A UV Complete Partially Composite-pNGB Higgs

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with Jamison Galloway and Adam Martin, to appear soon

Plan

- Introduction to Bosonic Technicolor (BTC)
- The UV theory
  - Minimal BTC with $SU(4)/Sp(4)$ coset
  - Discrete symmetries CP, G_{LR}
- The vacuum (mis)alignment
- The scalar spectrum
  - Higgs mass, phenomenology
- Vector resonances
  - $SU(2)_{L,R}$ parity doubling and the S parameter
  - phenomenology

Introduction to Bosonic Technicolor

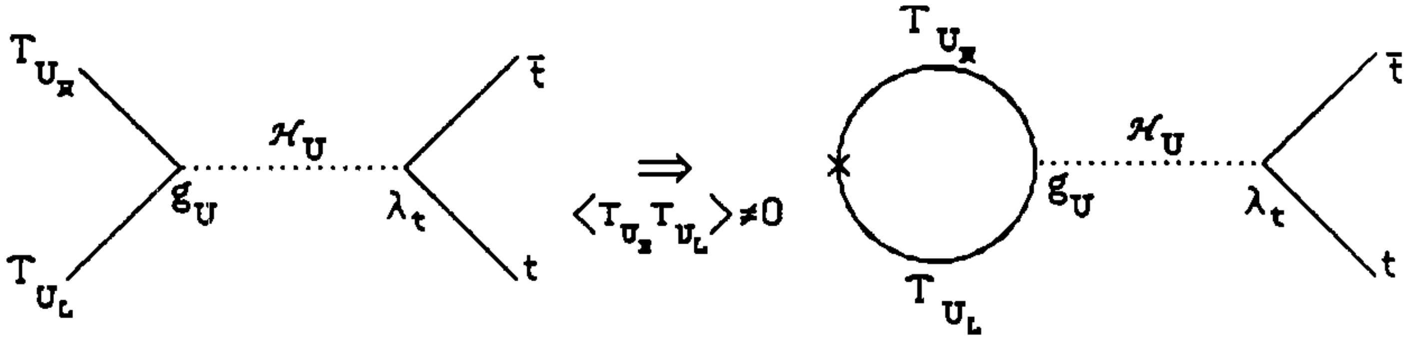
- BTC combines technicolor and supersymmetry Dine, A.K., Samuel '90; non-susy version: Simmons, '89
- technicolor condensates trigger electroweak symmetry breaking
- fundamental Higgs fields H_u, H_d give masses to quarks, leptons
- supersymmetry stabilizes the Higgs scalar masses
- Higgs VEV's via Yukawa couplings to technifermion condensates

$$\lambda_U \bar{U}_R T_L H_u + \lambda_D \bar{D}_R T_L H_d \Rightarrow \langle H_u \rangle \sim \lambda_U \frac{\langle \bar{U}_R U_L \rangle}{m^2_{H_u}}, \quad \langle H_d \rangle \sim \lambda_D \frac{\langle \bar{D}_R D_L \rangle}{m^2_{H_d}}$$

  - positive Higgs mass parameters, $m^2_{H_u}, m^2_{H_d} > 0 \Rightarrow$ no electroweak symmetry breaking in absence of TC

- W, Z receive masses both from technicolor condensates, HIggs VEV's

$$v_W^2 = (246 \text{ GeV})^2 \approx f^2_{\text{TC}} + f^2_u + f^2_d, \qquad \langle H_{u,d} \rangle \equiv f_{u,d}/\sqrt{2}$$



-Fermion mass generation in BTC via “Higgs scalar exchange", integrated out in heavy limit

-for light Higgs, use chiral Lagrangian approach Carone, Simmons; Carone, Georgi

- Minimal BTC = MSSM + $SU(N)_{\rm TC}$, with technifermion superfields

$$\hat{T}_L(2_{\rm TC}, 1_C, 2_L, 0), \quad \hat{U}_R(2_{\rm TC}, 1_C, 1_L, -1/2), \quad \hat{D}_R(2_{\rm TC}, 1_C, 1_L, +1/2),$$

  and Yukawa superpotential

$$W_{\rm Y} = \lambda_U \hat{U}_R \hat{T}_L \hat{H}_u + \lambda_D \hat{D}_R \hat{T}_L \hat{H}_d$$

  - $N_{\rm TC} = 2$ is minimal choice
  - $N_{\rm TC} = 3$ disfavored: stable fractionally charged technibaryons; $SU(2)_L$ anomaly
  - $N_{\rm TC} = 4$ disfavored by S parameter

- superpartner technigluino, technisquarks acquire masses $> \Lambda_{\rm TC}$, yielding a QCD-like technicolor theory at lower scales

