Hadronic light-by-light scattering in the muon $g-2$

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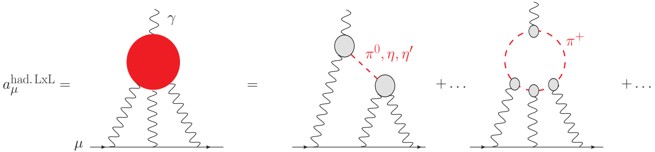
International Workshop on e^+e^- collisions from Phi to Psi 2017 (PhiPsi17)
Schloss Waldthausen (near Mainz), Germany, 26 - 29 June 2017

Outline

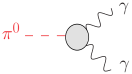
- Introduction: Hadronic light-by-light scattering (HLbL) in the muon $g-2$
- Current status: Model calculations
- Model-independent approaches:
  1. HLbL from dispersion relations (data driven approach)
     (Theory Talks on Thursday by Danilkin, Kubis, Pauk, Colangelo)
  2. HLbL from Lattice QCD
     (Talks on Thursday by Wittig, Lehner)
- Conclusions and Outlook

Hadronic light-by-light scattering

HLbL in muon $g - 2$ from strong interactions (QCD):



Coupling of photons to hadrons, e.g. π^0, via form factor:



- Relevant scales ($\langle VVVV \rangle$ with offshell photons): $0 - 2$ GeV $\gg m_\mu$ (hadronic resonance region)
- View before 2014: in contrast to HVP, no direct relation to experimental data $\rightarrow$ size and even sign of HLbL contribution to a_μ unknown !
- Approach: use hadronic model at low energies with exchanges and loops of resonances and some (dressed) "quark-loop" at high energies.
- Constrain models using experimental data (processes of hadrons with photons: decays, form factors, scattering) and theory (ChPT at low energies; short-distance constraints from pQCD / OPE at high momenta).
- Problems: Four-point function $\Pi_{\mu\nu\rho\sigma}(q_1, q_2, q_3)$ involves many Lorentz structures that depend on several invariant momenta $\Rightarrow$ distinction between low and high energies not as easy as for two-point function in HVP. Mixed regions: one loop momentum Q_1^2 large, the other Q_2^2 small and vice versa.

HLbL in muon $g-2$

- Frequently used estimates:

$$a_\mu^{\text{HLbL}} = (105 \pm 26) \times 10^{-11}$$ (Prades, de Rafael, Vainshtein '09) ("Glasgow consensus")

$$a_\mu^{\text{HLbL}} = (116 \pm 39) \times 10^{-11}$$ (AN '09; Jegerlehner, AN '09)

  Based almost on same input: calculations by various groups using different models for individual contributions. Error estimates are mostly guesses !
  For comparison:

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} \approx 300 \times 10^{-11} \qquad (3-4\ \sigma)$$

- Need much better understanding of complicated hadronic dynamics to get reliable error estimate of $\pm 20 \times 10^{-11}$ (δa_μ(future exp) $= 16 \times 10^{-11}$).

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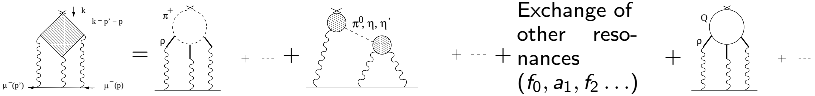
- Recent new proposal: Colangelo et al. '14, '15; Pauk, Vanderhaeghen '14: use dispersion relations (DR) to connect contribution to HLbL from presumably numerically dominant light pseudoscalars to in principle measurable form factors and cross-sections (no data yet !):

$$\gamma^*\gamma^* \quad \to \quad \pi^0, \eta, \eta'; \quad \pi^+\pi^-, \pi^0\pi^0$$

Could connect HLbL uncertainty to exp. measurement errors, like HVP.

- Future: HLbL from Lattice QCD (model-independent, first-principle). First steps and results: Blum et al. (RBC-UKQCD) '05, ..., '15, '16, '17. Work ongoing by Mainz group: Green et al. '15; Asmussen et al. '16, '17.

