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Resummation for jet processes

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Effective Field Theories for Collider Physics, Flavor Phenomena
and Electroweak Symmetry Breaking
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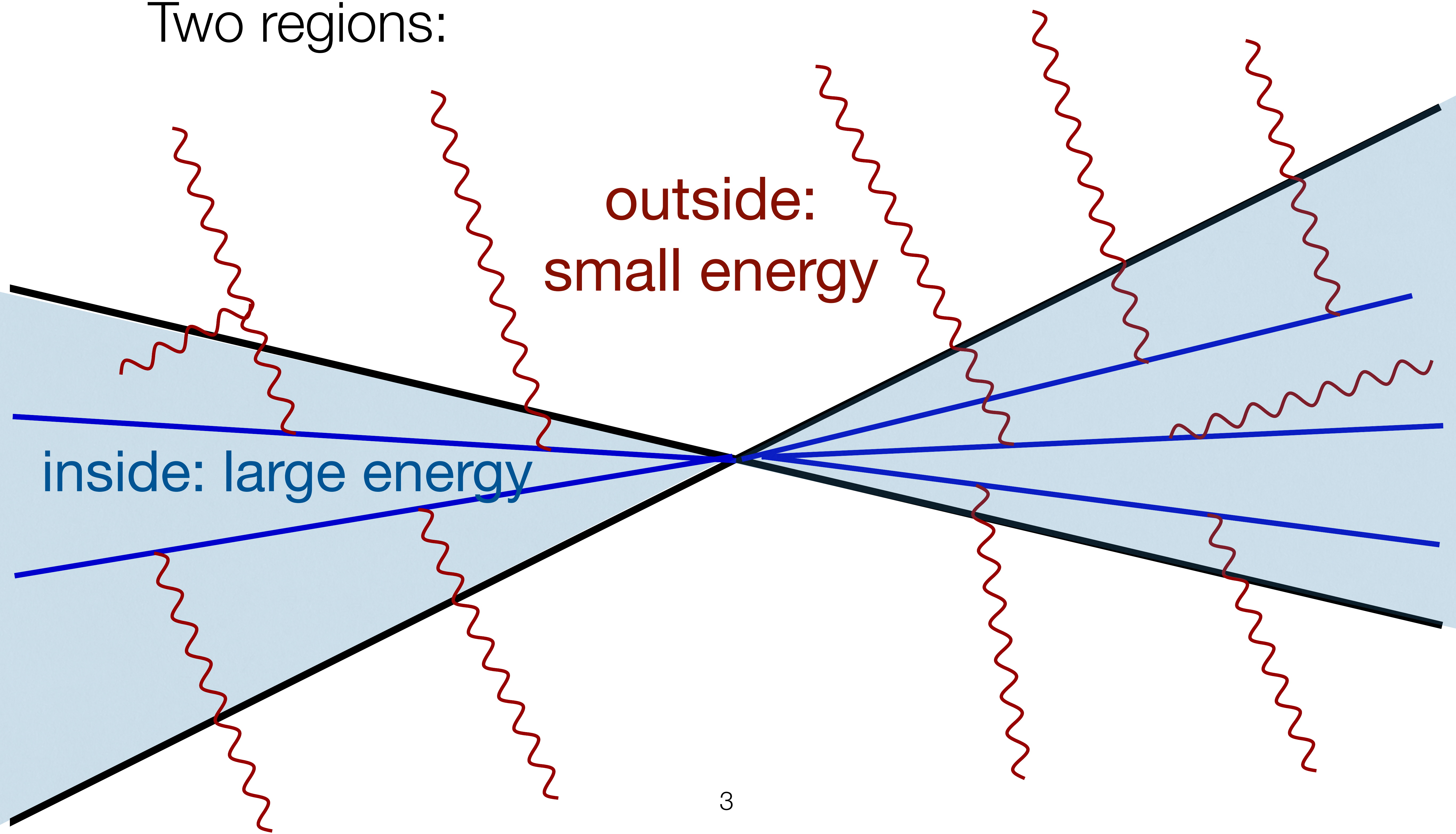
Long history of resummation of large logarithms in high-energy processes

- Sudakov ’56
- Yennie Frautschi Suura theory ‘61
- …
- Collins Soper Sterman (CSS) ‘84
- …

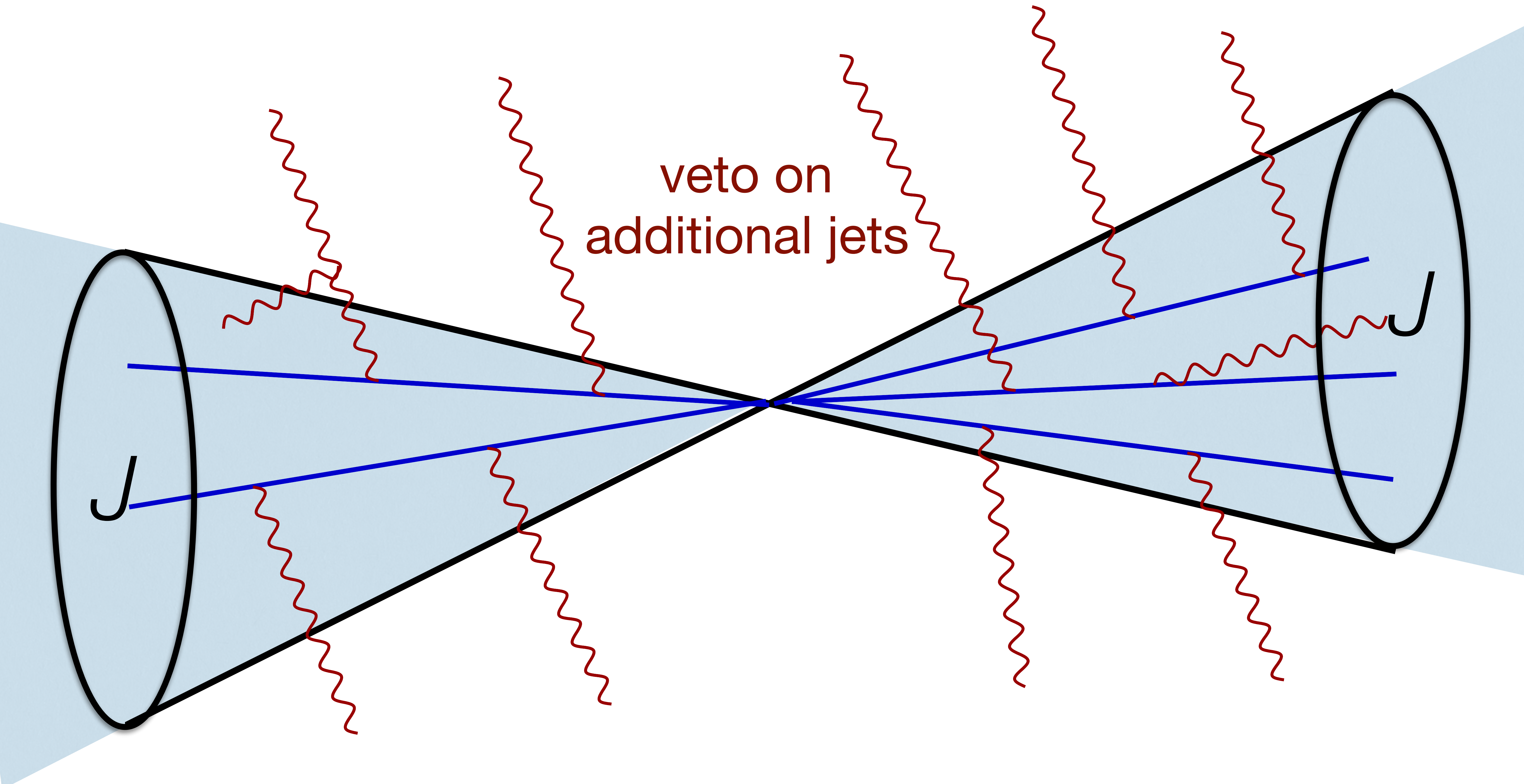
but despite modern EFT methods such as SCET, there are basic problems where the structure of higher-order logarithms is not known.

Consider the simplest collider-physics problem involving large logarithms.

