	-0.38
incl e.m. decays $\pi^0\gamma$ and $\eta\gamma$:	+ 5.29	+ 1.19(1)(4)	+ 2.06(2)(7)
missing $\phi \to \pi^+\pi^-\pi^0$:	+ 5.26	+ 1.35(4)	+ 2.78(8)
sum	14.71(1.5)	3.96(0.4)	4.46(0.4)

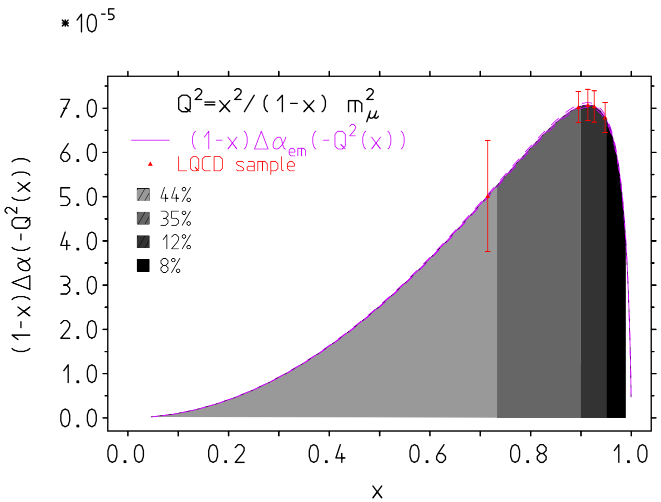
Alternative method: measure space-like $\alpha_{\rm QED,eff}(t)$

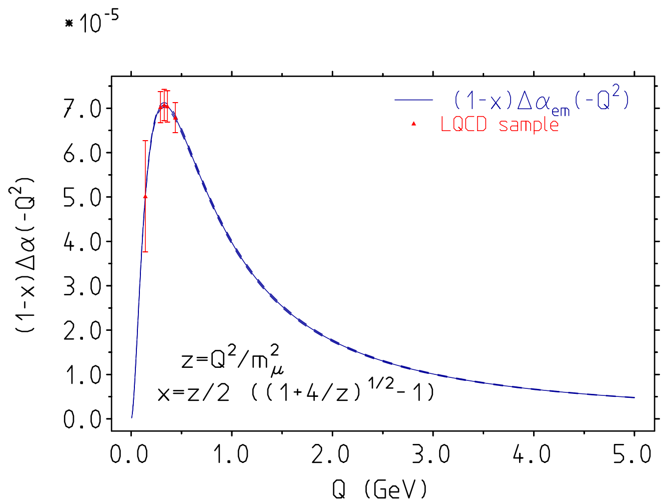
Newly proposed recently: [arXiv:1504.02228,1609.08987]
"A new approach to evaluate the leading hadronic corrections to the muon g-2"
Carloni Calame, Passera, Trentadue, Venanzoni 2015; Abbiendi et al. 2016

- space-like $\Delta\alpha_{\rm had}(-Q^2) = 1 - \frac{\alpha}{\alpha(-Q^2)} - \Delta\alpha^{\rm lep}(-Q^2)$ determines $a_\mu^{\rm had}$ via

$$\boxed{a_\mu^{\rm had} = \frac{\alpha}{\pi}\int\limits_0^1 {\rm d}x\,(1-x)\,\Delta\alpha_{\rm had}\left(-Q^2(x)\right)}$$

where $Q^2(x) \equiv \frac{x^2}{1-x}m_\mu^2$ is the space–like square momentum–transfer. Also in the Euclidean region the integrand is highly peaked, now around half of the ρ meson mass scale.



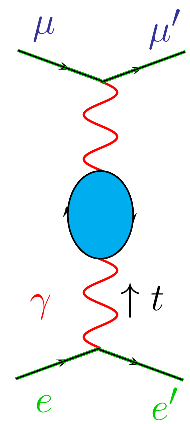


The integrand of a_μ^{had} integral as functions of x and Q. Strongly peaked at about 330 MeV.

