NNLO Differential Calculation from Soft-Collinear Effective Theory: WHH production at hadron colliders

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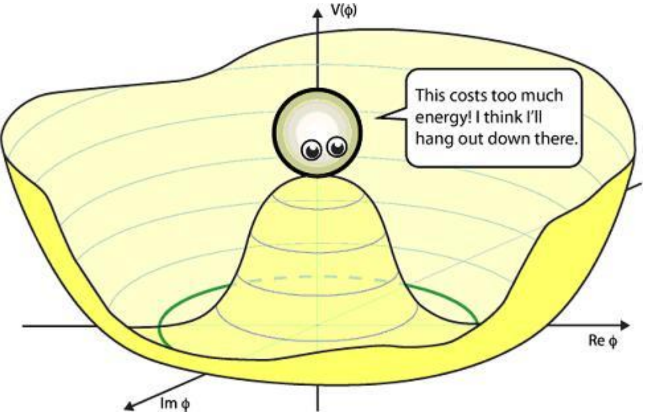
in collaboration with Hai Tao Li, arXiv:1607.06382

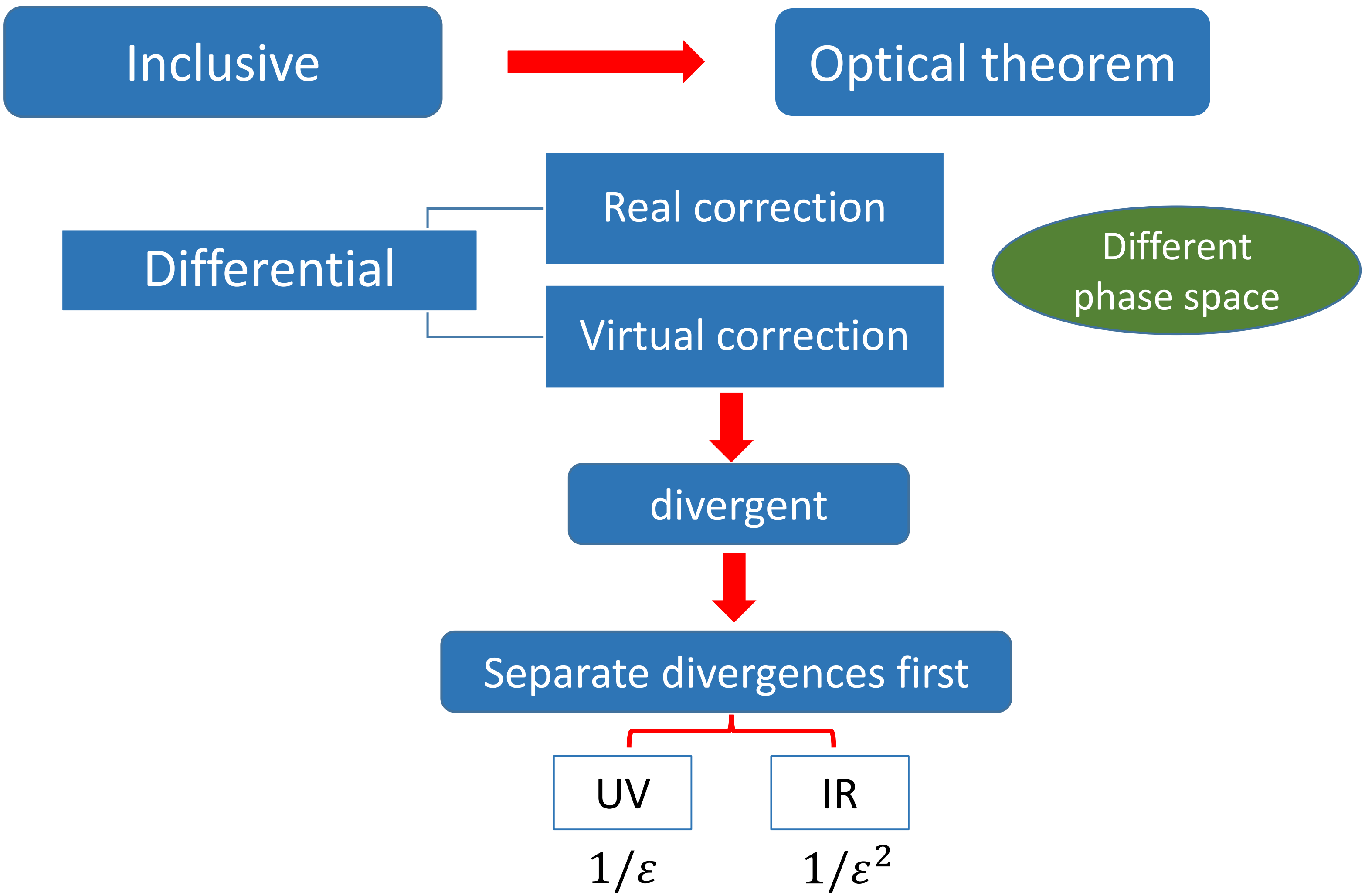
Eltville, 2016.09.14



Precise calculation in QCD

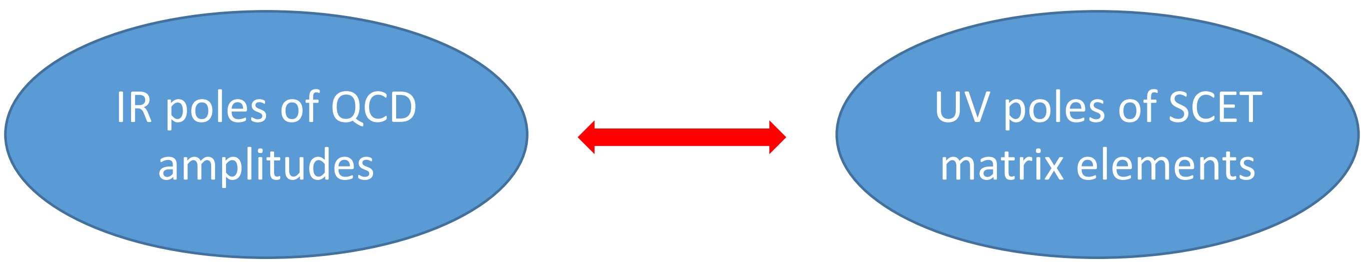
- LHC is a hadron collider. QCD radiations exist widespread.
- Kinematic distributions are different at higher orders.
- The description of jets is more accurate.
- The cross sections can be increased or decreased.
- The theoretical uncertainties can be reduced.
- NP may appear just as enhancement of some observables. It is necessary to understand the background precisely.