Two regions:



Arises in many situation, in particular in all exclusive **jet cross sections**

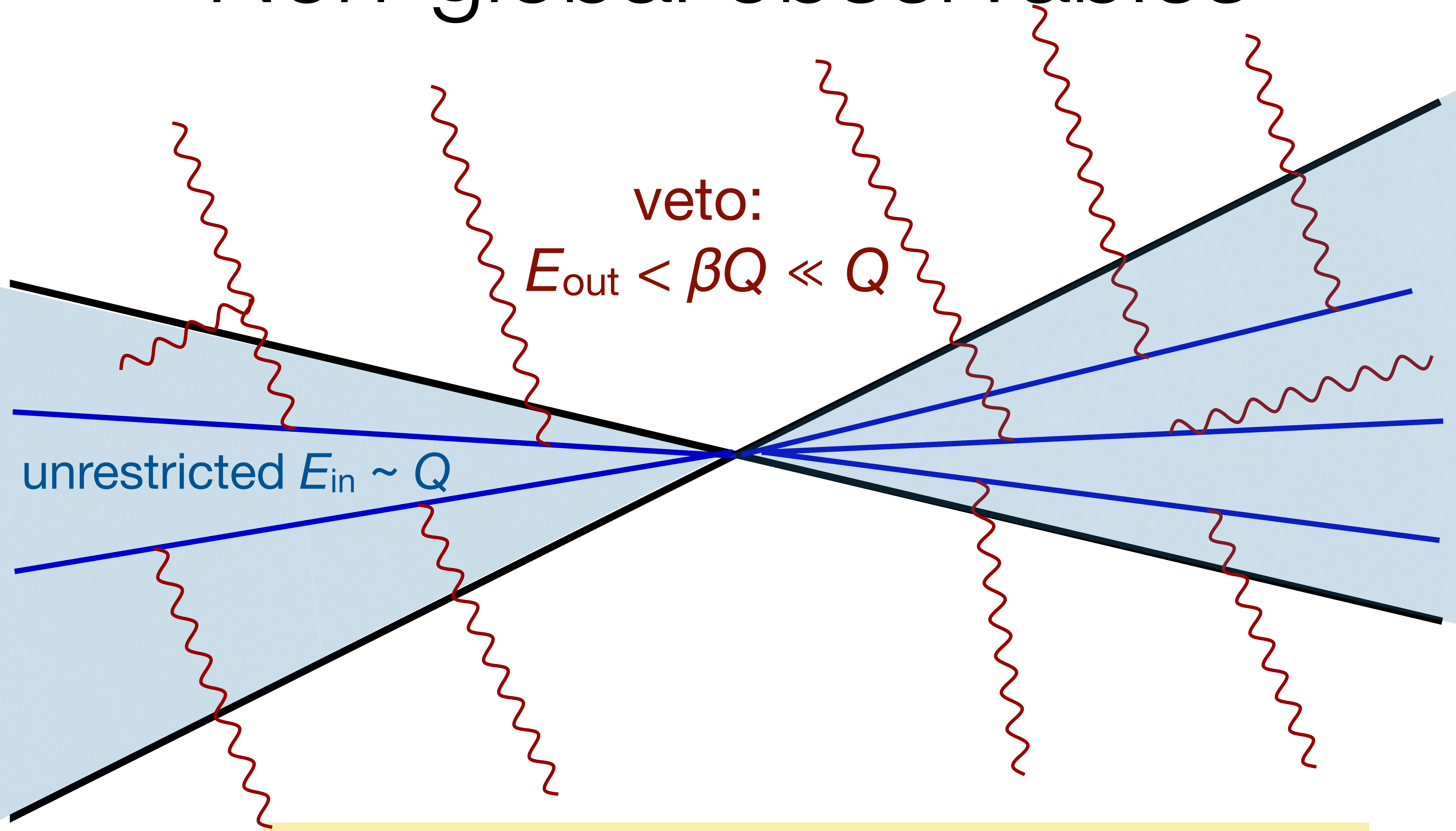


Many more examples

- jet vetoes (includes unrestricted radiation near the beam pipe)
- gaps between jets
- jet substructure
- isolated photons (veto on radiation near photon)
- event shapes such as the light-jet mass and narrow jet broadening
- …

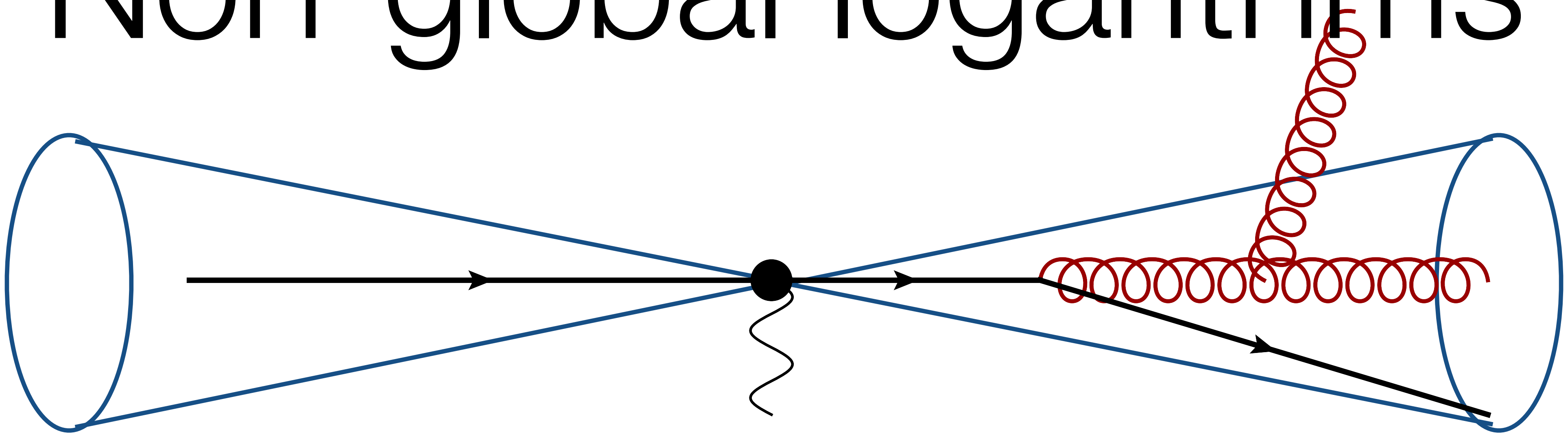
Such observables are called non-global, since they are insensitive to radiation inside certain regions of phase space.

Non-global observables



$\rightarrow$ large logs $\alpha_s^n \ln^n(E_{out} / E_{in}) \sim \alpha_s^n \ln^n(\beta)$

Non-global logarithms



Large logarithms $\alpha_s^n \ln^m(\beta)$ in non-global observables do not exponentiate Dasgupta and Salam '02.

Leading logarithms at large N_c can be obtained from non-linear integral equation

$$\partial_{\hat{L}} G_{kl}(\hat{L}) = \int \frac{d\,\Omega(n_j)}{4\pi} W_{kl}^j \left[\Theta_{\text{in}}^{n\bar{n}}(j)\, G_{kj}(\hat{L})\, G_{jl}(\hat{L}) - G_{kl}(\hat{L})\right]$$

Banfi, Marchesini, Smye '02

$\hat{L} \sim N_c\, \alpha_s \ln \beta$

$$W_{kl}^j = \frac{n_k \cdot n_l}{n_k \cdot n_j\, n_l \cdot n_j}$$ dipole radiator

LL resummation

- The leading logarithms arise from configurations in which the emitted gluons are strongly ordered

$$E_1 \gg E_2 \gg E_3 \gg \ldots \gg E_m$$

- Multi-gluon emission amplitudes become extremely simple in this limit, especially at large N_c

$$\left|\mathcal{M}_{ab}^{1\cdots m}\right|^2 = \left|\langle p_1 \cdots p_m \left| Y_a^\dagger Y_b \right| 0 \rangle\right|^2 = N_c^m g^{2m} \sum_{\text{perms of } 1\cdots m} \frac{(p_a \cdot p_b)}{(p_a \cdot p_1)\,(p_1 \cdot p_2) \cdots (p_m \cdot p_b)}$$

- Using their structure Banfi, Marchesini, Smye '02 derived an integral equation for resummation of leading logs at large N_c: BMS equation.

