Perturbative Unitarity in the Dark Sector

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Based on 1412.5660, 1501.03153 and ongoing work with Matthew Cahill-Rowley, Sonia El-Hedri, and Devin Walker

The Scale of New Physics

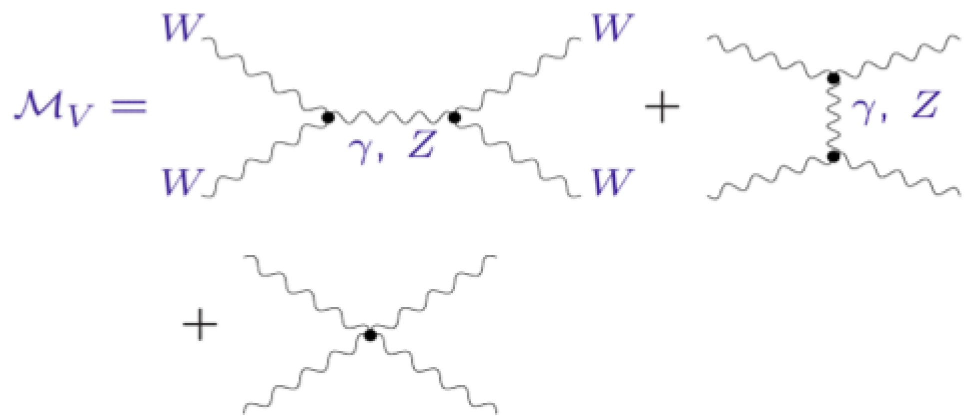
- Historically in HEP, we’ve often known where we were going
  - Fermi theory of weak decays needed new bosons
  - Precision measurements pointed to the top quark
  - Heavy bosons needed symmetry breaking
- After the Higgs discovery, we have no map
  - The Standard Model is stubbornly good
- Where are we going, and how far away is it?

The Scale of New Physics

- Naturalness
  - The Higgs mass is subject to corrections from new physics, these corrections are potentially huge
  - The SM as a UV theory requires cancellations in these corrections to 1 part in $10^{\sim 30}$
  - New symmetry could enforce this if it happens low enough, but bounds on scale depend on amount of tuning deemed acceptable
- New Phenomena
  - Neutrino masses need at least the Weinberg operator, but that can be at scales far beyond what we'll see
  - Dark matter is the other new particle we need experimentally

The LHC No-Lose Theorem

- The Higgs (or something else doing its job) had to be there
  - Symmetry has to break to make gauge bosons massive
- Why did it have to be within the LHC's reach?



This amplitude (missing a Higgs contribution) grows with energy, and predicts scattering probabilities greater than 1 beyond energies of about 800 GeV

Basics of Unitarity

- We start with a scattering matrix

$$S = 1 + iT$$

- Unitarity gives the optical theorem

$$S^\dagger S = I \Rightarrow \frac{1}{2}(T - T^\dagger) = |T_{ii}^2|$$

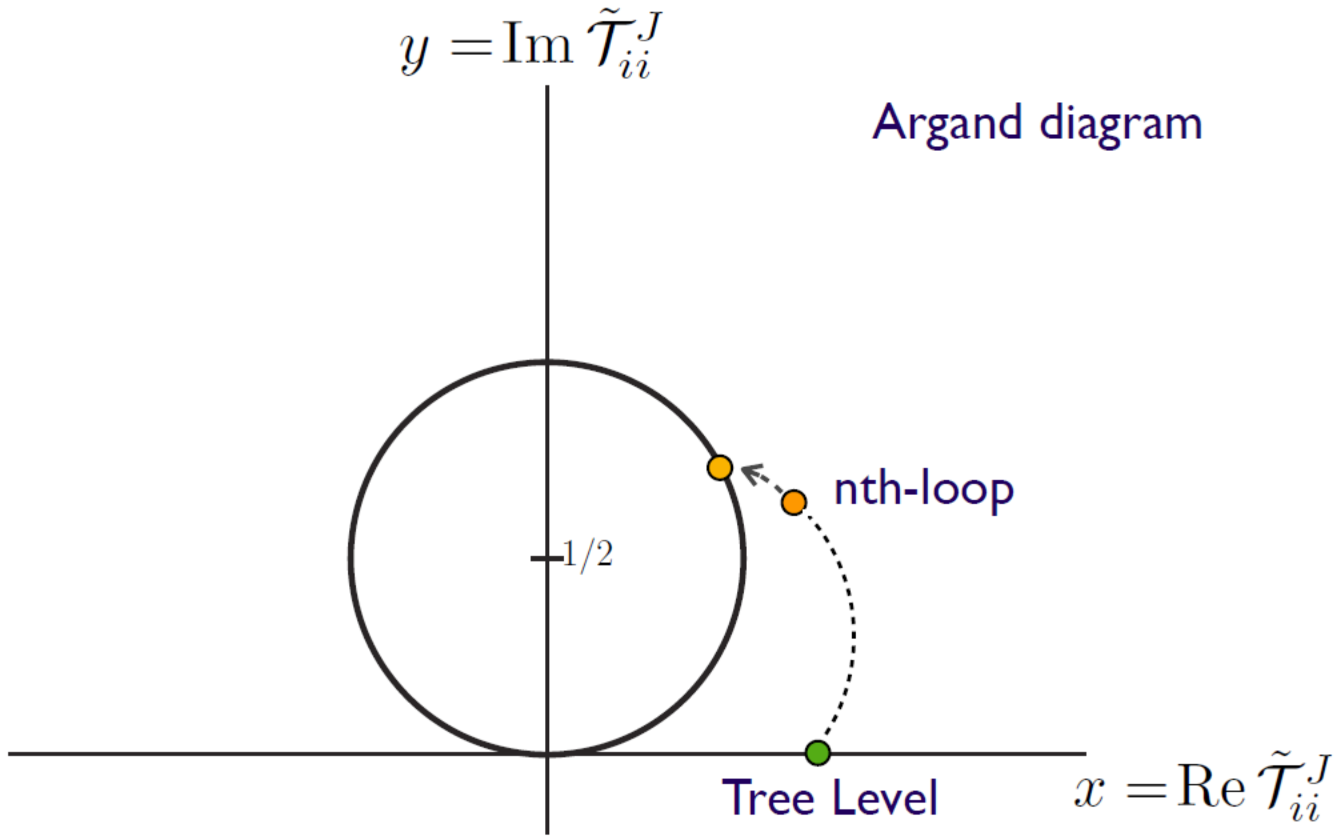
- Expanding in partial waves

$$\tilde{T}_{ij}^J = \frac{\lambda_i^{1/4}\lambda_f^{1/4}}{32\pi s}\int_{-1}^{1} T_{ij} P_J(\cos\theta) d\cos\theta$$

- We find that

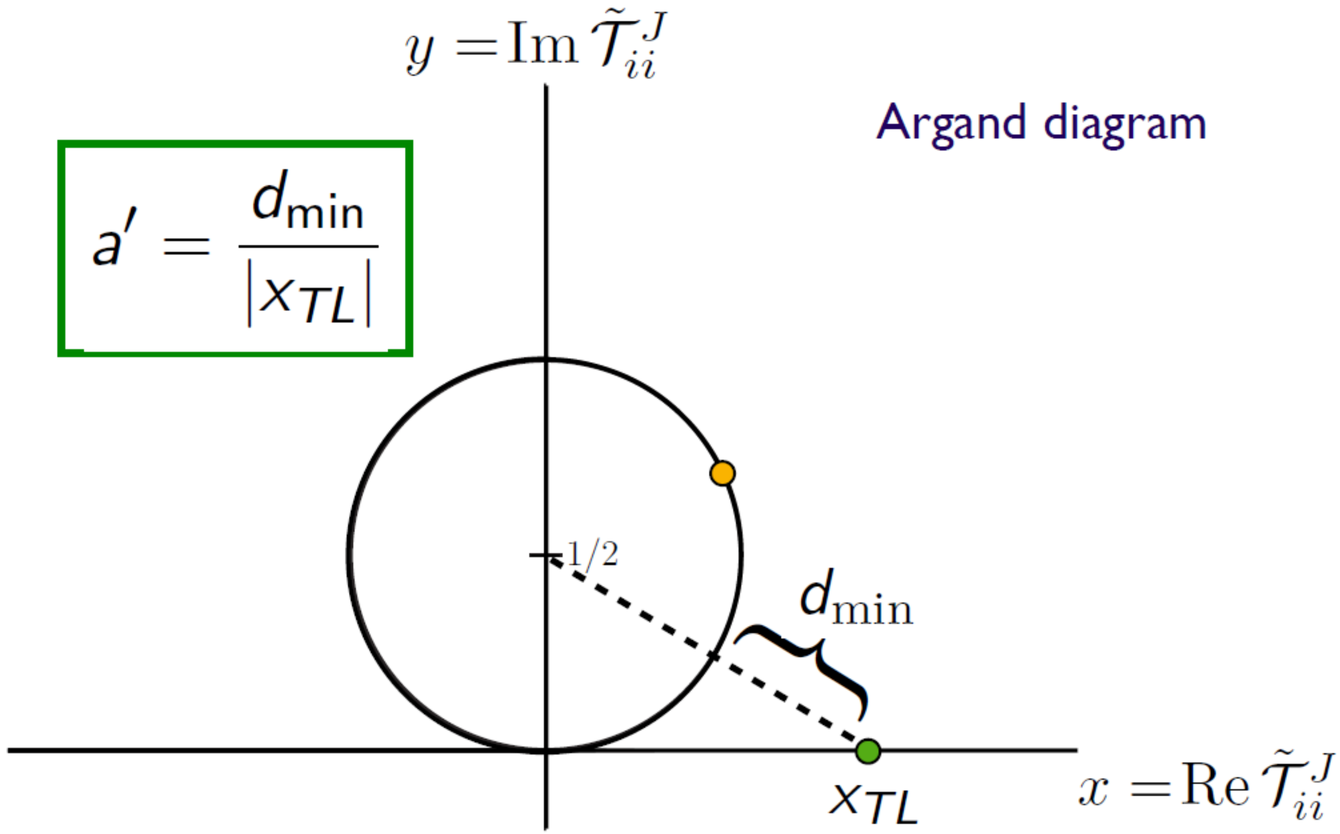
$$\mathrm{Im}\,\tilde{T}_{ii} = |\tilde{T}_{ii}|^2 \Rightarrow |\mathrm{Re}\,\tilde{T}_{ii}| < \frac{1}{2}$$

