

Workshop Effective Field Theories for Collider Physics, Flavor Phenomena and Electroweak Symmetry Breaking

Eltville, September 2016

Flavour Observables: Anomalies and correlations

Fulvia De Fazio INFN- Bari

- Tensions between SM & (some) exp results
- 331 Models
- Focus on  $\varepsilon'/\varepsilon$  and correlations with other observables

Based on

A.J. Buras, J. Girrbach, FDF, JHEP 1402 (2014) 112
A.J. Buras, J. Girrbach-Noe, FDF JHEP 1408 (2014) 039
A.J. Buras, FDF JHEP 1603 (2016) 010
A.J. Buras, FDF JHEP 1608 (2016) 115

Results for flavour observables deviating from SM predictions



## Observables in $B \to K^* \mu^+ \mu^-$ : LHCb results





Theory indications: NP contributions to C<sub>9</sub> should be large

LHCb JHEP 1602 (2016) 104

ε'/ε

World Ave NA48 + KTeV

$$(\varepsilon'/\varepsilon)_{\rm exp} = (16.6 \pm 2.3) \times 10^{-4}$$

Recent SM result using RBC-UKQCD lattice results for B parameters

$$\varepsilon'/\varepsilon = (1.9 \pm 4.5) \times 10^{-4}$$

A.J. Buras, M. Gorbahn, S. Jager, M. Jamin JHEP 11 (2015) 202

Using large – N results

A.J. Buras, J.-M. Gerard JHEP 12 (2015) 008

$$(\varepsilon'/\varepsilon)_{\rm SM} = (8.6 \pm 3.2) \times 10^{-4}$$

The main uncertainty in SM comes from the cancellation between QCD and EW penguin contribution

#### Aim of this talk:

Explore correlations between  $\epsilon'/\epsilon$  and these "problematic" observables in a well defined NP model

In the case of  $\epsilon'/\epsilon$  consider only the shift wrt SM

331 Models: general features

P. Frampton, PRL 69 (92) 2889 F. Pisano & V. Pleitez, PRD 46 (92) 410



Gauge group: $SU(3)_C \times SU(3)_L \times U(1)_X$ Spontaneously broken to $SU(3)_C \times SU(2)_L \times U(1)_X$ Spontaneously broken to $SU(3)_C \times U(1)_O$ 

Nice features:  requirement of anomaly cancelation + asympotic freedom of QCD implies number of generations= number of colors

 two quark generations transform as triplets under SU(3)<sub>L</sub>, one as an antitriplet this may allow to understand why top mass is so large

Fundamental relation:

$$Q = T_3 + \beta T_8 + X$$

Key parameter: defines the variant of the model

 $\beta = \pm 1/\sqrt{3}, \pm 2/\sqrt{3}$ 

- lead to interesting phenomenology
- for  $\beta = \pm 1/\sqrt{3}$  the new gauge bosons have integer charge





New heavy fermions

Extended Higgs sector

A new heavy Z' mediates tree level FCNC in the quark sector

D,S,T new heavy quarks

**E**<sub>1</sub> new heavy neutrinos (both L & R)

Three SU(3)<sub>L</sub> triplets, one sextet

 $\rho,\eta,\xi$ 





331 Models: gauge sector



331 Models: gauge sector

Mixing pattern among the neutral gauge bosons

# Mixing between Z and Z'

$$Z^{1}_{\mu} = \cos \xi Z_{\mu} + \sin \xi Z'_{\mu}, \qquad Z^{2}_{\mu} = -\sin \xi Z_{\mu} + \cos \xi Z'_{\mu}$$

$$\sin \xi \cong O(10^{-3})$$
  $\square$   $Z_{\mu}^{1} = Z_{\mu}$   $Z_{\mu}^{2} = Z_{\mu}'$ 

$$\sin \xi = \frac{c_W^2}{3} \sqrt{f(\beta)} \left( 3\beta \frac{s_W^2}{c_W^2} + \sqrt{3}a \right) \left[ \frac{M_Z^2}{M_{Z'}^2} \right] \equiv B(\beta, a) \left[ \frac{M_Z^2}{M_{Z'}^2} \right]$$
$$f(\beta) = \frac{1}{1 - (1 + \beta^2)s_W^2} > 0$$