Non-global logarithms

A lot of recent work on these types of logarithms

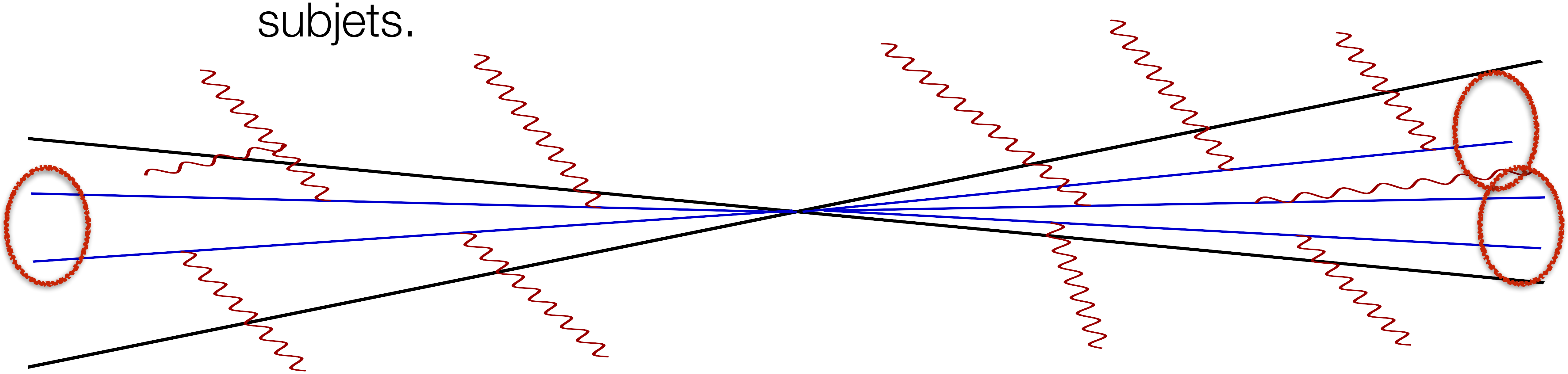
- Resummation of leading logs beyond large N_c Weigert '03, Hatta, Ueda '13 + Hagiwara '15; Caron-Huot '15.
  - Caron-Huot's functional RG has a close relation to our results
- Fixed-order results: 2 loops for $S(\omega_L,\omega_R)$. Kelley, Schwartz, Schabinger and Zhu '11; Hornig, Lee, Stewart, Walsh and Zuberi '11; with jet-cone Kelley, Schwartz, Schabinger and Zhu '11; von Manteuffel, Schabinger and Zhu '13, leading non-global log up to 5 loops by solving BMS equation Schwartz, Zhu '14, 5 loops and arbitrary N_c Delenda, Khelifa-Kerfa '15
- Approximate resummation of such logs, based on resummation for observables with *n* soft subjets. Larkoski, Moult and Neill '15

A systematic factorization of non-global observables was missing.

"Globalization"

Alternative SCET approach to observables with NGLs based on resummation for substructure. Larkoski, Moult, Neill '15

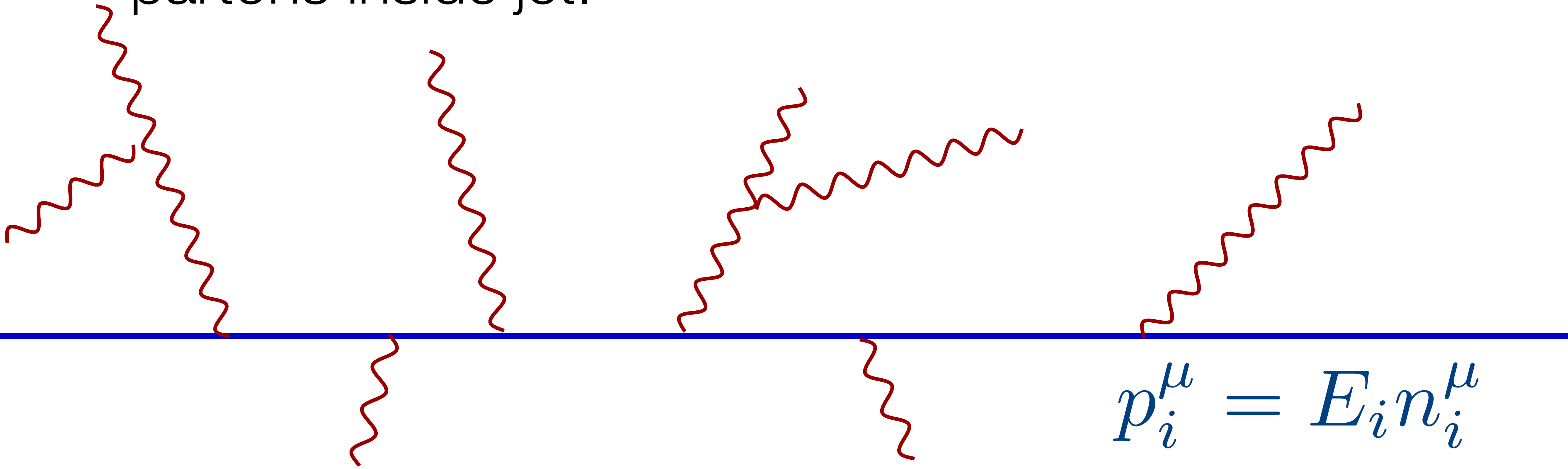
- Divide jet cross section into contributions from *n* sub-jets. Idea is to lower the hard scale in the NGLs by resolving the subjets.



- Resum global logarithms in subjet observables: "Dressed gluons".
- At leading-log level, this maps into iterative solution of BMS equation (talk by Ian Moult at LHC-ESI workshop)

Factorization for NGLs

Basic physics is soft radiation off energetic partons inside jet.



$$p_i^\mu = E_i n_i^\mu$$

Wilson line along direction of each hard parton inside the jet.

$$\boldsymbol{S}_i(n_i) = \mathbf{P} \exp\left(i g_s \int_0^\infty ds \, n_i \cdot A_s^a(s n_i) \, \boldsymbol{T}_i^a \right)$$

Wilson line and eikonal interaction

Consider one-gluon matrix element of Wilson line

$$\langle k,\lambda,b|\,\boldsymbol{S}_i\,|0\rangle = ig_s\,\boldsymbol{T}^a\int_0^\infty ds\,\langle k,\lambda,b|n_i\cdot A^a(sn_i)|0\rangle + \mathcal{O}(g_s^2)$$

$$= ig_s\,\boldsymbol{T}^a\int_0^\infty ds\,e^{isn_i\cdot k}\langle k,\lambda,b|n_i\cdot A^a_\mu(0)|0\rangle$$

$$= ig_s\,\boldsymbol{T}^b n_i\cdot\varepsilon(k,\lambda)\frac{e^{isn_i\cdot k}}{in_i\cdot k}\Bigg|_0^\infty$$

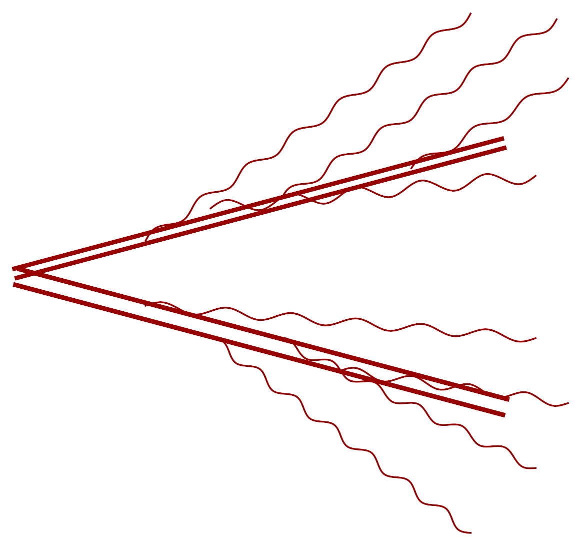
need small imaginary part n · k ≡ n · k + iε