- measuring directly low energy running $\alpha_{\mathrm{QED}}(s)$ in space-like region via
- very different paradigm: no VP subtraction issue!
- no exclusive channel collection
- even 1% level measurement can provide important independent information

$\mu^- e^-$ scattering

$$\mu^-(p_-)\;\; e^-(q_-) \to \mu^-(p'_-)\;\; e^-(q'_-)$$



Get a_μ^{had} from $\mu^- e^- \to \mu^- e^-$ process

G. Abbiendi et al. , arXiv:1609.08987 Luca Trentadue's Talk

$$\frac{d\sigma^{\mathrm{unpol.}}_{\mu^- e^- \to \mu^- e^-}}{dt} = 4\pi\, \alpha(t)^2\, \frac{1}{\lambda(s, m_e^2, m_\mu^2)} \left\{ \frac{\left(s - m_\mu^2 - m_e^2\right)^2}{t^2} + \frac{s}{t} + \frac{1}{2} \right\}$$

- The primary goal of [arXiv:1504.02228,1609.08987]: determining a_μ^{had}

in an alternative way

- $\Pi'_\gamma(Q^2) - \Pi'_\gamma(0) = -\Delta\alpha_{\rm had}(-Q^2) = \frac{\alpha}{\alpha(-Q^2)} + \Delta\alpha^{\rm lep}(-Q^2) - 1$

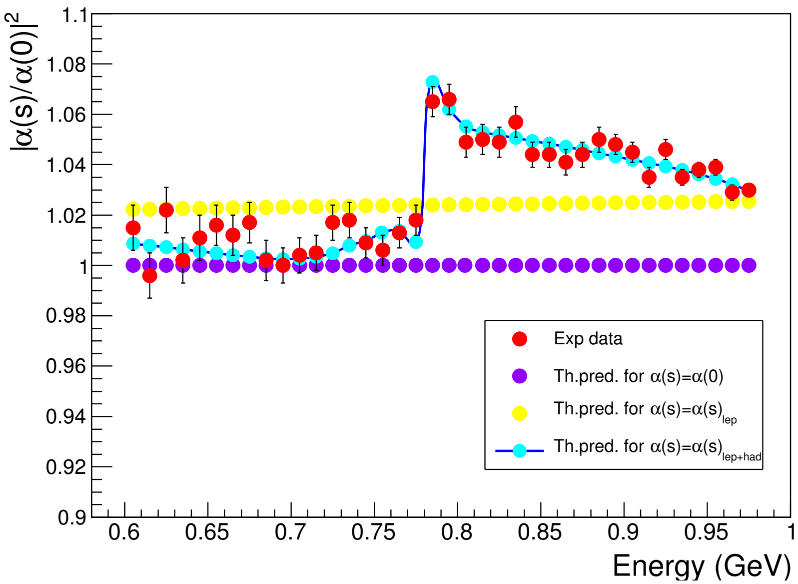
  directly checks lattice QCD data
- My proposal here: determine very accurately

$$\boxed{\Delta\alpha_{\rm had}\left(-Q^2\right) \text{ at } Q \approx 2.5 \text{ GeV}}$$

by this method (one single number!) as the non-perturbative part of $\Delta\alpha_{\rm had}\left(M_Z^2\right)$ as in “Adler function” approach.

