Bounds on Amplitudes and EFTs

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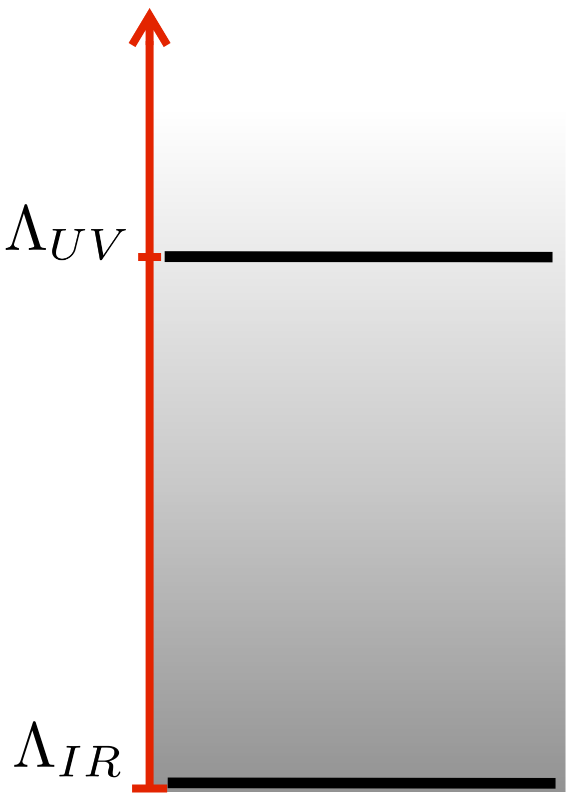
IPhT - CEA/Saclay

based on 1605.06111 and work in progress

Eltville, Burg Crass, ‘EFTs for collider physics, Flavor phenomena and EWSB’, Sept 15th 2016

Hierarchy of scales

a blessing…



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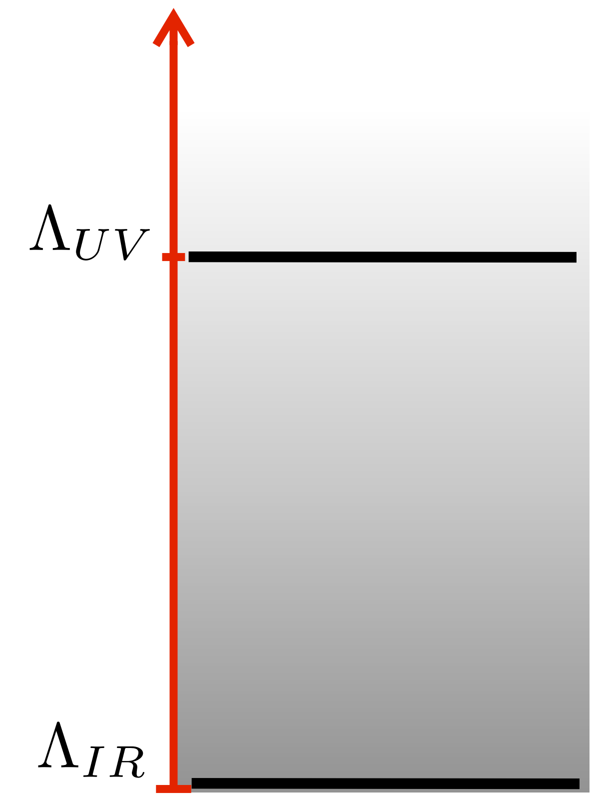

a blessing…

- small parameter E/Λ_{UV}
- emerging patterns
- suppress dangerous operators

$$\mathcal{L}_{\mathcal{IR}} = \mathcal{L}^{\Delta\leq 4} + \sum_{\mathcal{O}} \frac{\mathcal{O}(x)}{\Lambda_{UV}^{\Delta-4}}$$

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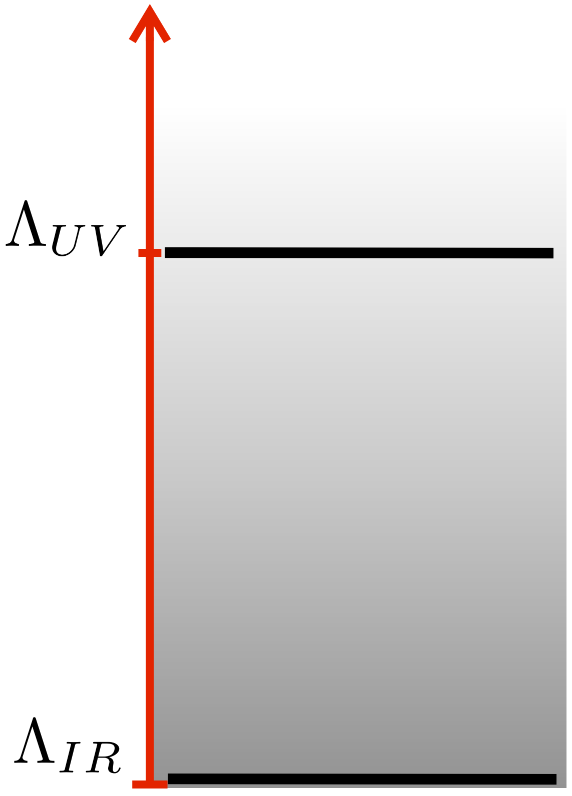

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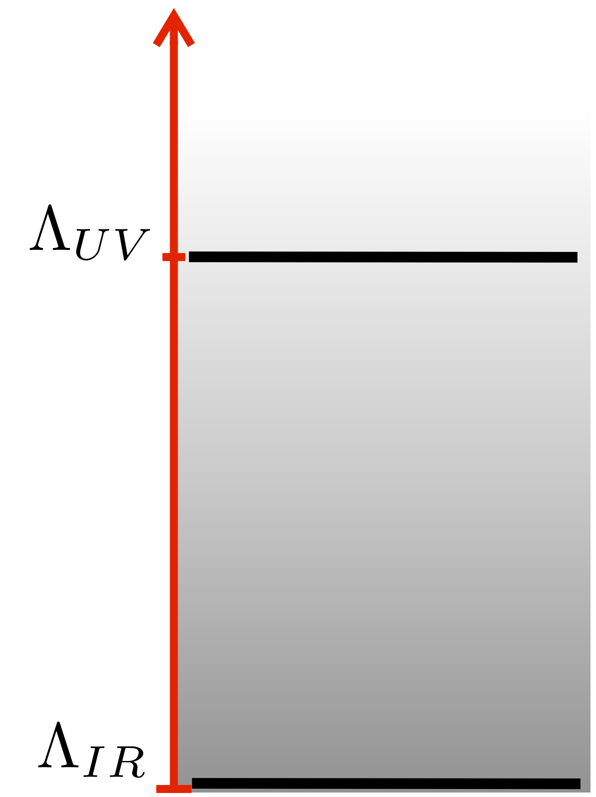

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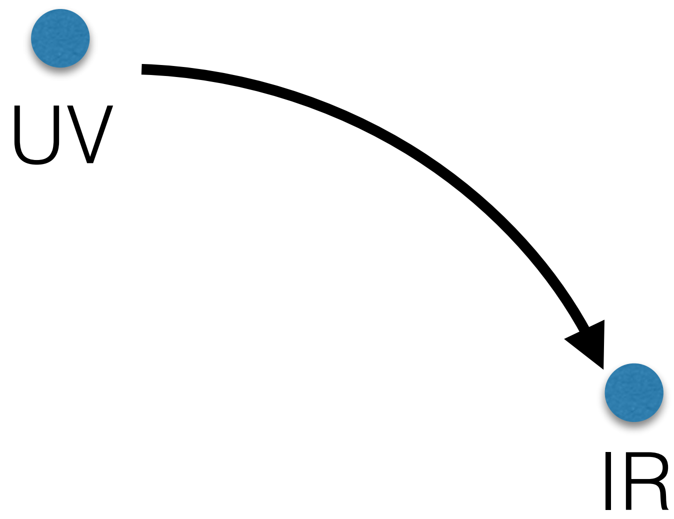
large couplings from a strong sector may help

e.g. in CHM: $\mathcal{L} = \frac{g_*^2}{m_*^2}(\partial H^2)^2$

The EFT paradigm



$$\mathcal{L}_{\mathcal{IR}} = \sum_i c_i \frac{\mathcal{O}_i(x)}{\Lambda_{UV}^{\Delta-4}}$$

EFT encodes UV-info via c_i

Finite set of C's is needed at any order in E/Λ_{UV}

Power counting = understanding = symmetries

Symmetry <—> Softness

Higher dim-operators may dominate the amplitude within EFT

just suppress relevant, marginal and less-irrelevant operators by symmetries

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1-to-1 amplitude dominated by a less-relevant operator

$$\mathcal{M}(1 \to 1) \sim \frac{1}{E} \qquad\qquad \epsilon \cdot m_* < E < m_*$$

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Example

(2) $\bar{\psi} i \partial \psi - g A_\mu \bar{\psi} \gamma^\mu \psi + \ldots$

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(2) $\bar{\psi} i \partial \psi - g A_\mu \bar{\psi} \gamma^\mu \psi + \ldots \xrightarrow[g \ll 1]{} \bar{\psi} i \partial \psi - \epsilon \cdot g_* A_\mu \bar{\psi} \gamma^\mu \psi + \frac{g_*^2}{m_*^2} (\bar{\psi} \gamma^\mu \psi)^2 + \ldots$

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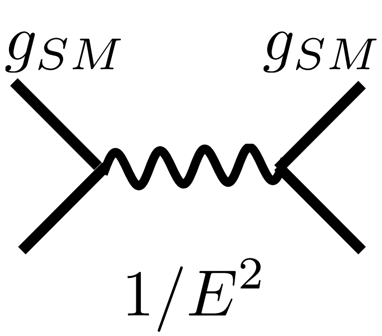
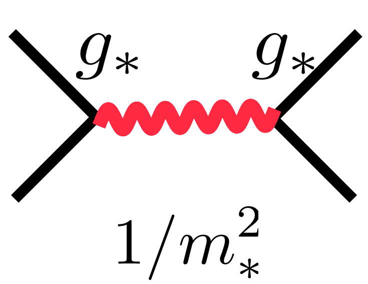
$$(2)\quad \bar\psi i\not\partial\psi - gA_\mu\bar\psi\gamma^\mu\psi + \dots \xrightarrow[g\ll 1]{} \bar\psi i\not\partial\psi - \epsilon\cdot g_* A_\mu\bar\psi\gamma^\mu\psi + \frac{g_*^2}{m_*^2}(\bar\psi\gamma^\mu\psi)^2 + \dots$$

dominated by dim-6 at intermediate energy $\qquad \epsilon\cdot m_* < E < m_*$





$$\mathcal{M}(2\to 2) = \frac{g_{SM}^2}{E^2}\left(1 + \frac{1}{\epsilon^2}\frac{E^2}{m_*^2}\right)$$

Amplitude runs fast within the validity of EFT

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Examples

(3) $(\partial\pi)^2 - m_*^2\pi^2 + g_*^2\pi^4 + \dots$

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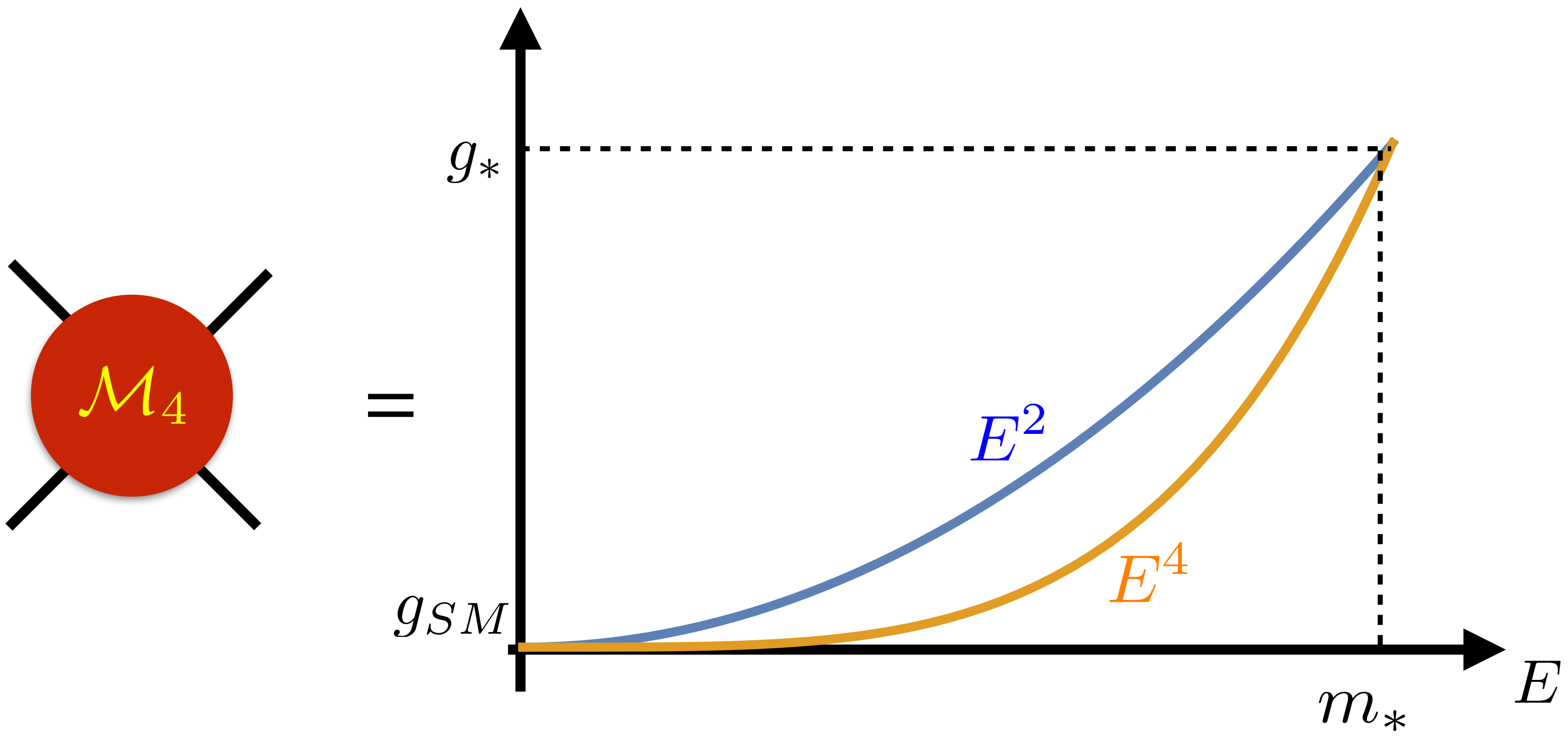
$$\epsilon \cdot m_* < E < m_*$$

