# Modern Particle Detectors

Bernhard Ketzer
Helmholtz-Institut für Strahlen- und Kernphysik

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Boppard

# Summary of Second Lecture

- Ionization detectors
  - Transport of charge: drift and diffusion
  - Gas amplification
  - Energy resolution
  - Signal formation: Ramo-Shockley Theorem
- Position-sensitive detectors
  - Resistive Plate Chambers (RPC)
  - Micropattern Gaseous Detectors (MPGD)
    - Micromegas
    - Gas Electron Multiplier

# Plan of the Lecture

# 4 Position Measurement

4.1 Resistive Plate Chambers

4.2 Micropattern Gaseous Detectors

4.3 Semiconductor Detectors

# New Challenges

**Larger active areas**

- Bulk Micromegas
- Single-mask GEMs

**Higher rates**

- Pixel readout
- Ion backflow suppression


**Aging, discharge protection**

- Materials
- Multi-stage amplification
- Segmentation
- Resistive coating

**Higher resolutions**

- $\mu$Pixel
- InGrid

**Special shapes**

- cylindrical
- spherical

# ATLAS Muon Chambers

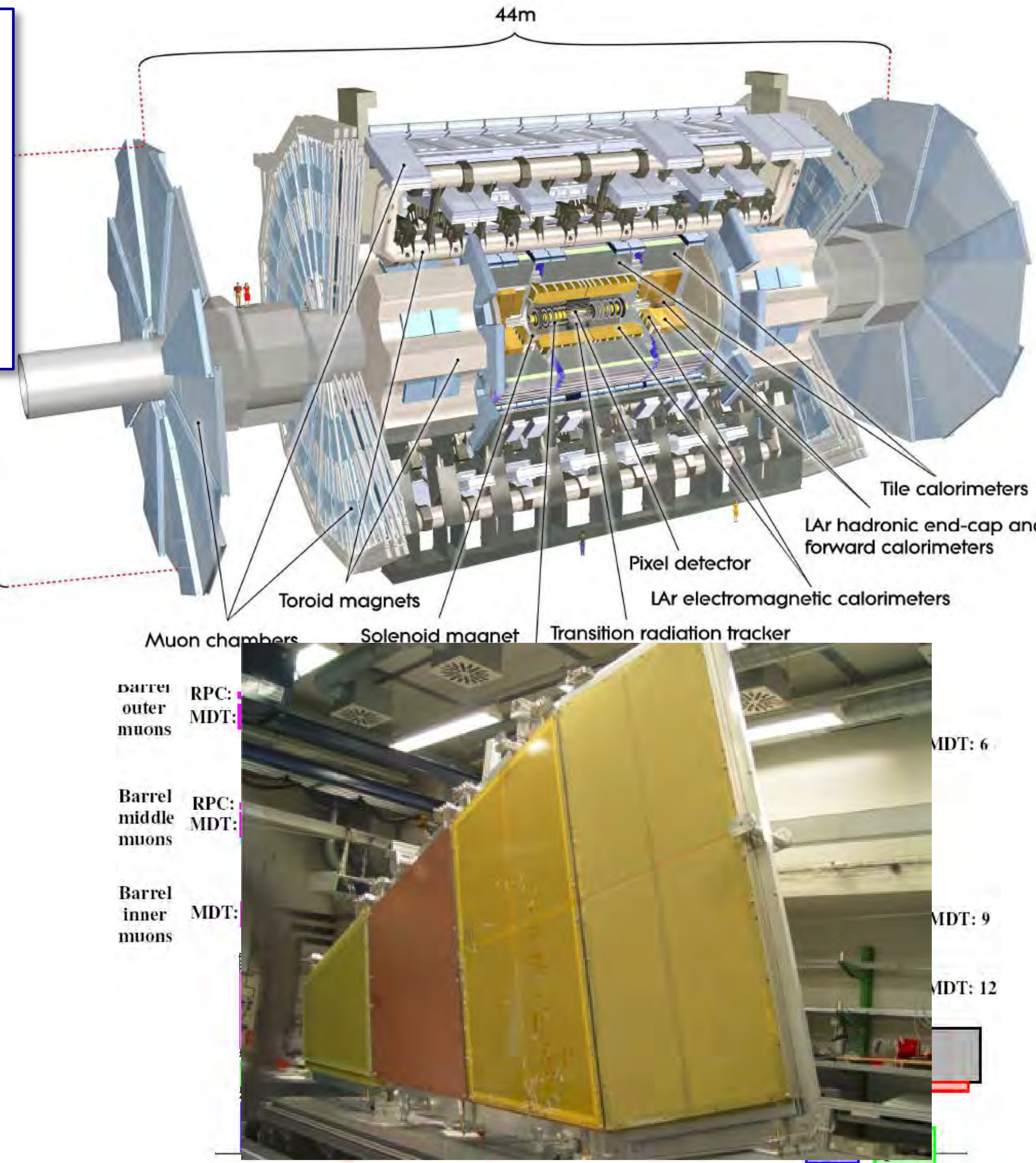
**LHC** LS2: $\mathcal{L} = 10 \cdot 10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$

⇨Rates too high for

- Endcap Inner: CSC, MDT, TGC
- Endcap Middle: MDT, TGC

Upgrade of Muon Spectrometer with **Micromegas**:

- Rate capability > 5 kHz/cm$^2$
- Spatial resolution ~100 μm
- Time resolution < 5 ns
- Efficiency > 98%
- Level-1 triggering capability
- Radiation hardness, no aging
- Area ~ 400 m$^2$



