Very rare, exclusive, hadronic decays in QCD factorization

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EFTs for Collider Physics, Flavor Phenomena and EWSB

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PRISMA | Cluster of Excellence
Precision Physics, Fundamental Interactions and Structure of Matter

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Exclusive hadronic decays can serve as probes for new physics, revealing more information when combined with “more conventional” searches!

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Price to pay: Very **small branching ratios** and difficult reconstruction!

Based on:

Exclusive Radiative Decays of W and Z Bosons in QCD Factorization
Yuval Grossman, MK, Matthias Neubert

JHEP 1504 (2015) 101, arXiv:1501.06569

Exclusive Radiative Z-Boson Decays to Mesons with Flavor-Singlet Components
SA, MK, Matthias Neubert

JHEP 1602 (2016) 162, arXiv:1512.09135

Exclusive Radiative Higgs Decays as Probes of Light-Quark Yukawa Couplings
MK, Matthias Neubert

JHEP 1508 (2015) 012, arXiv:1505.03870

Exclusive Weak Radiative Higgs Decays in the Standard Model and Beyond
SA, MK, Matthias Neubert

arXiv:160x.soon?

Outline

QCD-factorization
Derivation of the factorization formula

History

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The framework of QCD factorization was originally developed by Brodsky, Efremov, Lepage and Radyushkin in the beginning of the 1980's.

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[Brodsky, Lepage (*1980*), Phys. Rev. D 22, 2157]

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The factorization formula was **derived using light-cone perturbation theory**.

The derivation **can also be phrased in** the language of **soft-collinear effective theory**.

[Bauer et al. (*2001*), Phys. Rev. D 63, 114020]
[Bauer Pirjol, Stewart (*2002*), Phys. Rev. D 65, 054022]
[Beneke, Chapovsky, Diehl, Feldmann (*2002*), Nucl. Phys. B 643, 431]

QCD factorization: The **hadronization** happens well **after the hard scattering has taken place** $\rightarrow$ separation of scales.

An effective theory approach

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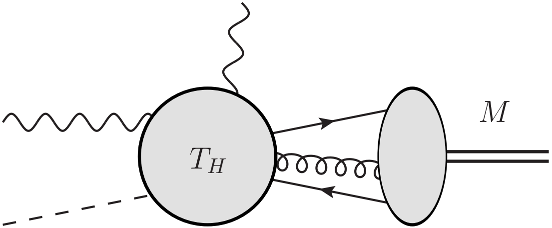

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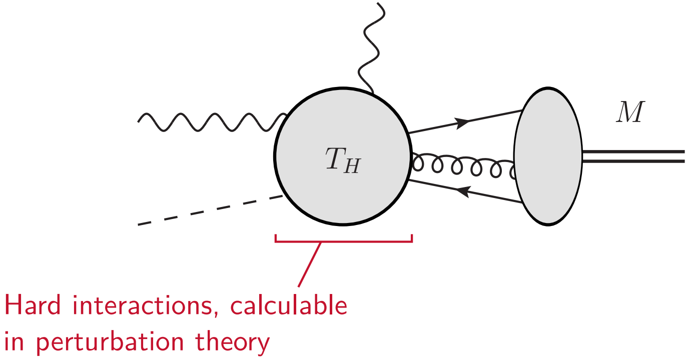

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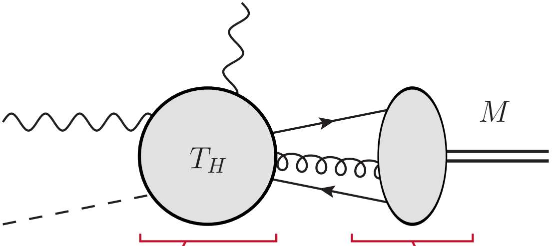

Hard interactions, calculable in perturbation theory

Non-perturbative physics, hadronic input

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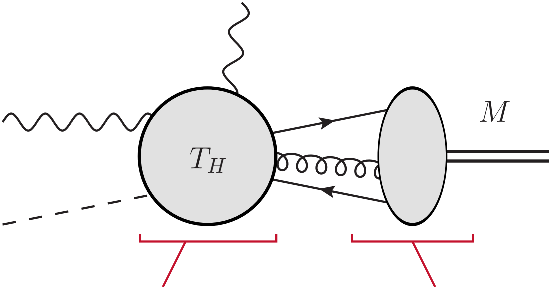

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The **scale separation** in the case at hand **calls for an effective theory** description!

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Strategy: Using SCET, write down **all effective operators** from **collinear partons** that can excite the meson from the QCD vacuum.

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The operators are bi-local along the light-like direction $\bar{n}$:

$$\begin{aligned} J &\sim \bar{q}_c(x) \dots q_c(x) \;+\; \bar{q}_c(x) \dots t(\bar{n} \cdot \partial) q_c(x) \;+\; \dots \\ &\to \bar{q}_c(x) \dots q_c(x + t\bar{n}) \end{aligned}$$

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Match **partonic diagrams** to these **current operators**.

The non-perturbative **hadronization** is **encoded in the matrix element** of the current operators between the **QCD vacuum** and the **hadronic final state** $\langle M \,|\, J \,|\, 0 \rangle$.