Running Coupling



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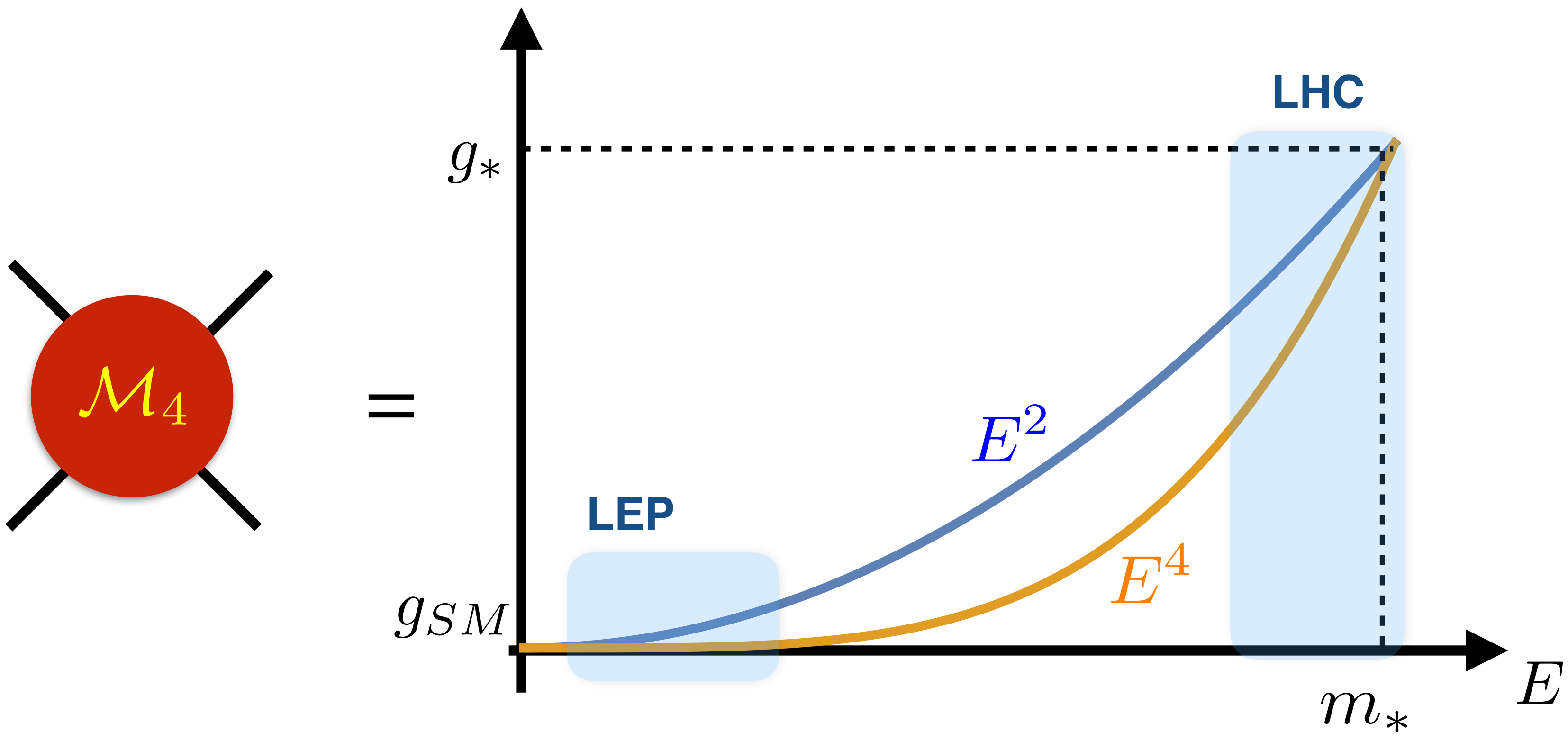

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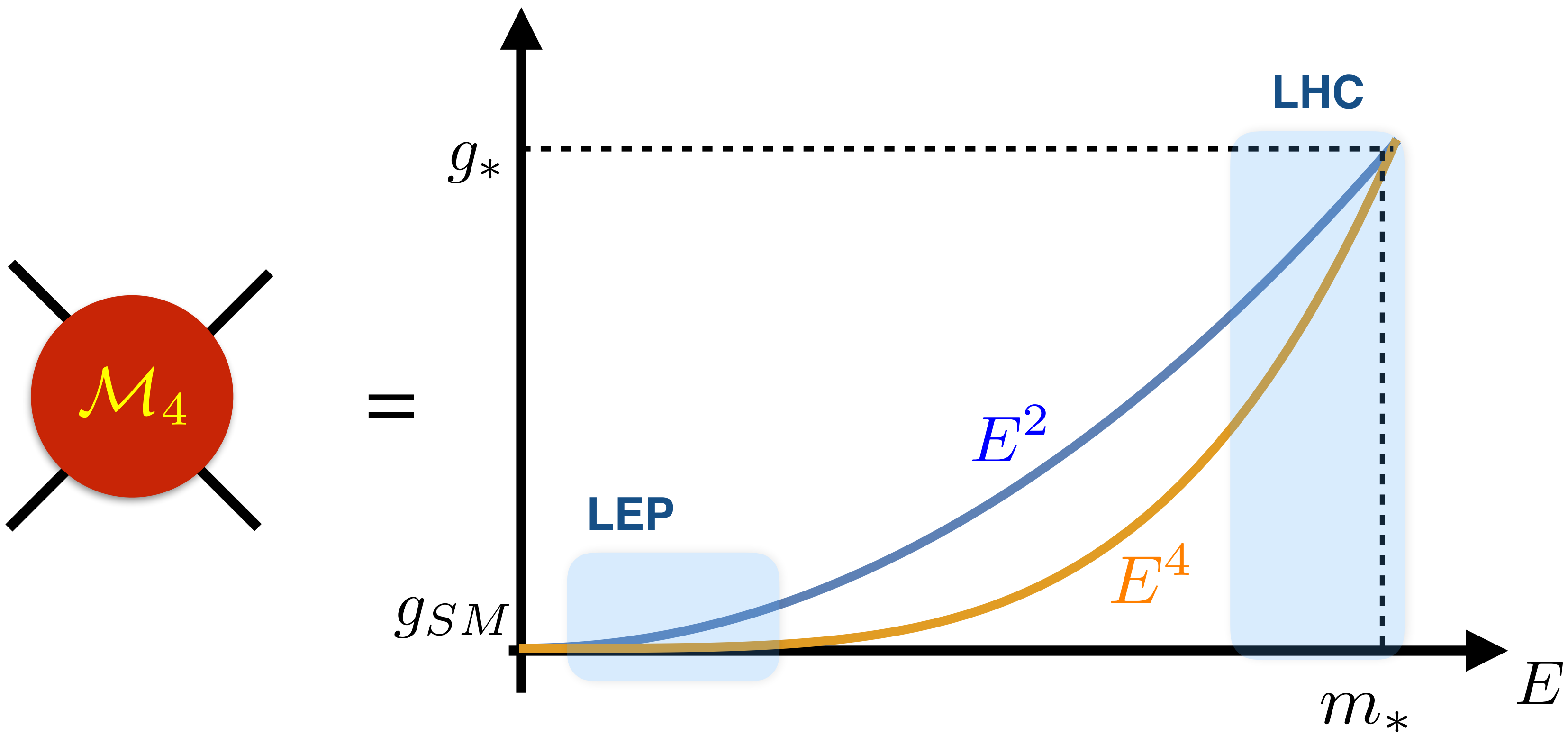
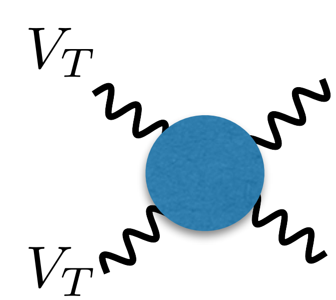


'remedios': strongly int. transv. vectors

1603.03064 Liu, Pomarol, Rattazzi, Riva

$\frac{g_*^2}{m_*^4} W_{\mu\nu}^4 \longrightarrow$



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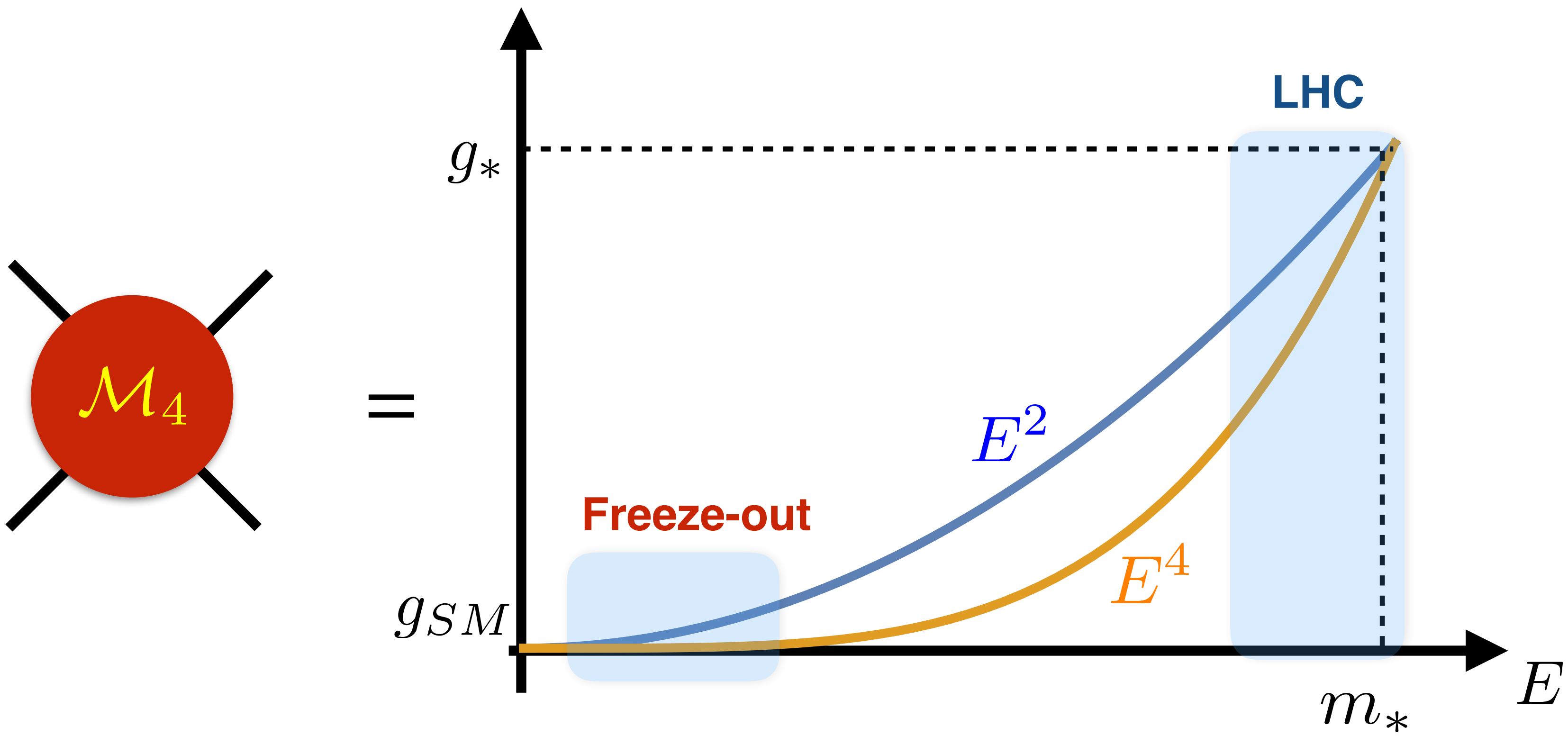

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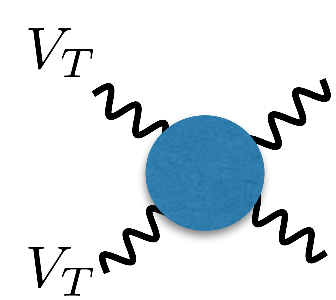

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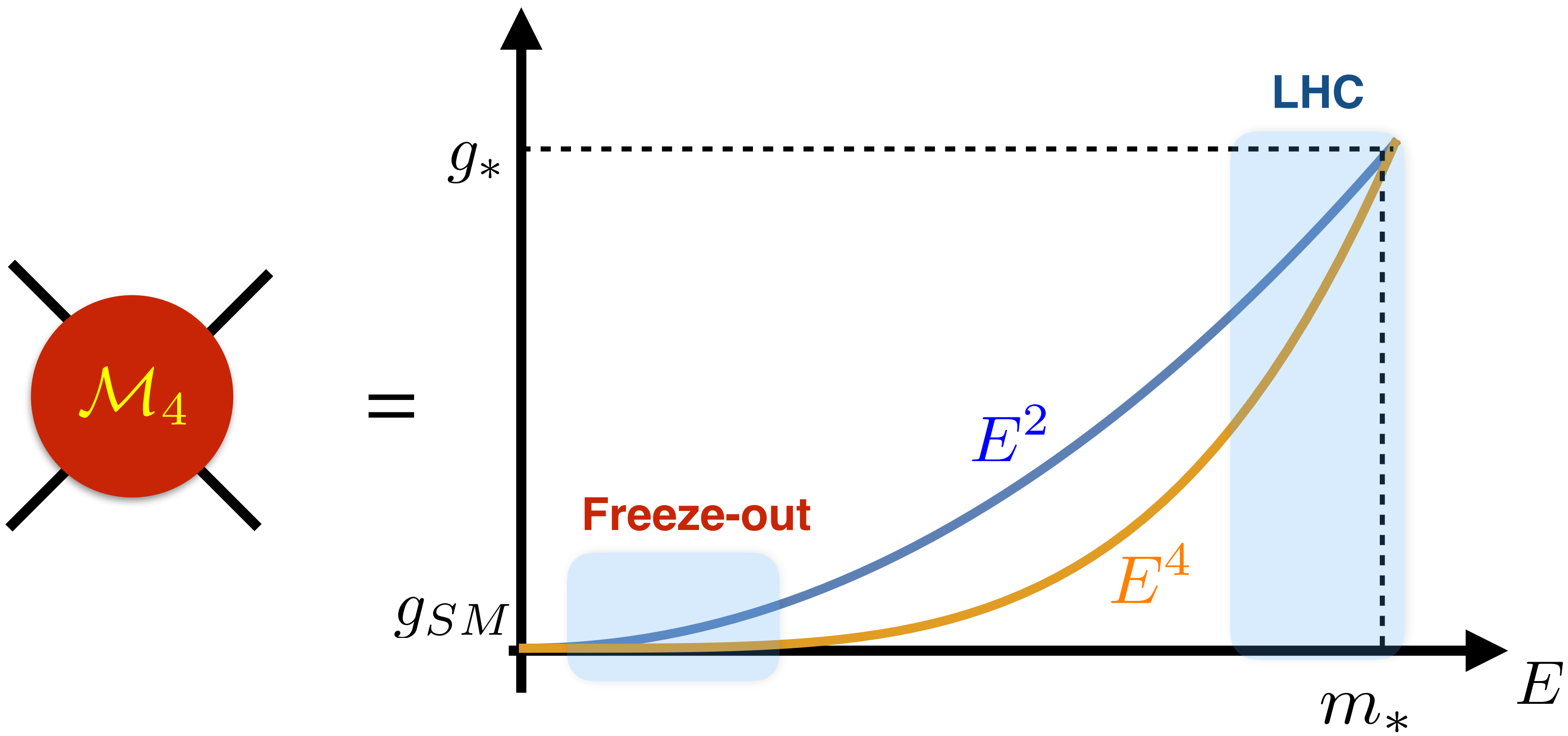

