Study of corrections to the eikonal approximation

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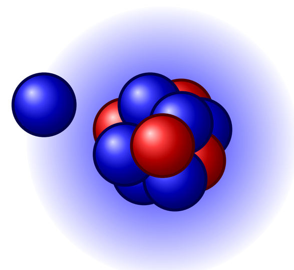
Motivation

- **Halo nuclei:** exotic structure with large matter radius due to one or two loosely-bound neutrons.

→ Seen as a **compact core** with one or two **valence neutrons**.

Ex: ^{11}Be $\equiv$ ^{10}Be + n
^{11}Li $\equiv$ ^{9}Li + n + n



Short-lived:

→ Cannot be studied through spectroscopic methods

→ Studied through **reaction processes:** elastic scattering, breakup reactions,...

⇒ **Need an accurate reaction model to infer reliable information**

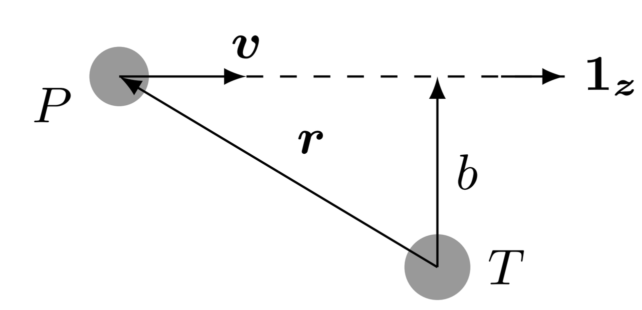
- The **eikonal model** is a quantal approximation which
  - ⊕ has a **reduced computational time** (factor of 8 gain from exact calculations)
  - ⊕ provides a **simple interpretation** of the reaction
  - ⊖ is **valid only at high energies**

⇒ **Aim of this study: extend its range of validity to low energies**

The eikonal approximation and its corrections

Assumptions:

- Central potential V simulating the projectile (P) - target (T) interaction
- In a first step, **elastic scattering** of structureless and spinless nuclei



Schrödinger equation: $\left(-\frac{\hbar^2}{2\mu}\Delta_{\boldsymbol{r}} + V(r)\right)\Psi(\boldsymbol{r}) = E\,\Psi(\boldsymbol{r})$,

where μ is the P-T reduced mass and E the energy.

The eikonal approximation: at high energy, the wave function ≈ plane wave [1]

$$\Psi(\boldsymbol{r}) = e^{ikz}\,\widehat{\Psi}(\boldsymbol{r}) \quad \text{with } |\Delta_{\mathbf{r}}\widehat{\Psi}| \ll k\left|\frac{\partial}{\partial z}\widehat{\Psi}\right|$$

$\Longrightarrow$ **Solutions:** $\boxed{\Psi(\boldsymbol{r}) = e^{ikz}e^{i\chi(b)}}$ with $\chi(b) = -\frac{1}{\hbar v}\int_{-\infty}^{z} V(b,z')dz'$ the **eikonal phase**

P is seen as following a **straight-line** upon which it accumulates a phase through its interaction with T.

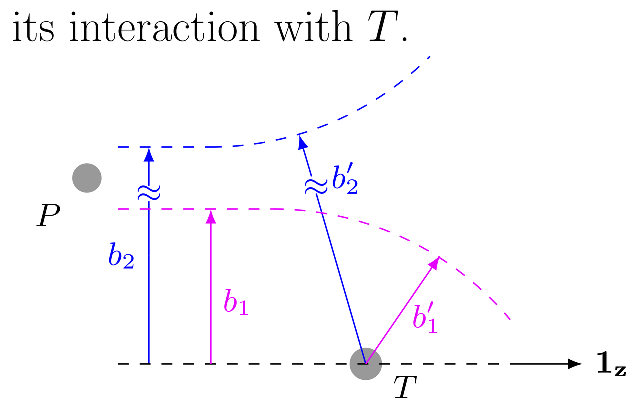
At low energy: P is deflected by T → eikonal approximation **not valid**.

⇒ **Two corrections to account for the deflection:**

① **Wallace's correction:** perturbation development of the T-matrix [2]
$T = (V + VgV) + VgNgV + ...$ where g is the eikonal propagator.