The art of precise calculation



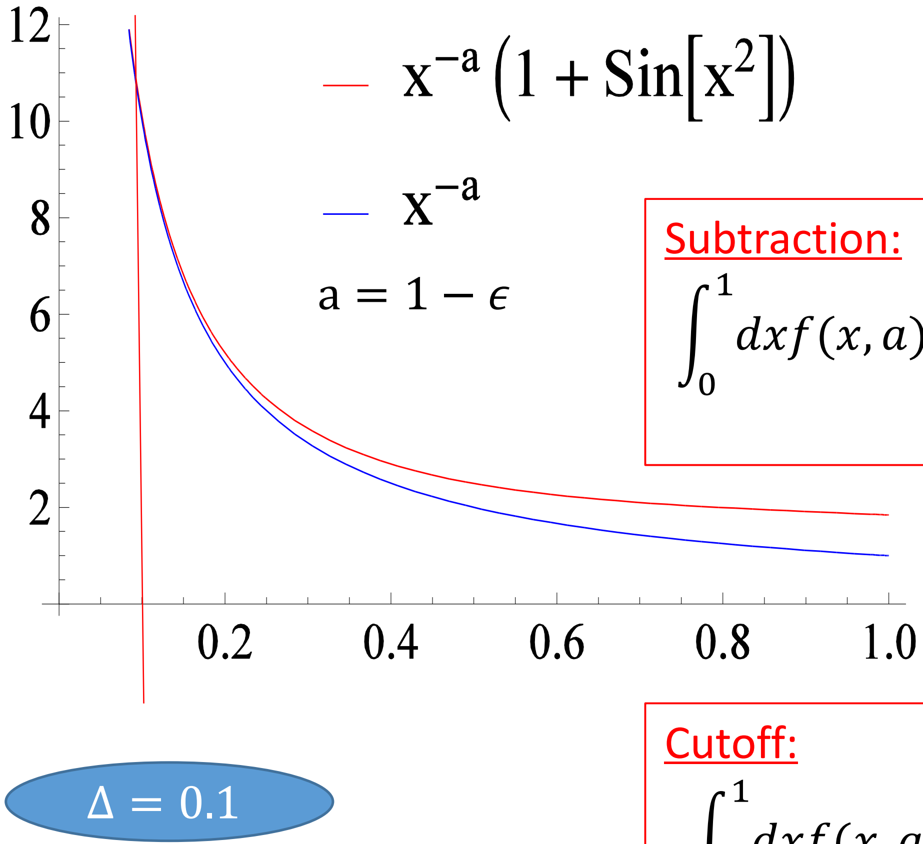
IR divergences

- Two-loop structure was predicted by Catani in ’98.
- And the complete results were obtained later. [Aybat, Dixon, Sterman ’06, Becher, Neubert ’09, Gardi and Magnea ‘09]
- Extension to massive amplitudes was achieved soon. [Ferroglia, Neubert, Pecjak, Yang ‘09]



However, only knowing poles of $1/\varepsilon^n$ is not enough to make the real corrections finite. We need to know the behavior of cross section near the poles.

The idea of subtraction and cutoff



Subtraction:

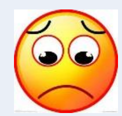
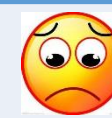
$$\int_0^1 dx f(x,a) = \int_0^1 dx\,[f(x,a) - f(0,a)] + \int_0^1 dx\, f(0,a)$$

Cutoff:

$$\int_0^1 dx f(x,a) = \int_0^\Delta dx\,[f(0,a)] + \int_\Delta^1 dx\, f(x,a)$$

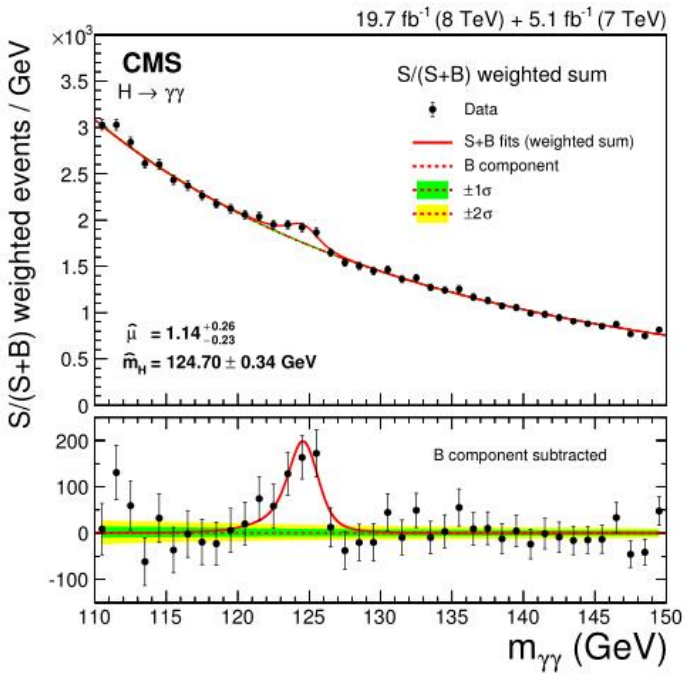
Near pole regions

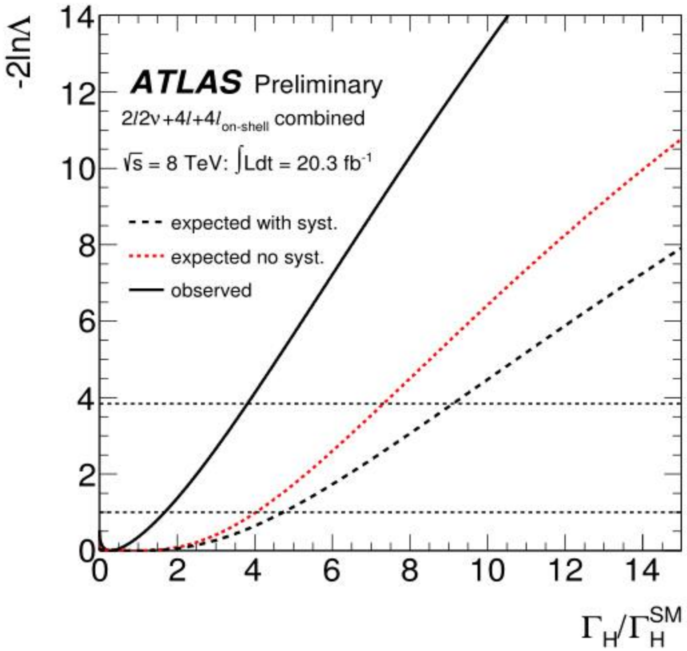
- Study the cross section near soft or collinear regions.
  $\sim x^{-1+\varepsilon}$

Near pole	Resummation scheme	Soft	Collinear	Extension to colored FS
$x = 1 - \frac{M^2}{s}$	Threshold resummation			
$x = \frac{\sum m_j^2}{s}$	N-jettiness resummation			
$x = \frac{q_T^2}{s}$	Transverse momentum resummation			

Catani, Grazzini, Phys. Rev. Lett. 98, 222002 (2007)
Boughezal, Focke, Liu, Petriello, Phys.Rev. Lett. 115, 062002 (2015)

