The study of nuclear structure far from stability

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Introduction

Stable nuclei are qualitatively described by “simple” models

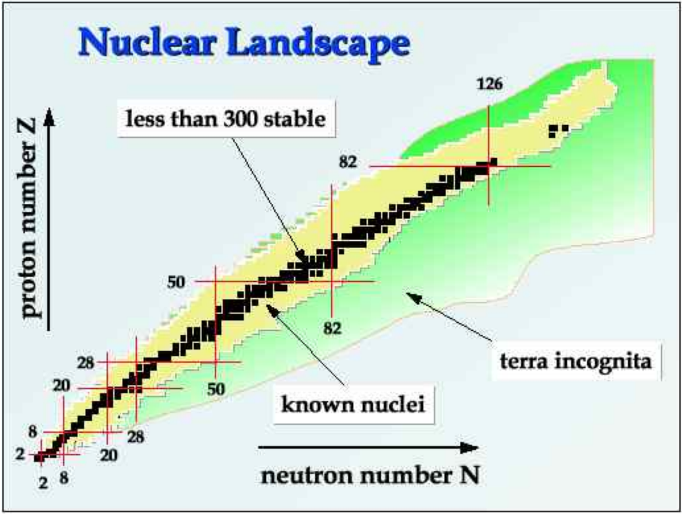
- (semi-empirical) liquid-drop model
- (basic) shell model

New techniques enable ab-initio methods (A-body models)

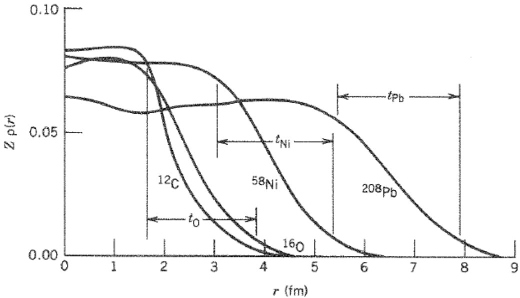
What happens far from stability ?
Experimentally, Radioactive-Ion Beams (RIB) available since 80s
$\Rightarrow$ study of structure far from stability
$\Rightarrow$ discovery of exotic structures

- super-heavy elements
- halo nuclei
- shell inversions

Nuclear Landscape



Charge distributions in (stable) nuclei



- constant density ρ_0 out to the surface (saturation)
- same skin thickness t

(Stable) nuclei look like liquid drops of radius $R \propto A^{1/3}$

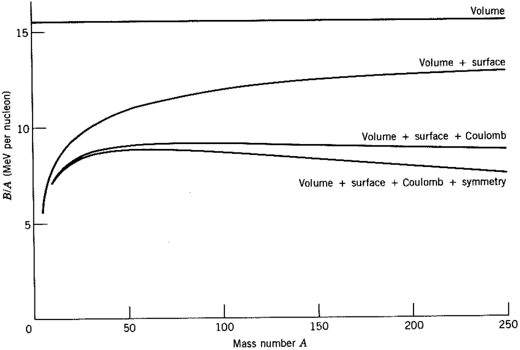
Liquid-drop model

Bethe-Weizsäcker semi-empirical mass formula

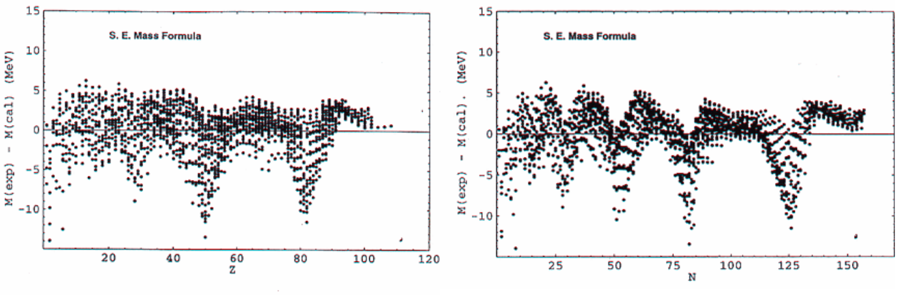
$$B(Z,N) = a_v A - a_s A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_{\mathrm{Sym}} \frac{(A-2Z)^2}{A}$$

Exoenergetic reactions :

- fission of heavy nuclei (nuclear power plants, atomic bomb)
- fusion of light nuclei (stars, thermonuclear weapons)