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With our effective operator $J_q(t) = \bar{q}_c(t\bar{n})\,\Gamma\,[t\bar{n},0]\,q_c(0)$ the amplitude for $X \to M + V$ is then given by:

$$i\mathcal{A} = \int \mathcal{C}(t,\dots)\langle M(k)|\, J_q(t,\dots)\,|0\rangle dt$$

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The **hadronic matrix element** defines a function analogous to the decay constants. In fact, these are just the local case $(t = 0)$ above. The generalization to our **bi-local current operator**

$$\langle M(k)|\,J_q(t,\dots)\,|0\rangle \sim f_M \int e^{i(t\bar{n})\cdot(xk)}\phi_M^q(x)dx$$

defines the light-cone distribution amplitude (**LCDA**), which encodes the non-perturbative physics in the exclusive hadronic final state.

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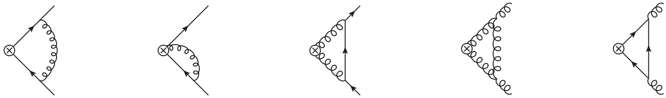
For mesons with a **flavor-singlet** component, there is an analogous **contribution from two gluons**.

QCD-factorization
Light-cone distribution amplitudes

Remember, we are dealing with a **huge scale hierarchy**: m_Z vs. Λ_{QCD}

⇒ Large logarithms $\alpha_s \log(m_Z/\Lambda_{\text{QCD}})$ need to be resummed.

Examples of corrections to the LCDAs at $\mathcal{O}(\alpha_s)$:



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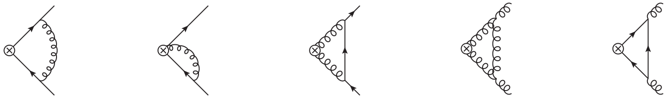

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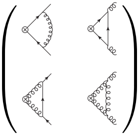
$$\begin{pmatrix} \phi_q^{\text{ren}} \\ \phi_g^{\text{ren}} \end{pmatrix} =$$



$$\otimes \begin{pmatrix} \phi_q^{\text{bare}} \\ \phi_g^{\text{bare}} \end{pmatrix}$$

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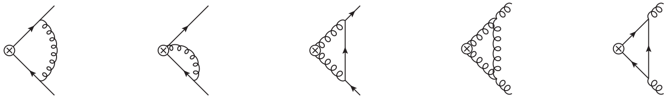

The LCDAs are renormalized according to:

$$\begin{pmatrix} \phi_q^{\rm ren}(x,\mu) \\ \phi_g^{\rm ren}(x,\mu) \end{pmatrix} = \int_0^1 \left[\mathbf{1}\cdot\delta(x-y) \;+\; \frac{\alpha_s(\mu)}{4\pi\epsilon} \begin{pmatrix} V_{qq}(x,y) & V_{qg}(x,y) \\ V_{gq}(x,y) & V_{gg}(x,y) \end{pmatrix} \right] \begin{pmatrix} \phi_q^{\rm bare}(y) \\ \phi_g^{\rm bare}(y) \end{pmatrix} dy$$

[Brodsky, Lepage (*1979*), Phys. Lett. B 87, 359]

[Terentev (*1981*), Sov. J. Nucl. Phys. 33, 911]

[Ohrndorf (*1981*), Nucl. Phys. B 186, 153]

[Shifman, Vysotsky (*1981*), Nucl. Phys. B 186, 475]

[Baier, Grozin (*1981*), Nucl.Phys. B192 476-488]

Renormalization

The LCDAs are expanded in the eigenfunctions of the evolution Kernels:

$$\phi_M^q(x,\mu) = 6x\,\bar{x}\left[1 + \sum_{n=1}^{\infty} a_n^M(\mu) C_n^{(3/2)}(2x-1)\right]$$

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$$\left[\mu\frac{d}{d\mu} + \frac{\alpha_s(\mu)}{4\pi}\begin{pmatrix}\gamma_n^{qq} & \gamma_n^{qg}\\ \gamma_n^{gq} & \gamma_n^{gg}\end{pmatrix}\right]\begin{pmatrix}a_n^M\\ b_n^M\end{pmatrix} + \mathcal{O}(\alpha_s^2) = 0$$

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At higher orders, moments of order n mix with moments of order $k < n$.

When scale-evolved to high scales, all Gegenbauer moments decrease:

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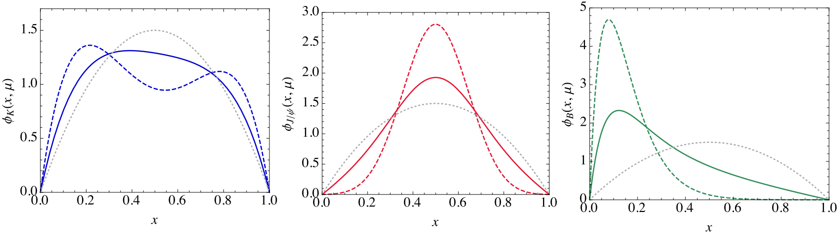

a) K LCDA   b) J/ψ LCDA   c) B LCDA

LCDAs for mesons at different scales, dashed lines: $\phi_M(x, \mu = \mu_0)$, solid lines: $\phi_M(x, \mu = m_Z)$, grey dotted lines: $\phi_M(x, \mu \to \infty)$

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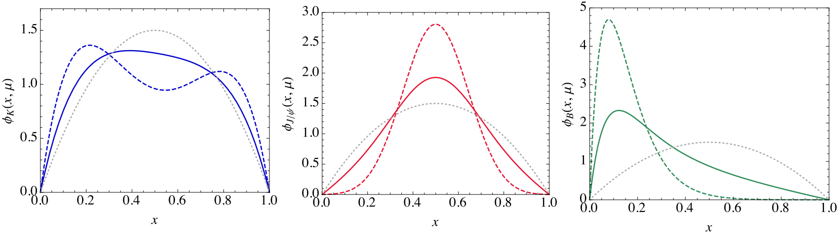

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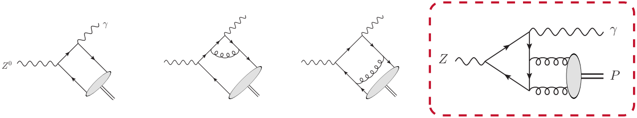
At high scales compared to $\Lambda_{\rm QCD}$ (e.g. $\mu \sim m_Z$) the sensitivity to poorly-known a_n^M, b_n^M is greatly reduced!