A Picture of Unitarity



Schuessler and Zeppenfeld 0710.5175
Aydemir, Anber, Donoghue 1203.5153

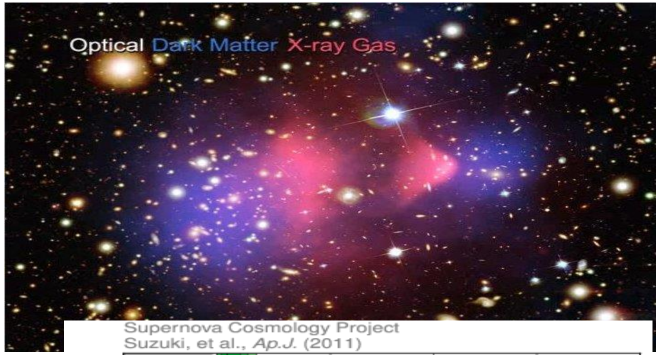
A Picture of Unitarity

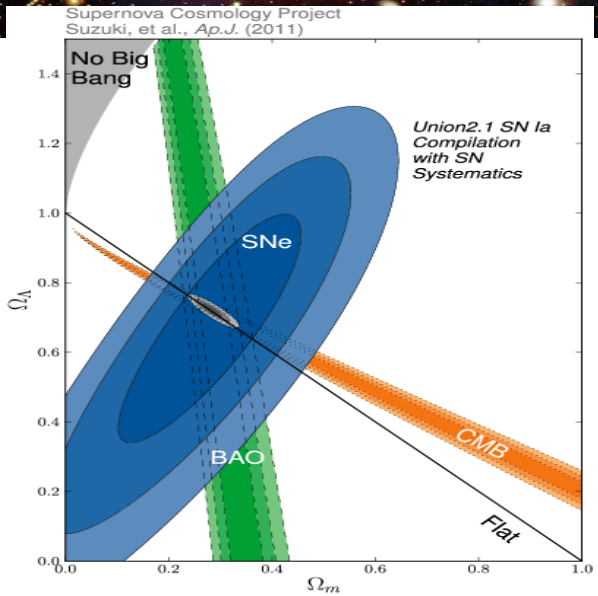


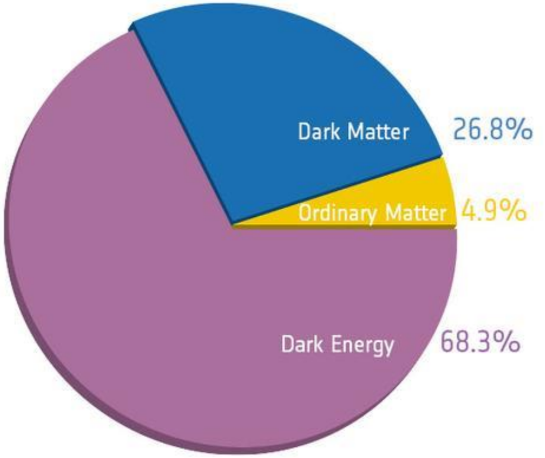
Schuessler and Zeppenfeld 0710.5175

Dark Matter

- The evidence for dark matter is myriad and well-known.
- This evidence is one of the only truly experimental signs that we must have physics beyond the Standard Model.
- Cosmological observations tell us how much dark matter is needed to match observations.
- From the particle physics perspective, we're left asking what dark matter is and how it fits into a microscopic understanding of nature.





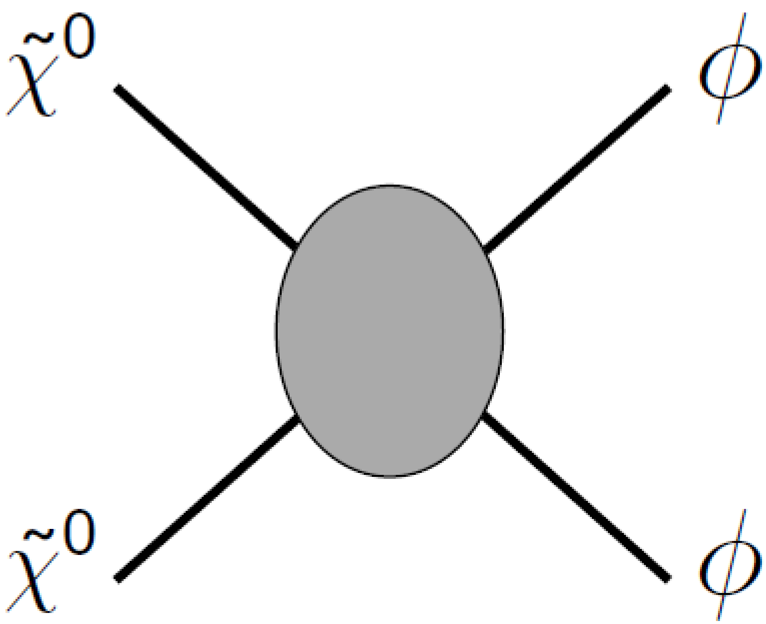


WIMP (Thermal) Dark Matter

- One of the most attractive proposals to explain dark matter is that it is a Weakly Interacting Massive Particle.
  - WIMPs naturally lead to the correct amount of dark matter in the universe.
  - WIMPs are automatic ingredients of many models of physics beyond the Standard Model, such as supersymmetric models.
- The one thing robustly predicted by all thermal models of dark matter is the annihilation cross-section.

Unitarity and Dark Matter

- By insisting on unitarity in a general dark matter scenario, we can bound dark matter to be lighter than 120 TeV for coupling below 4π



$$\propto \lambda^2$$

Griest and Kamionkowski, 1990

Dark Matter Models

- Here I will consider two different models of dark matter in turn
- First, a model using the 'gauge portal' of kinetic mixing between a new U(1) and hypercharge
- Second, a SUSY-type simplified model with new scalars charged under the SM

Gauge Portal Dark Matter

- This model is characterized by the Lagrangian

$$\mathcal{L}_{DM} \supset g' \bar{\chi}\gamma^{\mu}\gamma_5 Z'_{\mu}\chi - \lambda_{\chi}\bar{\chi}\Phi\chi$$

$$\mathcal{L}_{gauge} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}Z'_{\mu\nu}Z'^{\mu\nu} + \frac{\sin\delta}{2}Z'_{\mu\nu}B^{\mu\nu}$$

$$\mathcal{L}_{Higgs} \supset \left|D_{\mu}\Phi\right|^2 + V\left(H, \Phi; \lambda_1, \lambda_2, \lambda_3\right)$$

- With breaking of the new symmetry by

$$\Phi = \frac{1}{\sqrt{2}}(\mathbf{u} + \phi^0)$$

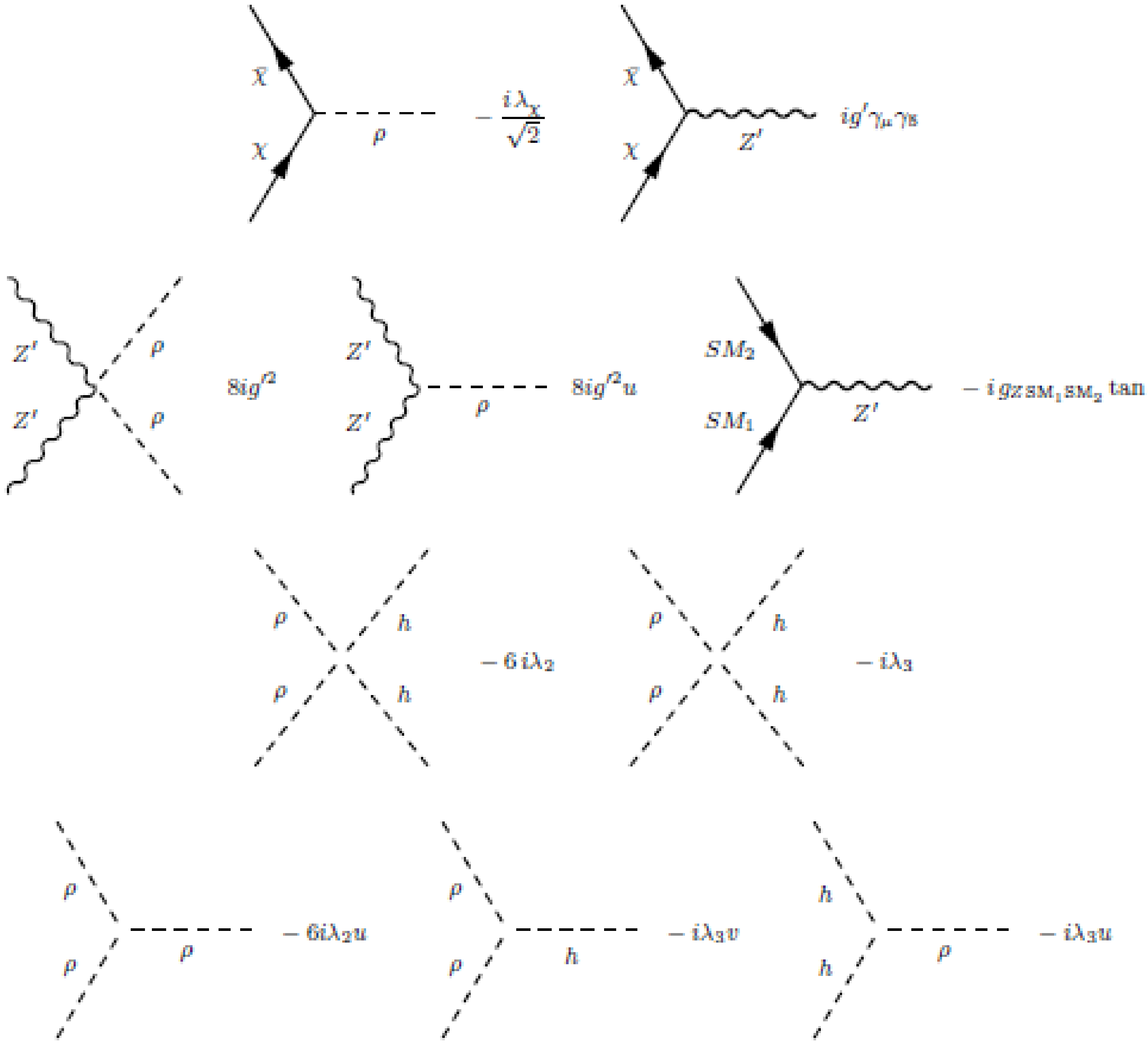
Babu, Kolda, March-Russell hep-ph/9710441

Gauge Portal Dark Matter

- This gives us 6 parameters and 1 new scale:

$$g', \lambda_\chi, \lambda_1, \lambda_2, \lambda_3, \sin\delta, \mathbf{u}$$

- The dimensionless couplings can be constrained directly from unitarity, but only ratios of scales can be constrained
- Here, the annihilation rate will set upper bounds on the scale of symmetry breaking



$\bar{\chi}$
χ
ρ
$-\frac{i\lambda_\chi}{\sqrt{2}}$
$\bar{\chi}$
χ
Z'
$ig'\gamma_\mu\gamma_5$
Z'
Z'
ρ
ρ
$8ig'^2$
Z'
Z'
ρ
$8ig'^2u$
SM_2
SM_1
Z'
$-i\,g_{Z\,SM_1 SM_2}\tan\delta$
ρ
ρ
h
h
$-6\,i\lambda_2$
ρ
ρ
h
h
$-i\lambda_3$
ρ
ρ
ρ
$-6i\lambda_2u$
ρ
ρ
h
$-i\lambda_3v$
h
h
ρ
$-i\lambda_3u$