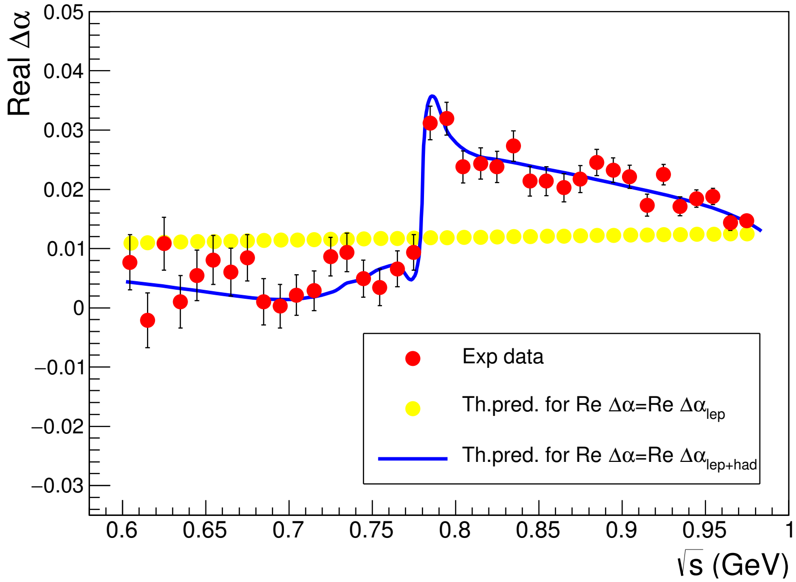
News on VP subtraction

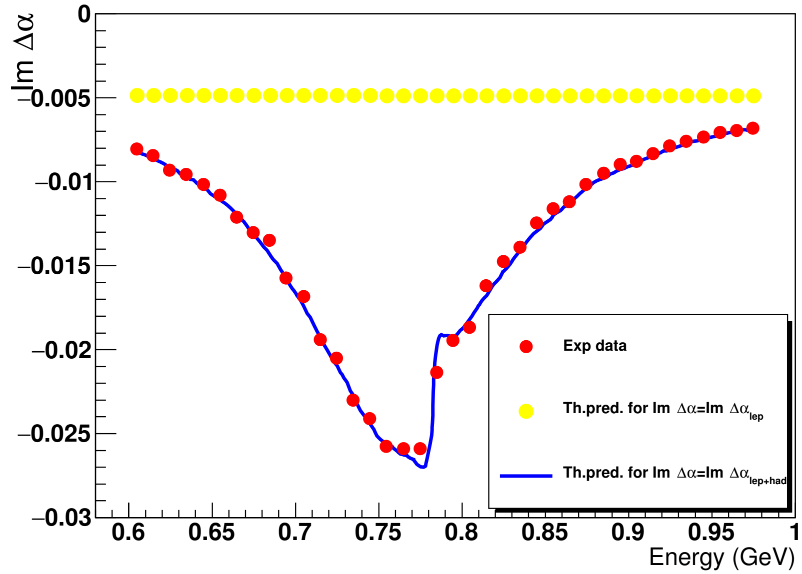
First measurement of complex VP function in ρ resonance region $\Leftrightarrow$ complex
$\Delta\alpha_{\rm QED}(s) = -[\Pi'_\gamma(s) - \Pi'_\gamma(0)]$
KLOE 2016, arXiv:1609.06631, Graziano Talk



$$\left|\frac{\alpha(s)}{\alpha(0)}\right|^2 = \left|1+\Pi'_{\gamma\,\rm ren}\right|^{-2} \; ;$$
$$\sigma^{(0)} = \sigma^{\rm data}\left|1+\Pi'_{\gamma\,\rm ren}\right|^{2} \; ;$$

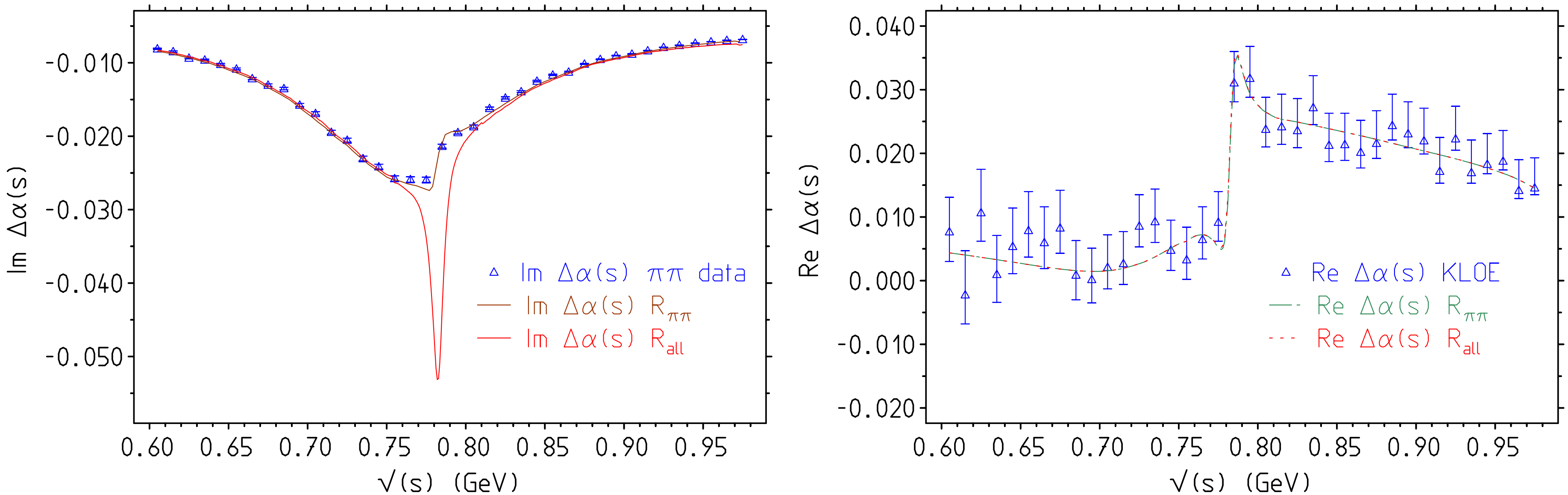
VP subtraction by locally measured quantity usually to be obtained via dispersion integral (as a global object)