Hadronic Z-boson decays

The $Z \to M\gamma$ amplitude

The decay amplitude is governed by diagrams:



singlets only!

Form factor decomposition:

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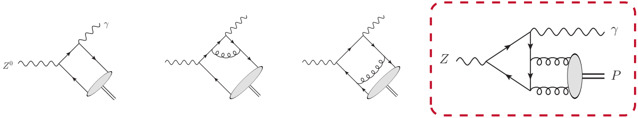

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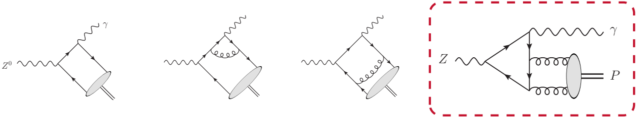
The form factors contain the convolution integrals:

$$F^M \sim \int_0^1 dx\, H(x,\mu)\phi_M(x,\mu) = \sum_n C_{2n}(\mu) a_{2n}^M(\mu)$$

$$C_n(\mu) = 1 + \frac{C_F\alpha_s(\mu)}{4\pi}\left\{ 3\log\frac{m_Z^2}{\mu^2} + \ldots \right\}$$

The $Z \to M\gamma$ amplitude

The decay amplitude is governed by diagrams:



singlets only!

Form factor decomposition:

$$i\mathcal{A} = \pm \frac{egf_M}{2\cos\theta_W}\left[i\epsilon_{\mu\nu\alpha\beta}\frac{k^\mu q^\nu \varepsilon_Z^\alpha \varepsilon_\gamma^{*\beta}}{k\cdot q} F_1^M - \left(\varepsilon_Z \cdot \varepsilon_\gamma^* - \frac{q\cdot \varepsilon_Z k \cdot \varepsilon_\gamma^*}{k\cdot q}\right) F_2^M \right]$$

The form factors contain the convolution integrals:

$$F^M \sim \int_0^1 dx\, H(x,\mu)\phi_M(x,\mu) = \sum_n C_{2n}(\mu) a_{2n}^M(\mu)$$

$$C_n(\mu) = 1 + \frac{C_F \alpha_s(\mu)}{4\pi}\left\{ 3\log\frac{m_Z^2}{\mu^2} + \dots \right\}$$

Evaluating the hard function at $\mu = m_Z$ and evolving it down to μ_{hadr} resums large logarithms $\left[\alpha_s \log(m_Z^2/\mu^2)\right]^n$.

The flavor of etas: Singlets and octets

The "singlet" in $\eta^{(\prime)}$ means $(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle)/\sqrt{3}$. **However**, at the factorization scale $\mu \approx m_Z$, a flavor singlet is rather

$$\frac{1}{\sqrt{5}} \left(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle + |c\bar{c}\rangle + |b\bar{b}\rangle \right).$$

⇒ Have to **rearrange operators at each threshold scale** into singlet and non-singlet combinations, which are **different for every $\mathbf{n_f}$**.

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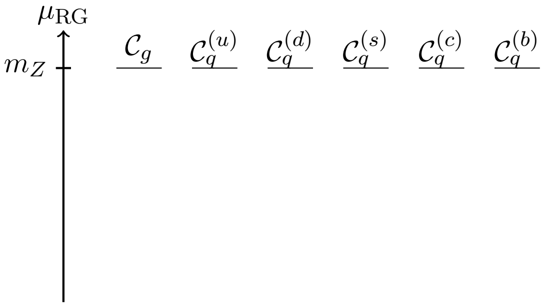

compute hard functions at high scale

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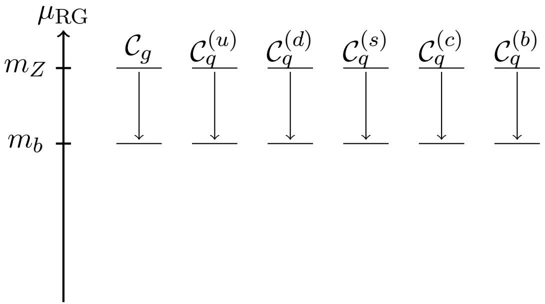

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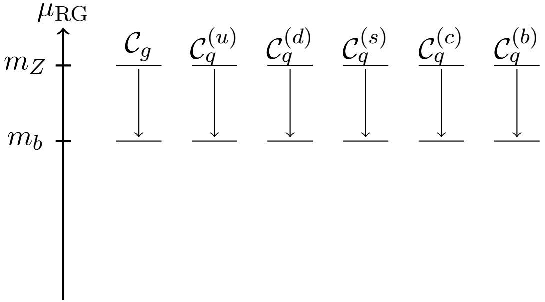

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singlet now consists of 4 flavors!