# Micromegas – Bulk Technology

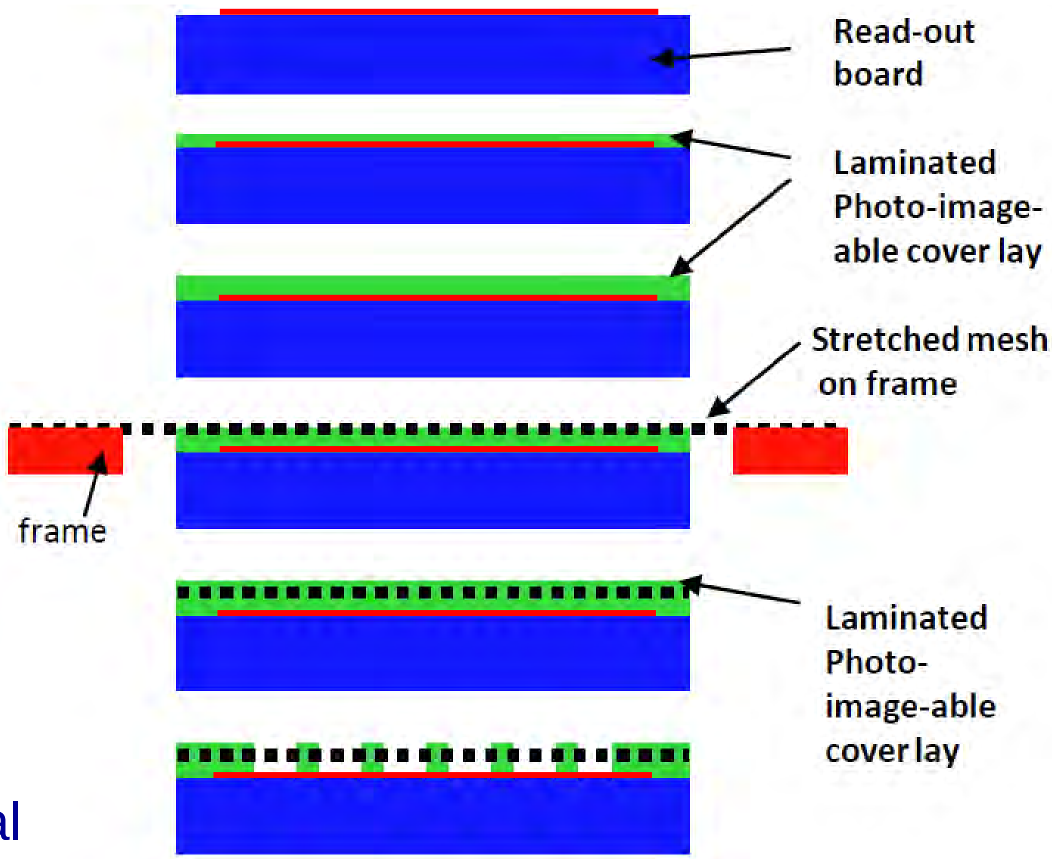
Anode PCB, e.g. FR4, with Cu

Photoresistive film, e.g. Vacrel

Woven wire mesh
- 2m × 40 m rolls
- Fe, Cu, Ti, stainless steel

Laminated together at high temp. and pressure

Etching of photoresistive material
⇨ pillars of 200 - 300 μm ø




[I. Giomataris et al., NIM A 560, 405 (2006)]

# Spark Protection

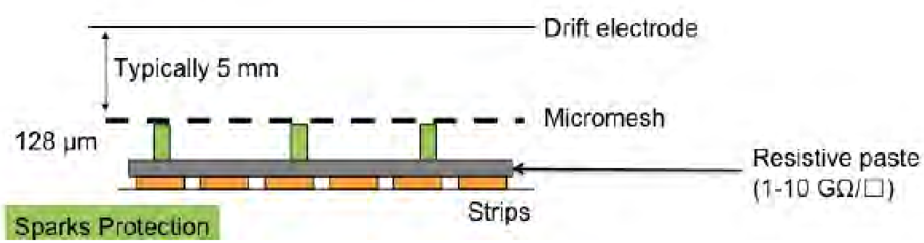


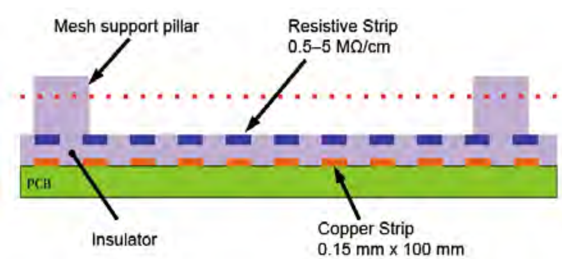
Consequences of sparks:

- Complete discharge of mesh ⇨ dead time: t ~ RC = 1MΩ * 1nF = 1ms
- Huge charge ⇨ protection of FE electronics necessary (diode clamping, AC)
- Destruction of strips, etc.

Strategy: limit impact on detector performance

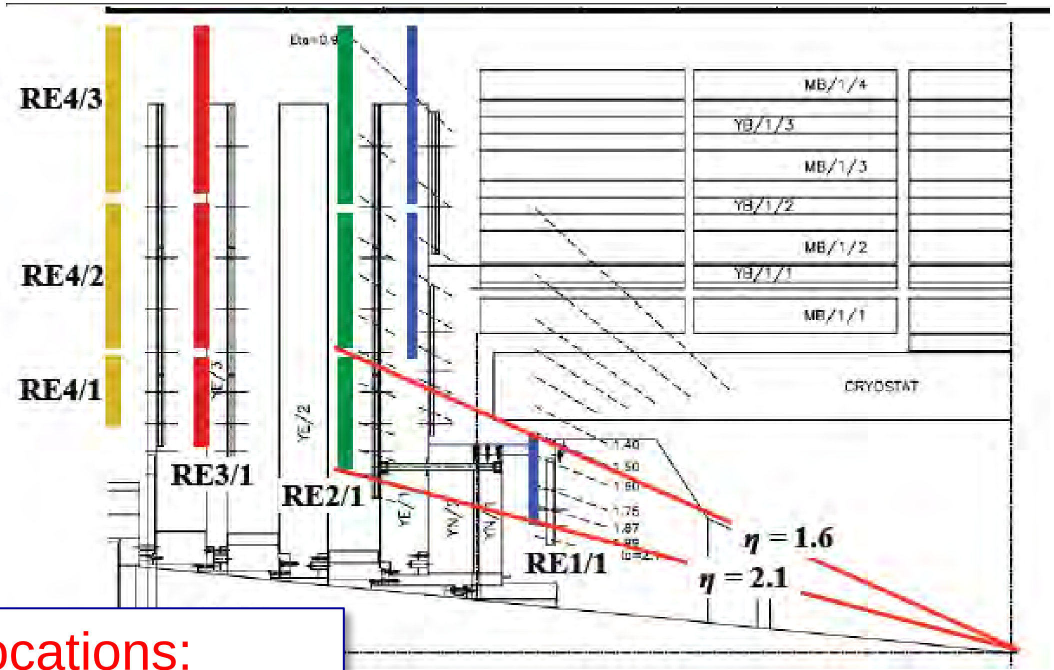
- Mesh segmentation ⇨ reduction of charge, localization of effect
- Resistive electrodes:
  - resistive layer on anode [T. Alexopoulos et al., JINST 5, P02003 (2010)] [T. Alexopoulos et al., RD51-Note-2010-006]
  - resistive mesh [R. de Oliveira et al., RD51-NOTE-2010-007]



## Present Muon System:

- Drift Tubes
- Cathode Strip Chambers
- RPC
- no coverage for $|\eta|>1.6$ (REi/1)




## S-LHC: Requirements for REi/1 locations:

- $\Lambda \sim 10^{34\text{-}35}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$
- Flux: several 10 $\mathrm{kHz/cm}^2$
- Total integrated charge: several 10 $\mathrm{C/cm}^2$
- Triggering capability

## Upgrade project: **GEM**

- triple GEM
- 990 × (220 - 455) $\mathrm{mm}^2$

[D. Abbaneo et al., RD51-Note-2010-005-1]

# GEM Single-Mask Process

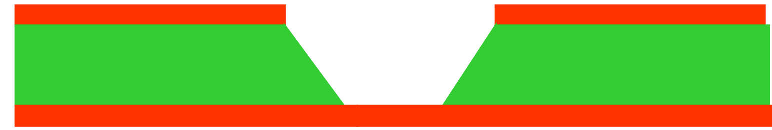
Photoresist deposition on base material

Photoresist hole patterning (single mask)