Linking $\Lambda_{\rm TC}$ and $m_{\rm susy}$

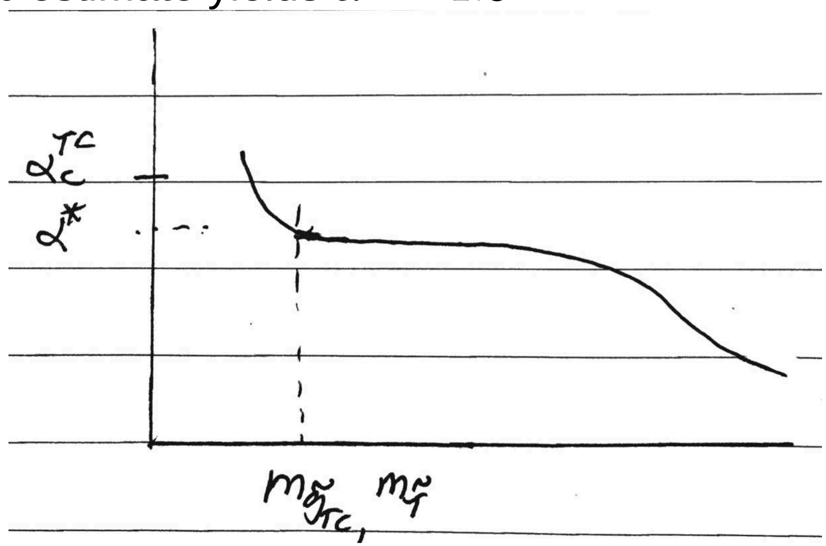
- BTC introduces two scales at low energies: (i) m_{susy}, the scale of superpartner masses; (ii) $\Lambda_{\rm TC}$, the scale of TC chiral symmetry breaking
- potential coincidence problem since, e.g. $m_{\rm susy}/\Lambda_{\rm TC} = O({\rm few})$
- when techni-superpartners acquire masses and “decouple", technicolor beta function becomes more negative.
  - more rapid increase in $\alpha_{\rm TC}$ below $m_{\rm susy}$ could link the two scales
    AK, Samuel ’91
- most attractive scenario Azatov, Galloway, Luty ’11:
  above $m_{\rm susy}$, $\alpha_{\rm TC}$ sits near a superconformal strong IR fixed point. Provides direct link between $m_{\rm susy}$ and $\Lambda_{\rm TC}$

- appealing realization Galloway, Martin, AK:

  SUSY $SU(2)$ with $n_f = 2$ and an adjoint matter superfield has a strong IR fixed point, with chiral symmetry unbroken in the supersymmetric theory Elitzur, Forge, Giveon, Rabinovici '95

  - R symmetric BTC has precisely this TC field content. Can yield the following running of α_{TC}:

    - A perturbative 2-loop estimate yields $\alpha^* \approx 1.8$



The minimal UV theory

- The model: asymptotically free $SU(2)_{\rm TC}$, confining at scale Λ.
  For simplicity consider the non-supersymmetric version

- The technifermion (TC-fermion) content is

	$SU(2)_{\rm TC}$	$SU(2)_W$	$U(1)_Y$
$\begin{pmatrix}\Psi^1\\ \Psi^2\end{pmatrix} \equiv T_{1,2}$	$\begin{pmatrix}\Box\\ \Box\end{pmatrix}$	$\Box$	0
$\Psi^3 \equiv U$	$\Box$	1	$-1/2$
$\Psi^4 \equiv D$	$\Box$	1	$+1/2$

- all fermions are treated as LH Weyl fields, transforming under the $(1/2, 0)$ representation of the Lorentz group $SU(2) \times SU(2) \sim SO(3,1)$

- With weak interactions turned off, the model possesses a global $SU(4)$ symmetry under which the four-component object Ψ is a fundamental, $\Psi \mapsto U\Psi$, $U \in SU(4)$

$$\Psi = \begin{pmatrix} T_1 & T_2 & U & D \end{pmatrix}^T$$

- The TC-fermion condensate

$$\langle \Psi^a \, \Psi^{T,b} \epsilon \, C^{-1} \rangle \propto \Phi^{ab}$$

is antisymmetric in the $SU(4)$ flavor indices a, b by Fermi-Dirac statistics, $\Phi^T = -\Phi$

- assume it breaks $SU(4)$ to its maximal vectorlike subgroup $Sp(4)$
$\Rightarrow SU(4)/Sp(4)$ coset structure $\cong SO(6)/SO(5)$

- most general $Sp(4)$ preserving condensate Galloway, Evans, Luty, Tacchi 1001.1361

$$\Phi = \begin{pmatrix} e^{i\alpha} \, \epsilon \cos\theta & \mathbf{1}_2 \sin\theta \\ -\mathbf{1}_2 \sin\theta & -e^{-i\alpha} \epsilon \cos\theta \end{pmatrix}, \quad \theta \in [0, \pi]$$

  - obtained by applying $SU(4)$ rotations to the canonical $Sp(4)$ preserving vacuum

$$\Phi = \begin{pmatrix} 0 & \mathbf{1}_2 \\ -\mathbf{1}_2 & 0 \end{pmatrix},$$

  - α is a CP violating phase: $\Phi \to -\Phi^\dagger$ under CP

- $\sin\theta = 0$: electroweak (EWK) symmetry is unbroken, "EWK vacuum"
  $\sin\theta = 1$: the condensate is purely $SU(2)_L$ breaking, "TC vacuum"
- The $Sp(4)$ vacuum degeneracy is lifted by the UV TC-fermion interactions
- previous BTC studies only included fundamental Higgs - technifermion Yukawa couplings, thus selecting the TC vacuum ($\theta = \pi/2$)
- we explore the benefits of small misalignment from the EWK vacuum:
  small to moderate $\sin\theta$
  - minimally accomplished by adding gauge singlet TC-fermion masses of $O(v_W)$
  - they can be linked to SUSY breaking, therefore to $\Lambda_{\rm TC}$
  - an appealing alternative, and a feature of RBTC: 4- technifermion operators

