Precise predictions for gauge-boson pair production processes at the LHC

Barbara Jäger

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what's the plan for today?

❋ gauge boson pair production at hadron colliders – an overview:

- why is this class of processes important?
- what has been done?
- what has not yet been done?

❋ electroweak corrections to $pp \to W^+W^-$ and ZZ:

- approximative calculations
- details of the full calculation
- phenomenological results

in collaboration with M. Billoni, B. Biedermann,
A. Denner, S. Dittmaier, L. Hofer, L. Salfelder

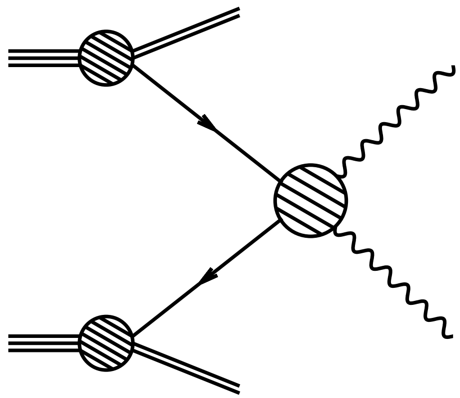
what's the plan for today?

WHY BOTHER WITH VV??

- gauge boson pair production at hadron colliders – an overview:
  - why is this class of processes important?
  - what has been done?
  - what has not yet been done?

- electroweak corrections to $pp \to W^+W^-$:
  - the on-shell approximation
  - going beyond: the double-pole approximation
  - details of the calculation
  - phenomenological results

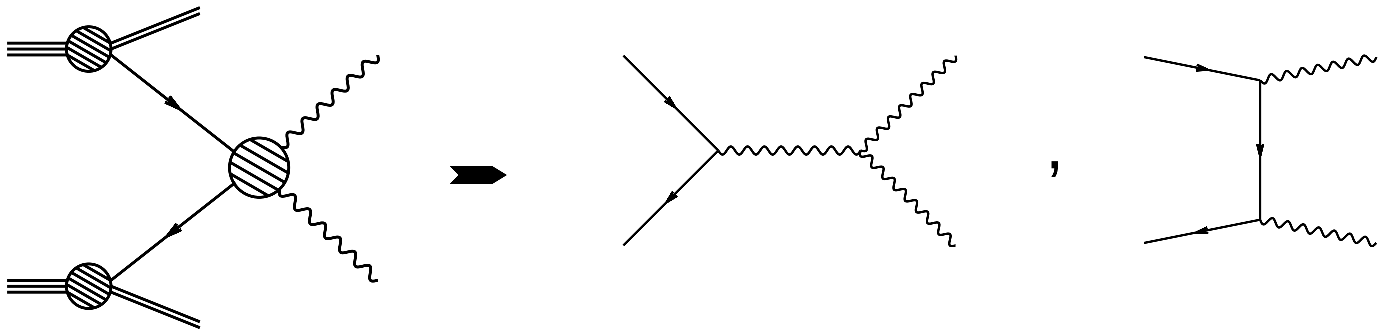
gauge-boson pair production



probe non-Abelian structure of the SM at high energies:

- (anomalous) triple-gauge-boson couplings
- dynamics of longitudinal massive gauge bosons

gauge-boson pair production



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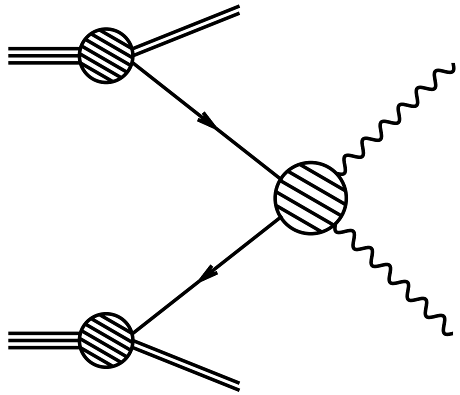
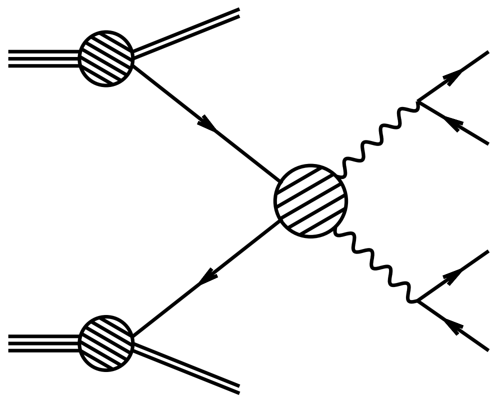


$$pp \to VV \to 4f$$

constitutes important class of
background processes to:

- the Higgs search in the mode $pp \to H \to VV \to 4f$
- new physics searches with leptons+$\not{E}_T$ signatures
(e.g. SUSY-particle pair production)

gauge-boson pair production



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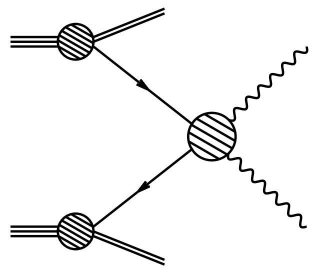
gauge-boson pair production @ NLO QCD

$h_1 h_2 \to ZZ$:

Ohnemus, Owens (1991) / Mele, Nason, Ridolfi (1991)

$h_1 h_2 \to W^{\pm} Z$:

Ohnemus (1991) / Frixione, Nason, Ridolfi (1992)



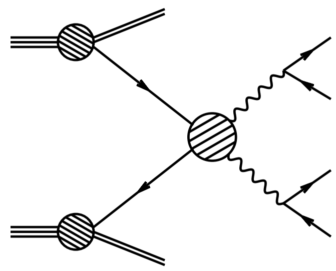
$h_1 h_2 \to W^+ W^-$:

Ohnemus (1991) / Frixione (1993)

including leptonic decays:

analytical expressions:

Dixon, Kunszt, Signer (1998) / Baur, Han, Ohnemus (1996)



implementation in public code `MCFM`:

Campbell, Ellis (1999)

gauge-boson pair production @ NLO QCD

$$pp \to W^+(\to e^+\nu_e)W^-(\to \mu^-\overline{\nu}_\mu)$$

$\sqrt{s}$ [TeV] and cuts	σ^{LO} [fb]	σ^{NLO} [fb]	K-factor
7 (basic)	144	249	1.73
7 (Higgs)	7.14	15.19	2.13
14 (basic)	296	566	1.91
14 (Higgs)	13.7	34.7	2.53

numbers taken from `MCFM`: *Campbell, Ellis, Williams (2011)*

gauge-boson pair production @ NLO QCD

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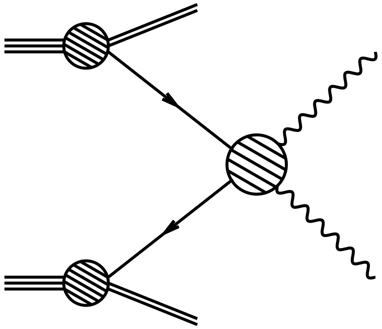
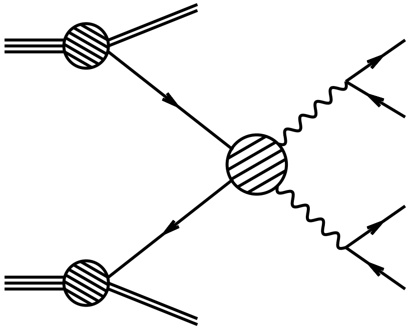
- size of NLO-QCD corrections is large and cut-dependent
- not expected from variation of central scale $M_W/2 \leq \mu_f \leq 2M_W$ at LO (← qg channels)

gauge-boson pair production & parton showers

NLO-QCD calculations matched with multi-purpose parton-shower programs PYTHIA, HERWIG, SHERPA





MC@NLO:

Frixione, Webber (2002)

POWHEG:

Nason, Ridolfi (2006)

POWHEG in HERWIG++:

Hamilton (2010)

POWHEG in SHERPA:

Höche, Krauss, Schönherr, Siegert (2010)

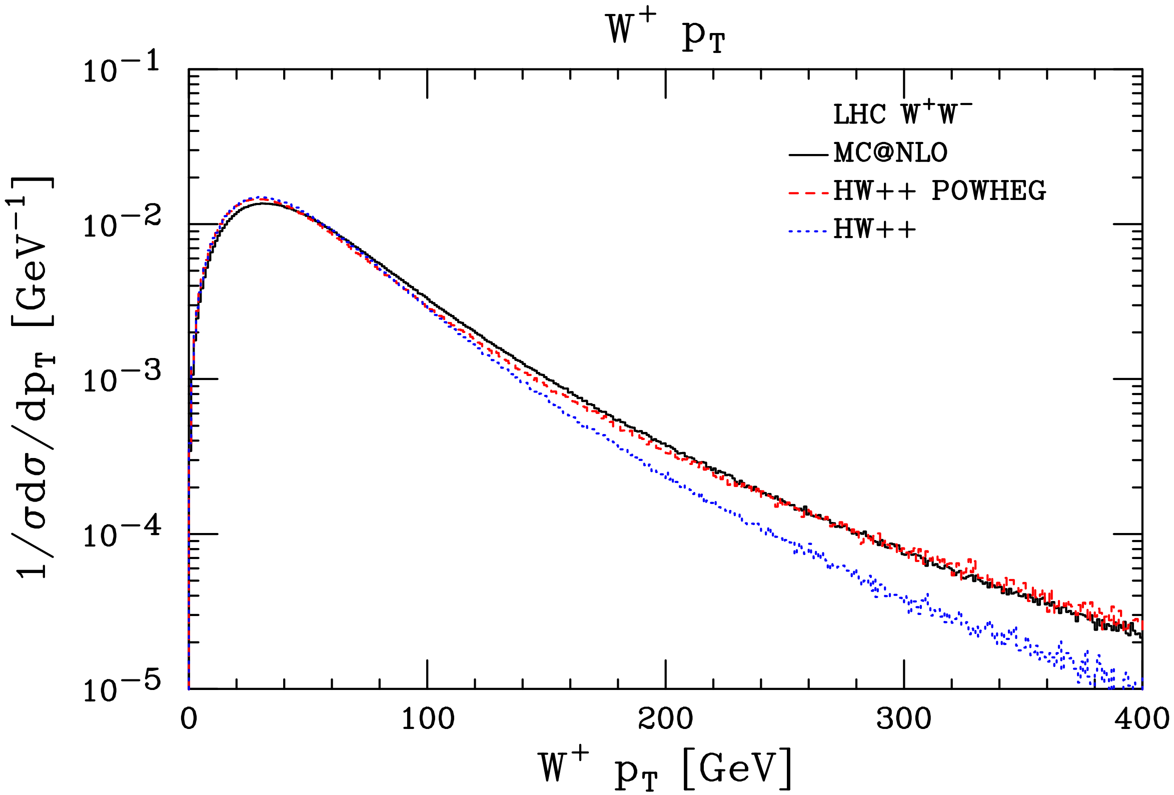
POWHEG-BOX:

Melia, Nason, Röntsch, Zanderighi (2011)

aMC@NLO:

Frederix et al. (2011)

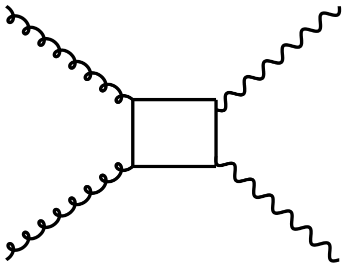
gauge-boson pair production & parton showers



Hamilton (2010)

- high-p_T tails: NLO+PS deviate from LO+PS results ($\leftarrow qg$)
- mostly: agreement between different NLO+PS simulations
- deviations between `MC@NLO` and `POWHEG` in distributions sensitive to extra jet emission

gauge-boson pair production – loop contributions



gluon-induced contributions
first occur at one-loop level

considered first by

Dicus, Kao, Repko (1987); Glover, van der Bij (1989)

phenomenological study for the LHC:

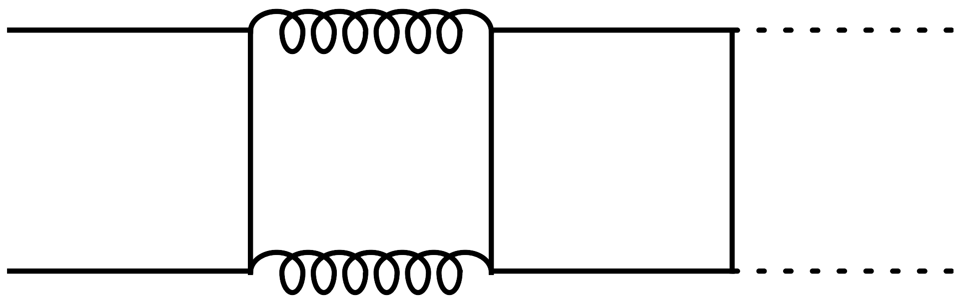
Dührssen, Jakobs, van der Bij, Marquard (2005)

inclusion of off-shell effects and heavy-quark loops:

Binoth, Ciccolini, Kauer, Krämer (2005,2006);
Binoth, Kauer, Mertsch (2008)