❒ $\left|\frac{\alpha(s)}{\alpha(0)}\right|^2 = \frac{\sigma(e^+e^- \to \mu^+\mu^-)}{\sigma(e^+e^- \to \mu^+\mu^-)_{\rm pt}}$ ❒ $R(s) = \frac{\sigma(e^+e^- \to \pi^+\pi^-)}{\sigma(e^+e^- \to \mu^+\mu^-)} \Rightarrow$ Re $\alpha(s)$, Im $\alpha(s)$

We mention that in the imaginary part is only included the $\pi\pi$ part measured in the same experiment (KLOE). The 3π channel could have been added from other experiments which have measured that channel. The effect is illustrated in the Figure



Including the missing 3π channel changes Im $\Delta\alpha_{\rm had}$ substantially at the ω resonance, which is not included in KLOE paper

In contrast plot for $|\alpha(s)/\alpha(0)|^2$ includes the effects from all channels and effect buried in errors of $\mathrm{Real}\Delta\alpha(s)$.

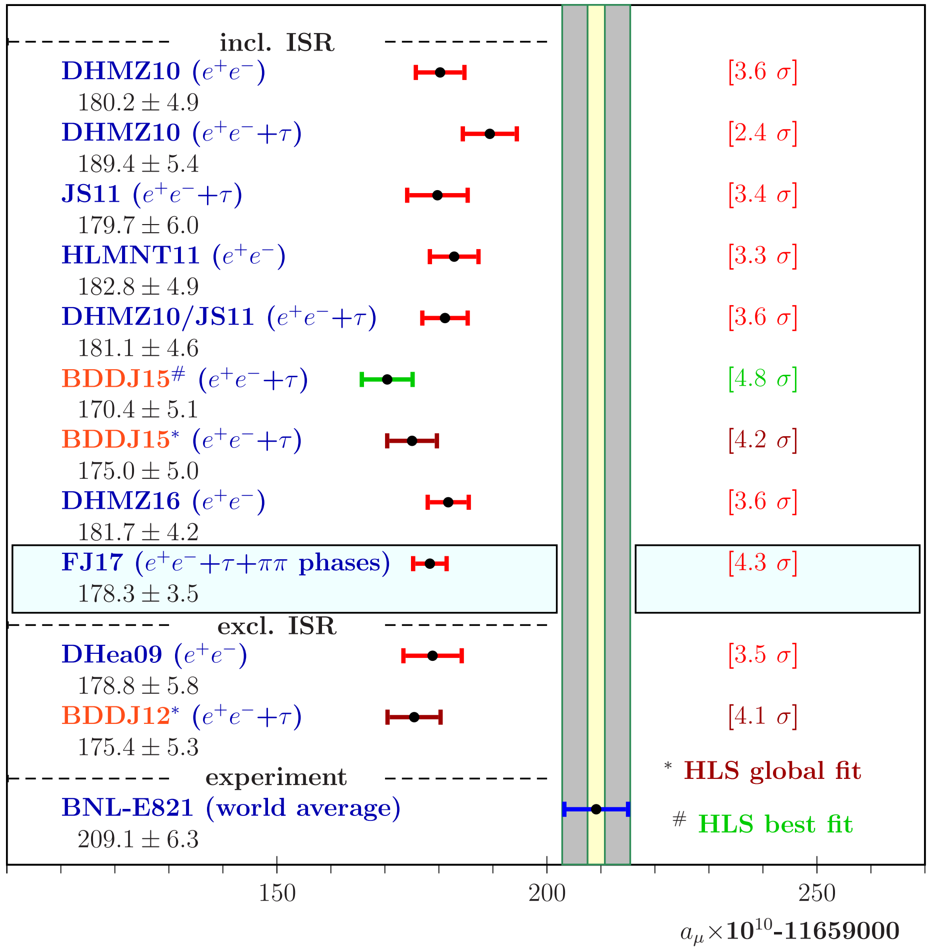
HVP for the muon anomaly

Thomas Teubner, Zhiqing Zhang's Talks

$a_\mu^{\mathrm{had}(1)} = (689.46 \pm 3.25)[688.77 \pm 3.38][688.07 \pm 1.14]\ 10^{-10}$ (LO)
$a_\mu^{\mathrm{had}(2)} = (-99.27 \pm 0.67)\ 10^{-10}$ (NLO)
$a_\mu^{\mathrm{had}(3)} = (\quad 1.224 \pm 0.010)\ 10^{-10}$ (NNLO) Kurz et al 2014
$a_\mu^{\mathrm{had,LbL}} = 10.34 \pm 2.88 \times 10^{-10}$ (HLbL)
$a_\mu^{\mathrm{weak}} = (15.36 \pm 0.11[m_H, m_t] \pm 0.023[\mathrm{had}])\ 10^{-10}$ (LO+NLO)
all e^+e^-–data based [2017 update]

The QED prediction of a_μ is given by Laporta 2017, Aoyama et al. 2012/14

