$(g\text{-}2)_\mu$ overview

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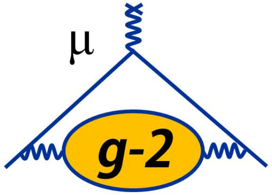


- Introduction
- QED and weak contributions
- a_μ^{had} : HVP (and HLbL) status, KNT17 HVP update
- BSM?
- Outlook

Introduction & motivation:

SM `too' successful, but incomplete:

- ν masses (small) and mixing point towards some high-scale (GUT) physics
- Need to explain dark matter & dark energy
- Not enough CP violation in the SM for matter-antimatter asymmetry
- And: $a_\mu^{EXP} - a_\mu^{SM}$ at ~ 3-4 σ plus other deviations e.g. in the flavour sector

Is there a common New Physics (NP) explanation for all these puzzles?

- Uncoloured leptons are particularly clean probes to establish and constrain/ distinguish NP, complementary to high energy searches at the LHC

- No direct signals for NP from LHC so far:
  - some models like CMSSM are in trouble already when trying to accommodate LHC exclusion limits and to solve muon g-2
  - is there any TeV scale NP out there? Or unexpected new low scale physics?

The key may be provided by low energy observables
including precision QED, EDMs and LFV → see talks by R. Szafron and A. de Gouvea

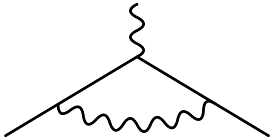
Introduction

- Dirac equation (1928): g is 2 for fundamental fermions $\vec{\mu} = g \frac{Qe}{2m} \vec{s}$
- 1947: small deviations from predictions in hydrogen and deuterium hyperfine structure; Kusch & Foley propose explanation with $g_s = 2.00229 \pm 0.00008$

- 1948: Schwinger calculates the famous radiative correction:

  that g = 2 (1+a), with

  a = (g-2)/2 = α/(2π) = 0.001161



`` If you can't join 'em, beat 'em "

This explained the discrepancy and was a crucial step in the development of perturbative QFT and QED

- The anomaly a (Anomalous Magnetic Moment) is from the Pauli term:

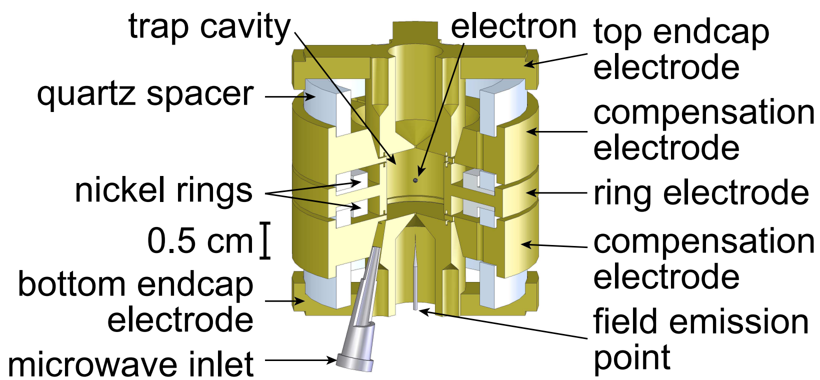
$$\delta \mathcal{L}_{\rm eff}^{\rm AMM} = -\frac{Qe}{4m} a \bar{\psi}(x) \sigma^{\mu\nu} \psi(x) F_{\mu\nu}(x)$$

This is a dimension 5 operator, non-renormalisable and hence not part of the fundamental (QED) Lagrangian. But it occurs through radiative corrections and is calculable in perturbation theory.

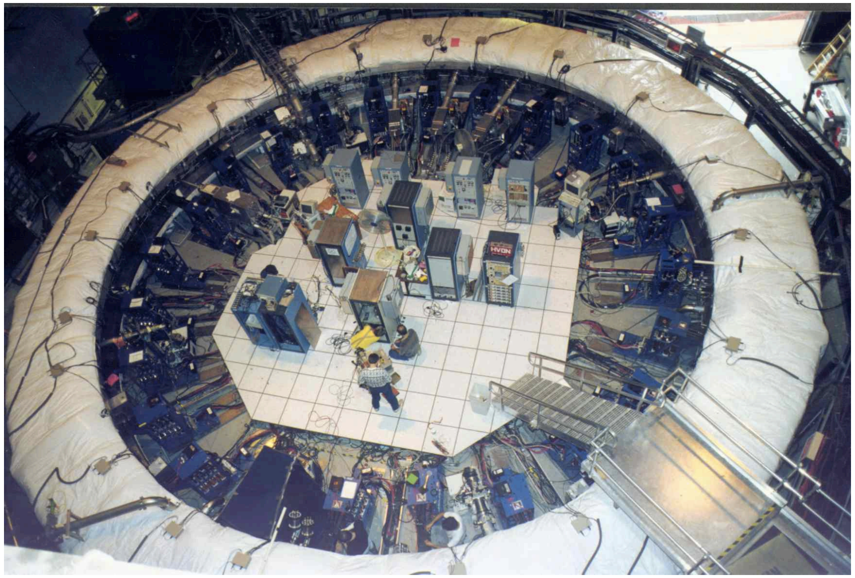
Magnetic Moments: a_e vs. a_μ

a_e= 1 159 652 180.73 (0.28) 10^{-12} [0.24ppb]
Hanneke, Fogwell, Gabrielse, PRL 100(2008)120801

a_μ= 116 592 089(63) 10^{-11} [0.54ppm]
Bennet et al., PRD 73(2006)072003



one electron quantum cyclotron



- a_e^{EXP} more than 2000 times more precise than a_μ^{EXP}, but for e^- loop contributions come from very small photon virtualities, whereas muon `tests' higher scales

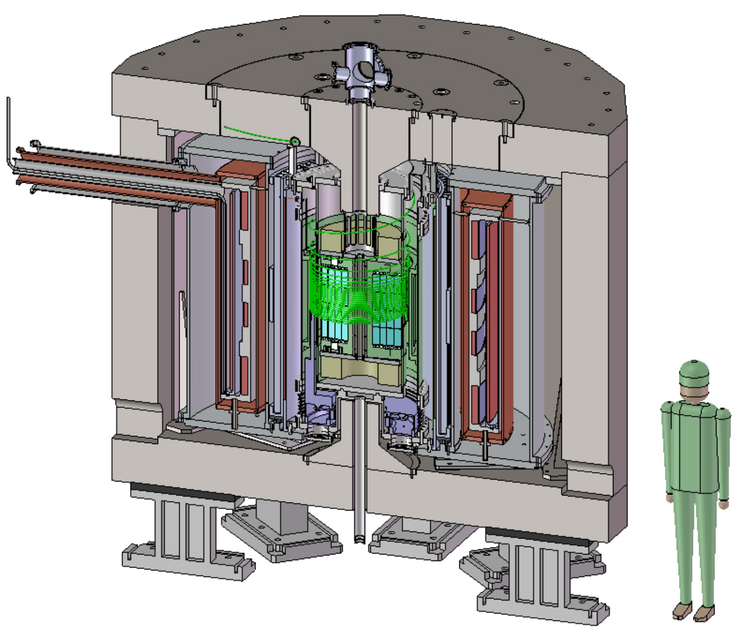
- dimensional analysis: sensitivity to NP (at high scale Λ_{NP}): $a_\ell^{\rm NP} \sim \mathcal{C}\, m_\ell^2/\Lambda_{\rm NP}^2$