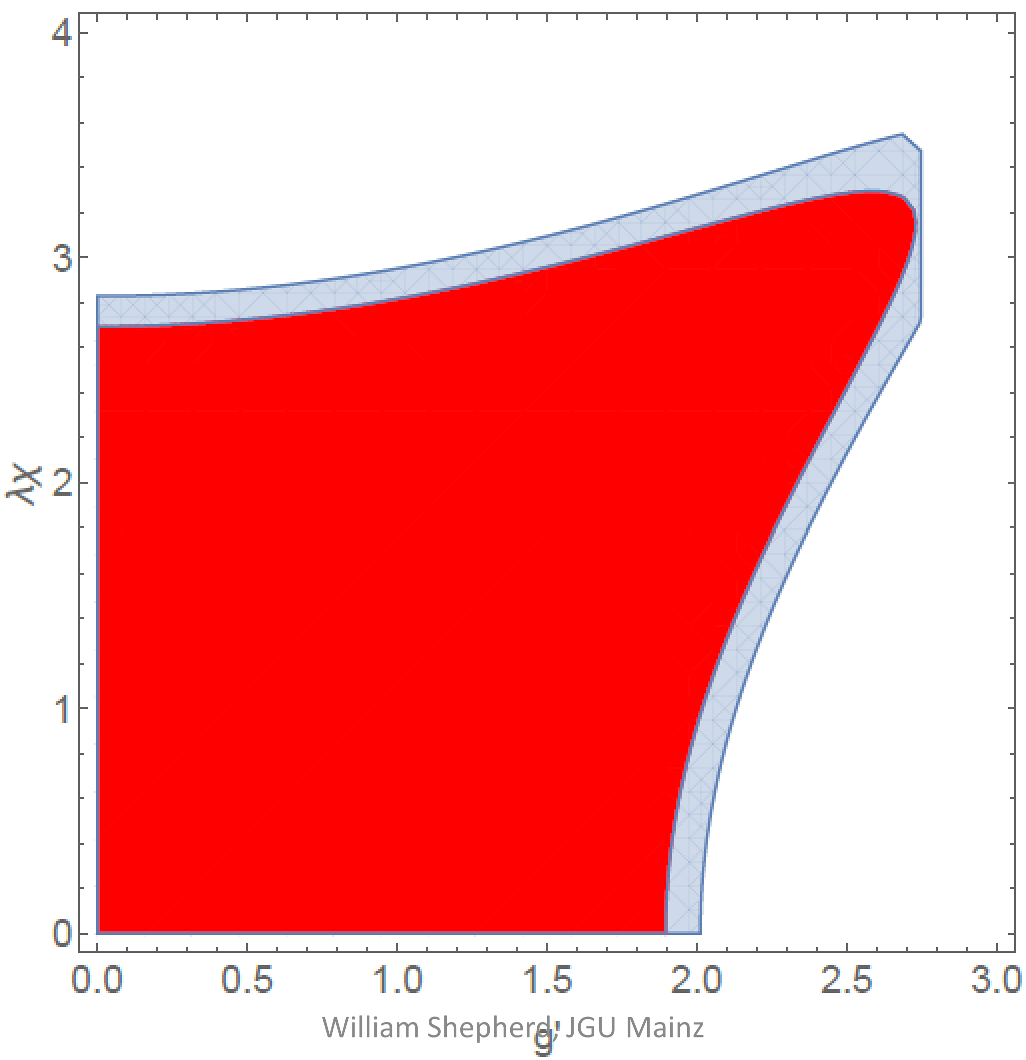
Unitarity Constraints

- We calculate in the high-mass limit and the gauge basis the p-wave scattering matrix

$$(\chi_+\bar{\chi}_+, \chi_-\bar{\chi}_-, \chi_+\bar{\chi}_-, \chi_-\bar{\chi}_+, \rho Z', \rho Z, h Z, h Z')$$

$$\mathcal{T} = \frac{-1}{64\sqrt{2}} \begin{pmatrix} -6g'^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -6g'^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4g'^2 & 2g'^2 & \frac{16}{3}g'^2 - 4\lambda_\chi^2 & 0 & 0 & 0 \\ 0 & 0 & 2g'^2 & 4g'^2 & -\frac{16}{3}g'^2 + 4\lambda_\chi^2 & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3}g'^2 - 4\lambda_\chi^2 & -\frac{16}{3}g'^2 + 4\lambda_\chi^2 & 2\lambda_2 + 5g'^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2\lambda_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_3 \end{pmatrix}$$

Dark Matter Coupling Bounds



Additional Constraints

- Unitarity isn't the only concern in this model
  - Vacuum stability from the scalar potential

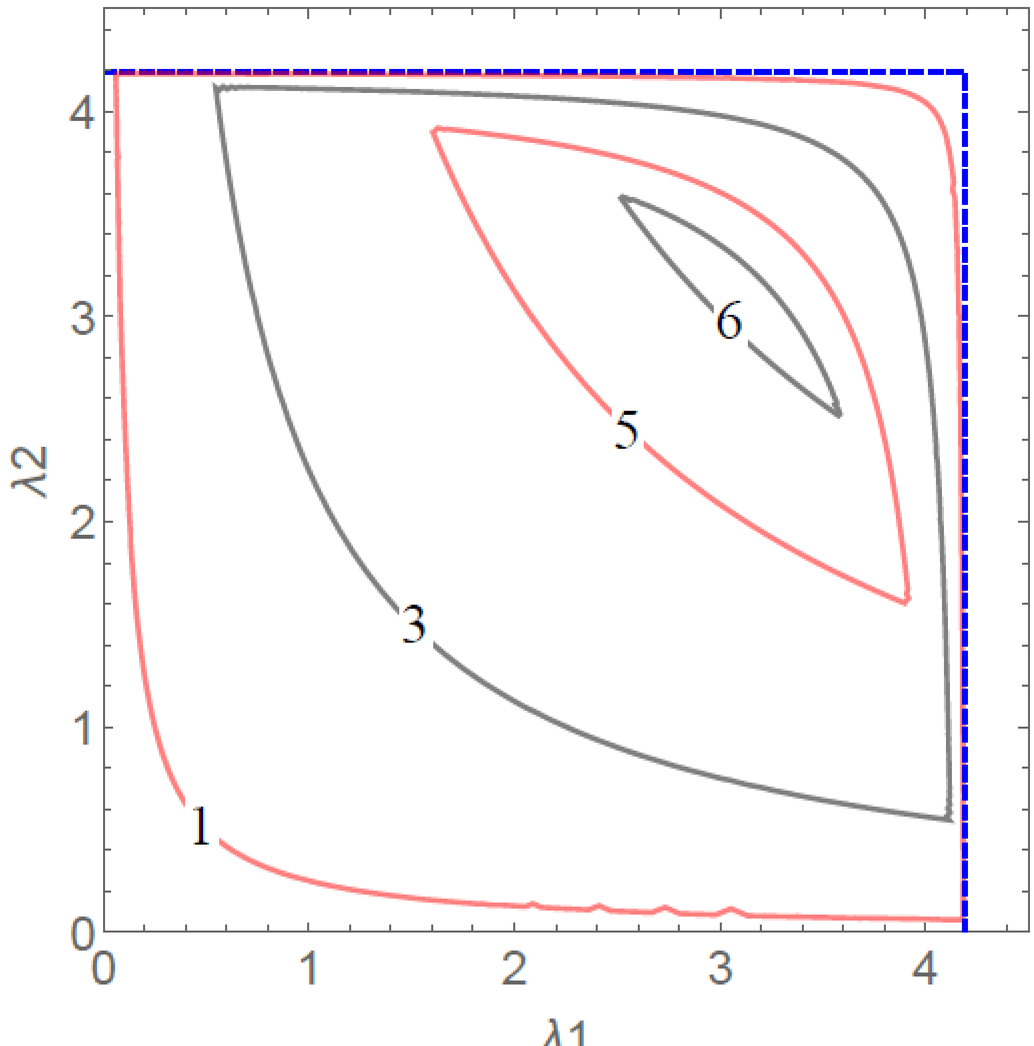
$$V(H,\Phi) = \lambda_1 \left(H^\dagger H - \frac{v^2}{2}\right)^2 + \lambda_2 \left(\Phi^\dagger \Phi - \frac{u^2}{2}\right)^2 + \lambda_3 \left(H^\dagger H - \frac{v^2}{2}\right)\left(\Phi^\dagger \Phi - \frac{u^2}{2}\right)$$

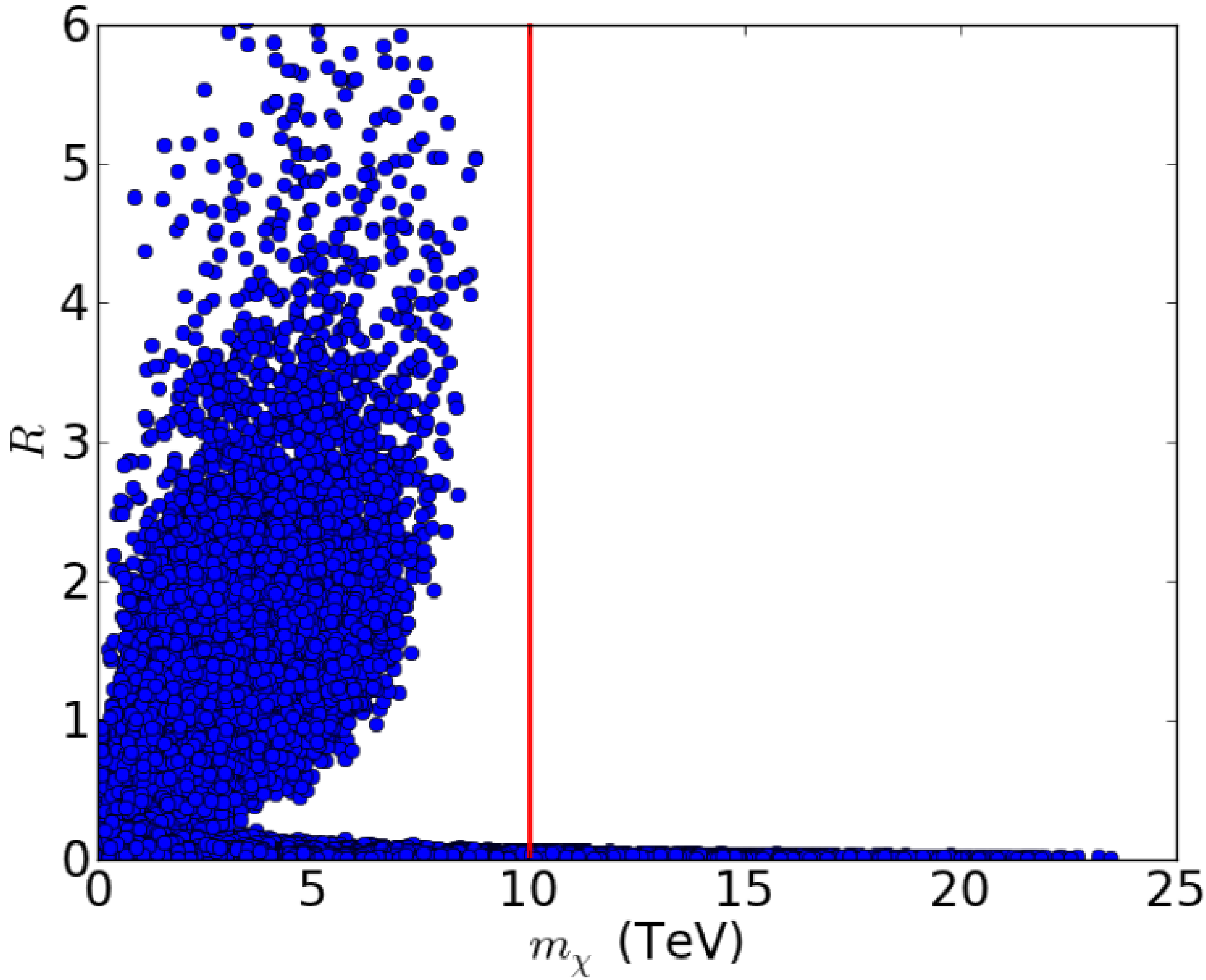
  - Electroweak precision constraints

$$\Upsilon = \left(\frac{\tan\delta}{0.1}\right)^2 \left(\frac{250\text{GeV}}{m_{Z_2}}\right)^2$$