$$\begin{aligned} a_\mu^{\mathrm{QED}} &= \frac{\alpha}{2\pi} + 0.765\,857\,423(16)\left(\frac{\alpha}{\pi}\right)^2 \\ &+24.050\,509\,82(28)\left(\frac{\alpha}{\pi}\right)^3 + 130.8734(60)\left(\frac{\alpha}{\pi}\right)^4 + 751.917(932)\left(\frac{\alpha}{\pi}\right)^5 . \end{aligned}$$



Dependence of a_μ predictions on recent evaluations of $a_\mu^{\mathrm{had,LO}}$

Standard model theory and experiment comparison

Contribution	Value $\times 10^{10}$	Error $\times 10^{10}$	Reference
QED incl. 4-loops + 5-loops	11 658 471.886	0.003	Aoyama12,Laporta17
Hadronic LO vacuum polarization	689.46	3.25	
Hadronic light–by–light	10.34	2.88	BPP,HK,KN,MV,JN
Hadronic HO vacuum polarization	-8.70	0.06	FJ17
Weak to 2-loops	15.36	0.11	Gnendiger et al. 13
Theory	11 659 178.3	3.5	–
Experiment	11 659 209.1	6.3	BNL
The. - Exp. 4.3 standard deviations	-30.6	7.2	–

HVP for the electron anomaly

$a_e^{\mathrm{had}(1)} = (184.90 \pm 1.08)\ 10^{-14}$ (LO)
$a_e^{\mathrm{had}(2)} = (-22.13 \pm 0.12)\ 10^{-14}$ (NLO)
$a_e^{\mathrm{had}(3)} = (\quad 2.80 \pm 0.02)\ 10^{-14}$ (NNLO) Kurz et al 2014
$a_e^{\mathrm{had,LbL}} = 3.7(5) \times 10^{-14}$ (HLbL)
$a_e^{\mathrm{weak}} = (3.053 \pm 0.002[m_H, m_t] \pm 0.023[\mathrm{had}])\ 10^{-14}$ (LO+NLO)
all e^+e^-–data based [2017 update]

The QED prediction of a_e is given by Laporta 2017, Aoyama et al. 2012/14

$$a_e^{\mathrm{QED}} = \frac{\alpha}{2\pi} - 0.328\,478\,444\,002\,54(33)\left(\frac{\alpha}{\pi}\right)^2 + 1.181\,234\,016\,816(11)\left(\frac{\alpha}{\pi}\right)^3 - 1.91134(182)\left(\frac{\alpha}{\pi}\right)^4 + 7.791(580)\left(\frac{\alpha}{\pi}\right)^5 .$$