HLbL in muon $g-2$: model calculations (summary of selected results)



de Rafael '94:

Chiral counting:	p^4	p^6	p^8	p^8
N_C-counting:	1	N_C	N_C	N_C

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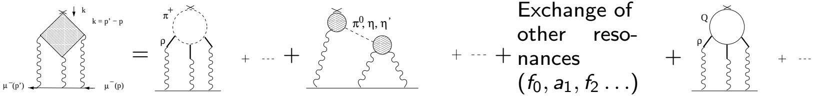

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Chiral counting:		p^4	p^6	p^8	p^8
N_C-counting:		1	N_C	N_C	N_C

Contribution to $a_\mu \times 10^{11}$:

BPP:	+83 (32)	-19 (13)	+85 (13)	-4 (3) [f_0, a_1]	+21 (3)
HKS:	+90 (15)	-5 (8)	+83 (6)	+1.7 (1.7) [a_1]	+10 (11)
KN:	+80 (40)		+83 (12)		
MV:	+136 (25)	0 (10)	+114 (10)	+22 (5) [a_1]	0
2007:	+110 (40)				
PdRV:	+105 (26)	-19 (19)	+114 (13)	+8 (12) [f_0, a_1]	+2.3 [c-quark]
N,JN:	+116 (39)	-19 (13)	+99 (16)	+15 (7) [f_0, a_1]	+21 (3)
		ud.: -45	ud.: $+\infty$		ud.: +60

ud. = undressed, i.e. point vertices without form factors

Recall (in units of 10^{-11}): $\delta a_\mu(\text{HVP}) \approx 30-40$; $\delta a_\mu(\text{exp [BNL]}) = 63$; $\delta a_\mu(\text{future exp}) = 16$

Pseudoscalars π^0, η, η': numerically dominant contribution (according to most models !). Other contributions not negligible. Cancellation between (dressed) π-loop and (dressed) quark-loop !

BPP = Bijnens, Pallante, Prades '96, '02; HKS = Hayakawa, Kinoshita, Sanda '96, '98, '02; KN = Knecht, AN '02; MV = Melnikov, Vainshtein '04; 2007 = Bijnens, Prades; Miller, de Rafael, Roberts; PdRV = Prades, de Rafael, Vainshtein '09 (compilation; Glasgow consensus); N,JN = AN '09; Jegerlehner, AN '09 (compilation)

HLbL in muon $g-2$: model calculations (continued)

Contribution	BPP	HKS, HK	KN	MV	BP, MdRR	PdRV	N, JN
π^0, η, η'	85 ± 13	82.7 ± 6.4	83 ± 12	114 ± 10	–	114 ± 13	99 ± 16
axial vectors	2.5 ± 1.0	1.7 ± 1.7	–	22 ± 5	–	15 ± 10	22 ± 5
scalars	-6.8 ± 2.0	–	–	–	–	-7 ± 7	-7 ± 2
π, K loops	-19 ± 13	-4.5 ± 8.1	–	–	–	-19 ± 19	-19 ± 13
π, K loops +subl. N_C	–	–	–	0 ± 10	–	–	–
quark loops	21 ± 3	9.7 ± 11.1	–	–	–	2.3 (c-quark)	21 ± 3
Total	83 ± 32	89.6 ± 15.4	80 ± 40	136 ± 25	110 ± 40	105 ± 26	116 ± 39

BPP = Bijnens, Pallante, Prades '95, '96, '02; HKS = Hayakawa, Kinoshita, Sanda '95, '96; HK = Hayakawa, Kinoshita '98, '02; KN = Knecht, AN '02; MV = Melnikov, Vainshtein '04; BP = Bijnens, Prades '07; MdRR = Miller, de Rafael, Roberts '07; PdRV = Prades, de Rafael, Vainshtein '09; N = AN '09, JN = Jegerlehner, AN '09