Top copper etching

Resist stripping

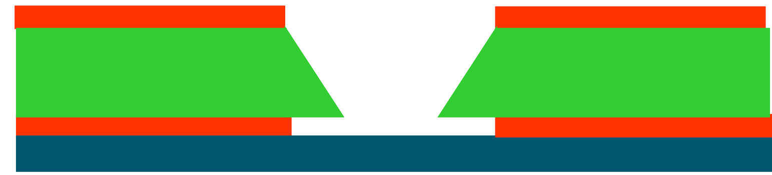
Polyimide anisotropic etching



Bottom resist protection deposition

Bottom copper etching

Top copper protected by galvanic connection

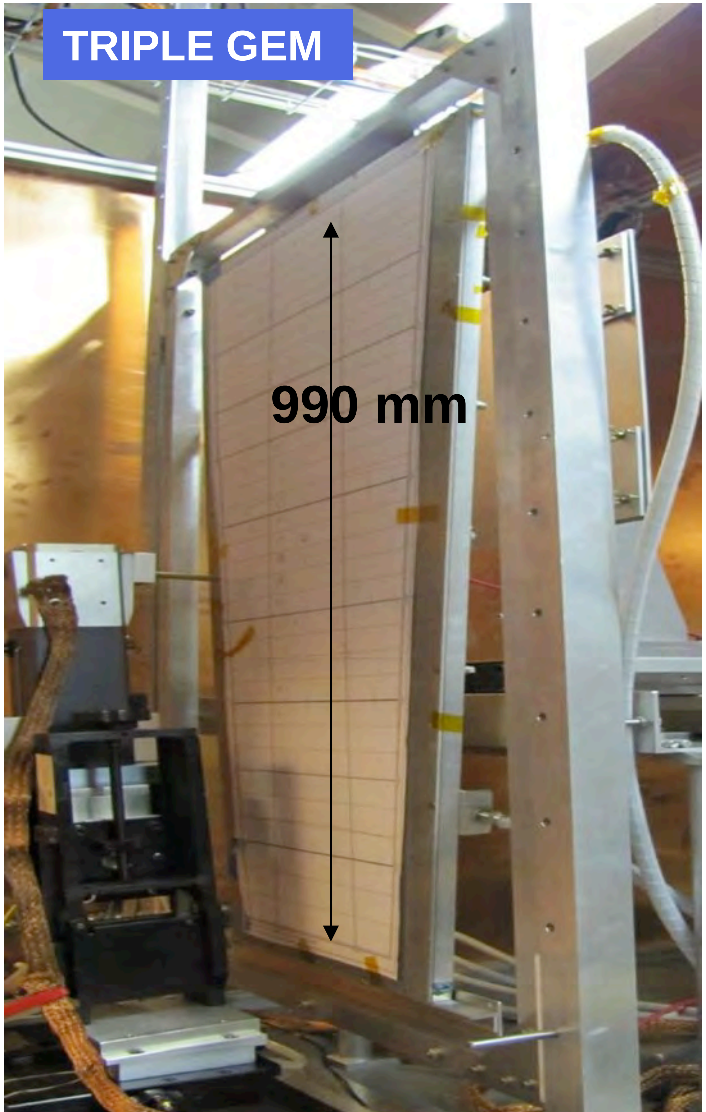


Resist stripping

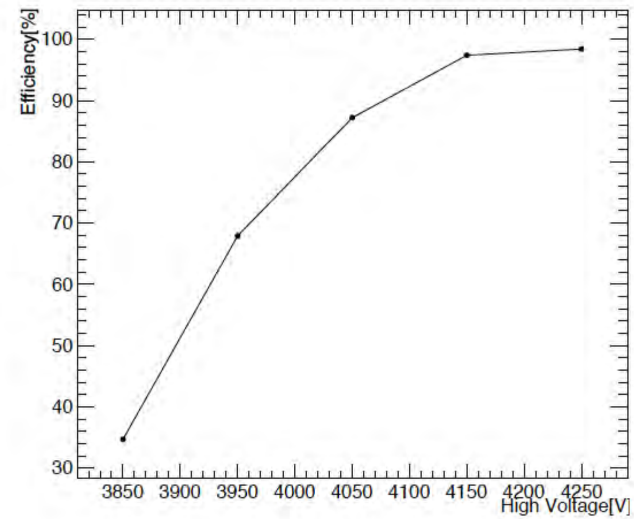
Soft polyimide etching


[M. Villa et al., arXiv 1007.1131v1]

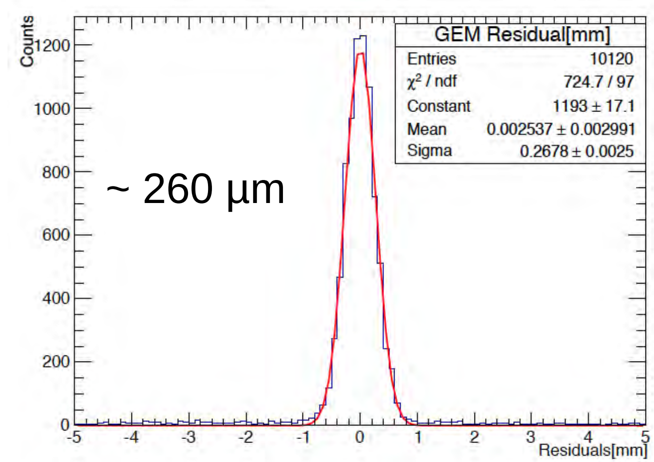
# CMS Muon Chambers




EFFICIENCY




POSITION ACCURACY




***D. Abbaneo et al, JINST 9(2014)C01053***

# New Challenges

**Larger active areas**

- Bulk Micromegas
- Single-mask GEMs

**Higher rates**

- Pixel readout
- Ion backflow suppression


**Aging, discharge protection**

- Materials
- Multi-stage amplification
- Segmentation
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**Higher resolutions**

