AUTOMATING CALCULATIONS IN SOFT-COLLINEAR EFFECTIVE THEORY

[GUIDO BELL]

based on: GB, R. Rahn and J. Talbert, arXiv:1512.06100
GB, R. Rahn and J. Talbert, work in progress

UNIVERSITÄT SIEGEN

OUTLINE

SOFT-COLLINEAR EFFECTIVE THEORY

MOMENTUM SCALES

RESUMMATION BEYOND RG EQUATIONS

AUTOMATED RESUMMATIONS

NNLL INGREDIENTS

TWO-LOOP SOFT ANOMALOUS DIMENSION

RESULTS

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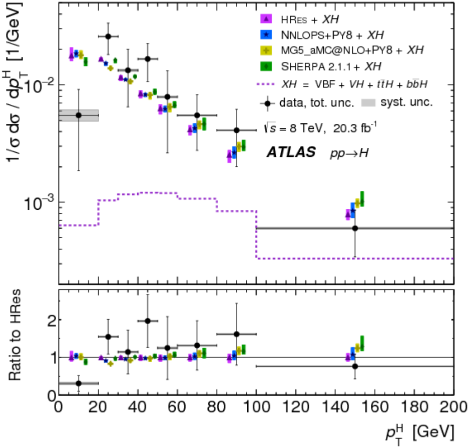
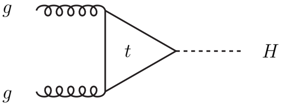
Higgs p_T spectrum





$m_t \simeq 175$ GeV

$m_H \simeq 125$ GeV

$p_T \simeq 0 - 200$ GeV

$\Lambda_{QCD} \simeq 0.5$ GeV

Factorisation

For $\Lambda_{QCD} \ll p_T, m_H, m_t$ the cross section factorises

$$d\sigma \simeq \sum_{i,j} f_{i/p}(\Lambda_{QCD}, \mu) \otimes f_{j/p}(\Lambda_{QCD}, \mu) \otimes d\hat{\sigma}_{ij\to HX}(p_T, m_H, m_t, \mu)$$

- universal parton-distribution functions $f_{i/p}$
- perturbative partonic cross section $d\hat{\sigma}_{ij\to HX}$

Factorisation scale μ separates short- and long-distance dynamics

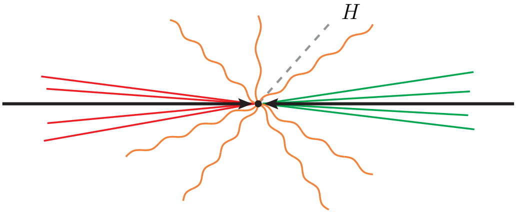
- single-logarithmic evolution controlled by DGLAP equations

$$\frac{df_{i/p}(\mu)}{d\ln\mu} = \sum_j P_{ij}(\alpha_s) \otimes f_{j/p}(\mu)$$

Small p_T

For $p_T \ll m_H, m_t$ the partonic cross section factorises further

$$d\hat{\sigma} \simeq H(m_H, m_t, \mu)\; J_1(p_T, \mu) \otimes J_2(p_T, \mu) \otimes S(p_T, \mu)$$



- hard function H
- jet (beam) functions J_i
- soft function S

} perturbative

double-logarithmic RG evolution

$\Rightarrow$ Sudakov logarithms $\alpha_s^n \ln^{2n} \frac{m_H}{p_T}$

Soft-Collinear Effective Theory

[Bauer, Fleming, Pirjol, Stewart 00;
Beneke, Chapovsky, Diehl, Feldmann 02]

Effective field theory for energetic massless particles

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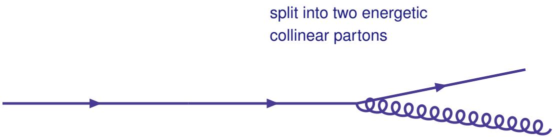