② **The semi-classical correction:** $\chi(b) \rightarrow \frac{b'}{b}\chi(b')$,

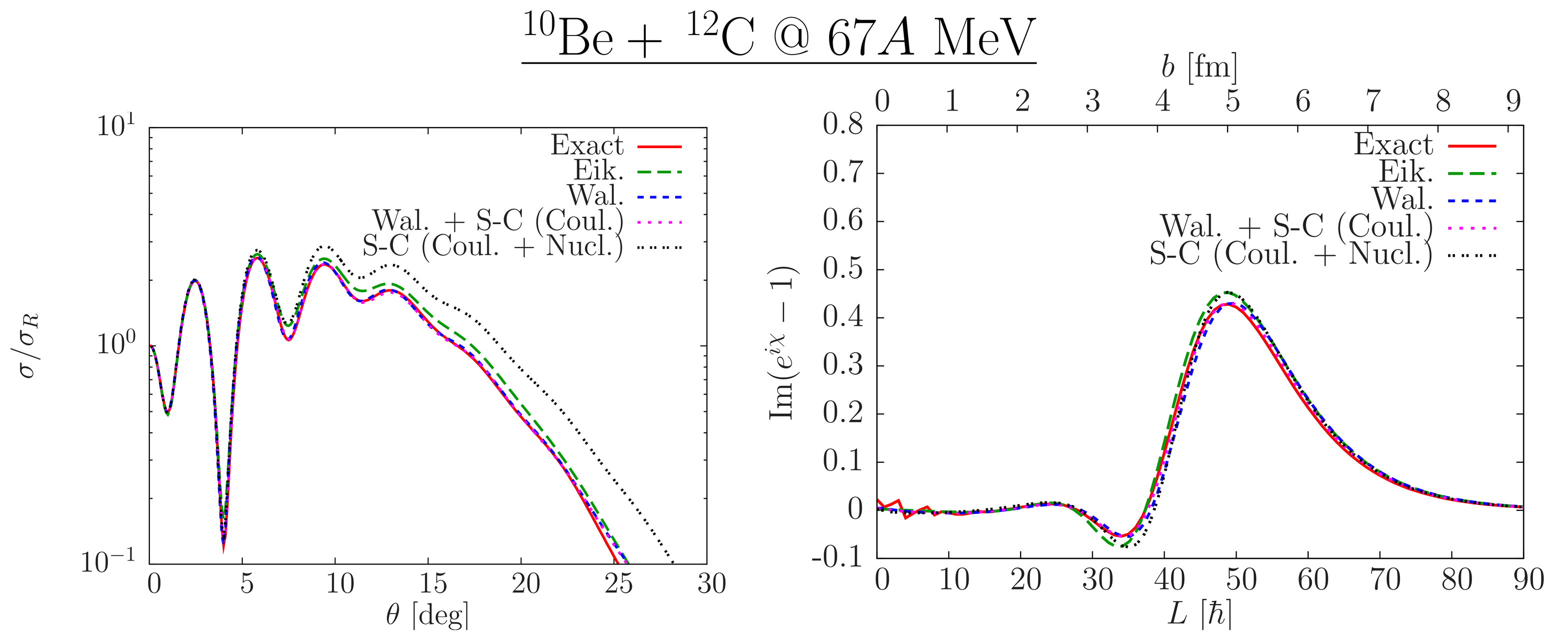
where b' is the P-T distance of closest approach computed using the **real part of the potential** [3,4].