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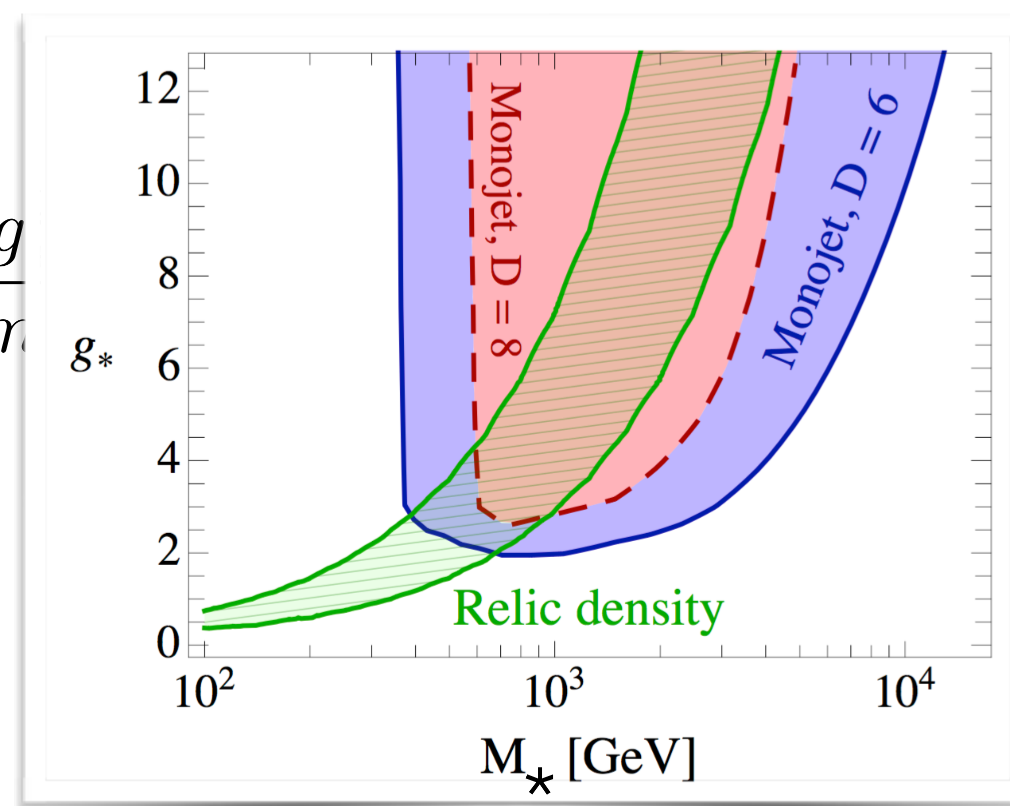

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DM as light pseudo-goldstino

1607.02474 Brugisser, Riva, Urbano

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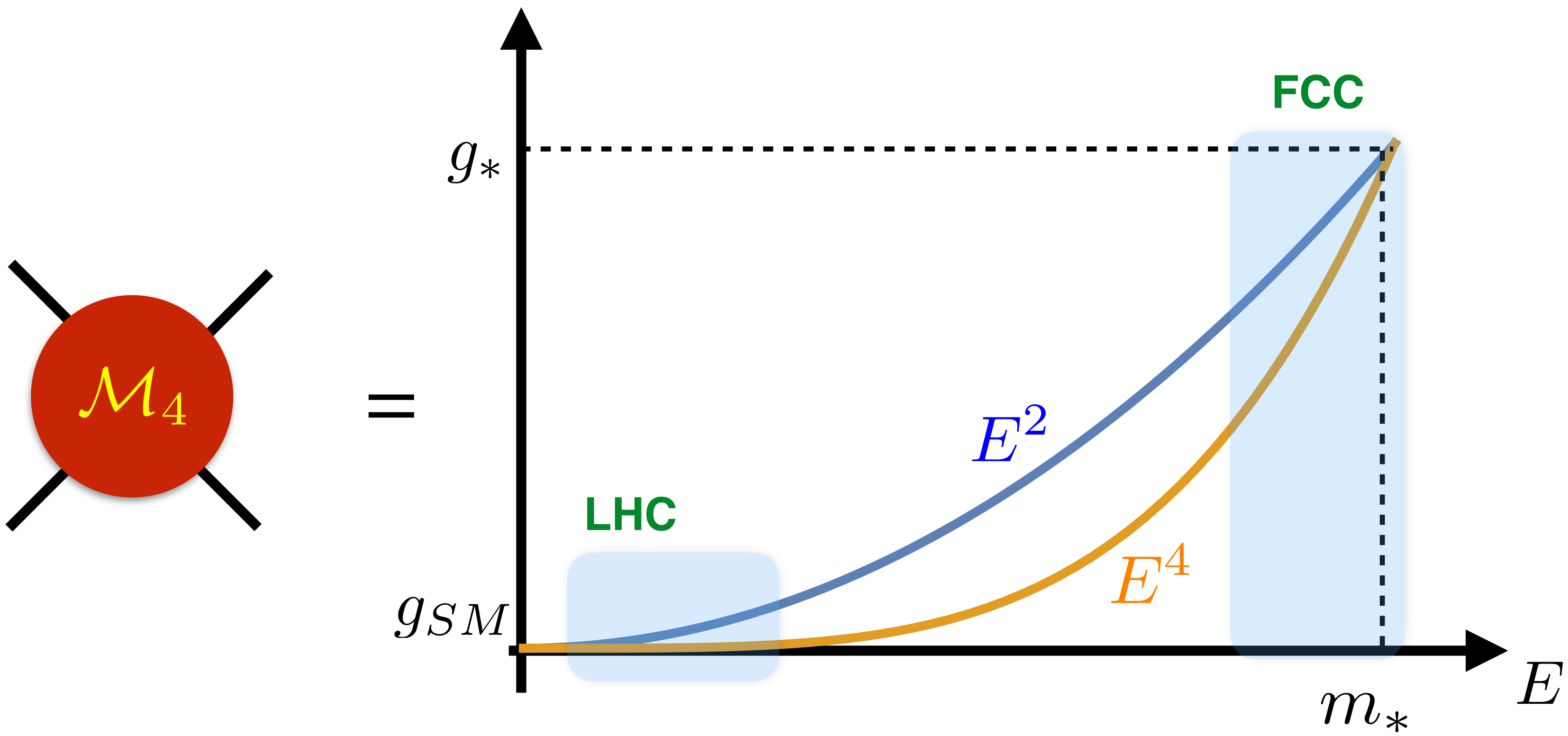

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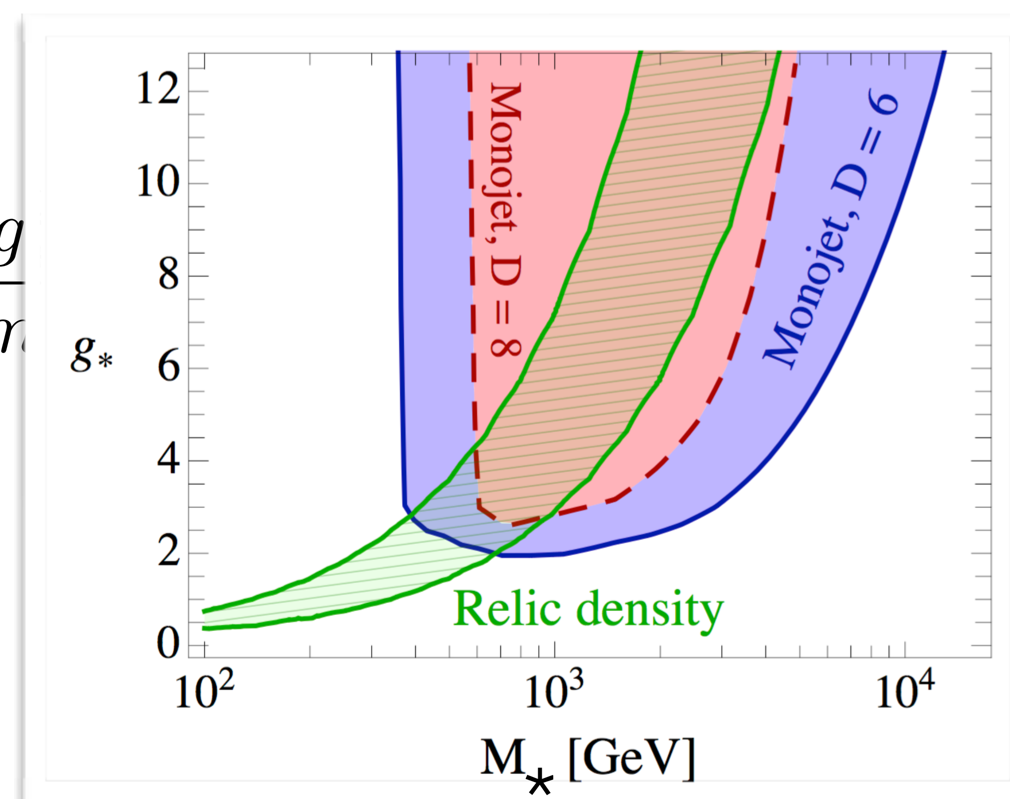

How fast?

Goldstones	$(\partial\pi)^2\pi^2$	$\sim E^2$
4-Fermions	$(\bar{\psi}\gamma^\mu\psi)^2$	
	$\ldots$	

can amplitudes be softer than E^4 ?

(within an EFT)

dilaton	$(\partial\sigma)^4$	$\sim E^4$
Goldstino	$\bar{\psi}^2\Box\psi^2$	
remedios	$F_{\mu\nu}^4$	
	$\ldots$	

UV-IR connection

(well known)

e.g. hep-th/0602178

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi

For spin-0 particles the answer is: No!

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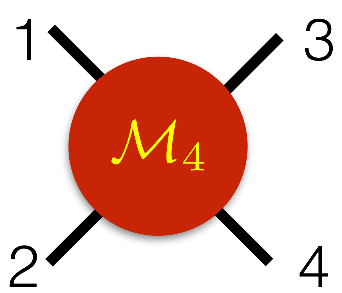

Analyticity, Crossing, and Unitarity

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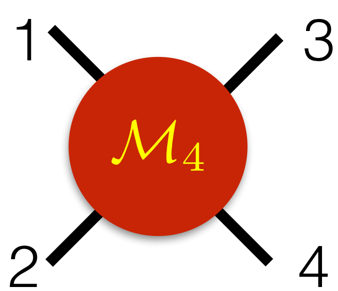

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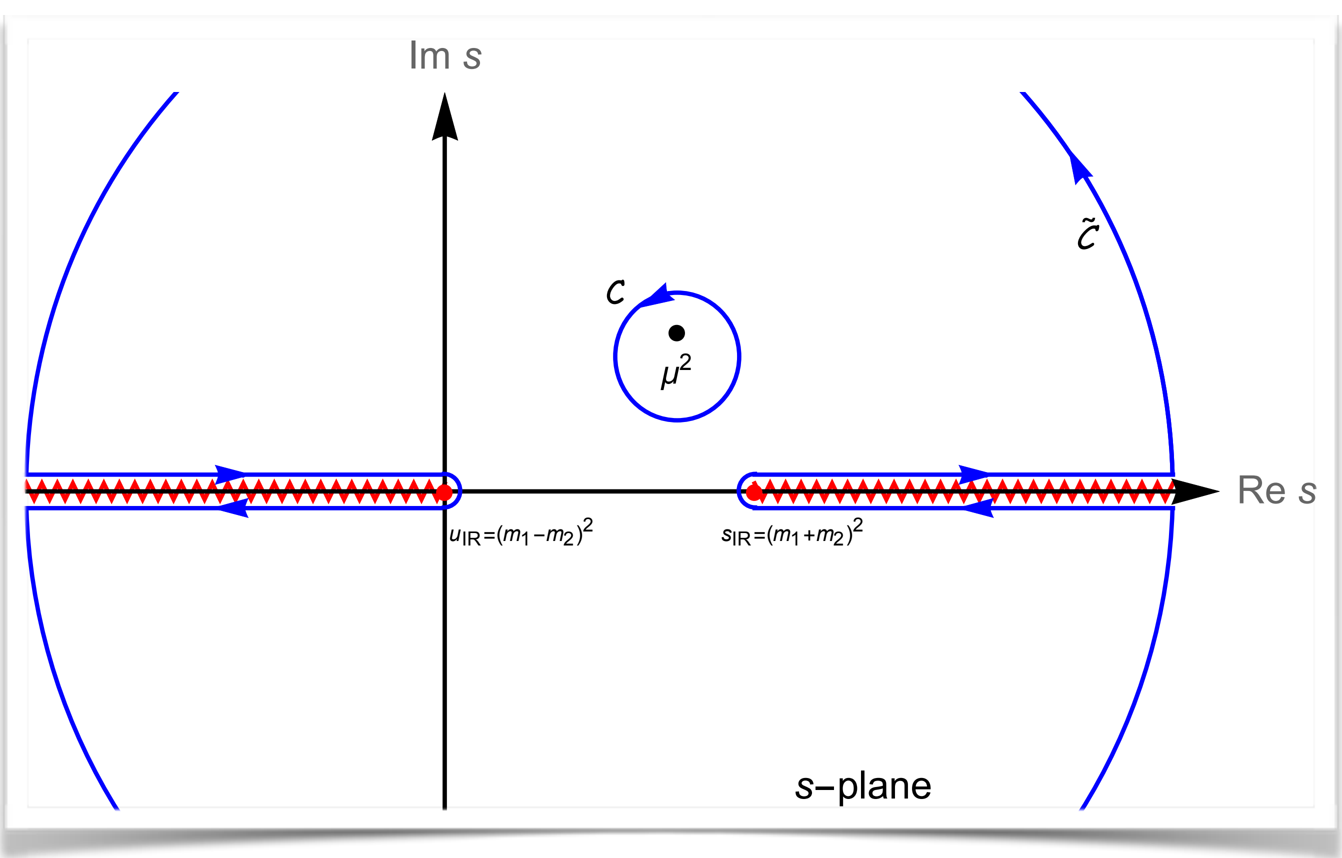

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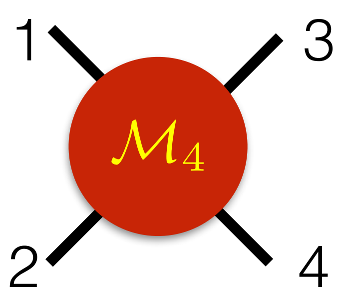