- Minimal UV potential In $SU(4)$ notation

$$V_{UV} = -\Psi^T \epsilon C^{-1}(M+\lambda)\Psi + h.c. + m_H^2|H|^2 + \lambda_h|H|^4$$

  - $C^{-1} = \mathrm{diag}[i\sigma_2, i\sigma_2, i\sigma_2, i\sigma_2]$ acts on LH Weyl spinors in Ψ,
  - ϵ acts on TC indices
  - H is SM Higgs doublet with $m_H^2 > 0$

- M, λ are 4×4 matrices containing singlet masses, Yukawas: $m_{1,2}$, $\lambda_{U,D}$, $(M+\lambda)$ is an $SU(4)$ breaking spurion, $(M+\lambda) \mapsto U^*(M+\lambda)U^T$

$$M = \frac{1}{2}\begin{pmatrix} m_1\,\epsilon & 0 \\ 0 & -m_2\,\epsilon \end{pmatrix}, \quad \lambda = \frac{1}{2}\begin{pmatrix} 0 & -H_\Lambda \\ H_\Lambda^T & 0 \end{pmatrix},$$

$$H_\Lambda = \frac{1}{\sqrt{2}}\begin{pmatrix} \lambda_U(\sigma_h + v^* - i\pi_h^3) & \lambda_D(-i\pi_h^1 + \pi_h^2) \\ -\lambda_U(i\pi_h^1 + \pi_h^2) & \lambda_D(\sigma_h + v + i\pi_h^3) \end{pmatrix}.$$

σ_h ($\vec{\pi}_h$) are the scalar (pseudoscalar) components of H, $\quad v \equiv \langle H \rangle$

- the TC-fermion masses: $m_1 T_2 T_1 + m_2 U D + m_U T_1 U + m_D T_2 D$

$$m_U = \lambda_U v^* / \sqrt{2}, \quad m_D = \lambda_D v / \sqrt{2}$$

- The gauge-kinetic term for Ψ, including EWK and TC

$$\mathcal{L}_{\mathrm{KE}} = i\Psi^\dagger \, \bar{\sigma}^\mu (\partial_\mu - -i\mathcal{A}_\mu - iG^a_\mu \tau^a/2 \; 1_4) \, \Psi, \qquad \bar{\sigma}_\mu = (1, -\vec{\sigma}^i_\mu)$$

  - EWK gauge interaction embedding in $SU(4)$

$$\mathcal{A}_\mu = \begin{pmatrix} g_2 \, W^a_\mu \, \frac{1}{2}\tau^a & 0 \\ 0 & -g_1 B_\mu \, \frac{1}{2}\tau^3 \end{pmatrix}$$

Discrete symmetries in the UV: CP

- CP is the only discrete symmetry of the TC interactions lying outside of $SU(4)$

$$CP: \ \Psi(x^\mu) \mapsto i\,\epsilon\,C^{-1}\,\Psi^*(x_\mu)\,.$$

  - pseudoreality of the $SU(2)_{\rm TC}$ fundamental $\Rightarrow$ P and C are separately unphysical, only being defined up to arbitrary $SU(4)$ rotations
  - fundamental d.o.f. are the four LH Weyl spinors in Ψ. A true discrete symmetry must rotate among them.
    - for a single LH Weyl fermion, P exchanges $(1/2, 0) \leftrightarrow (0, 1/2)$, which proceeds via conjugation to construct the RH field, bringing in C to recover the LH one

- For simplicity, assume CP-invariant V_{UV} $\Rightarrow$ $m_{1,2}$, λ_{UD} are real
  - checked, to $O(p^4)$, that minimizing the IR potential then yields $\alpha = \arg(v) = 0$, which we assume holds to all orders (no spontaneous CPV)

- CP invariance of the EWK interactions $\Rightarrow$ $\mathcal{A}_\mu \mapsto \mathcal{A}_\mu^T$ (usual gauge boson transformations)

G_{LR}: $SU(2)_L \leftrightarrow SU(2)_R$ interchange

- G_{LR}-parity interchanges the generators of $SU(2)_L$ and $SU(2)_R$ (also see Franzosi, et al. 1605.01363)

- it resides in $SU(4)$, and the left-over $Sp(4)$ symmetry (it has nothing to do with spacetime parity)

- $\Psi \mapsto \mathcal{G}_{LR}\Psi$ where, up to an overall phase

$$\mathcal{G}_{LR} = -\begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}$$

  - To see this, extend to the left-right symmetric gauge group, and require that G_{LR} exchanges top and bottom components of Ψ and $g_{2L}W_L \leftrightarrow g_{2R}W_R$

- under $G_{LR} \in SU(4)$, $\quad M + \lambda \rightarrow \mathcal{G}^T_{LR}\,(M+\lambda)\,\mathcal{G}_{LR}$

$$\Rightarrow \; m_{1\,(2)} \rightarrow m_{2\,(1)}, \;\; m_{U\,(D)} \rightarrow m_{D\,(U)}, \;\; \lambda_{U\,(D)}h \rightarrow \lambda_{D\,(U)}h, ...$$

  Thus, G_{LR} invariance of V_{UV} would require $m_1 = m_2$, $\lambda_U = \lambda_D$