- μPixel
- InGrid

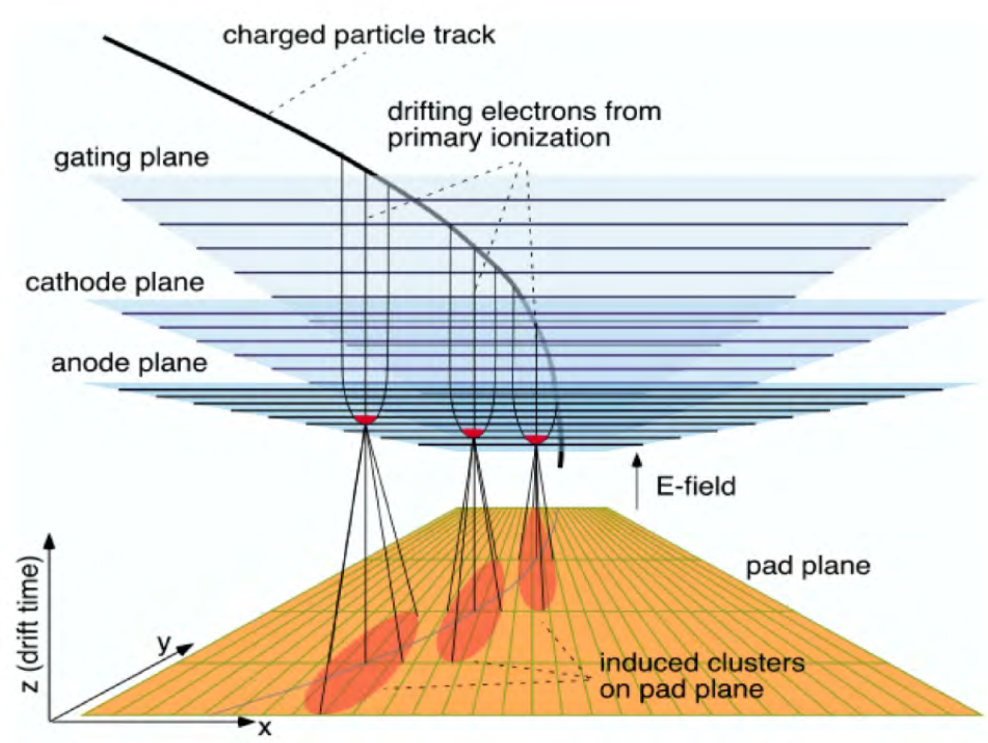
**Special shapes**

- cylindrical
- spherical

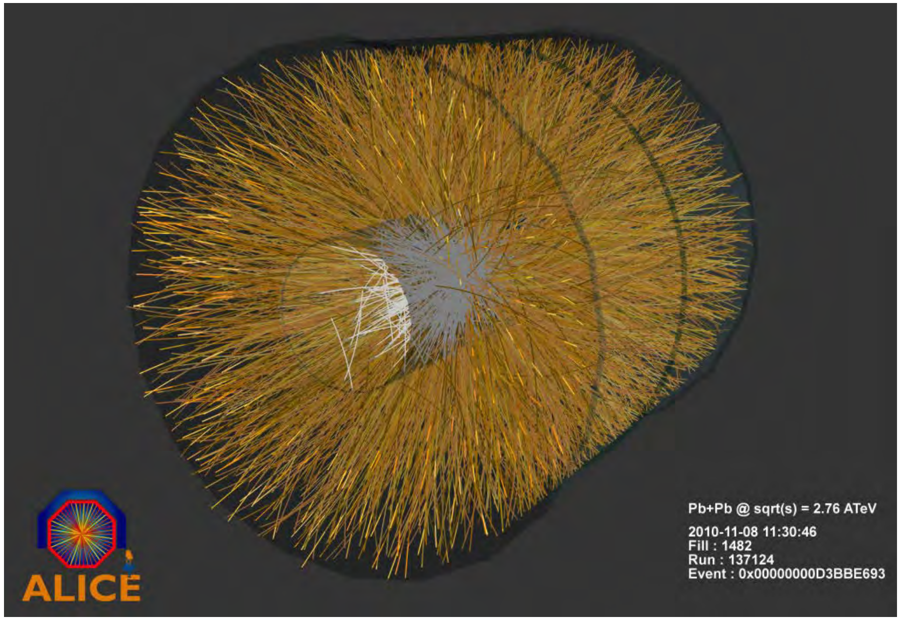
**Time Projection Chamber** [D.R. Nygren et al., Phys. Today 31, 46 (1978)]

Combination of MWPC and Drift Chamber: 3-D tracking device

- long drift path (~m) in gas-filled volume ⇨ z coordinate
- MWPC + pads perpendicular to drift path ⇨ x,y coordinates



# Time Projection Chamber




An (almost) ideal tracking detector:

- Large acceptance
- Large active volume
- Low material budget
- **3D picture** of event
  - ⇨ simple pattern recognition
- Extremely high particle densities
  - ⇨ heavy ion experiments
- Good momentum resolution
- Particle identification

Limitations:

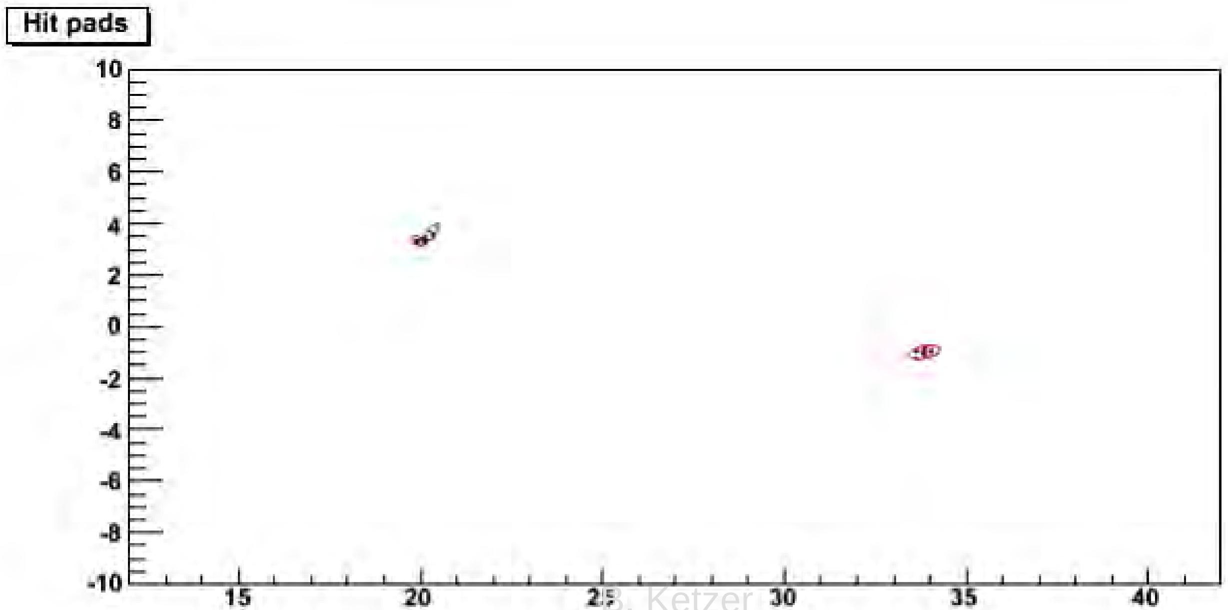
- No redundancy
- Calibration very demanding
- Drift distortions due to ion backflow
- Gating ⇨ low trigger rates

Examples:

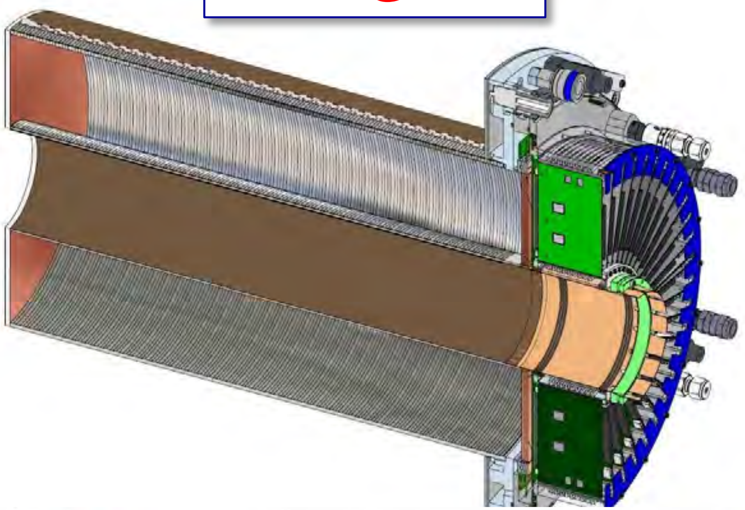
- STAR (420 cm × 400 cm)
- ALICE (500 cm × 500 cm)

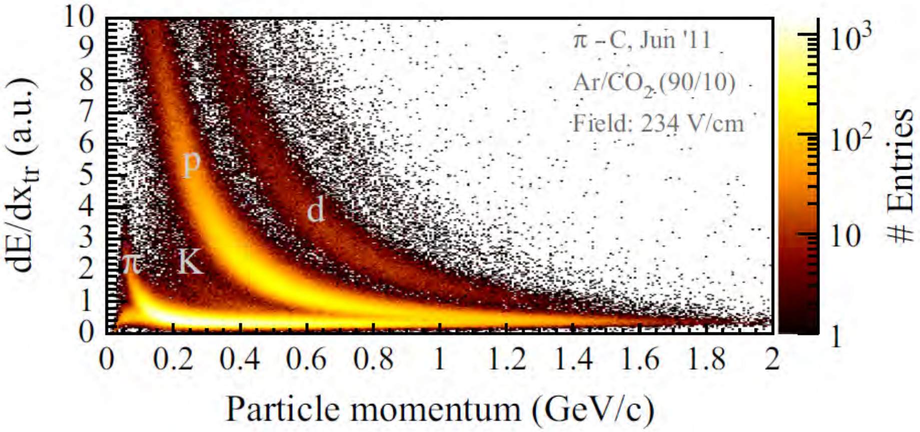
# TPC Operation at High Rates

- **High rates:** drift time > 1 / (event rate) ⇨ overlapping events
- **Goal:** operate TPC continuously
  - ⇨ analog event pipeline
  - ⇨ **3D "Movie"**
- **ALICE:** Operate at high luminosity $\mathcal{L} = 6 \cdot 10^{27}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$
  - ⇨ Record all minimum bias events
  - ⇨ 50 kHz in Pb-Pb collisions, i.e. 100× higher than present