→ impact depends on cuts;
can be large

towards NNLO QCD for $pp \to VV$



✔ 2-loop master integrals for $\bar{q}q \to VV$

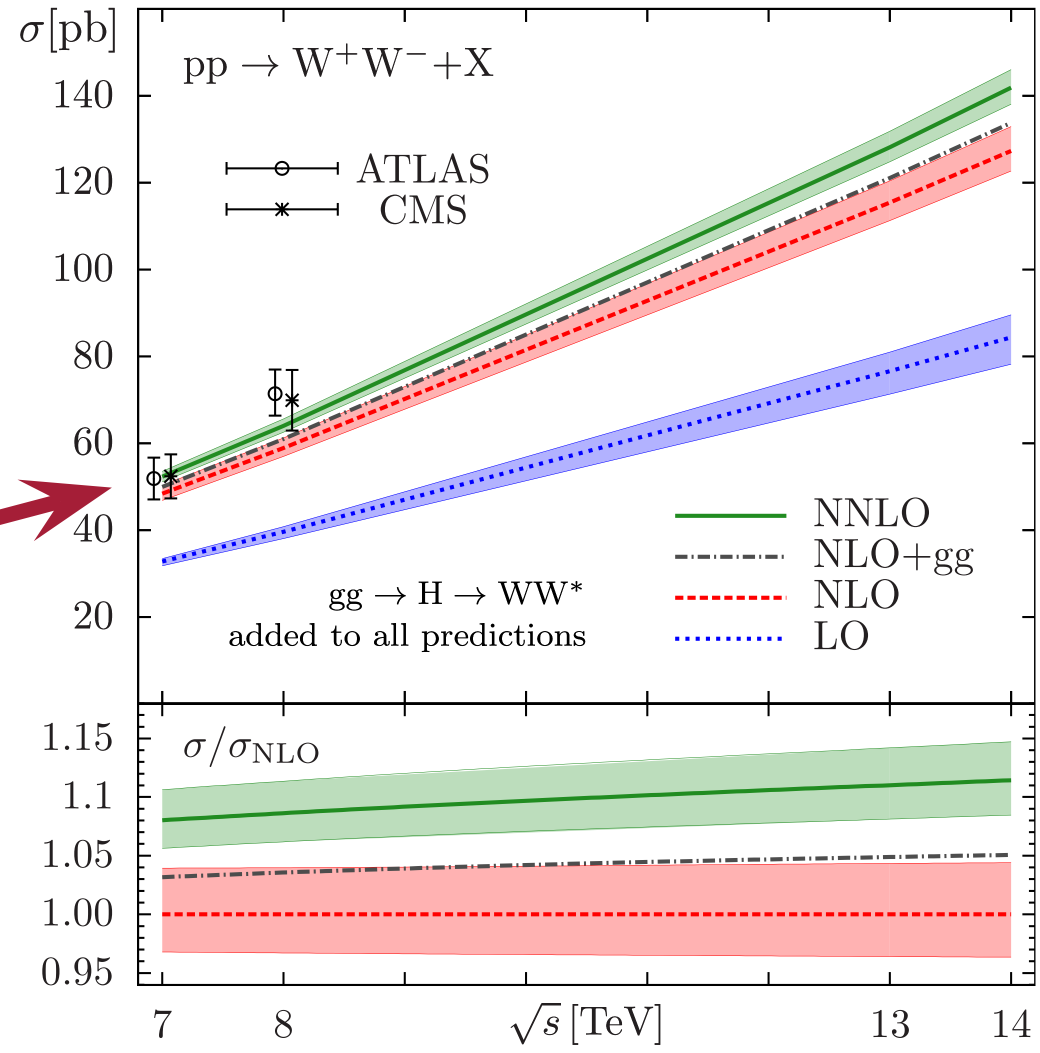
Gehrmann, Tancredi, Weihs (2013)

Gehrmann, von Manteuffel, Tancredi, Weihs (2014)

$pp \to WW$ @ NNLO QCD!

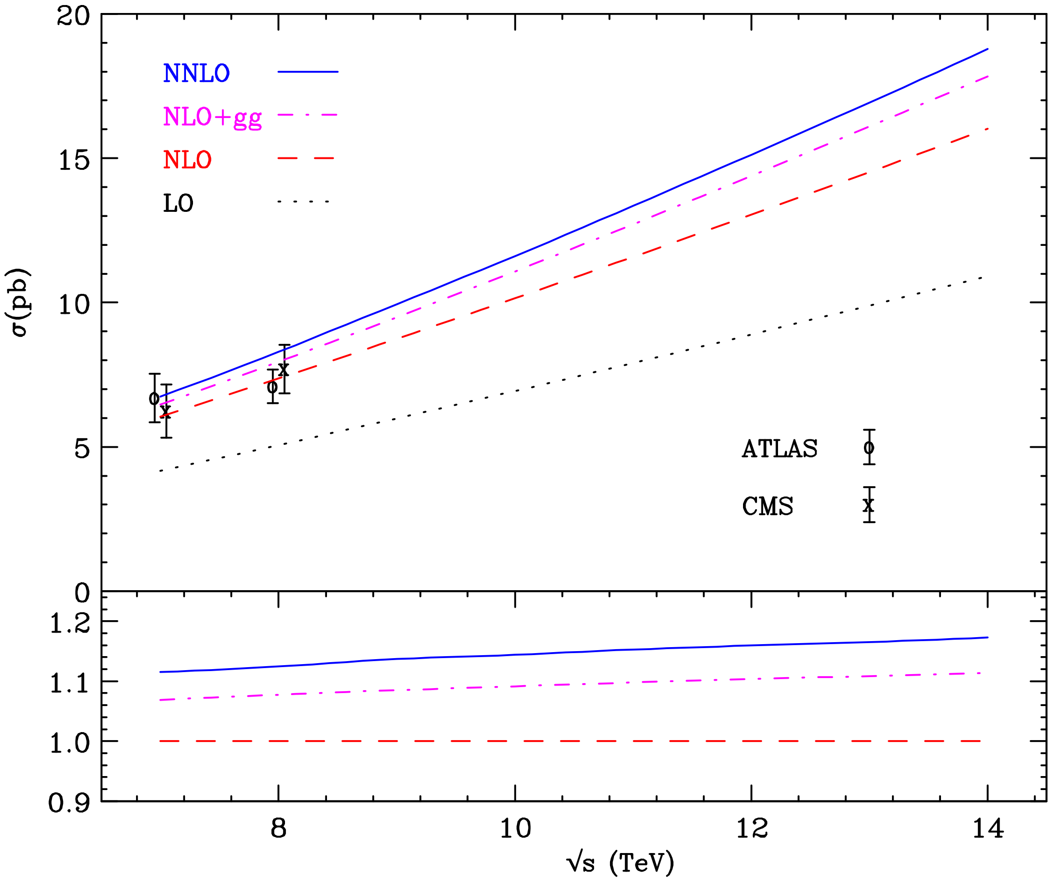
Gehrmann et al. (08/2014)

note:
improved agreement with LHC data



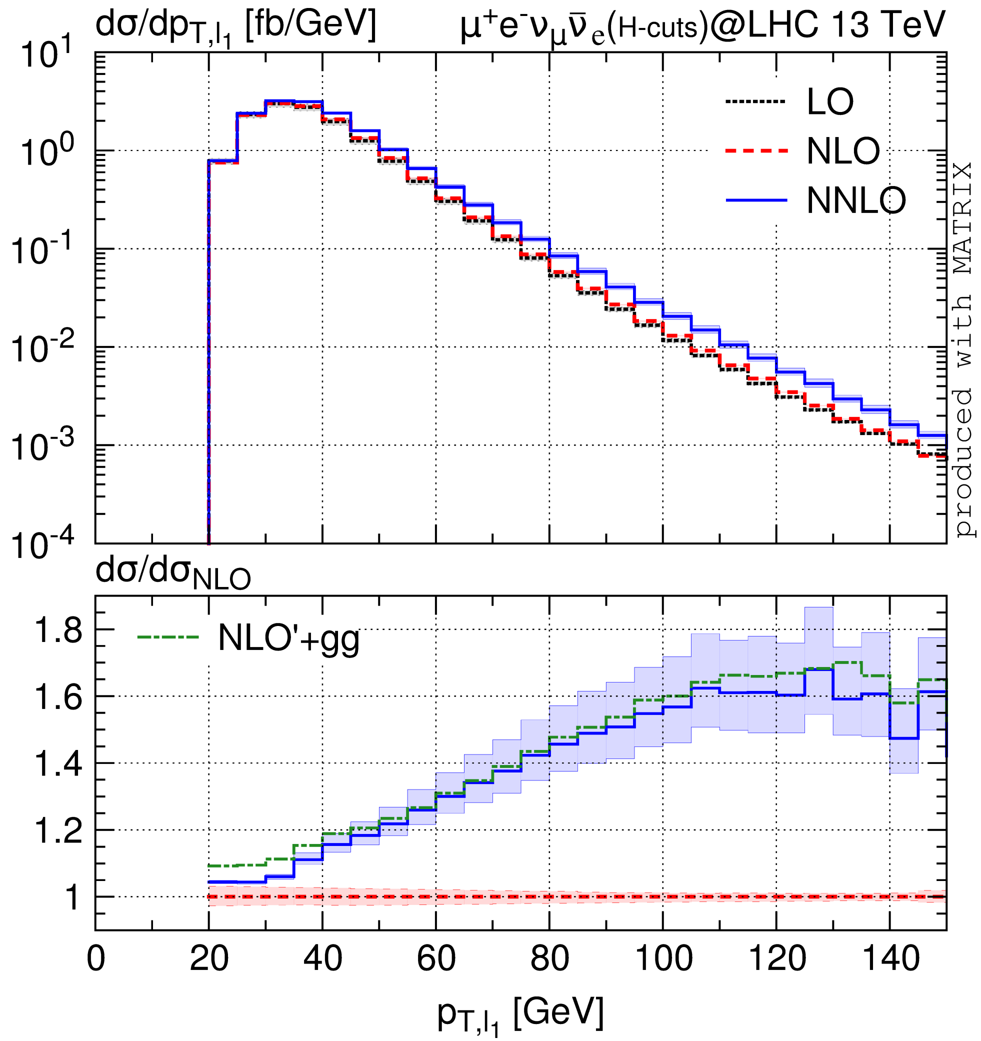
$pp \to ZZ$ @ NNLO QCD!

Cascioli et al. (05/2014)



$pp \to WW$ @ NNLO QCD: going differential

Grazzini et al. (05/2016)



fully differential Monte Carlo:

allows for arbitrary cuts
and distributions/correlations of
leptonic decay products

✔ realistic predictions possible

EW corrections: generic features

naive expectation:

$\alpha \sim \alpha_s^2 \to$ NLO EW $\sim$ NNLO QCD ?

but: systematic enhancements possible, e.g.:

- kinematic effects
- photon emission $\to$ mass-singular logs, e.g. $\frac{\alpha}{\pi} \ln\left(\frac{Q}{m_\mu}\right)$
- high energies $\to$ EW Sudakov logs, e.g. $\frac{\alpha}{\pi} \ln^2\left(\frac{Q}{M_W}\right)$

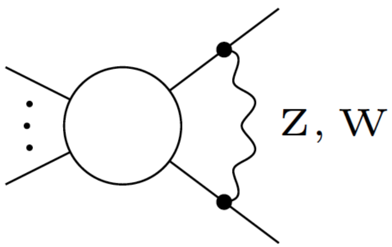
EW corrections: Sudakov logarithms

typical $2 \to 2$ process: at high energy
EW corrections enhanced by large logs

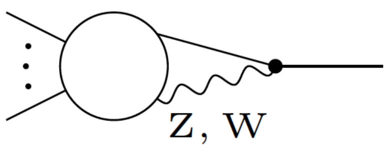
$$\ln^2\left(\frac{Q^2}{M_W^2}\right) \sim 25$$ @ energy scale of 1 TeV

universal origin of leading EW logs:

mass singularities in virtual corrections related to external lines



soft and collinear virtual gauge bosons: $\to$ double logs



soft or collinear virtual gauge bosons: $\to$ single logs

EW corrections: Sudakov logarithms

compare to QED / QCD:

IR singularities of virtuals canceled
by real-emission contributions

electroweak bosons massive

$\longrightarrow$ real radiation experimentally distinguishable

non-Abelian charges of W, Z are open

$\longrightarrow$ Bloch-Nordsieck theorem not applicable

M. Ciafaloni, P. Ciafaloni, Comelli; Beenakker, Werthenbach;
Denner, Pozzorini; Kühn et al., Baur; . . .

EW effects in PDFs

consistent calculation at NLO EW requires PDFs including
$\mathcal{O}(\alpha)$ corrections and new photon PDF

MRST2004QED: first PDF set with $\mathcal{O}(\alpha)$ corrections

NNPDF2.3QED (2013): NNPDF set with $\mathcal{O}(\alpha)$ corrections

- 2013: best PDF prediction at (N)NLO QCD + NLO QED
- PDF samples for error estimate provided
- photon PDF fitted to DIS and Drell-Yan data $(10^{-5} \lesssim x \lesssim 10^{-1})$
  (note lack of experimental information for large x)
- being updated; currently: NNPDF3.0QED

new physics effects in VV production

general contribution to Lagrangian for WWV interaction,
compatible with C and P conservation:

$$\mathcal{L}_{WWV} = g_{WWV}\Big[ig_1^V(W^*_{\mu\nu}W^\mu V^\nu - W_{\mu\nu}W^{*\,\mu}V^\nu) + i\kappa^V W^*_\mu W_\nu V^{\mu\nu} + i\frac{\lambda^V}{M_W^2}W^*_{\rho\mu}W^\mu_{\ \nu}V^{\nu\rho}\Big]$$

supplied by form factors to tame unitarity violations at high energies:

$$\Delta g \to \frac{\Delta g}{(1+M_{VV}^2/\Lambda^2)^2}$$

LEP bounds:

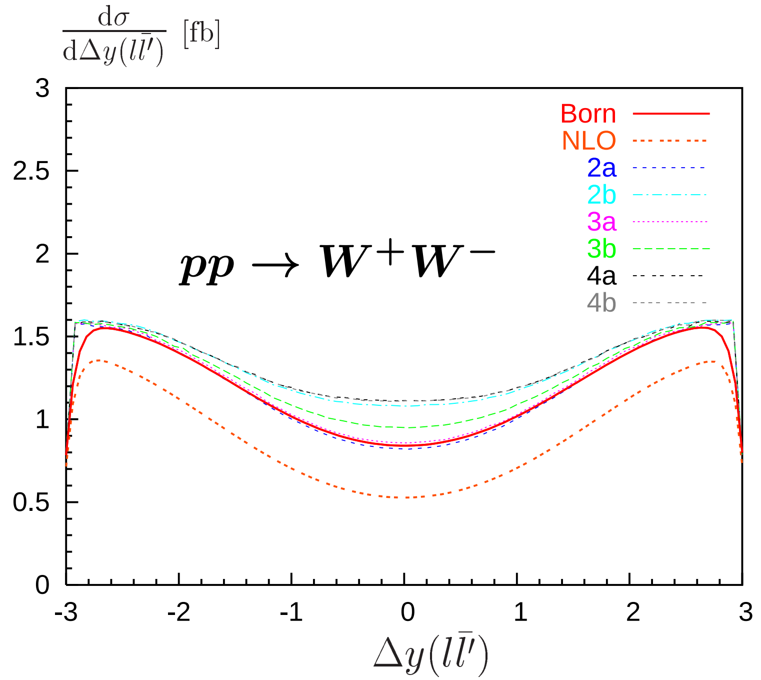
$$\Delta g_1^Z = (-0.054, 0.028),\ \ \Delta\kappa^\gamma = (-0.117, 0.067),$$
$$\Delta\lambda^Z = \Delta\lambda^\gamma = (-0.07, 0.012)$$

(SM: $g_1^V = \kappa^V = 1$ and $\lambda^V = 0$)

higher order or new physics effects?

parameterize new physics by anomalous triple gauge boson couplings λ, $\Delta\kappa_\gamma$, Δg_1^Z

Accomando, Kaiser (2005)



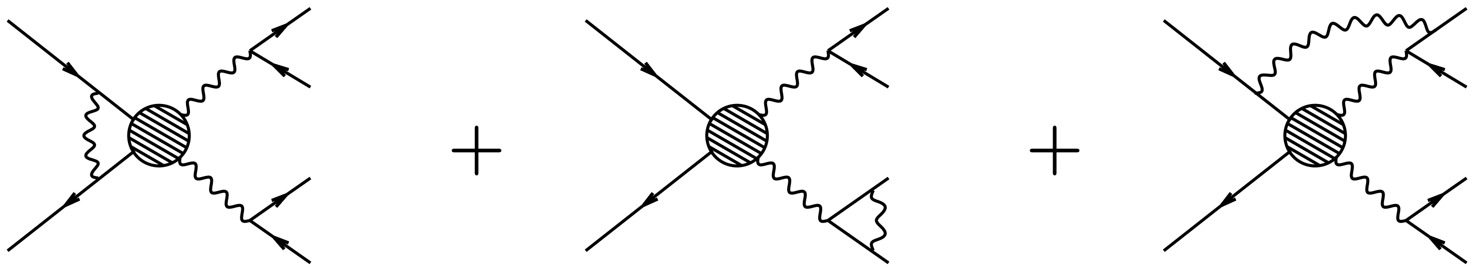
Scenario	λ	Δg_1^Z	$\Delta\kappa_\gamma$
Born/NLO EW	0	0	0
2a/2b	0	± 0.02	0
3a/3b	0	0	± 0.04
4a/4b	± 0.02	0	0

missing EW corrections can fake anomalous triple-gauge boson couplings

gauge-boson pair production beyond LO EW

$pp \to VV \to$ 4 leptons: $\mathcal{O}(\alpha)$ corrections

more challenging than QCD corrections:

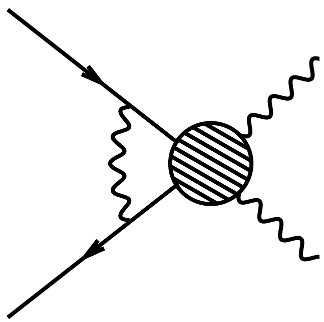


$\longrightarrow$ first step: employ approximations:

- retain only universal logarithms that are large at high energies
- double pole approximation for gauge bosons

Accomando, Denner, Pozzorini, Kaiser (2001-2004)

on-shell gauge-boson pair production @ NLO EW



$\mathcal{O}(\alpha)$ corrections to

$pp \to VV$

Bierweiler, Kasprzik, Kühn, Uccirati (2012-2013)

Baglio, Ninh, Weber (2013)

$\rightarrow$ EW corrections negative and small for inclusive x-secs,

but can be large and negative in tails of distributions

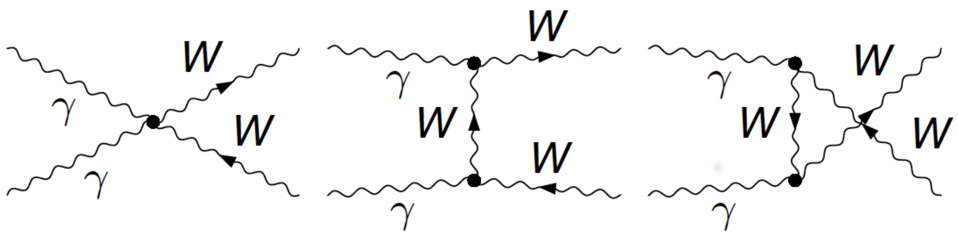
(universal Sudakov logarithms)

photon-induced contributions to $pp \to VV$

non-vanishing PDFs for photons in proton

$\to$ need to consider sub-processes

of type $\gamma\gamma \to VV$ at LO

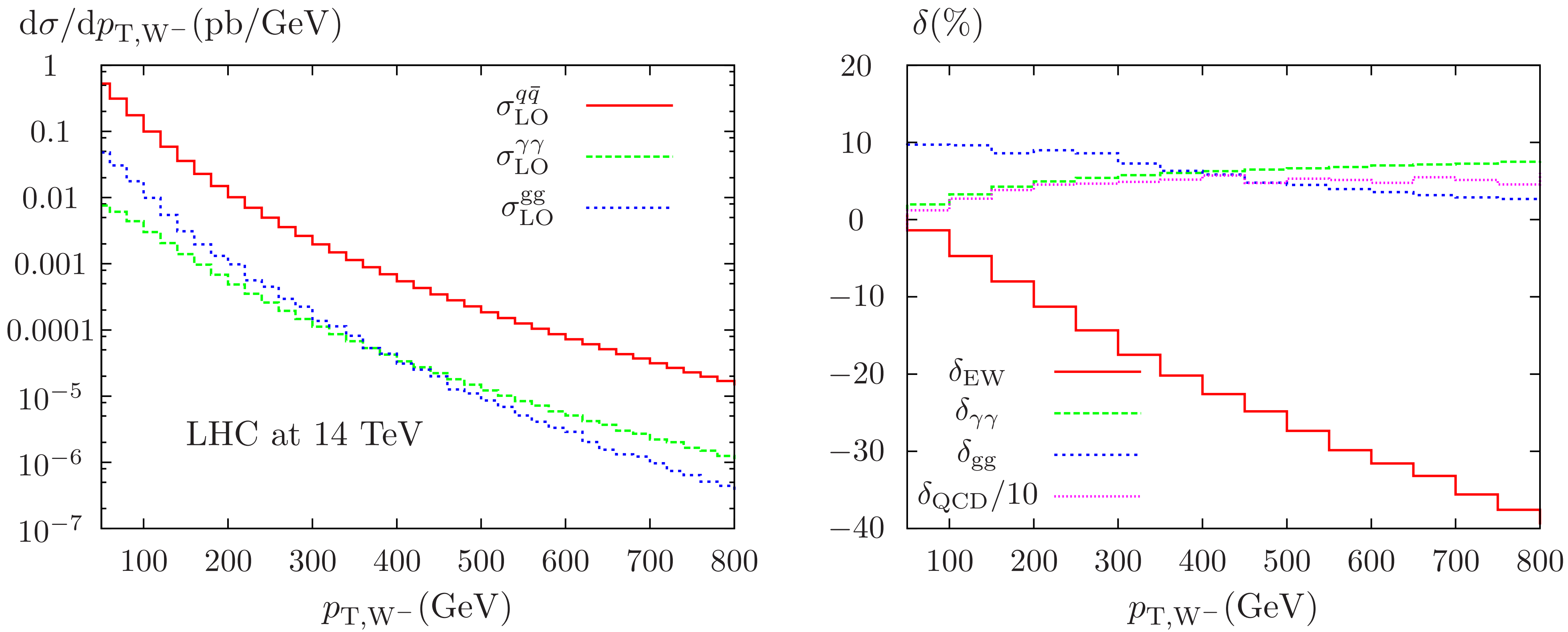


effects are small for inclusive x-secs, but up to several tens of percent for some distributions relative to dominant $q\bar{q}$ processes at LO ,

$\to$ can be of the same size as EW corrections to $\bar{q}q \to VV$, but opposite in sign

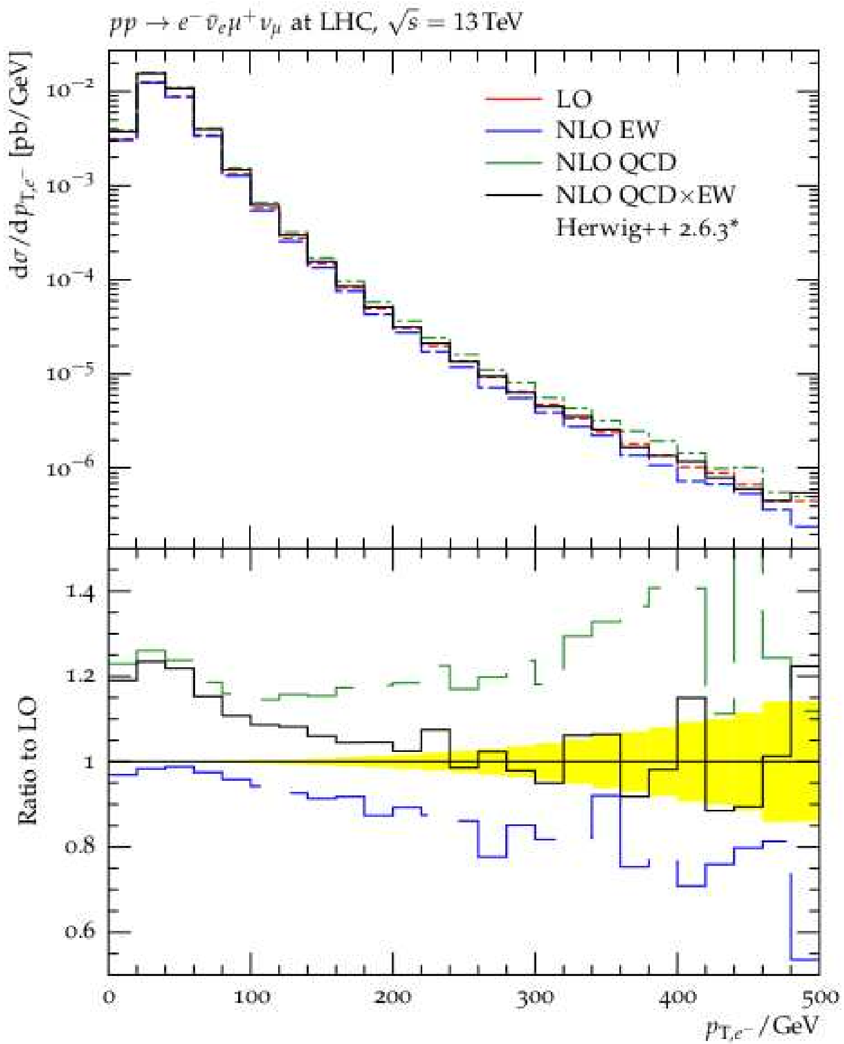
on-shell gauge-boson pair production @ NLO EW

Bierweiler, Kasprzik, Kühn, Uccirati (2012)