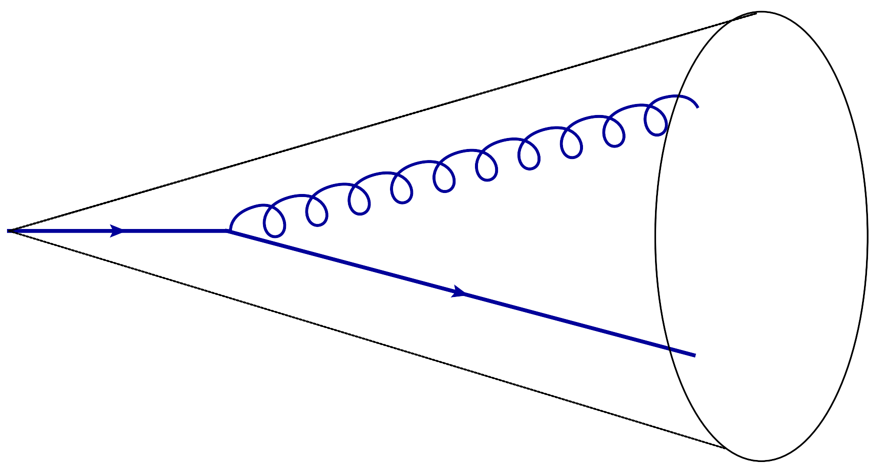
$$= -g_s\boldsymbol{T}^b\frac{n_i\cdot\varepsilon(k,\lambda)}{n_i\cdot k} = \boxed{-g_s\boldsymbol{T}^b\frac{p_i\cdot\varepsilon(k,\lambda)}{p_i\cdot k}}$$

eikonal interaction

Soft emissions in process with m energetic particles are obtained from the matrix elements of the operator

$$\boldsymbol{S}_1(n_1)\,\boldsymbol{S}_2(n_2)\;\ldots\;\boldsymbol{S}_m(n_m)|\mathcal{M}_m(\{\underline{p}\})\rangle$$





soft Wilson lines along the directions of the energetic particles / jets (color matrices)

soft particles can be inside or outside

hard scattering amplitude with *m* particles (vector in color space)

energetic partons must be inside

For a jet of several (nearly) collinear energetic particles, one can combine

$$\boldsymbol{S}_1(n)\,\boldsymbol{S}_2(n) = \mathbf{P}\exp\left(ig_s\int_0^\infty ds\, n\cdot A_s^a(sn)\,(\boldsymbol{T}_1^a+\boldsymbol{T}_2^a)\right)$$

into a single Wilson line with the total color charge.

For non-global observables one cannot combine the soft Wilson lines → complicated structure of logs!

- For a wide-angle jet, the energetic particles are not collinear.
- For a narrow-angle jets (see later), we find that small-angle soft radiation plays an important role. Resolves directions of individual energetic partons!

Factorization theorem

TB, Neubert, Rothen, Shao ’15 ’16, see also Caron-Huot ‘15

Hard function.
m hard partons along
fixed directions $\{n_1, \ldots, n_m\}$

Soft function
with *m* Wilson lines

$$\sigma(\beta) = \sum_{m=2}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \mu) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q\beta, \mu) \rangle \,,$$

color trace

integration over the *m* directions

First all-order factorization theorem for non-global observable. Achieves scale separation!

Comments

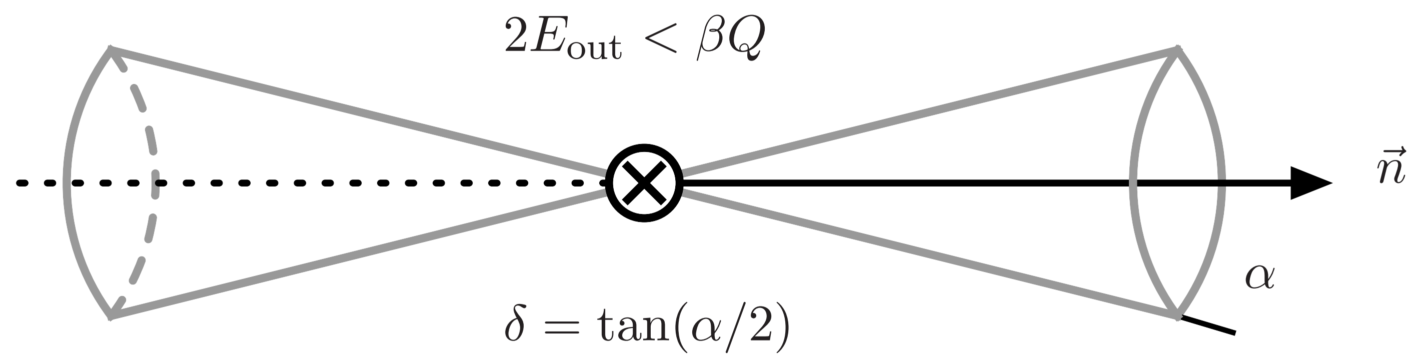
- Infinitely many operators $\mathcal{S}_m$, mix under RG
- Also for narrow-cone jets, the same type of structure is relevant TB, Neubert, Rothen, Shao '15 '16

$$\boldsymbol{\mathcal{H}}_m \otimes \boldsymbol{\mathcal{S}}_m \longrightarrow \boldsymbol{\mathcal{J}}_m \otimes \boldsymbol{\mathcal{U}}_m$$

collinear

"coft"
soft+collinear

- **Check:** Have computed all ingredients for cone cross section at NNLO. Obtain full logarithmic structure at this order.



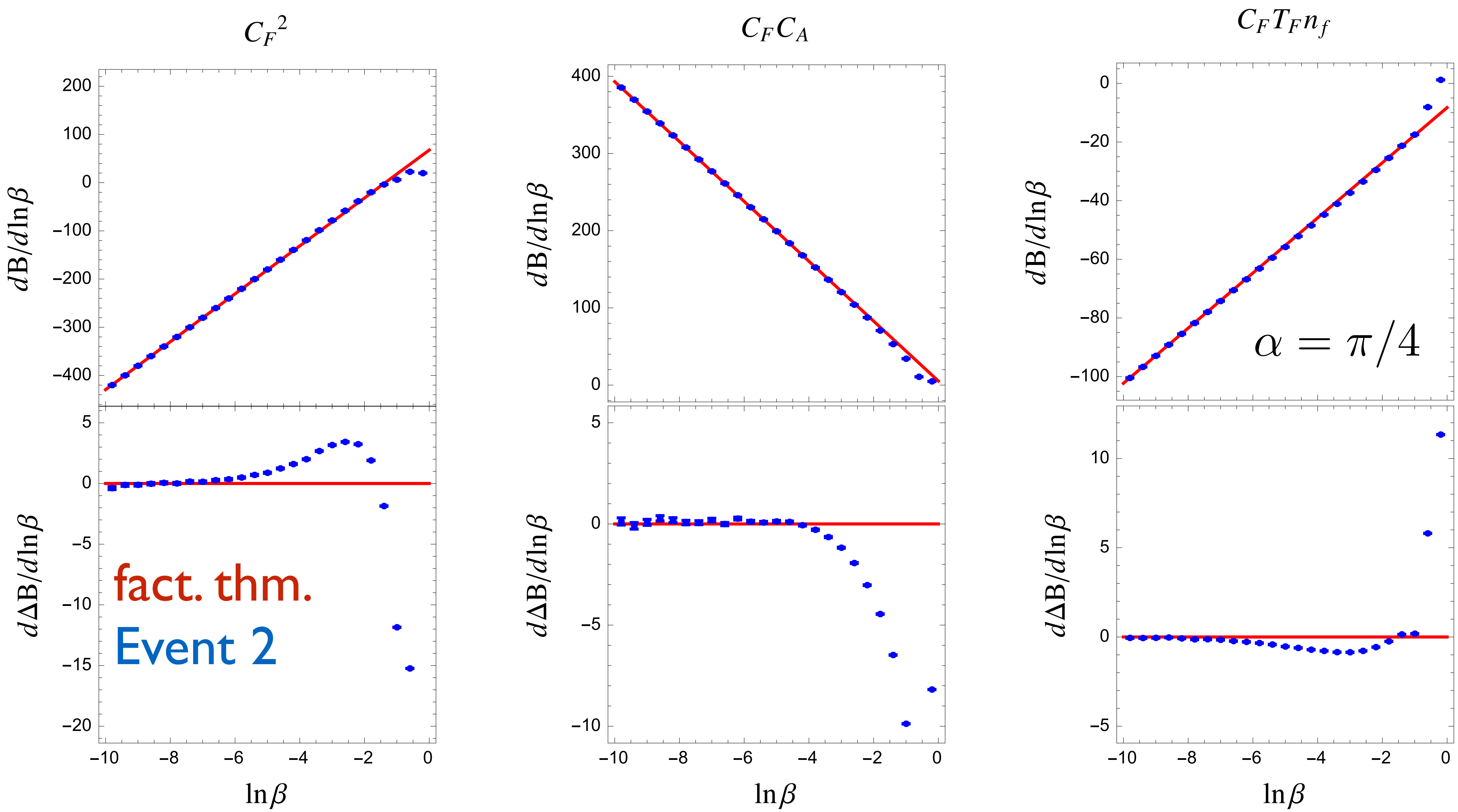
$$\frac{\sigma(\beta,\delta)}{\sigma_0} = 1 + \frac{\alpha_s}{2\pi} A(\beta,\delta) + \left(\frac{\alpha_s}{2\pi}\right)^2 B(\beta,\delta) + \dots .$$