FOPI @ GSI



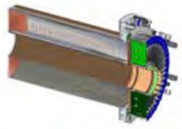


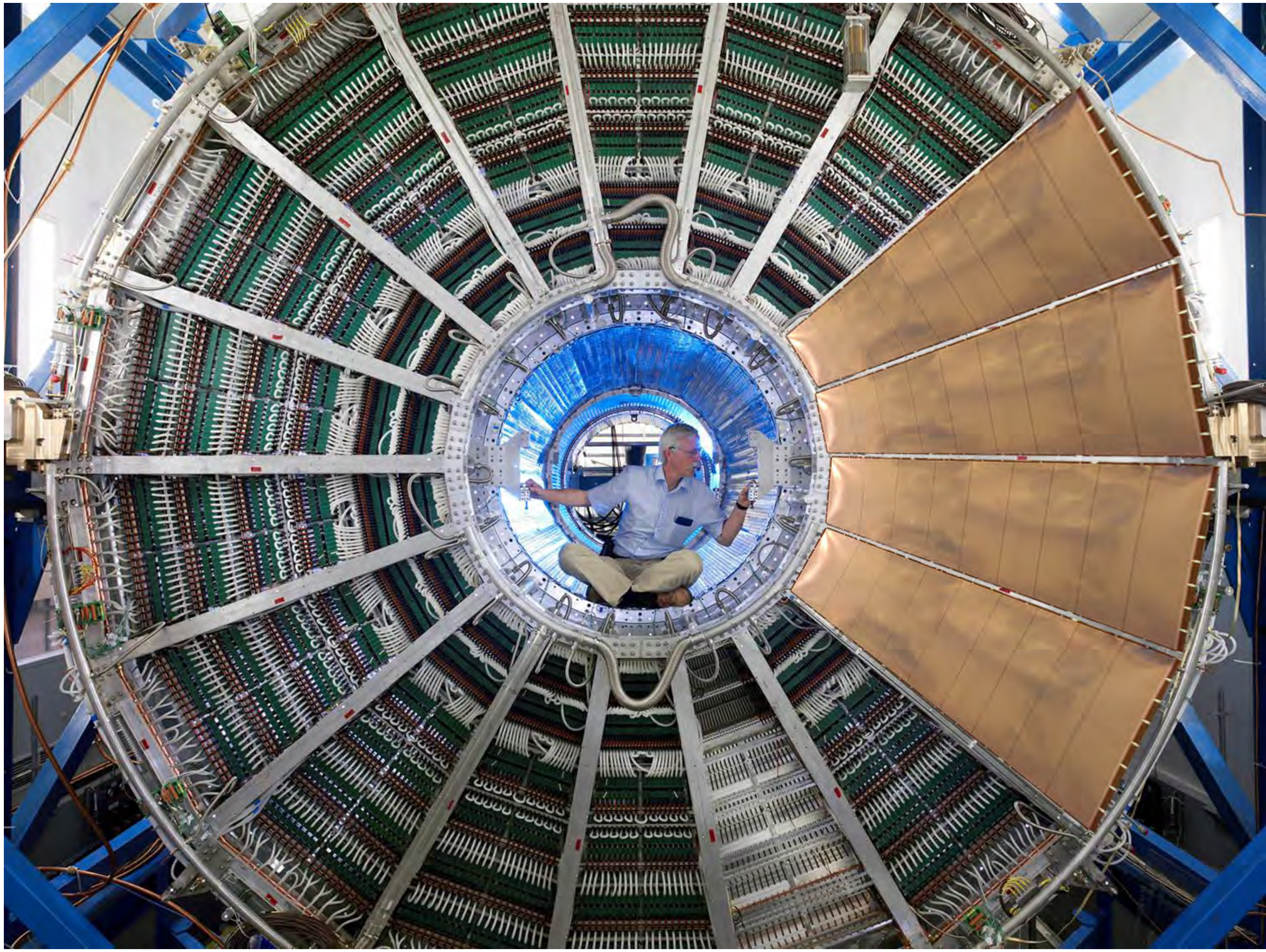

[B. Ketzer et al., NIM A 732, 237 (2013),
M. Ball et al., arXiv1207.0013, 2012,
F.V. Böhmer et al., NIM A 737, 214 (2014)]

# GEM-TPC

FOPI @ GSI

ALICE @ LHC

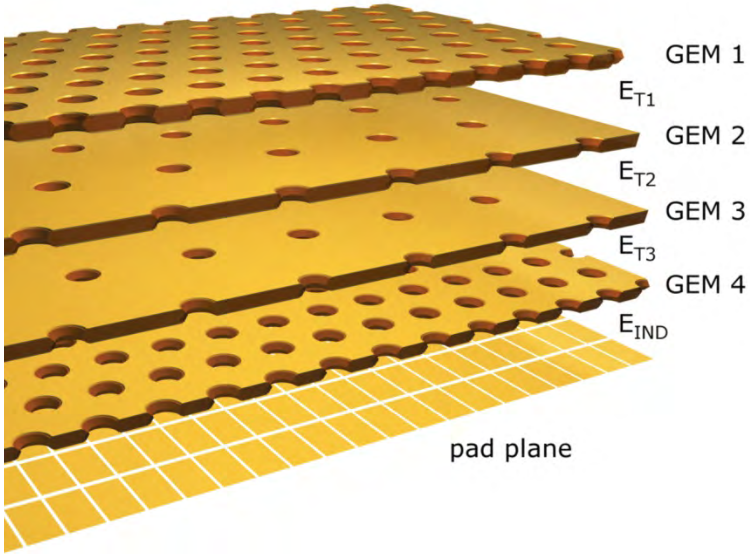




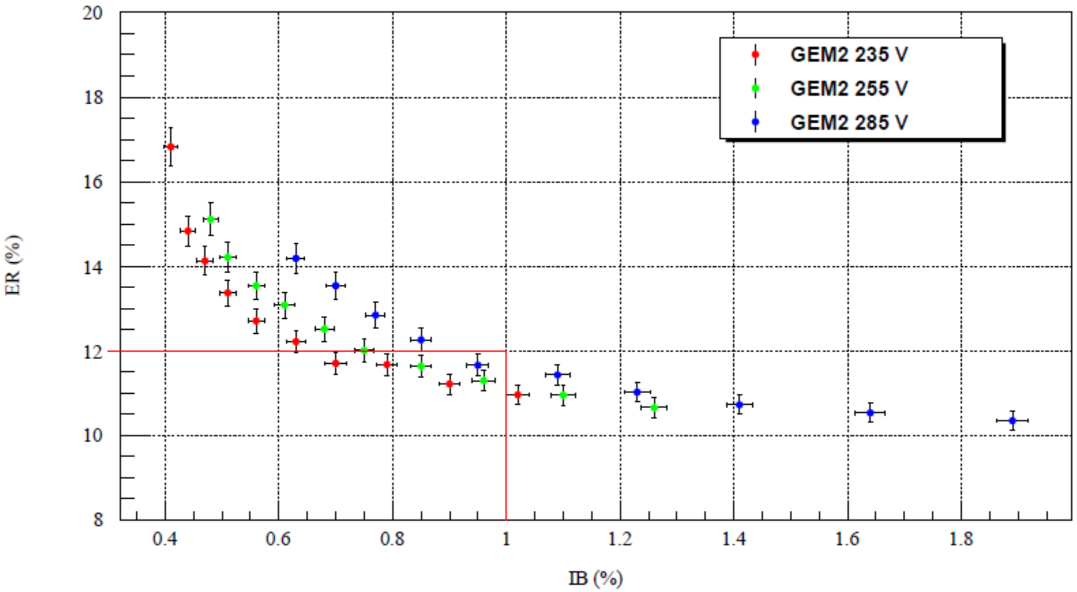
[B. Ketzer et al., NIM A 732, 237 (2013)]