$pp \to VV$ and parton shower in HERWIG++

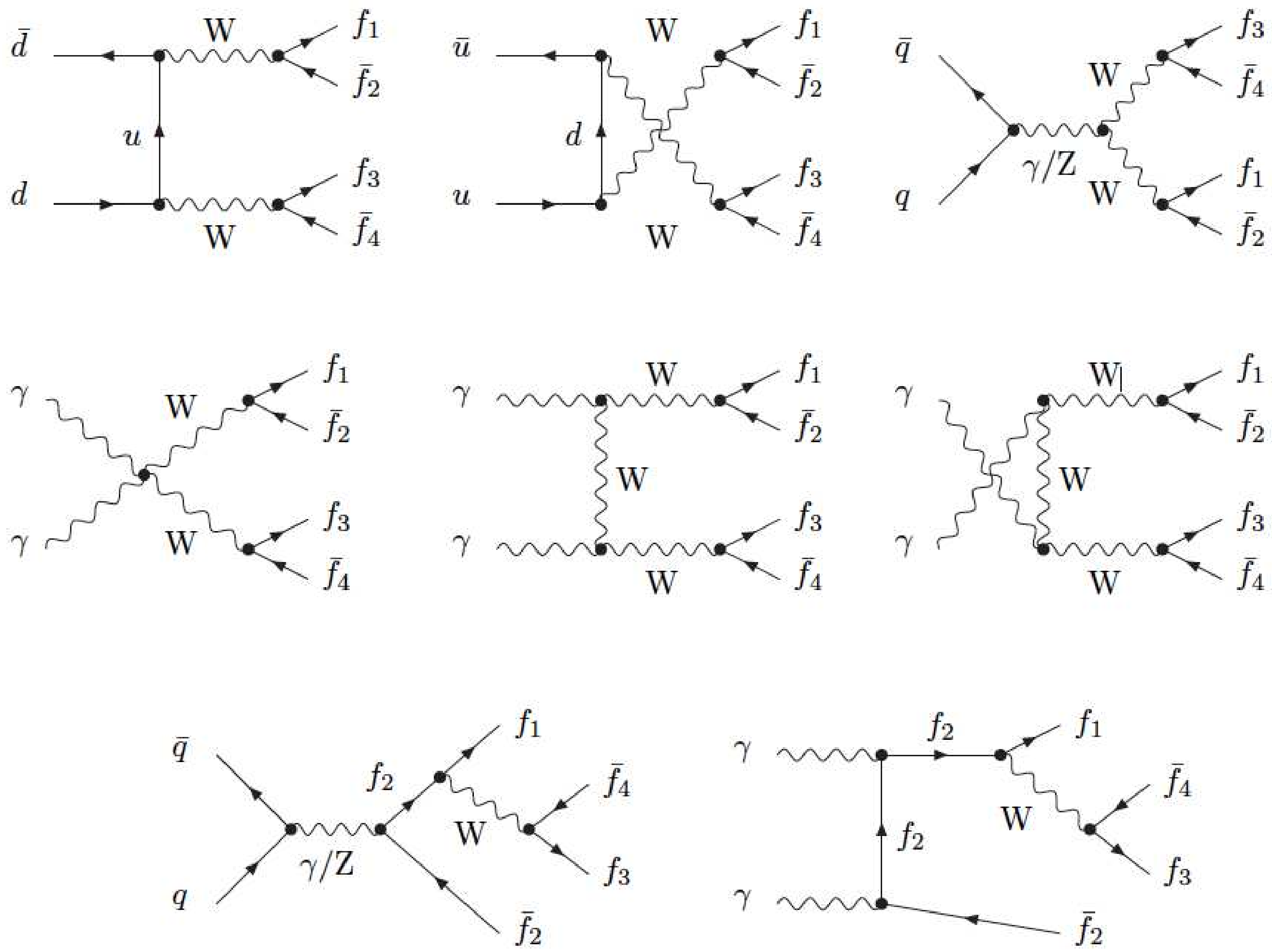
Gieseke Kasprzik, Kühn (2014)



- combination of fixed-order calculation for $pp \to VV$ with parton shower
- leptonic decays are handled by HERWIG++
- QCD and EW effects combined

$$d\sigma_{\mathrm{QCD}\times\mathrm{EW}} = K_{\mathrm{weak}}(\hat{s}, \hat{t}) \times d\sigma_{\mathrm{QCD}}$$

beyond the on-shell approximation at tree-level



resonant contributions of type

$$\bar{q}q \to W^+W^- \to \ell^+\nu\ell^-\bar{\nu}$$

$$\gamma\gamma \to W^+W^- \to \ell^+\nu\ell^-\bar{\nu}$$

non-resonant contributions in all channels

NLO-EW beyond the on-shell approximation

leading order: full off-shell calculation

- light quark contributions ($q = u, d, c, s$)
- $b\bar{b}$-induced contributions ($< 2\%$)
- photon-induced contributions ($< 1\%$)

real-emission and virtual contributions:

for light quark channels use full off-shell calculation or double pole approximation

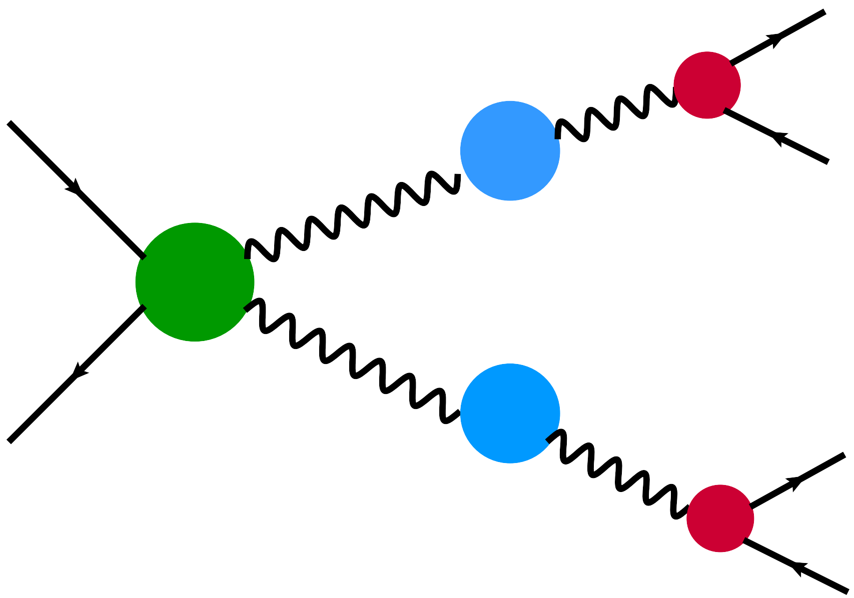
(analogous to Racoon approach for $e^+e^- \to 4$ fermions *[Denner, Dittmaier, Roth, Wackeroth (1999-2002)]*)

the double pole approximation (DPA)

full EW corrections to $pp \to 4$ fermions challenging

$\to$ compute tree-level contributions exactly,
resort to double pole approximation for virtuals

(analogous to Racoon approach for $e^+e^- \to 4$ fermions
[Denner, Dittmaier, Roth, Wackeroth (1999-2002)])

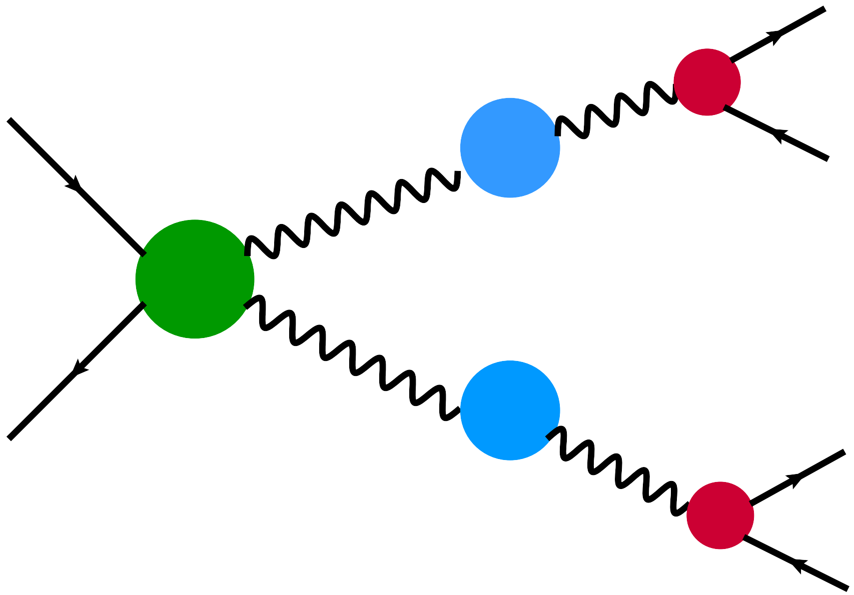


on-shell production

on-shell decay

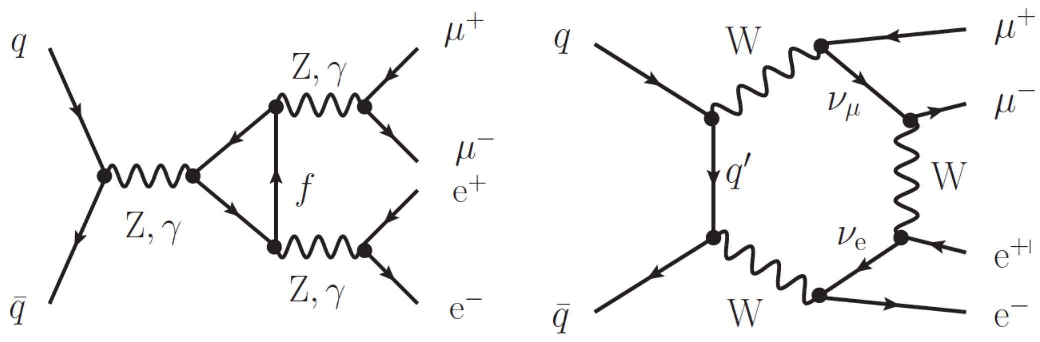
the double pole approximation (DPA)

- doubly-resonant diagrams fully considered
- $\hat{=}$ expansion around poles
- expect error $\sim \Gamma_W / M_W$ w.r.t full EW calculation
- structure of corrections simpler $\longrightarrow$ faster code



on-shell production　　　　　　　on-shell decay

the full off-shell calculation: $pp \to 4\ell$



- ❋ all resonant and non-resonant diagrams contributing to $\bar{q}q \to 4\ell$ fully considered
- ❋ complex-mass scheme for weak boson resonances:

$$m_V^2 \to m_V^2 + i m_V \Gamma_V$$

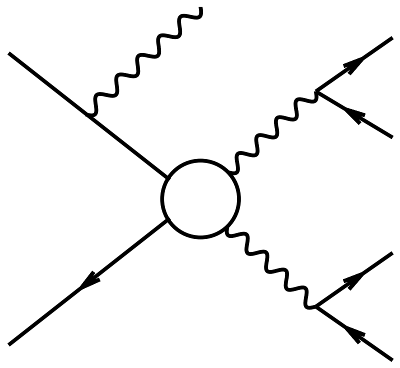
$\to$ applicable and gauge-invariant everywhere in phase space

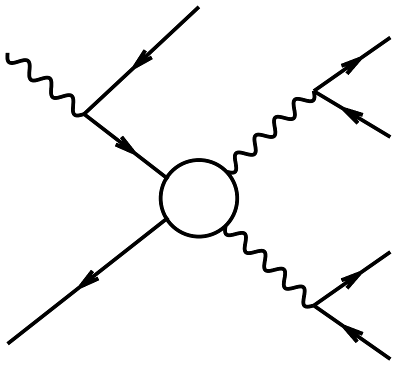
- ❋ tensor loop integrals (up to hexagons) evaluated with `COLLIER`
- ❋ per channel: $\sim 10^3$ diagrams $\to$ CPU intensive

real emission contributions

...full matrix elements for two classes of processes

$\bar{q}q \to \ell^+\nu\ell^-\bar{\nu}\ \gamma$ $\qquad\qquad$ $\gamma q \to \ell^+\nu\ell^-\bar{\nu}\ q$





...encounter IR divergences that need to be handled with care

$\to$ Catani-Seymour type subtraction procedure

adapted for EW corrections *[Dittmaier (1999)]*

some more details on the calculation

- phase-space integration:

  multi-channel integrator based on
  Monte-Carlo for $\gamma\gamma \to 4$ fermions
  [Bredenstein, Dittmaier, Roth (2005)]

- matrix elements computed via

  in-house `Mathematica` routines,
  converted into `Fortran` code

- all leading-order and real emission amplitudes

  compared with `MadGraph`

- independent calculation based on `RECOLA`

  (“recursive computation of one-loop amplitudes”)

EW input parameter scheme

❋ EW parameters obtained from G_μ, M_W, M_Z via

$$\cos\theta_W = M_W/M_Z\,, \alpha_{G_\mu} = \frac{\sqrt{2}G_\mu M_W^2 \sin^2\theta_W}{\pi} \quad (G_\mu \text{ scheme})$$

☞ accounts for higher order corrections associated with running coupling and universal top-mass corrections to ρ parameter

❋ contributions involving photon radiation effects:

use instead $\alpha(0)$ as effective coupling

[c. f. Denner (1993)]

$pp \to WW \to e^+\nu_e\mu^-\bar{\nu}_\mu$: phenomenological setup

NNPDF2.3qed

factorization scale $\mu_F = M_W$

photon recombination

minimal cuts:

$p_{T,\ell} > 20$ GeV, $|y_\ell| < 2.5$

jet veto: $p_{T,j} > 100$ GeV

ATLAS cuts:

$p_{T,\ell} > 20$ GeV, $|y_\ell| < 2.5$

$p_{T,\ell}^{\text{leading}} > 25$ GeV, $E_T^{\text{miss}} > 25$ GeV,

$R_{e\mu} > 0.1$, $M_{e\mu} > 10$ GeV

jet veto: not jets with $p_{T,j} > 25$ GeV

$pp \to W^+W^-$: cross section contributions

	$\sigma^{\text{LO}}_{\bar{q}q}$ [fb]	$\delta^{\text{NLO}}_{\bar{q}q}$ [%]	$\delta^{q\neq b}_{q\gamma}$ [%]	$\delta_{\gamma\gamma}$ [%]	$\delta_{b\gamma}$ [%]
LHC8	238.65(3)	-3.28	0.44	0.84	1.81
LHC13	390.59(3)	-3.41	0.49	0.73	2.30
ATLAS8	165.24(1)	-3.56	-0.26	1.01	0.18
ATLAS13	271.63(1)	-3.71	-0.27	0.87	0.23

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ATLAS13	271.63(1)	$\mathbf{-3.71}$	-0.27	0.87	0.23