Higgs mass and width



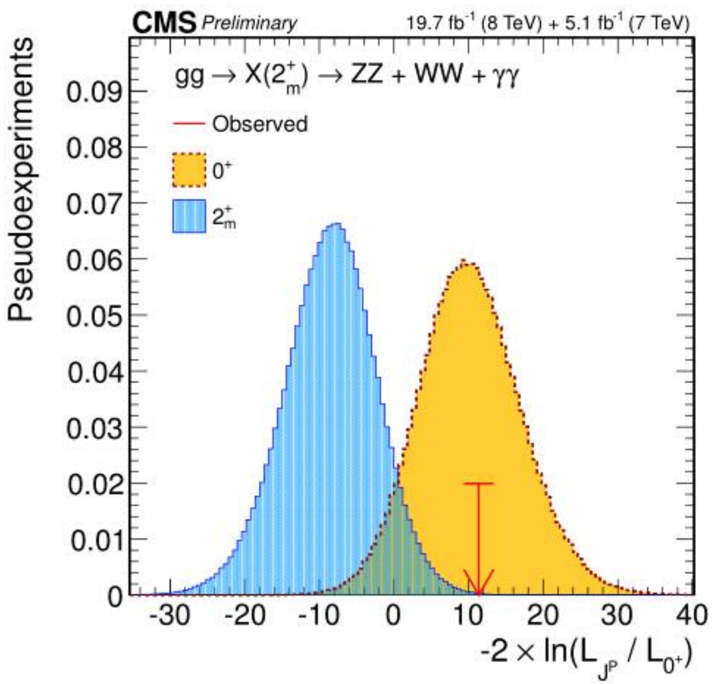


$$\sigma_{peak}^{sig} \propto \frac{g_i^2 g_f^2}{\Gamma_h}$$

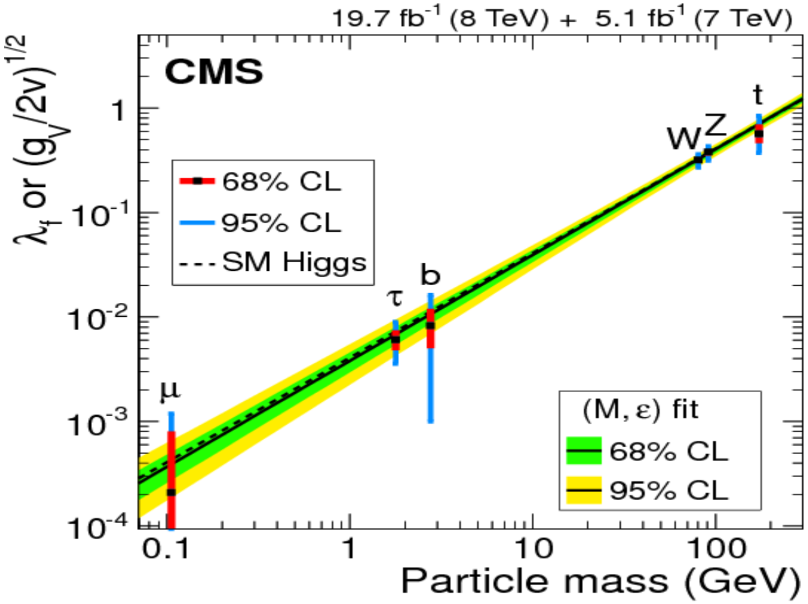
Caola, Melnikov ‘13

$$\sigma_{off}^{sig} \propto g_i^2 g_f^2 \qquad \sigma_{off}^{int} \propto g_i \ g_f$$

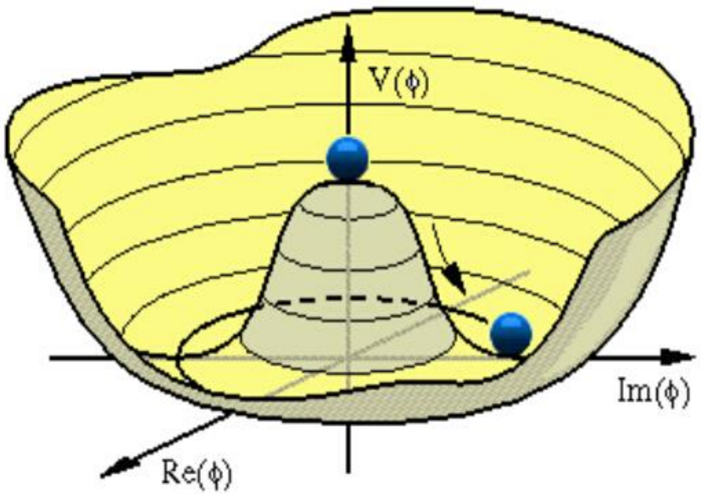
Higgs spin, CP and couplings



CP-even spin 0 hypothesis strongly preferred.
No significant deviations from SM couplings.
Data up to now are consistent with a SM Higgs boson.

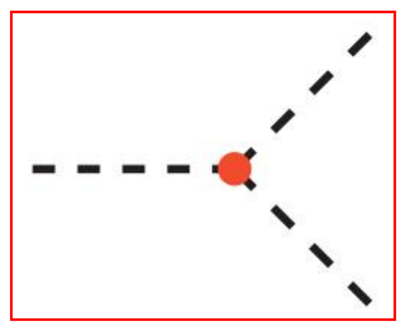


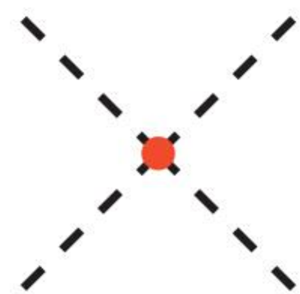
Higgs potential



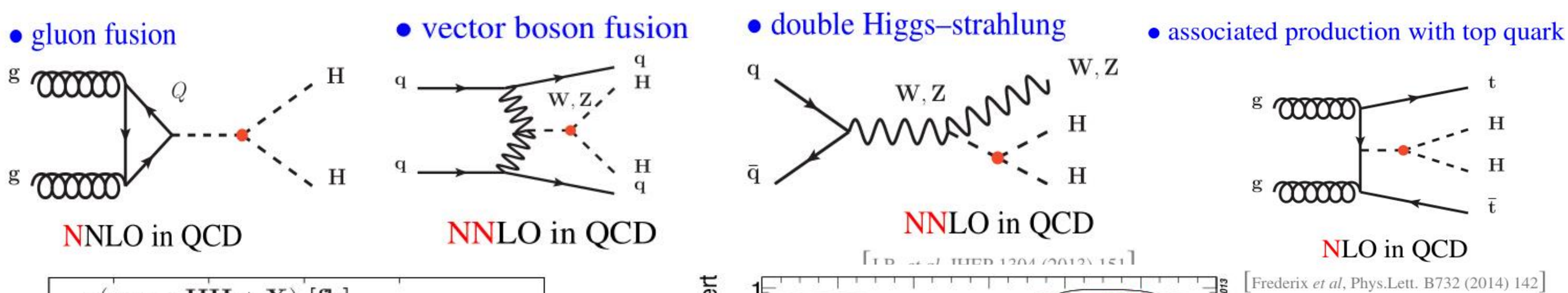
$$V(\phi) = -m^2|\phi|^2 + \lambda|\phi|^4$$

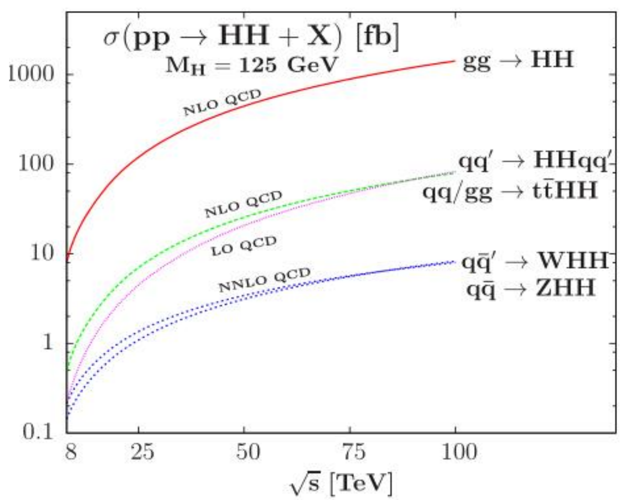
$$\phi = \begin{pmatrix} 0 \\ \frac{v + H(x)}{\sqrt{2}} \end{pmatrix} \Rightarrow V(H) = \frac{1}{2}M_H^2 H^2 + \mathbf{\frac{1}{2}\frac{M_H^2}{v}H^3} + \frac{1}{8}\frac{M_H^2}{v^2}H^4$$