# Ion Backflow – ALICE Solution

universitätbonn




- Triple-GEM setup not sufficient
- New chambers: 4-GEM setup with standard (S) and large pitch (LP)
- Field configuration optimized to provide
  - IB < 1%
  - $\sigma_E/E$ < 12% (for $^{55}Fe$ X-rays)
  - Discharge stability



# New Challenges

**Larger active areas**
- Bulk Micromegas
- Single-mask GEMs

**Higher rates**
- Pixel readout
- Ion backflow suppression


**Aging, discharge protection**
- Materials
- Multi-stage amplification
- Segmentation
- Resistive coating

**Higher resolutions**
- µPixel
- GridPix

**Special shapes**
- cylindrical
- spherical

# MPGD readout by Pixel ASIC

**Ultimate resolution:**

- Only diffusion-limited
- Granularity = primary ionization cluster spread: pixel size < 100μm

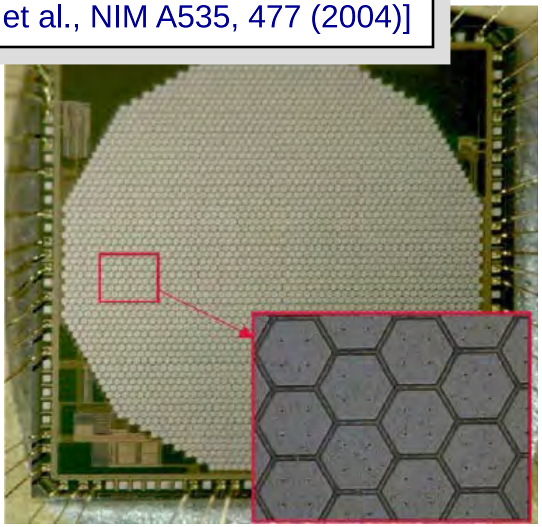
**`Naked' CMOS pixel chip:**

- charge collecting pad
- preamplifier, shaper
- discriminator / sample&hold
- data readout (MUX)

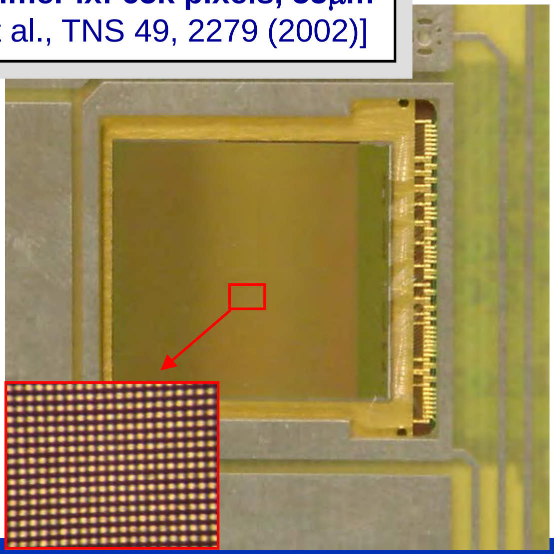
⬇

Resolve individual electrons!

- pattern recognition & track fitting in dense environment
- d*E*/d*x* resolution for TPC