$$A(\beta,\delta) = C_F \left[-8\ln\delta\ln\beta - 1 + 6\ln 2 - 6\ln\delta - 6\delta^2 + \left(\frac{9}{2} - 6\ln 2\right)\delta^4 + 4\,\mathrm{Li}_2(\delta^2) - 4\,\mathrm{Li}_2(-\delta^2)\right]$$

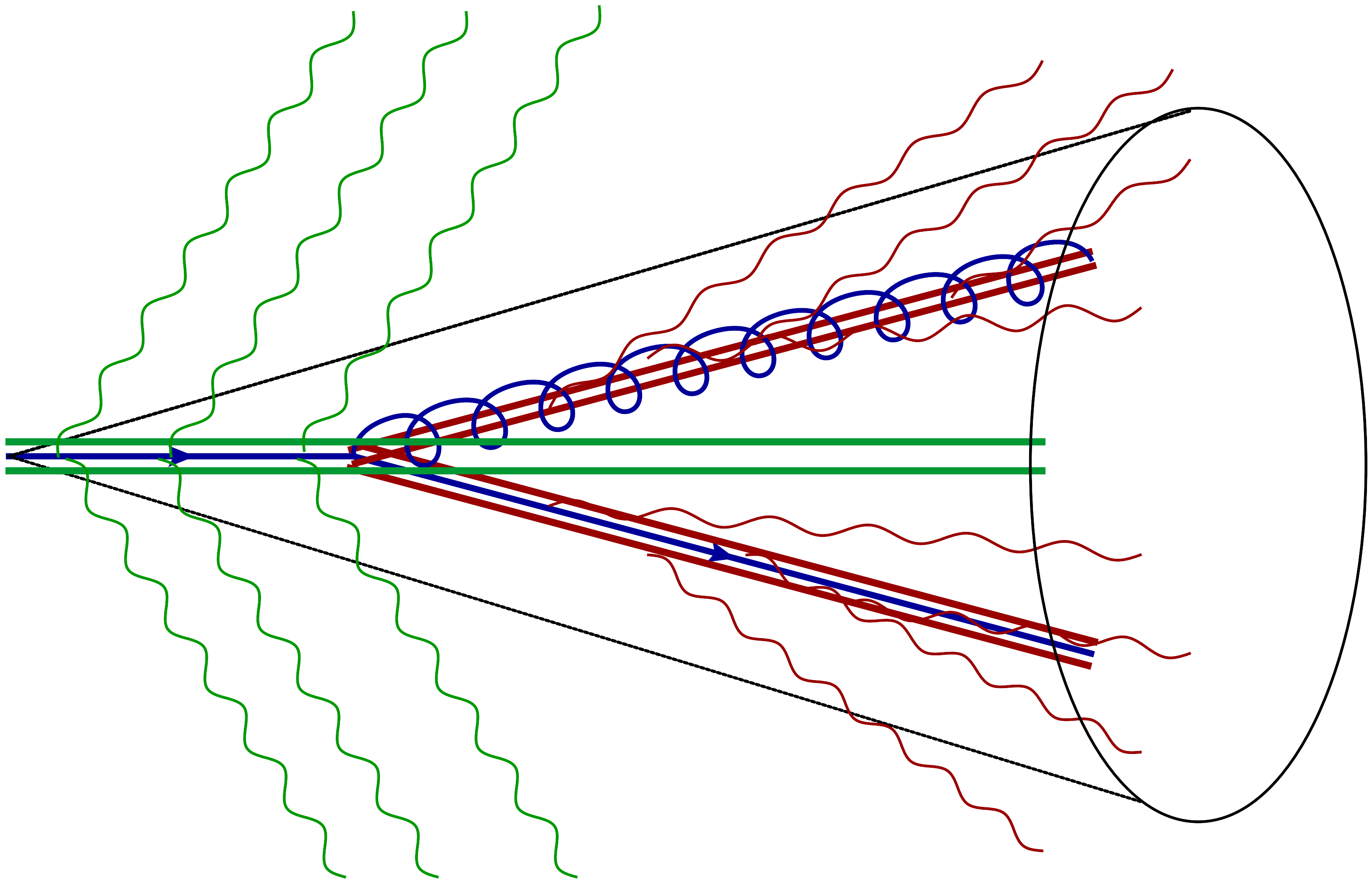
$$B(\beta,\delta) = C_F^2 B_F + C_F C_A B_A + C_F T_F n_f B_f$$

$$\begin{aligned}
B_A = {}& \frac{4}{3}\left[11\ln\delta - \frac{\pi^2}{2} + 3\,\mathrm{Li}_2(\delta^4)\right]\ln^2\beta + \frac{4}{3}\Bigg[11\ln^2\delta - \frac{67\ln\delta}{3} + \frac{4\delta^4\ln\delta}{(1-\delta^4)^2} + \frac{1}{1-\delta^4} \\
& + 36\ln\delta\ln^2\left(1-\delta^2\right) - 12\ln\delta\ln^2\left(1+\delta^2\right) + 22\ln\delta\ln\left(1-\delta^2\right) - 5\pi^2\ln\left(1-\delta^2\right) \\
& + 22\ln\delta\ln\left(1+\delta^2\right) - \pi^2\ln\left(1+\delta^2\right) - 4\ln^3\left(1+\delta^2\right) + 33\,\mathrm{Li}_2\left(-\delta^2\right) + 22\,\mathrm{Li}_2\left(\delta^2\right) \\
& + 48\ln\delta\,\mathrm{Li}_2\left(-\delta^2\right) - 12\ln\left(1-\delta^2\right)\mathrm{Li}_2\left(-\delta^2\right) - 36\ln\left(1+\delta^2\right)\mathrm{Li}_2\left(-\delta^2\right) \\
& + 12\ln 2\,\mathrm{Li}_2\left(-\delta^2\right) + 24\ln\delta\,\mathrm{Li}_2\left(\delta^2\right) + 24\ln\left(1-\delta^2\right)\mathrm{Li}_2\left(\delta^2\right) + 12\ln 2\,\mathrm{Li}_2\left(\delta^2\right) \\
& + 12\ln\left(1-\delta^4\right)\mathrm{Li}_2\left(1-\delta^2\right) - 6\,\mathrm{Li}_3\left(1-\delta^4\right) + 24\,\mathrm{Li}_3\left(1-\delta^2\right) - 36\,\mathrm{Li}_3\left(-\delta^2\right) \\
& - 36\,\mathrm{Li}_3\left(\delta^2\right) + 24\,\mathrm{Li}_3\left(\frac{\delta^2}{1+\delta^2}\right) - 12\,\zeta_3 - \frac{11\pi^2}{12} - \frac{1}{2} - \pi^2\ln 2 - \frac{3}{8}M_A^{[1]}(\delta)\Bigg]\ln\beta \\
& + c_2^A(\delta)\,,
\end{aligned}$$

Numerical check against **Event2**



- Works: agreement for small β.
- Reproduce all logs, not only the leading ones!