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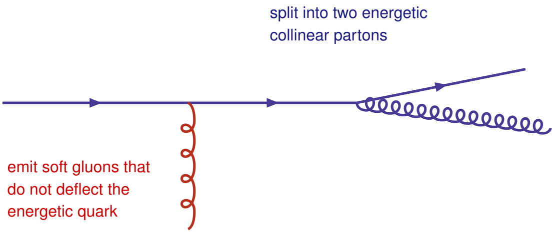

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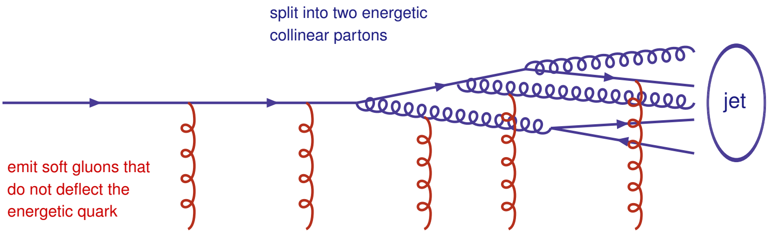
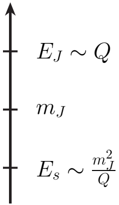


$\Rightarrow$ jet of collinear particles $m_J^2 \ll E_J^2$

soft large-angle radiation $E_s \ll E_J$

SCET-1

Three-scale problem: $E_s^2 \ll m_J^2 \ll E_J^2$



$$\boxed{d\hat{\sigma} \simeq H(Q,\mu)\; J(m_J,\mu) \otimes S(m_J^2/Q,\mu)}$$

$$\ln^2 \frac{Q^2}{m_J^2} = \frac{1}{2}\ln^2 \frac{Q^2}{\mu^2} - \ln^2 \frac{m_J^2}{\mu^2} + \frac{1}{2}\ln^2 \frac{m_J^4/Q^2}{\mu^2}$$

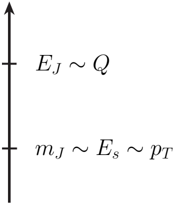
Sudakov resummation with standard EFT techniques

$$\frac{dH(Q,\mu)}{d\ln\mu} = \left[2\,\Gamma_{\rm cusp}(\alpha_s)\,\ln\frac{Q^2}{\mu^2} + 4\gamma_H(\alpha_s)\right] H(Q,\mu)$$

$$\Rightarrow H(Q,\mu) = H(Q,\mu_h)\; U_H(\mu_h,\mu)$$

SCET-2

Two-scale problem: $E_s^2 \sim m_J^2 \ll E_J^2$



$$\boxed{d\hat{\sigma} \simeq H(Q,\mu)\ J(p_T,\mu) \otimes S(p_T,\mu)}$$

$$\ln^2 \frac{Q^2}{p_T^2} = \ln^2 \frac{Q^2}{\mu^2} - \ln^2 \frac{p_T^2}{\mu^2} + \quad ?$$

Jet and soft functions are not well-defined in dimensional regularisation

$$k^\mu = k_- \frac{n^\mu}{2} + k_+ \frac{\bar{n}^\mu}{2} + k_\perp^\mu \qquad \Rightarrow \qquad J \sim \int_0^Q \frac{dk_+}{k_+} \qquad S \sim \int_{E_s}^\infty \frac{dk_+}{k_+}$$

$\Rightarrow$ in light-cone coordinates DR is attached to the transverse space $d^{d-2}k_\perp$

Collinear anomaly

Need additional regulator that distinguishes modes by their LC components [Becher, GB 11]

$$\int d^4k\ \delta(k^2)\ \theta(k^0) \quad \Rightarrow \quad \int d^dk\ \left(\frac{\nu}{k_+}\right)^\alpha \delta(k^2)\ \theta(k^0)$$

$$\left.\begin{aligned} \Rightarrow\ S &\sim +\frac{1}{\alpha} + \ln\frac{\nu}{p_T} \\ J &\sim -\frac{1}{\alpha} - \ln\frac{\nu}{Q} \end{aligned}\right\}\ \ln\frac{Q}{p_T} \quad \Rightarrow \quad \ln^2\frac{Q^2}{p_T^2} = \ln^2\frac{Q^2}{\mu^2} - \ln^2\frac{p_T^2}{\mu^2} - 2\ln\frac{p_T^2}{\mu^2}\ \ln\frac{Q^2}{p_T^2}$$