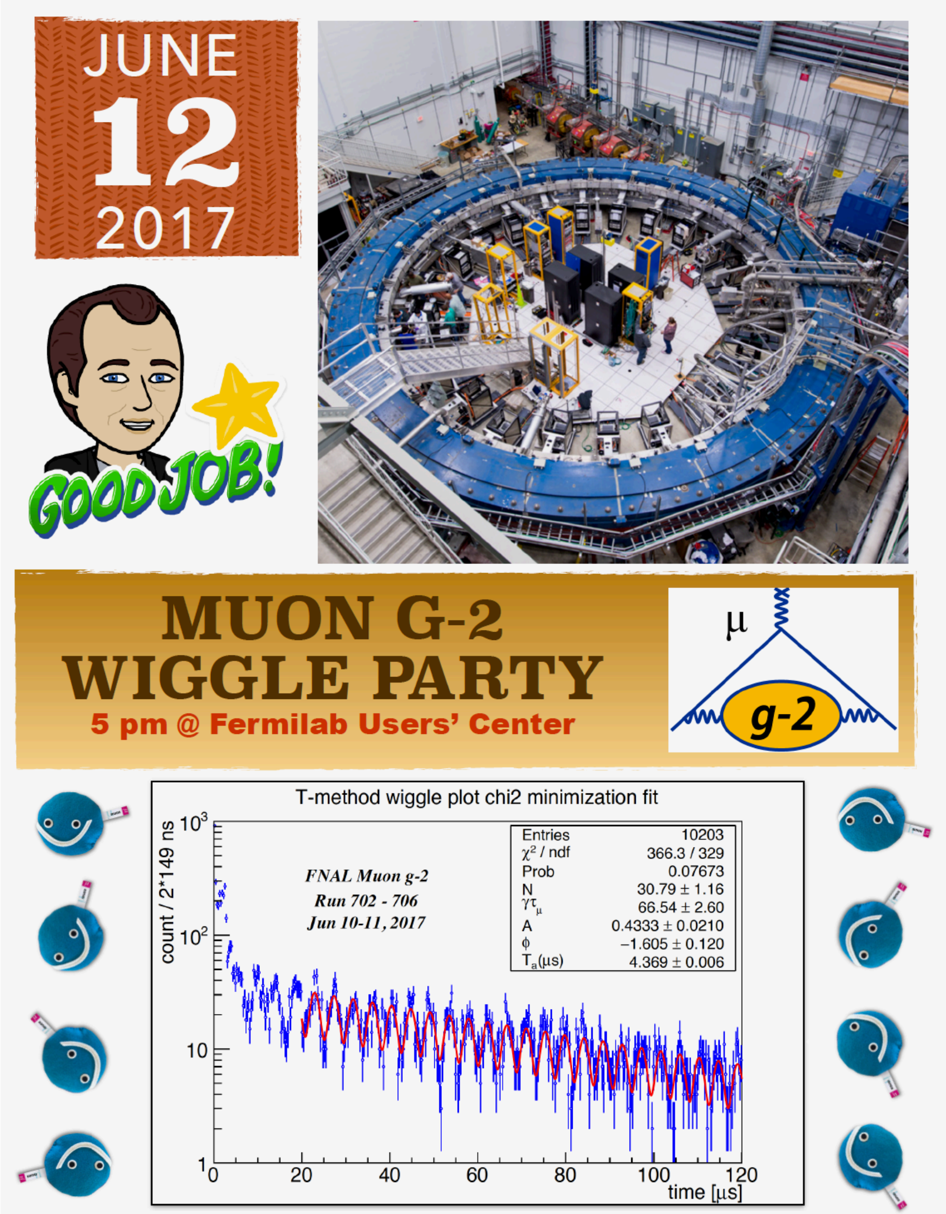
→ µ wins by $m_\mu^2/m_e^2 \sim 43000$ for NP, but a_e provides best determination of α

a_μ: back to the future

- CERN started it nearly 40 years ago
- Brookhaven delivered 0.5ppm precision
- E989 at FNAL and J-PARC's g-2/EDM experiments are happening and should give us certainty

→ talks by Lee Roberts and Tsutomu Mibe





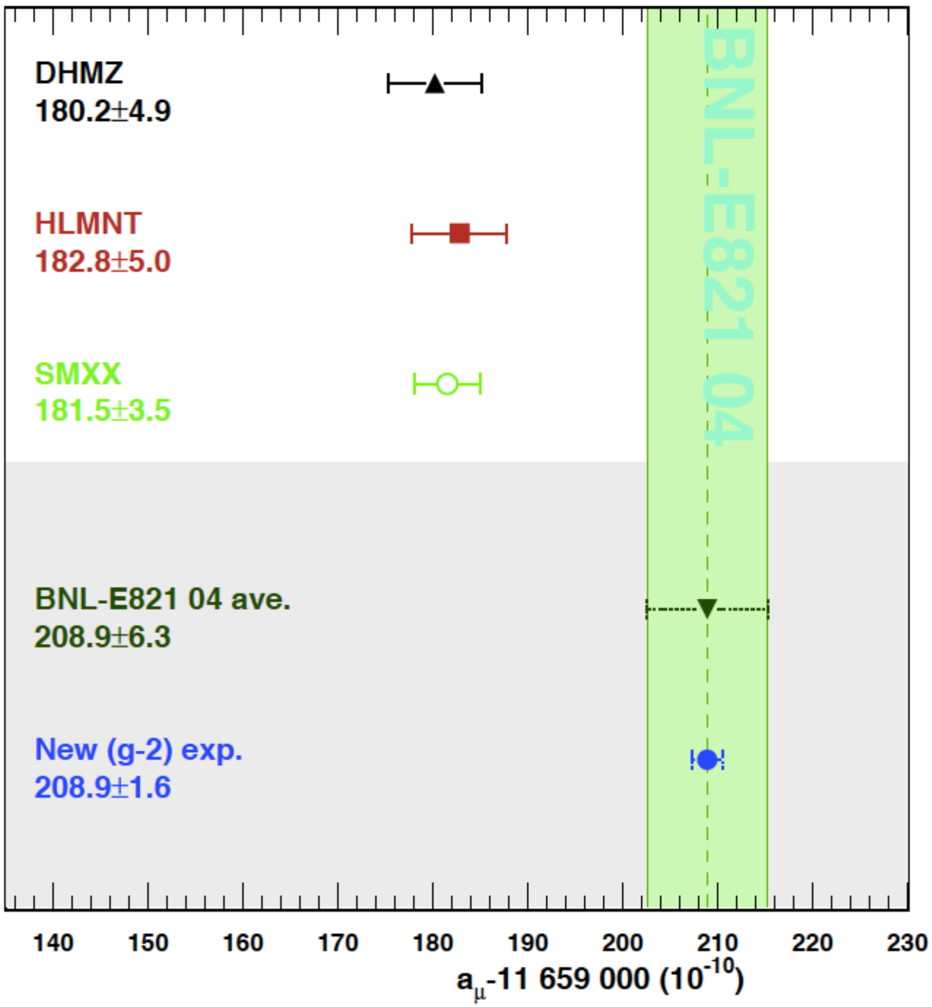
a_μ: Status and future projection → charge for SM TH

$$a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm hadronic} + a_\mu^{\rm NP?}$$