Variation from the semi-empirical mass formula



More bound systems at Z or $N = 2, 8, 28, 50, 82, 126$

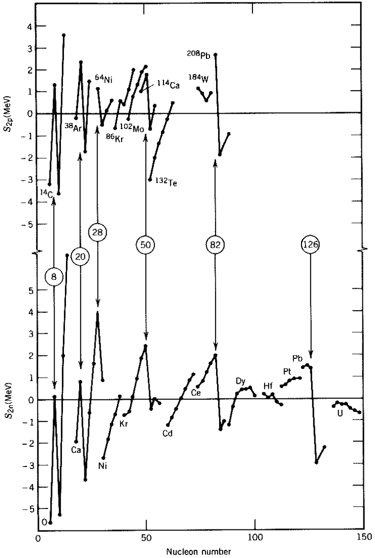
magic numbers

⇒ shell structure in nuclei as in atoms ?

Two-nucleon separation energy

Same magic numbers in S_{2p} and S_{2n}

$\Rightarrow$ more bound at shell closure
cf. ionisation energies of atoms

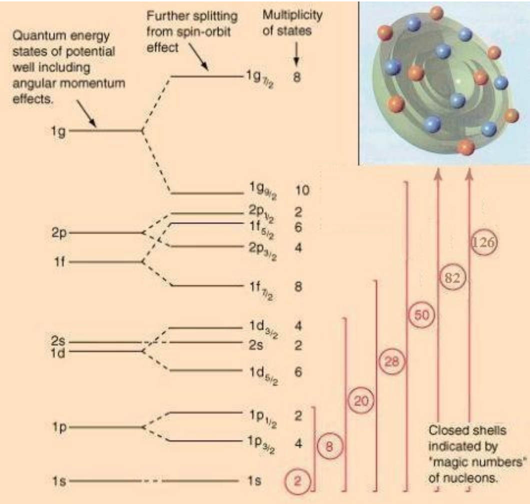


Shell model

Developed in 1949 by M. Goeppert Mayer, H. Jensen and E. Wigner (NP 1963)

As electrons in atoms, nucleons in nuclei feel a mean field and arrange into shells

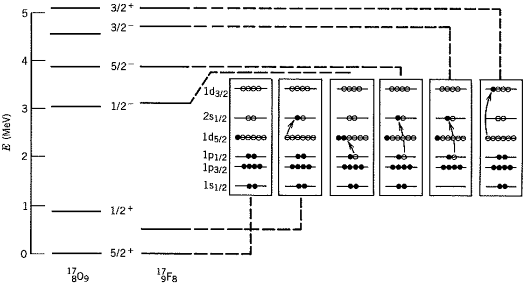
Spin-orbit coupling is crucial to get right ordering of shells



Example

Shell model explains the higher stability at some Z and N

It predicts the spin and parity of ground state of most nuclei and some of their excited levels, e.g. ^{17}O and ^{17}F



Nowadays

Can we go beyond these models ?

Can we build ab-initio models ?
i.e. based on first principles

- nucleons as building blocks
- realistic N-N interaction

What happens away from stability ?

- Is nuclear density similar for radioactive nuclei ?
- Is there a tetraneutron ?
- Are magic numbers conserved ?

A-body Hamiltonian

Nuclear-structure calculations : A nucleons (Z protons+N neutrons)

Relative motion described by the A-body Hamiltonian

$$H = \sum_{i=1}^{A} T_i + \sum_{j>i=1}^{A} V_{ij}$$

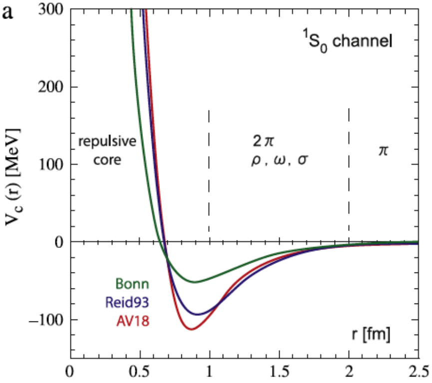
$\Rightarrow$ solve the A-body Schrödinger equation