$\Rightarrow$ induces rapidity logarithms that cannot be resummed with RG techniques

Rapidity logarithms exponentiate (in position space) [Becher, Neubert 10; Chiu, Jain, Neill, Rothstein 11]

$$\mathcal{J}(x_T,\mu)\ \mathcal{S}(x_T,\mu) \ = \ \left(Q^2 x_T^2\right)^{-F(x_T,\mu)}\ W(x_T,\mu)$$

- anomaly exponent $F(x_T,\mu)$, remainder function $W(x_T,\mu)$

Applications

e^+e^- event-shape variables

- Thrust (N^{3}LL)
  [Becher, Schwartz 08]
- Heavy jet mass (N^{3}LL)
  [Chien, Schwartz 10]
- C-parameter (N^{3}LL)
  [Hoang, Kolodrubetz, Mateu, Stewart 14]
- Jet broadenings (NNLL)
  [Becher, GB 12]
- Angularities (NNLL)
  [GB, Hornig, Lee, Talbert, in progress]

hadron collider observables

- Threshold Drell-Yan (N^{3}LL)
  [Becher, Neubert, Xu 07]
- $W/Z/H$ at large p_T (N^{3}LL)
  [Becher, GB, Lorentzen, Marti 13,14]
- Higgs at small p_T (NNLL)
  [Becher, Neubert, Wilhelm 12]
- jet veto (NNLL)
  [Becher et al 13; Stewart et al 13]
- N-jettiness (NNLL)
  [Berger et al 10; Jouttenus et al 13]

Can we automate these calculations?

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Automated resummation codes: CAESAR (NLL) [Banfi, Salam, Zanderighi 04]

ARES (NNLL, e^+e^-) [Banfi, McAslan, Monni, Zanderighi 14]

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- MOMENTUM SCALES
- RESUMMATION BEYOND RG EQUATIONS

AUTOMATED RESUMMATIONS

- NNLL INGREDIENTS
- TWO-LOOP SOFT ANOMALOUS DIMENSION
- RESULTS

Resummation ingredients

	Γ_{cusp}	$\gamma_H, \begin{cases} \gamma_J, \gamma_S \\ F \end{cases}$	$c_H, \begin{cases} c_J, c_S \\ W \end{cases}$
LL	1-loop	–	–
NLL	2-loop	1-loop	tree
NNLL	3-loop	2-loop	1-loop
N^3LL	4-loop	3-loop	2-loop

- Γ_{cusp} is known to 3-loops [Moch, Vermaseren, Vogt 04]
- γ_H is known to 3-loops ("dipole formula") [Becher, Neubert 09]
- 1-loop c_H from fixed-order QCD calculations
- 1-loop c_J, c_S, W are rather straight-forward
- $\gamma_J = -\gamma_H - \gamma_S$ fixed by RG invariance

$\Rightarrow$ the non-trivial NNLL ingredient is 2-loop γ_S / F !

Generic dijet soft functions

Dijet soft functions are of the generic form

$$S(\tau,\mu) = \frac{1}{N_c} \sum_X \mathcal{M}(\tau;\{k_i\}) \, \mathrm{Tr} \, \langle 0|S_{\bar{n}}^\dagger S_n|X\rangle\langle X|S_n^\dagger S_{\bar{n}}|0\rangle$$