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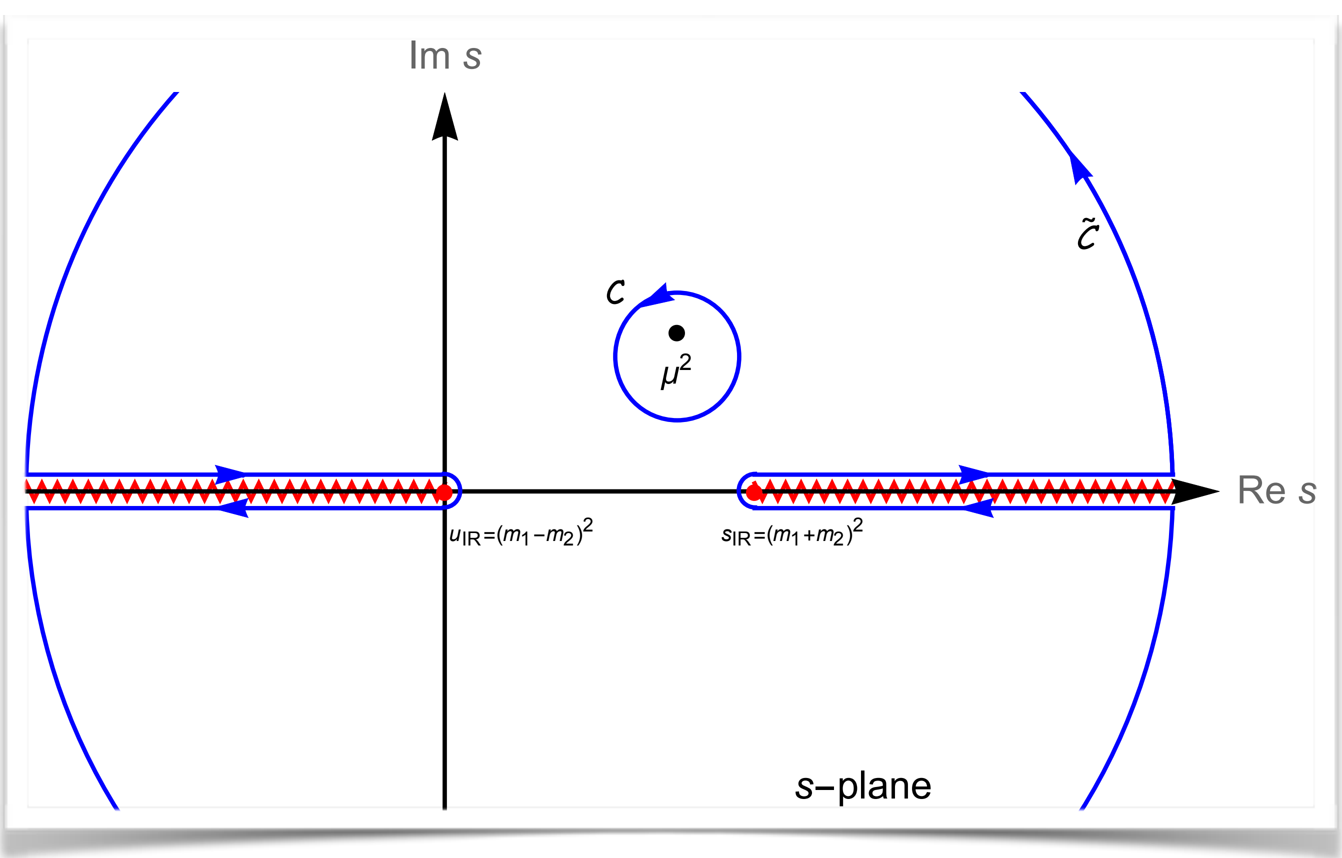
small circle=big circle



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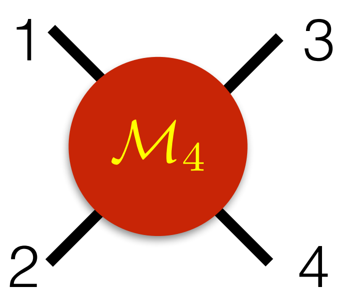

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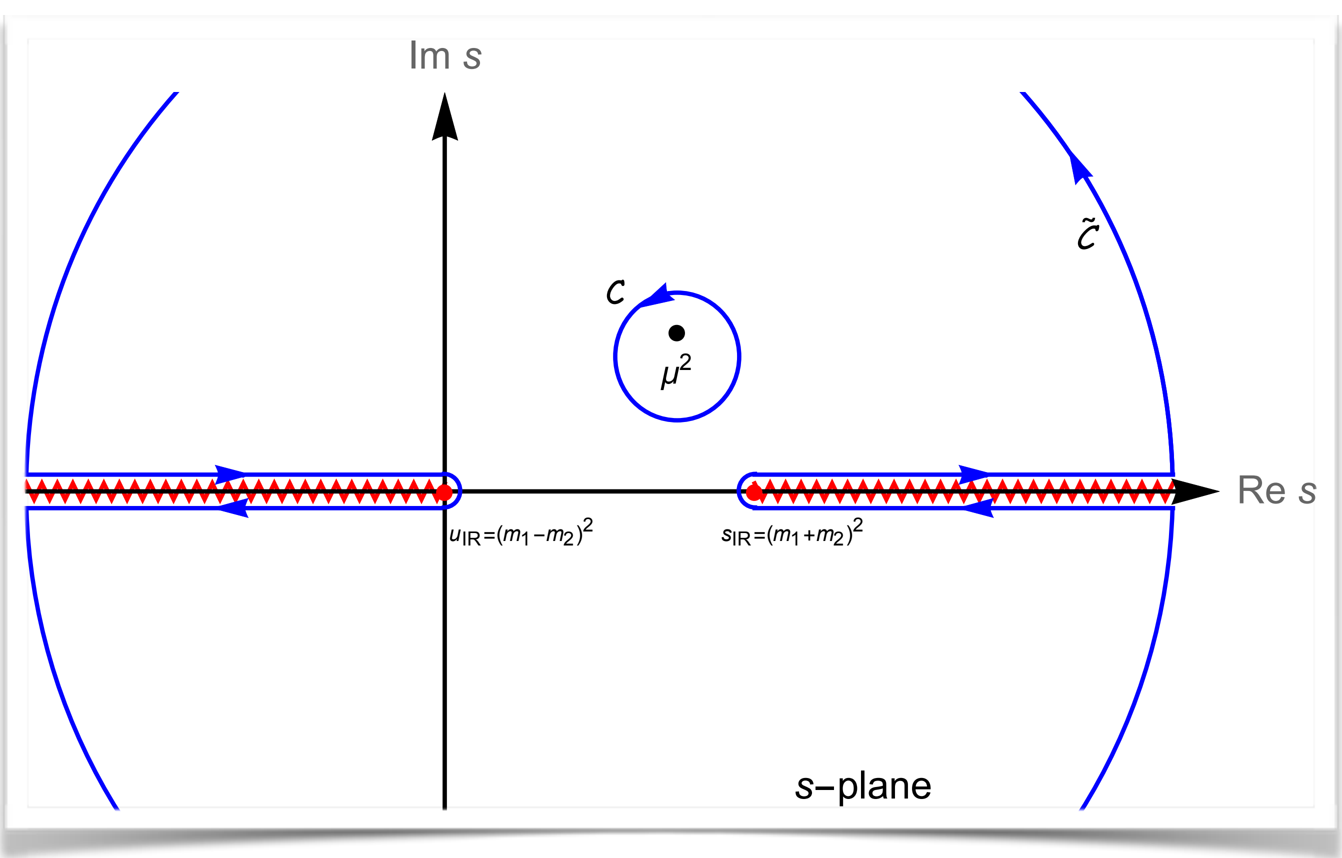
$$s \leftrightarrow u$$
$$Disc_s = Disc_u$$



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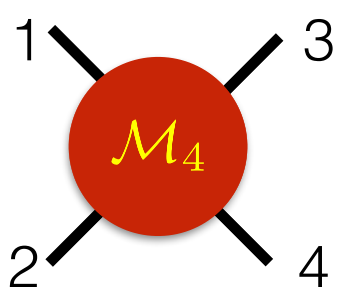

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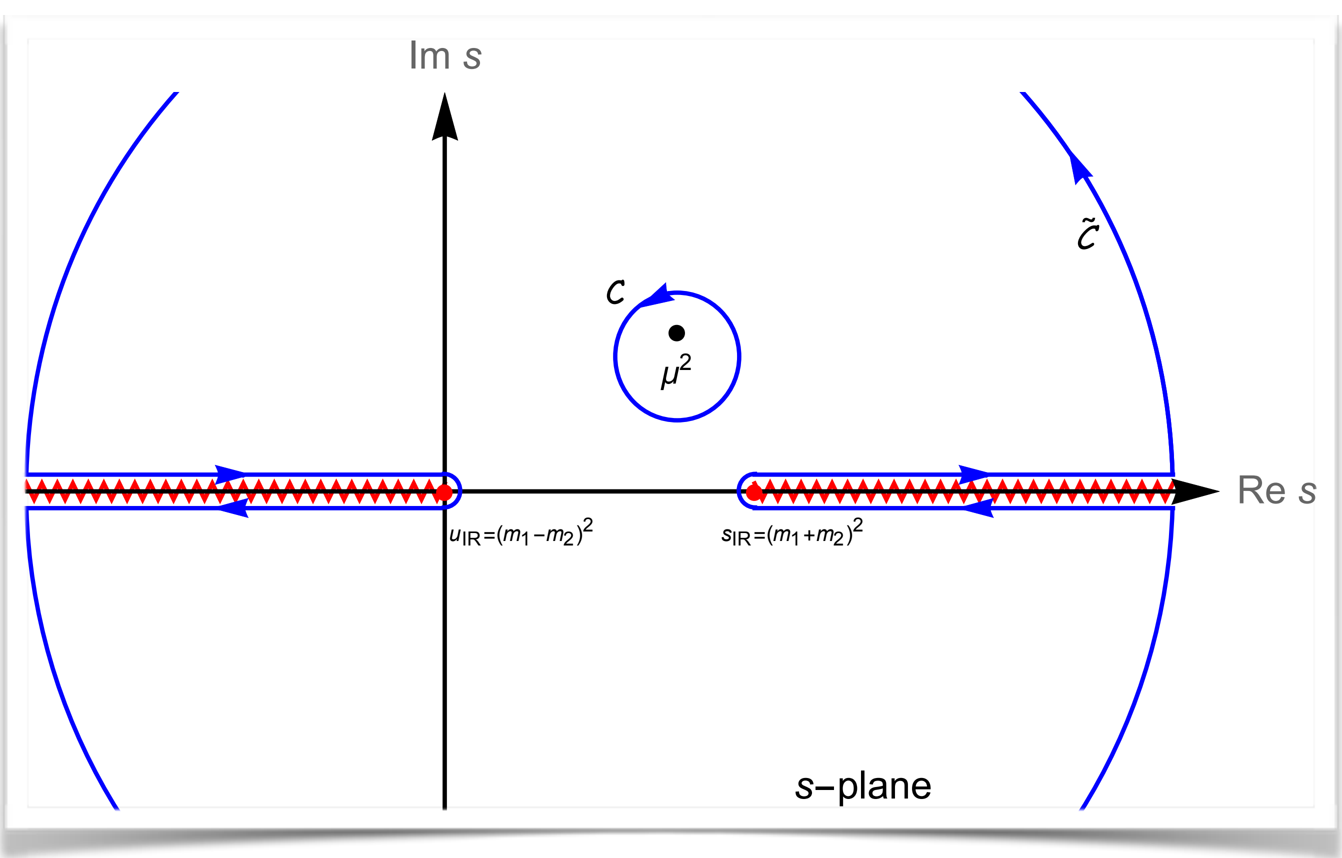
t=0



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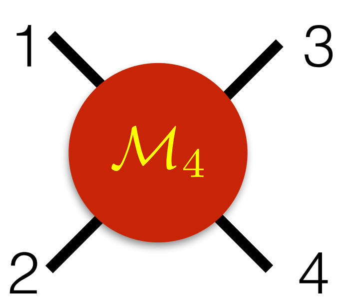

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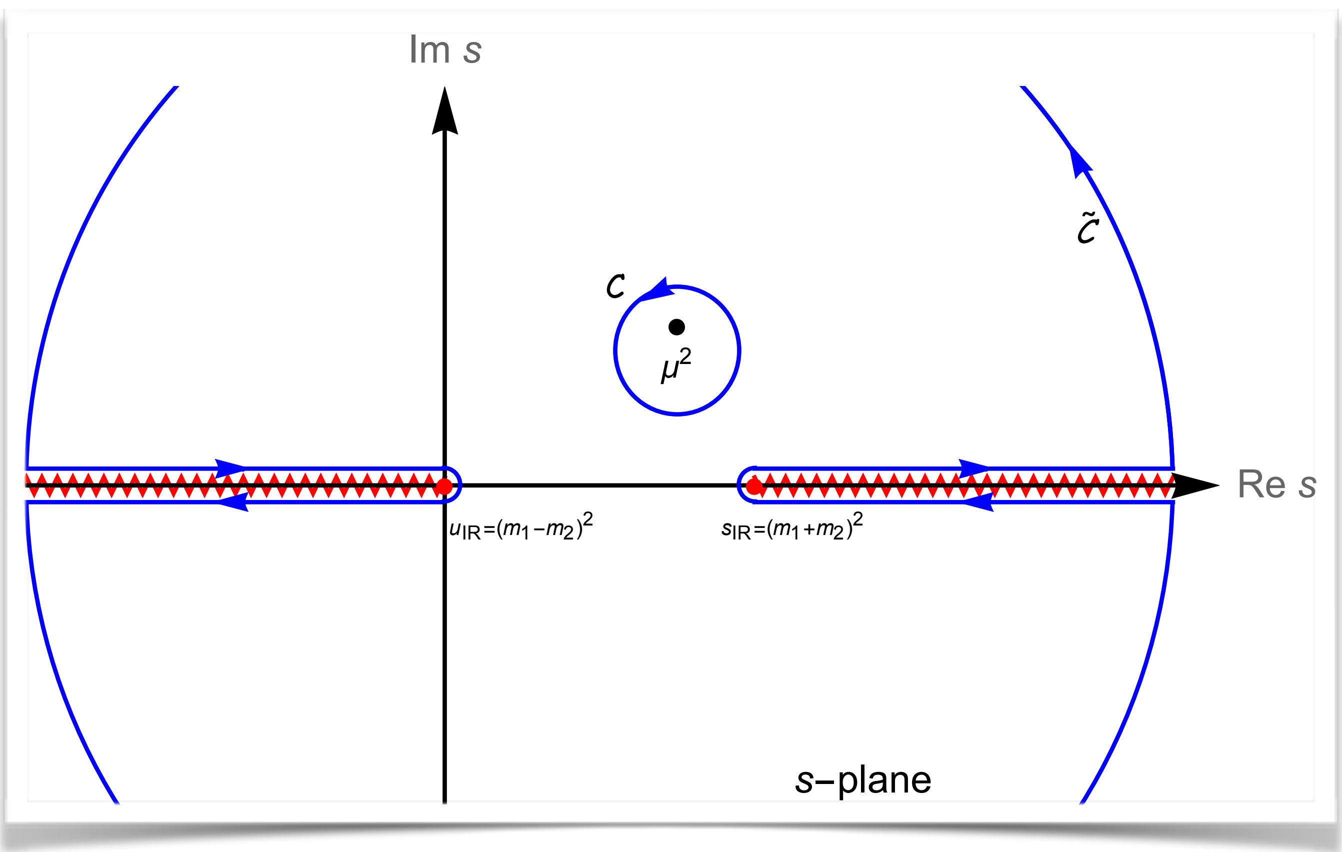

$$\mathcal{M}''(2 \to 2)\big|_{IR} = \int_0^\infty \frac{ds}{s^3}\sigma_{12\to\text{anything}}(s) > 0$$

IR-side   UV-side

E^4 -terms are strictly positive

Example

$$\pi \to \pi + \text{const}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \pi)^2 + \frac{c}{\Lambda^4}(\partial_\mu \pi)^4 + \ldots$$

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$\pi\pi \to \pi\pi$

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This interacting theory can't be softer than E^4

Spinning Particles?