- PdRV (Glasgow consensus): Do not consider dressed light quark loops as separate contribution. Assume it is already taken into account by using short-distance constraint of MV '04 on pseudoscalar-pole contribution (no form factor at external vertex). Added all errors in quadrature.
- N, JN: New evaluation of pseudoscalar exchange contribution imposing new short-distance constraint on off-shell form factors. Took over most values from BPP, except axial vectors from MV. Added all errors linearly.
- Note that recent reevaluations of axial vector contribution lead to much smaller estimates than in MV: $a_\mu^{\mathrm{HLbL;axial}} = (8\pm3)\times10^{-11}$ (Pauk, Vanderhaeghen '14; Jegerlehner '14, '15). This would shift central values of compilations downwards:

$$a_\mu^{\mathrm{HLbL}} = (98\pm26)\times10^{-11} \quad \text{(PdRV)}$$
$$a_\mu^{\mathrm{HLbL}} = (102\pm39)\times10^{-11} \quad \text{(N, JN)}$$

Model calculations of HLbL: other recent developments

- First estimate for tensor mesons (Pauk, Vanderhaeghen '14):

$$a_\mu^{\mathrm{HLbL;tensor}} = 1 \times 10^{-11}$$

- Dressed pion-loop
Potentially important effect from pion polarizability and a_1 resonance
(Engel, Patel, Ramsey-Musolf '12; Engel '13; Engel, Ramsey-Musolf '13)
Maybe large negative contribution, in contrast to BPP '96, HKS '96:

$$a_\mu^{\mathrm{HLbL;\pi-loop}} = -(11 - 71) \times 10^{-11}$$

Not confirmed by recent reanalysis by Bijnens, Relefors '15, '16. Essentially get again old central value from BPP, but smaller error estimate:

$$a_\mu^{\mathrm{HLbL;\pi-loop}} = (-20 \pm 5) \times 10^{-11}$$

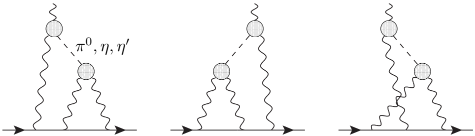
Hopefully new dispersive approaches can settle the issue without the use of models (Colangelo et al.; Danilkin, Pauk, Vanderhaeghen).

- Dressed quark-loop
Dyson-Schwinger equation approach (Fischer, Goecke, Williams '11, '13)
Large contribution, no damping seen, in contrast to BPP '96, HKS '96:

$$a_\mu^{\mathrm{HLbL;quark-loop}} = 107 \times 10^{-11} \qquad \text{(Incomplete calculation !)}$$

More general questions: How to get proper matching of models with perturbative QCD ? How to avoid double-counting of dressed quark-loop with hadronic contributions ?

Pion-pole contribution to $a_\mu^{\mathrm{HLbL};\pi^0}$ (analogously for η, η')



$$a_\mu^{\mathrm{HLbL};\pi^0} = \left(\frac{\alpha_e}{\pi}\right)^3 \left(a_\mu^{\mathrm{HLbL};\pi^0(1)} + a_\mu^{\mathrm{HLbL};\pi^0(2)}\right)$$

where [Jegerlehner, AN '09]

$$a_\mu^{\mathrm{HLbL};\pi^0(1)} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \; w_1(Q_1, Q_2, \tau) \; \mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_1^2, -(Q_1+Q_2)^2) \; \mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_2^2, 0)$$

$$a_\mu^{\mathrm{HLbL};\pi^0(2)} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \; w_2(Q_1, Q_2, \tau) \; \mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_1^2, -Q_2^2) \; \mathcal{F}_{\pi^0\gamma^*\gamma^*}(-(Q_1+Q_2)^2, 0)$$

$w_{1,2}(Q_1, Q_2, \tau)$ are model-independent weight functions which are concentrated at small momenta [AN '16]. Multiply the double- and single-virtual pion transition form factors (TFF's).