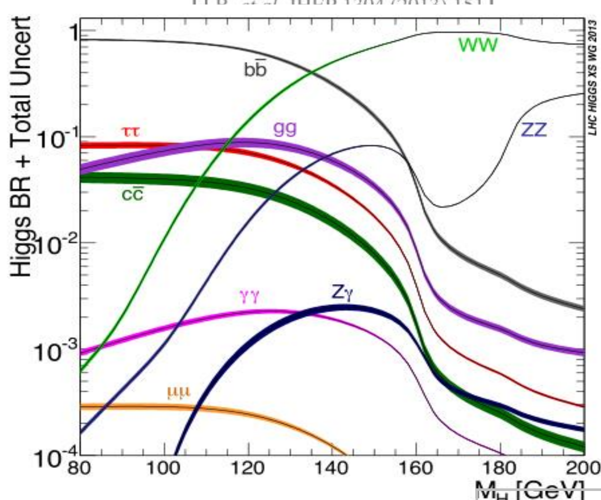




Higgs pair production



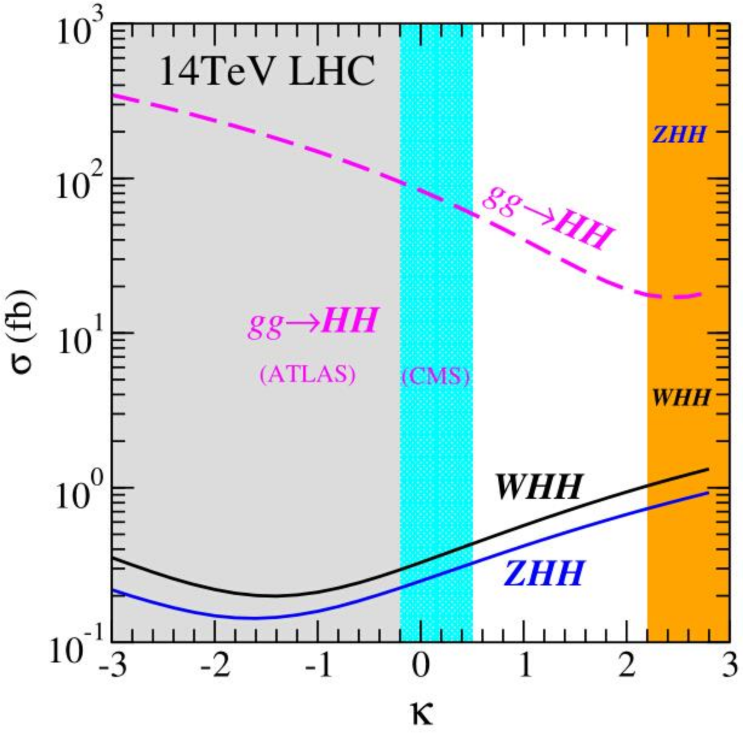




$b\bar{b}WW$
$b\bar{b}\gamma\gamma$
$b\bar{b}\tau\bar{\tau}$
VBF
$t\bar{t}HH$

Baur, Plehn, and Rainwater ‘04,
Dolan, Englert, and Spannowsky ’12,
Papaefstathiou, Yang, and Zurita ’13,
Liu and Zhang ’14,
……….

Different channels



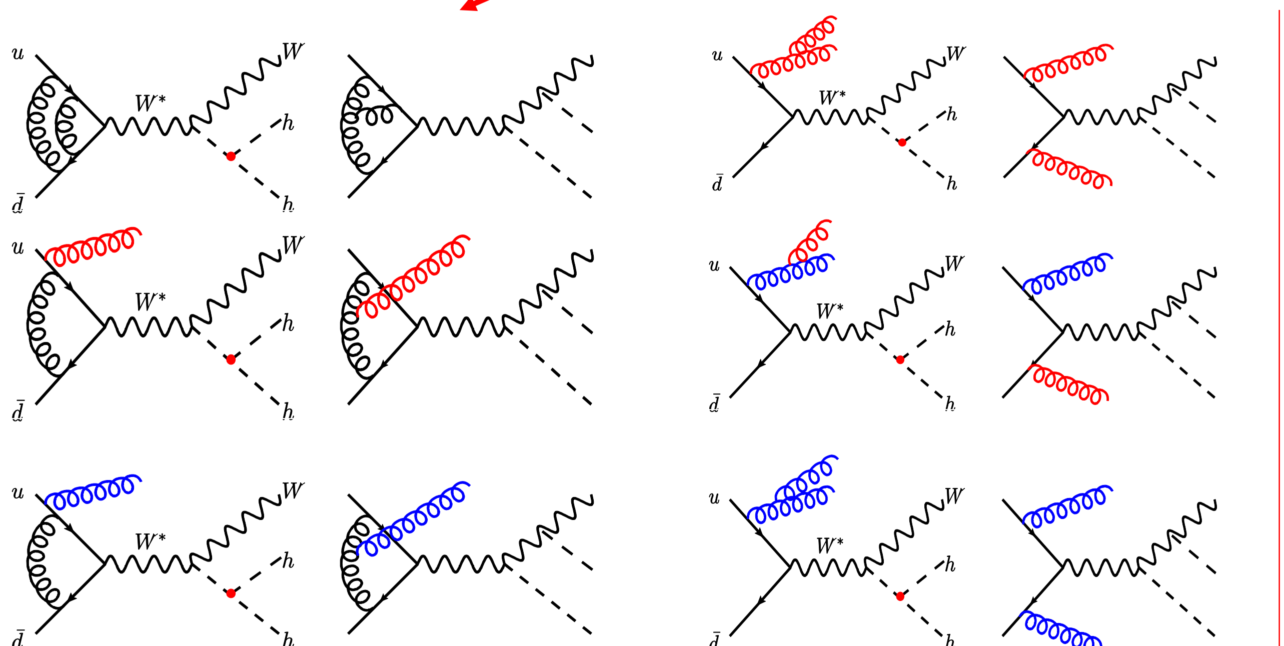
$$\sigma(W^\pm HH)\times \mathrm{BR}(W^\pm \to \ell^\pm \nu_\ell, HH \to b\bar{b}b\bar{b}) = 0.042\mathrm{fb},$$
$$\sigma(ZHH)\times \mathrm{BR}(Z \to \nu\bar{\nu}, HH \to b\bar{b}b\bar{b}) = 0.028\mathrm{fb},$$
$$\sigma(gg \to HH)\times \mathrm{BR}(HH \to \gamma\gamma b\bar{b}) = 0.053\mathrm{fb},$$

The different channels are complementary to each other and deserve discussion on the same footing.

Qing-Hong Cao, Yandong Liu, Bin Yan,arXiv:1511.03311

NNLO prediction for $pp \to VHH$

$$\left.\frac{d\sigma_3}{d\Phi_3 dy}\right|_{\text{NNLO}} = \underbrace{\int_0^{q_T^{\text{cut}}} dq_T \frac{d\sigma_3}{d\Phi_3 dy dq_T}}_{\text{SCET}} + \underbrace{\int_{q_T^{\text{cut}}}^{q_T^{\max}} dq_T \frac{d\sigma_{3+j}}{d\Phi_3 dy dq_T}}_{\text{NLO calculations}}$$