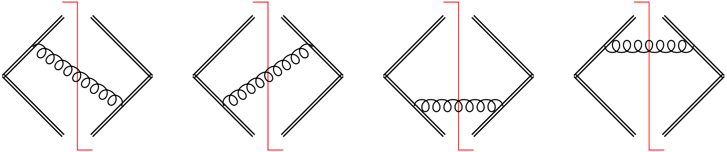
- light-like Wilson lines $S_n, S_{\bar{n}}$
- generic measurement function $\mathcal{M}(\tau;\{k_i\})$
- relevant for $e^+e^- \to 2$ jets, $e^-p \to 1$ jet, $pp \to 0$ jets
- but provides 2-loop γ_S / 2-loop F also for $pp \to 2$ jets [Becher, Garcia, Piclum 15]

Structure of divergences is independent of the observable

⇒ isolate singularities with universal phase-space parametrisation

⇒ compute observable-dependent integrations numerically

NLO calculation



NLO calculation

Real emission

$$S_1 \sim \int d^dk \, \left(\frac{\nu}{k_+ + k_-}\right)^{\alpha} \delta(k^2)\,\theta(k^0)\; \mathcal{M}(\tau;k)\; |\mathcal{A}(k)|^2$$

- $n \leftrightarrow \bar{n}$ symmetrised version of phase-space regulator
- $|\mathcal{A}(k)|^2 \sim \dfrac{1}{k_+ k_-}$

Generic measurement function in Laplace space

$$\boxed{\mathcal{M}(\tau;k) = \exp\Big(-\tau\, k_T\, y^{n/2}\, f(y,\theta)\Big)} \qquad\qquad y = \frac{k_+}{k_-} \in \{0,1\}$$

- k_T dependence fixed on dimensional grounds
- θ is angle between $\vec{k}_\perp$ and measurement vector $\vec{v}_\perp$
- $f(y,\theta)$ finite and non-zero in collinear limit $y \to 0$

Measurement function

$$\mathcal{M}(\tau; k) = \exp\Big(- \tau \, k_T \, y^{n/2} \, f(y, \theta) \Big)$$

Observable	n	$f(y, \theta)$
Thrust	1	1
Angularities	$1 - A$	1
Recoil-free broadening	0	$1/2$
C-parameter	1	$1/(1+y)$
Threshold Drell-Yan	-1	$1 + y$
W@large p_T	-1	$1 + y - 2\sqrt{y}\,\cos\theta$
e^+e^- transverse thrust	1	$\frac{1}{s\sqrt{y}}\left(\sqrt{\left(c\cos\theta + \left(\frac{1}{\sqrt{y}} - \sqrt{y}\right)\frac{s}{2}\right)^2 + 1 - \cos^2\theta} - \left\lvert c\cos\theta + \left(\frac{1}{\sqrt{y}} - \sqrt{y}\right)\frac{s}{2}\right\rvert\right)$

NLO master formula

After performing the observable-independent integrations one has

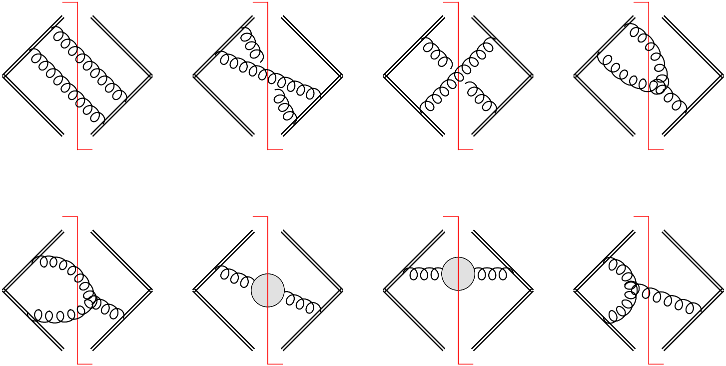
$$S_1 \sim \Gamma(-2\varepsilon - \alpha) \int_0^1 dy \, \frac{y^{-1+n\varepsilon+\alpha/2}}{(1+y)^\alpha} \int_{-1}^1 d\cos\theta \;\; \sin^{-1-2\varepsilon}\theta \;\; \big[f(y,\theta)\big]^{2\varepsilon+\alpha}$$