- the isospin rotation in $\mathcal{G}_{LR}$ is reminsicent of G-parity

The $SU(4)/Sp(4)$ coset for arbitrary θ **Galloway et al.**

- 5 broken $SU(4)$ generators, X^i, in 5 of $Sp(4) \cong SO(5)$, satisfying $X\Phi - \Phi X^T = 0$
  $s \equiv \sin\theta$, $c \equiv \cos\theta$

$$X^1 = \frac{1}{2\sqrt{2}} \begin{pmatrix} s\sigma_1 & -c\sigma_3 \\ -c\sigma_3 & s\sigma_1 \end{pmatrix}, X^2 = \frac{1}{2\sqrt{2}} \begin{pmatrix} s\sigma_2 & ic1_2 \\ -ic1_2 & -s\sigma_2 \end{pmatrix}, X^3 = \frac{1}{2\sqrt{2}} \begin{pmatrix} s\sigma_3 & c\sigma_1 \\ c\sigma_1 & s\sigma_3 \end{pmatrix},$$

$$X^4 = \frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}, X^5 = \frac{1}{2\sqrt{2}} \begin{pmatrix} c1_2 & -s\epsilon \\ s\epsilon & -c1_2 \end{pmatrix}$$

- 10 unbroken $Sp(4)$ generators, T^i, in 10 (adjoint) of $Sp(4)$, satisfying $T\Phi + \Phi T^T = 0$

$$T^1 = \frac{1}{2\sqrt{2}} \begin{pmatrix} \sigma_1 & 0 \\ 0 & -\sigma_1 \end{pmatrix}, T^2 = \frac{1}{2\sqrt{2}} \begin{pmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix}, T^3 = \frac{1}{2\sqrt{2}} \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix},$$

$$T^4 = \frac{1}{2\sqrt{2}} \begin{pmatrix} c\sigma_1 & s\sigma_3 \\ s\sigma_3 & c\sigma_1 \end{pmatrix}, T^5 = \frac{1}{2\sqrt{2}} \begin{pmatrix} c\sigma_2 & -is1_2 \\ is1_2 & -c\sigma_2 \end{pmatrix}, T^6 = \frac{1}{2\sqrt{2}} \begin{pmatrix} c\sigma_3 & -s\sigma_1 \\ -s\sigma_1 & c\sigma_3 \end{pmatrix}, \ldots$$

- subgroup structure: $Sp(4) \supset SU(2)_1 \times SU(2)_2$,
  - $SU(2)_{1,2}$ identified with generaotrs $(T^a \pm T^{a+3})/\sqrt{2},\ a = 1, 2, 3$
  - reduce to $SU(2)_{L,R}$ in $\theta \to 0$ limit
- isospin group $SU(2)_V = SU(2)_{L+R} = SU(2)_{1+2}$, with generators $T^{1,2,3}$
- under $SU(2)_1 \times SU(2)_2$

$$5 = (2, 2) + (1, 1)$$
$$10 = (3, 1) + (1, 3) + (2, 2)$$

- we follow the CCWZ prescription, arranging the 5 NGBs into

$$\xi = \exp(\sqrt{2} i \, \vec{\pi} \cdot \vec{X}/f) \mapsto U \xi V^\dagger ,$$

  where the transformation applies to global rotations with $U \in SU(4)$ and $V \in Sp(4)$ $\Rightarrow V \Phi V^T = \Phi$

- The transformations of the pions under CP, G_{LR} obtained by considering the transformations of the corresponding vector currents, $\Psi^\dagger \bar{\sigma}_\mu \, X^a \Psi$

- The eaten NGB's are linear combinations of the CP-odd $\pi^{1,2,3}$ and $\pi_h^{1,2,3}$

- π^4 is the CP-even component of a composite $SU(2)_L$ Higgs doublet

$$\frac{1}{\sqrt{2}} \begin{pmatrix} \pi^1 + i\pi^2 \\ \pi^4 + i\pi^3 \end{pmatrix}$$

- π^5 is a CP-odd isosinglet

- $\pi^{1,2,3,5}$, $\pi_h^{1,2,3}$ are G_{LR} odd; π^4, σ_h are G_{LR} even

Chiral Lagrangian for scalars and vacuum alignment

- kinetic terms expressed in terms of $C_\mu = i\xi^\dagger D_\mu \xi$ (following composite Higgs notation, e.g. Contino et al., Panico and Wulzer)
  - project onto broken and unbroken directions ($C_\mu = d_\mu + E_\mu$)

$$d_\mu = 2\mathrm{tr}(C_\mu X^a)X^a \mapsto V d_\mu V^\dagger \qquad (5-\mathrm{plet}),$$
$$E_\mu = 2\mathrm{tr}(C_\mu T^a)T^a \mapsto V(E_\mu + \partial_\mu)V^\dagger \quad (10-\mathrm{plet}),\ ,$$

- spurion building blocks

$$\chi_\pm = \xi^T(M+\lambda)\xi\Phi \pm \mathrm{H.c.}, \qquad \chi_\pm \mapsto V\chi_\pm V^\dagger$$

- $O(p^2)$ Lagrangian

$$\mathcal{L}^{(2)} = \frac{f^2}{2}\mathrm{tr}(d_\mu\, d^\mu) + 4\pi f^3 Z_2\, \mathrm{tr}(\chi_+),$$

  - $Z_2 \approx 1.47,$ from $N_c = n_f = 2$ lattice study Pica et al. 1602 .06559

- TC and fundamental Higgs H gauge kinetic terms yield EWK scale

$$v_W^2 = (246\ \mathrm{GeV})^2 = f^2 \sin^2\theta + v^2 \,.$$

  - associate $f \sin\theta$ with VEV of composite pNGB Higgs π^4

- The $O(p^2)$ potential

$$V_{\rm eff}^{(2)} = 8\pi f^3 Z_2 (m_{12}\cos\theta - \lambda_{UD} v \sin\theta/\sqrt{2}) + m_H^2 v^2/2$$

$m_{12} \equiv m_1 + m_2, \quad \delta m_{12} \equiv m_1 - m_2,$ etc.