$$H\,|\Psi_n\rangle = E_n|\Psi_n\rangle$$

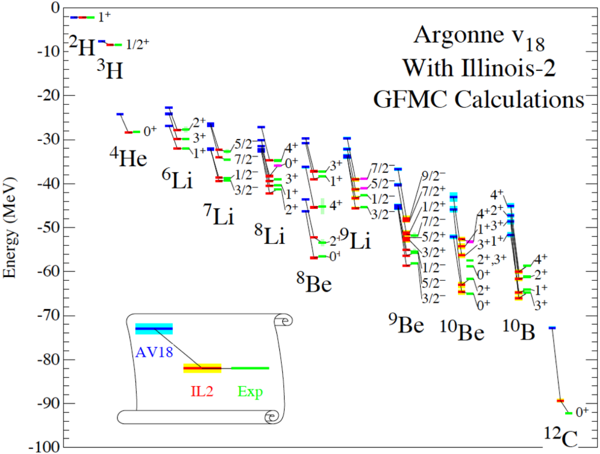
$\{E_n\}$ is the nucleus spectrum

Realistic N-N interactions

V_{ij} not (yet) deduced from QCD
$\Rightarrow$ phenomenological potentials fitted on N-N observables :
d binding energy,
N-N phaseshifts
Ex. : Argonne V18, CD-Bonn,...



Light nuclei calculations



[R. Wiringa, Argonne]

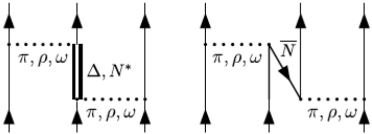
Three-body force

Need three-body forces to get it right…

$$H = \sum_{i=1}^{A} T_i + \sum_{j>i=1}^{A} V_{ij} + \sum_{k>j>i=1}^{A} V_{ijk} + \cdots$$

But there is no such thing as three-body force…

They simulate the non-elementary character of nucleons
$\Rightarrow$ include virtual Δ resonances, $\bar{\mathrm{N}}$…



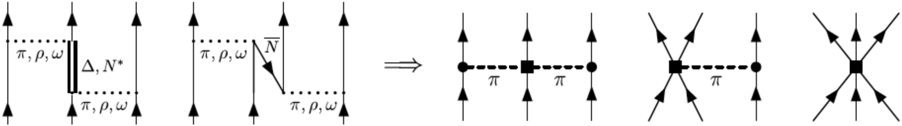
Phenomenological 3-body interaction fitted on $A > 2$ levels : IL2
Alternatively, derived from EFT

Effective Field Theory

EFT is an effective quantum field theory based on QCD symmetries with resolution scale Λ that selects appropriate degrees of freedom : nuclear physics is not built on quarks and gluons, but on nucleons and mesons

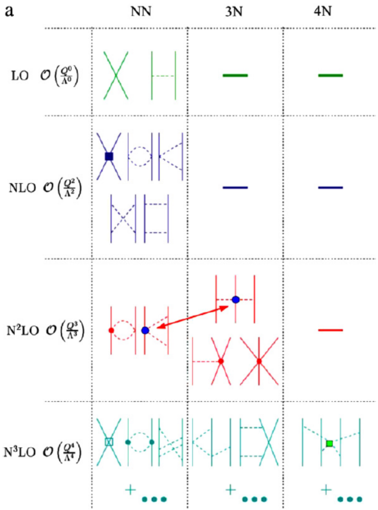
EFT provides the nuclear force with a systematic expansion in Q/Λ

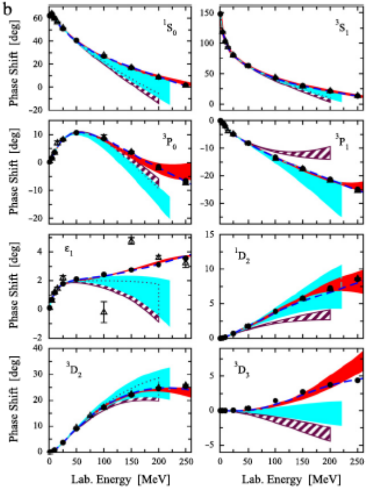
- gives an estimate of theoretical uncertainty
- naturally includes many-body forces



[see E. Eppelbaum's talk on Tuesday morning and S. Bacca's on Thursday morning]

Expansion of the EFT force





Solving the Schrödinger equation

$$H\,|\Psi_n\rangle = E_n|\Psi_n\rangle$$

Ψ usually developed on a basis $\{|\Phi_{[\nu]}\rangle\}$:

$$|\Psi_n\rangle = \sum_{[\nu]} \langle \Phi_{[\nu]}|\Psi_n\rangle \; |\Phi_{[\nu]}\rangle$$