  - Dark Matter relic density and searches

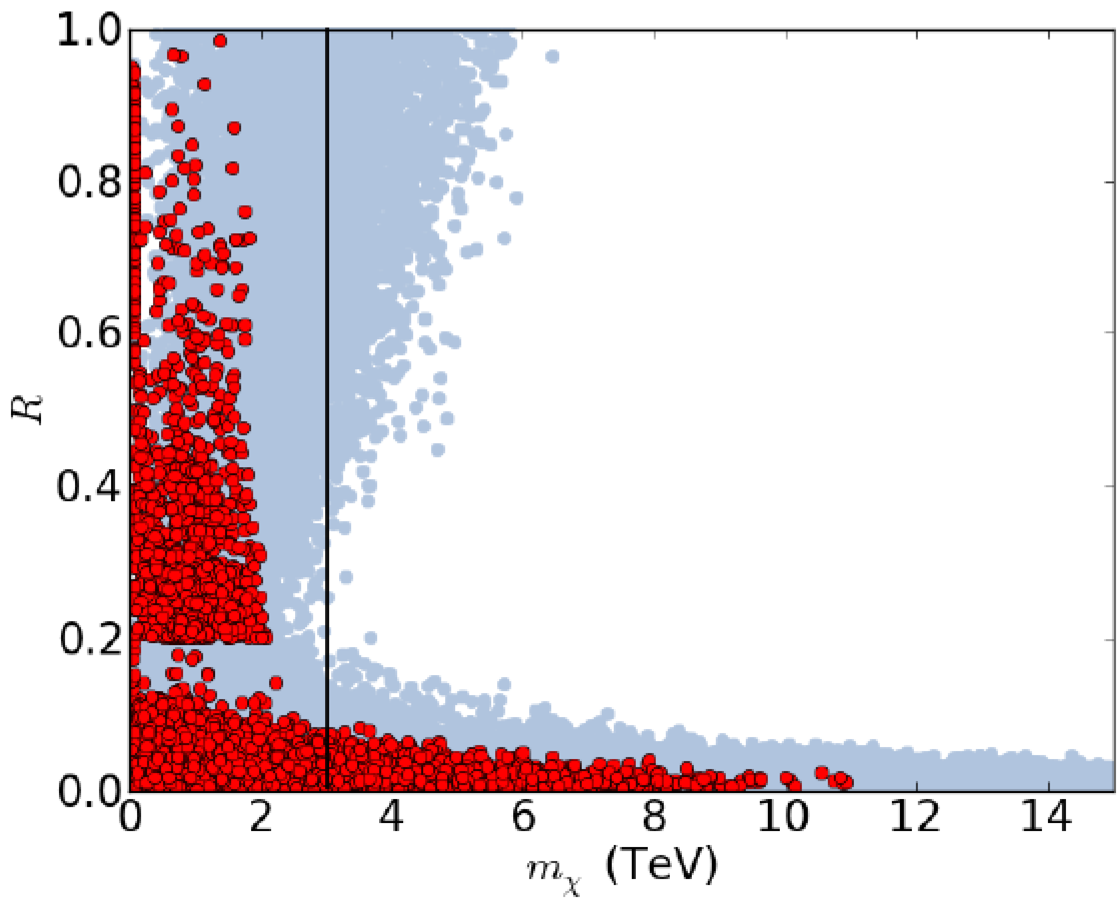
Scalar Coupling Constraints

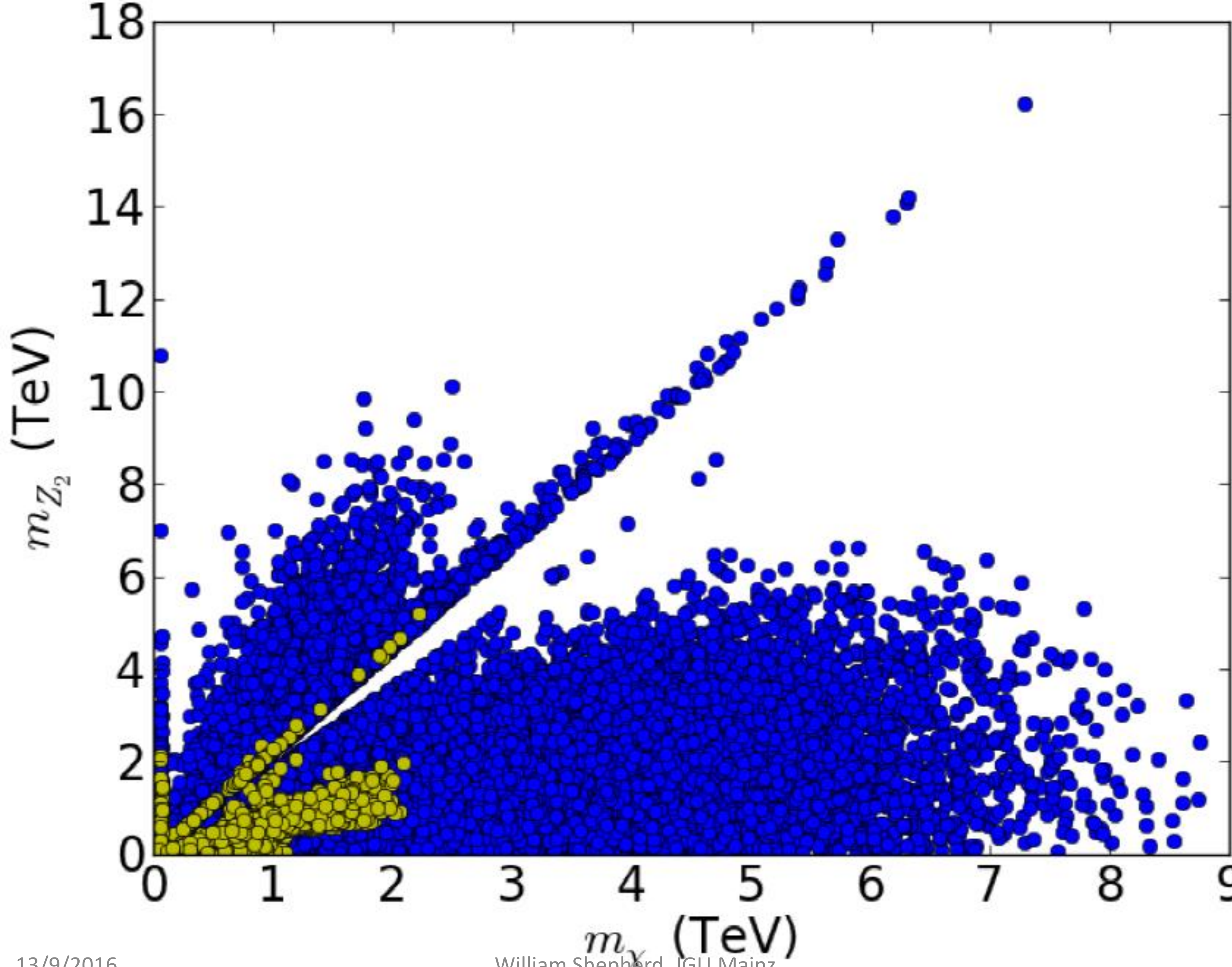




$$R = \min\left\{\frac{|2m_\chi - m_{Z'}|}{m_\chi}, \frac{|2m_\chi - m_{\phi^0}|}{m_\chi}\right\}$$

‘Pure Vector’ Interactions




18
16
14
12
10
8
6
4
2
0
m_{Z_2} (TeV)
0
1
2
3
4
5
6
7
8
9
m_χ (TeV)

Colored Scalars and Dark Matter

- In a SUSY-inspired model, we add

$$\tilde{u}_R = (\tilde{u}_R, \tilde{c}_R, \tilde{t}_R)$$

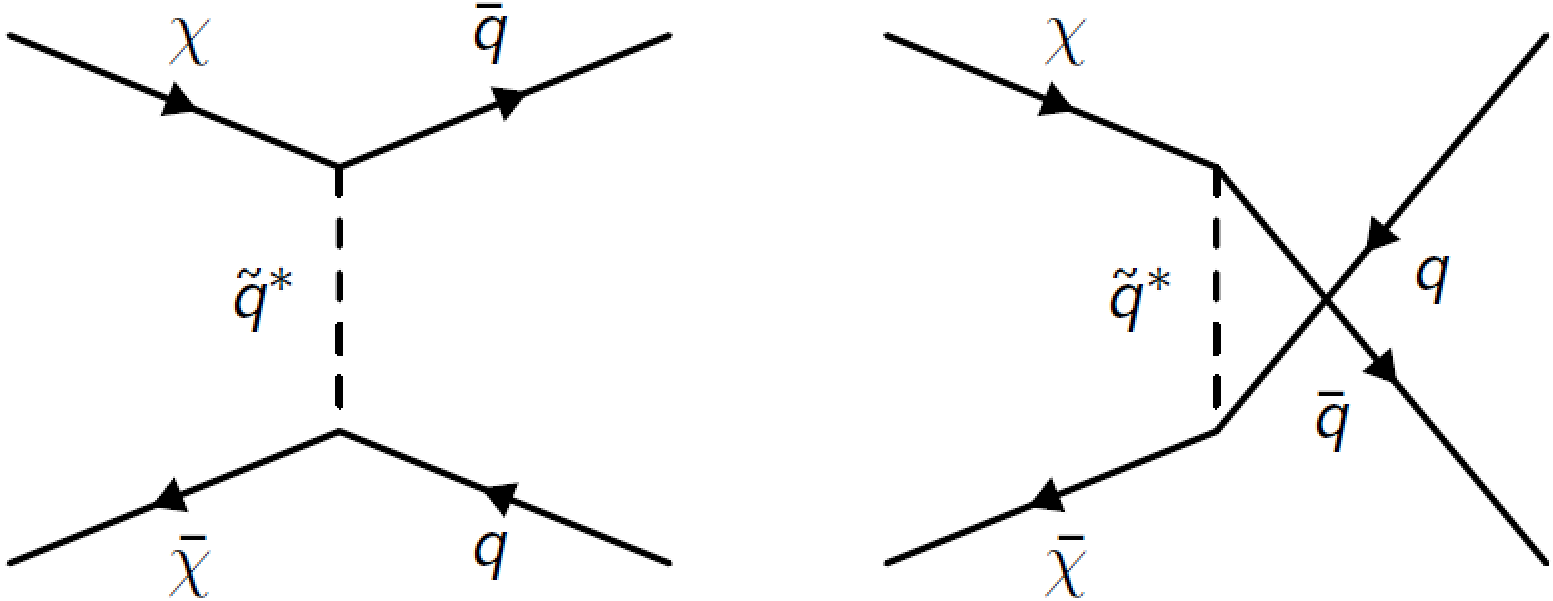
- And the Lagrangian terms

$$\mathcal{L} \supset \frac{1}{2} M_\chi \bar{\chi}\chi + \frac{1}{2} {M_{\tilde{u}}}^2 \tilde{u}^* u + \lambda_{\mathrm{dark}} \tilde{u}^* \bar{\chi} P_R u$$