  - For simplicity, ignore quartic - also motivated by SUSY where it is a small perturbation

  - EWK, top Yukawa loop effects, usually considered in composite Higgs models, are a negligible perturbation

- minimizing $V_{\rm eff}^{(2)}$ obtain vacuum solution ($m_{UD}, m_{12} > 0, \quad \theta \in [\pi/2, \pi]$)

$$\tan\theta = -\frac{m_{UD}}{m_{12}}, \qquad v = \frac{4\sqrt{2}\,\lambda_{UD}\ \sin\theta\, f^3 \pi Z_2}{m_H^2}$$

$$\Rightarrow \quad \sin\theta = \sqrt{1 - \frac{m_{12}^2}{\lambda_{UD}^4}\,\frac{m_H^4}{16\pi^2 f^6 Z_2^2}}.$$

  - can show $\tan\theta = -m_{UD}/m_{12}$ to all orders

- for given f or $\Lambda_{\rm TC}$, largest tuning of $\sin\theta$ due to variation of λ_{UD}

$$|d\log(\sin\theta)/d\ \log(\lambda_{UD})| = 2\cot^2\theta$$

  - For example, $\sin\theta \sim 1/3 - 1/2$ is tuned at $\sim 6\% - 17\%$
    (moderate $\sin\theta$ is fine phenomenologically e.g. Higgs, precision EWK)
  - in principle, f more tuned, but f could be linked to m_H or m_{12} in SUSY theory
  - 4-technifermion operators, e.g. due to exchange of TC-adjoint scalar with mass $m_A \gtrsim \Lambda_{\rm TC}$, offer an alternative in which $m_{1,2} \propto \langle \Psi\Psi^T \rangle \propto f^3$, like v.

Vacuum misalignment and scalar spectrum

- To elucidate the structure of the vacuum and scalar spectrum, it is useful to project $(M+\lambda)$ onto the $Sp(4)$ singlet ($\propto \Phi$) and vector directions, for vacuum rotation θ

$$(M+\lambda) = -\frac{1}{2}\left(\hat{m} + \frac{\lambda_{UD}\sigma_h + i\,\delta\lambda_{UD}\,\pi_h^3}{2\sqrt{2}} s_\theta\right)\Phi + \frac{i}{2}\Phi\,\left(\lambda_{UD}\,\chi_\theta^a + i\delta\lambda_{UD}\;\;{\chi_\theta'}^a\right)X^a$$

  - the fermion mass $\hat{m}$ and $Sp(4) \cong SO(5)$ vectors χ_θ, χ'_θ are

$$\hat{m} \equiv \frac{1}{2}(-m_{12}\,c_\theta + m_{UD}\,s_\theta) = \frac{1}{2}(m_{12}^2 + m_{UD}^2) = 2\pi f^3 Z_2 \lambda_{UD}^2/m_H^2\,|_{\theta<\pi}$$

$$\vec{\chi}_\theta = (\pi_h^1,\,\pi_h^2,\,\pi_h^3,\sigma_h c_\theta + v\,c_\theta + \sqrt{2}\,m_{12}s_\theta/\lambda_{UD},0) = (\pi_h^1,\,\pi_h^2,\,\pi_h^3,\sigma_h c_\theta,0)$$

$$\vec{\chi}'_\theta = (-\pi_h^2,\,\pi_h^1,\,\sigma_h + v,\pi_h^3\,c_\theta,\,\delta m_{12}/\delta\lambda_{UD}),$$

  - the $O(4)$ components of $\vec{\chi}_\theta$, $\vec{\chi}'_\theta$ have opposite CP, generalizing the $O(4)$ vectors of Gasser and Leutwyler (Ann Phys) for the $SU(2)\times SU(2)/SU(2)$ coset

  - constant term in $\chi_\theta^4 \supset v\,c_\theta + \sqrt{2}\,m_{12}s_\theta/\lambda_{UD}$
    must cancel to avoid terms in $V_{\rm eff}$ of form constant $\times\pi^4$
    from operators $\propto \vec{\chi}_\theta\cdot\vec{\pi} \Rightarrow\ \tan\theta = -m_{UD}/m_{12}$ to all orders

- compare the $O(4) \subset Sp(4)$ vectors for the rotated vacuum $(\theta \neq 0)$

$$\vec{\pi} = (\pi^1, \pi^2, \pi^3, \pi^4), \quad \vec{\chi}_\theta = (\pi_h^1,\, \pi_h^2,\, \pi_h^3, \sigma_h c_\theta)$$