Solving the Schrödinger equation reduces to matrix diagonalisation

$$\begin{aligned}\langle \Phi_{[\mu]}|H|\Psi_n\rangle &= \sum_{[\nu]} \langle \Phi_{[\mu]}|H|\Phi_{[\nu]}\rangle\langle \Phi_{[\nu]}|\Psi_n\rangle \\ &= E_n \, \langle \Phi_{[\mu]}|\Psi_n\rangle\end{aligned}$$

$\Rightarrow$ need to build an efficient set of basis states $\{|\Phi_{[\nu]}\rangle\}$

Clear short review paper : [Bacca EPJ Plus **131**, 107 (2016)]

No-Core Shell Model

One should be able to account for the fermion nature of nucleons
⇒ wave function must be antisymmetric
⇒ basis states built as Slater determinants
of 1-body mean-field wave functions ϕ_{ν_i}

$$\begin{aligned}\langle \xi_1 \xi_2 \ldots \xi_A | \Phi_{[\nu]} \rangle &= \mathcal{A}\, \phi_{\nu_1}(\xi_1)\, \phi_{\nu_2}(\xi_2) \ldots \phi_{\nu_A}(\xi_A) \\ &= \frac{1}{A!} \begin{vmatrix} \phi_{\nu_1}(\xi_1) & \phi_{\nu_1}(\xi_2) & \cdots & \phi_{\nu_1}(\xi_A) \\ \phi_{\nu_2}(\xi_1) & \phi_{\nu_2}(\xi_2) & \cdots & \phi_{\nu_2}(\xi_A) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{\nu_A}(\xi_1) & \phi_{\nu_A}(\xi_2) & \cdots & \phi_{\nu_A}(\xi_A) \end{vmatrix}\end{aligned}$$

The shell model uses harmonic-oscillator wave functions for ϕ_{ν_i}
The basis size increases with A ⇒ limited to light nuclei

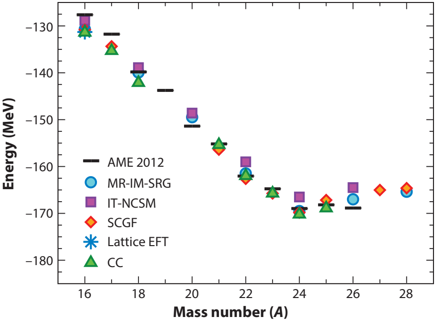
Coupled-cluster theory

Instead of building a huge basis and diagonalise the Hamiltonian, one can start from a reference Slater determinant Φ_0 and build the wave function as

$$
\begin{aligned}
\Psi &= e^{T}\ \Phi_0 \\
\text{where}\quad T &= T_1 \quad \left(T_1 = \sum_{ia} t_i^a\ a_a^\dagger a_i\right) \qquad (1p\text{-}1h \text{ excitation}) \\
&+ T_2 \quad \left(T_2 = \sum_{ijab} t_{ij}^{ab}\ a_a^\dagger a_b^\dagger a_j a_i\right) \qquad (2p\text{-}2h \text{ excitation}) \\
&+ \ldots
\end{aligned}
$$

Converges quickly ⇒ less computational expensive than NCSM
⇒ available for heavier nuclei

Example : oxygen isotopes



[Hebeler *et al.* Annu. Rev. Nucl. Part. Sci. **65**, 457 (2015)]

Different *ab initio* models predict similar result
All require $3N$ forces to reproduce the dripline at ^{24}O

What happens far from stability ?

Liquid-drop and shell models are fair models of stable nuclei

What happens away from stability ?

In 80s Radioactive-Ion Beams were developed
Enable study of nuclear structure

[see A. Cowley's talk on Thursday]

- are radioactive nuclei compact ?
- are shells conserved far from stability ?

Reactions involving radioactive nuclei useful in astrophysics

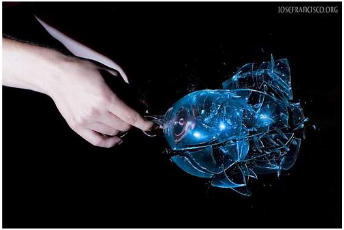
[see 2nd part, J. Jose's talk on Monday,
M. Aliotta's talk on Wednesday]

How ?