For extracting α_{QED} based on the SM prediction

$$a_e^{\mathrm{SM}} = a_e^{\mathrm{QED}} + 1.723(12) \times 10^{-12} \text{ (hadronic \& weak)}$$

$$\alpha^{-1}(a_e) = 137.035\,999\,1550(331)(0)(27)(14)[333]$$

Contributions to $a_e(h/M)$ in units 10^{-6}

contribution	$\alpha(h/M_{\mathrm{Cs06}})[8.0\,\mathrm{ppb}]$ $\alpha^{-1} = 137.03600000(110)$	$\alpha(h/M_{\mathrm{Rb11}})[0.66\,\mathrm{ppb}]$ $\alpha^{-1} = 137.035999037(91)$
universal	1159.652 169 15(929)(0)(4)	1159.652 177 28(77)(0)(4)
μ–loops	0.000 002 738 (0)	0.000 002 738 (0)
τ–loops	0.000 000 009 (0)	0.000 000 009 (0)
hadronic	0.000 001 693 (13)	0.000 001 693 (13)
weak	0.000 000 030 (0)	0.000 000 030 (0)
theory	1159.652 173 59(929)	1159.652 181 73(77)
experiment	1159.652 180 73 (28)	1159.652 180 73 (28)
$a_e^{\mathrm{exp}} - a_e^{\mathrm{the}}$	$7.14(9.30) \times 10^{-12}$	$-1.00(0.82) \times 10^{-12}$

Backup Slides

Muon – neutral channel: missing effects in lattice QCD simulations performed in the isospin limit $m_d = m_u$ and without QED effects. Tabulated are the effects δa_μ in units 10^{-10}, integrated from 300 MeV to 1 GeV

Correction type	GS model	shift
$I = 1$ NC: GS fit of e^+e^- data, ω switched off	489.21*	
$\omega - \rho$ mixing	491.89	+2.68
FSR of ee $I = 1 + 0$	496.11	+4.22
$\gamma - \rho$ mixing	486.47	-2.74
elmag. shift $m_{\pi^0} \to m_{\pi^\pm}$		shift of *
$I = 1$ NC $m_\pi = m_{\pi^0}$ in $R(s)$ vs. $\|F_\pi\|^2$ [$\|F_\pi\|^2$ fixed]	502.01	+12.81
$I = 1$ NC m_π physical in $\|F_\pi\|^2$ [BW ρ FF]	455.89	
$I = 1$ NC $m_\pi = m_{\pi^0}$ in $\|F_\pi\|^2$	441.97	-13.92
combined $m_\pi = m_{\pi^0}$	500.91	
physical $m_\pi = m_{\pi^\pm}$ plus e.m. shift in mass&width	489.20	1.12
elmag. channels HLS12		
$\pi^0\gamma$	4.64 ± 0.04	
$\eta\gamma$	0.65 ± 0.01	
$\pi^+\pi^-\pi^0$ missing disconnected ?	5.26 ± 0.15	

Electron – neutral channel: missing effects in lattice QCD simulations performed in the isospin limit $m_d = m_u$ and without QED effects. Tabulated are the effects δa_μ in units 10^{-14}, integrated from 300 MeV to 1 GeV

Correction type	GS model	shift
$I=1$ NC: GS fit of e^+e^- data, ω switched off	134.49*	
$\omega-\rho$ mixing	135.24	+0.75
FSR of ee $I=1+0$	136.41	+1.17
$\gamma-\rho$ mixing	133.99	-0.50
elmag. shift $m_{\pi^0} \to m_{\pi^\pm}$		shift of *
$I=1$ NC $m_\pi = m_{\pi^0}$ in $R(s)$ vs. $\|F_\pi\|^2$ [$\|F_\pi\|^2$ fixed]	138.21	+3.72
$I=1$ NC m_π physical in $\|F_\pi\|^2$ [BW ρ FF]	125.76	
$I=1$ NC $m_\pi = m_{\pi^0}$ in $\|F_\pi\|^2$	121.85	-3.91
combined $m_\pi = m_{\pi^0}$	500.91	
physical $m_\pi = m_{\pi^\pm}$ plus e.m. shift in mass&width	489.20	+0.19
elmag. channels HLS12		
$\pi^0\gamma$	1.05 ± 0.04	
$\eta\gamma$	0.14 ± 0.01	
$\pi^+\pi^-\pi^0$ missing disconnected ?	5.26 ± 0.15	