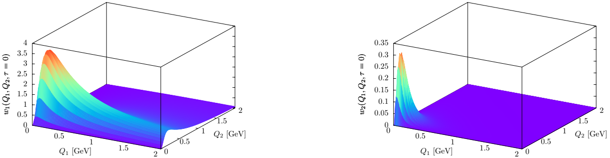
3-dim. integration over lengths $Q_i = |(Q_i)_\mu|, i = 1, 2$ of the two Euclidean momenta and angle θ between them: $Q_1 \cdot Q_2 = Q_1 Q_2 \cos\theta$ with $\tau = \cos\theta$. Note in arguments of form factors: $(Q_1 + Q_2)^2 \equiv Q_1^2 + 2Q_1 \cdot Q_2 + Q_2^2 = Q_1^2 + 2Q_1 Q_2 \tau + Q_2^2$.

Generalization to full HLbL tensor (Master formula from Colangelo et al. '15):

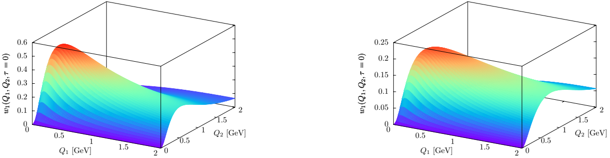
$$a_\mu^{\mathrm{HLbL}} = \frac{2\alpha^3}{3\pi^2} \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \sqrt{1-\tau^2}\, Q_1^3 Q_2^3 \sum_{i=1}^{12} T_i(Q_1, Q_2, \tau)\, \tilde{\Pi}_i(Q_1, Q_2, \tau)$$

$a_\mu^{\text{HLbL;P}}$: relevant momentum regions and results from models

Model-independent weight functions $w_{1,2}(Q_1, Q_2, \tau)$ for π^0 with $\theta = 90^\circ (\tau = 0)$:



Model-independent weight functions $w_1(Q_1, Q_2, \tau)$ for η (left) and η' (right):



- Relevant momentum regions below 1 GeV for π^0, below $1.5 - 2$ GeV for η, η'.
- w_1: behavior for large Q_1 corresponds to OPE limit in first TFF $\sim 1/Q_1^2$.
- Most model calculations for light pseudoscalars (poles or exchanges) agree at level of 15%, but full range of estimates (central values) much larger:

$$a_\mu^{\text{HLbL};\pi^0} = (50 - 80) \times 10^{-11} = (65 \pm 15) \times 10^{-11} \quad (\pm 23\%)$$
$$a_\mu^{\text{HLbL;P}} = (59 - 114) \times 10^{-11} = (87 \pm 27) \times 10^{-11} \quad (\pm 31\%)$$

- Hopefully soon DR approach to TFF's (Hoferichter et al. '14; Talk Kubis), data on $\gamma^*\gamma^* \to \pi^0, \eta, \eta'$ (AN '16; Talk Redmer) and lattice QCD calculations of TFF's (Gérardin, Meyer, AN '16) can give precise and model-independent results.

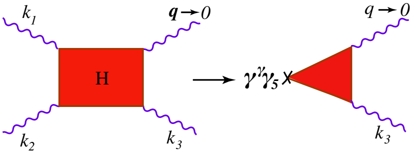
Short-distance constraint from OPE on HLbL in $g-2$

Melnikov; Vainshtein '04, further explanations in Prades, de Rafael, Vainshtein '09.

In HLbL contribution to $g-2$ consider OPE limit $k_1^2 \approx k_2^2 \gg k_3^2$ and $k_1^2 \approx k_2^2 \gg m_\rho^2$:

$$\int d^4x_1 \int d^4x_2\, \mathrm{e}^{-ik_1\cdot x_1 - ik_2\cdot x_2}\, j_\nu(x_1)\, j_\rho(x_2) = \frac{2}{\hat{k}^2}\, \epsilon_{\nu\rho\delta\gamma} \hat{k}^\delta \int d^4z\, \mathrm{e}^{-ik_3\cdot z}\, j_5^\gamma(z) + \mathcal{O}\left(\frac{1}{\hat{k}^3}\right)$$

$j_5^\gamma = \sum_q Q_q^2\, \bar{q}\gamma^\gamma\gamma_5 q$ is the axial current, $\hat{k} = (k_1 - k_2)/2 \approx k_1 \approx -k_2$