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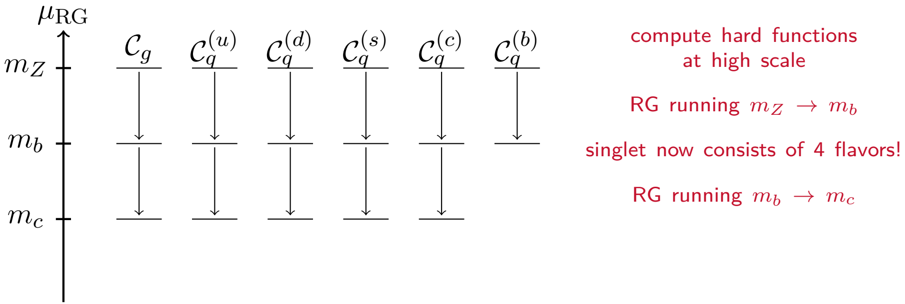

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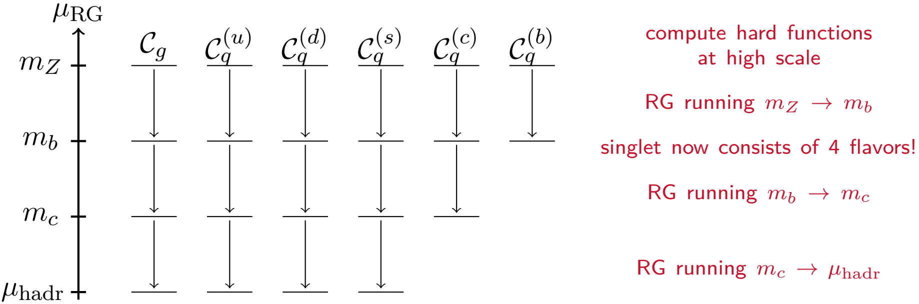

Results for $Z \to M\gamma$

For the branching ratios $\mathrm{BR}(Z \to M\gamma)$ we find:

$Z \to \ldots$	Branching ratio		asym.	LO
$\pi^0\gamma$	$(9.80\ {}^{+0.09}_{-0.14}{}_{\mu}\ \pm 0.03_f\ \pm 0.61_{a_2} \pm 0.82_{a_4})$	$\cdot 10^{-12}$	7.71	14.67
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$\omega\gamma$	$(2.89\ {}^{+0.03}_{-0.05}{}_{\mu}\ \pm 0.15_f\ \pm 0.29_{a_2} \pm 0.25_{a_4})$	$\cdot 10^{-8}$	2.54	3.84
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↑ **scale dependence**

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JG|U

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decay constant

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scale dependence — **decay constant** — **LCDA shape**

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obtained when using only asymptotic form of LCDA

$$\phi_M(x) = 6x(1-x)$$

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The form factors become:

$$
\begin{aligned}
\mathrm{Re}\,F_1^M &= \mathcal{Q}_M \left[0.94 + 1.05\,a_2^M(m_Z) + 1.15\,a_4^M(m_Z) + 1.22\,a_6^M(m_Z) + \dots\right] \\
&= \mathcal{Q}_M \left[0.94 + 0.41\,a_2^M(\mu_h) + 0.29\,a_4^M(\mu_h) + 0.23\,a_6^M(\mu_h) + \dots\right]
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$$= \mathcal{Q}_M \left[0.94 + 0.41\, a_2^M(\mu_h) + 0.29\, a_4^M(\mu_h) + 0.23\, a_6^M(\mu_h) + \ldots\right]$$

$\to$ RGE from high to low scale reduces sensitivity to a_n^M!

Hadronic Higgs decays
Radiative hadronic Higgs decays

Idea: Use hadronic Higgs decays to probe non-standard Higgs couplings.

[Isidori, Manohar, Trott (*2014*), Phys. Lett. B 728, 131]
[Bodwin, Petriello, Stoynev, Velasco (*2013*), Phys. Rev. D 88, no. 5, 053003]
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$$+ \frac{\alpha}{4\pi v} \left(\kappa_{\gamma\gamma} h F_{\mu\nu} F^{\mu\nu} - \tilde{\kappa}_{\gamma\gamma} h F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{2\kappa_{\gamma Z}}{s_W c_W} h F_{\mu\nu} Z^{\mu\nu} - \frac{2\tilde{\kappa}_{\gamma Z}}{s_W c_W} h F_{\mu\nu} \tilde{Z}^{\mu\nu} \right)$$

blue terms: $\to 1$ in SM, **red terms**: $\to 0$ in SM!

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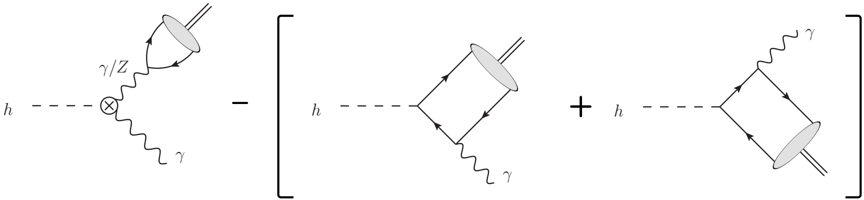
$\to$ Provides a model-independent analysis of NP effects in $h \to V\gamma$ decays!