- singularities from $k_T \to 0$ and $y \to 0$ are factorised
- additional regulator is needed only for $n = 0$ ($\to$ SCET-2)

Isolate singularities with standard subtraction techniques

$$\int_0^1 dx \;\, x^{-1+n\varepsilon} \; f(x) \; = \int_0^1 dx \;\, x^{-1+n\varepsilon} \; \Big[\underbrace{f(x) - f(0)}_{\text{finite}} + \underbrace{f(0)}_{1/\varepsilon} \Big]$$

NNLO calculation



NNLO calculation

Double real emission

$$S_2^{RR} \sim \int d^d k \left(\frac{\nu}{k_+ + k_-} \right)^{\alpha} \delta(k^2)\,\theta(k^0) \int d^d l \left(\frac{\nu}{l_+ + l_-} \right)^{\alpha} \delta(l^2)\,\theta(l^0)\;\; \mathcal{M}(\tau; k, l)\;\; |\mathcal{A}(k,l)|^2$$

- non-trivial matrix element

$$|\mathcal{A}(k,l)|^2 \sim C_F T_F n_f \; \frac{2k \cdot l\,(k_- + l_-)\,(k_+ + l_+) - (k_- l_+ - k_+ l_-)^2}{(k_- + l_-)^2\,(k_+ + l_+)^2\,(2k \cdot l)^2} + C_F C_A \;\dots + C_F^2 \;\dots$$

⇒ complex singularity structure with overlapping divergences

Phase-space parametrisation

$$p_- = k_- + l_- \qquad a = \sqrt{\frac{k_- l_+}{k_+ l_-}} \qquad\qquad p_T = \sqrt{p_+ p_-}$$

$$\Rightarrow$$

$$p_+ = k_+ + l_+ \qquad b = \sqrt{\frac{k_- k_+}{l_- l_+}} = \frac{k_T}{l_T} \qquad\qquad y = \frac{p_+}{p_-}$$

Measurement function

Generic form

$$\boxed{\mathcal{M}(\tau; k, l) = \exp\Big(-\tau\, p_T\, y^{n/2}\, F(a, b, y, \theta_k, \theta_l, \theta_{kl})\Big)} \qquad a, b, y \in \{0, 1\}$$

- p_T dependence fixed on dimensional grounds
- three angles in transverse plane: $\theta_k \sphericalangle (\vec{k}_\perp, \vec{v}_\perp)$, $\theta_l \sphericalangle (\vec{l}_\perp, \vec{v}_\perp)$, $\theta_{kl} \sphericalangle (\vec{k}_\perp, \vec{l}_\perp)$
- $y^{n/2}$ as in NLO $\Rightarrow$ $F(a, b, y, \theta_k, \theta_l, \theta_{kl})$ finite and non-zero for $y \to 0$

Constraints from infrared-collinear safety

- soft limit ($k^\mu \to 0$): $F(a, 0, y, \theta_k, \theta_l, \theta_{kl}) = f(y, \theta_l)$
- collinear limit ($k^\mu \propto l^\mu$): $F(1, b, y, \theta_l, \theta_l, \theta_{kl}) = f(y, \theta_l)$

Strategy

Disentangle overlapping singularities with sector decomposition algorithm [Binoth, Heinrich 00]

- `SecDec-3.0` [Borowka, Heinrich, Jones, Kerner, Schlenk, Zirke 15]

  simple interface to NNLO master formula

  Cuba library for numerical integrations

  currently limited to SCET-1 observables (additional rapidity regulator in development)

- in-house C++ code

  designed for dijet soft functions (SCET-1 and SCET-2)

  Cuba library for numerical integrations

- analytic approach

  compact integral representations for γ_S and F

Soft anomalous dimension

RG equation in Laplace space

$$\frac{d\,S(\tau,\mu)}{d\ln\mu} = -\frac{1}{n}\left[4\,\Gamma_{\rm cusp}(\alpha_s)\,\ln(\mu\bar{\tau}) - 2\gamma^S(\alpha_s)\right] S(\tau,\mu)$$