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s, t, u
+**polarizations** $\begin{cases} \mathbf{1} \\ u_\alpha^\sigma \quad v_\alpha^\sigma \\ \varepsilon_\mu^\sigma \\ \dots \end{cases}$

(not amplitudes squared)

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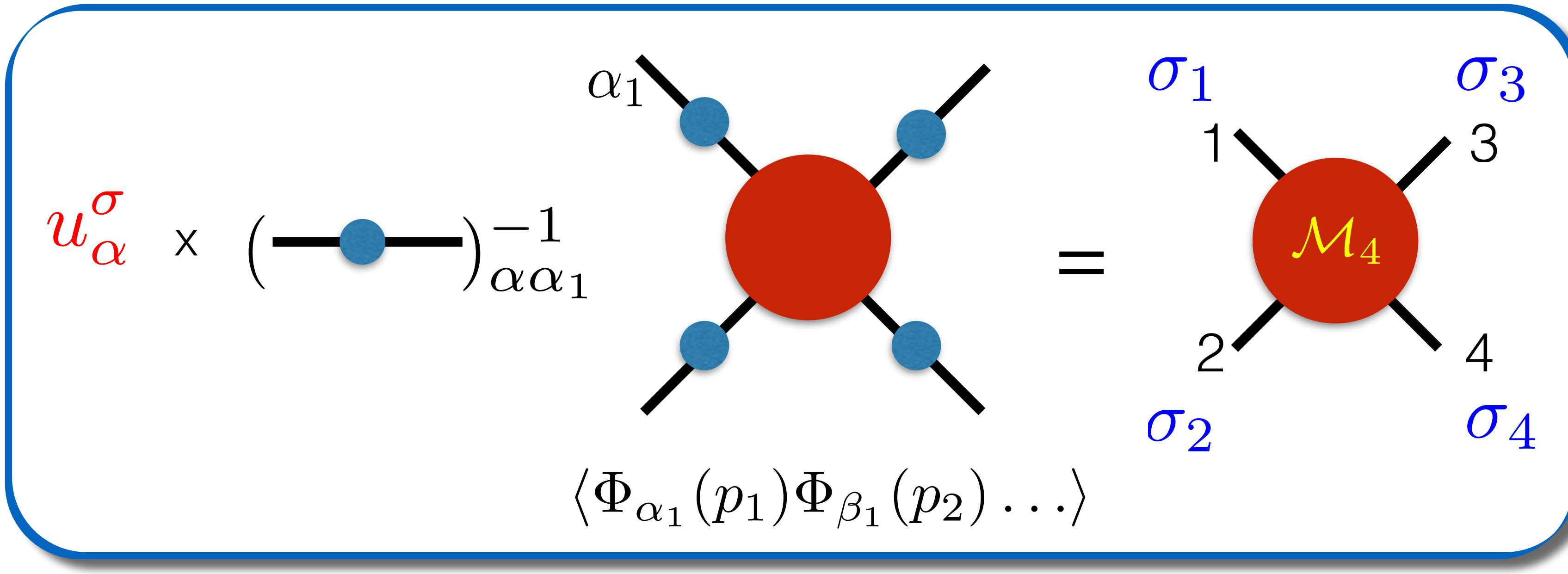

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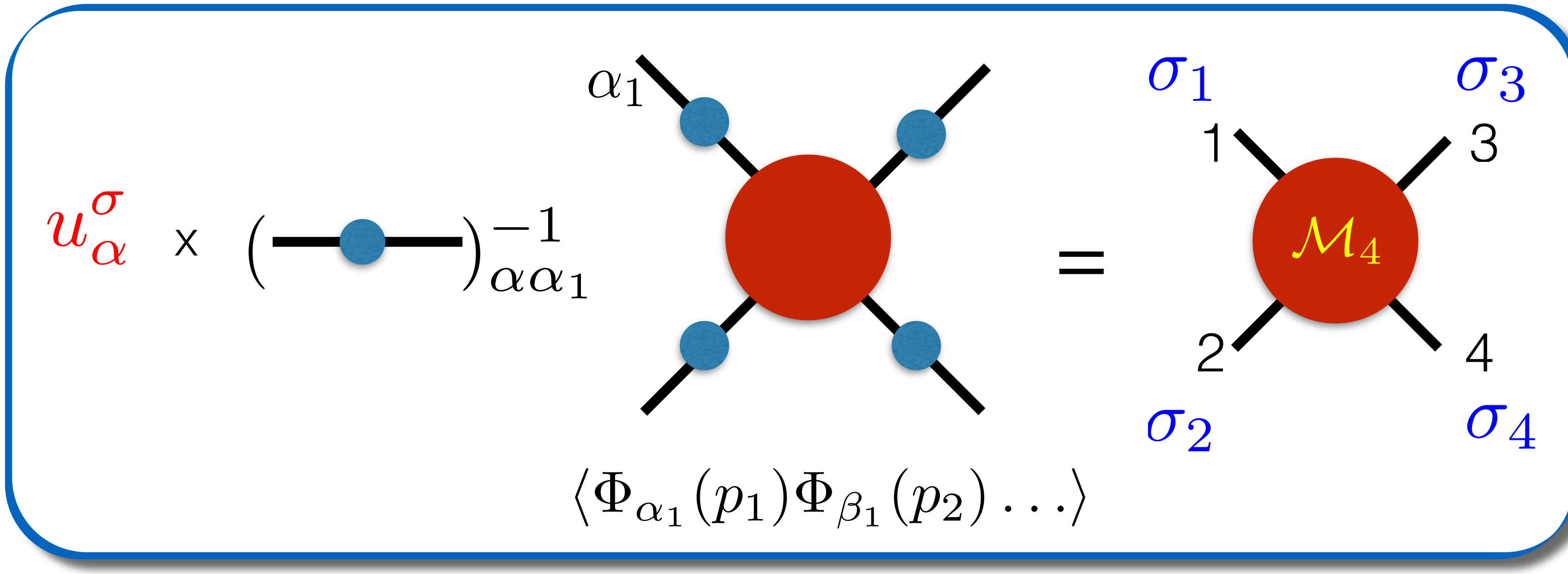

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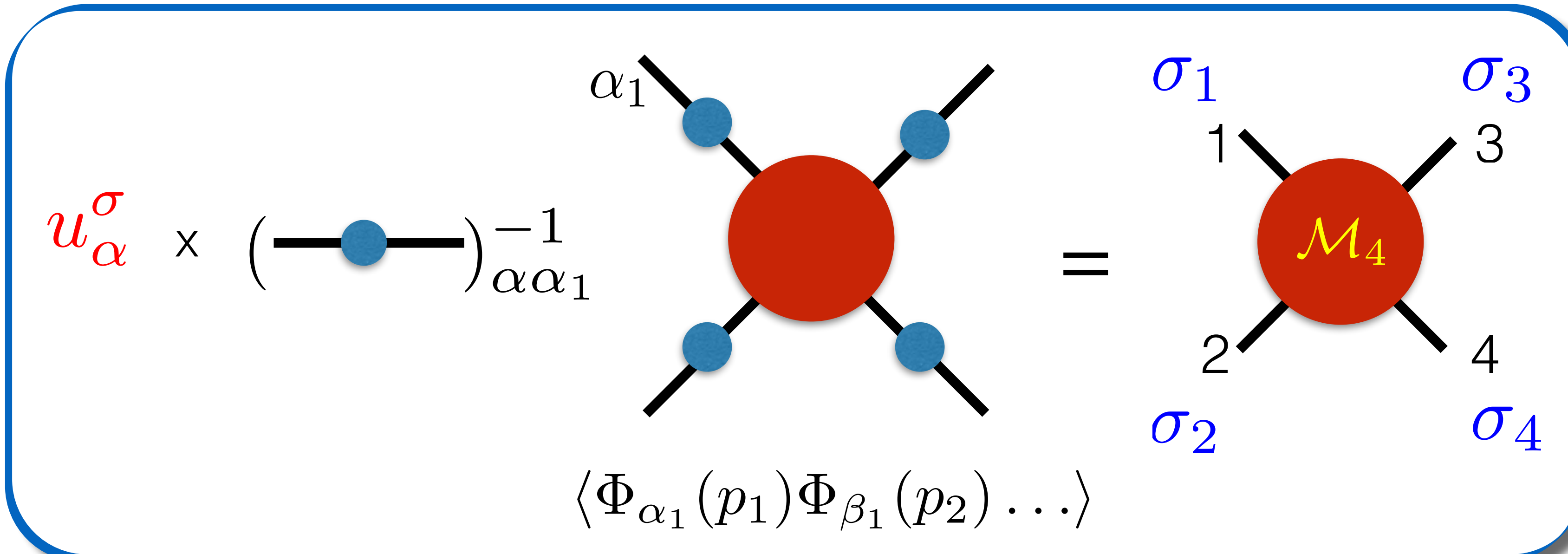

(2) polarizations carry non-analyticities

$$\left\{ \begin{array}{l} \epsilon^L_\mu(\mathbf{p}) \sim (p_z, 0, 0, \sqrt{p_z^2 + m^2})^T \\ u(\mathbf{p}) \sim \sqrt{p_\mu \sigma^\mu} \\ \ldots \end{array} \right.$$

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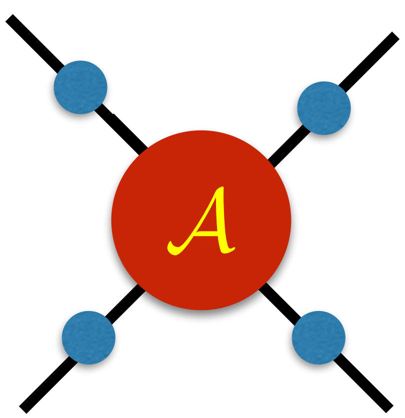

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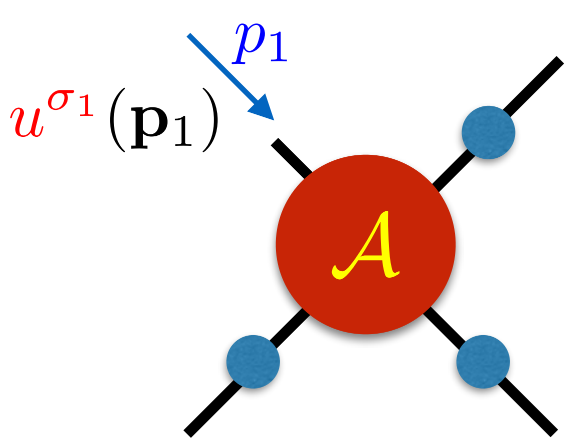

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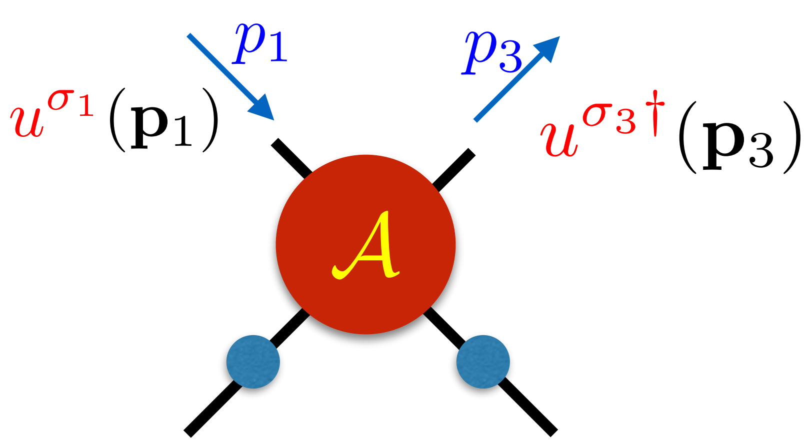

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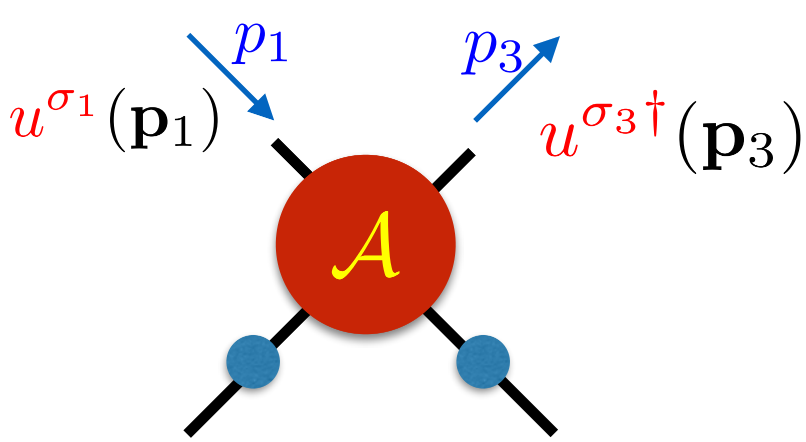

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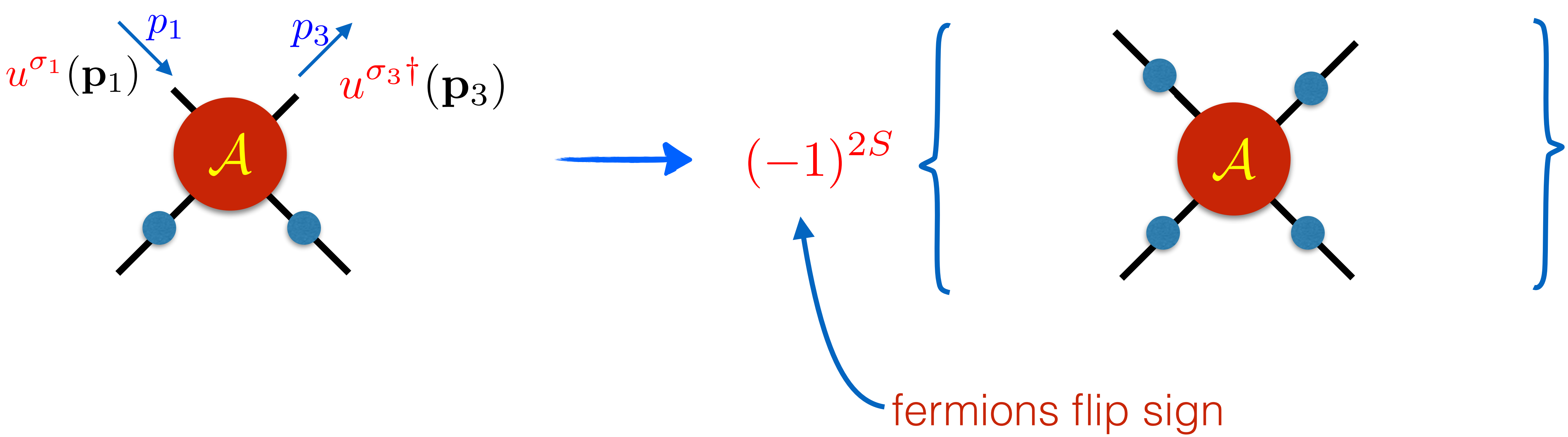

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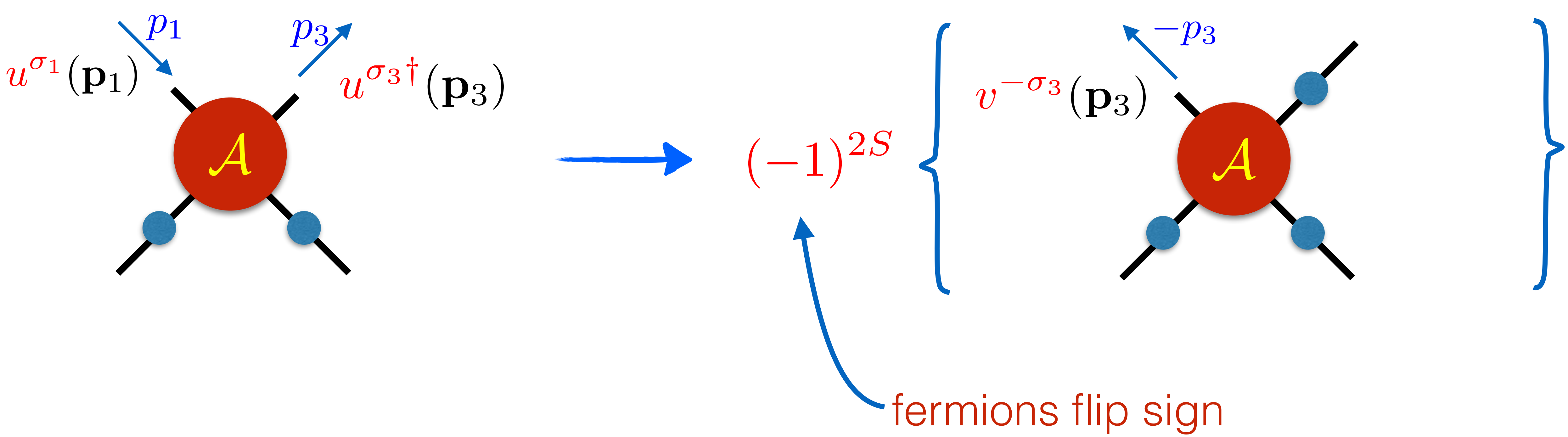