  - after EWK symmetry breaking, the fundamental and composite $O(3)$ vectors $(\pi_h^1,\, \pi_h^2,\, \pi_h^3)$ and $(\pi^1,\, \pi^2,\, \pi^3)$ remain aligned
    - $SU(2)_{1+2}(\theta \neq 0) = SU(2)_{1+2}(\theta = 0) = SU(2)_{L+R} = O(3)$
  - the composite Higgs π^4 is rotated by by θ relative to σ_h [and $\pi^4(\theta = 0)$, as in composite Higgs]
    - σ_h in $(M + \lambda)$: $\quad \propto \sigma_h(-s_\theta \mathbf{1}_4 + i c_\theta 2\sqrt{2} X^4)$, i.e. it is rotated by θ in the $Sp(4)$ singlet direction

- in terms of $SU(4)$ matrix representations (expand $\xi \Phi \xi^T \Rightarrow \pi^i \propto X^i \Phi$)

$$\pi^i = \pi_h^i = \frac{1}{2}\begin{pmatrix} 0 & i\sigma^i \\ -i\sigma^{iT} & 0 \end{pmatrix};\ \pi^4 = \frac{1}{2}\begin{pmatrix} i\sigma^2 \sin\theta & -\cos\theta\, \mathbf{1_2} \\ \cos\theta\, \mathbf{1_2} & -i\sigma^2 \sin\theta \end{pmatrix};\ \ \sigma_h = \frac{1}{2}\begin{pmatrix} 0 & -\mathbf{1_2} \\ \mathbf{1_2} & 0 \end{pmatrix}$$

- the fundamental Higgs doublet mass decomposes as

$$m_H^2 |H|^2 = m_H^2 \vec{\chi}_0 \cdot \vec{\chi}_0 = m_H^2 [\vec{\chi}_\theta \cdot \vec{\chi}_\theta + (\sigma_h s_\theta)^2]$$

The scalar mass matrices

- for given m_{12}, if $\lambda_{UD} > \lambda_{UD}^* \Rightarrow \theta \neq 0$, and

$$M_{\pi^+}^2 = m_H^2 \begin{pmatrix} 1 & -t_\beta \\ -t_\beta & t_\beta^2 \end{pmatrix},$$

$$M_h^2 = m_H^2 \begin{pmatrix} c_\theta^2 & -c_\theta t_\beta \\ -c_\theta t_\beta & t_\beta^2 \end{pmatrix} + \begin{pmatrix} m_H^2 s_\theta^2 & 0 \\ 0 & 0 \end{pmatrix}$$

in the bases (π_h^+, π^+) and (h, π^4), $\qquad t_\beta \equiv \tan\beta = v/(f \sin\theta)$

  - (22) entries are the GMOR relation for fermion mass $\hat{m}$:
    $m_\pi^2 = m_H^2 t_\beta^2 = 16\pi f Z_2 \, \hat{m}$

- $M_{\pi^+}^2$ and first matrix in M_h^2 are related by $Sp(4)$ invariance:
  (11), (12), (22) entries $\propto \vec{\chi}_\theta \cdot \vec{\chi}_\theta, \quad \vec{\chi}_\theta \cdot \vec{\pi}, \quad \vec{\pi} \cdot \vec{\pi}$

  - therefore, both have massless eigenstates: the “eaten" NGB’s and would-be Higgs h

  - the Higgs mass is lifted by the second matrix in M_h^2, corresponding to the $Sp(4)$ singlet’s mass, $m_H^2(\sigma_h s_\theta)^2$

- charged pion ($a=\pm$) and Higgs mass eigenstates: $\tan 2\alpha = \cos\theta\ \tan 2\beta$

$$G^a = s_\beta\,\pi_h^a + c_\beta\,\pi^a, \quad \tilde{\pi}^a = -c_\beta\,\pi_h^a + s_\beta\,\pi^a,$$
$$h = c_\alpha\,h - s_\alpha\,\pi^4, \quad \mathcal{H} = s_\alpha\,h + c_\alpha\,\pi^4,$$

- non-zero masses

$$m_{\tilde{\pi}}^2 = m_H^2/c_\beta^2, \quad m_{h,\mathcal{H}}^2 = m_H^2\left(1 \mp \sqrt{1 - s_\theta^2\, s_{2\beta}^2}\right)/2c_\beta^2\,.$$

  - in limit $s_\theta^2 c_\beta^2 << 1$ the light Higgs mass is (up to small quartic shift $\approx \lambda_h v^2$)

$$m_h^2 = m_H^2 \sin^2\theta$$

  - Higgs is dominantly fundamental, with admixture of composite pNGB π^4
  - Higgs mass is associated with misalignment $\propto \sin\theta$, as in composite Higgs
  - small $\sin\theta$ offers opportunity to raise the fundamental Higgs mass m_H^2, or SUSY scale

- the $Sp(4)$ singlet radial σ mode's mass mixing with σ_h is $\sim m_H^2 \sin\theta \tan\beta$, thus Higgs mass $\sin^2\theta$ suppression persists; same true of higher orders in the chiral expansion

Higgs phenomenology

- hVV ($V = W^\pm, Z$) and $h\bar{f}f$ couplings normalized to SM: κ_V and κ_F, and $s_\theta^2 << 1$ limits

$$\kappa_V = c_\alpha s_\beta - s_\alpha\, c_\beta\, c_\theta \;\mapsto\; 1 - c_\beta^2\, s_\theta^2\,/2,$$
$$\kappa_F = c_\alpha / s_\beta \;\mapsto\; 1 - c_{2\beta}\, c_\beta^2\, s_\theta^2\,/2\,.$$