Idea : break a heavy nuclei into pieces to produce exotic isotopes

- ISOL : Fire a proton at a heavy nucleus

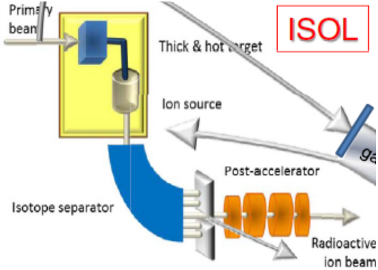
- In-flight : Smash a heavy nucleus on a target



Where ?


CANADA
UNITED STATES
NORTH PACIFIC OCEAN
NORTH ATLANTIC OCEAN
RUSSIA
SIBERIA
KAZAKHSTAN
MONGOLIA
CHINA
INDIA
IRAN
ALGERIA
LIBYA
EGYPT
SAHARA
SUDAN
BRAZIL
SOUTH PACIFIC OCEAN
SOUTH ATLANTIC OCEAN
INDIAN OCEAN
AUSTRALIA
INDONESIA
SOUTHERN OCEAN
Antarctica

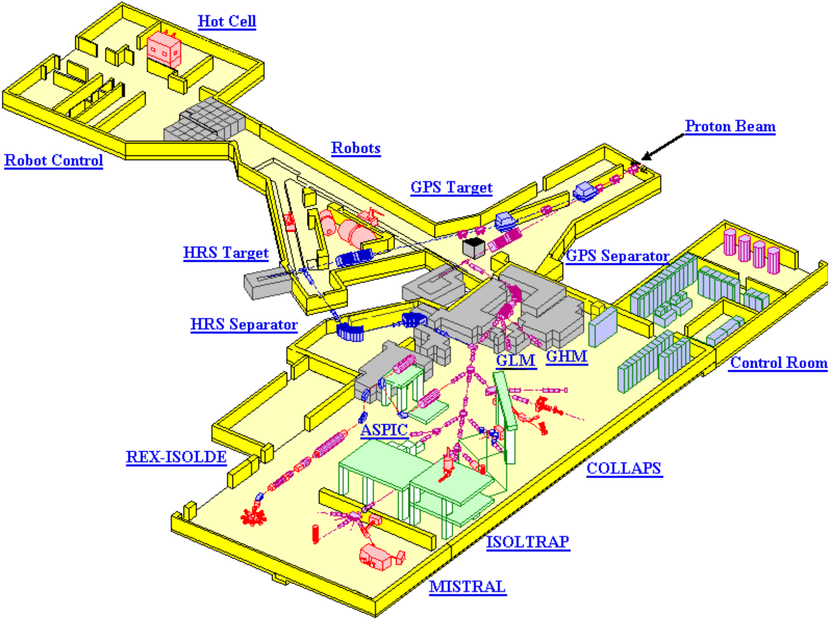
ISOL : Isotope Separation On Line



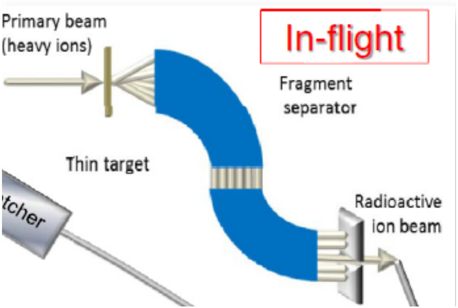
high-energy/intensity primary beam of light nuclei (e.g. protons) on thick target of heavy elements (Ta or UC_x)
$\Rightarrow$ spallation/fragmentation produces exotic fragments
Diffuse in the target and effuse to an ion source
Then selected using dipole magnet (A/Q)
Either used directly (mass measurement, radioactive decay. . .) or post-accelerated for reactions (e.g. astrophysical energy)

Examples : ISOLDE (CERN), TRIUMF, SPIRAL (GANIL)

ISOLDE @ CERN


Hot Cell
Proton Beam
Robots
Robot Control
GPS Target
HRS Target
GPS Separator
HRS Separator
GLM
GHM
Control Room
ASPIC
REX-ISOLDE
COLLAPS
ISOLTRAP
MISTRAL