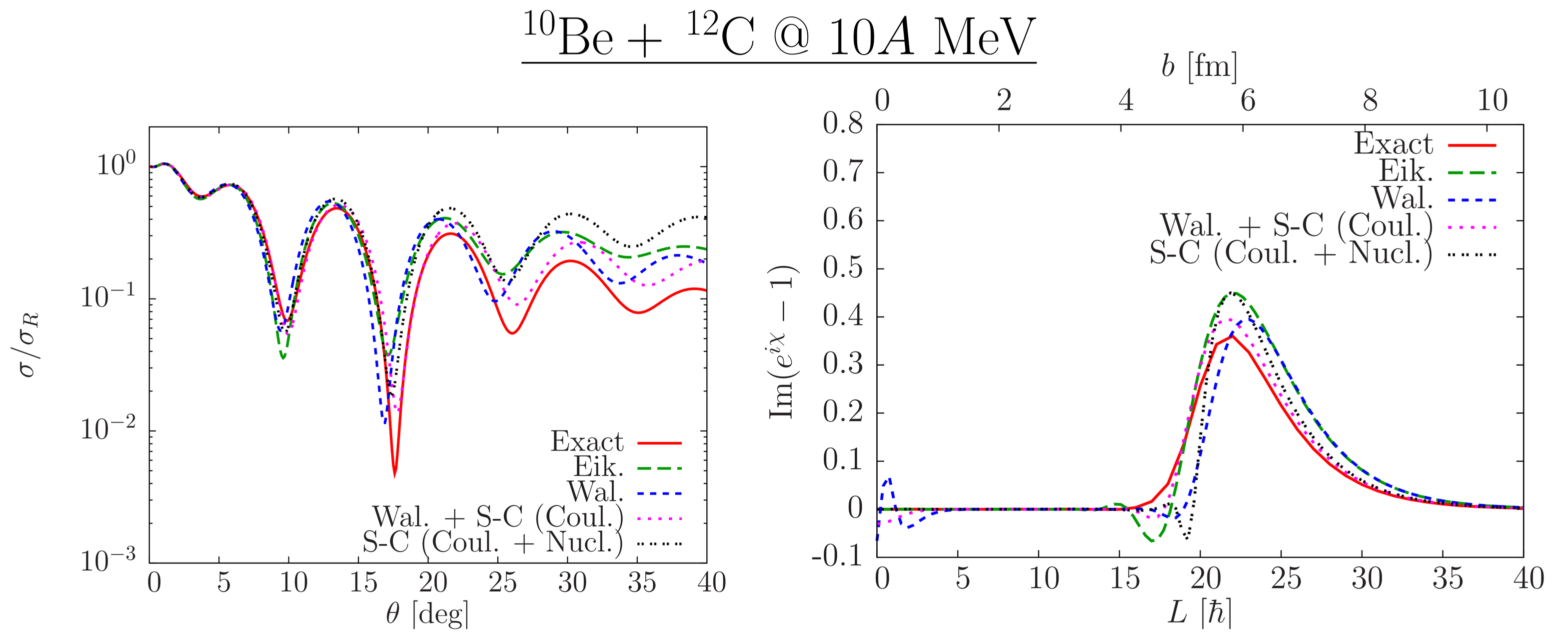
Deflection of the trajectories at two b: the trajectory at b_1 is nuclear dominated at b_2 is Coulomb dominated

Cross sections and eikonal phases of the elastic scattering of ^{10}Be off ^{12}C

^{10}Be + ^{12}C @ 67A MeV



^{10}Be + ^{12}C @ 10A MeV



① **Wallace's correction** (Wal.):

- nearly exact results @ 67A MeV but less effective @ 10A MeV
- better reproduction of the oscillation pattern of the cross sections
- shift of the results: σ to forward angles
  $e^{i\chi}$ to large b

\+ **Semi-classical correction** to only the **Coulomb** interaction (Wal. + S-C (Coul.)):

- compensation of the shift: σ to large angles
  $e^{i\chi}$ to small b

⊕ **better agreement with the exact results**

⊖ **inaccurate at low energies and large angles**

⊖ **lack of absorption at small** b

② **The semi-classical correction** to the **whole interaction** (S-C (Coul. + Nucl.)):

- more accurate results at forward scattering angles
- better reproduction of the oscillations

⊖ **insufficient at large angles**

⊖ **lack of absorption at small** b

Conclusions and prospects

①**Wallace's correction**: small extension to low energies
not sufficient at very low energies and large angles

②**The semi-classical correction:** not accurate at large angles

⇒**Lack of absorption** at small b and large angles

- **To increase absorption: use a complex distance of closest approach computed with the whole optical potential**
- **Generalization to two- and three-body projectile and breakup reactions**

References

[1] R. J. Glauber, *High energy collision theory*, Lecture in theoretical physics, (1959).
[2] S. J. Wallace, Ann. Phys. **78**, 190 (1972).
[3] C. E. Aguiar, F. Zardi and A. Vitturi, Phys. Rev. C **56**, 1511 (1997).
[4] T. Fukui, K. Ogata, and P. Capel. Phys. Rev. C **90**, 034617 (2014).

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