Narrow-angle jets

Soft emissions from a narrow jet

For a narrow jet $\delta \to \mathbf{0}$ in direction *n* one would expect that one could combine

$$\boldsymbol{S}_1(n_1)\,\boldsymbol{S}_2(n_2) \approx \mathbf{P}\exp\left(ig_s\int_0^\infty ds\,n\cdot A_s^a(sn)\,(\boldsymbol{T}_1^a+\boldsymbol{T}_2^a)\right)$$

since $n_1 \approx n_2 \approx n$.

Doing so, one ends up with a single Wilson line per jet and a simple form of the soft radiation.

- Works for global observables such as thrust, broadening, …

Soft emissions from a narrow jet

Consider the emission of single soft a gluon from energetic particles with momenta p_i inside a narrow jet:

$$\sum_i Q_i \frac{p_i \cdot \varepsilon}{p_i \cdot k} = Q_{\rm tot} \frac{n \cdot \varepsilon}{n \cdot k} + \dots$$

Approximation: $p_i^\mu \approx E_i \, n^\mu$

This approximation breaks down when the soft emission has a small angle, i.e. when $k^\mu \approx \omega \, n^\mu$!

Small region of phase space, but it turns out that it gives a leading contribution to jet rates!

Momentum modes for jet processes

TB, Neubert, Rothen, Shao, 1508.06645; Chien, Hornig and Lee 1509.04287

	Region	Energy	Angle	Inv. Mass
standard SCET	Hard	Q	1	Q
	Collinear	Q	δ	Qδ
	Soft	βQ	1	βQ
new	Coft	βQ	δ	βδQ

Full jet cross section is recovered after adding the contributions from all regions ("method of regions")

- Additional coft mode has **very low characteristic scale βδQ**! Jets are less perturbative than they seem!
- Effective field theory has additional "coft" degree of freedom.

Momentum modes again (for experts)

Split momenta into light-cone components

$$p^\mu = p_+ \frac{n^\mu}{2} + p_- \frac{\bar{n}^\mu}{2} + p_\perp^\mu$$

Scaling of the momentum components ($\beta \sim \delta^2$)

$$
\begin{array}{rl}
 & \quad\;\; (\; p_+ \;,\; p_- \;,\; p_\perp \;) \\
\text{collinear:} & p_c \sim Q\,(\; 1 \;,\; \delta^2 \;,\; \delta \;) \\
\text{soft:} & p_s \sim Q\,(\; \beta \;,\; \beta \;,\; \beta \;) \\
\text{coft:} & p_t \sim \beta Q\,(\; 1 \;,\; \delta^2 \;,\; \delta \;)
\end{array}
$$

Note: every component of coft mode is smaller than the corresponding collinear one. Different than SCET_I , SCET_II, $\text{SCET}_{1.5}$, SCET_n , SCET_+, …

Method of regions expansion

To isolate the different contributions, one expands the amplitudes as well as the phase-space constraints in each momentum region.

- Generic soft mode has O(1) angle: after expansion, it is always outside the jet.
- Collinear mode has large energy $E \gg \beta Q$. Can never go outside the jet.
- Coft mode can be inside or outside, but its contribution to the momentum inside the jet is negligible.

Expansion is performed on the integrand level: the full result is obtained after combining the contributions from the different regions.

Factorization for two-jet cross section

TB, Neubert, Rothen, Shao, arXiv:1508.06645

Laplace space
$\tau \leftrightarrow \beta$

color trace

integration over angles

$$\widetilde{\sigma}(\tau) = \sigma_0 \, H(Q) \, \widetilde{S}(Q\tau) \left[\sum_{m=1}^{\infty} \left\langle \mathcal{J}_m(Q\delta) \otimes \widetilde{\mathcal{U}}_m(Q\delta\tau) \right\rangle \right]^2$$

Coft functions with
m Wilson lines

Soft function

Hard function

Jet functions with *m* partons
at fixed direction

Checks against wide-angle result and fixed-order event generator.

All-order resummation

$$\sigma(\beta) = \sum_{m=2}^{\infty} \big\langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \mu) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q\beta, \mu) \big\rangle \, ,$$

High-*E* physics
Wilson coefficients

Low-*E* physics
EFT Operator

- Renormalization of hard Wilson coefficients

$$\boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \delta, \epsilon) = \sum_{l=2}^{m} \boldsymbol{\mathcal{H}}_l(\{\underline{n}\}, Q, \delta, \mu)\, \boldsymbol{Z}^H_{lm}(\{\underline{n}\}, Q, \delta, \epsilon, \mu)$$

  - Same *Z*-factor must render $\mathcal{S}_\mathrm{m}$ finite!
  - Associated anomalous dimension $\boldsymbol{\Gamma}^H$

$$\frac{d}{d\ln\mu}\, \boldsymbol{Z}^H_{km}(\{\underline{n}\}, Q, \delta, \epsilon, \mu) = \sum_{l=k}^{m} \boldsymbol{Z}^H_{kl}(\{\underline{n}\}, Q, \delta, \epsilon, \mu)\, \hat{\otimes}\, \boldsymbol{\Gamma}^H_{lm}(\{\underline{n}\}, Q, \delta, \mu)$$

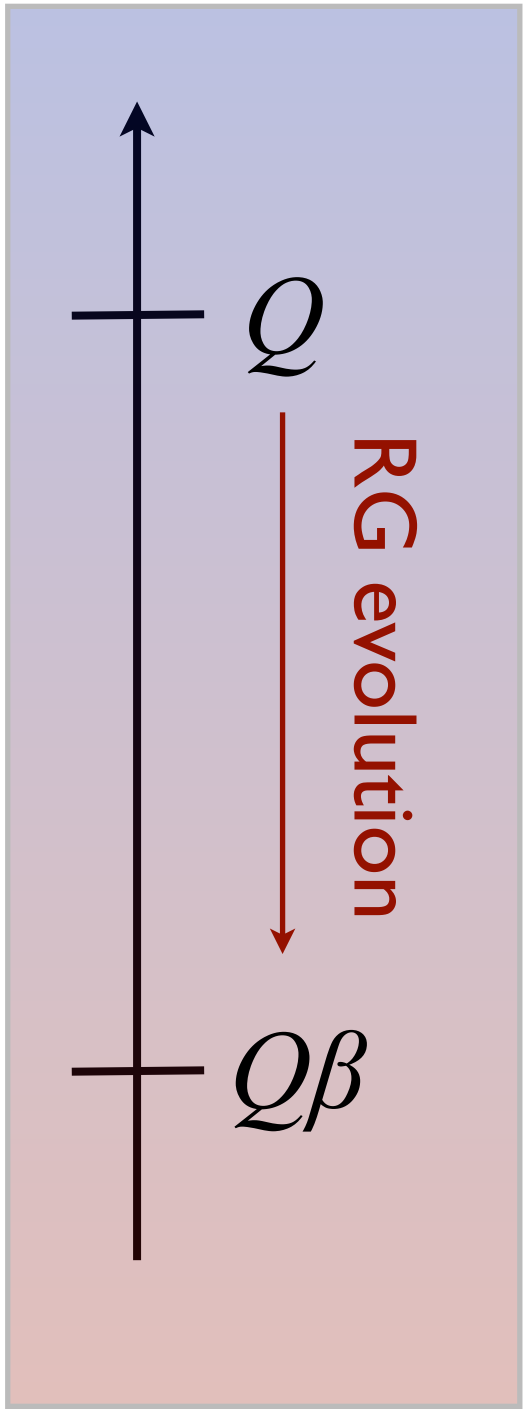
Resummation by RG evolution

Wilson coefficients fulfill renormalization group (RG) equations

$$\frac{d}{d\ln\mu}\boldsymbol{\mathcal{H}}_m(Q,\mu) = -\sum_{l=2}^{m}\boldsymbol{\mathcal{H}}_l(Q,\mu)\,\boldsymbol{\Gamma}^H_{lm}(Q,\mu)$$

1. Compute $\mathcal{H}_{\mathrm{m}}$ at a characteristic high scale $\mu_h \sim Q$
2. Evolve $\mathcal{H}_{\mathrm{m}}$ to the scale of low energy physics $\mu_l \sim Q\beta$

Avoids large logarithms $\alpha_s{}^n \ln^n(\beta)$ of scale ratios which can spoil convergence of perturbation theory.