The $h \to V\gamma$ decays

Several different diagram topologies:



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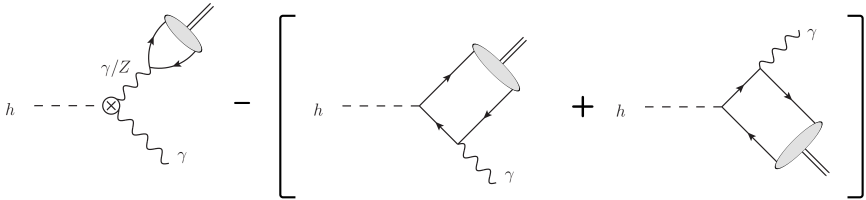

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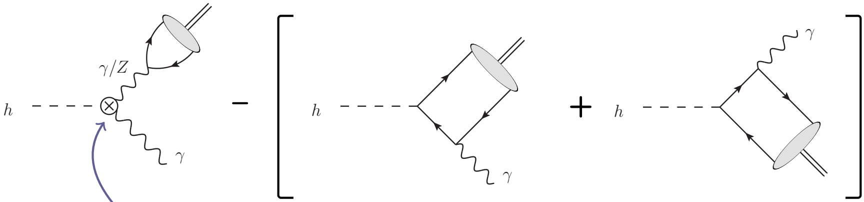

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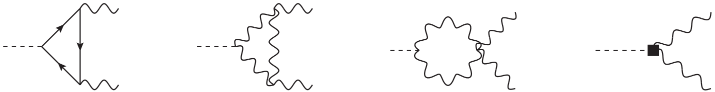

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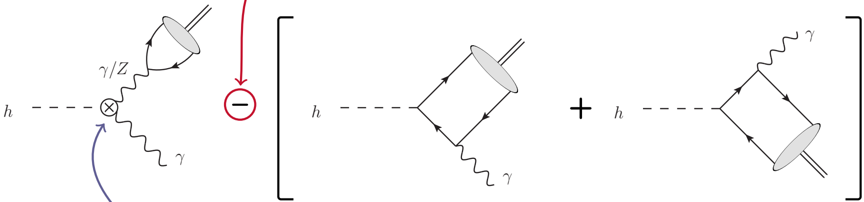
Several different diagram topologies:

destructive interference!



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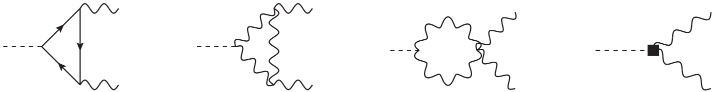

The $h \to V\gamma$ decays

Form factor decomposition:

$$i\mathcal{A}(h \to V\gamma) = -\frac{e f_V}{2}\left[\left(\varepsilon_V^* \cdot \varepsilon_\gamma^* - \frac{q \cdot \varepsilon_V^* k \cdot \varepsilon_\gamma^*}{k \cdot q}\right) F_1^V - i\epsilon_{\mu\nu\alpha\beta}\frac{k^\mu q^\nu \varepsilon_V^{*\alpha} \varepsilon_\gamma^{*\beta}}{k \cdot q} F_2^V\right]$$

Contributions from both diagram topologies, the **direct** contributions ($h \to (q\bar{q} \to V)\gamma$) and the **indirect** contributions ($h \to (Z/\gamma \to M)\gamma$).

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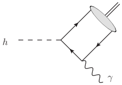

$$F_{1,\text{direct}}^V \propto \kappa_q \frac{f_V^\perp(\mu)}{f_V}\left[1 - \frac{C_F \alpha_s(\mu)}{\pi} \log \frac{m_h^2}{\mu^2}\right]\left(\sum_{n=0}^{\infty} C_{2n}(m_h, \mu) a_{2n}^{V_\perp}(\mu)\right)$$

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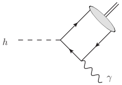

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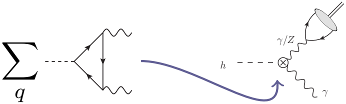
The **indirect** form factors however, are proportional to all κ_X in the Lagrangian!



There could be NP in **any** of these contributions leading to deviations from the SM prediction for our amplitudes!

The $h \to V\gamma$ decays

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To reduce the theoretical uncertainty, we **normalize the branching ratio to the $h \to \gamma\gamma$ branching ratio**, which also makes our prediction insensitive to the total Higgs width:

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corrections from the indirect contributions due to off-shellness

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$\to$ only very weak sensitivity to the indirect contributions!

Nummerical results

Assuming SM couplings of all particles, we find:

$$\mathrm{BR}(h \to \rho^0\gamma) = (1.68 \pm 0.02_f \pm 0.08_{h\to\gamma\gamma}) \cdot 10^{-5}$$
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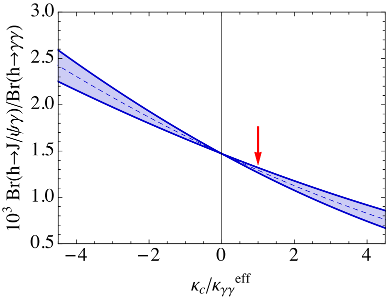
But: What is wrong with the Υ-channels?