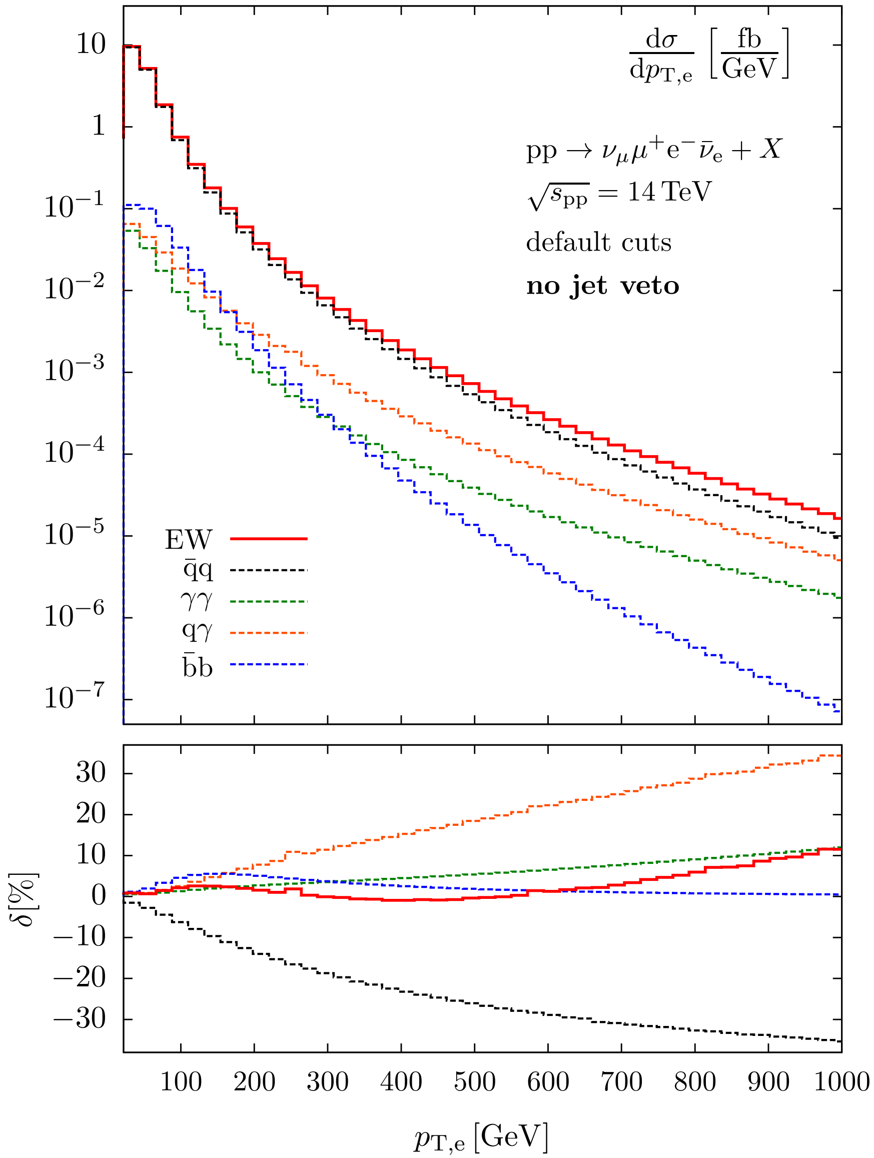
full calculation very well reproduced by DPA:

$$\text{LHC13 (DPA)}: \boldsymbol{\delta^{\text{NLO}}_{\bar{q}q} = -2.91\%}$$

$$\text{ATLAS13 (DPA)}: \boldsymbol{\delta^{\text{NLO}}_{\bar{q}q} = -3.18\%}$$

transverse-momentum distribution (DPA)

Billoni et al. (2013)



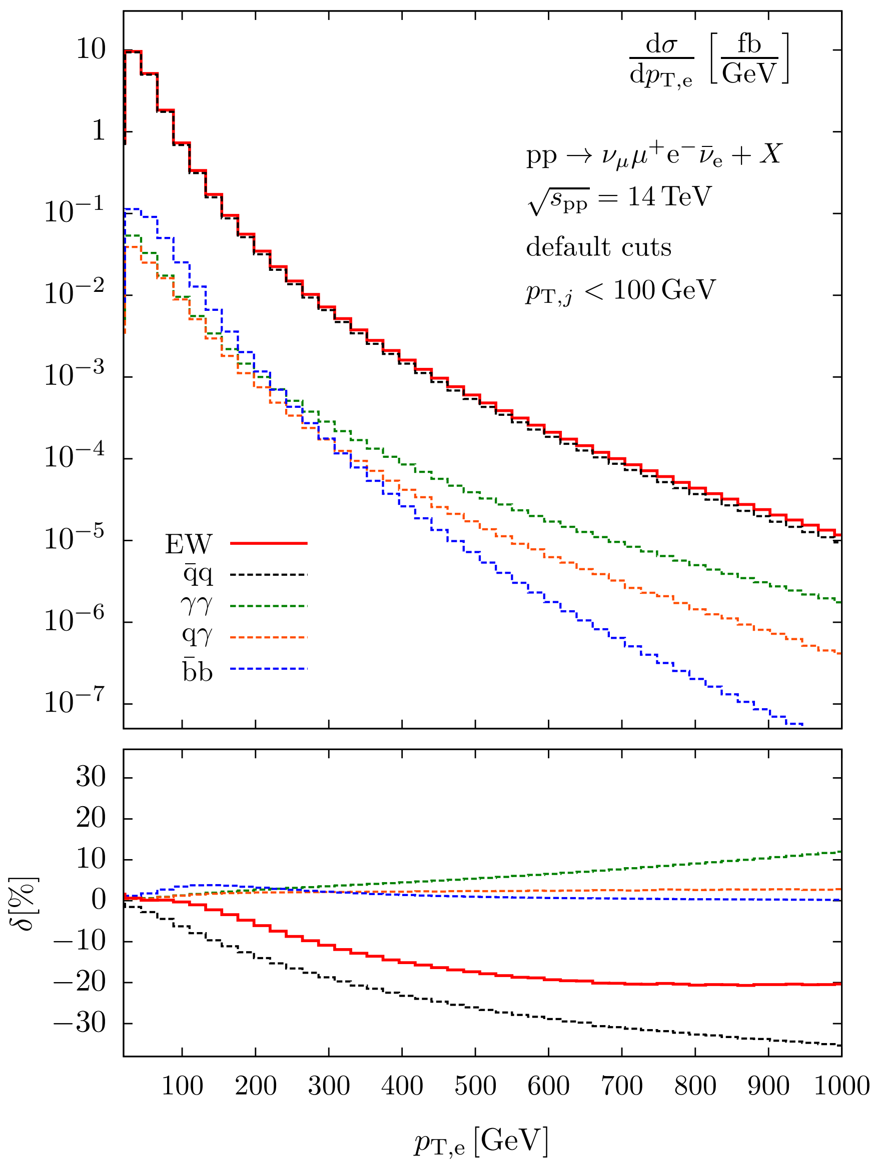
no jet veto:

- $\boldsymbol{\delta_{\bar{q}q} = -30\%}$ for $\boldsymbol{p_{T,e} = 900}$ GeV (Sudakov logs)
- $\boldsymbol{\delta_{\gamma\gamma}}$ = up to +10 %
- $\boldsymbol{\delta_{\gamma q}}$ large due to soft $\boldsymbol{W}$ emission (same effect in QCD corrections leads to huge K factors)

$\rightarrow$ apply jet veto

transverse-momentum distribution (DPA)

Billoni et al. (2013)



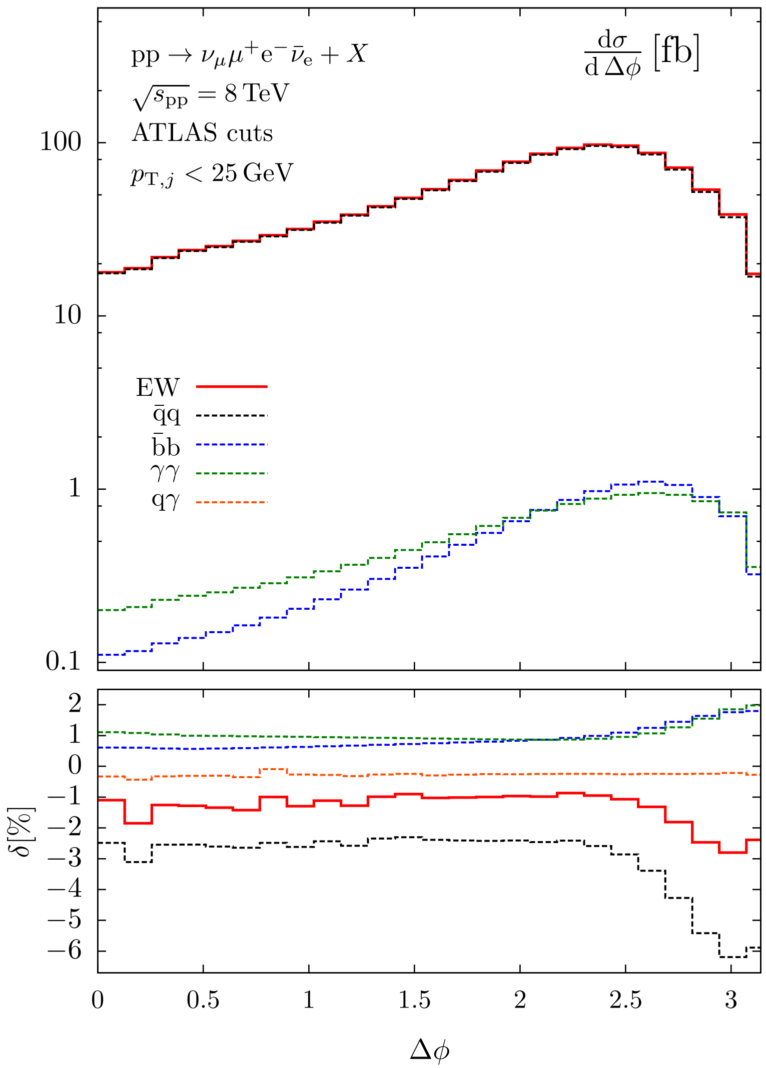
no jet veto:

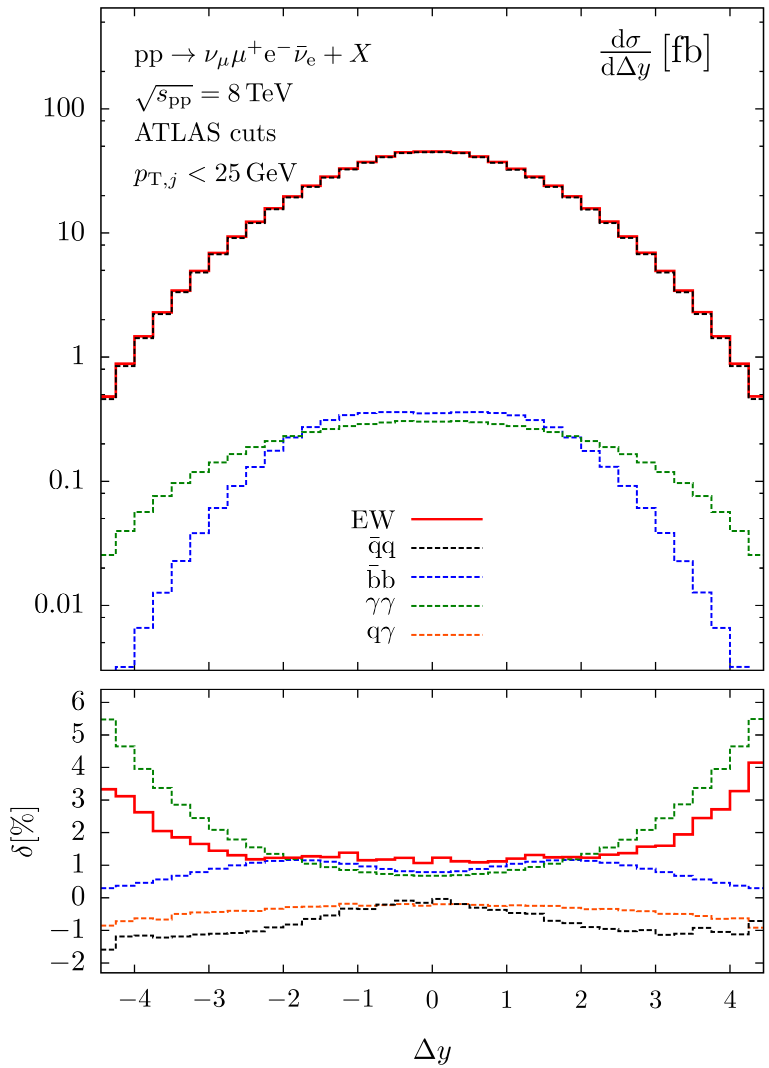
- $\delta_{\bar{q}q}$ up to –30 %
- $\delta_{\gamma\gamma}$ up to +10 %
- $\delta_{\gamma q}$ up to +30 %

$\rightarrow$ apply jet veto:

- $\delta_{\gamma q} < 5\%$ even at high p_T
- δ_{EW} = –20% for $p_{T,e} = 900$ GeV

angular distributions (DPA) . . .