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$$f(\beta) = \frac{1}{1 - (1 + \beta^2)s_W^2} > 0$$

Consequences of mixing:

flavour violating couplings of Z to quarks are generated

$$\Delta_L^{ij}(Z) = \sin \xi \, \Delta_L^{ij}(Z') \qquad (i \neq j)$$

 flavour diagonal couplings to SM fermions differ from SM Z couplings



Corresponding right-handed quarks are singlets







Corresponding right-handed quarks are singlets





Quark mass eigenstates defined upon rotation through two unitary matrices  $U_L \otimes V_L$ 

$$V_{\rm CKM} = U_L^{\dagger} V_L$$

In contrast to SM only one of them can be traded for VCKM, the other one enters in Z' couplings to quarks

$$V_{L} = \begin{pmatrix} \tilde{c}_{12}\tilde{c}_{13} & \tilde{s}_{12}\tilde{c}_{23}e^{i\delta_{3}} - \tilde{c}_{12}\tilde{s}_{13}\tilde{s}_{23}e^{i(\delta_{1}-\delta_{2})} & \tilde{c}_{12}\tilde{c}_{23}\tilde{s}_{13}e^{i\delta_{1}} + \tilde{s}_{12}\tilde{s}_{23}e^{i(\delta_{2}+\delta_{3})} \\ -\tilde{c}_{13}\tilde{s}_{12}e^{-i\delta_{3}} & \tilde{c}_{12}\tilde{c}_{23} + \tilde{s}_{12}\tilde{s}_{13}\tilde{s}_{23}e^{i(\delta_{1}-\delta_{2}-\delta_{3})} & -\tilde{s}_{12}\tilde{s}_{13}\tilde{c}_{23}e^{i(\delta_{1}-\delta_{3})} - \tilde{c}_{12}\tilde{s}_{23}e^{i\delta_{2}} \\ -\tilde{s}_{13}e^{-i\delta_{1}} & -\tilde{c}_{13}\tilde{s}_{23}e^{-i\delta_{2}} & \tilde{c}_{13}\tilde{c}_{23} \end{pmatrix}$$

331 Models: Z' couplings to quarks The case of  $B_d$ ,  $B_s$ , K systems



 $\begin{array}{c} 331 \text{ Models: } Z' \text{ couplings to quarks}\\ \text{The case of } \mathsf{B}_{\mathsf{d}}, \mathsf{B}_{\mathsf{s}}, \mathsf{K} \text{ systems} \end{array}$   $\begin{array}{c} z' \\ & & \\ &$ 

Summary of the variants of 331 Models

- value of  $\beta$ :  $\beta = \pm 1/\sqrt{3}, \pm 2/\sqrt{3}$
- fermion representation: F1 or F2
- Z-Z' mixing parameter:  $\tan \overline{\beta}=0.2$ ,  $\tan \overline{\beta}=1$ ,  $\tan \overline{\beta}=5.0$



### 24 possibilities to be explored!

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1	MI	scen.	β	$ anar{eta}$	MI	scen.	β	$ anar{eta}$	MI	scen.	β	$ anar{eta}$
N	<b>M</b> 1	$F_1$	$-2/\sqrt{3}$	1	M9	$F_2$	$-2/\sqrt{3}$	1	M17	$F_1$	$-2/\sqrt{3}$	0.2
N	M2	$F_1$	$-2/\sqrt{3}$	5	M10	$F_2$	$-2/\sqrt{3}$	5	M18	$F_2$	$-2/\sqrt{3}$	0.2
N	M3	$F_1$	$-1/\sqrt{3}$	1	M11	$F_2$	$-1/\sqrt{3}$	1	M19	$F_1$	$-1/\sqrt{3}$	0.2
N	M4	$F_1$	$-1/\sqrt{3}$	5	M12	$F_2$	$-1/\sqrt{3}$	5	M20	$F_2$	$-1/\sqrt{3}$	0.2
N	M5	$F_1$	$1/\sqrt{3}$	1	M13	$F_2$	$1/\sqrt{3}$	1	M21	$F_1$	$1/\sqrt{3}$	0.2
N	M6	$F_1$	$1/\sqrt{3}$	5	M14	$F_2$	$1/\sqrt{3}$	5	M22	$F_2$	$1/\sqrt{3}$	0.2
N	M7	$F_1$	$2/\sqrt{3}$	1	M15	$F_2$	$2/\sqrt{3}$	1	M23	$F_1$	$2/\sqrt{3}$	0.2
N	<b>M</b> 8	$F_1$	$2/\sqrt{3}$	5	M16	$F_2$	$2/\sqrt{3}$	5	M24	$F_2$	$2/\sqrt{3}$	0.2