Two-loop solution with $L = \ln(\mu\bar{\tau})$

$$S(\tau,\mu) = 1 + \left(\frac{\alpha_s}{4\pi}\right)\left\{-\frac{2\Gamma_0}{n}L^2 + \frac{2\gamma_0^S}{n}L + c_1^S\right\} + \left(\frac{\alpha_s}{4\pi}\right)^2\left\{\frac{2\Gamma_0^2}{n^2}L^4 - 4\Gamma_0\left(\frac{\gamma_0^S}{n^2} + \frac{\beta_0}{3n}\right)L^3\right.$$
$$\left. -2\left(\frac{\Gamma_1}{n} - \frac{(\gamma_0^S)^2}{n^2} - \frac{\beta_0\gamma_0^S}{n} + \frac{\Gamma_0 c_1^S}{n}\right)L^2 + 2\left(\frac{\gamma_1^S}{n} + \frac{\gamma_0^S c_1^S}{n} + \beta_0 c_1^S\right)L + c_2^S\right\}$$

Assuming non-abelian exponentiation, the results will be of the form

$$\gamma_1^S = \gamma_1^{C_A}\;C_F C_A + \gamma_1^{n_f}\;C_F T_F n_f \qquad\qquad c_2^S = c_2^{C_A}\;C_F C_A + c_2^{n_f}\;C_F T_F n_f + \frac{1}{2}(c_1^S)^2$$

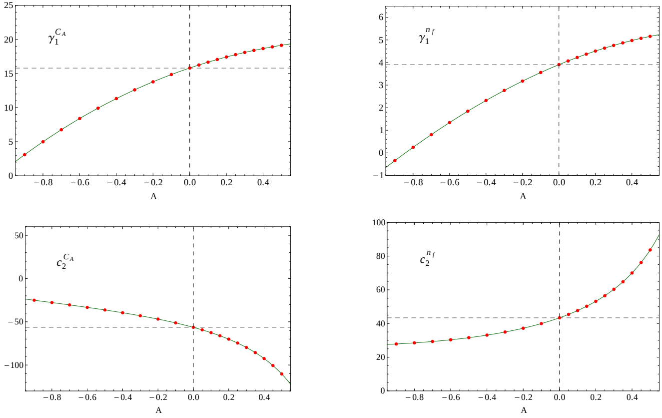
SCET-1 results

Observable	$\gamma_1^{C_A}$	$\gamma_1^{n_f}$	$c_2^{C_A}$	$c_2^{n_f}$
Thrust [Kelley et al, Monni et al 11]	15.7945 (15.7945)	3.90981 (3.90981)	-56.4992 (-56.4990)	43.3902 (43.3905)
C-parameter [Hoang et al 14]	15.7947 (15.7945)	3.90980 (3.90981)	-57.9754 (-58.16 ± 0.26)	43.8179 (43.74 ± 0.06)
Threshold Drell-Yan [Belitsky 98]	15.7946 (15.7945)	3.90982 (3.90981)	6.81281 (6.81287)	-10.6857 (-10.6857)
W@large p_T [Becher et al 12]	15.7947 (15.7945)	3.90981 (3.90981)	-2.65034 (-2.65010)	-25.3073 (-25.3073)
Transverse thrust [Becher, Garcia 15]	-158.278 (-148^{+20}_{-30})	19.3955 (18^{+2}_{-3})	—	—

- upper numbers: `SecDec` in a few hours on a single 8-core machine
- lower numbers: analytic (black) or fit to fixed-order code (gray)

Angularities

e^+e^- event-shape that interpolates between thrust ($A = 0$) and broadening ($A = 1$)