Tau – neutral channel: missing effects in lattice QCD simulations performed in the isospin limit $m_d = m_u$ and without QED effects. Tabulated are the effects δa_μ in units 10^{-8}, integrated from 300 MeV to 1 GeV

Correction type	GS model	shift
$I = 1$ NC: GS fit of e^+e^- data, ω switched off	167.66*	
$\omega - \rho$ mixing	168.39	+0.73
FSR of ee $I = 1 + 0$	169.80	+1.41
$\gamma - \rho$ mixing	165.14	-2.52
elmag. shift $m_{\pi^0} \to m_{\pi^\pm}$		shift of *
$I = 1$ NC $m_\pi = m_{\pi^0}$ in $R(s)$ vs. $\lvert F_\pi\rvert^2$ [$\lvert F_\pi\rvert^2$ fixed]	171.22	+3.56
$I = 1$ NC m_π physical in $\lvert F_\pi\rvert^2$ [BW ρ FF]	154.23	
$I = 1$ NC $m_\pi = m_{\pi^0}$ in $\lvert F_\pi\rvert^2$	150.05	-4.18
combined $m_\pi = m_{\pi^0}$	500.91	
physical $m_\pi = m_{\pi^\pm}$ plus e.m. shift in mass&width	489.20	+0.62
elmag. channels HLS12		
$\pi^0\gamma$	1.77 ± 0.07	
$\eta\gamma$	0.29 ± 0.01	
$\pi^+\pi^-\pi^0$ missing disconnected ?	5.26 ± 0.15	

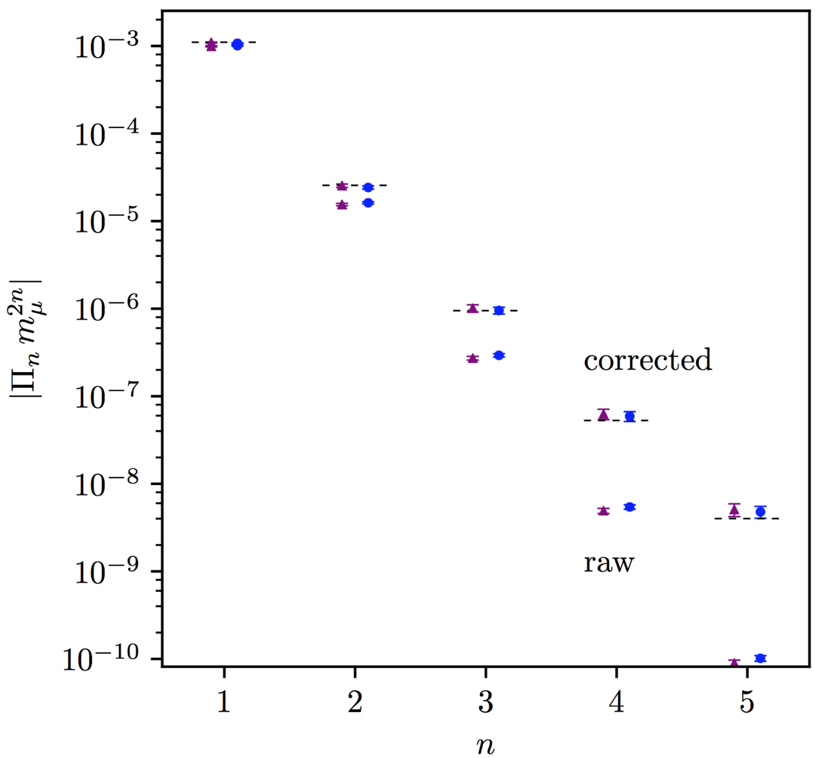
Based on the identity

$$-\frac{\Pi(Q^2)}{Q^2} = \frac{\alpha}{3\pi} \int\limits_{4m_\pi^2}^{\infty} \frac{\mathrm{d}s}{s} \frac{R(s)}{s+Q^2} \;.$$

the Taylor coefficients can be calculated dispersively in the time–like approach via

$$\Pi_{n+1} = (-1)^{(n+1)} \frac{\alpha}{3\pi} \int\limits_{4m_\pi^2}^{\infty} \frac{\mathrm{d}s}{s} \frac{R(s)}{s^{n+1}} \;.$$