### Preliminary selection among the variants of 331 Models: constraints from EWPO

Quantity	Input Data	SMfit	Pull
$\Gamma_Z$	2.4952(23)	2.4954(14)	0.09
$\sigma_h$ [nbarn]	41.540(37)	41.479(14)	-1.65
$R_{\ell}$	20.767(25)	20.740(17)	-1.08
$\mathcal{A}^\ell_{\mathrm{FB}}$	0.0171(10)	0.01627(2)	-0.83
$\mathcal{A}_{\ell}(\text{LEP})$	-0.1465(33)	-0.1472(7)	-0.2
$\mathcal{A}_\ell(\mathrm{SLD})$	-0.1513(21)	-0.1472(7)	1.95
$\mathcal{A}_{\ell}$	-0.1499(18)	-0.1472(7)	1.50
$\sin^2 \theta_{ m eff}^l$	0.2324(12)	0.23148(10)	-0.7
$\mathcal{A}_{c}$	-0.670(27)	-0.6679(3)	0.07
$\mathcal{A}_b$	-0.923(20)	-0.93464(5)	-0.58
$\mathcal{A}^c_{ ext{FB}}$	0.0707(35)	0.0738(4)	0.88
$\mathcal{A}^b_{ ext{FB}}$	0.0992(16)	0.1032(5)	2.5
$R_c$	0.1721(30)	0.17223(6)	0.04
$R_b$	0.21629(66)	0.21548(5)	-1.23

Selection can be done comparing the performance of 331 models to that of SM through the pulls for a generic observable  $O_i$ :

$$P_i^{\rm SM} = \frac{\rm SMfit_i - (\rm Input \ Data)_i}{\sigma_{\rm exp}^i}, \qquad P_i^{331} = \frac{\rm SMfit_i + \delta \mathcal{O}_i - (\rm Input \ Data)_i}{\sigma_{\rm exp}^i}.$$

$$\Omega^{\rm SM} = \sum_{i} \left( P_i^{\rm SM} \right)^2 = 15.72 \qquad \qquad \Omega^{331} = \sum_{i} \left( P_i^{331} \right)^2$$

Models with smallest  $\Omega^{331}$  are favoured

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1	M8	$F_1$	$2/\sqrt{3}$	5	M16	$F_2$	$2/\sqrt{3}$	5	M24	$F_2$	$2/\sqrt{3}$	0.2

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M8	$F_1$	$2/\sqrt{3}$	5	M16	$F_2$	$2/\sqrt{3}$	5				



$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{331} = \left(\frac{\varepsilon'}{\varepsilon}\right)_{\rm SM} + \left(\frac{\varepsilon'}{\varepsilon}\right)_{Z'} + \left(\frac{\varepsilon'}{\varepsilon}\right)_{Z} \equiv \left(\frac{\varepsilon'}{\varepsilon}\right)_{\rm SM} + \Delta(\varepsilon'/\varepsilon)$$

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{V} = \frac{\omega}{|\varepsilon_{K}|\sqrt{2}} \frac{\mathrm{Im}A_{2}(V)}{\mathrm{Re}A_{2}}, \qquad \omega = \frac{\mathrm{Re}A_{2}}{\mathrm{Re}A_{0}} = (4.46) \times 10^{-2} \qquad A_{I} \equiv \langle (\pi\pi)_{I} | \mathcal{H}_{\mathrm{eff}} | K \rangle$$