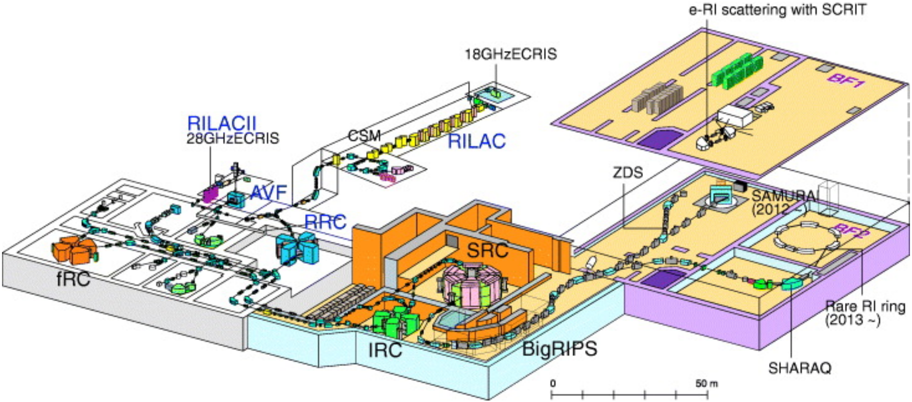
In-flight projectile fragmentation



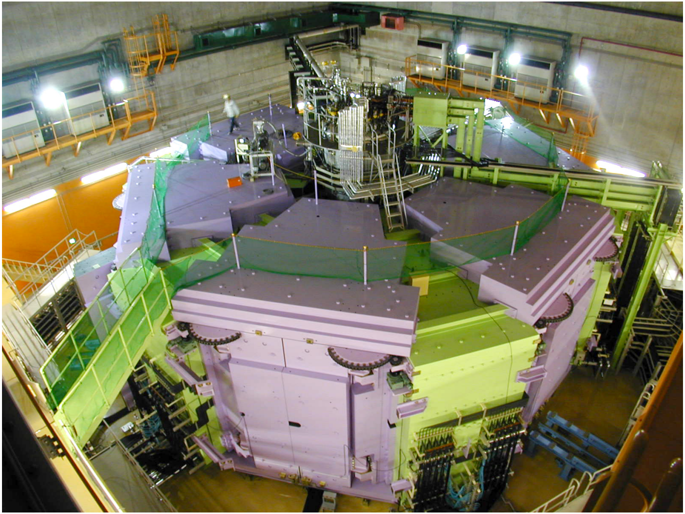
high-energy primary beam of heavy ions (e.g. ^{18}O, ^{48}Ca, U...)
on thin target of light elements (Be or C)
$\Rightarrow$ fragmentation/fission produces many exotic fragments at $\approx v_{\text{beam}}$
Sorted in fragment separator
Used for high-energy reactions (KO, breakup...)

Examples : RIKEN, NSCL (MSU), GSI, GANIL

RIBF @ RIKEN


e-RI scattering with SCRIT
18GHzECRIS
BF1
RILACII
28GHzECRIS
CSM
RILAC
ZDS
AVF
SAMURAI
(2012 ~)
RRC
BF2
SRC
fRC
Rare RI ring
(2013 ~)
IRC
BigRIPS
SHARAQ
0
50 m

Superconducting Ring Cyclotron



Largest superconducting cyclotron in the world
Delivers a U beam at $350A$MeV

Properties

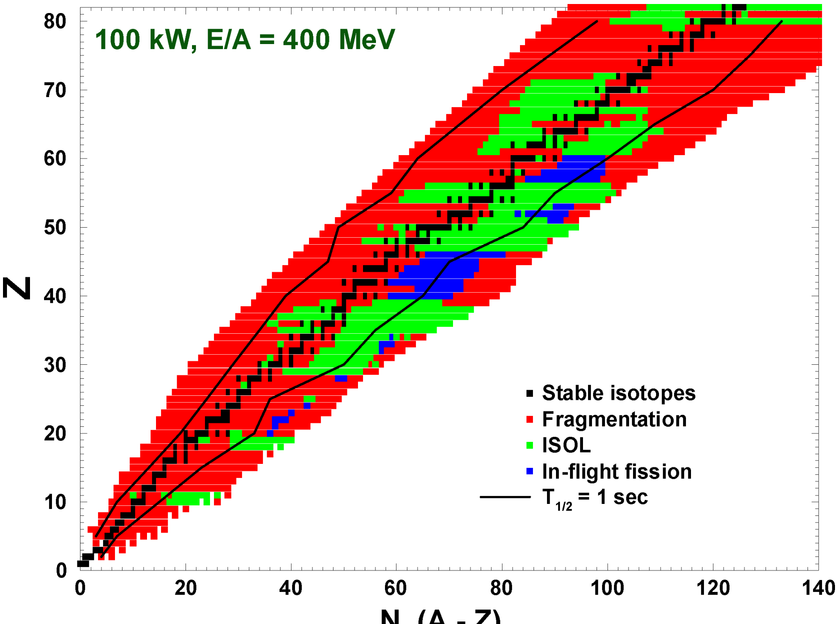
ISOL

- Low beam energy
  may require post-acceleration
- Low beam intensity
- Not all elements produced
  - Slow
  - Chemically limited
- Good beam quality :
  can use chemistry and atomic physics to select fragments