- This introduces the new parameter and scales

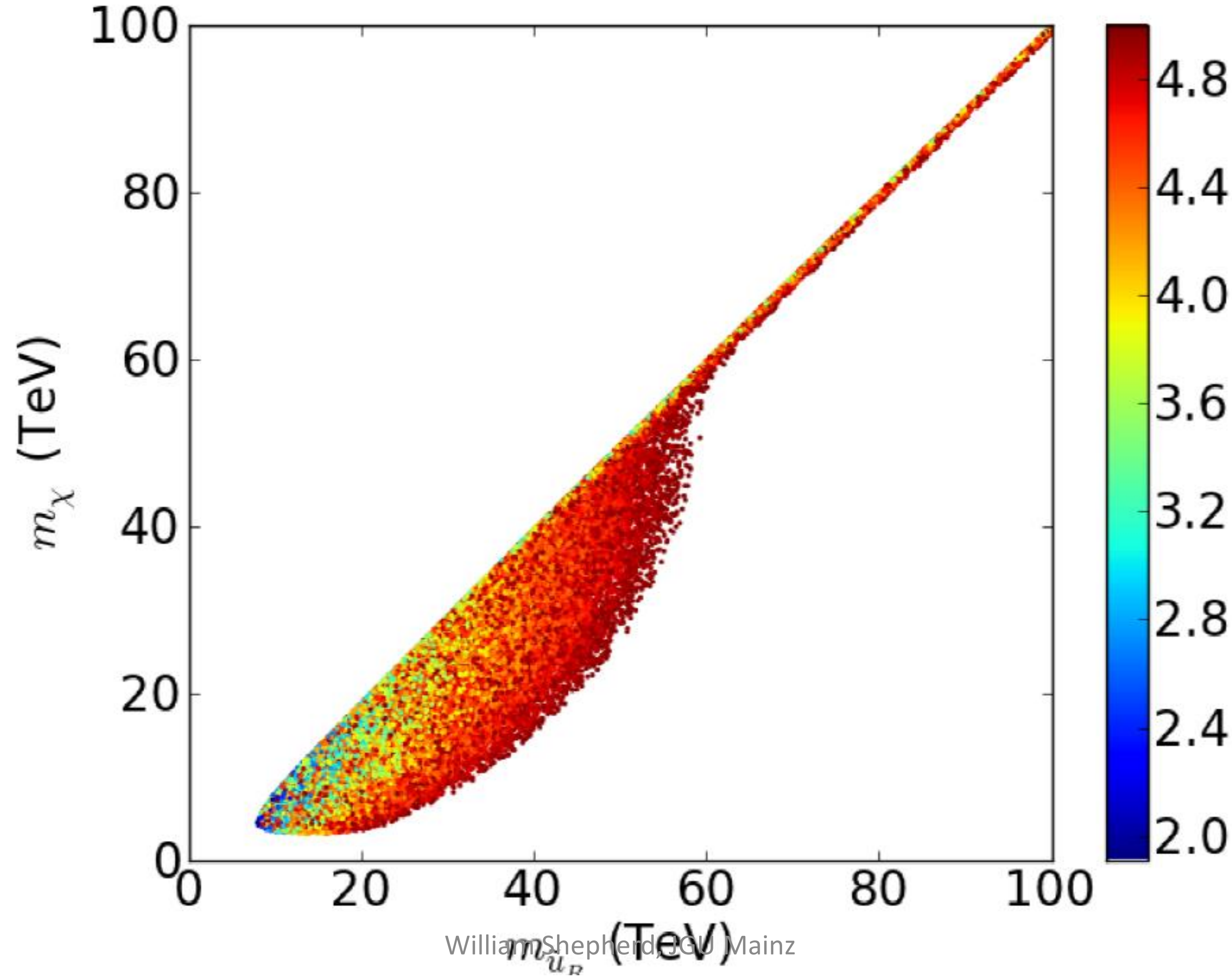
$$\lambda_{\mathrm{dark}},\ M_\chi,\ M_{\tilde{u}}$$

Dirac Dark Matter with 'Squarks'

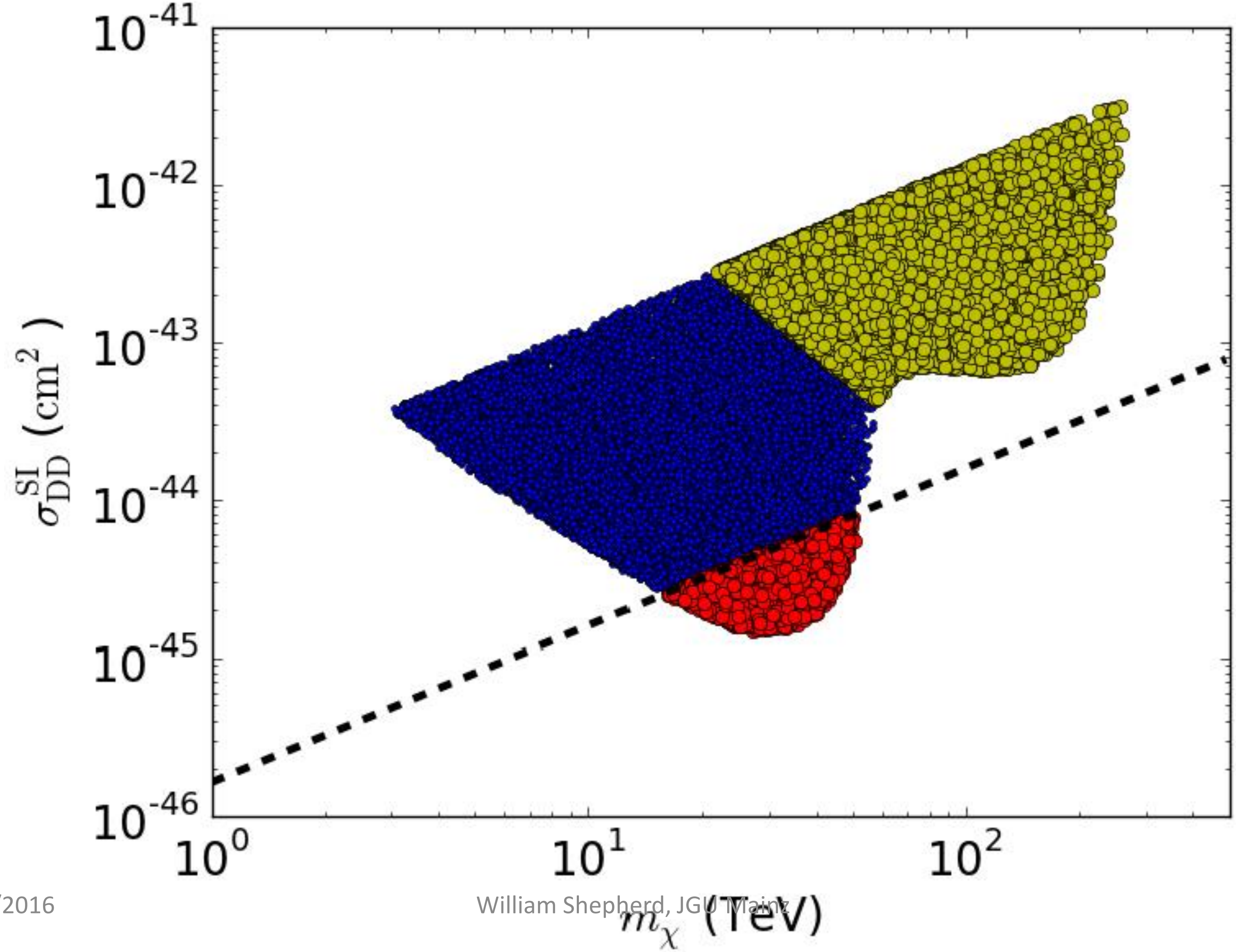


$$\langle \sigma v \rangle_{\mathrm{ann}}^{v=0} \propto \frac{\lambda_{\mathrm{dark}}^4}{(M_\chi^2 + M_{\tilde{u}}^2)^4}$$