The others with $q_T > q_T^{cut}$

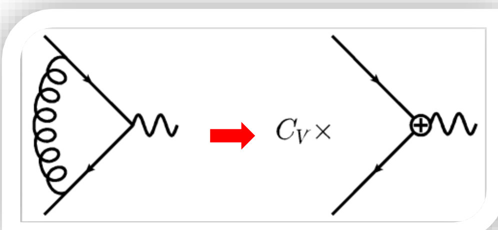
NNLO prediction for $pp \to VHH$

$$\frac{d\sigma_3}{d\Phi_3 dy}\bigg|_{\mathrm{NNLO}} = \underbrace{\int_0^{q_T^{\mathrm{cut}}} dq_T \frac{d\sigma_3}{d\Phi_3 dy dq_T}}_{\mathrm{SCET}} + \underbrace{\int_{q_T^{\mathrm{cut}}}^{q_T^{\mathrm{max}}} dq_T \frac{d\sigma_{3+j}}{d\Phi_3 dy dq_T}}_{\text{NLO calculations}}$$

$$\frac{d\sigma}{dq_T^2 dy} = \frac{1}{2s}\sum_{i,j=q\bar{q}g}\int_{\zeta_1}^1 \frac{dz_1}{z_1}\int_{\zeta_2}^1 \frac{dz_2}{z_2}\int d\Phi_3 \boxed{H_{q\bar{q}}(M,\mu)} f_{i/N_1}(\zeta_1/z_1,\mu) f_{j/N_2}(\zeta_1/z_2,\mu) C_{q\bar{q}\leftarrow ij}(z_1,z_2,q_T,M,\mu) + \mathcal{O}(\frac{q_T^2}{M^2})$$

Becher, Neubert, Wilhelm, `11

Becher, Neubert, Xu, `07



NNLO:

$$C_F\left(\frac{\alpha_s}{4\pi}\right)^2 [C_F H_F + C_A H_A + T_F n_f H_f]$$

$$\begin{aligned}
H_F &= \frac{L^4}{2} - 3L^3 + \left(\frac{25}{2} - \frac{\pi^2}{6}\right)L^2 + \left(-\frac{45}{2} - \frac{3\pi^2}{2} + 24\zeta_3\right)L + \frac{255}{8} + \frac{7\pi^2}{2} - \frac{83\pi^4}{360} - 30\zeta_3\,,\\
H_A &= \frac{11}{9}L^3 + \left(-\frac{233}{18} + \frac{\pi^2}{3}\right)L^2 + \left(\frac{2545}{54} + \frac{11\pi^2}{9} - 26\zeta_3\right)L\\
&\quad - \frac{51157}{648} - \frac{337\pi^2}{108} + \frac{11\pi^4}{45} + \frac{313}{9}\zeta_3\,,\\
H_f &= -\frac{4}{9}L^3 + \frac{38}{9}L^2 + \left(-\frac{418}{27} - \frac{4\pi^2}{9}\right)L + \frac{4085}{162} + \frac{23\pi^2}{27} + \frac{4}{9}\zeta_3\,,
\end{aligned} \qquad \text{(B.2)}$$

$$\frac{d}{d\ln\mu} C_V(-M^2,\mu) = \left[\Gamma_{\mathrm{cusp}}^F(\alpha_s)\ln\frac{-M^2}{\mu^2} + 2\gamma^q(\alpha_s)\right] C_V($$

$$C_V(-M^2,\mu) = 1 + \frac{C_F\alpha_s(\mu)}{4\pi}\left(-L^2 + 3L - 8 + \frac{\pi^2}{6}\right)$$

NNLO prediction for $pp \to VHH$

$$\frac{d\sigma_3}{d\Phi_3 dy}\bigg|_{\rm NNLO} = \underbrace{\int_0^{q_T^{\rm cut}} dq_T \frac{d\sigma_3}{d\Phi_3 dy dq_T}}_{\rm SCET} + \underbrace{\int_{q_T^{\rm cut}}^{q_T^{\rm max}} dq_T \frac{d\sigma_{3+j}}{d\Phi_3 dy dq_T}}_{\rm NLO\ calculations}$$

$$\frac{d\sigma}{dq_T^2 dy} = \frac{1}{2s}\sum_{i,j=q\bar{q}g}\int_{\zeta_1}^1 \frac{dz_1}{z_1}\int_{\zeta_2}^1 \frac{dz_2}{z_2}\int d\Phi_3 H_{q\bar{q}}(M,\mu) \boxed{f_{i/N_1}(\zeta_1/z_1,\mu) f_{j/N_2}(\zeta_1/z_2,\mu)}\, \boxed{C_{q\bar{q}\leftarrow ij}(z_1,z_2,q_T,M,\mu)} + \mathcal{O}\left(\frac{q_T^2}{M^2}\right)$$

Parton Distribution Function

$$C_{q\bar{q}\to Whh}(z_1,z_2,q_T,M,\mu) = \frac{1}{4\pi}\int d^2x_\perp e^{-ix_\perp\cdot q_\perp}\left(\frac{x_T^2 M^2}{b_0^2}\right)^{F_{q\bar{q}}(x_T^2,\mu)} I_{q\leftarrow i}(z_1,L_\perp,\alpha_s) I_{\bar{q}\leftarrow j}(z_1,L_\perp,\alpha_s)$$

$$B_{i/N}(\xi,x_T^2,\mu) = \sum_j \int_\xi^1 \frac{dz}{z} I_{i\leftarrow j}(z,x_T^2,\mu)\,\phi_{j/N}(\xi/z,\mu) + \mathcal{O}(\Lambda_{\rm QCD}^2\, x_T^2)$$

NNLO: Gehrmann, Luebbert, and Yang Phys.Rev.Lett.109.242003; JHEP, 06(2014)155

NNLO prediction for $pp \to VHH$

$$\left.\frac{d\sigma_3}{d\Phi_3 dy}\right|_{\text{NNLO}} = \underbrace{\int_0^{q_T^{\text{cut}}} dq_T \frac{d\sigma_3}{d\Phi_3 dy dq_T}}_{\text{SCET}} + \underbrace{\int_{q_T^{\text{cut}}}^{q_T^{\max}} dq_T \frac{d\sigma_{3+j}}{d\Phi_3 dy dq_T}}_{\text{NLO calculations}}$$

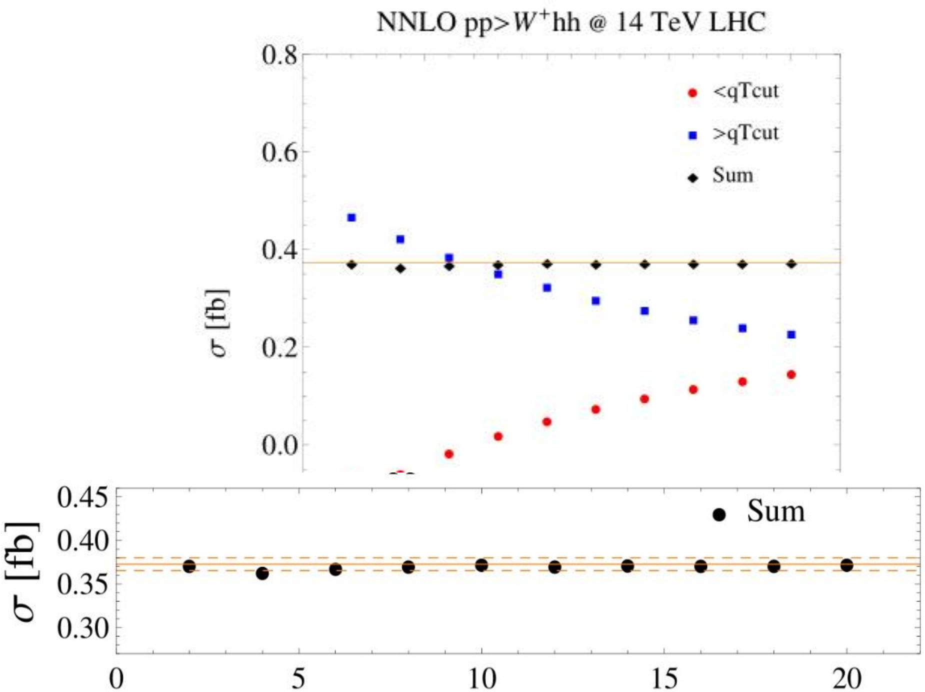
$$\frac{d\sigma}{dq_T^2 dy} = \frac{1}{2s} \sum_{i,j=q\bar{q}g} \int_{\zeta_1}^1 \frac{dz_1}{z_1} \int_{\zeta_2}^1 \frac{dz_2}{z_2} \int d\Phi_3 H_{q\bar{q}}(M,\mu)$$
$$f_{i/N_1}(\zeta_1/z_1,\mu) f_{j/N_2}(\zeta_1/z_2,\mu) C_{q\bar{q}\leftarrow ij}(z_1,z_2,q_T,M,\mu)$$