  - small percent level deviations from SM
  - note additional c_β^2 in deviations compared to composite Higgs

- For $h\gamma\gamma, h\gamma Z$ and vector resonance discussion introduce $Sp(4)$ covariant field strengths Galloway etal

$$\mathcal{D}_{\mu\nu} = \nabla_{[\mu} d_{\nu]}, \quad \mathcal{F}_{\mu\nu} = -i[\nabla_\mu, \nabla_\nu]\,.$$

∇_μ is $Sp(4)$ covariant derivative

  - $\mathcal{D}_{\mu\nu},\ \mathcal{F}_{\mu\nu}$ tranform homogeneously under $Sp(4)$
  - $\mathcal{F}_{\mu\nu}$ is a 10 of $Sp(4)$, $\mathcal{D}_{\mu\nu}$ is a 5 of $Sp(4)$

- effective operator for $h\gamma\gamma$

$$\mathcal{L}_{\chi\mathcal{F}\mathcal{F}} = \frac{\lambda_\chi \sec\beta \sin\theta}{64\pi^3 v_W} \,\mathrm{tr}(\chi^+ \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu}), \quad \lambda_\chi = O(1)\mathrm{in\ NDA}$$

- induced $h\gamma\gamma$ coupling

$$\mathcal{L} = c_\gamma^{\mathrm{TC}} \frac{\alpha}{\pi v_W} h A_{\mu\nu} A^{\mu\nu}, \quad c_\gamma^{\mathrm{TC}} = \frac{\lambda_\chi \lambda_{UD} c_\alpha}{32\sqrt{2}\pi c_\beta} s_\theta^2 ,$$

compared to $c_\gamma^{\mathrm{SM}} \simeq .23$.

- Including modified Higgs couplings to t, W

$$\Gamma_{\gamma\gamma}/\Gamma_{\gamma\gamma}^{SM} \simeq 1.52 \,|\kappa_F c_\gamma^{\mathrm{SM}} - 1.04\kappa_V + c_\gamma^{\mathrm{TC}}|^2 .$$

$\Rightarrow$ TC shifts in $\Gamma_{\gamma\gamma,VV,\bar{f}f}$ are suppressed by s_θ^2, deviations are small, percent level.

The vector resonances

- all resonances appear in representations of the unbroken subgroup, $Sp(4)$
- consider the lowest lying 10- and 5-plet vectors (also see Franzosi et al. 1605.01363)

$$\hat{R}_{10} = R_{10}^a T^a, \quad \hat{R}_5 = R_5^a X^a \,, \qquad \hat{R} \mapsto V \hat{R} V^\dagger$$

  - $R_{10}^a, \quad R_{10}^{a+3}, \quad R_5^a$, $a = 1, 2, 3$ are triplets of $SU(2)_V$
  - $R_{10}^{1..3}$ are G_{LR} even; $R_{10}^{4..6}$, $R_5^{1..3}$ are G_{LR} odd
  - $R_{10}^{a\pm} = (R_{10}^a \pm R_{10}^{a+3})/\sqrt{2}$, a=1..3 are triplets of $SU(2)_{1,2}$, interchanged under G_{LR}
  - based on the vector currents $\Psi^\dagger \bar{\sigma}_\mu T^a \Psi$ at $\theta = \pi/2$, $\hat{R}_{10}$ and $\hat{R}_5$ generalize the QCD $\vec{\rho}$ and $\vec{a}_1$ triplets, respectively
  - however, R_{10}^a and R_{10}^{a+3}, a=1..3, are the G_{LR} "parity doubling partners"

Vector Lagrangian

- employ antisymmetric tensor formalism Gasser and Leutwyler; Ecker etal
- The kinetic terms are (M_R^2 is the mass in the chiral limit)

$$\mathcal{L}_{\rm kin} = -\frac{1}{2}\,{\rm tr}(\nabla^\lambda \hat{R}_{\lambda\mu} \nabla_\nu \hat{R}^{\nu\mu} - \frac{1}{2} M_R^2 \hat{R}_{\mu\nu} \hat{R}^{\mu\nu})\,,$$

  - A related object, $R_\mu = -M_R^{-1}\,\nabla^\nu R_{\nu\mu}$
    satisfies Proca equation for massive vector field

- Most general $O(p^2)$ interaction Lagrangian, linear in $R_{5,10}$

$$\mathcal{L}_R^{(2)} = {\rm tr}\left(i\,G_{10} \hat{R}_{10,\mu\nu} d^\mu d^\nu + \frac{F_{10}}{\sqrt{2}} \hat{R}_{10,\mu\nu} \mathcal{F}^{\mu\nu} + \frac{F_5}{\sqrt{2}} \hat{R}_{5,\mu\nu} \mathcal{D}^{\mu\nu} \right)$$

  - $F_{10,5}$ are the vector decay constants,

$$\langle R^a_{10(5)} | \Psi^\dagger\, \bar{\sigma}_\mu\, T^a (X^a)\, \Psi | 0 \rangle = -iF_{10(5)} M_{10(5)} \epsilon^*_\mu$$

  - $G_{10} = -2\sqrt{2} f^2 / F_{10}$ in vector meson dominance (VMD) approximation
    (VMD $\rho\pi\pi$ coupling, $g_{\rho\pi\pi} = -m_\rho / f_\rho$, is 16% below exp.; $\phi K K$ is within a few $\%$)