Allowing deviations of the κ_q and no CP-odd couplings:



Ratio of BR for J/ψ

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Phenomenology

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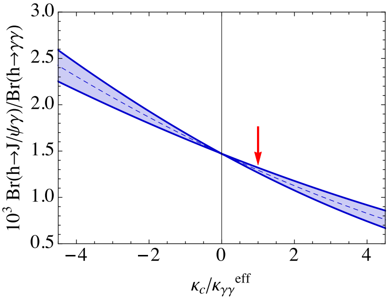

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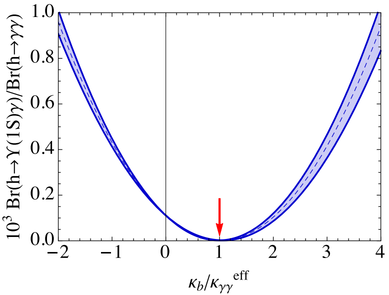
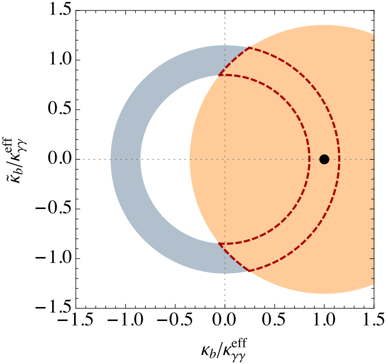


Ratio of BR for $\Upsilon(1S)$

Usually, the **indirect contributions** are the **dominant** ones, however **for the Υ**, the **direct contribution** is **comparable**, leading to a **cancellation** between the two.

$\Rightarrow$ This leads to a **strong sensitivity to NP effects**!

Possible future scenarios:



Blue: direct measurements of $h \to b\bar{b}$ constrain $\kappa_b^2 + \tilde{\kappa}_b^2$
Orange: measurements of $h \to \Upsilon\gamma$ constrain $(1-\kappa_b)^2 + \tilde{\kappa}_b^2$

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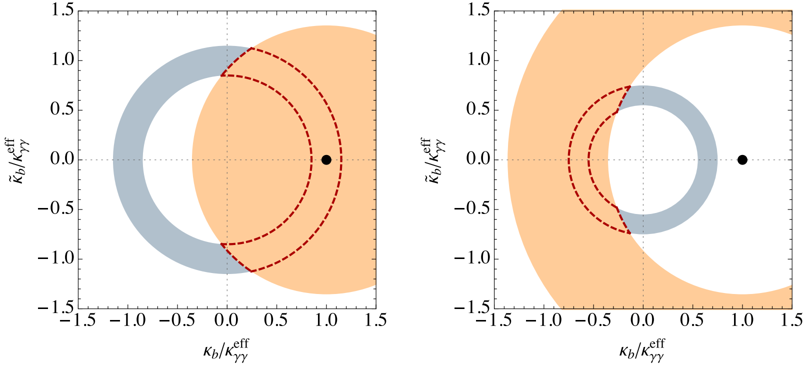

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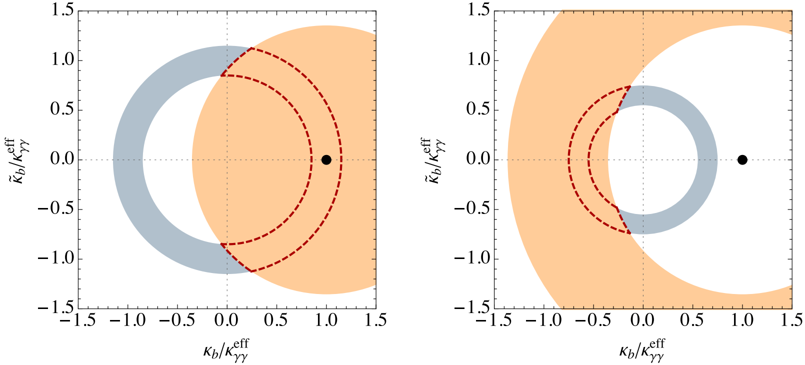

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$\Rightarrow$ From the **overlap** one can find information on the CP-odd coupling, **even the sign** of the CP-even coupling!

Hadronic Higgs decays

Weak radiative hadronic Higgs decays

Higgs decay to a meson and a Z-boson

For select mesons, literature exists on these modes.

[Isidori, Manohar, Trott (*2014*), Phys.Lett. B728 131-135]

[Gao (*2014*), Phys.Lett. B737 366-368]

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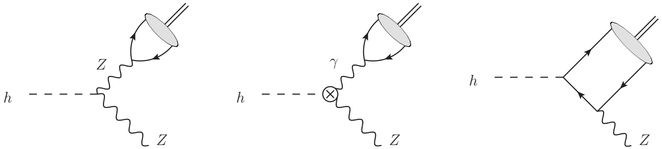

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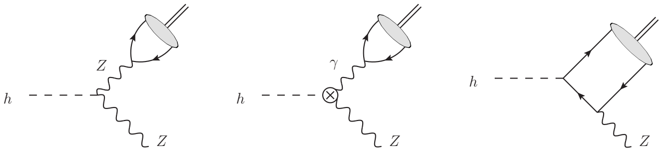

While the diagrams $h \to Z(\gamma^* \to V)$ are **loop-suppressed**, the photon is off-shell only by m_V^2, **lifting** the **suppression**.

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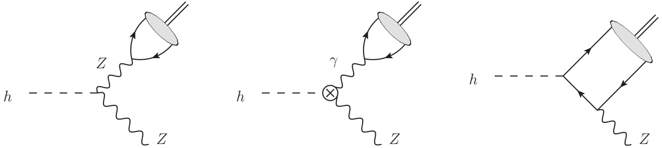

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The direct contributions are only important for **heavy quarkonia**.