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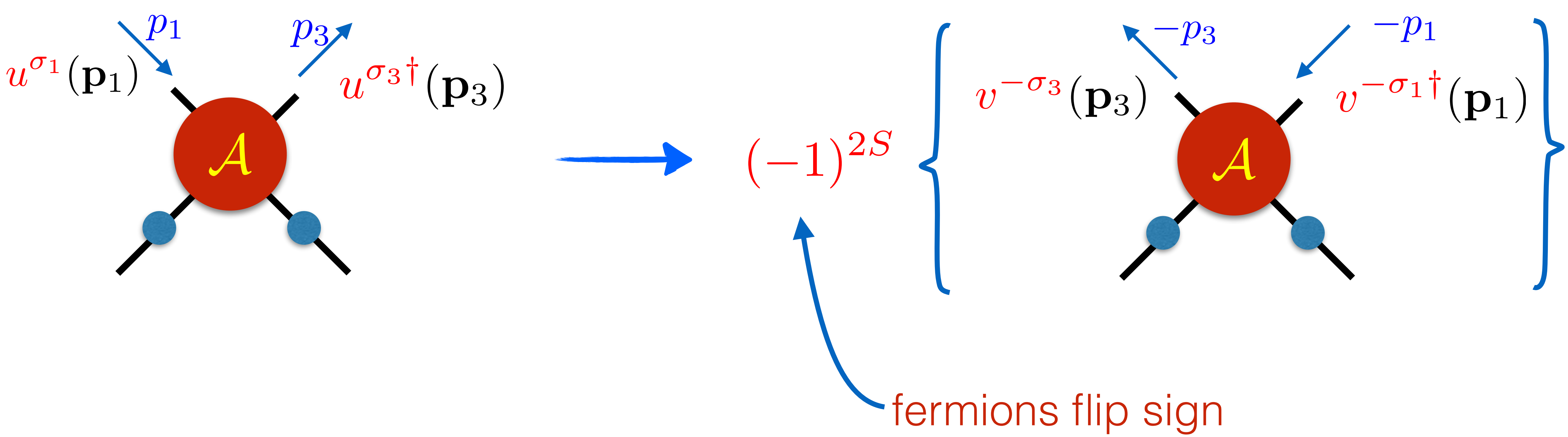

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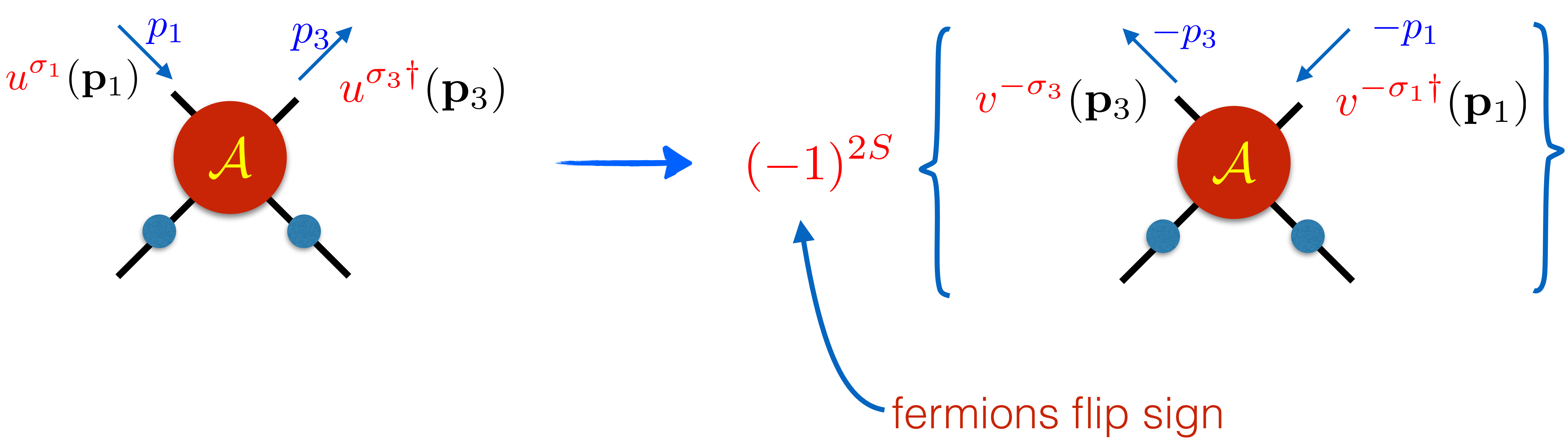

$$p_1 \leftrightarrow -p_3 \qquad\qquad u^{\sigma}(\mathbf{p}_1) \leftrightarrow v^{-\sigma}(\mathbf{p}_3)$$

Forward elastic scattering is special!

All previous issues cancel against each other out

Elastic and Forward

(1) **Lorentz Invariance**

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forward elastic amp. is invariant

$$\mathcal{M} = \mathcal{M}(s)$$

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$\longrightarrow$

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$$k_3^{\sigma_3} \to k_1^{\sigma_1} \qquad\qquad \mathcal{M}(k_1^{\sigma_1}\ldots \to k_1^{\sigma_1}\ldots) = \left[u_\alpha^{\sigma_1}(\mathbf{k}_1)u_\beta^{\sigma_1\,\dagger}(\mathbf{k}_1)\ldots\right]\mathcal{A}_{\alpha\beta\ldots}(k_1,\ldots)$$

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$\rho(-k_1)^{\sigma_1}$?

if so, crossing in elastic scattering **t=0** would be become again **s ⟷ u=-s**

$$\mathcal{M}_{\text{particles}}(s) = \mathcal{M}_{\text{antiparticles}}(u = -s)$$

Elastic and Forward

(4) **Locality**

analytically continue density matrices off-shell

$$u^\sigma(\mathbf{k})u^{\sigma\,\dagger}(\mathbf{k}) \equiv \rho(\mathbf{k}) \quad \longrightarrow \quad \rho^\sigma(\mathbf{k}) \to \rho^\sigma(k)$$

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$$\langle T\psi(x_1)\psi^\dagger(x_2)\rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ikx_{12}} \frac{\rho^{\sigma}(k)}{k^2 - i\epsilon}$$

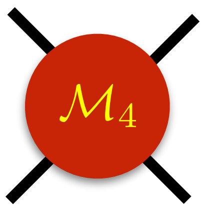
propagator's numerator:
its parity fixed by Spin-Statistics

Bound on Softness

can amplitudes be **softer** than E^4 ?

(within an EFT)

Bound on Softness



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Universal statement irrespectively spins

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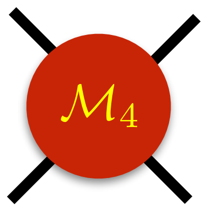

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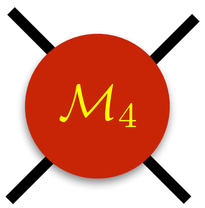

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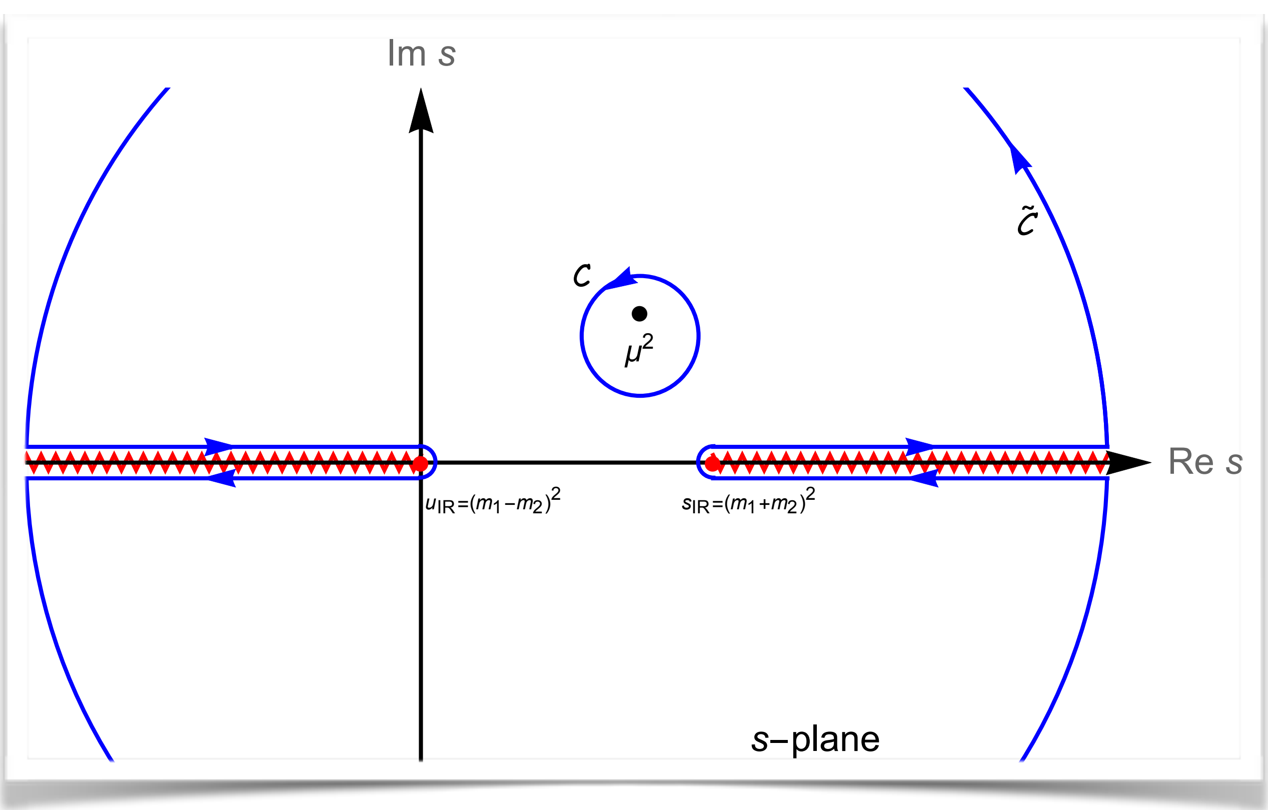

$$\mathcal{M}''(2\to 2)\big|_{IR} = \int_0^\infty \frac{ds}{s^3}\sigma_{12\to\text{anything}}(s) > 0$$

IR-side UV-side

E^4 -terms are strictly positive

Examples

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doesn't admit a local unitary UV completion

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up to field red. →

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→

$$a_\psi > 0 \qquad a_A > 0 \qquad a_\pi > 0$$

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constrained superfields Komargodski Seiberg 0907.2441

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$$w_R < \frac{1}{2} f_a F$$

$$\longrightarrow \quad \Gamma(a \to GG) < \frac{1}{32\pi} \left(\frac{m_a^5}{F^2} \right)$$

Examples

(4) SM fermions as pseudo-Goldstini

(revive old idea Bardeen and Visnjic NPB 1982)

w/ F. Riva, J. Serra and F. Sgarlata

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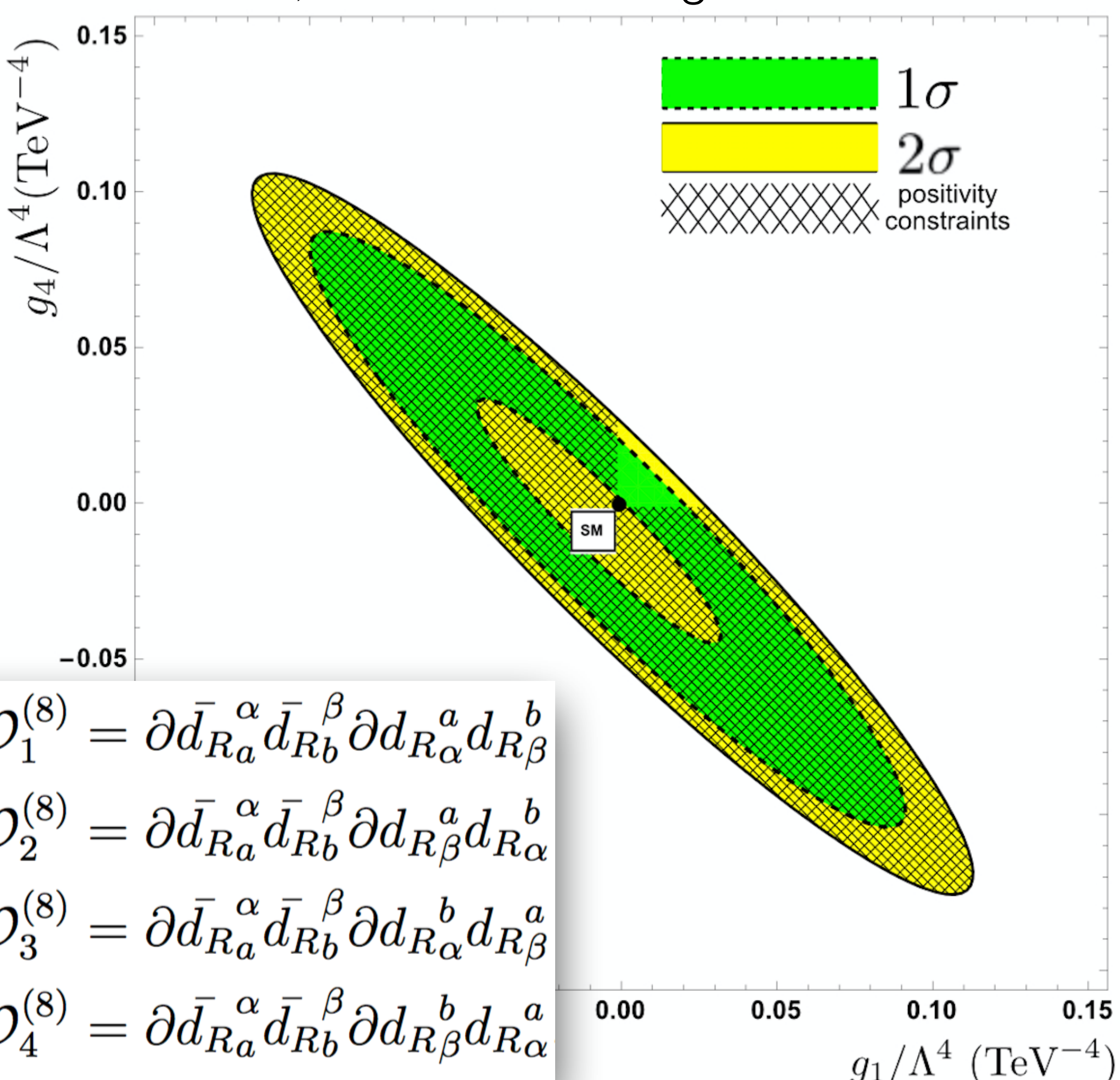