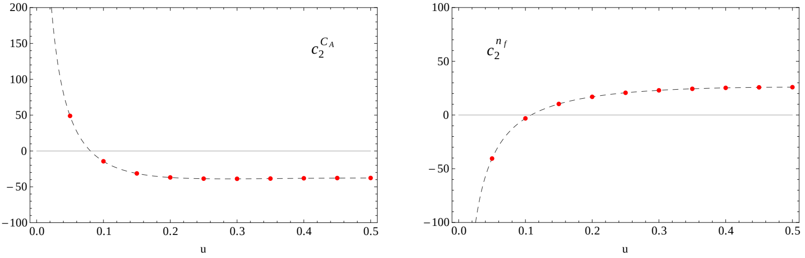


$\Rightarrow$ last missing ingredient for NNLL resummation [GB, Hornig, Lee, Talbert, in progress]

Hemisphere masses

Double differential event-shape variable

$$\frac{d^2\sigma}{dM_L dM_R} \quad \Rightarrow \quad \frac{d^2\sigma}{d\tau_L d\tau_R} \qquad\qquad \tau = \tau_L + \tau_R \qquad\qquad u = \frac{\tau_L}{\tau_L + \tau_R}$$



⇒ perfect agreement with analytic result

[Kelley, Schwartz, Schabinger, Zhu 11]

Anomaly exponent

RG equation in Laplace space

$$\frac{d\,F(\tau,\mu)}{d\ln\mu} = 2\,\Gamma_{\rm cusp}(\alpha_s)$$

Two-loop solution with $L = \ln(\mu\bar{\tau})$

$$F(\tau,\mu) = \left(\frac{\alpha_s}{4\pi}\right)\left\{2\Gamma_0 L + d_1\right\} + \left(\frac{\alpha_s}{4\pi}\right)^2\left\{2\Gamma_0\beta_0 L^2 + 2\,(\Gamma_1 + \beta_0 d_1)\,L + d_2\right\}$$

Results will be presented in the form

$$\boxed{d_2 = d_2^{C_A}\;C_F C_A + d_2^{n_f}\;C_F T_F n_f}$$

SCET-2 results

Observable	$d_2^{C_A}$	$d_2^{n_f}$
Recoil-free broadening [Becher, GB 12]	7.03595 (7.03605)	-11.5393 (-11.5393)
p_T resummation [Becher, Neubert 10]	-3.73389 (-3.73167)	-8.29610 (-8.29630)
E_T resummation	15.9804 ($-$)	-18.7370 ($-$)
Transverse thrust [Becher et al 15]	208.098 (208.0 ± 0.1)	-37.1766 (-37.191 ± 0.006)

- do not confirm QCD result for E_T resummation

$$B_g^{(2)} = \frac{1}{16}\left(d_2 + 2\gamma_1^g + \beta_0 e_1^g\right) = \begin{cases} 33.0081 & \text{(our result)} \\ -5.1 \pm 1.6 & \text{[Grazzini et al 14]} \end{cases}$$

Conclusions

SCET can be used to resum Sudakov logarithms for collider observables

- resummation beyond RG techniques

First step towards automated NNLL resummations

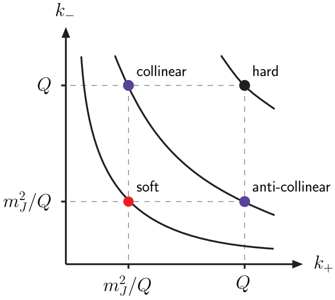
- automated setup to compute two-loop soft anomalous dimensions
- applies to SCET-1 and SCET-2 observables
- next steps: relax non-abelian exponentiation assumption,

  more than two jet directions, massive particles

Backup slides

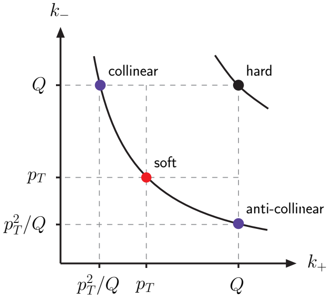
Momentum modes

SCET-1



$k_s^2 \ll k_c^2$

SCET-2



$k_s^2 \sim k_c^2$

In SCET-2 one cannot distinguish soft from collinear mode when radiated into jet direction

$\Rightarrow$ need additional regulator that distinguishes modes by their rapidities