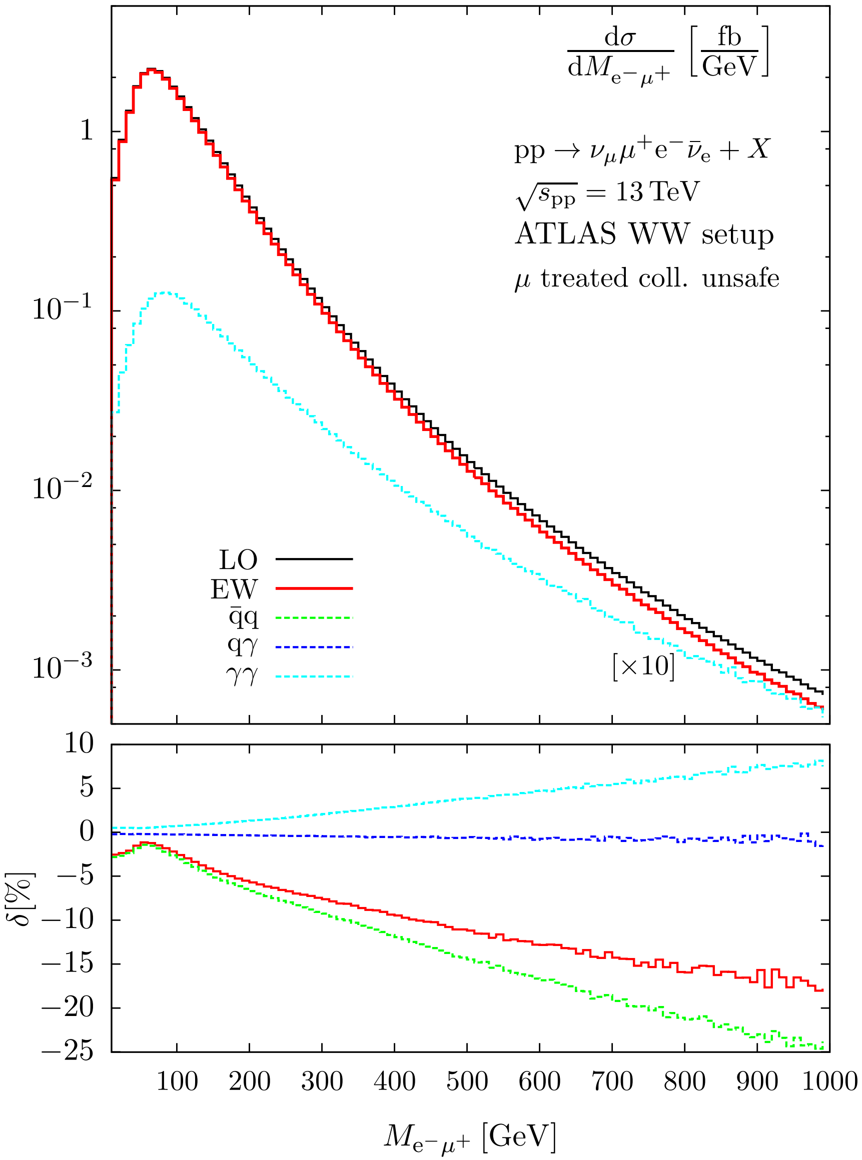




. . . in general only marginally affected by EW corrections

invariant mass distribution

Biedermann et al. (2016)



- ❋ large negative corrections in $\bar{q}q$ channel,
- ❋ positive contributions from $\gamma\gamma$ channel

→ sum of corrections moderate even at high values of $M_{e\mu}$ ($< 10\%$)

error estimate of the approximation

- error estimate of the NLO EW calculation

  (impact of missing 2-loop EW corrections):

$$\boldsymbol{\Delta \sim (\delta_{\mathrm{EW}})^2}$$

- error estimate of the DPA:

$$\boldsymbol{\Delta_{\mathrm{DPA}} \sim \max\left\{ (\delta_{\mathrm{EW}}^{\mathrm{DPA}})^2,\ \frac{\alpha}{\pi}\frac{\Gamma_W}{m_W}\ln(\ldots),\ |\delta_{\mathrm{EW}}^{\mathrm{DPA}}| \times \frac{\sigma_{\mathrm{LO}} - \sigma_{\mathrm{LO}}^{\mathrm{DPA}}}{\sigma_{\mathrm{LO}}^{\mathrm{DPA}}} \right\}}$$

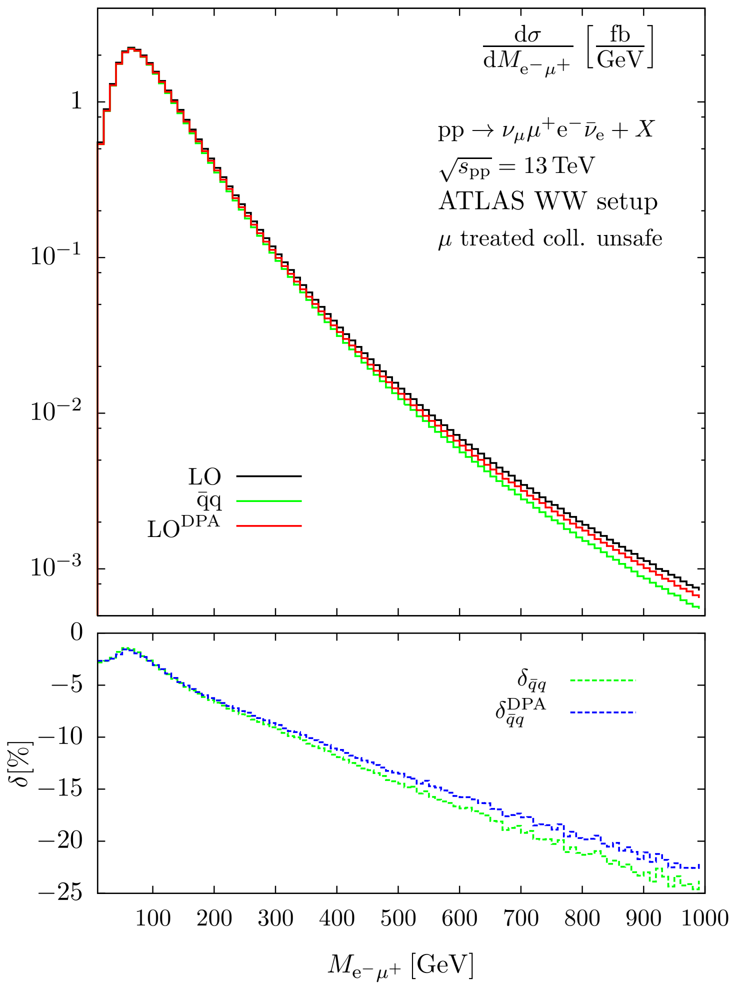
(1) missing 2-loop EW corrections

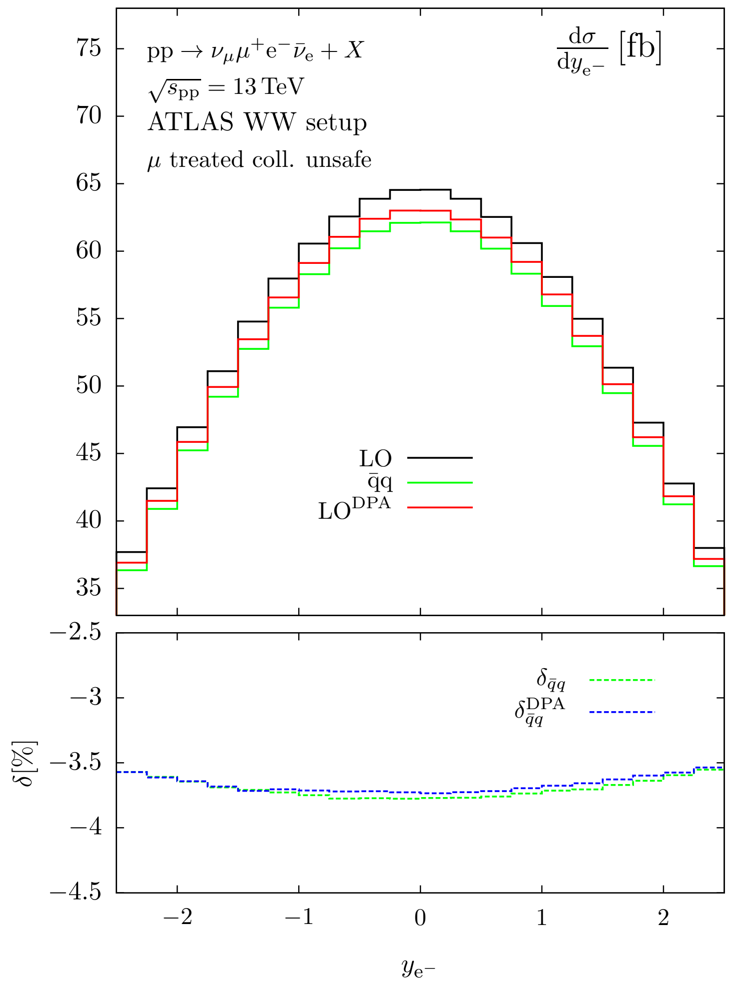
(2) missing off-shell contributions in regions where the DPA applies

(3) change of NLO EW corrections due to failure of DPA

DPA versus full calculation

Biedermann et al. (2016)

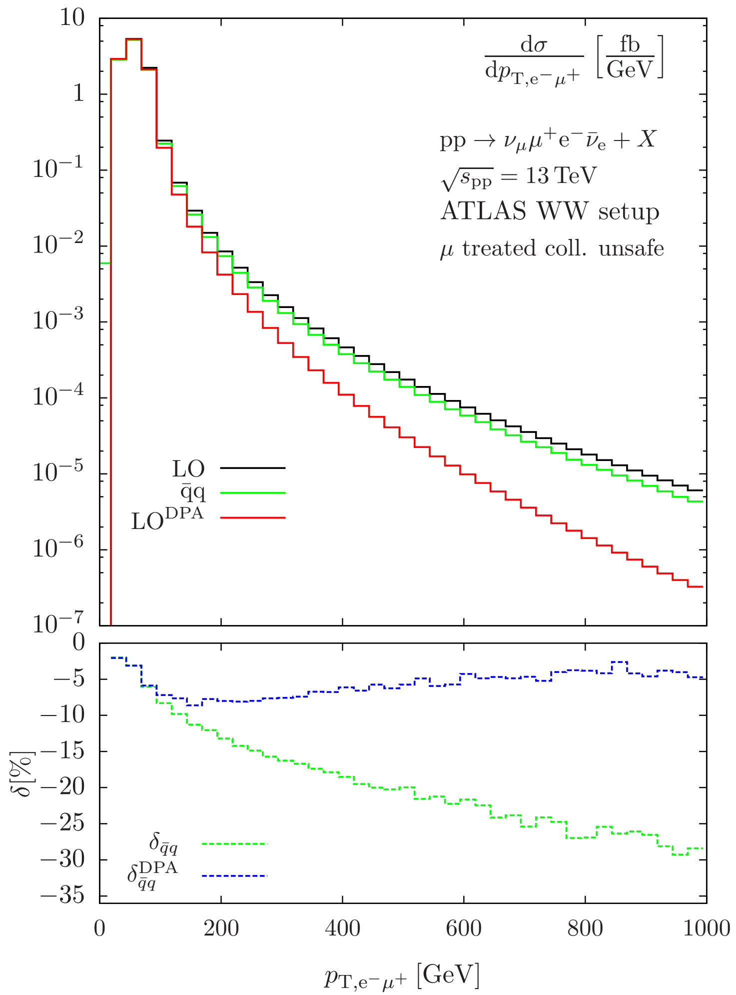


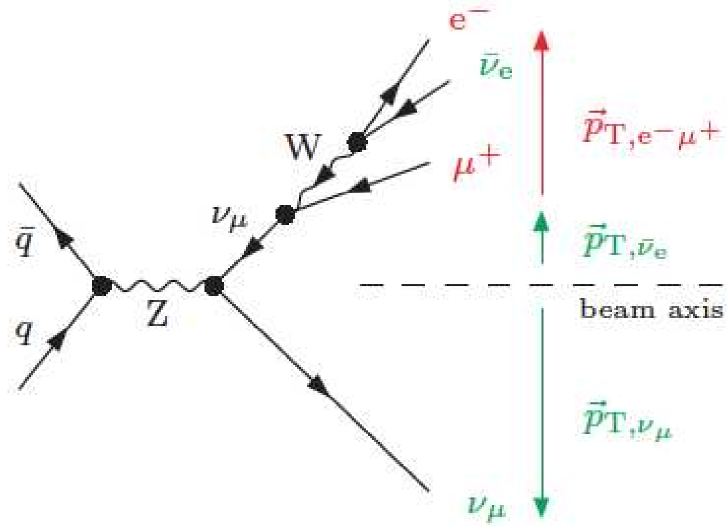


rapidity and invariant-mass distributions:
good agreement between DPA and full calculation

DPA versus full calculation

Biedermann et al. (2016)





- doubly-resonant diagrams strongly suppressed
- singly-resonant diagrams dominate: $(e\mu)$ pair recoils against $(\nu_\mu\bar{\nu}_e)$ pair

☞ poor agreement between DPA and full calculation for transverse-momentum of lepton pair

$pp \to ZZ \to \mu^+\mu^-e^+e^-$: phenomenological setup

NNPDF2.3qed

factorization scale $\mu_F = M_Z$

Higgs-search specific cuts:

$p_{T,\ell} > 6$ GeV, $|y_\ell| < 2.5$,

$\Delta R_{\ell\ell} > 0.2$,

40 GeV$<$ $M_{\ell_1^+\ell_1^-}$ $<$120 GeV ,

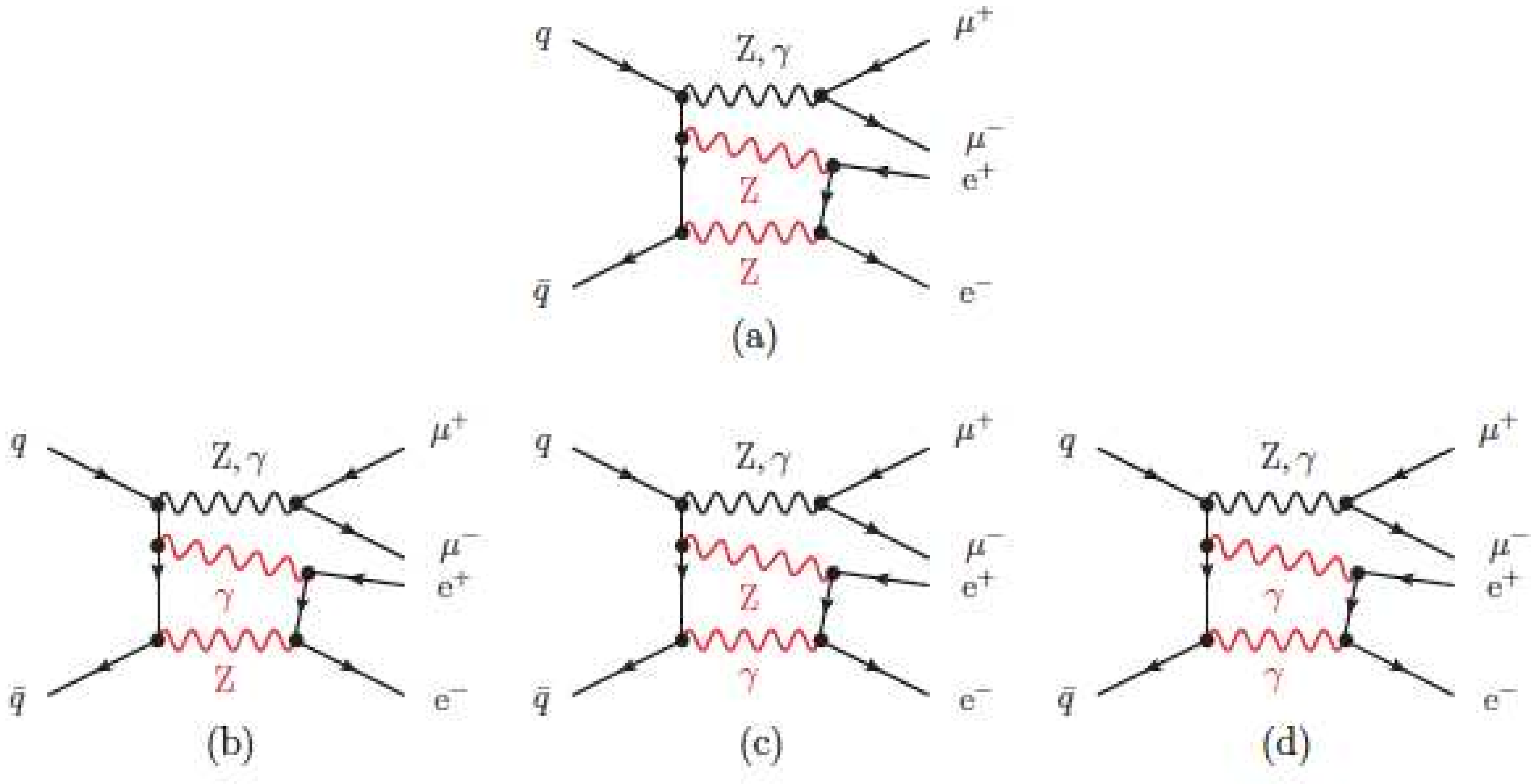
12 GeV$<$ $M_{\ell_2^+\ell_2^-}$ $<$120 GeV ,

$M_{4\ell}$ $>$100 GeV

$pp \to \mu^+\mu^-e^+e^-$: weak and photonic corrections

process without charged currents at LO

$\to$ can perform gauge-invariant decomposition
into weak and photonic corrections



$pp \to ZZ \to \mu^+\mu^-e^+e^-$: cross sections

$\sqrt{s}$ [TeV]	$\sigma^{\mathrm{LO}}_{\bar{q}q}$ [fb]	$\delta^{\mathrm{EW}}_{\bar{q}q}$ [%]	$\delta^{\mathrm{weak}}_{\bar{q}q}$ [%]
7	**7.3293(4)**	**−3.4**	**−3.3**
8	**8.4704(2)**	**−3.5**	**−3.4**
13	**13.8598(3)**	**−3.6**	**−3.6**
14	**14.8943(8)**	**−3.6**	**−3.6**

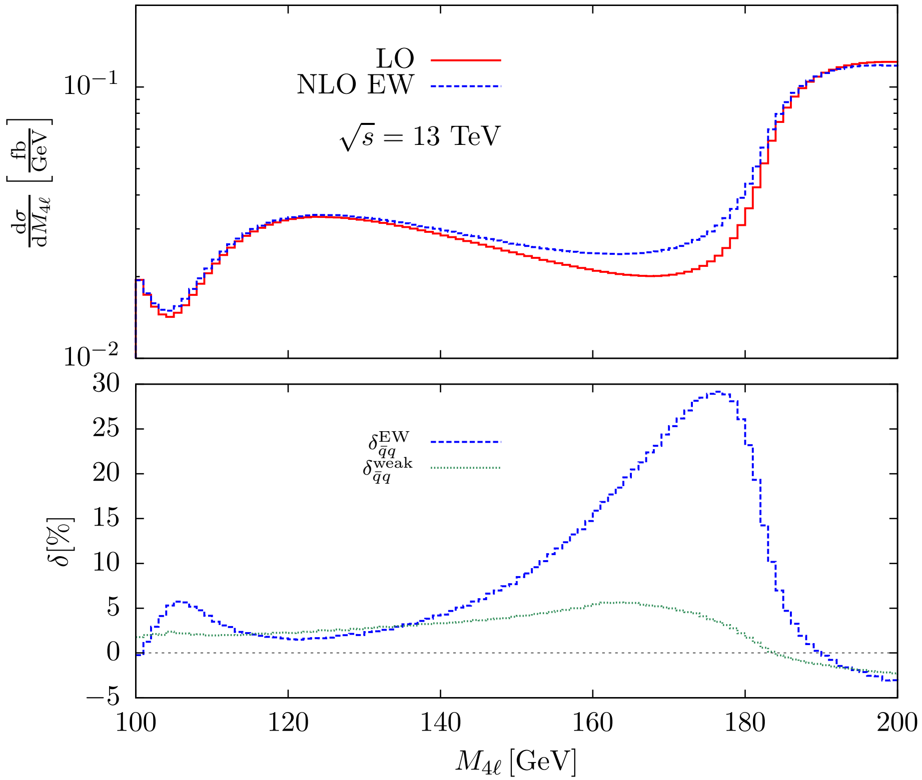
(recall: Higgs-search specific setup)

total xsec dominated by ZZ on-shell production

- weak corrections moderate
- photonic corrections negligible

$pp \to ZZ \to \mu^+\mu^-e^+e^-$ as a Higgs background

[Biedermann et al. (2016)]



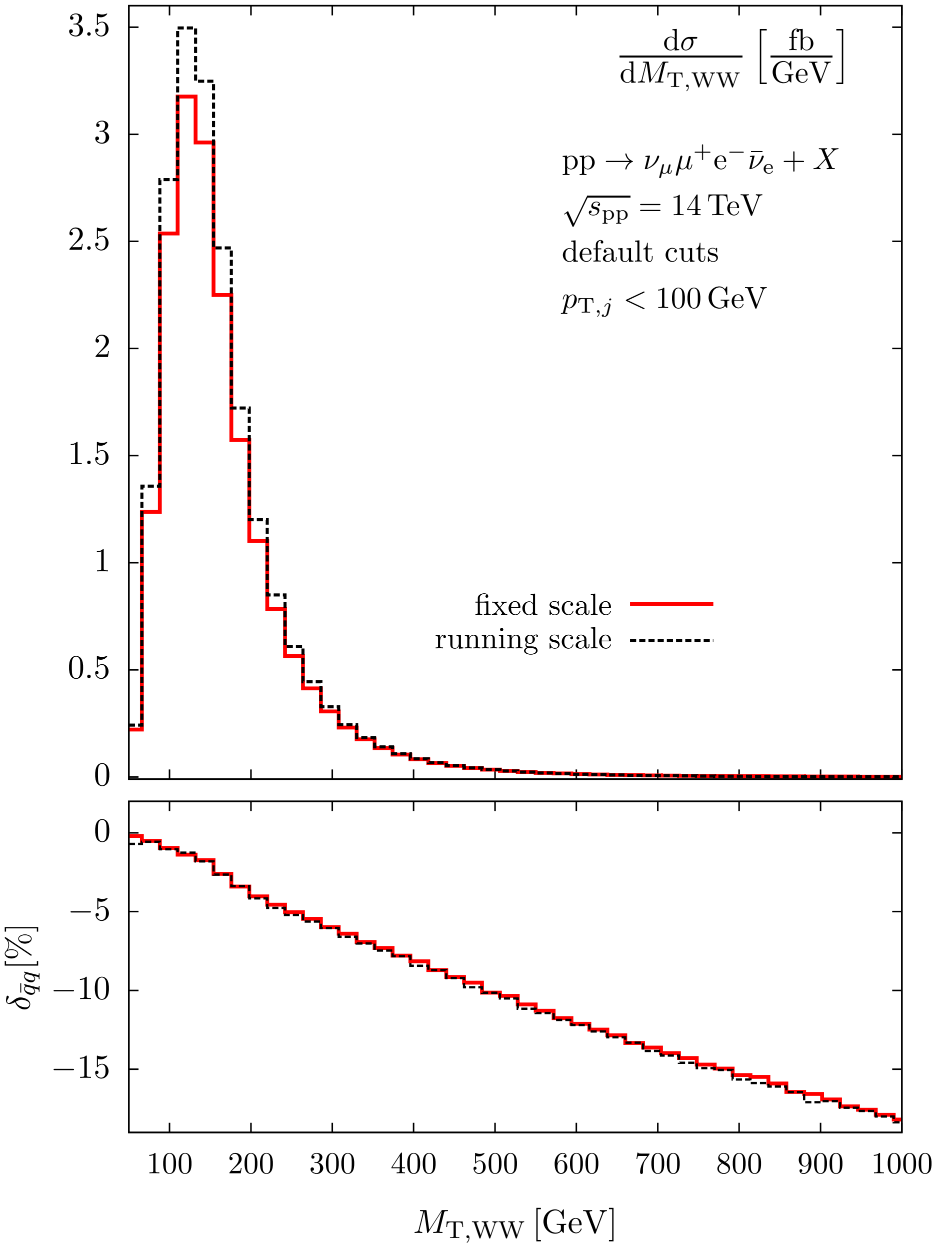
- radiative tails below thresholds and peaks

  (caused by cuts and mass spectrum)

- weak corrections change sign at ZZ threshold

  $\to$ approximation based on global rescaling factor does not work

scale dependence



choice of factorization scale:

fixed scale: $\mu_F = \xi M_W$
dynamical scale: $\mu_F = \xi M_{WW}$

vary ξ in range (0.5 , 2)

$\rightarrow$ overall change of x-sec : $\sim 8\%$

(mostly PDF effect)

combination of QCD and EW corrections

EW corrections insensitive to scale choice

☞ combination with QCD corrections via factorization ansatz:

$$d\sigma^{\text{best}} = d\sigma_{qq}^{\text{QCD}} \times \left(1 + \delta_{qq}^{\text{EW}}\right) + d\sigma_{gg} + d\sigma_{\gamma\gamma} + d\sigma_{q\gamma}$$

summary

first computation of EW corrections to $pp \to 4$ leptons

that gives full access to leptonic final state:

- EW corrections to integrated x-sec small
- sizable effects in tails of distributions (Sudakov logarithms)
- $\gamma\gamma$ induced contributions non-negligible
- γq induced contributions can be suppressed by jet veto
- scale dependence small

conclusions

- weak boson pair production processes provide powerful probes of the structure of the Standard Model

  e.g. triple gauge boson couplings

- serve as important backgrounds

  . . . to searches for the Higgs boson
  . . . to searches for new physics

- impact of radiative corrections can be large and dependent on experimental selection criteria

  $\longrightarrow$ to achieve precision required by experiment:

  - consider QCD and EW corrections
  - disregard (on-shell, high-energy, . . .) approximations

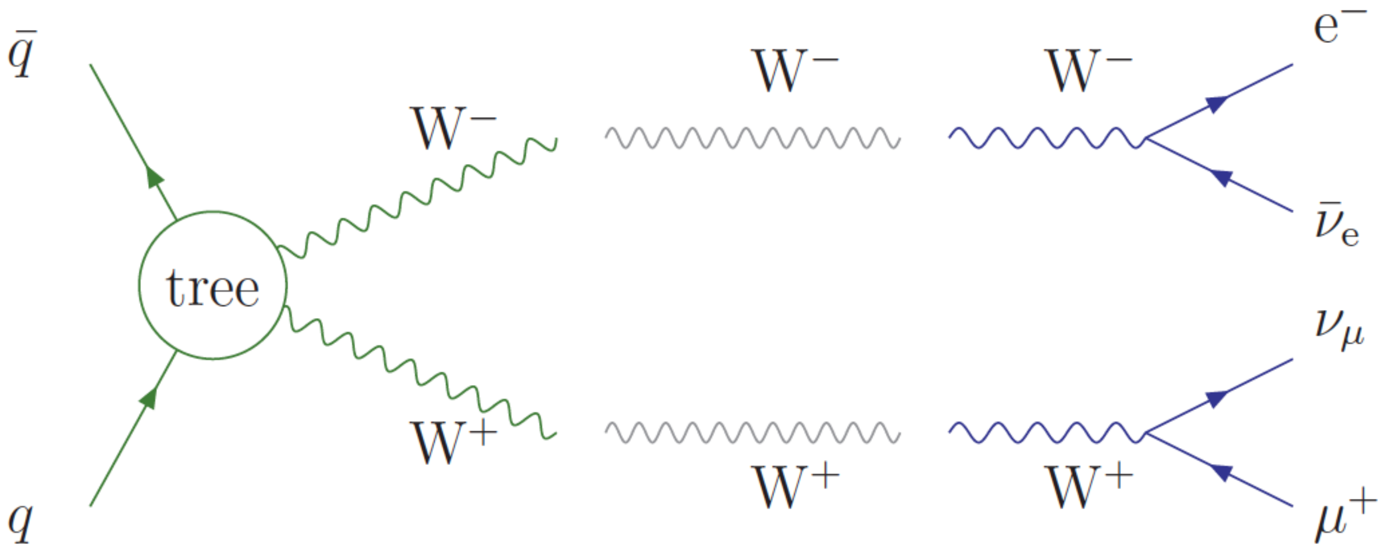
Thank

You.

backup slides . . .