$$\mathcal{O}_1^{(8)} = \partial \bar{d}_{R_a}^{\ \alpha} \bar{d}_{R_b}^{\ \beta} \partial d_{R_\alpha}^{\ a} d_{R_\beta}^{\ b}$$
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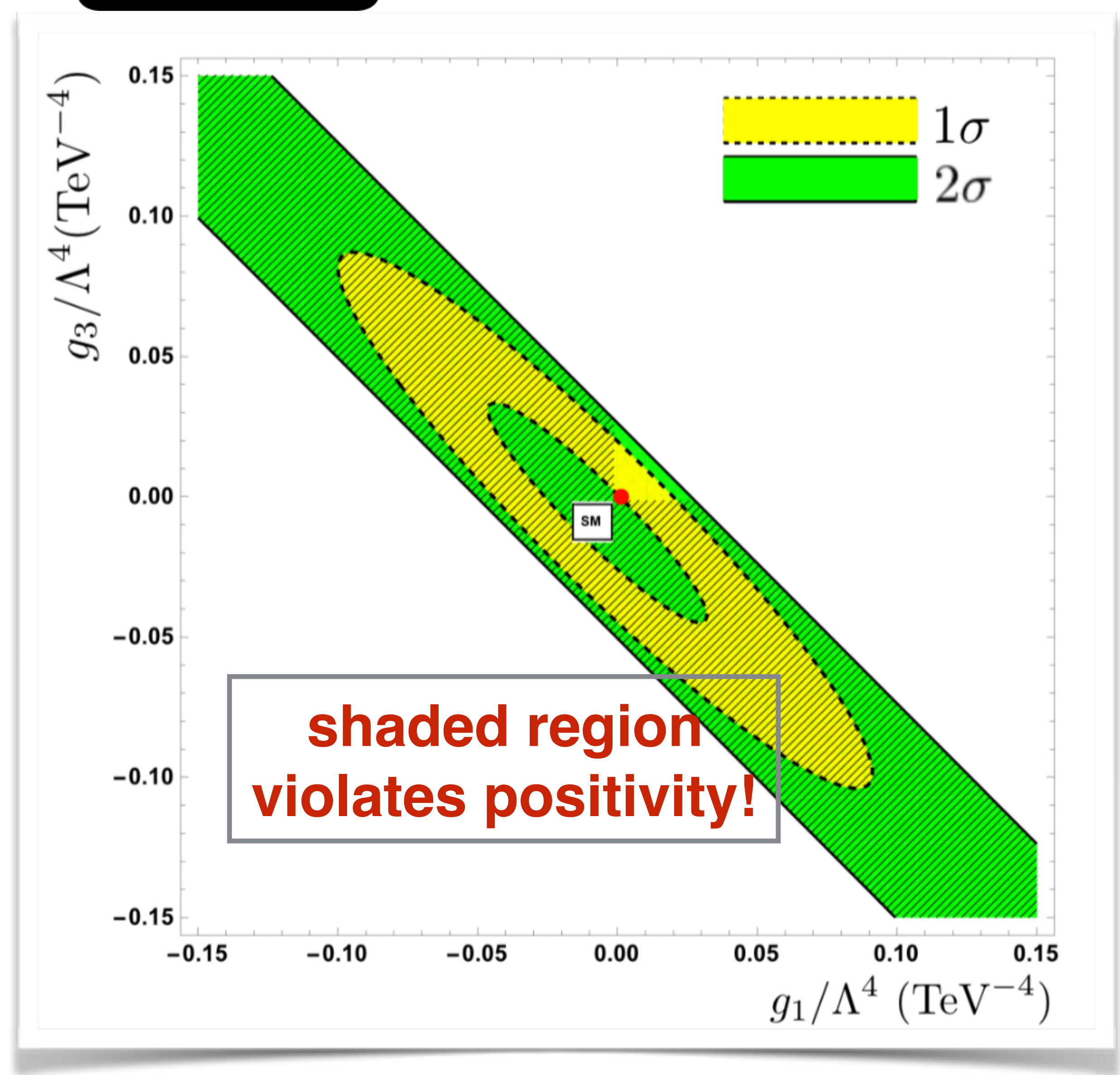
$$\mathcal{O}_4^{(8)} = \partial \bar{d}_{R_a}^{\ \alpha} \bar{d}_{R_b}^{\ \beta} \partial d_{R_\beta}^{\ b} d_{R_\alpha}^{\ a}$$

for illustration only!
pedestrian bounds



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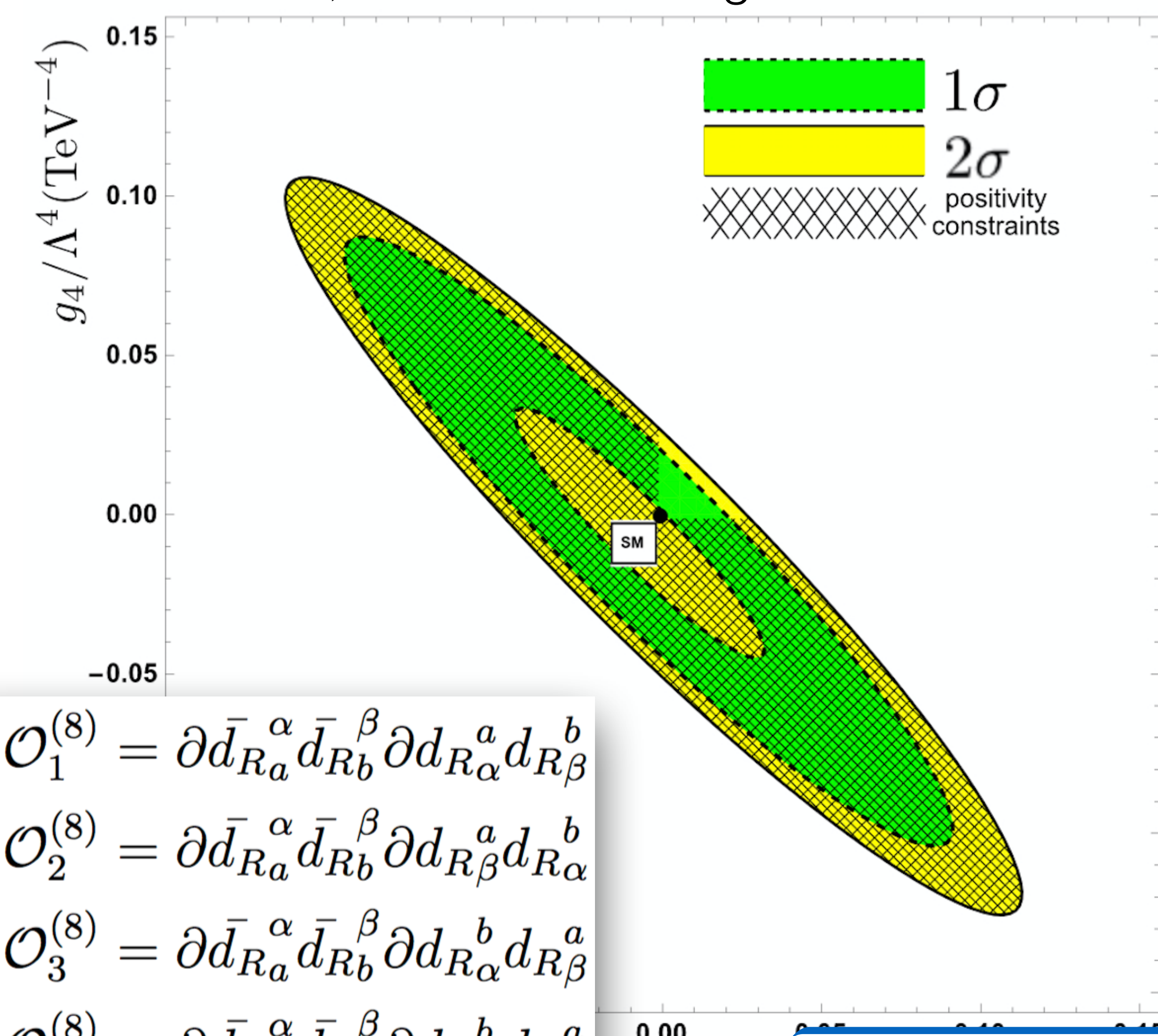
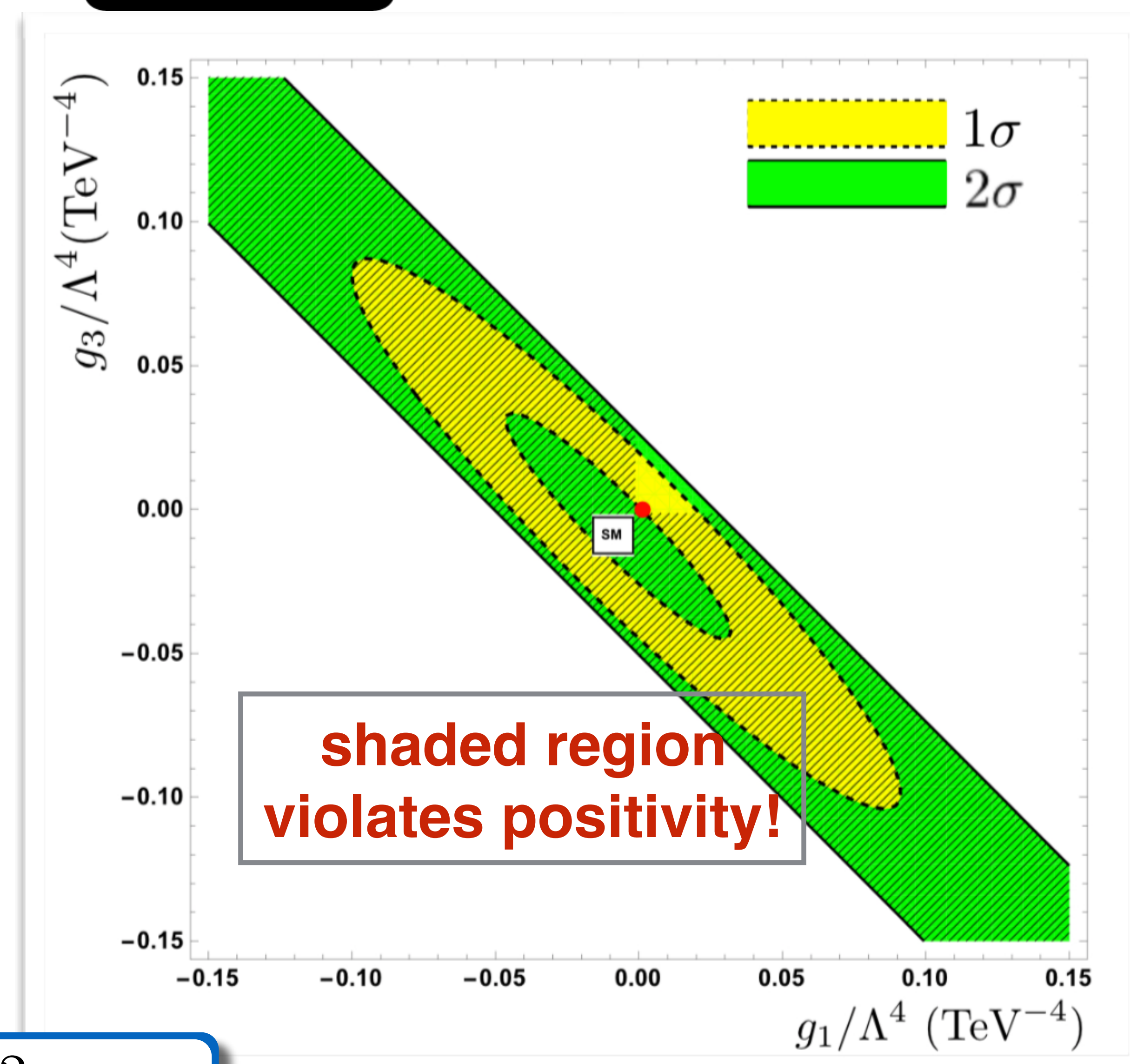


$$\mathcal{O}_1^{(8)} = \partial \bar{d}_{Ra}^{\ \alpha} \bar{d}_{Rb}^{\ \beta} \partial d_{R\alpha}^{\ a} d_{R\beta}^{\ b}$$

$$\mathcal{O}_2^{(8)} = \partial \bar{d}_{Ra}^{\ \alpha} \bar{d}_{Rb}^{\ \beta} \partial d_{R\beta}^{\ a} d_{R\alpha}^{\ b}$$

$$\mathcal{O}_3^{(8)} = \partial \bar{d}_{Ra}^{\ \alpha} \bar{d}_{Rb}^{\ \beta} \partial d_{R\alpha}^{\ b} d_{R\beta}^{\ a}$$

$$\mathcal{O}_4^{(8)} = \partial \bar{d}_{Ra}^{\ \alpha} \bar{d}_{Rb}^{\ \beta} \partial d_{R\beta}^{\ b} d_{R\alpha}^{\ a}$$