- Strong constraints from the $\langle AVV \rangle$ triangle diagram (non-renormalization theorems: Adler, Bardeen '69; 't Hooft '80; Vainsthein '03; Knecht et al. '04).
- At large k_1^2, k_2^2 the pseudoscalar and axial-vector exchanges dominate in HLbL.
- Constraints seem to imply that in pion-pole contribution there is no pion transition form factor at external vertex $\Rightarrow$ enhanced contribution.
- Saturation of this QCD short-distance constraint by pion-pole alone as suggested by Melnikov, Vainshtein '04 is, however, only a model ansatz !
- Only sum of all contributions to HLbL, i.e. exchanges and loops of resonances, has to match the QCD constraint from OPE / pQCD (global quark-hadron duality). See also Dorokhov, Broniowski '08; Greynat, de Rafael '12.

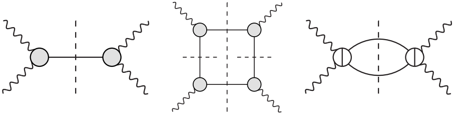
Data-driven approach to HLbL using dispersion relations

Strategy: Split contributions to HLbL into two parts:

- I: Data-driven evaluation using DR (hopefully numerically dominant):
  - (1) π^0, η, η' poles
  - (2) $\pi\pi$ intermediate state
- II: Model dependent evaluation (hopefully numerically subdominant):
  - (1) Axial vectors (3π-intermediate state), ...
  - (2) Quark-loop, matching with pQCD

Error goals: Part I: 10% precision (data driven), Part II: 30% precision.
To achieve overall error of about 20% ($\delta a_\mu^{\text{HLbL}} = 20 \times 10^{-11}$).

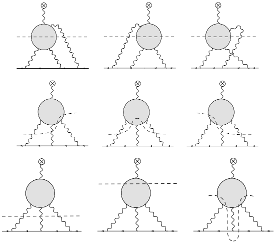
Colangelo et al. '14, '15:
Classify intermediate states in 4-point function. Then project onto $g-2$.



Colangelo et al. '17: pion-box contribution (middle diagram) using precise information on pion vector form factor and S-wave $\pi\pi$-rescattering effects from pion-pole in left-hand cut (LHC) (part of right diagram):

$$\begin{aligned} a_\mu^{\pi-\text{box}} &= -15.9(2) \times 10^{-11} \\ a_{\mu,J=0}^{\pi\pi,\pi-\text{pole LHC}} &= -8(1) \times 10^{-11} \\ \text{Sum of the two} &= -24(1) \times 10^{-11} \end{aligned}$$

Pauk, Vanderhaeghen '14:
DR directly for Pauli FF $F_2(k^2)$.

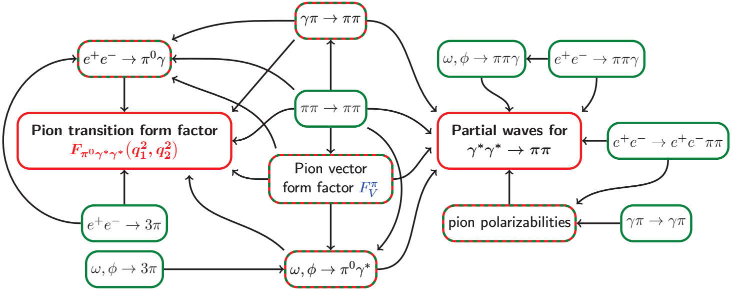


HLbL sum rules to get constraints from experimental data or lattice QCD on models: Pascalutsa, Vanderhaeghen '10; Pascalutsa, Pauk, Vanderhaeghen '12; Green et al. '15; Danilkin, Vanderhaeghen '17

Data-driven approach to HLbL using dispersion relations (continued)

Hadronic light-by-light: a roadmap

GC, Hoferichter, Kubis, Procura, Stoffer arXiv:1408.2517 (PLB '14)

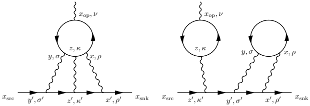


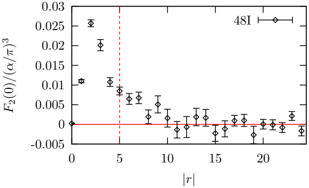
Artwork by M. Hoferichter

A reliable evaluation of the HLbL requires many different contributions by and a collaboration among theorists and experimentalists