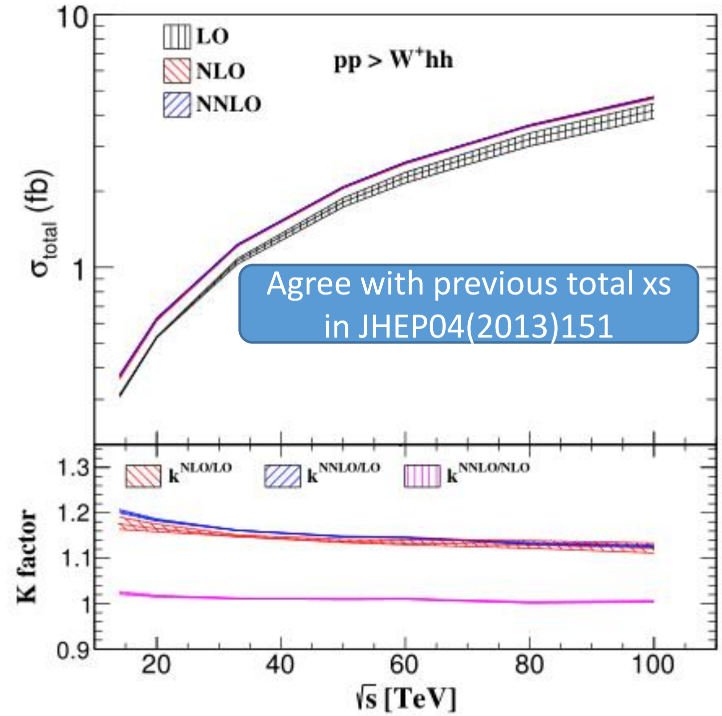
MadGraph5_aMC@NLO

No jet algorithm

$$C_{q\bar{q}\to Whh}(z_1,z_2,q_T,M,\mu) = \frac{1}{4\pi}\int d^2x_\perp e^{-ix_\perp\cdot q_\perp}\left(\frac{x_T^2 M^2}{b_0^2}\right)^{F_{q\bar{q}}(x_T^2,\mu)} I_{q\leftarrow i}(z_1,L_\perp,\alpha_s) I_{\bar{q}\leftarrow j}(z_1,L_\perp,\alpha_s),$$