In-flight

- High beam energy
  $v_{\text{fragments}} \approx v_{\text{beam}}$
- High beam intensity
- Efficient production
  - Fast
  - Chemically independent
- Many fragments in beam
  $\Rightarrow$ need ion ID

Choose according what you want to measure

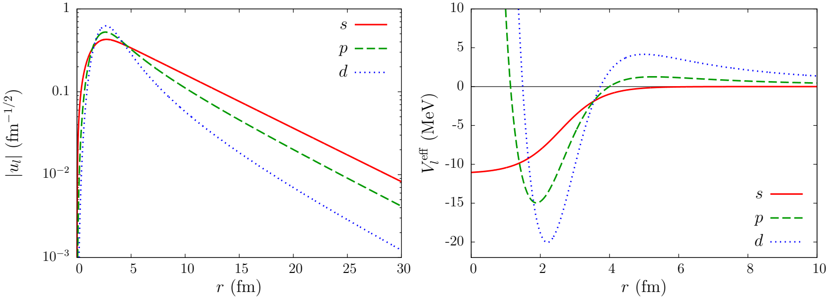


Halo structure

Seen as core + one or two neutrons at large distance
[P. G. Hansen and B. Jonson, Europhys. Lett. 4, 409 (1987)]

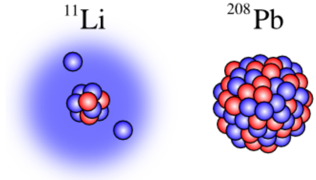
Peculiar structure of nuclei due to small $S_{\rm n}$ or $S_{\rm 2n}$
$\Rightarrow$ neutrons tunnel far from the core to form a halo

Halo only appears for low centrifugal barrier (low ℓ)



Halo nuclei

- Light, neutron-rich nuclei
- small S_{n} or $S_{2\mathrm{n}}$
- low-ℓ orbital



One-neutron halo

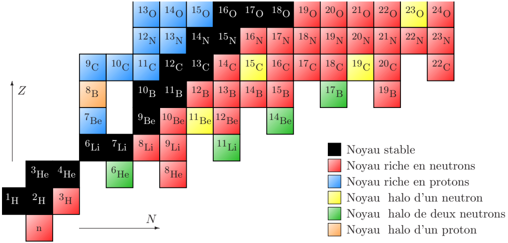
^{11}Be ≡ ^{10}Be + n

^{15}C ≡ ^{14}C + n

Two-neutron halo

^{6}He ≡ ^{4}He + n + n

^{11}Li ≡ ^{9}Li + n + n



Two-neutron halo nuclei are Borromean. . .

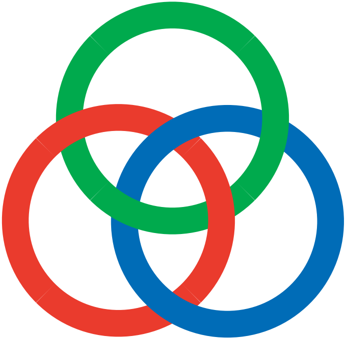
c+n+n is bound but not two-body subsystems

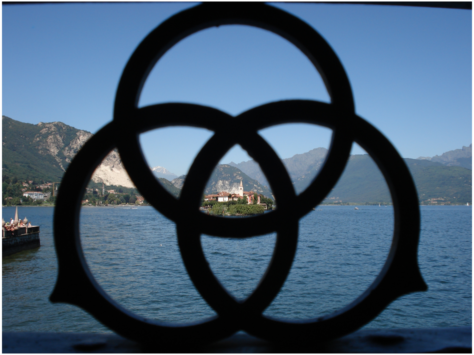
e.g. ^{11}Li bound but not ^{10}Li nor ^{2}n [see A. Cowley's talk on Thursday]

Borromean nuclei

Named after the Borromean rings. . .