$$\Lambda \gtrsim 9 \left(\frac{g_*}{4\pi}\right)^{1/2} \mathrm{TeV}$$

for $g_* > 3$

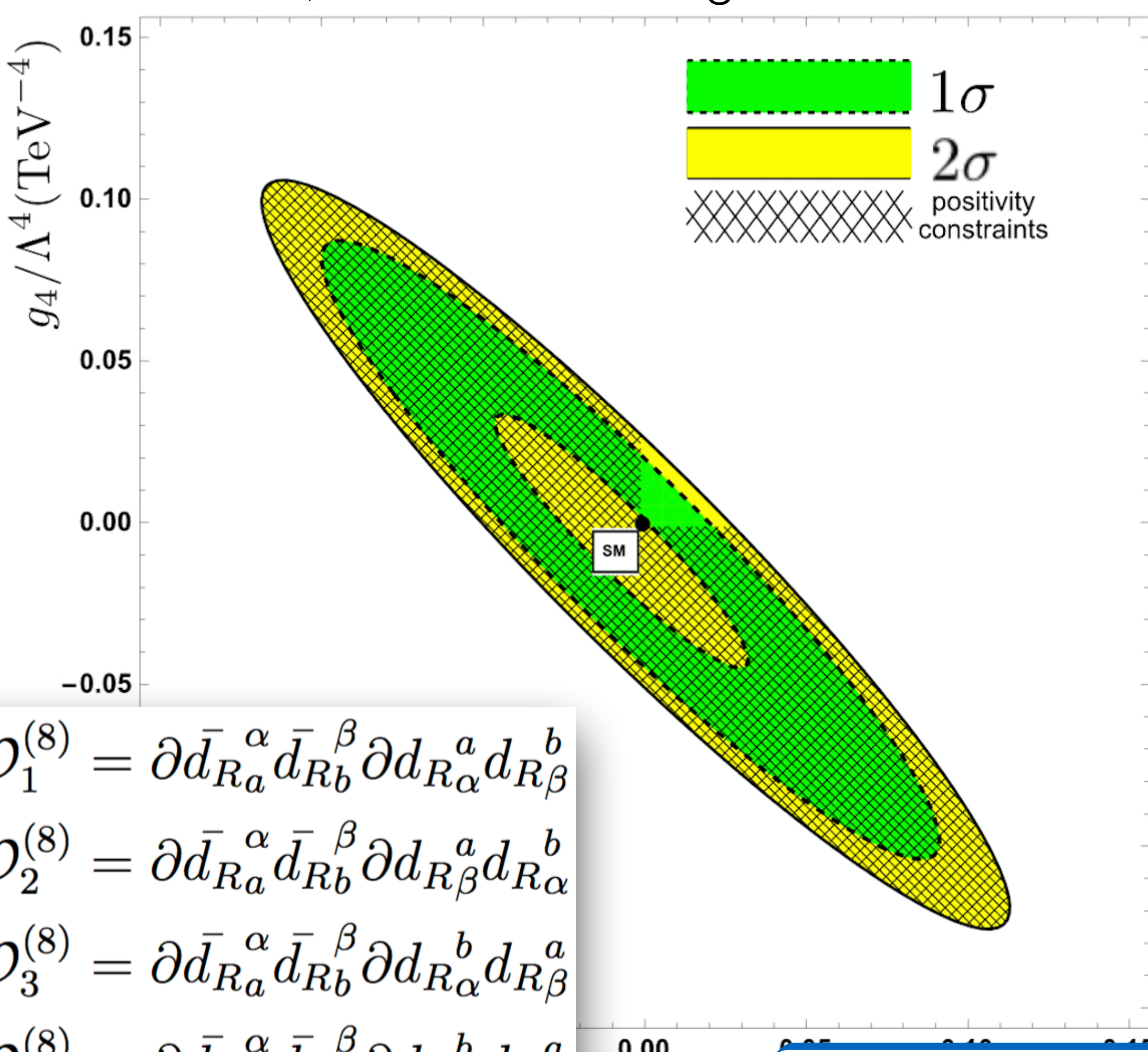
Examples

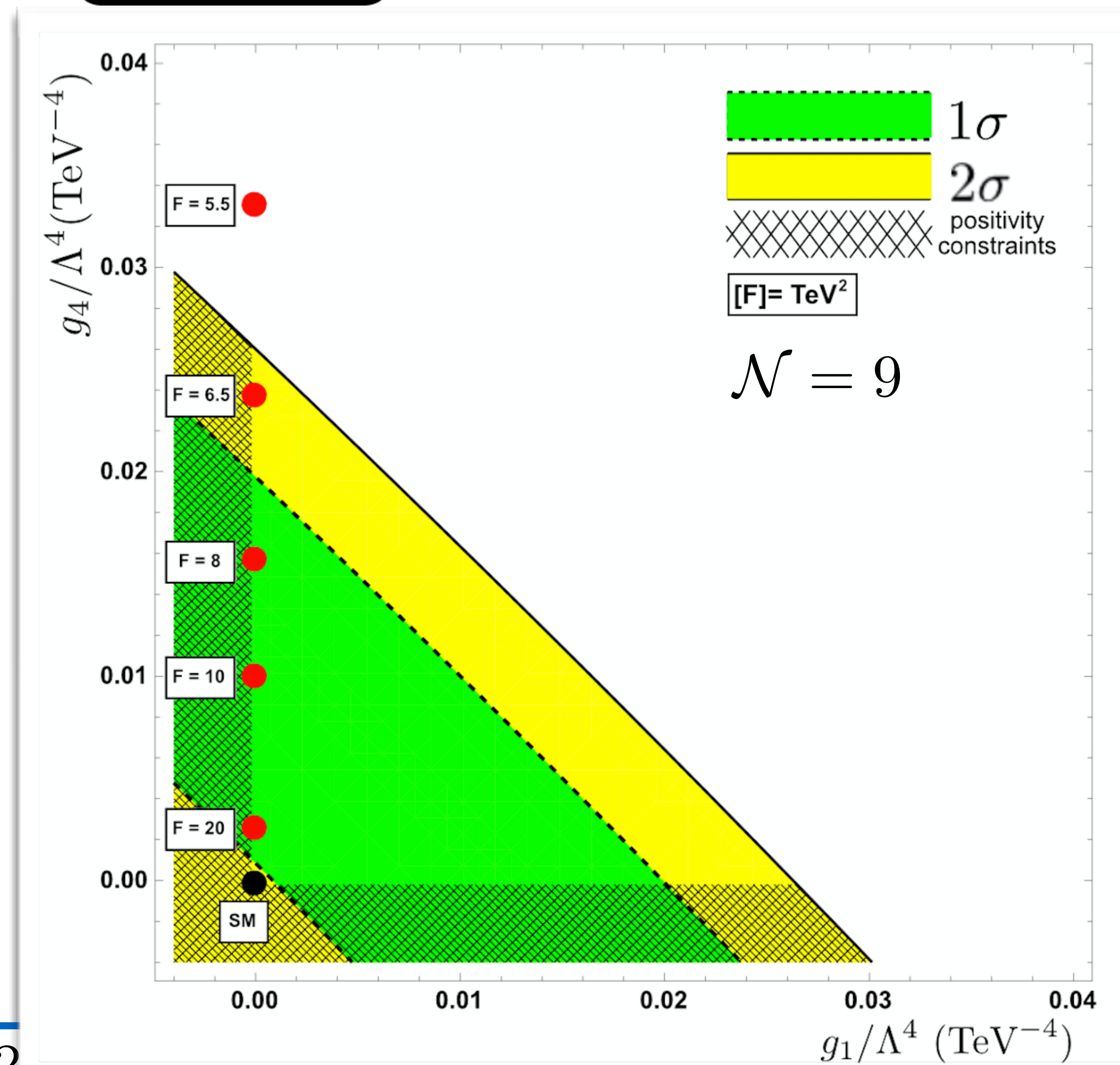
(4) SM fermions as pseudo-Goldstini

(revive old idea Bardeen and Visnjic NPB 1982)

w/ F. Riva, J. Serra and F. Sgarlata

for illustration only!
pedestrian bounds





$$\mathcal{O}_1^{(8)} = \partial \bar{d}_{Ra}^{\ \alpha} \bar{d}_{Rb}^{\ \beta} \partial d_{R\alpha}^{a} d_{R\beta}^{b}$$
$$\mathcal{O}_2^{(8)} = \partial \bar{d}_{Ra}^{\ \alpha} \bar{d}_{Rb}^{\ \beta} \partial d_{R\beta}^{a} d_{R\alpha}^{b}$$
$$\mathcal{O}_3^{(8)} = \partial \bar{d}_{Ra}^{\ \alpha} \bar{d}_{Rb}^{\ \beta} \partial d_{R\alpha}^{b} d_{R\beta}^{a}$$
$$\mathcal{O}_4^{(8)} = \partial \bar{d}_{Ra}^{\ \alpha} \bar{d}_{Rb}^{\ \beta} \partial d_{R\beta}^{b} d_{R\alpha}^{a}$$

$$\Lambda \gtrsim 9 \left(\frac{g_*}{4\pi}\right)^{1/2} \mathrm{TeV}$$

for $g_* > 3$

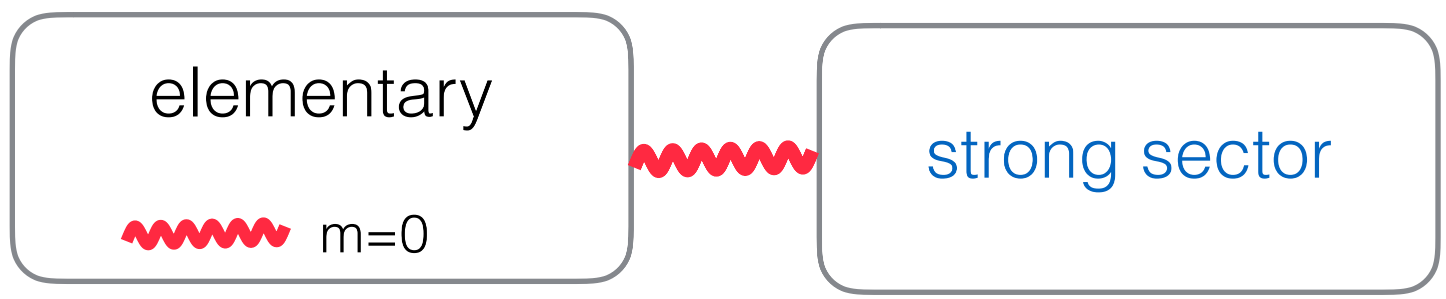
Conclusions

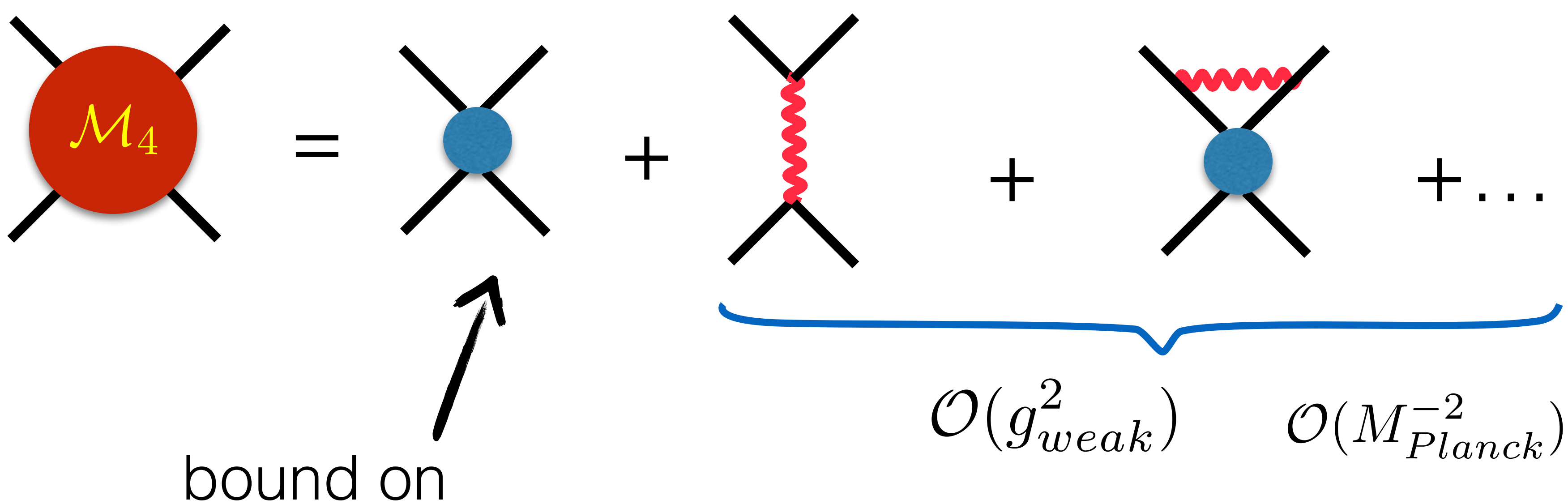
- *Not all EFTs are created equal:*
  some live in the swampland and can't be UV-completed
- *Universal bounds on softness and positivity of 2-to-2 amplitudes, for arbitrary spins*
- *Unitarity crossing, and analyticity work as usual only in the forward elastic scattering*
- *2-to-2 Amplitudes can't run arbitrarily fast*
  (low-energy constraints can't be made arbitrarily irrelevant).
- *Non-trivial constraints on EFTs beyond symmetries*
  (R-axion, Goldstinos, fermionic shift sym…)
- *Boundaries from positivity must be included whenever dim-8 operators are relevant*

thank you!

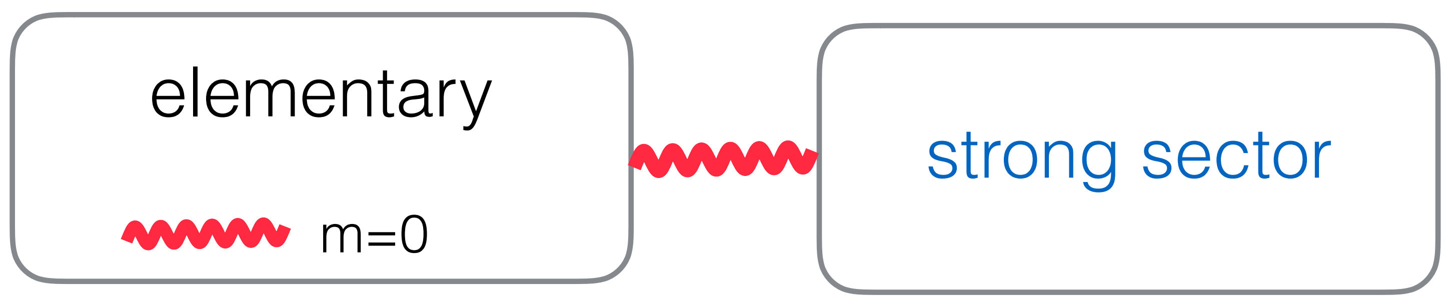
backup slides

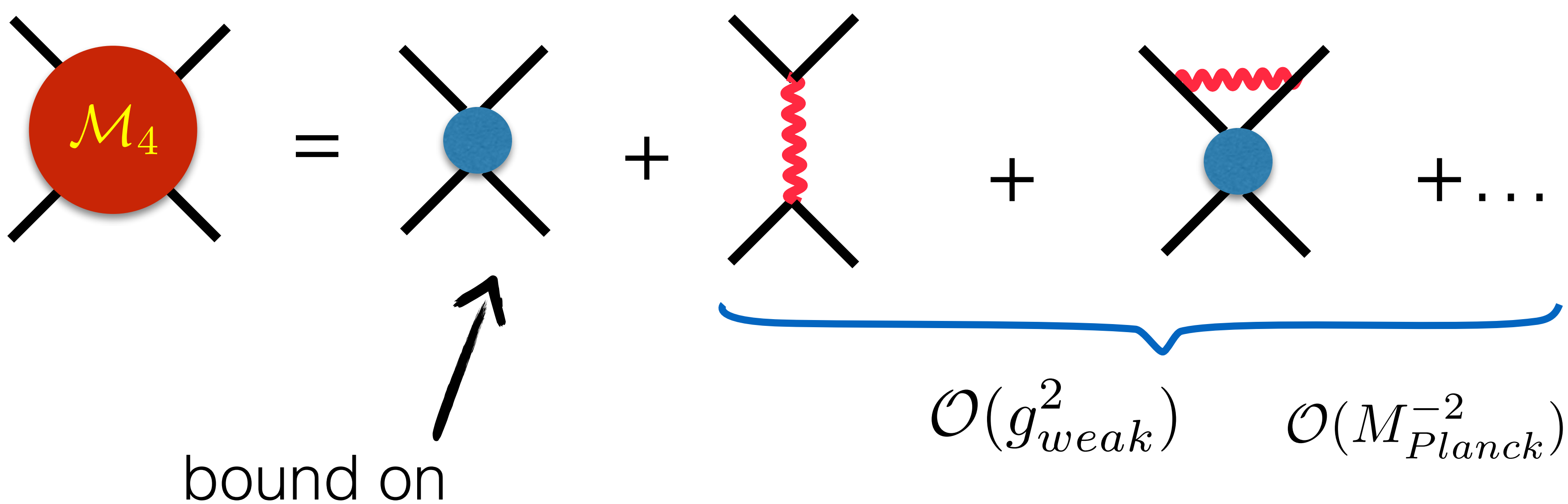
MASSLESS HIGHER-SPIN STATES





MASSLESS HIGHER-SPIN STATES





(if ~~~~ = —— $\langle JJJJ \rangle_{strong}$ YM

$\langle TTTT \rangle_{strong}$ gravity

bound on higher powers (Froissart relaxed))