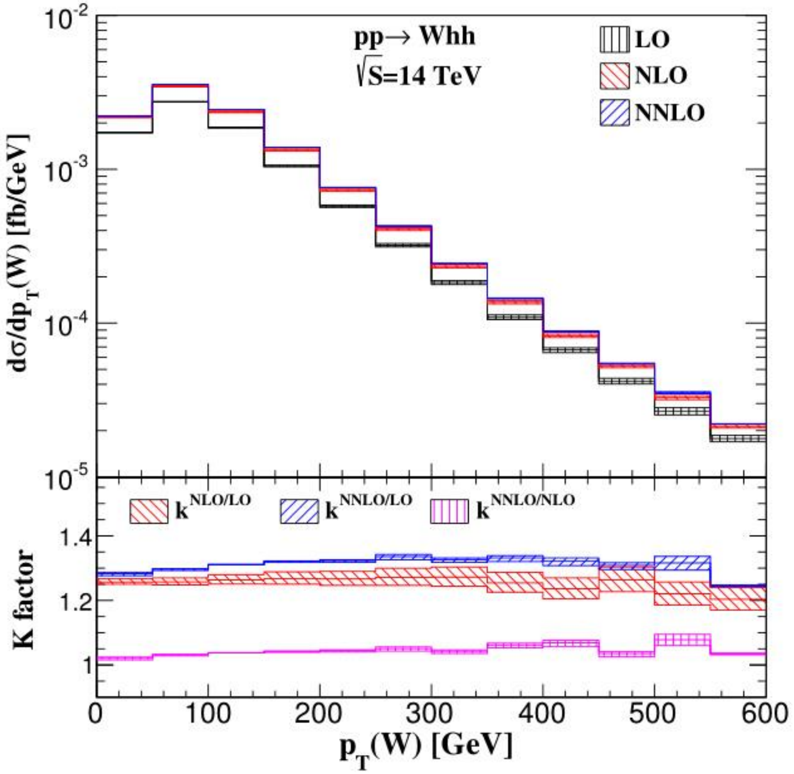
Numerical results

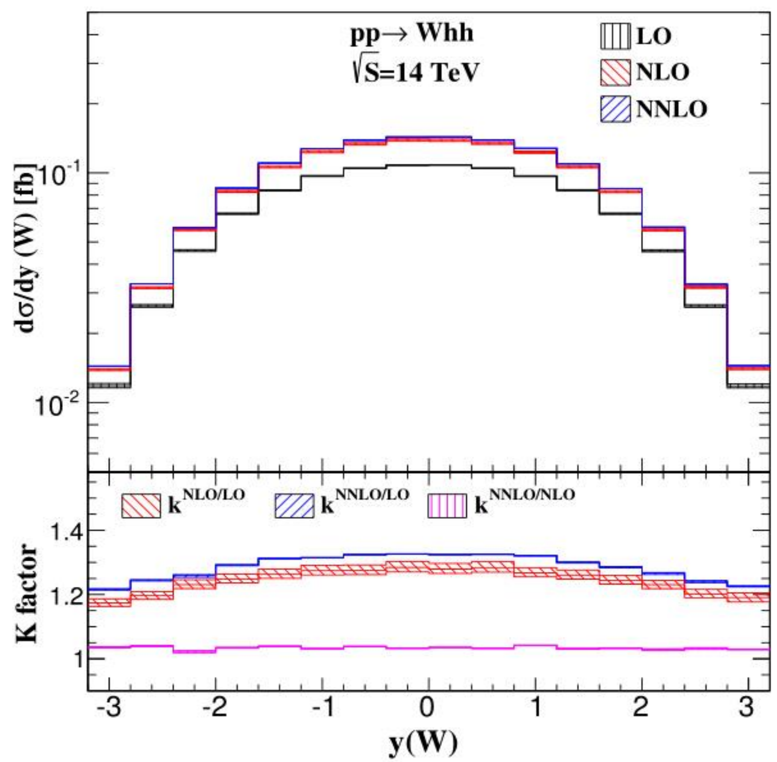




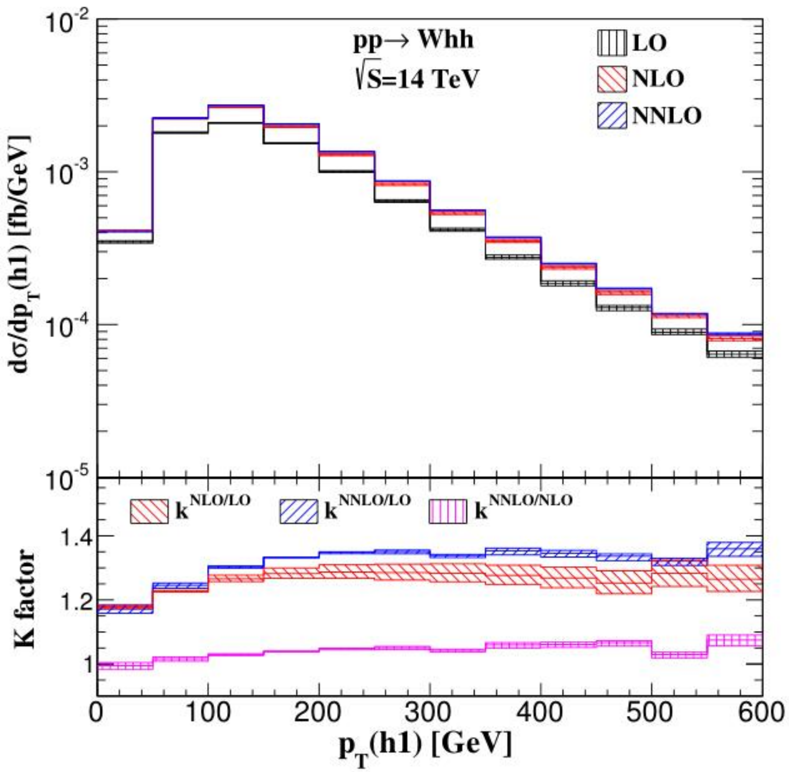
Agree with previous total xs in JHEP04(2013)151