- if mean values stay and with no a_μ^{SM} improvement: 5σ discrepancy

- if also EXP+TH can improve a_μ^{SM} `as expected' (consolidation of L-by-L on level of Glasgow consensus, about factor 2 for HVP): NP at 7-8σ

- or, if mean values get closer, very strong exclusion limits on many NP models (extra dims, new dark sector, xxxSSSM)…

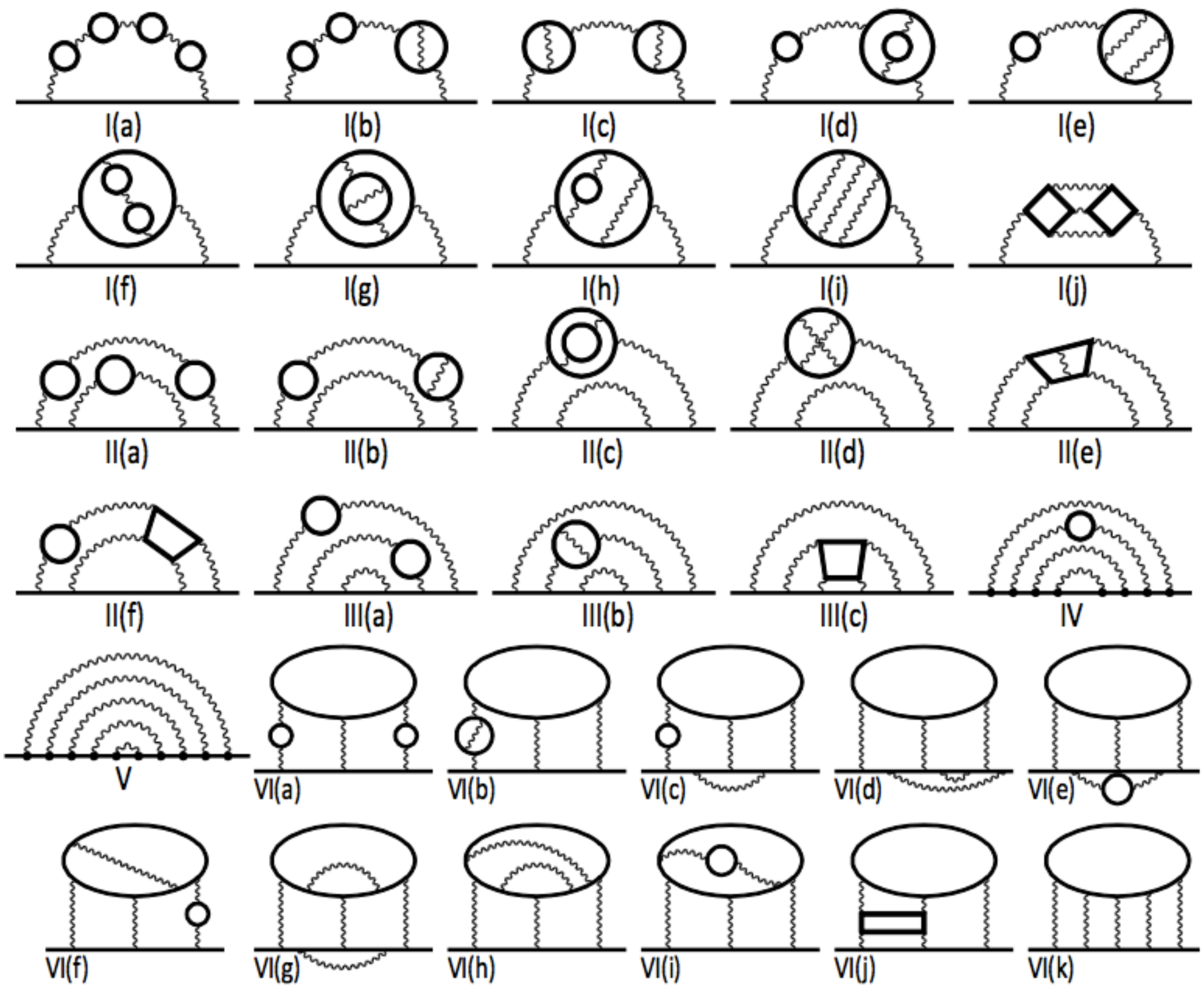


a_μ^{QED} Kinoshita et al.: g-2 at 1, 2, 3, 4 & 5-loop order

T. Aoyama, M. Hayakawa,
T. Kinoshita, M. Nio (PRLs, 2012)

A triumph for perturbative QFT and computing!

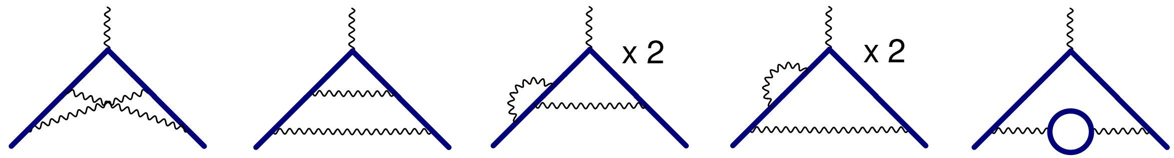
10th
12672
diagrams



- code-generating code, including
- renormalisation
- multi-dim. numerical integrations

a_μ^{QED}

- Schwinger 1948: 1-loop $a = (g-2)/2 = \alpha/(2\pi) = 116\ 140\ 970 \times 10^{-11}$
- 2-loop graphs:



- 72 3-loop and 891 4-loop diagrams …
- Kinoshita et al. 2012: 5-loop completed numerically (12672 diagrams):

$a_\mu^{QED} = 116\ 584\ 718.951\ (0.009)\ (0.019)\ (0.007)\ (0.077) \times 10^{-11}$

errors from: lepton masses, 4-loop,   5-loop,   α from ^{87}Rb

- QED extremely accurate, and the series is stable: $a_\mu^{\rm QED} = C_\mu^{2n} \sum_n \left(\frac{\alpha}{\pi}\right)^n$

$$C_\mu^{2,4,6,8,10} = 0.5,\ 0.765857425(17),\ 24.05050996(32),\ 130.8796(63),\ 753.29(1.04)$$

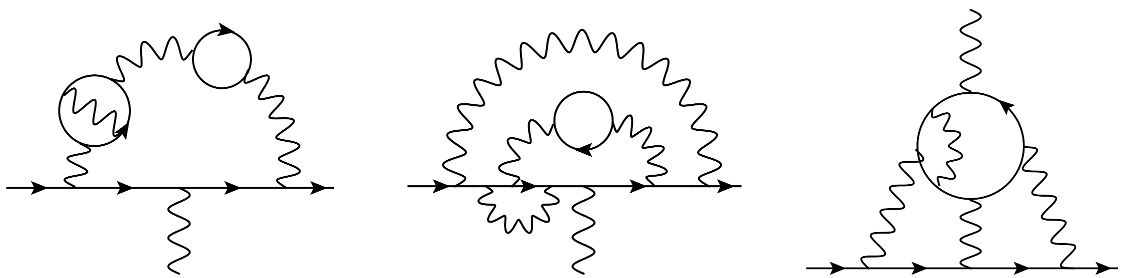
- Could a_μ^{QED} still be wrong?
  Some classes of graphs known analytically (Laporta; Aguilar, Greynat, deRafael),

a_μ^{QED}

- … but 4-loop and 5-loop rely heavily on numerical integrations

- Recently several independent checks of 4-loop and 5-loop diagrams:
  Baikov, Maier, Marquard [NPB 877 (2013) 647], Kurz, Liu, Marquard, Smirnov AV+VA, Steinhauser [NPB 879 (2014) 1, PRD 92 (2015) 073019, 93 (2016) 053017]:

- all 4-loop graphs with internal lepton loops now calculated independently, e.g.