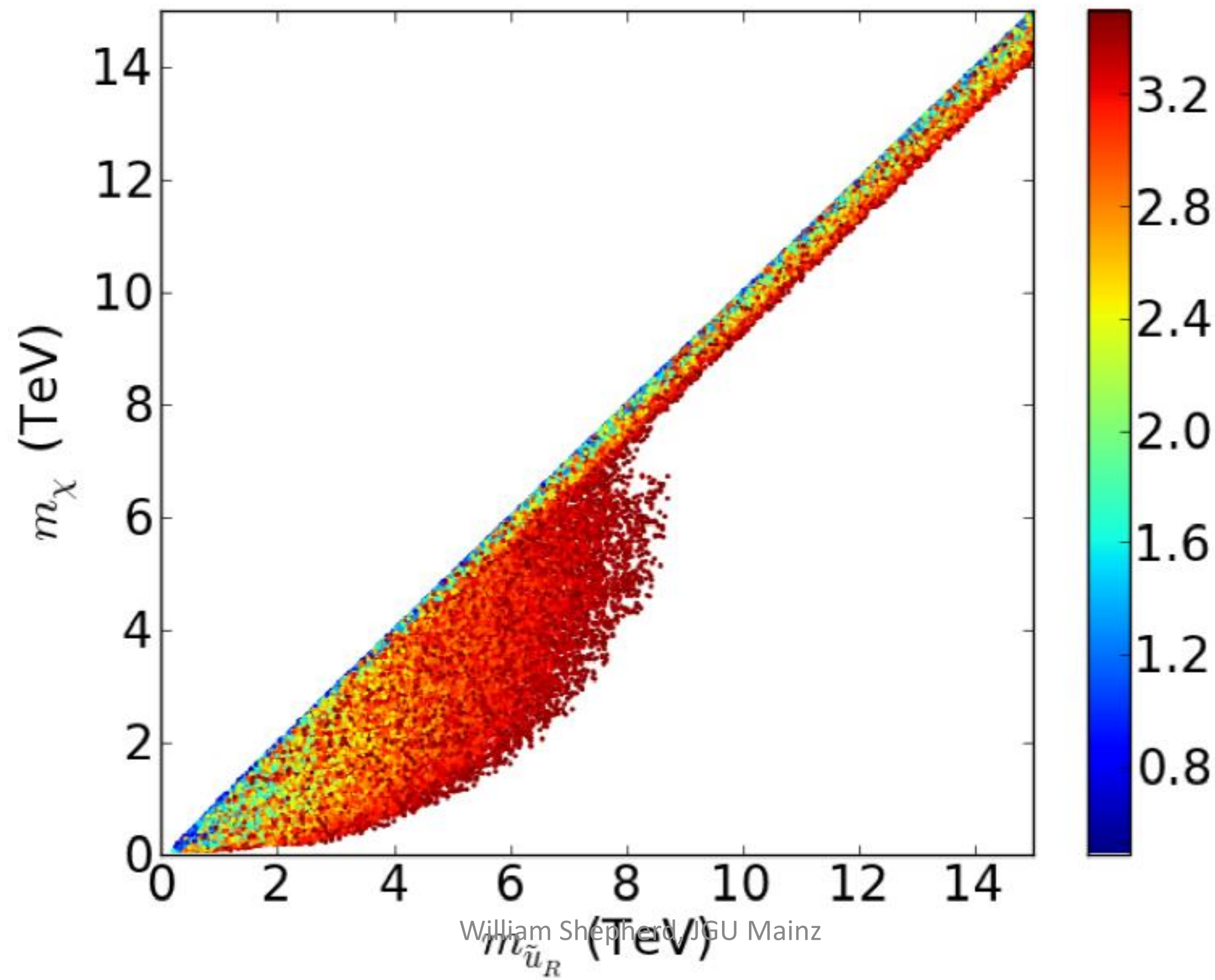
Dirac Dark Matter



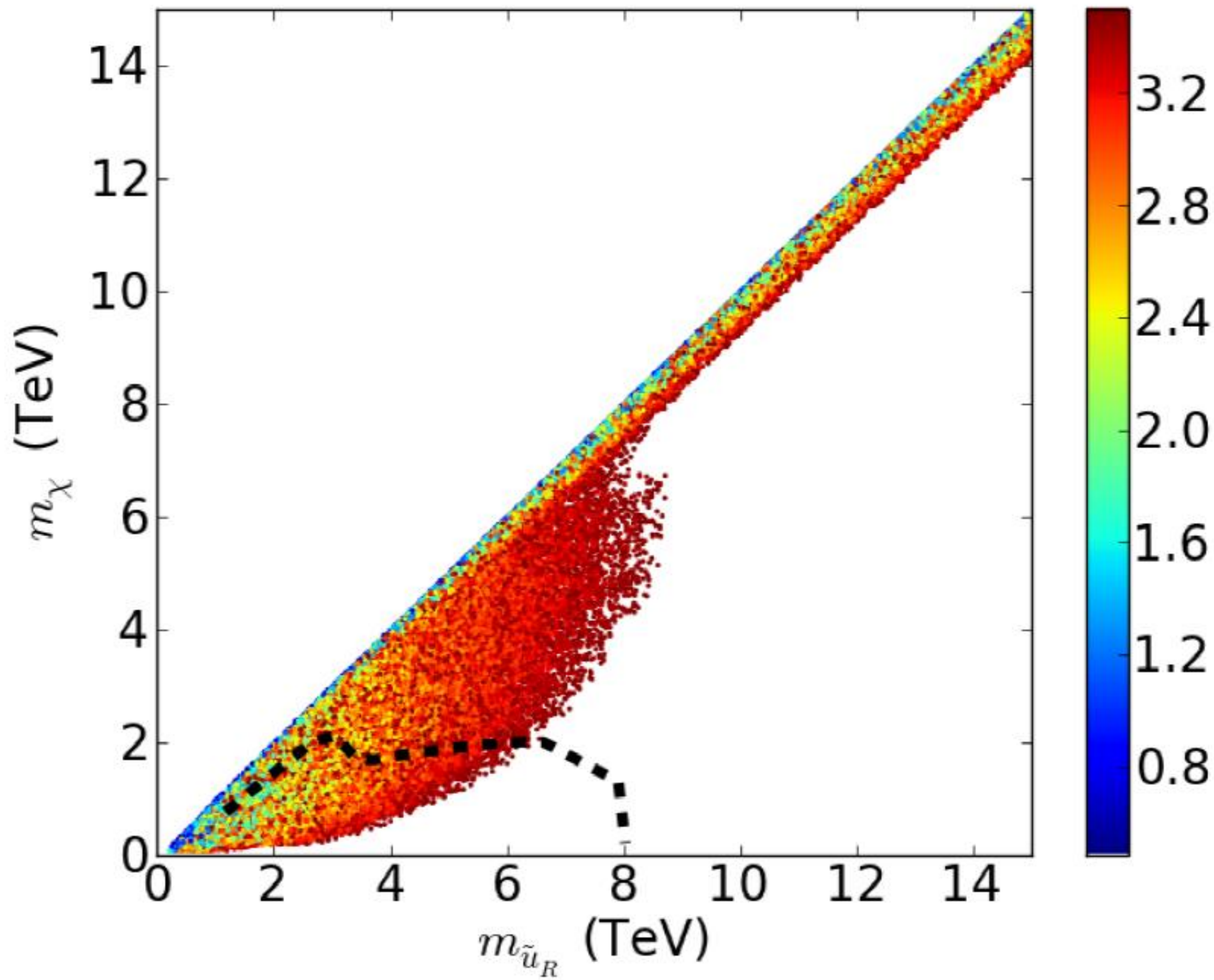
Direct Detection



Majorana Dark Matter



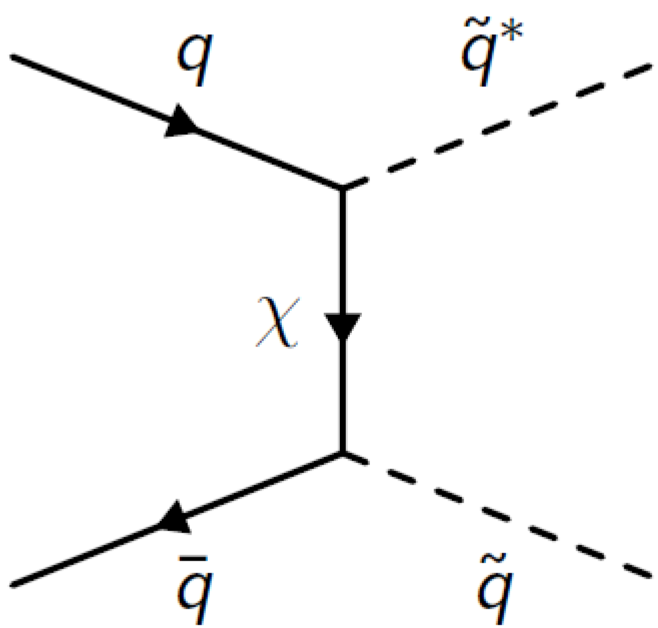
FCC Squark Searches



Cohen, Golling, Hance, Henrichs, Howe, Loyal, Padhi, Wacker [arXiv:1311.6480]

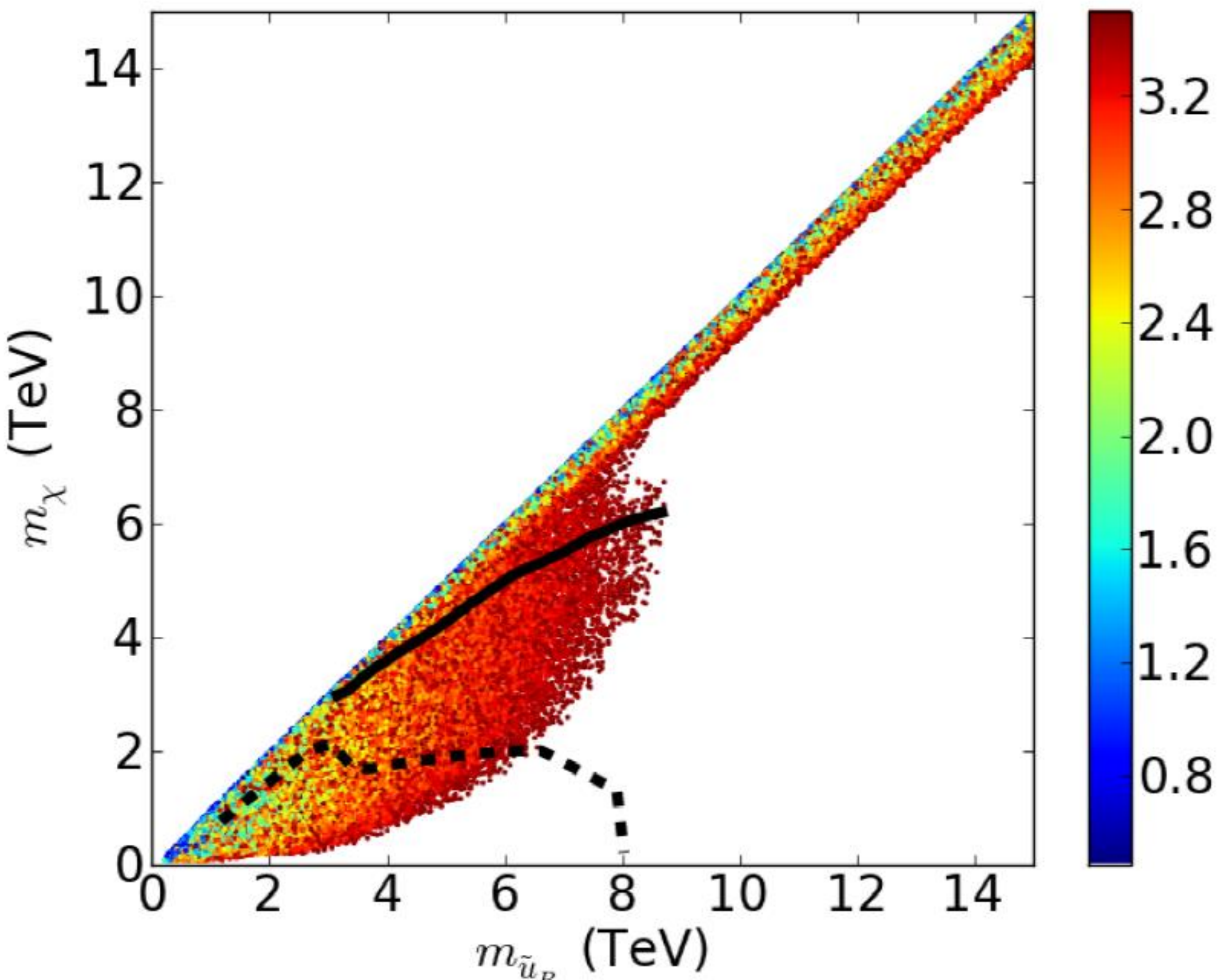
New Production Mechanism

- Here we don't have the SUSY constraints on couplings, though
  - Increased DM couplings give increased production



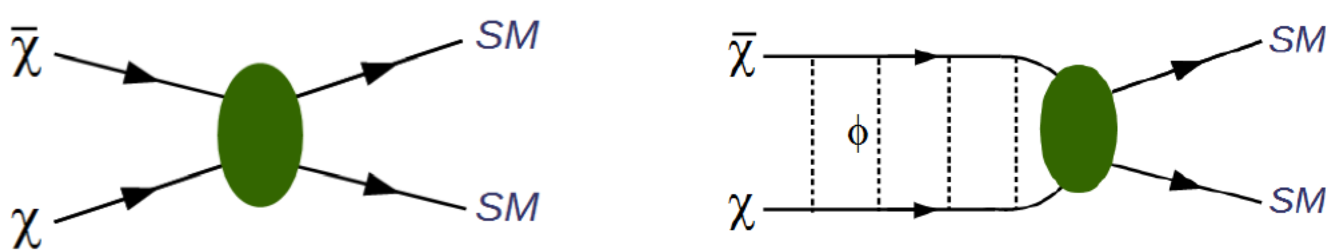
$$\propto \quad \lambda_{\text{dark}}^4$$