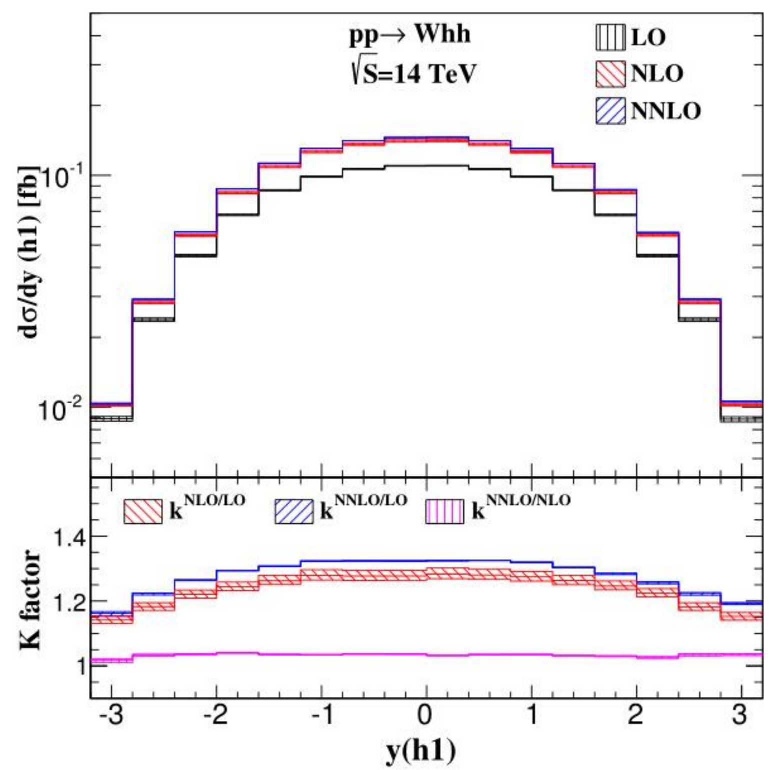
Kinematic distributions



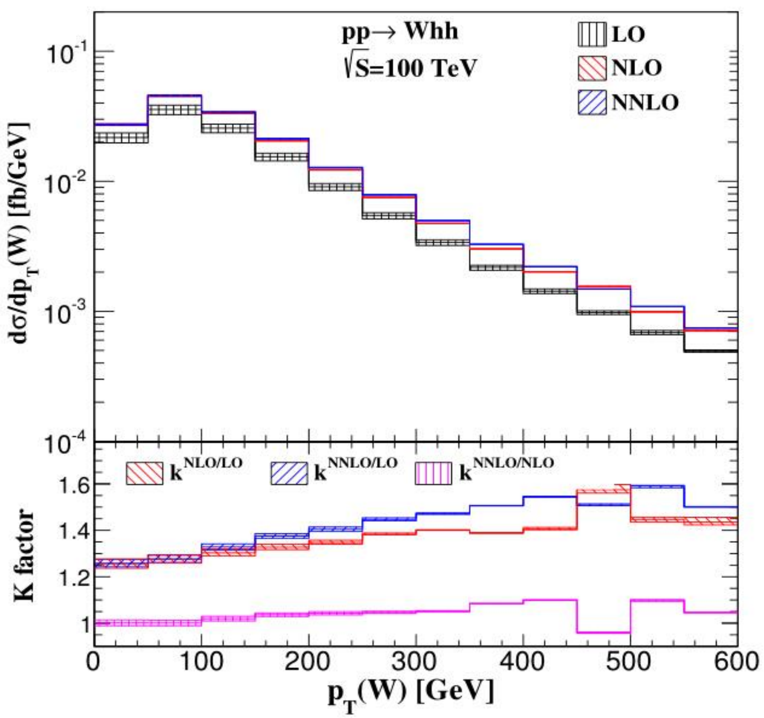


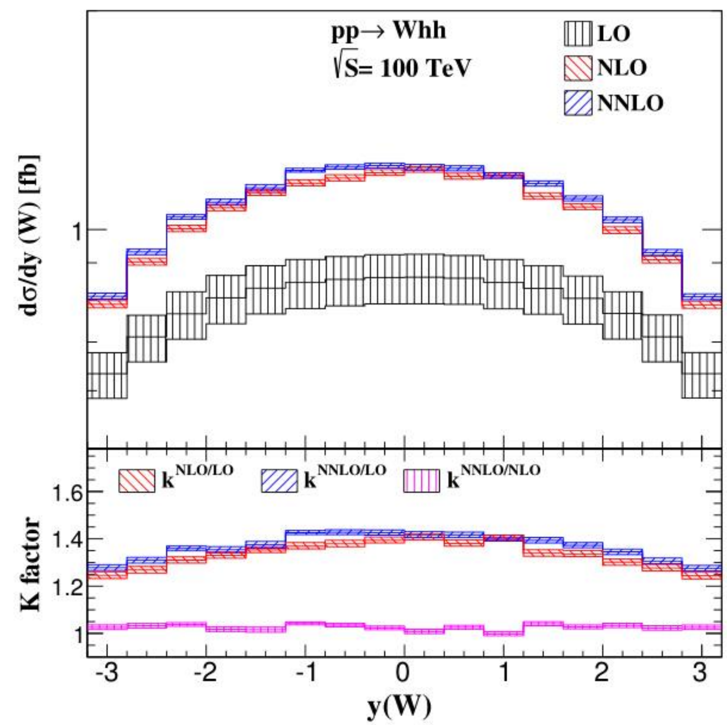
Kinematic distributions



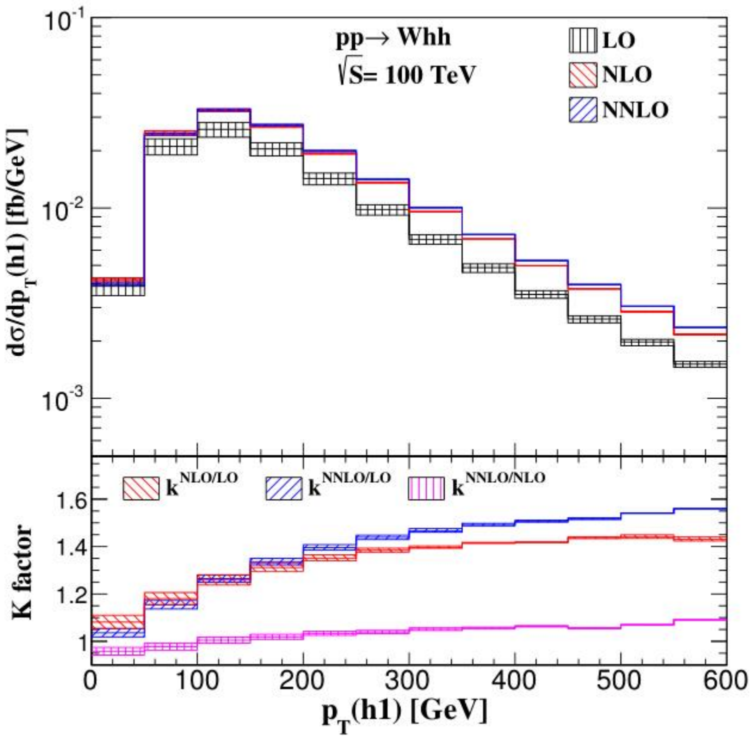


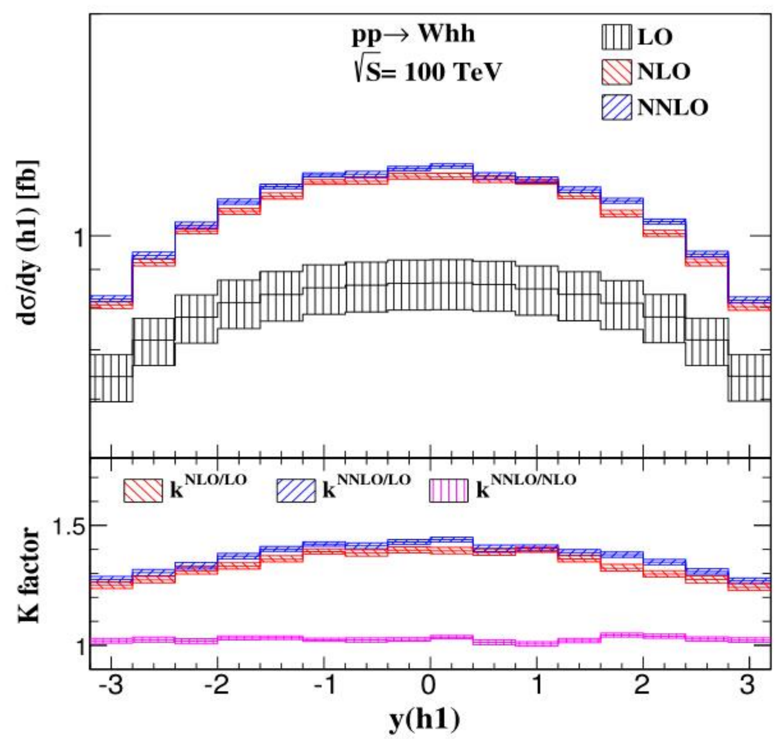
Kinematic distributions





Kinematic distributions





Cross sections after cuts

σ [fb]	boosted region	jet veto
LO	$0.271^{+3.0\%}_{-3.5\%}$	$6.30^{+7.1\%}_{-7.7\%}$
NLO	$0.360^{+0.5\%}_{-0.1\%}$	$3.76^{+6.3\%}_{-5.7\%}$
NNLO	$0.382^{+0.7\%}_{-0.5\%}$	$3.04^{+2.7\%}_{-2.2\%}$
$K^{\mathrm{NLO/LO}}$	1.33	0.60
$K^{\mathrm{NNLO/LO}}$	1.41	0.48
$K^{\mathrm{NNLO/NLO}}$	1.06	0.81

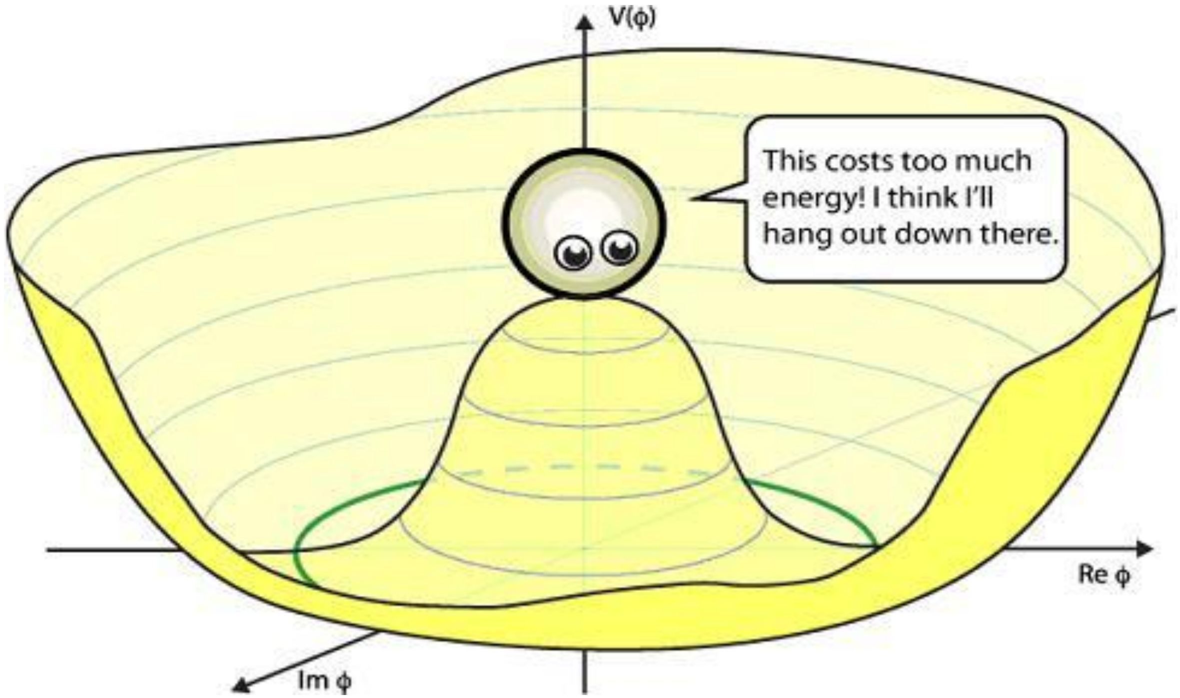
$p_T(W) > 200$ GeV, $|y(W)| < 2.4$,
$p_T(h) > 200$ GeV, $|y(h)| < 2.4$

$p_T(\mathrm{jet}) > 30$ GeV
$|\eta(\mathrm{jet})| < 3.5$
$R = 0.7$

Conclusions

- It is essential to measure the Higgs self-couplings after its discovery.
- This can be achieved by studying the Higgs pair production at colliders.
- We present the QCD NNLO prediction on the total cross section as well as the various kinematic distributions of this process based on q_T subtraction.
- The NNLO effects reduce the scale uncertainties significantly, and are sizable in the large transverse momentum region or jet-vetoed cross section.
- These theoretical results can be utilized in future experimental analysis.

Thank You!


V(ϕ)
This costs too much energy! I think I'll hang out down there.
Re ϕ
Im ϕ