Numerical Results

Mode	SM Branching ratio $[10^{-6}]$
$h \to \pi^0 Z$	$(2.30 \pm 0.01_f \pm 0.09_\Gamma)$
$h \to \eta Z$	$(0.83 \pm 0.08_f \pm 0.03_\Gamma)$
$h \to \eta' Z$	$(1.24 \pm 0.12_f \pm 0.05_\Gamma)$
$h \to \rho^0 Z$	$(7.19 \pm 0.09_f \pm 0.28_\Gamma)$
$h \to \omega Z$	$(0.56 \pm 0.01_f \pm 0.02_\Gamma)$
$h \to \phi Z$	$(2.42 \pm 0.05_f \pm 0.09_\Gamma)$
$h \to J/\psi Z$	$(2.30 \pm 0.06_f \pm 0.09_\Gamma)$
$h \to \Upsilon(1S) Z$	$(15.38 \pm 0.21_f \pm 0.60_\Gamma)$
$h \to \Upsilon(2S) Z$	$(7.50 \pm 0.14_f \pm 0.29_\Gamma)$
$h \to \Upsilon(3S) Z$	$(5.63 \pm 0.10_f \pm 0.22_\Gamma)$

Idea: Use these decays to probe $\kappa_{\gamma Z}$

The bound on $\kappa_{\gamma Z}$ from CMS is:

$$\sqrt{|\kappa_{\gamma Z} - 2.395|^2 + |\tilde{\kappa}_{\gamma Z}|^2} < 7.2$$

[CMS (*2013*), Phys.Lett. B726 587-609]

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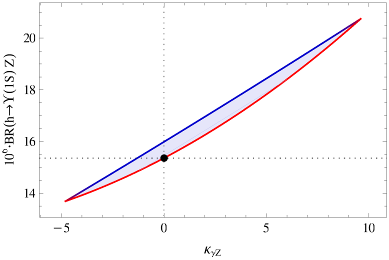

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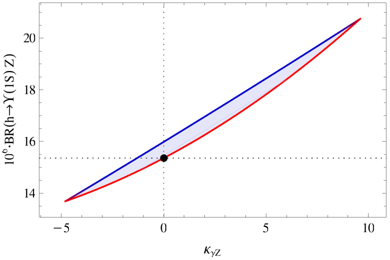

The decays $h \to \Upsilon(1S)Z$ can serve as **complementary probes** of $\kappa_{\gamma Z}$

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Backup slides

$Z \to M + \gamma$ decays as BSM probes

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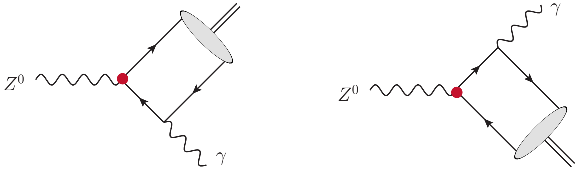

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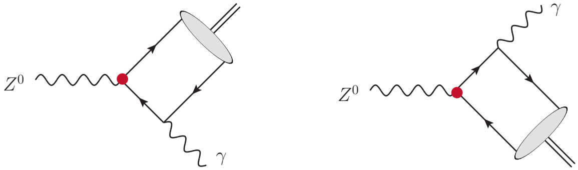

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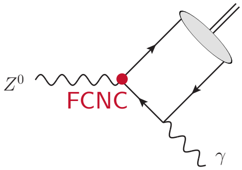

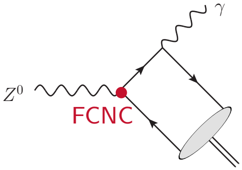

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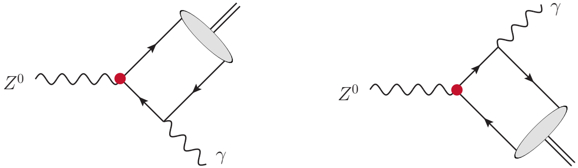
Introducing FCNC couplings allows the production of flavor off-diagonal mesons



Model independent predictions for flavor off-diagonal mesons:

Decay mode	Branching ratio	SM background
$Z^0 \to K^0\gamma$	$\left[(7.70 \pm 0.83)\,\lvert v_{sd}\rvert^2 + (0.01 \pm 0.01)\,\lvert a_{sd}\rvert^2\right] \cdot 10^{-8}$	$\frac{\lambda}{\sin^2\theta_W}\,\frac{\alpha}{\pi} \sim 2 \cdot 10^{-3}$
$Z^0 \to D^0\gamma$	$\left[(5.30\,^{+0.67}_{-0.43})\,\lvert v_{cu}\rvert^2 + (0.62\,^{+0.36}_{-0.23})\,\lvert a_{cu}\rvert^2\right] \cdot 10^{-7}$	$\frac{\lambda}{\sin^2\theta_W}\,\frac{\alpha}{\pi} \sim 2 \cdot 10^{-3}$
$Z^0 \to B^0\gamma$	$\left[(2.08\,^{+0.59}_{-0.41})\,\lvert v_{bd}\rvert^2 + (0.77\,^{+0.38}_{-0.26})\,\lvert a_{bd}\rvert^2\right] \cdot 10^{-7}$	$\frac{\lambda^3}{\sin^2\theta_W}\,\frac{\alpha}{\pi} \sim 8 \cdot 10^{-5}$
$Z^0 \to B_s\gamma$	$\left[(2.64\,^{+0.82}_{-0.52})\,\lvert v_{bs}\rvert^2 + (0.87\,^{+0.51}_{-0.33})\,\lvert a_{bs}\rvert^2\right] \cdot 10^{-7}$	$\frac{\lambda^2}{\sin^2\theta_W}\,\frac{\alpha}{\pi} \sim 4 \cdot 10^{-4}$