From talk by Colangelo at Radio Monte Carlo Meeting, Frascati, May 2016

HLbL in muon $g-2$ from Lattice QCD: RBC-UKQCD approach





- Blum et al. '05, ..., '15: First attempts: Put QCD + (quenched) QED on the lattice. Subtraction of lower-order in α HVP contribution needed, very noisy.
- Jin et al. '15, '16, '17: Step by step improvement of method to reduce statistical error by one or two orders of magnitude and remove some systematic errors.
- Calculate $a_\mu^{\text{HLbL}} = F_2(q^2 = 0)$ via moment method in position-space (no extrapolation to $q^2 = 0$ needed).
- Use exact expression for all photon propagators. Treat $r = x - y$ stochastically by sampling points x, y. Found empirically: short-distance contribution at small $|r| < 0.6$ fm dominates.

Results (for $m_\pi = m_{\pi,\text{phys}}$, lattice spacing $a^{-1} = 1.73$ GeV, $L = 5.5$ fm):

$$a_\mu^{\text{cHLbL}} = (116.0 \pm 9.6) \times 10^{-11} \quad \text{(quark-connected diagrams)}$$
$$a_\mu^{\text{dHLbL}} = (-62.5 \pm 8.0) \times 10^{-11} \quad \text{(leading quark-disconnected diagrams)}$$
$$a_\mu^{\text{HLbL}} = (53.5 \pm 13.5) \times 10^{-11}$$

Beware ! Statistical error only ! Missing systematic effects:

- Expect large finite-volume effects from QED $\sim 1/L^2$. Blum et al. '17: use infinite volume, continuum QED (like Mainz approach: Asmussen et al. '16).
- Expect large finite-lattice-spacing effects.
- Omitted subleading quark-disconnected diagrams (10% effect ?).

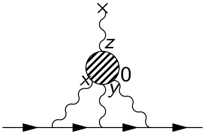
HLbL in muon $g-2$ from Lattice QCD: Mainz approach

Developed independently (Asmussen, Green, Meyer, AN '15, '16, '17)

Master formula in position-space:

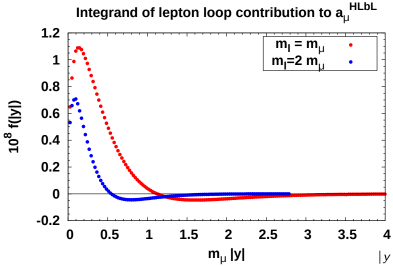
$$a_\mu^{\rm HLbL} = \frac{me^6}{3}\int d^4y \left[\int d^4x \underbrace{\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)}_{\rm QED} \underbrace{i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y)}_{\rm QCD}\right]$$

$$i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) = -\int d^4z\, z_\rho \left\langle j_\mu(x) j_\nu(y) j_\sigma(z) j_\lambda(0)\right\rangle$$



- Semi-analytical calculation of QED kernel.
- QED kernel computed in continuum and in infinite volume (Lorentz covariance manifest, no power law $1/L^2$ finite-volume effects).
- Kernel parametrized by 6 weight functions (and derivatives thereof), calculated on 3D grid in $|x|, |y|, x \cdot y$ and stored on disk.

- Test of QED kernel function: pion-pole contribution, lepton loop.
- Result for pion-pole contribution with VMD model agrees with known results for $m_\pi > 300$ MeV. Need already rather large lattice size > 4 fm for smaller pion masses.
- Analytical result for lepton loop (QED) with $m_{\rm loop} = m_\mu, 2m_\mu$ is reproduced at the percent level.



$|y| = 7.5$ fm

Conclusions and Outlook

- a_μ: Test of Standard Model, potential window to New Physics.
- Current situation:

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (309 \pm 82) \times 10^{-11} \qquad [3.8\ \sigma]$$

Hadronic effects ? Sign of New Physics ?