(from Steinhauser et al., PRD 93 (2016) 053017)

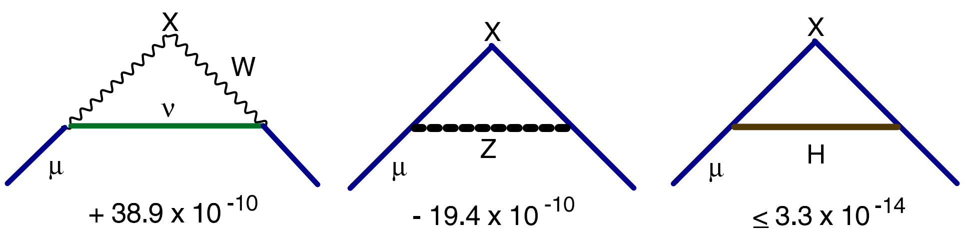
→ talk by Matthias Steinhauser

- 4-loop universal (massless) term calculated semi-analytically to 1100 digits (!) by Laporta, arXiv:1704.06996, also new numerical results by Volkov, 1705.05800

- all agree with Kinoshita et al.'s results, so QED is on safe ground ✓

a_μ Electro-Weak

- Electro-Weak 1-loop diagrams:



$a_\mu^{EW(1)} = 195\times10^{-11}$

- known to 2-loop (1650 diagrams, the first EW 2-loop calculation):
  Czarnecki, Krause, Marciano, Vainshtein; Knecht, Peris, Perrottet, de Rafael

- agreement, a_μ^{EW} relatively small, 2-loop relevant: $a_\mu^{EW(1+2\ loop)} = (154\pm2)\times10^{-11}$

- Higgs mass now known, update by Gnendiger, Stoeckinger, S-Kim,
  PRD 88 (2013) 053005

$$a_\mu^{EW(1+2\ loop)} = (153.6\pm1.0)\times10^{-11}$$ ✓

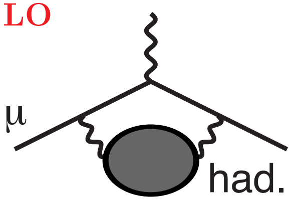
compared with $a_\mu^{QED} = 116\ 584\ 718.951\ (80) \times10^{-11}$

a_μ^{hadronic}

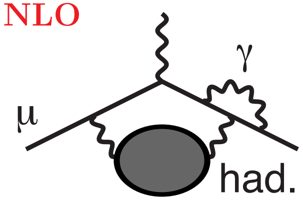
- Hadronic: non-perturbative, the limiting factor of the SM prediction ✗ ➠ ✓

$$a_\mu^{\rm had} = a_\mu^{\rm had,VP\ LO} + a_\mu^{\rm had,VP\ NLO} + a_\mu^{\rm had,Light-by-Light}$$

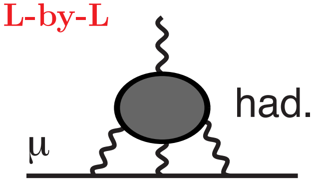
LO



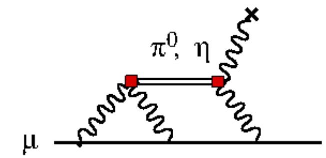
NLO



L-by-L



e.g.



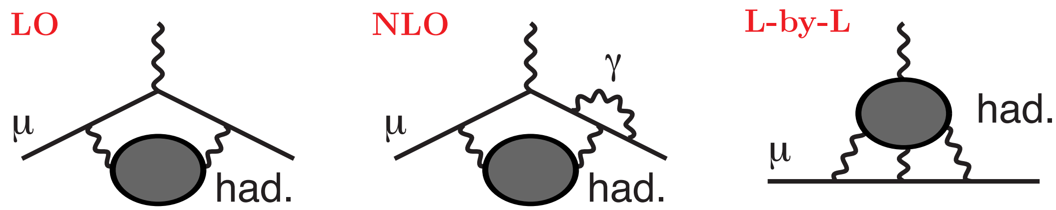
- L-by-L: - so far use of model calculations (+ form-factor data and pQCD constraints),
  → overview talk by A. Nyffeler
  - also good news from lattice QCD, and → talks by H. Wittig and C. Lehner
  - new dispersive approaches → talks by B. Kubis, V. Pauk and G. Colangelo

- Below I will use the `updated Glasgow consensus':
  (original by Prades+deRafael+Vainshtein) $a_\mu^{\text{had,L-by-L}} = (98 \pm 26) \times 10^{-11}$

- so far no indication for a big surprise
- expect that L-by-L prediction can be improved further
- with new results & progress, tell politicians/sceptics: L-by-L _can_ be predicted!

$a_\mu^{\text{had, VP}}$: Hadronic Vacuum Polarisation

$$a_\mu^{\text{had}} = a_\mu^{\text{had,VP LO}} + a_\mu^{\text{had,VP NLO}} + a_\mu^{\text{had,Light-by-Light}}$$

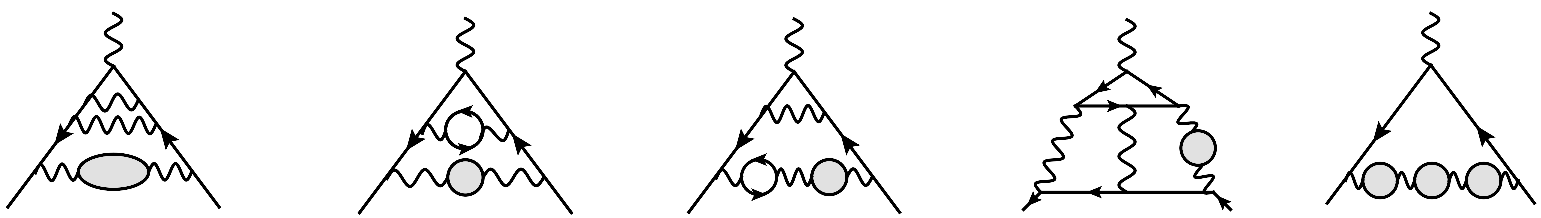


HVP: - most precise prediction by using e^+e^- hadronic cross section (+ tau) data and well known dispersion integrals → for space-like HVP see talk by L. Trentadue

- done at LO and NLO (see graphs) → see HVP talks by Z. Zhang and F. Jegerlehner

- and recently at NNLO [Steinhauser et al., PLB 734 (2014) 144, also F. Jegerlehner]

  $a_\mu^{\text{HVP, NNLO}} = + 1.24 \times 10^{-10}$ not so small, from e.g.:

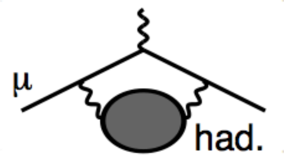


- Alternative: lattice QCD, but need QED and iso-spin breaking corrections

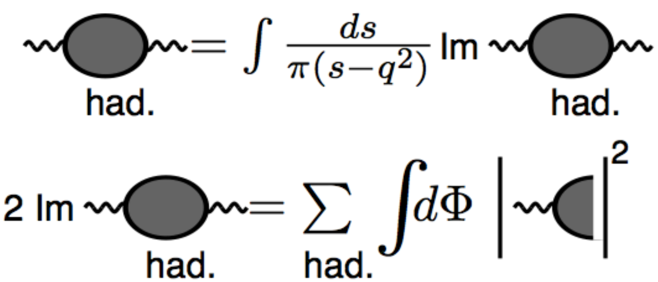
  Lots of activity by several groups, errors coming down, QCD+QED started

Hadronic Vacuum Polarisation, essentials:

Use of data compilation for HVP:



pQCD not useful. Use the dispersion relation and the optical theorem.



$$a_\mu^{\rm had,LO} = \frac{m_\mu^2}{12\pi^3} \int_{s_{\rm th}}^{\infty} ds\, \frac{1}{s} \hat{K}(s) \sigma_{\rm had}(s)$$

- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

$\Longrightarrow$ Lower energies more important

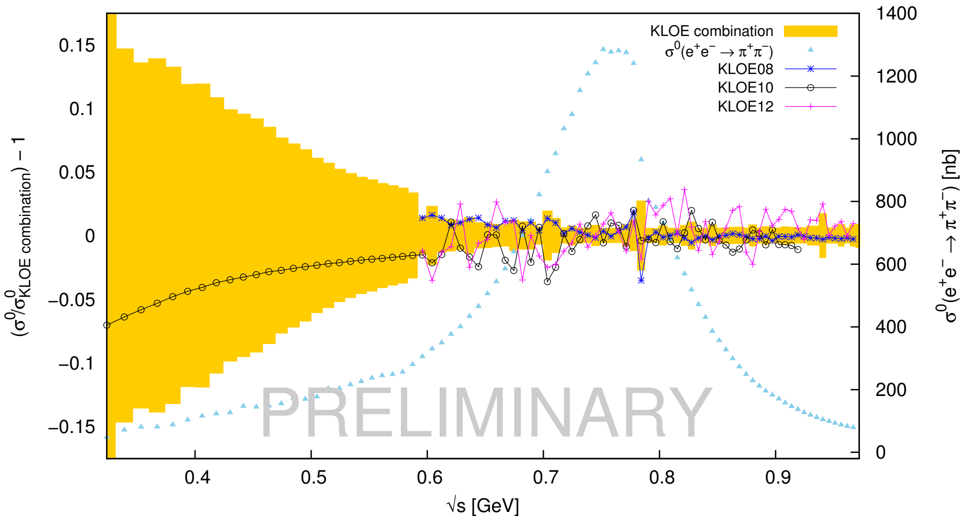
$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\rm had,LO}$

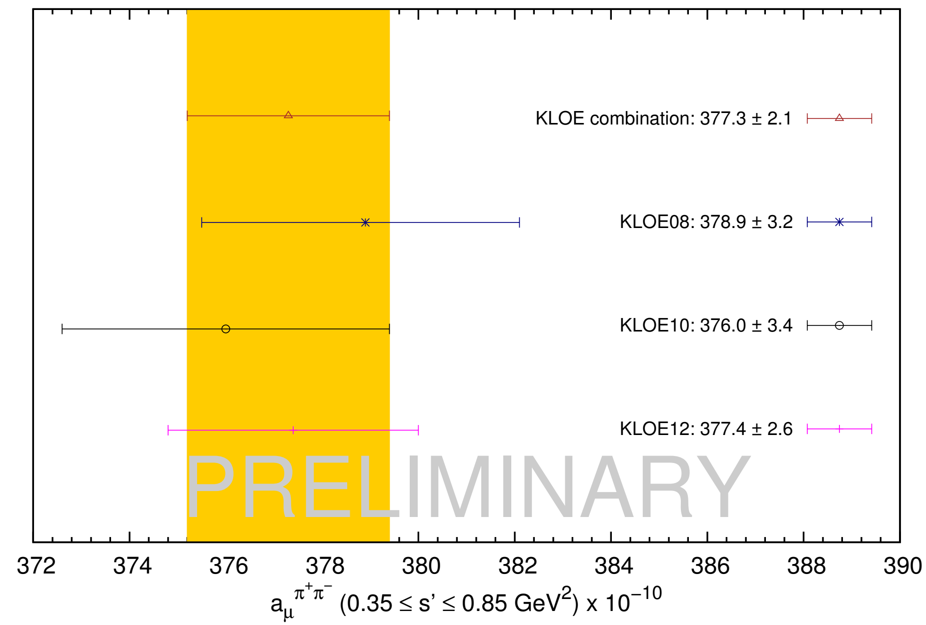
How to get the most precise $\sigma^0_{\rm had}$? e^+e^- data:

- Low energies: sum ~30 exclusive channels, 2π, 3π, 4π, 5π, 6π, KK, KKπ, KKππ, ηπ, …, use iso-spin relations for missing channels
- Above ~1.8 GeV: can start to use pQCD (away from flavour thresholds), supplemented by narrow resonances (J/Ψ, Υ)
- Challenge of data combination (locally in √s): many experiments, different energy bins, stat+sys errors from different sources, correlations; must avoid inconsistencies/bias
- traditional `direct scan' (tunable $e^+e^-$ beams) vs. `Radiative Return' [+ τ spectral functions]
- $\sigma^0_{\rm had}$ means `bare' σ, but WITH FSR: RadCorrs [HLMNT '11: $\delta a_\mu^{\rm had,\,RadCor\,VP+FSR} = 2 \times 10^{-10}$!]

HVP: KLOE 2π combination [preliminary]

⇒ Combination of KLOE08, KLOE10 and KLOE12 gives 85 distinct bins between $0.1 \leq s \leq 0.95$ GeV^2





→ Covariance matrix now correctly constructed

⇒ a positive semi-definite matrix

→ Non-trivial influence of correlated uncertainties on resulting mean value

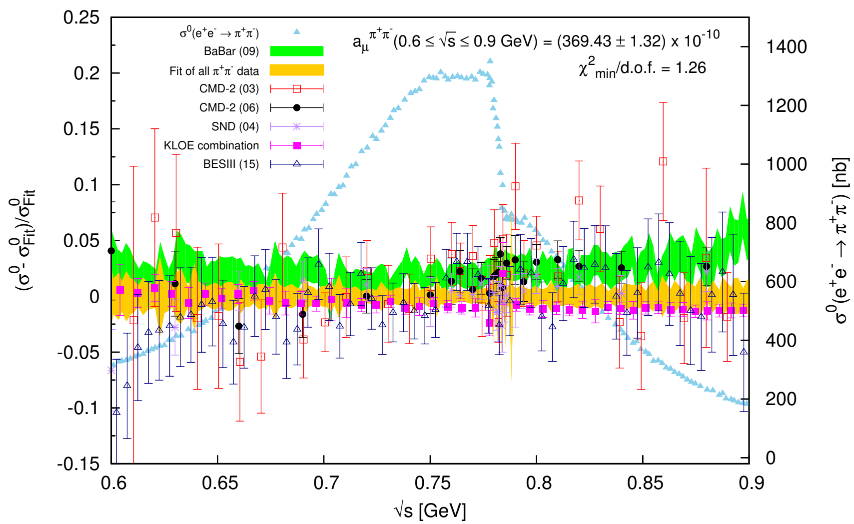
$$a_\mu^{\pi^+\pi^-}(0.1 \leq s' \leq 0.95\ \text{GeV}^2) = (489.9 \pm 2.0_{\text{stat}} \pm 4.3_{\text{sys}}) \times 10^{-10}$$