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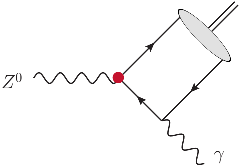

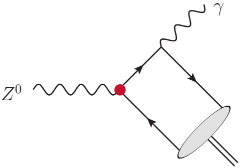

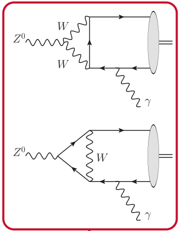

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$Z \to M + \gamma$ decays as FCNC probes

FCNCs would induce tree-level neutral-meson mixing, strongly constrained:

$\left\lvert \text{Re}\left[(v_{sd} \pm a_{sd})^2\right]\right\rvert$	$< 2.9 \cdot 10^{-8}$	$\left\lvert \text{Re}\left[(v_{sd})^2 - (a_{sd})^2\right]\right\rvert$	$< 3.0 \cdot 10^{-10}$
$\left\lvert \text{Im}\left[(v_{sd} \pm a_{sd})^2\right]\right\rvert$	$< 1.0 \cdot 10^{-10}$	$\left\lvert \text{Im}\left[(v_{sd})^2 - (a_{sd})^2\right]\right\rvert$	$< 4.3 \cdot 10^{-13}$
$\left\lvert (v_{cu} \pm a_{cu})^2 \right\rvert$	$< 2.2 \cdot 10^{-8}$	$\left\lvert (v_{cu})^2 - (a_{cu})^2 \right\rvert$	$< 1.5 \cdot 10^{-8}$
$\left\lvert (v_{bd} \pm a_{bd})^2 \right\rvert$	$< 4.3 \cdot 10^{-8}$	$\left\lvert (v_{bd})^2 - (a_{bd})^2 \right\rvert$	$< 8.2 \cdot 10^{-9}$
$\left\lvert (v_{bs} \pm a_{bs})^2 \right\rvert$	$< 5.5 \cdot 10^{-7}$	$\left\lvert (v_{bs})^2 - (a_{bs})^2 \right\rvert$	$< 1.4 \cdot 10^{-7}$

[Bona et al. (*2007*), JHEP 0803, 049]

[Bertone et al. (*2012*), JHEP 1303, 089]

[Carrasco et al. (*2013*), JHEP 1403, 016]

These bounds push our branching ratios down to 10^{-14}, rendering them unobservable.

Experimental considerations

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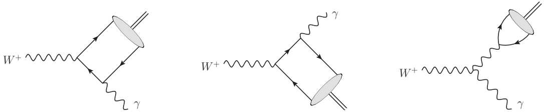
- **However**: Future lepton machines like ILC or TLEP might produce $10^{12} Z$'s and $10^{7} W$'s at the corresponding thresholds $\to$ This enables an experimental program to **test QCDF in a theoretically clean environment**!

Results for $W \to M\gamma$

The analysis in the case for $W \to M\gamma$ is almost the same, only this time, an indirect diagram exists involving the local matrix element:



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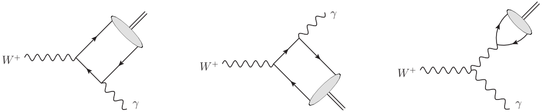

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$\pi^\pm\gamma$	$(4.00\,^{+0.06}_{-0.11}\,{}_\mu \pm 0.01_f \pm 0.49_{a_2} \pm 0.66_{a_4}) \cdot 10^{-9}$	2.45	8.09
$\rho^\pm\gamma$	$(8.74\,^{+0.17}_{-0.26}\,{}_\mu \pm 0.33_f \pm 1.02_{a_2} \pm 1.57_{a_4}) \cdot 10^{-9}$	6.48	15.12
$K^\pm\gamma$	$(3.25\,^{+0.05}_{-0.09}\,{}_\mu \pm 0.03_f \pm 0.24_{a_1} \pm 0.38_{a_2} \pm 0.51_{a_4}) \cdot 10^{-10}$	1.88	6.38
$K^{*\pm}\gamma$	$(4.78\,^{+0.09}_{-0.14}\,{}_\mu \pm 0.28_f \pm 0.39_{a_1} \pm 0.66_{a_2} \pm 0.80_{a_4}) \cdot 10^{-10}$	3.18	8.47
$D_s\gamma$	$(3.66\,^{+0.02}_{-0.07}\,{}_\mu \pm 0.12_{\text{CKM}} \pm 0.13_f\,^{+1.47}_{-0.82}\,{}_\sigma) \cdot 10^{-8}$	0.98	8.59
$D^\pm\gamma$	$(1.38\,^{+0.01}_{-0.02}\,{}_\mu \pm 0.10_{\text{CKM}} \pm 0.07_f\,^{+0.50}_{-0.30}\,{}_\sigma) \cdot 10^{-9}$	0.32	3.42
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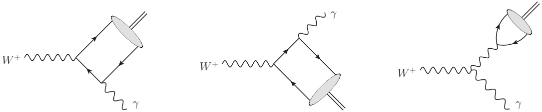

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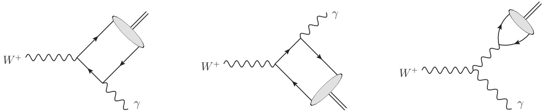

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