- $\mathcal{L}_R^{(2)}$ yields the bilinears ($a=1,2,3$)

$$\mathcal{L}_{\rm bilinear} = -\frac{1}{4} F_{10}\, R^a_{10} \left(g_2 W^a + g_1 B\, \delta^{a3}\right) - \frac{1}{4}\left(F_{10}\, c_\theta\, R^{a+3}_{10} - F_5\, s_\theta\, R^a_5\right)\left(g_2 W^a - g_1 B\, \delta^{a3}\right)$$

  - they induce the couplings to SM fermions responsible for vector Drell-Yan production, via the substitutions

$$W^a_\mu \to W^a_\mu - \frac{g_2 F_{10}}{2M_{10}}\left(R^a_{10,\mu} + R^{a+3}_{10,\mu} c_\theta\right) + \frac{g_2 F_5}{2M_5} R^a_{5,\mu} s_\theta$$

$$B_\mu \to B_\mu - \frac{g_1 F_{10}}{2M_{10}}\left(R^3_{10,\mu} - R^6_{10,\mu} c_\theta\right) - \frac{g_1 F_5}{2M_5} R^3_{5,\mu} s_\theta\,.$$

- leading R_{10} decays originate from the trilinears

$$-\frac{G_{10} M_{10}}{2\sqrt{2} f^2}\left(\epsilon^{abc} R^a_{10,\mu} \pi^b \partial^\mu \pi^c + R^{a+3}_{10,\mu}\left[\pi^5 \partial^\mu \pi^a - \pi^a \partial^\mu \pi^5\right]\right) + \ldots..,$$

- typically, we are far form the chiral limit, $\hat{m} \lesssim f$, closing the decay channels $R_{10}^{1..3,(4..6)} \to \tilde{\pi}\tilde{\pi}, (H\tilde{\pi})$.
  - Therefore $R_{10}^{1..3,(4..6)} \to \tilde{\pi} W_L/Z_L, (\tilde{\pi} h, H W_L/Z_L)$ dominate

S-parameter

- tree-level $R_{10}^{3,6}$, R_5^3 exchange yields

$$\Delta S_{\rm tree} = 4\pi \left(F_{10}^2/M_{10}^2 - F_5^2/M_5^2\right) \sin^2\theta$$

  - s_θ^2 suppression is a general feature of misalignment in composite Higgs
    Barbieri, Bellazzini et al; Contino et al; Panic, Wulzer...
  - here the origin of s_θ^2 in $\Delta S_{\rm tree}$ is explicit: $R_{10}^{3,6}$ parity doubling cancelation $\propto 1 - c_\theta^2$; and s_θ suppression of the R_5^3 couplings

- To estimate $\Delta S_{\rm tree}$ we use the $N_c = n_f = 2$ lattice results for M_{10}, f_π (full decay constant) away from the chiral limit
  - estimate F_{10}/f_π by fitting to f_V/f_P vs m_q in QCD
  - bound the contribution of R_5 via approximate upper and lower bounds, $M_5 < M_{10}\, m_{a_1}/m_\rho$ and $F_5 > f_{a_1} f/f_\pi^{\rm qcd}$
    - Take $f_{a_1} = 152$ MeV based on a phenomenological determination using ${\rm Br}(\tau^+ \to \nu_\tau \pi^+\pi^+\pi^-)$

- obtain $\Delta S_{\rm tree}/s_\theta^2 < [0.11, 0.09]$ ([0.19, 0.13] for R_{10}) for $\hat{m}_\theta/f = [0, 1.5]$
  - exhibits an expected (especially for R_{10}) decrease away from the chiral limit
  - implies $\Delta S \subset 0.10 \pm 0.08$ [Rome], 0.00 ± 0.08 [PDG] (1σ) is reasonable
- scalar loops in S are log divergent, due to c_θ factor in the π^4 gauge couplings Barbieri, Bellazzini et al 0706.0432. After SM Higgs subtraction

$$\Delta S_{\rm loop} = \frac{1}{24\pi}\left(s_\theta^2\left[s_\alpha^2 \log\frac{\Lambda^2}{m_h^2} + c_\alpha^2 \log\frac{\Lambda^2}{m_{\mathcal{H}}^2}\right] + F_{\rm fin}\right),$$

where $F_{\rm fin}$ is a lengthy expression from finite loop contributions

  - the large log (1st term) is additionally suppressed by s_α^2 relative to the usual composite Higgs, from projection of π^4 onto h
  - for cut-off $\Lambda < 8\pi f$ find $\Delta S_{\rm loop} < 0.01$ in our examples

T parameter

At one loop, T mainly arises from

- scalar loops with isospin breaking entering via $\tilde{\pi}^3 - \pi^4$, and $\tilde{\pi}^3 - \eta$ mixings, and $\tilde{\pi}^3 – \tilde{\pi}^+$ mass splitting
  - these loops vanish in the limit $(\lambda_U - \lambda_D) \to 0$, so their size can be controlled
- G^+ wave function renormalization via $B - R^{1,2,4,5}_{5,10}$ loops.
  - non-trivial to estimate rychkov; kamenik; pich.
    however, G^+ is dominantly fundamental $\Rightarrow$ projection of the vertices onto G^+ suppresses these effects by c_β^2, or $O(10)$
- so we conclude that misaligned BTC should reasonably live within the (S, T) plane's allowed 1σ ellipse