True FCC Reach

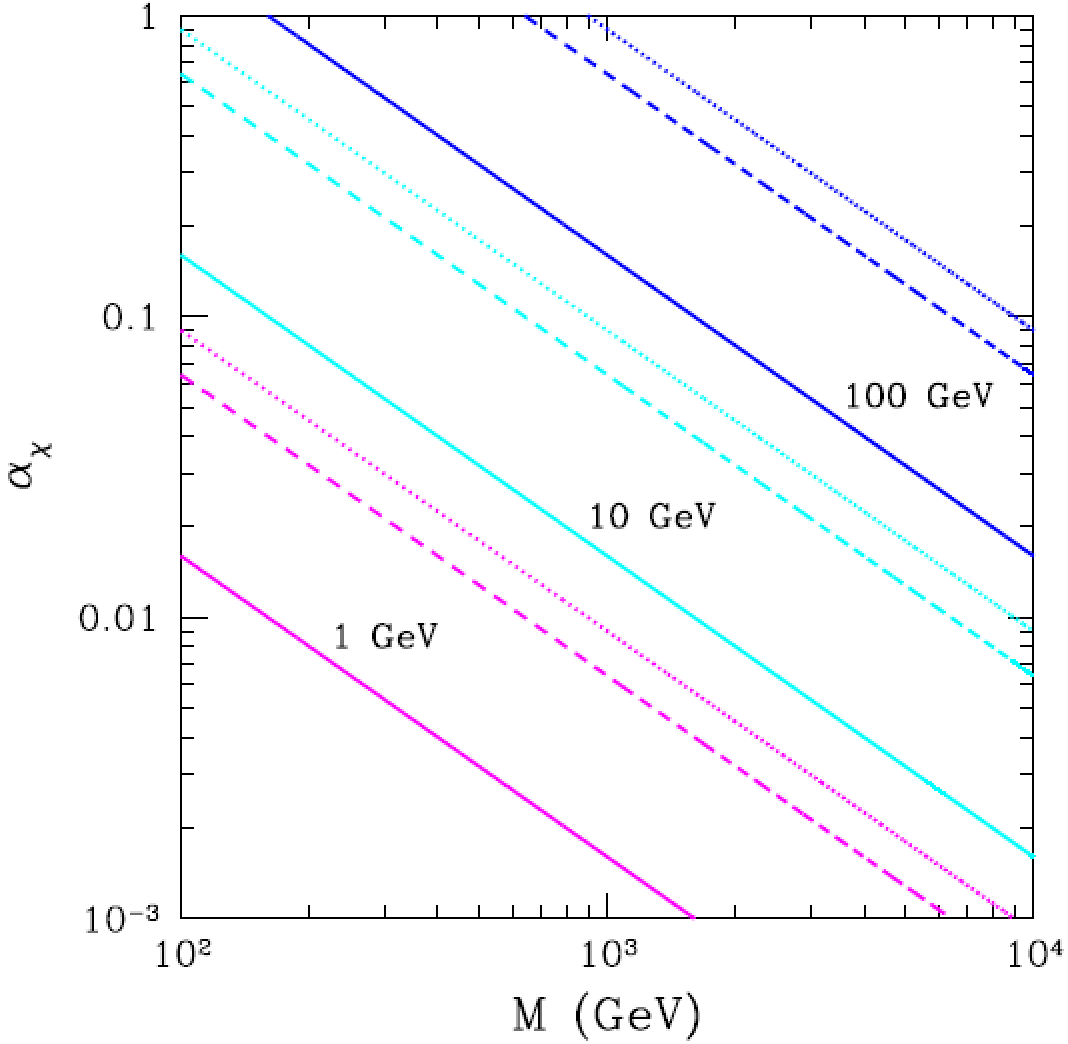


Strong Couplings and Bound States

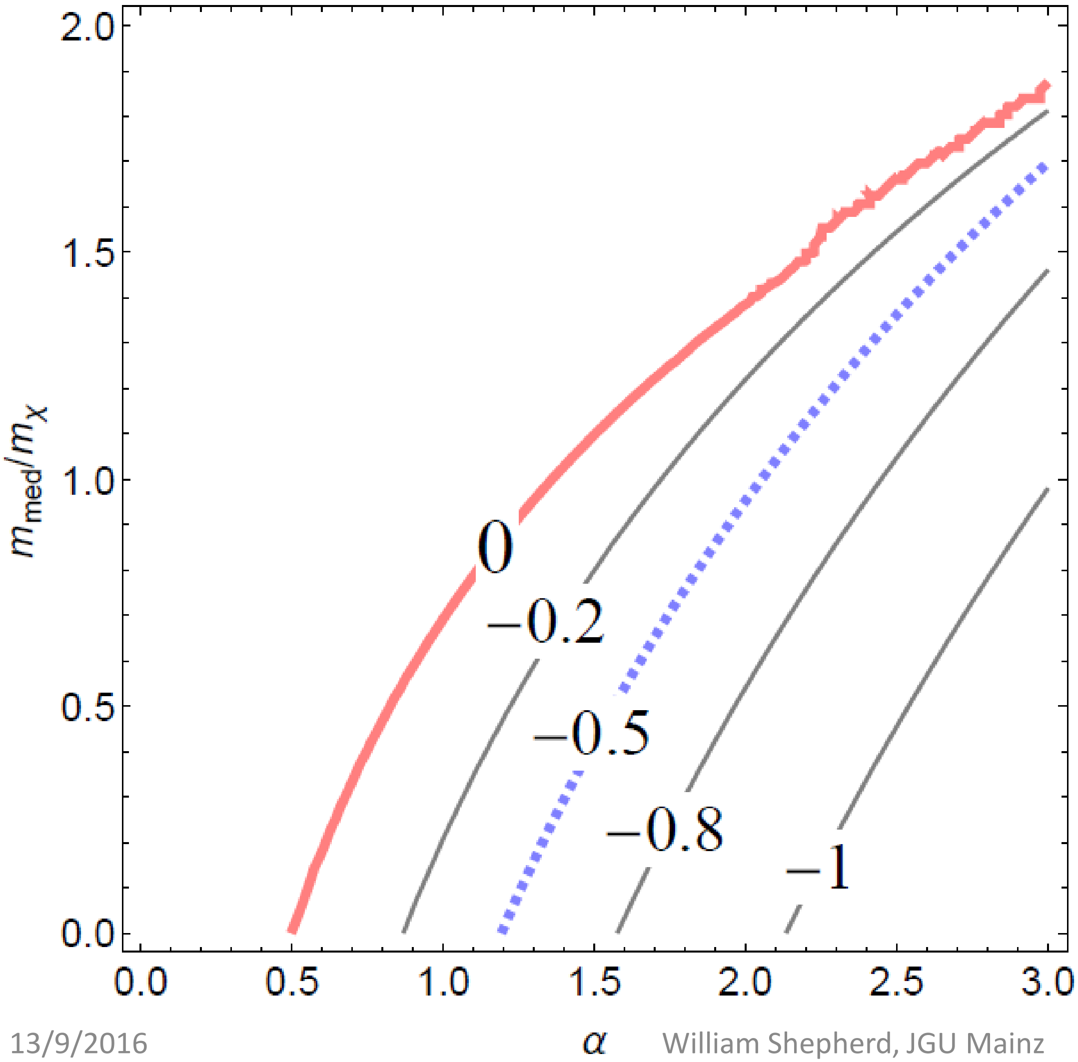
- All of this analysis has focused on the case of very strong couplings to get high allowed mass
- These large couplings can also lead to other effects that may be important
  - Sommerfeld enhancements
  - Dark matter bound states