RG = Parton Shower

- Ingredients for LL

$$\boldsymbol{\mathcal{H}}_2(\mu = Q) = \sigma_0$$
$$\boldsymbol{\mathcal{H}}_m(\mu = Q) = 0 \text{ for } m > 2$$
$$\boldsymbol{\mathcal{S}}_m(\mu = \beta Q) = 1$$

$$\boldsymbol{\Gamma}^{(1)} = \begin{pmatrix} \boldsymbol{V}_2 & \boldsymbol{R}_2 & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_3 & \boldsymbol{R}_3 & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_4 & \boldsymbol{R}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

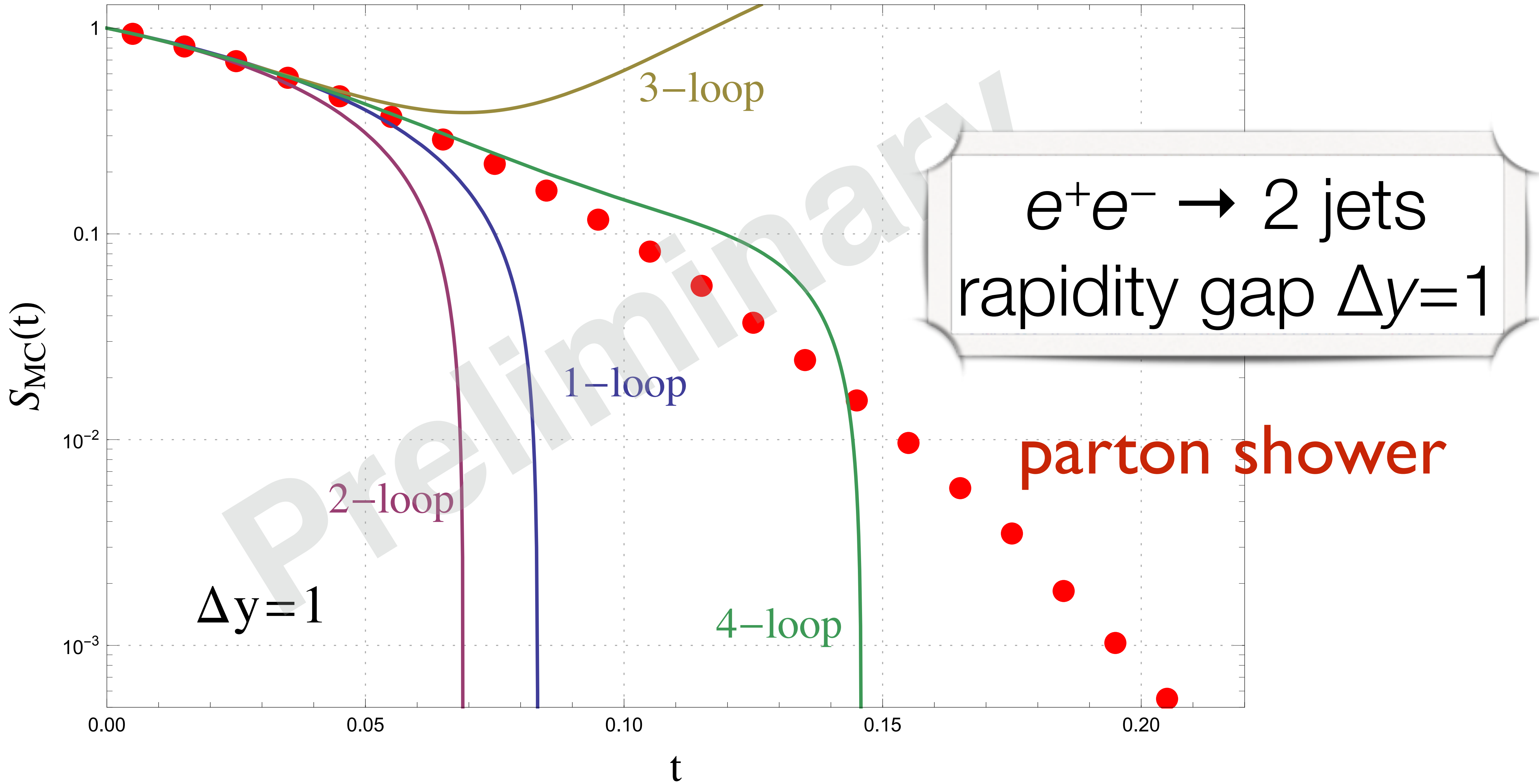
- RG

$$\frac{d}{dt}\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t)\boldsymbol{V}_m + \boldsymbol{\mathcal{H}}_{m-1}(t)\boldsymbol{R}_{m-1} \,.$$

$$t = \int_{\alpha(\mu)}^{\alpha(Q)} \frac{d\alpha}{\beta(\alpha)}\,\frac{\alpha}{4\pi}$$

- Equivalent to parton shower equation

$$\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t_1)e^{(t-t_1)V_n} + \int_{t_1}^{t} dt'\boldsymbol{\mathcal{H}}_{m-1}(t')\boldsymbol{R}_{m-1}e^{(t-t')\boldsymbol{V}_n}$$



- Equivalent to the dipole shower used by Dasgupta and Salam '02.
- For higher-log accuracy we will need to include corrections to $\mathcal{H}_m$, $\mathcal{S}_m$, $\boldsymbol{\Gamma}_{mn}$ into the shower.

Conclusion and Outlook

- Have obtained factorization formulae for non-global observables. Key features
  - Multi-Wilson line structure of soft radiation
  - Resummation of NGLs from RG evolution
- Are developing MC formalism for higher-log resummation
- Have applied formalism to hemisphere soft function and light-jet mass TB, Pecjak, Shao, in preparation
  - factorization theorems have same general structure as the ones for jet cross sections
- Applications …
- Interplay with Glauber gluons? Superleading logs?

Extra slides

Hemisphere soft function

- Most past studies of NGLs were performed for hemisphere soft function

$$S(\omega_L, \omega_R) = \frac{1}{N_c} \sum_X \mathrm{Tr} \langle 0 | S(\bar{n}) S^\dagger(n) | X \rangle \langle X | S(n) S^\dagger(\bar{n}) | 0 \rangle \delta(\omega_R - n \cdot P_R)\, \delta(\omega_L - \bar{n} \cdot P_L)$$

  - Leading logs are related to the ones arising in light-jet mass event shape

- Factorization formula for $\omega_L \ll \omega_R$

$$S(\omega_L, \omega_R) = \sum_{m=0}^{\infty} \langle \boldsymbol{\mathcal{H}}_m^S(\{\underline{n}\}, \omega_R) \otimes \boldsymbol{\mathcal{S}}_{m+1}(\{n, \underline{n}\}, \omega_L) \rangle$$