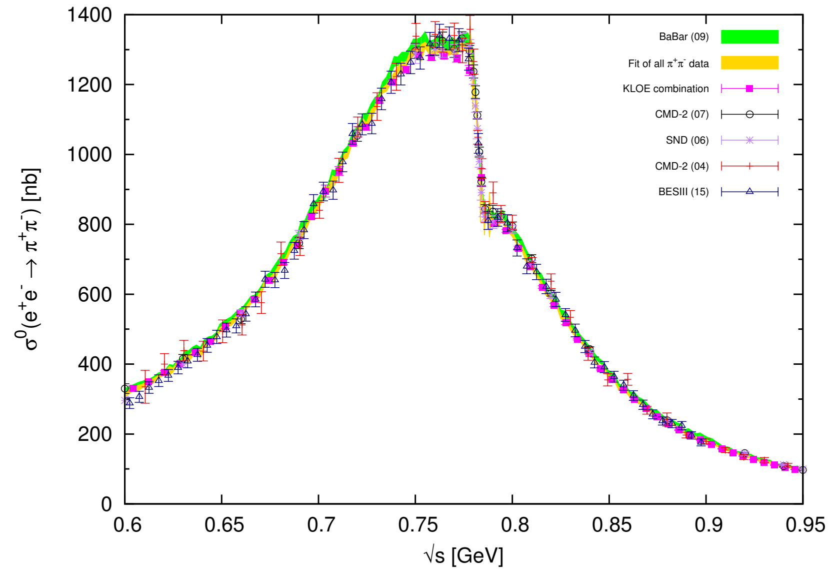
Publication by KLOE-2, Grazinao Venanzoni, Alex Keshavarzi, Stefan Mueller and TT under review

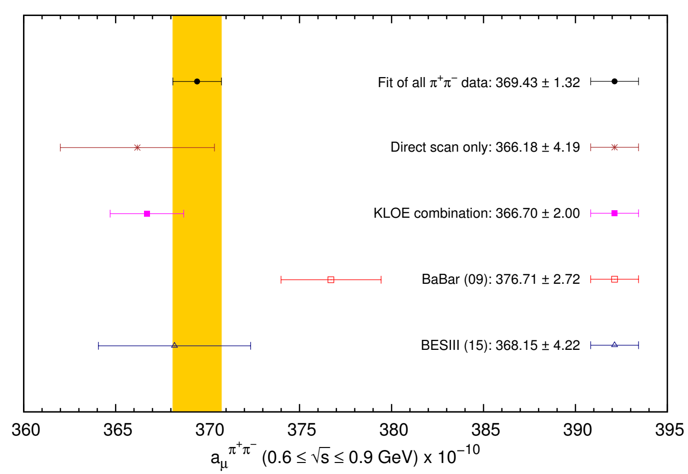
HVP: complete 2π combination by Keshavarzi+Nomura+T

$\Rightarrow$ Large improvement for 2π estimate

$\rightarrow$ BESIII [Phys.Lett. B753 (2016) 629-638] and KLOE combination provide downward influence to mean value







$\Rightarrow$ Correlated & experimentally corrected $\sigma^0_{\pi\pi(\gamma)}$ data now entirely dominant

$$a_\mu^{\pi^+\pi^-}(0.305 \le \sqrt{s} \le 2.00 \text{ GeV}):$$

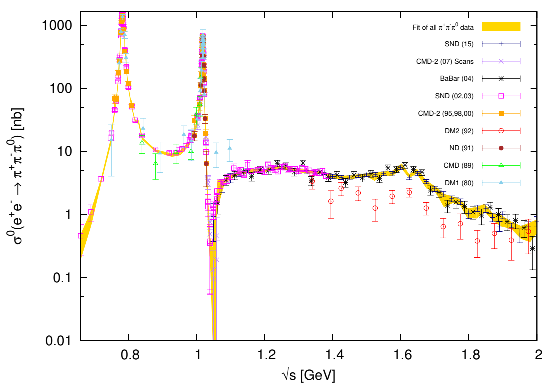
$$\text{HLMNT11: } 505.77 \pm 3.09$$

$$\text{KNT17: } 502.85 \pm 1.93 \text{ (!!)}$$

(no radiative correction uncertainties)