Yukawa Potential Bound States



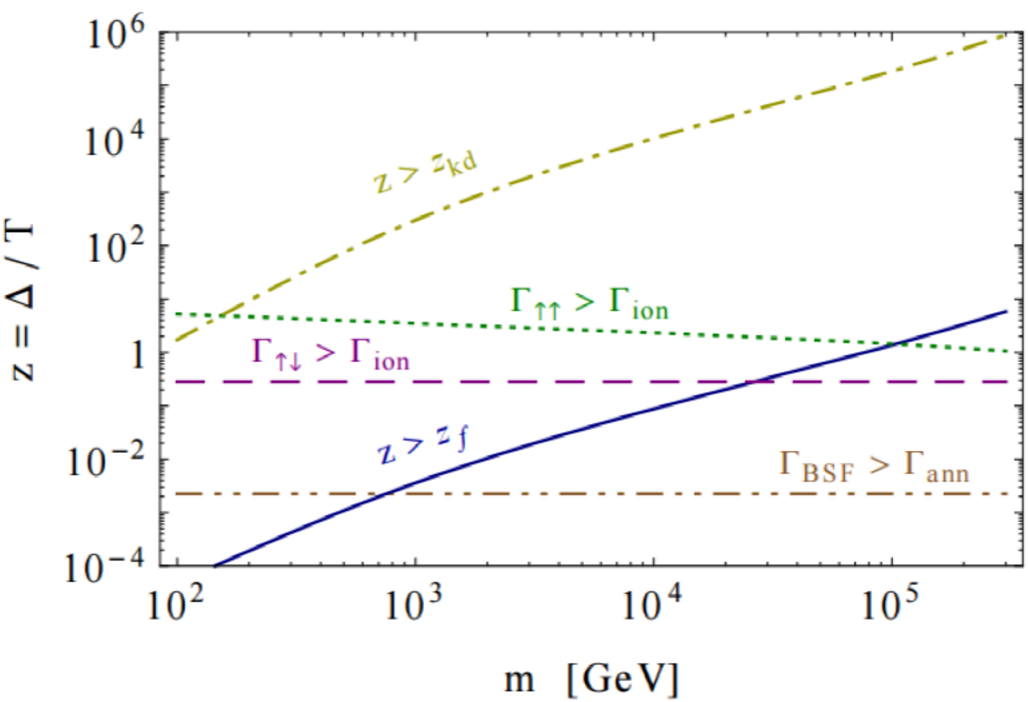
Yukawa Potential Bound States



$$V(r) = \alpha \frac{e^{-m_{\rm med} r}}{r}$$

$$T_{freeze} \sim \frac{m_\chi}{20}$$

Cosmological Rates



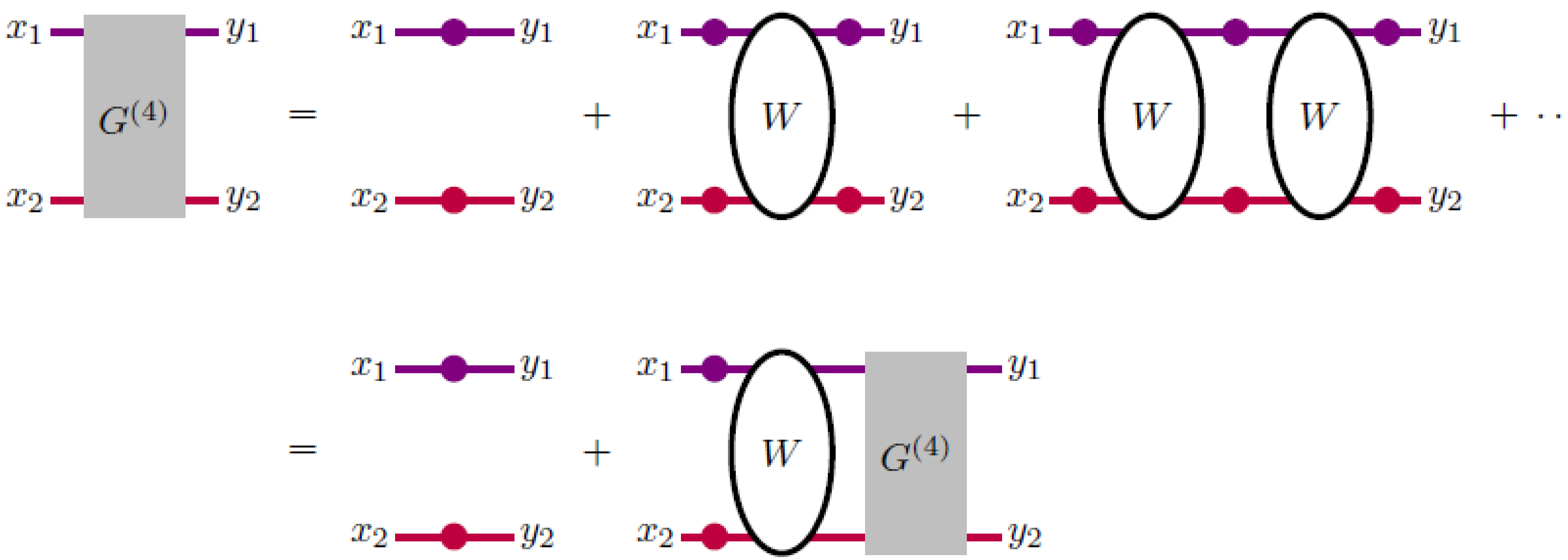
$$\Delta = \frac{m\alpha^2}{4}$$

↑↓ and ↑↑ bound states

von Harling and Petraki, 1407.7874

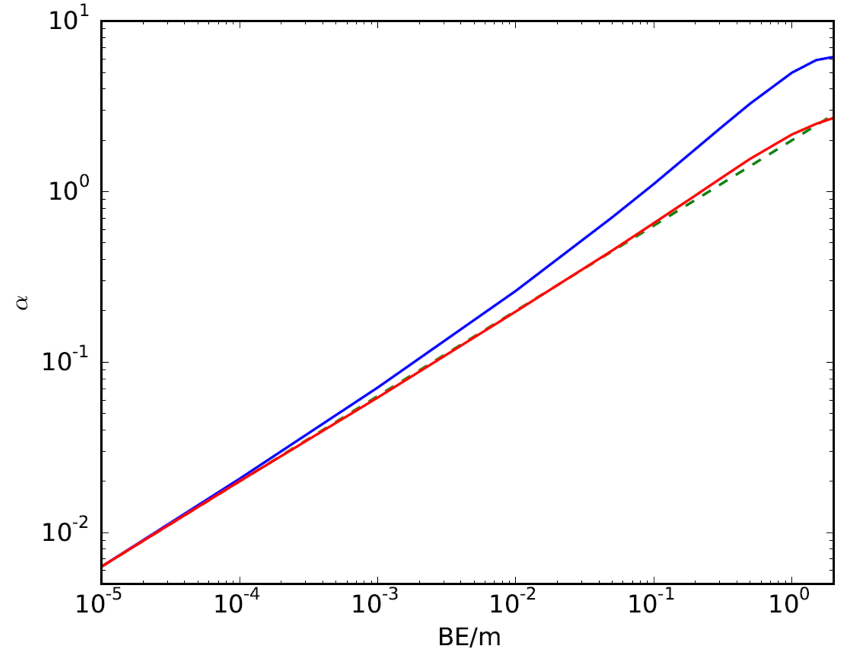
Bethe-Salpeter Equation

- States that are strongly bound enough to matter will have momenta high enough to require relativistic treatment
- If ladder diagrams are the dominant contribution to the binding the Bethe-Salpeter equation describes the physics



Relativistic Corrections

- States with binding energies of M/10 or larger require relativistic corrections to the coupling of a factor of 2 or more
- This will be an important shift in the cosmological implications of strong coupling



Rates for Relativistic States

- Rates for formation, ionization, and decay of bound states are calculated using the wavefunctions

$$\mathcal{M}_{\mathbf{k}\to n} = \sqrt{Z_\varphi(\mathbf{P}_\varphi)} \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \frac{\tilde{\Psi}^\star_{\mathbf{P},n}(p)}{S(p;P)} \frac{\tilde{\Phi}_{\mathbf{K},\mathbf{k}}(q)}{S(q;K)} \times \mathcal{C}^{(5)}_{\varphi-\mathrm{amp}}(P_\varphi, \eta_1 P + p, \eta_2 P - p;\ \eta_1 K + q, \eta_2 K - q)$$

- B-S equation is most easily solved in Euclidean space
  - Wick rotation gives correct on-shell results
  - Scattering, BSF involve off-shell processes

Wick Rotation

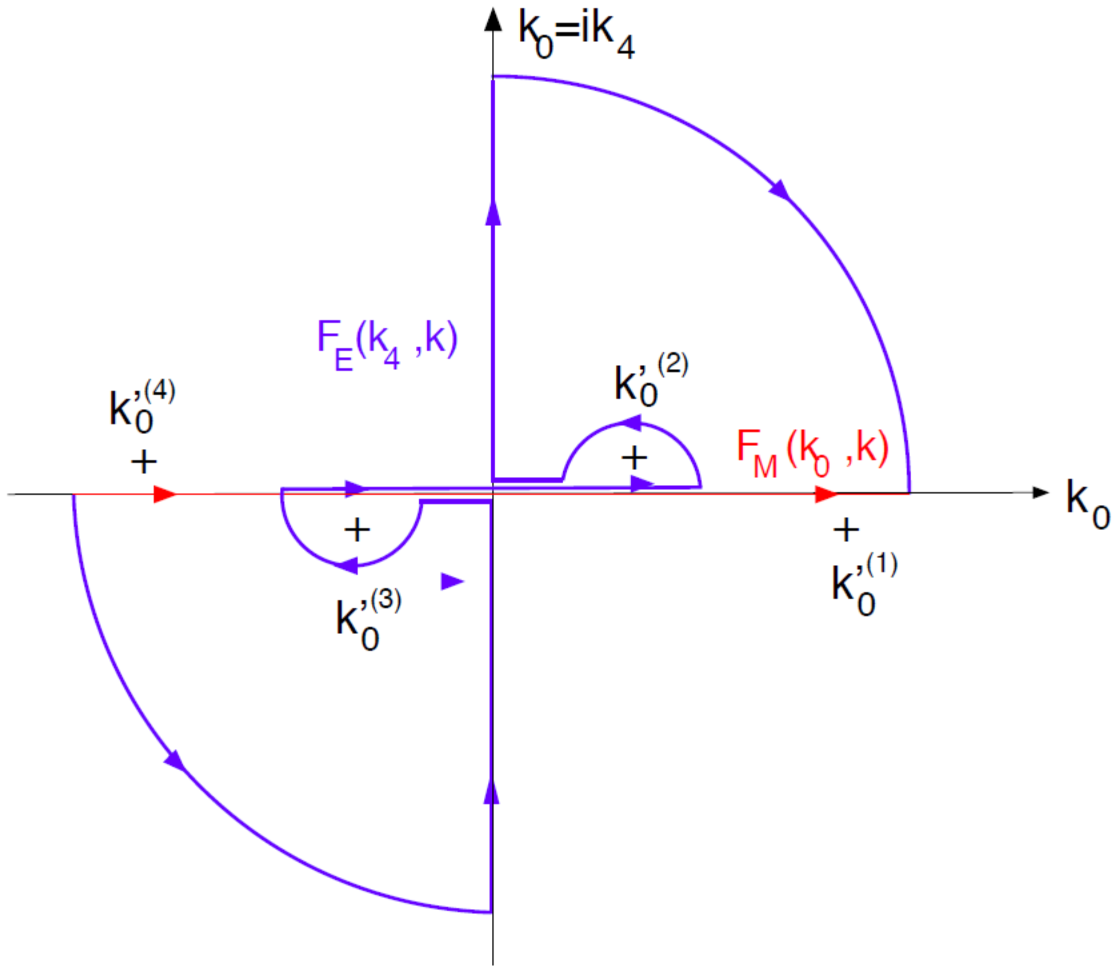


Image credit J. Carbonell

Minkowski Space B-S

- Many singularities to resolve
  - DM propagators and interaction kernel, minimally
- Treating these analytically yields a problem amenable to numerical treatment

$$
\begin{aligned}
F_0(k_0,k) &= F_0^B(k_0,k) + \frac{i\pi^2 k_s}{8\varepsilon_{k_s}} W_0^S(k_0,k,0,k_s) F_0(0,k_s) \\
&+ \frac{\pi}{2M}\int_0^\infty \frac{dk'}{\varepsilon_{k'}(2\varepsilon_{k'}-M)}\left[k'^2 W_0^S(k_0,k,a_-,k')F_0(|a_-|,k') - \frac{2{k_s}^2\varepsilon_{k'}}{\varepsilon_{k'}+\varepsilon_{k_s}} W_0^S(k_0,k,0,k_s)F_0(0,k_s)\right] \\
&- \frac{\pi}{2M}\int_0^\infty \frac{k'^2 dk'}{\varepsilon_{k'}(2\varepsilon_{k'}+M)} W_0^S(k_0,k,a_+,k')F_0(a_+,k') \\
&+ \frac{i}{2M}\int_0^\infty \frac{k'^2 dk'}{\varepsilon_{k'}}\int_0^\infty dk'_0 \left[\frac{W_0^S(k_0,k,k'_0,k')F_0(k'_0,k') - W_0^S(k_0,k,a_-,k')F_0(|a_-|,k')}{{k'}_0^2 - a_-^2}\right] \\
&- \frac{i}{2M}\int_0^\infty \frac{k'^2 dk'}{\varepsilon_{k'}}\int_0^\infty dk'_0 \left[\frac{W_0^S(k_0,k,k'_0,k')F_0(k'_0,k') - W_0^S(k_0,k,a_+,k')F_0(a_+,k')}{{k'}_0^2 - a_+^2}\right] \qquad (28)
\end{aligned}
$$

Carbonell and Karmanov, 1408.3761

Beyond Yukawa Potentials

- Constraining to consider just a pure pseudoscalar, the leading potential is

$$V(r) = V_S(r)\sigma_1.\sigma_2 + V_T(r)S_{12}(\hat{r})$$

$$V_S = \frac{\alpha_A}{12}\left(\frac{m_\phi}{m_\chi}\right)^2 \frac{e^{-m_\phi r}}{r}$$

$$V_T = \frac{\alpha_A}{12}\left(\frac{m_\phi}{m_\chi}\right)^2 \left(1 + \frac{3}{m_\phi r} + \frac{3}{(m_\phi r)^2}\right) \frac{e^{-m_\phi r}}{r}$$

- This gives very different states from the Yukawa or Coulomb ansätze
  - Rates will also be affected by mediator mass

Outlook

- Perturbativity arguments can be made fully rigorous through unitarity considerations
- These unitarity bounds provide strong constraints on dark matter dynamics
- Combined with collider searches we will be able to place strong limits on WIMPs
- Models with strong coupling like these may already be affected by new phenomena due to bound state formation
  - Investigations of cosmological impact of bound state dynamics are in progress