[M. V. Zhukov *et al.* Phys. Rep. 231, 151 (1993)]





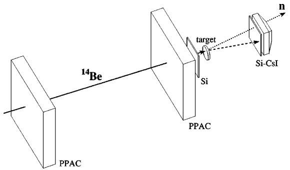
Tetraneutron

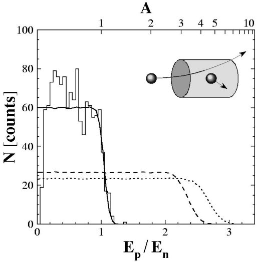
Can 4 neutrons be bound together ? or form a resonance ?
It would be a strong test of nuclear models far from stability

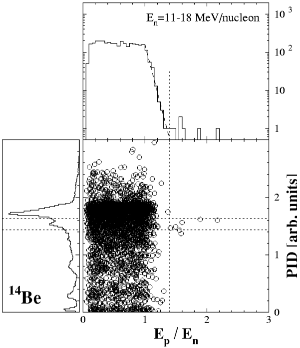
Various experiments have been performed to look for such a state
In 2002, Marqués *et al.* have reported to have found a bound ^{4}n using the breakup of the (very) neutron rich ^{14}Be :

$$^{14}\mathrm{Be} + \mathrm{C} \rightarrow \ ^{10}\mathrm{Be} + \ ^4\mathrm{n} + \mathrm{C} \qquad @\ 35A\text{MeV (GANIL)}$$

[Marqués *et al.* PRC **65**, 044006 (2002)]
They found 6 unexplained counts
⇒ possible ^{4}n







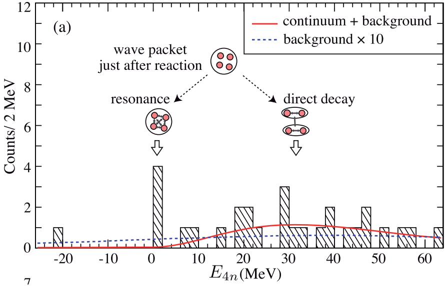
Double-charge exchange reaction

More recently, Kisamori *et al.* have measured

$$^{8}\mathrm{He} + {}^{4}\mathrm{He} \rightarrow 2\alpha + {}^{4}\mathrm{n} \qquad @186A\mathrm{MeV}\ \text{(RIKEN)}$$

[Kisamori *et al.* PRL 116, 052501 (2016)]

They measure 2 α in coincidence
and deduce $E_{4\mathrm{n}}$ by the missing-mass method

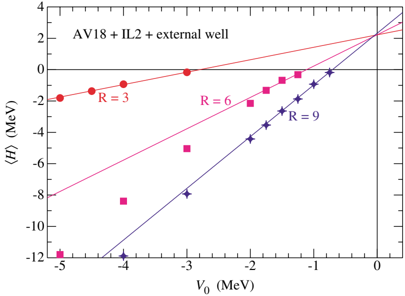


⇒ low-energy ^{4}n resonance ($E_{4\mathrm{n}} = 0.83 \pm 0.65$ MeV $\Gamma_{4n} < 2.6$ MeV)

On the theory side…

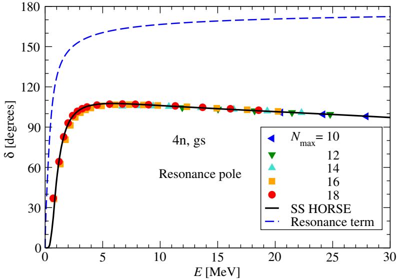
Since Marqués' measurement, theoretical models have been tested

Within GFMC Pieper predicted a ^{4}n resonance at ~ 2 MeV



[Pieper PRL **90**, 252501 (2003)]

Within NCSM Shirikov *et al.* find $E_{4n} = 0.844$ MeV $\Gamma = 1.378$ MeV



[Shirikov *et al.* PRL **117**, 182502 (2016)]

Summary

Liquid-drop and shell model describe qualitatively stable nuclei
Nowadays ab-initio nuclear-structure models from first principles

RIB enable study nuclear structure far from stability
Low intensities require new experimental techniques :
KO reactions, in-beam γ spectroscopy,...

- discovery of halo nuclei
  diffuse halo around a compact core
- shell inversions or shell collapse

RIB can be used to study reactions of astrophysical interest