HVP: other notable exclusive channels

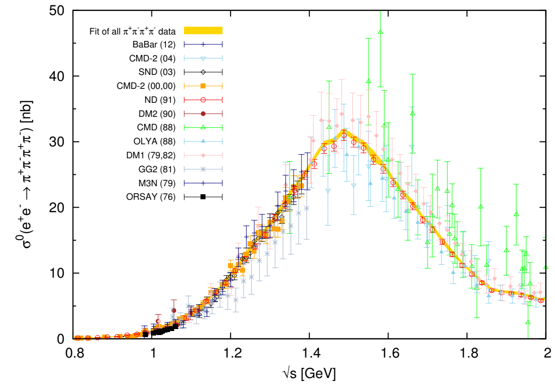
$\pi^+\pi^-\pi^0$



HLMNT11: 47.51 ± 0.99

KNT17: 47.68 ± 0.70

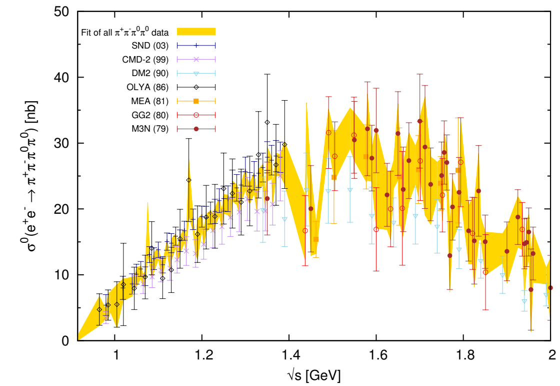
$\pi^+\pi^-\pi^+\pi^-$



HLMNT11: 14.65 ± 0.47

KNT17: 15.18 ± 0.14

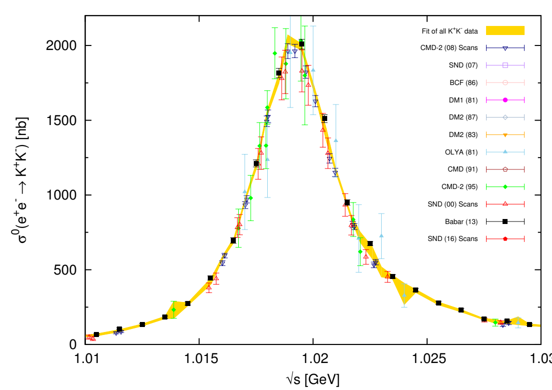
$\pi^+\pi^-\pi^0\pi^0$



HLMNT11: 20.37 ± 1.26

KNT17: 20.07 ± 1.19

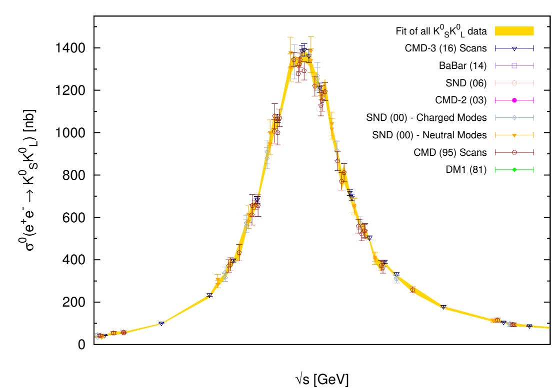
K^+K^-



HLMNT11: 22.15 ± 0.46

KNT17: 22.76 ± 0.22

$K_S^0 K_L^0$

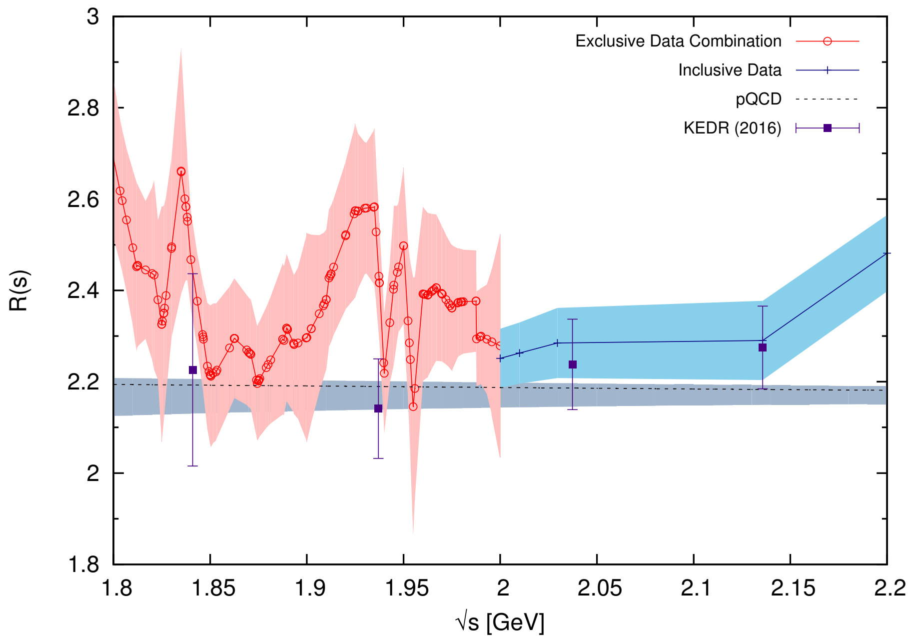


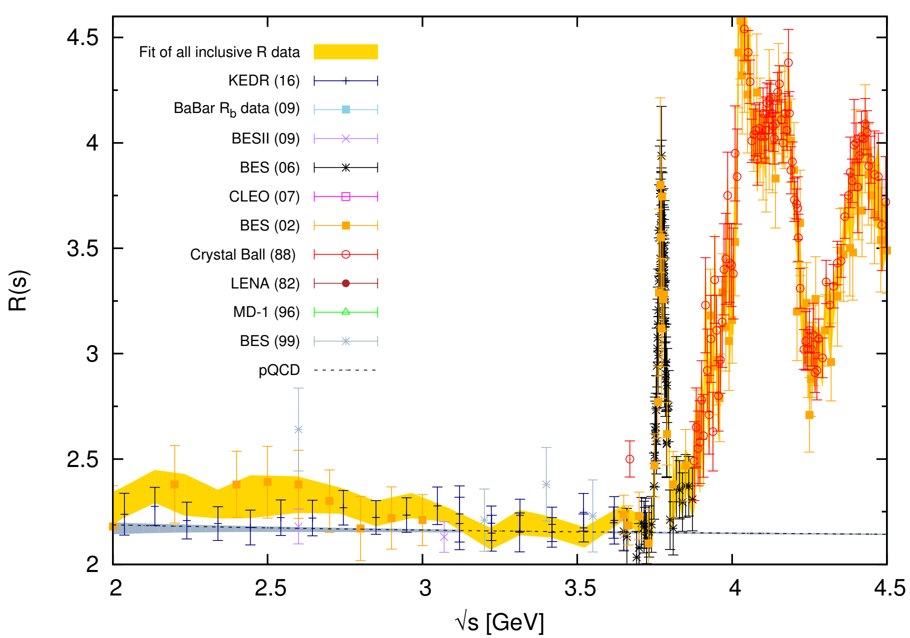
HLMNT11: 13.33 ± 0.16

KNT17: 13.09 ± 0.12

HVP: region of inclusive data/pQCD

⇒ New KEDR inclusive R data ranging $1.84 \leq \sqrt{s} \leq 3.05$ GeV [Phys.Lett. B770 (2017) 174-181] and $3.12 \leq \sqrt{s} \leq 3.72$ GeV [Phys.Lett. B753 (2016) 533-541]





$a_\mu^{\text{had, LOVP}}(1.84 \leq \sqrt{s} \leq 2.00 \text{ GeV})$:

pQCD : 6.42 ± 0.03

Data : 6.88 ± 0.25

$a_\mu^{\text{had, LOVP}}(2.60 \leq \sqrt{s} \leq 3.73 \text{ GeV})$:

pQCD (inflated errors) : 10.82 ± 0.38

Data : 11.20 ± 0.14

$\Longrightarrow$ **Choose to adopt entirely data driven estimate from threshold to 11.2 GeV**

a_μ^{SM}: update HLMNT11 → KNT17 presented @ TGM2

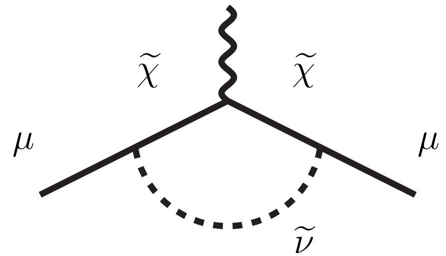
	2011		2017	*to be discussed
QED	11658471.81 (0.02)	$\longrightarrow$	11658471.90 (0.01)	[Phys. Rev. Lett. 109 (2012) 111808]
EW	15.40 (0.20)	$\longrightarrow$	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL	10.50 (2.60)	$\longrightarrow$	9.80 **(2.60)**	[EPJ Web Conf. 118 (2016) 01016]*
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]*
	HLMNT11		**KNT17**	
LO HVP	694.91 (4.27)	$\longrightarrow$	692.23 **(2.54)**	this work*
NLO HVP	-9.84 (0.07)	$\longrightarrow$	-9.83 (0.04)	this work*
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144] *
Theory total	11659182.80 (4.94)	$\longrightarrow$	11659181.00 (3.62)	this work
Experiment			11659209.10 (6.33)	world avg
Exp - Theory	26.1 (8.0)	$\longrightarrow$	28.1 (7.3)	this work
Δa_μ	3.3σ	$\longrightarrow$	3.9σ	this work

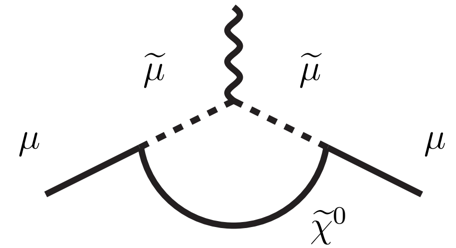
a_μ: New Physics?