mode with $p_\mu \sim \omega_R$

mode with $p_\mu \sim \omega_L$

Factorization theorem for left-jet mass

- Heavy jet mass is global, light jet mass non-global

$$\text{heavy jet mass:}\ \ \rho_h = \frac{1}{Q^2}\max(M_L^2, M_R^2)$$

$$\text{light jet mass:}\ \ \rho_\ell = \frac{1}{Q^2}\min(M_L^2, M_R^2)$$

- Relation to left-jet mass ρ_L

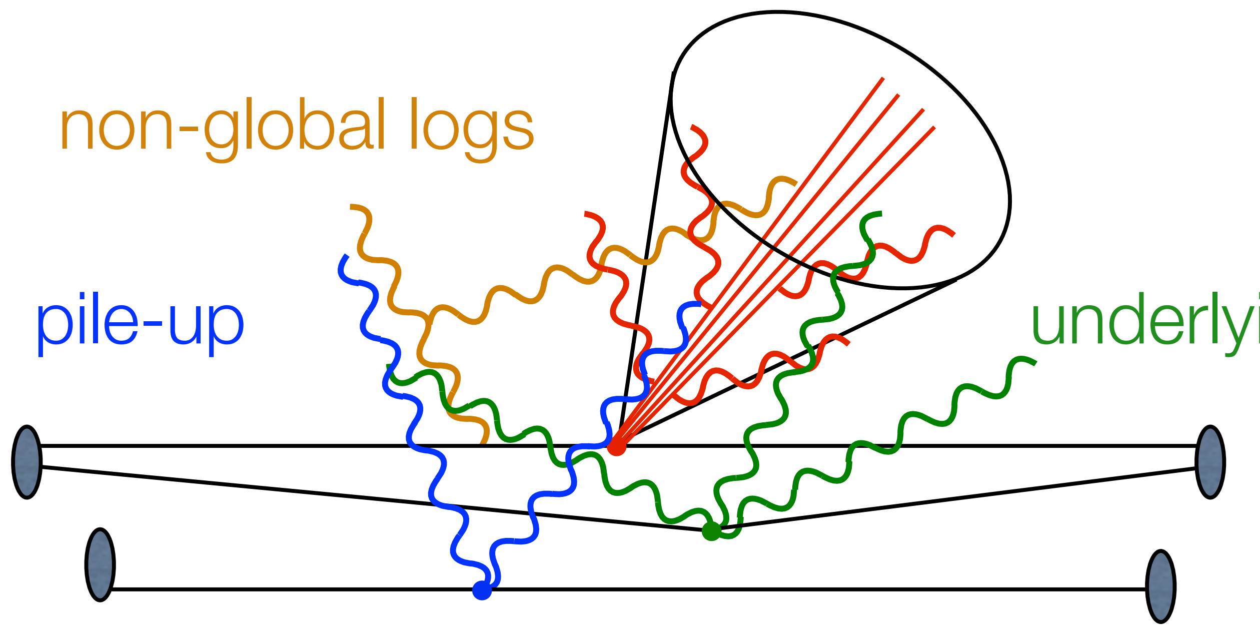
$$\frac{d\sigma}{d\rho_\ell} = 2\frac{d\sigma}{d\rho_L} - \frac{d\sigma}{d\rho_h}\bigg|_{\rho_L=\rho_h=\rho_\ell}$$

- Factorization formula

$$\frac{d\sigma}{dM_L^2} = \sum_{i=q,\bar{q},g}\int_0^\infty d\omega_R\, J_i(M_L^2 - Q\,\omega_L)\sum_{m=1}^{\infty}\langle \boldsymbol{\mathcal{H}}_m^i(\{\underline{n}\}, Q)\otimes \boldsymbol{\mathcal{S}}_m(\{n, \underline{n}\}, \omega_L)\rangle$$

Jet substructure: m_J in $pp \to Z + j$

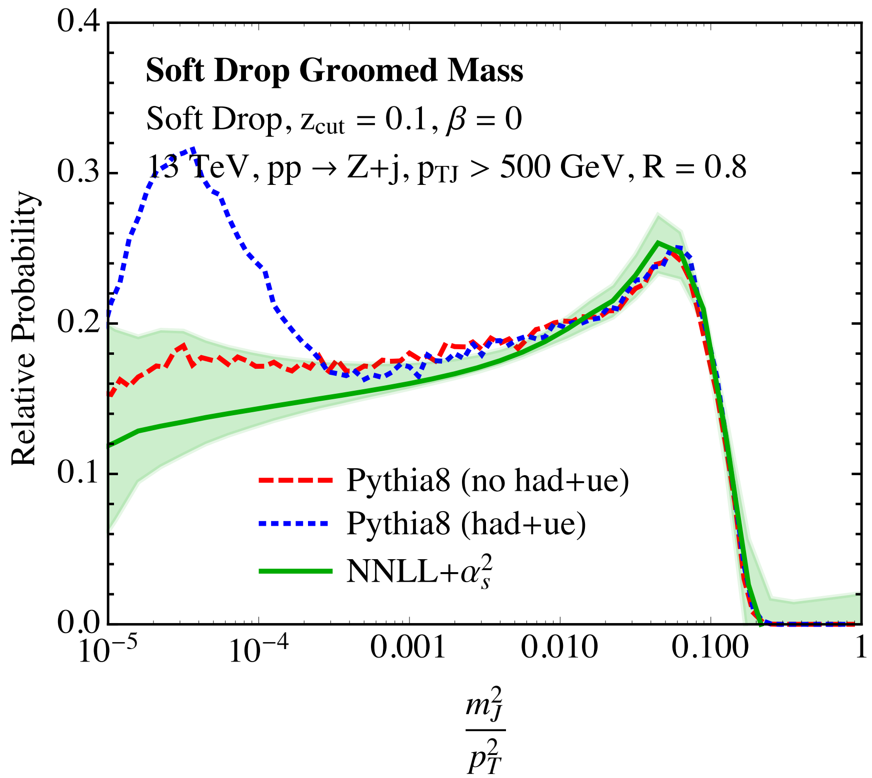
Challenges and contaminations

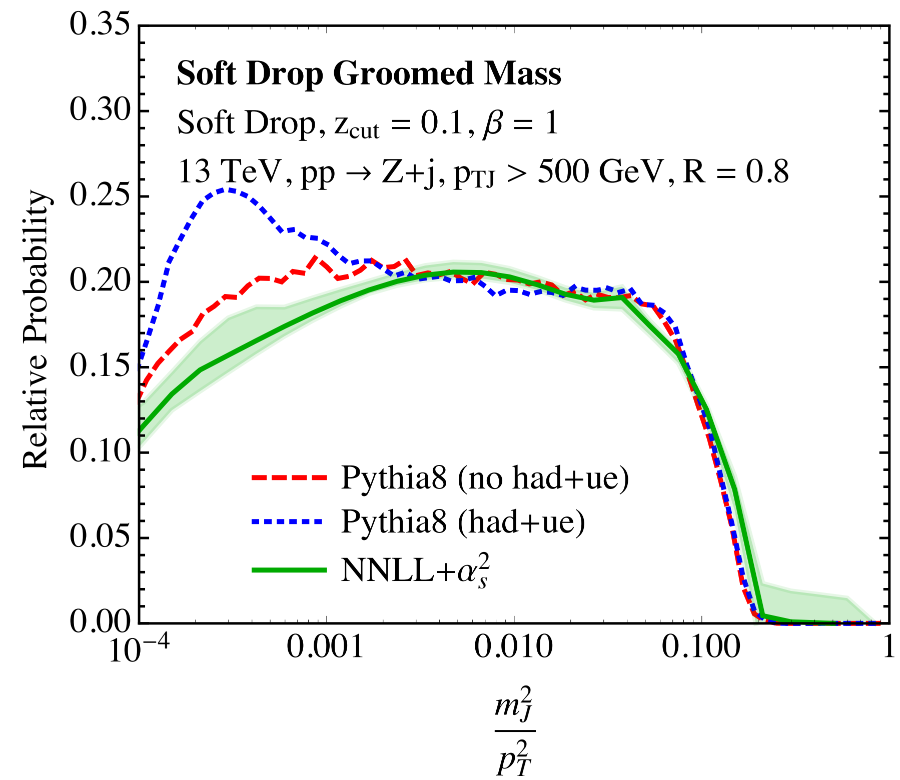


- Grooming can mitigate these problems
- mMDT also eliminates NGLs in m_J
- Analytical NLL Dasgupta, Fregoso, Marzani, Salam '13, Larkoski, Marzani, Soyez,Thaler '14

NNLL + $O(\alpha_s^2)$ for jet mass

Frye, Larkoski, Schwartz, Yan'16





Based on factorization

$$m_J^2 \ll z_{\rm cut} p_{TJ}^2 \ll p_{TJ}^2$$

$$\frac{d\sigma^{\rm resum}}{dm_J^2} = \sum_{k=q,\bar{q},g} D_k(p_T, z_{\rm cut}, R) S_{C,k}(z_{\rm cut} m_J^2) \otimes J_k(m_J^2)$$

sum over jet flavor

includes pdfs, emissions that were groomed away, out-of-jet radiation,...

collinear-soft radiation

hard collinear radiation