. . . for details and supplementary material

the double-pole approximation (DPA)



on-shell production

off-shell propagators

on-shell decay

the double-pole approximation (DPA)

$$\mathcal{M}_{\mathrm{DPA}} \sim \sum_{pol} \frac{1}{k^2_{W^+} - M^2_W + iM_W\Gamma_W} \frac{1}{k^2_{W^-} - M^2_W + iM_W\Gamma_W}$$

$$\times \mathcal{M}^{\bar{q}q\to W^+W^-} \times \mathcal{M}^{W^+\to\nu\ell^+} \times \mathcal{M}^{W^-\to\bar{\nu}\ell^-}$$

on-shell production

on-shell decay

off-shell propagators

on-shell projection in DPA

kinematics:

$$\boldsymbol{a(p_a) + b(p_b) \to W^+(k_+) + W^-(k_-)}$$

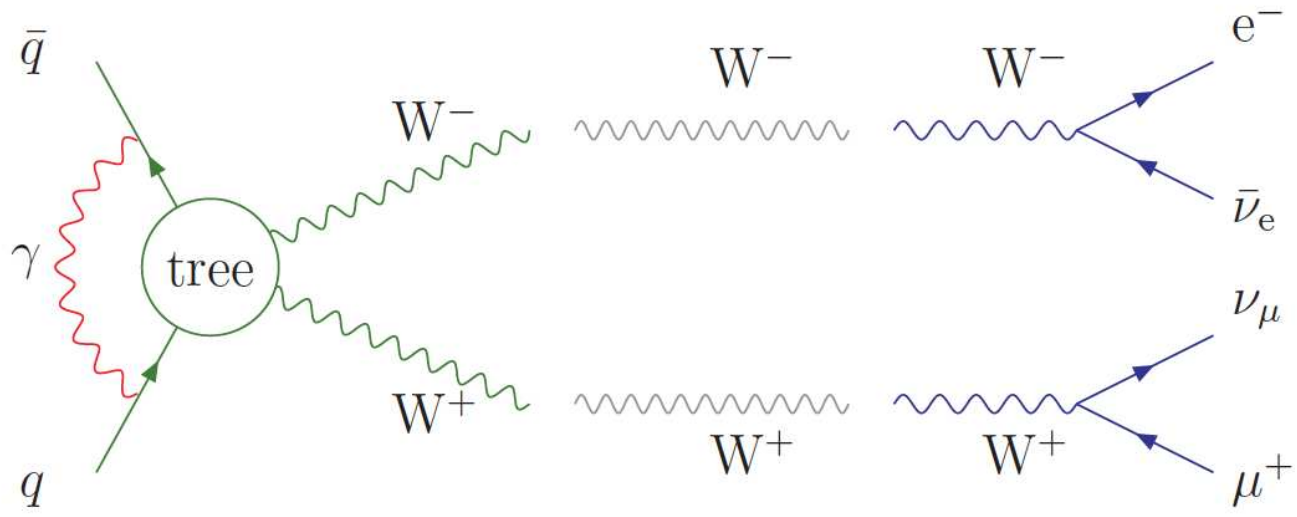
$$\boldsymbol{\to f_1(k_1) + \bar{f}_2(k_2) + f_3(k_3) + \bar{f}_4(k_4)}$$

gauge invariance requires on-shell kinematics
in production and decay amplitudes

$\to$ need to replace off-shell $\boldsymbol{W}$ momenta
with on-shell projections such that

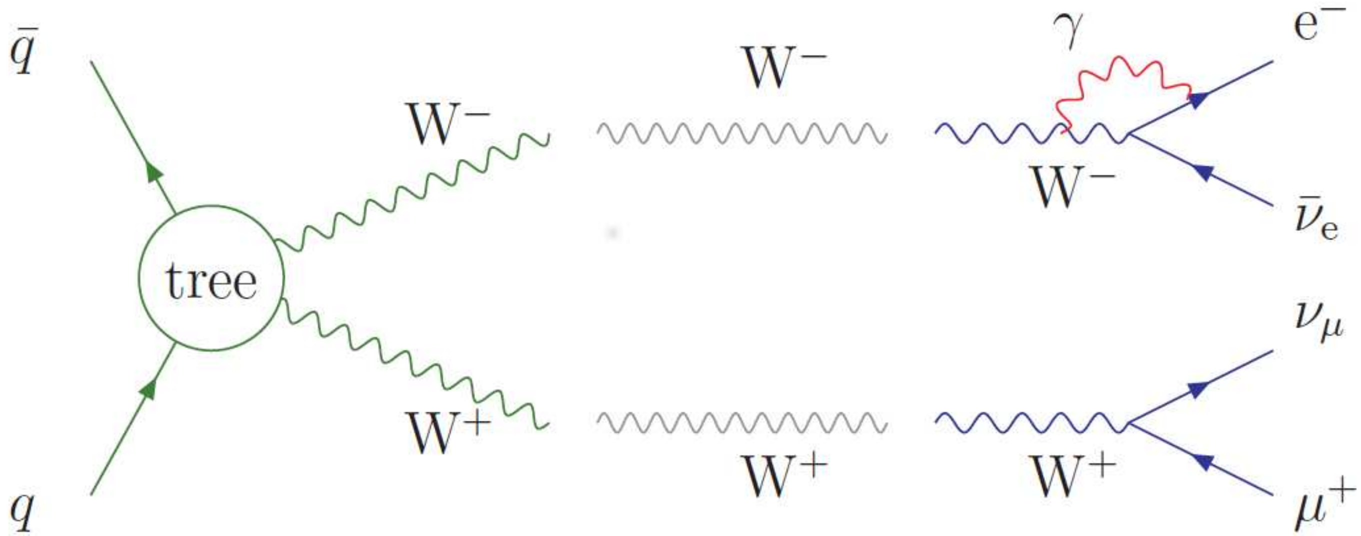
$$\boldsymbol{\hat{k}_W^2 = M_W^2}$$

virtual corrections in DPA



- factorizable corrections to production
- factorizable corrections to decay of W^-
- factorizable corrections to decay of W^+
- non-factorizable corrections (soft photon exchange)

virtual corrections in DPA



- factorizable corrections to production
- factorizable corrections to decay of W^-
- factorizable corrections to decay of W^+
- non-factorizable corrections (soft photon exchange)

factorizable virtual corrections in DPA

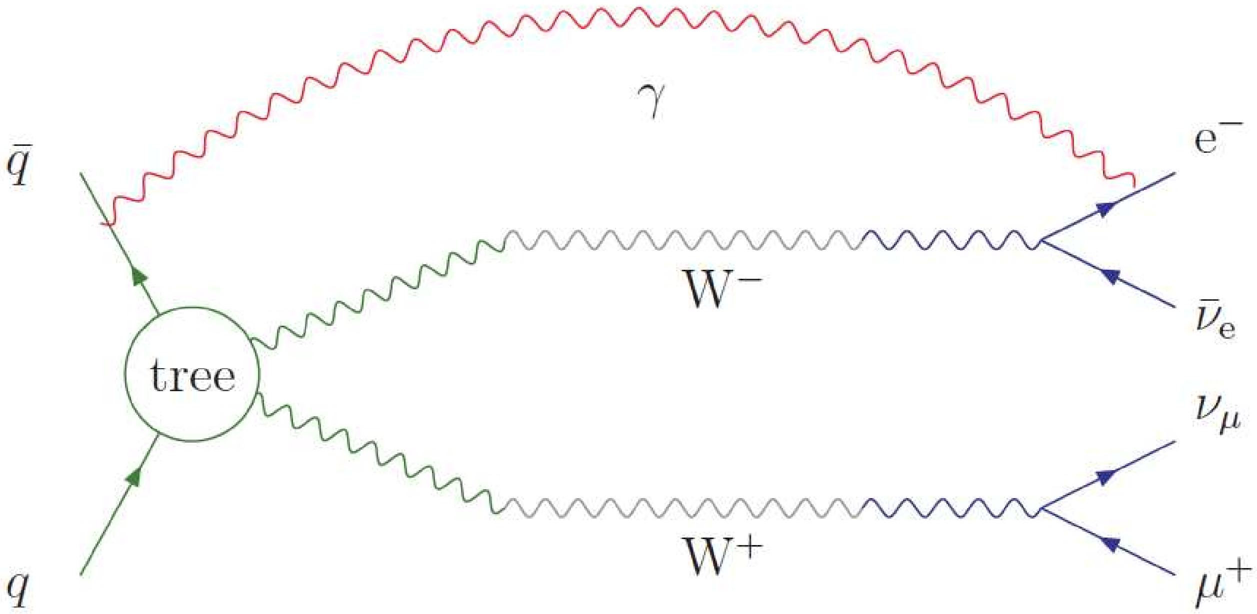
$$\mathcal{M}_{\mathrm{DPA}}^{\mathrm{virt,fact}} \sim \sum_{pol} \frac{1}{\left(k_{W^+}^2 - M_W^2 + iM_W\Gamma_W\right)} \cdot \frac{1}{\left(k_{W^-}^2 - M_W^2 + iM_W\Gamma_W\right)}$$

$$\times \Big\{ \delta\mathcal{M}^{\bar{q}q\to W^+W^-} \times \mathcal{M}^{W^+\to\nu\ell^+} \times \mathcal{M}^{W^-\to\bar{\nu}\ell^-}$$

$$+ \mathcal{M}^{\bar{q}q\to W^+W^-} \times \delta\mathcal{M}^{W^+\to\nu\ell^+} \times \mathcal{M}^{W^-\to\bar{\nu}\ell^-}$$

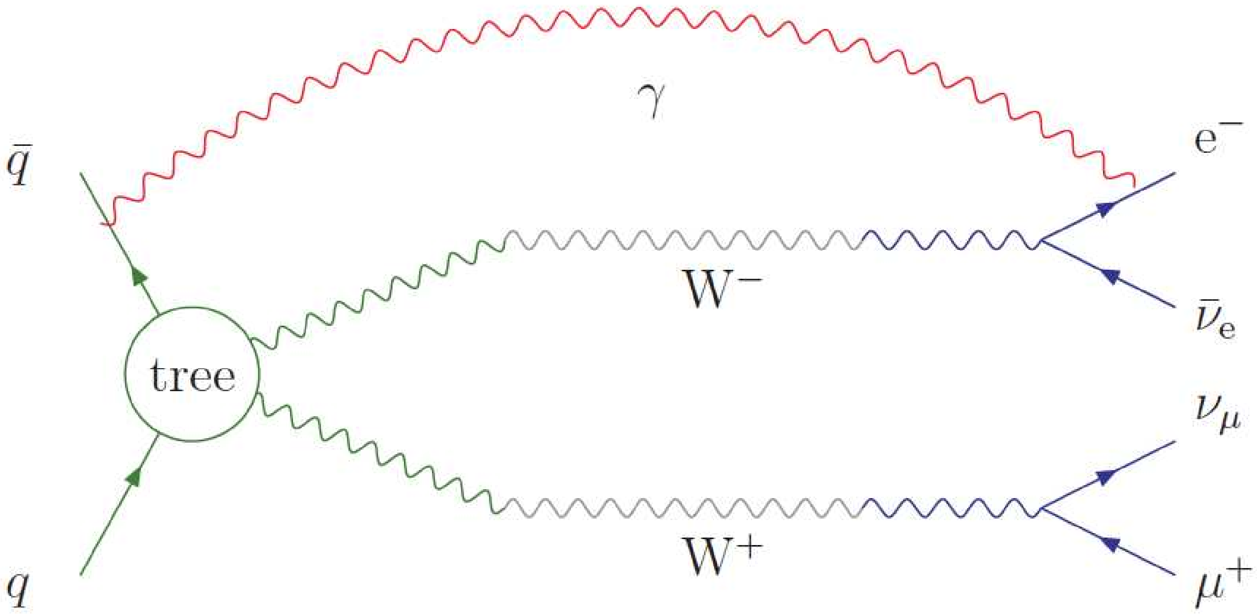
$$+ \mathcal{M}^{\bar{q}q\to W^+W^-} \times \mathcal{M}^{W^+\to\nu\ell^+} \times \delta\mathcal{M}^{W^-\to\bar{\nu}\ell^-} \Big\}$$

virtual corrections in DPA



- factorizable corrections to production
- factorizable corrections to decay of W^-
- factorizable corrections to decay of W^+
- non-factorizable corrections (soft photon exchange)

non-factorizable virtual corrections in DPA



$$\mathcal{M}_{\mathbf{DPA}}^{\mathbf{virt,non-fact}} \sim \mathcal{M}_{\mathbf{LO,DPA}}^{\bar{q}q\to W^+W^-\to\ell^+\nu\ell^-\bar{\nu}} \times \delta_{\mathbf{nfact}}^{\mathbf{virt}}$$

- non-factorizable corrections (soft photon exchange)

improved Born approximation

for $M_{WW} < 2m_W + \Delta_m$:

replace DPA with improved Born approximation
(captures dominant parts of virtual corrections)

[Denner, Dittmaier, Roth, Wackeroth (2001)]

$$d\sigma_{\bar{q}q}^{\mathrm{IBA}} \sim d\mathrm{PS} \left| \mathcal{M}_{\mathrm{IBA}}^{\bar{q}q \to \ell^+ \nu \ell^- \bar{\nu}} \right|^2 \cdot \left[1 + \delta_{\mathrm{Coul}} \right] \cdot g_{\mathrm{damp}}$$

Born type amplitudes
with adjusted couplings

Coulomb singularity
(damped away from threshold)