- Many BSM studies use g-2 as constraint or even motivation

- SUSY could easily explain g-2

  - Main 1-loop contributions:





  - Simplest case:

$$a_\mu^{\rm SUSY} \simeq sgn(\mu)\, 130 \times 10^{-11}\, \tan\beta \left(\frac{100\,{\rm GeV}}{\Lambda_{\rm SUSY}}\right)^2$$

  - Needs $\mu>0$, \`light' SUSY-scale Λ and/or large tan β to explain 281×10^{-11}

  - This is **already excluded by LHC searches** in the simplest SUSY scenarios (like CMSSM); causes large χ^2 in simultaneous SUSY-fits with LHC data and g-2

  - However: * SUSY does not have to be minimal (w.r.t. Higgs),
    * could have large mass splittings (with lighter sleptons),
    * be hadrophobic/leptophilic,
    * or not be there at all, but don't write it off yet…

New Physics? just a few of many recent studies

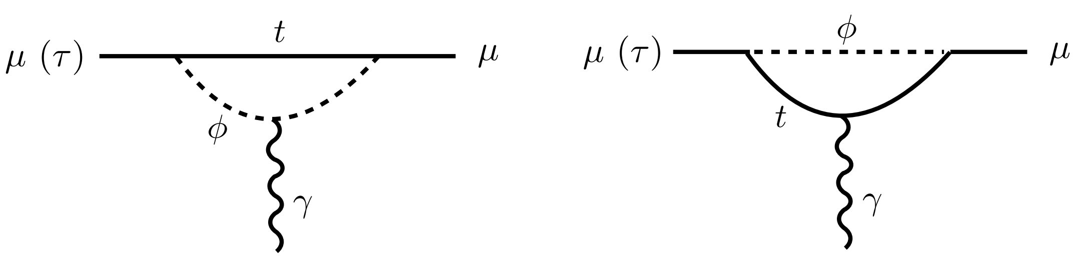
- Don't have to have full MSSM (like coded in GM2Calc [by Athron, …, Stockinger et al., EPJC 76 (2016) 62], which includes all latest two-loop contributions), and

- extended Higgs sector could do, see, e.g. Stockinger et al., JHEP 1701 (2017) 007, `The muon magnetic moment in the 2HDM: complete two-loop result'

➔ lesson: 2-loop contributions can be highly relevant in both cases; one-loop analyses can be misleading

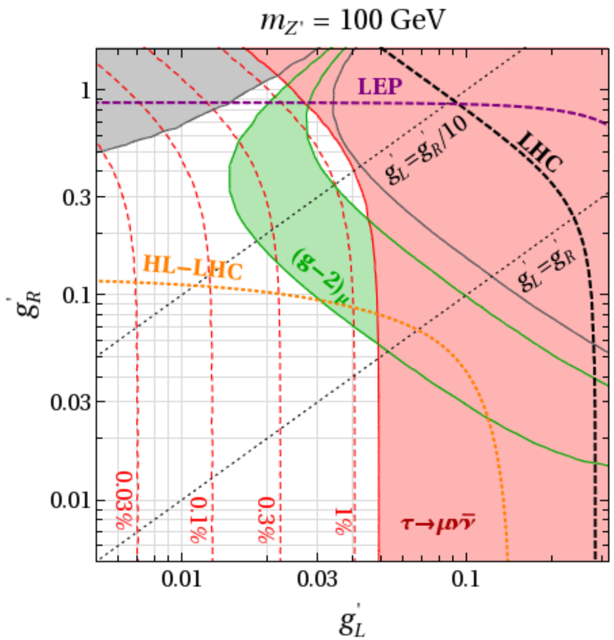
- 1 TeV Leptoquark Bauer + Neubert, PRL 116 (2016) 141802

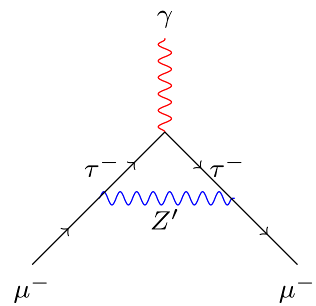
one new scalar could explain several anomalies seen by BaBar, Belle and LHC in the flavour sector (e.g. violation of lepton universality in B -> Kll, enhanced B -> Dτν) and solve g-2, while satisfying all bounds from LEP and LHC

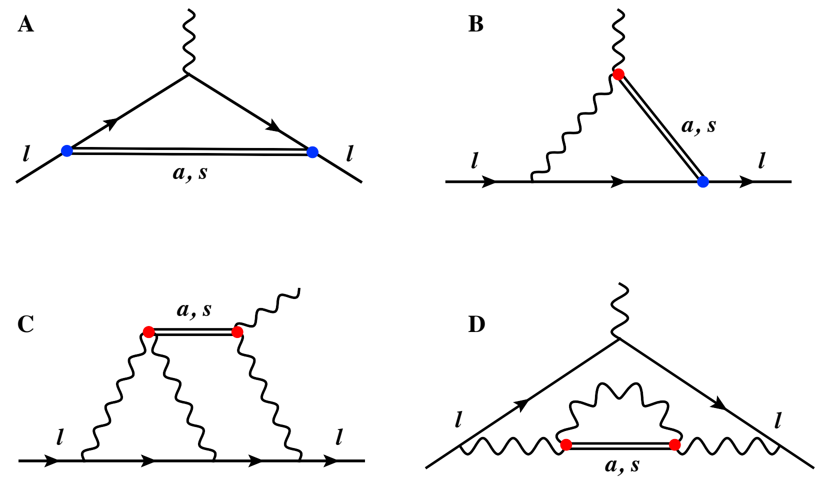


New Physics? just a few of many recent examples

- light Z' can evade many searches involving electrons by non-standard couplings preferring heavy leptons (but see BaBar's direct search limits in a wide mass range, PRD 94 (2016) 011102), or invoke flavour off-diagonal Z' to evade constraints [Altmannshofer et al., PLB 762 (2016) 389]







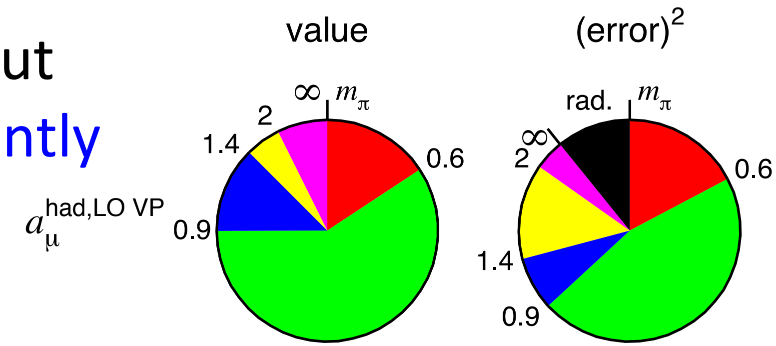
- axion-like particle (ALP), contributing like π^0 in HLbL [Marciano et al., PRD 94 (2016) 115033]

- `dark photon' - like fifth force particle [Feng et al., PRL 117 (2016) 071803]

→ see talks by M. Pospelov, A. Filippi, I. Jaegle

Conclusions/Outlook:

- All sectors of the Standard Model prediction of g-2 have been scrutinised a lot in recent years
- The basic picture has not changed, but recent data, many from ISR, significantly improve the prediction for a_μ^{HVP}



- Discrepancy ~ 3-4σ is consolidated
- With further hadronic data in the pipeline, also on FFs for HLbL, and efforts from lattice, the goal of squeezing Δa_μ^{SM} is in reach
- Many approaches to explain the discrepancy with NP, linking g-2 with other precision observables, the flavour sector, dark matter and direct searches, but so far NP only (con)strained…