Using the present world average (WA) compilation of the e^+e^- data (as available from hvpfunction) and the corresponding HLS model best fit one finds Benayoun 16



n	WA compilation		
0	1.01962131E+01	±	6.693577E-02
1	2.38190432E-01	±	1.257508E-03
2	8.89142868E-03	±	5.749533E-05
3	4.99117005E-04	±	4.018536E-06
4	3.78809709E-05	±	3.333508E-07
5	3.53345581E-06	±	2.971228E-08
6	4.06928851E-07	±	2.867316E-09

Contributions to the hadronic vacuum polarization $\hat{\Pi}(q^2)$ at $q^2 = -m_\mu^2$ coming from individual Taylor coefficients Π_n with $n = 1, 2, \ldots, 5$. Results are show for corrected (above) and uncorrected (raw , below) coefficients coming from lattice QCD simulations with physical sea-quark masses from two different lattices HPQCD Chakraborty et al. 16

Comment:
The recent analysis Davier et al. 15 reports 516.2 ± 3.5 for $e^+e^- + \tau$ in comparison to 506.9 ± 2.6 for e^+e^- for the range from threshold to 1.8 GeV. As below about 1 GeV the $\gamma - \rho$ mixing correction can be evaluated reliably via VMD II + sQED, it is determined by the electronic width of the ρ solely, we get 511.1 ± 3.5 and $a_\mu^{\mathrm{had\,LO}} = 692.6 \pm 3.3 \times 10^{-10}$ for e^+e^- becomes $a_\mu^{\mathrm{had\,LO}} = 696.8 \pm 4.0 \times 10^{-10}$ for $e^+e^- + \tau$ after the $\gamma - \rho$ mixing correction.

Complex vs. real α VP correction

- Usually adopted VP subtraction corrections: $\alpha(s) \to \alpha$

  $R(s)$ corrected by $(\alpha/\alpha(s))^2 = |1 + \mathrm{Re}\,\Pi'(s)|^2$ ($\Pi'(0)$ subtracted)

- more precisely, should subtract $|1 + \Pi'(s)|^2 = \alpha/|\alpha_c(s)|)^2$
  where $\alpha_c(s)$ complex version of running α

- complex version what the Novosibirsk CMD-2 Collaboration has been using in more recent analyzes [code available from Fedor Ignatov ⋙]

- Typically, corrections

$$\boxed{1 - |1 + \Pi'(s)|^2/(\alpha/\alpha(s))^2}$$

- non-resonance regions corrections $\lesssim$ 0.1 %

- at resonances where corrections $\sim 1/\Gamma_R$

Note: imaginary parts from narrow resonances, $\mathrm{Im}\,\Pi'(s)) = \frac{\alpha}{3}R(s) = \frac{3}{\alpha}\frac{\Gamma_{ee}}{\Gamma}$ at peak, are sharp spikes and are obtained correctly only by appropriately high resolution scans. For example,

$$\boxed{|1+\Pi'(s)|^2 - (\alpha/\alpha(s))^2 = (\mathrm{Im}\,\Pi'(s))^2}$$

at $\sqrt{s} = M_R$ is given by

1.23×10^{-3} $[\rho]$, 2.76×10^{-3} $[\omega]$, 1.56×10^{-2} $[\phi]$, 594.81 $[J/\psi]$, 9.58 $[\psi_2]$, 2.66×10^{-4} $[\psi_3]$, 104.26 $[\Upsilon_1]$, 30.51$[\Upsilon_2]$, 55.58 $[\Upsilon_3]$

Outlook

Good progress in collecting e^+e^- data and lattice QCD comes closer. Lattice already provided important information not available from elsewhere:

- concerning flavor separation $SU(2)$ correlators Harvey Meyer et al.
  $\Rightarrow$ non-perturbative part on running $\sin^2\theta$
- doubly virtual $\pi^0 \to \gamma^*\gamma^*$ Gerardin et al. rules out simple VMD transition form factor LMD+V type singled out!
- Extracting HPV from data: errors estimated are very progressive, data not very consistent $\Rightarrow$ more data still desperately needed
  $\Rightarrow$ please experimental friends, do not relax and continue your heroic efforts!
- Waiting for Fermilab and J-PARC new $g-2$ measurements: an exciting time to come.

Good luck for a successful new attack on the SM – new puzzles for theoreticians in sight?

$\boxed{\text{Thank you for your attention!}}$