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for V=Z, Z'

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$$A_{2}(V) = C_{8}(m_{c}, V) \langle Q_{8}(m_{c}) \rangle_{2}$$

$$\langle Q_{8}(m_{c}) \rangle_{2} = \sqrt{2} \left[\frac{m_{K}^{2}}{m_{s}(m_{c}) + m_{d}(m_{c})}\right]^{2} F_{\pi} B_{8}^{(3/2)} = 0.862 B_{8}^{(3/2)} \mathrm{GeV}^{3}.$$

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for V=Z, Z'

$$\begin{split} \left(\frac{\varepsilon'}{\varepsilon}\right)_{V} &= \frac{\omega}{|\varepsilon_{K}|\sqrt{2}} \frac{\mathrm{Im}A_{2}(V)}{\mathrm{Re}A_{2}}, \qquad \omega = \frac{\mathrm{Re}A_{2}}{\mathrm{Re}A_{0}} = (4.46) \times 10^{-2} \end{split} \qquad A_{I} &\equiv \langle (\pi\pi)_{I} | \mathcal{H}_{\mathrm{eff}} | K \rangle \\ A_{2}(V) &= C_{8}(m_{c}, V) \langle Q_{8}(m_{c}) \rangle_{2} \end{split} \\ \langle Q_{8}(m_{c}) \rangle_{2} &= \sqrt{2} \left[ \frac{m_{K}^{2}}{m_{s}(m_{c}) + m_{d}(m_{c})} \right]^{2} F_{\pi} B_{8}^{(3/2)} = 0.862 B_{8}^{(3/2)} \mathrm{GeV}^{3} \end{split}$$

$$\Delta(\varepsilon'/\varepsilon) = (1 + R_{\varepsilon'}) \left(\frac{\varepsilon'}{\varepsilon}\right)_{Z'}$$

depends on  $\beta$ , on Z-Z' mixing, on the fermion representation



First study of  $\epsilon'/\epsilon$  in 331 Models: results

A.J. Buras, FDF JHEP 1603 (2016) 010

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only 3 survive!



• Similar results for M8 & M9, differences in M16

2.2

2.1

2.3

 $|\epsilon_K| \, [10^{-3}]$ 

2.4

2.5

• other correlations should be investigated

2.5

-6

2.2

2.3

 $|\epsilon_K| \, [10^{-3}]$ 

2.4





- Similar results for M8 & M9, differences in M16
- other correlations should be investigated

the shift in  $\epsilon'/\epsilon$  is larger when  $\epsilon_\kappa$  is enhanced in all the 3 cases

#### correlations with B physics observables



M16 produces the largest new contribution to  $C_9$  -> can help softening the angular anomaly

correlations with B physics observables: in a few words...

- B Physics can distinguish between (M8,M9) and M16
- (M8,M9) perform better with suppression of  $B(B_s \rightarrow \mu^+ \mu^-)$
- M16 could help with the angular anomaly in B -> K<sup>\*</sup>  $\mu^+ \mu^-$

Dependence on V<sub>cb</sub>



Dependence on V<sub>cb</sub>



Varying also M<sub>7</sub>





NP effects in  $B(B_s \rightarrow \mu^+ \mu^-)$  and in  $Re(C_9^{NP})$  decrease

Varying also  $V_{ub}$ 





New SM results for  $\epsilon'/\epsilon$  are significantly below data

331 models:

- among 24 considered variants only 7 survive EPWO test
- only 3 can provide  $\Delta(\epsilon'/\epsilon) > 4.0 \ 10^{-4}$  (for M<sub>z'</sub>=3 TeV)
- M8 & M9 can also suppress B(B<sub>s</sub> ->  $\mu^+ \mu^-$ ) but cannot solve angular anomaly in B  $\rightarrow K^* \mu^+ \mu^-$
- M16 performs the opposite
- For larger  $M_{z'}$  (but below 50 TeV)  $\epsilon'/\epsilon$  can still be accomodated but effects in B physics are negligible
- In 331 models  $V_{cb}$ =0.040 favoured over  $V_{cb}$ =0.042