- Frequently-used model-estimates for HLbL (updated axial-vectors):

$$a_\mu^{\text{HLbL}} = (98 \pm 26) \times 10^{-11} \qquad \text{(PdRV ("Glasgow consensus"))}$$
$$a_\mu^{\text{HLbL}} = (102 \pm 39) \times 10^{-11} \qquad \text{(N, JN)}$$

- Two new $g-2$ experiments at Fermilab (E989) and J-PARC (E34) with goal of $\delta a_\mu^{\text{exp}} = 16 \times 10^{-11}$ (factor 4 improvement)
- Theory for HLbL needs to match this precision !
- Concerted effort needed of experiments (measuring processes with hadrons and photons), phenomenology / theory (data-driven using dispersion relations, matching with QCD short-distance constraints and modelling) and lattice QCD to improve HLbL estimate with reliable uncertainty.
- Muon $g-2$ Theory Initiative (Working Group on HLbL) (Talk El-Khadra) First Workshop recently near Fermilab, more to come in future.

Backup slides

Muon $g-2$: current status

Contribution	$a_\mu \times 10^{11}$	Reference
QED (leptons)	116 584 718.853 $\pm$ 0.036	Aoyama et al. '12
Electroweak	153.6 $\pm$ 1.0	Gnendiger et al. '13
HVP: LO	6889.1 $\pm$ 35.2	Jegerlehner '15
NLO	-99.2 $\pm$ 1.0	Jegerlehner '15
NNLO	12.4 $\pm$ 0.1	Kurz et al. '14
HLbL	102 $\pm$ 39	Jegerlehner '15 (JN '09)
NLO	3 $\pm$ 2	Colangelo et al. '14
Theory (SM)	116 591 780 $\pm$ 53	
Experiment	116 592 089 $\pm$ 63	Bennett et al. '06
Experiment - Theory	309 $\pm$ 82	3.8 σ

Discrepancy a sign of New Physics ?

Hadronic uncertainties need to be better controlled in order to fully profit from future $g-2$ experiments at Fermilab (E989) and J-PARC (E34) with $\delta a_\mu = 16 \times 10^{-11}$.

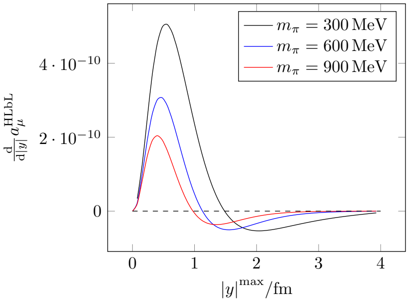
Numerical test of QED kernel: Pion-pole contribution to a_μ^{HLbL}

VMD model for pion transition form factor for illustration. Result for arbitrary pion mass can easily be obtained from 3-dimensional momentum-space representation (Jegerlehner + AN '09).

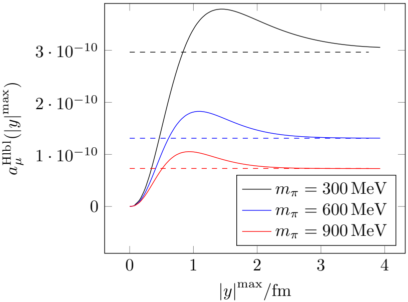
3-dim. integration in position space:

- $\int_y \to 2\pi^2 \int_0^\infty \mathrm{d}|y||y|^3$
- $\int_x \to 4\pi \int_0^\infty \mathrm{d}|x|\,|x|^3 \int_0^\pi \mathrm{d}\beta \sin^2\beta$ (cutoff for x integration: $|x|^{\mathrm{max}} = 4.05$ fm)

Integrand after integration over $|x|, \beta$:



Result for $a_\mu^{\mathrm{HLbL}}(|y|^{\mathrm{max}})$:



- All 6 weight functions contribute to final result, some only at the percent level.
- $|x|^{\mathrm{max}}, |y|^{\mathrm{max}} > 4$ fm needed for $m_\pi < 300$ MeV.
- For the physical pion mass, one needs to go to very large values of $|x|$ and $|y|$, i.e. very large lattice volumes, to reproduce known result of $5.7 \cdot 10^{-10}$.