**CMOS ASIC: 2100 pixels, 80μm**
[R. Bellazzini et al., NIM A535, 477 (2004)]

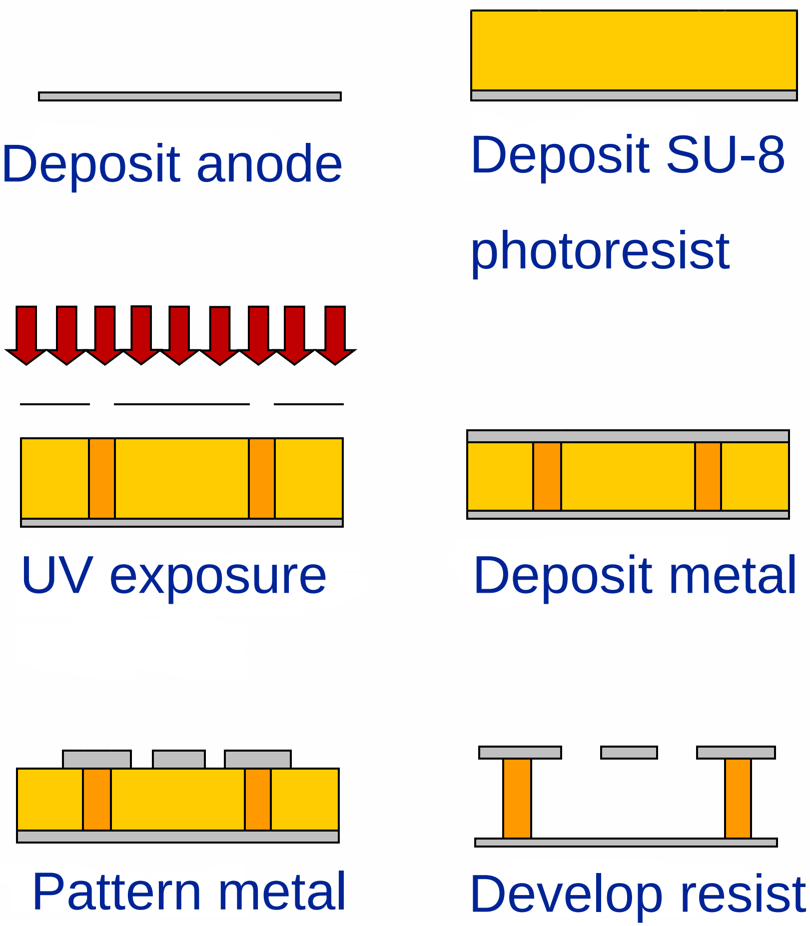


**MediPix2 / TimePix: 65k pixels, 55μm**
[X. Llopart et al., TNS 49, 2279 (2002)]

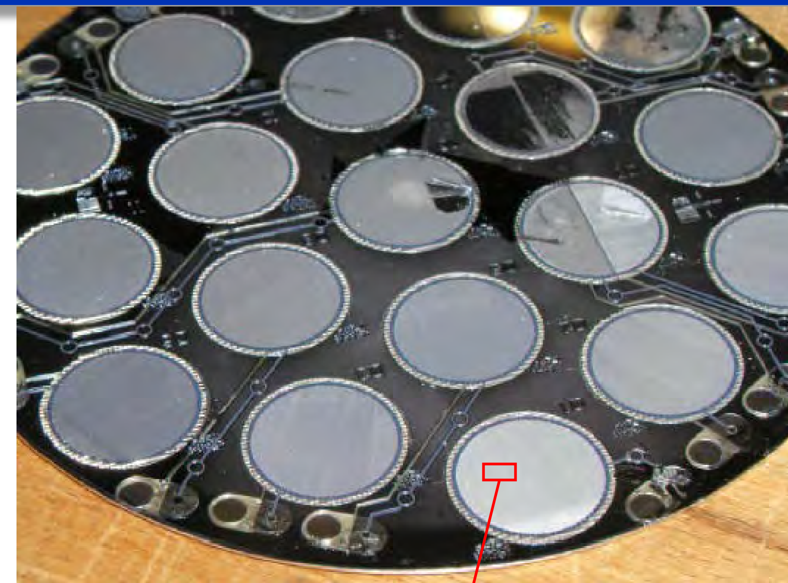
# InGrid

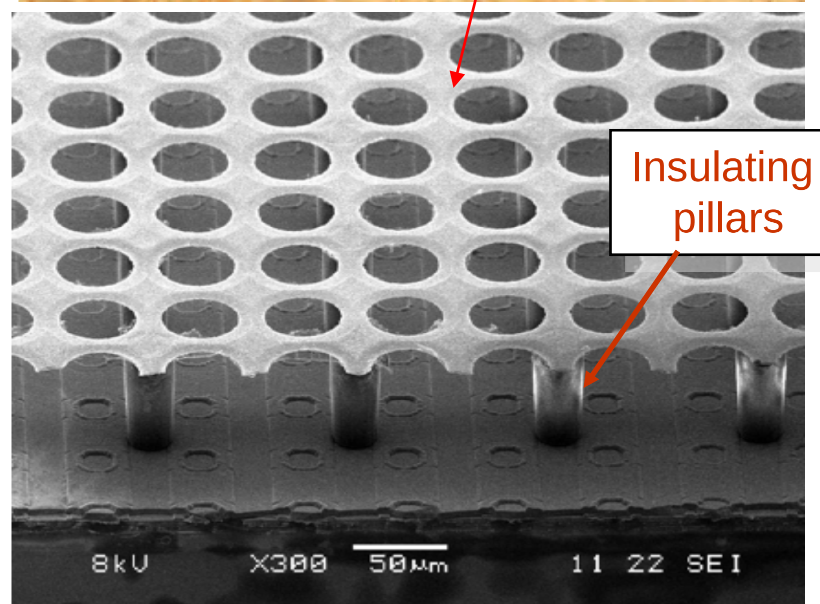
Integrated Micromegas and Pixel Sensor

(Si wafer post-processing & MEMS)




[J. Timmermans, VCI 2007]

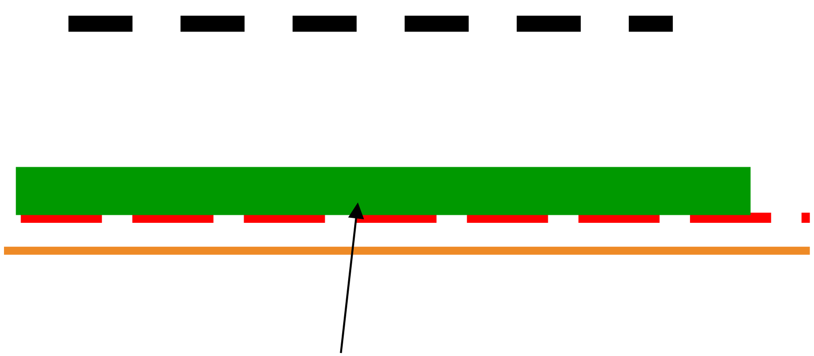




# Discharge Protection

Deposit of resistive layer on pixels

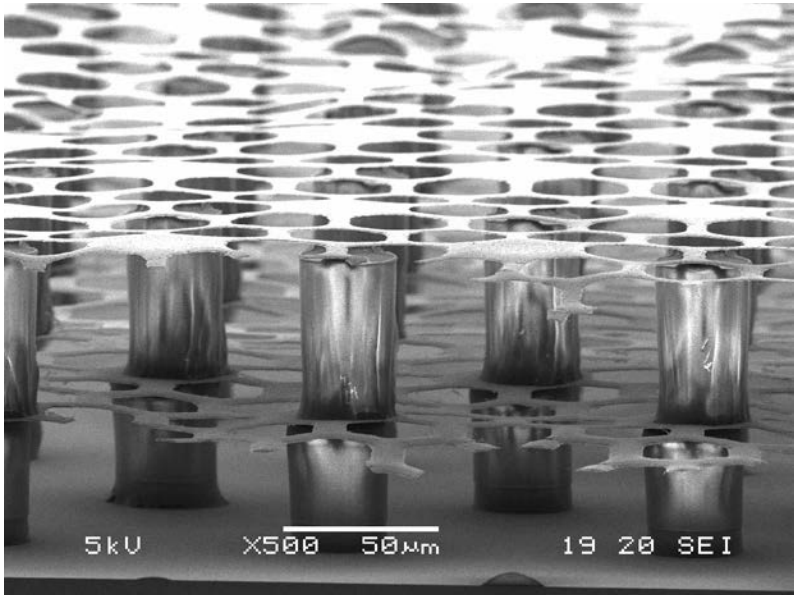
⇨ local reduction of E field



- 20 µm a-Si, $\rho = 2\ 10^8\ \Omega$ cm
- 7 µm $Si_3N_4$, $\rho = 10^8 - 10^{15}\ \Omega$ cm

SiNProt

Double amplification



# 4 Position Measurement

4.1 Resistive Plate Chambers

4.2 Micropattern Gaseous Detectors

4.3 Semiconductor Detectors

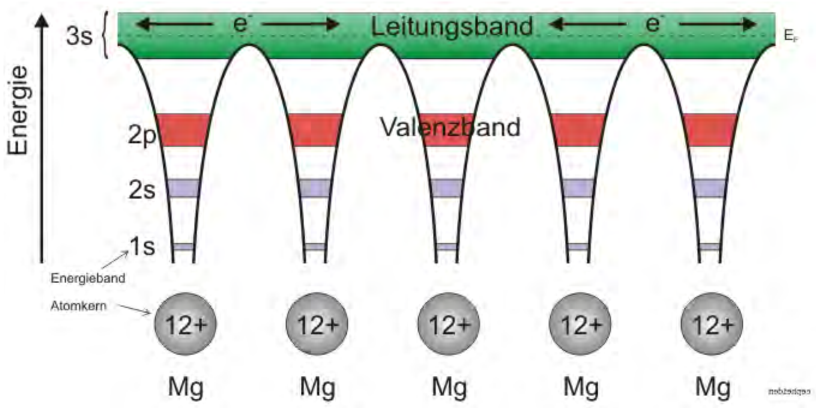
4.4 Track reconstruction

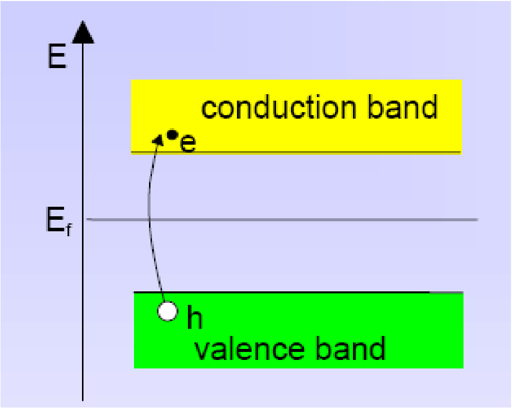
# 4.4 Semiconductor Detectors



**Principle:** solid state ionization chamber

Energy band structure due to periodic arrangement of atoms in crystal




Absorbed energy

➩ $e^-$-h pair

Applications:

- Nuclear physics: $\gamma$ energy measurement (spectroscopy)
- Particle physics:
  - – high-resolution vertex and tracking detectors
  - – photodetectors (calorimeters)

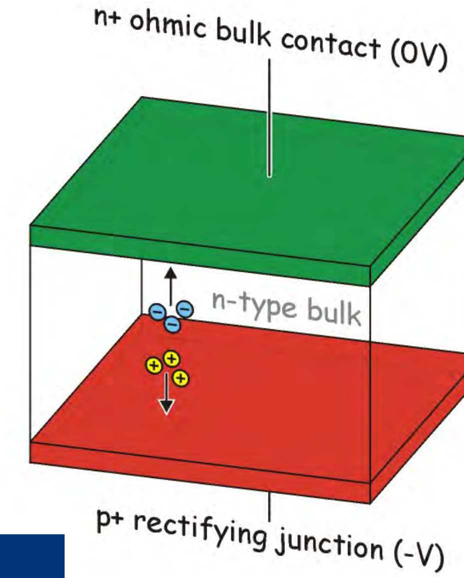
# pn Junction

pn junction as ionization detector:

- free charge carriers, produced by ionizing radiation in depletion zone, are collected by electric field and can be detected
- charge produced in the non-depleted region recombines

⇨ larger signal for larger depletion zone

⇨ apply external potential $V_{\text{bias}}$ with same polarity as $V_D$

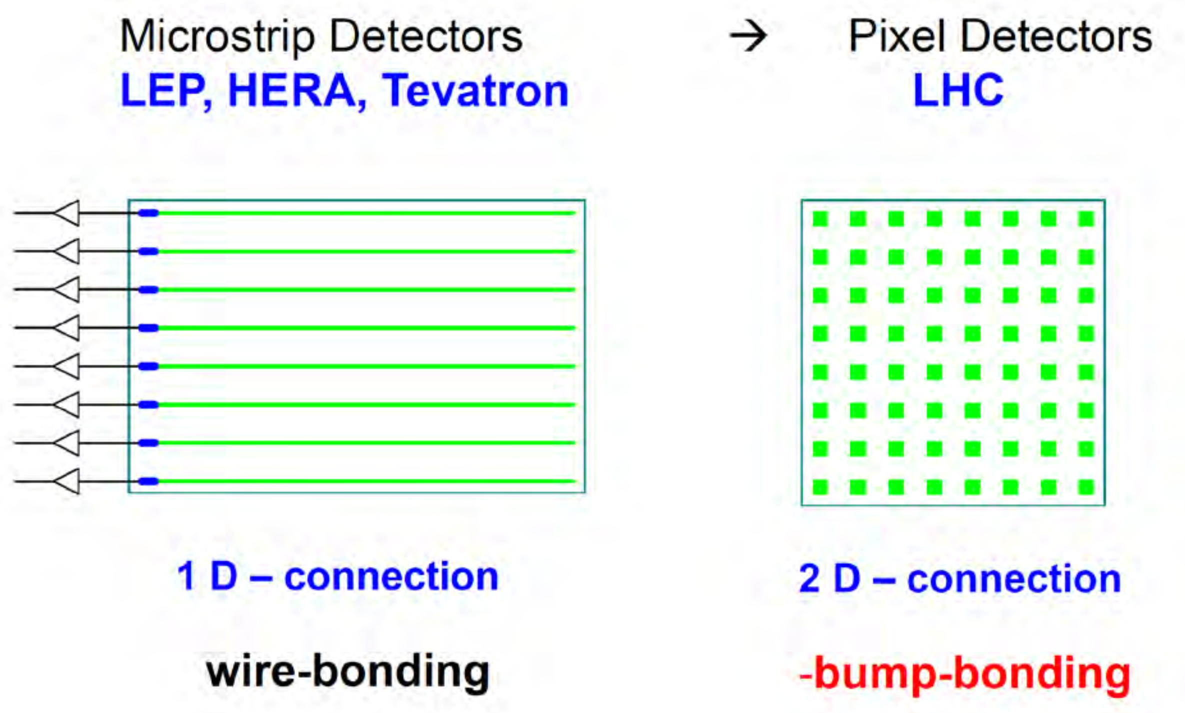



[A. Peisert, World Scientific, Singapore 1992]

| | Dopant conc. ($cm^{-3}$) | Resistivity | Process |
|---|---|---|---|
| Detector quality | $10^{12}$ (n) – $10^{15}$ ($p^+$) | ~ 5 kΩcm | Float zone |
| IC quality | $10^{17}$ – $10^{18}$ | ~ 1 Ωcm | Czochralski |

# Silicon Pixel Detectors

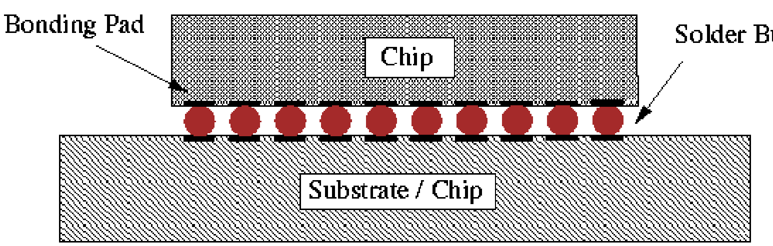
- LHC et al.: very high rate / multiplicity ($10^5$ × LEP/HERA)
- Too many ambiguities for strip detectors
- Requires 2D segmentation: pixels
- Connection to readout electronic chips: bump bonding

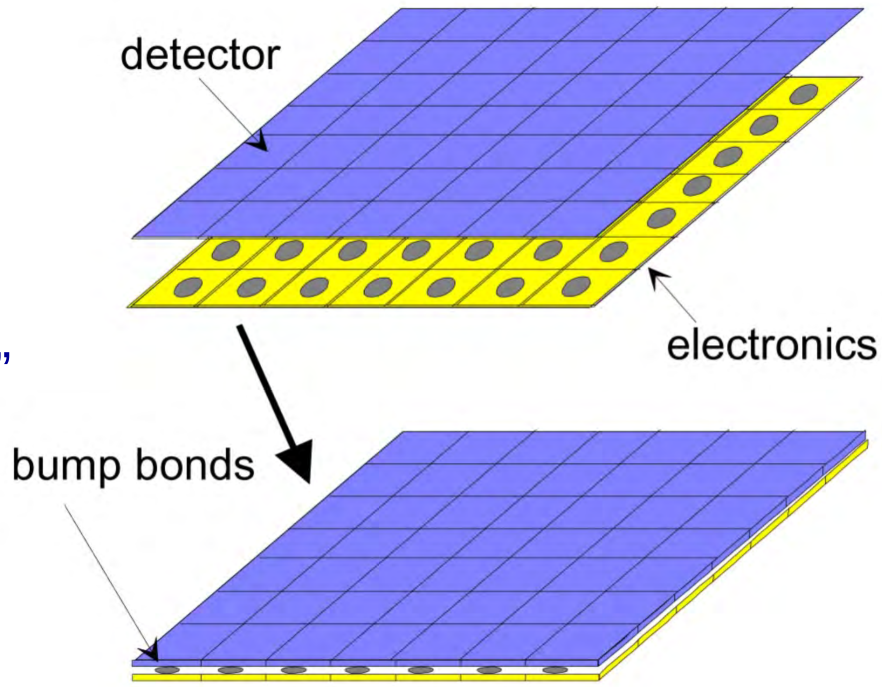


# Silicon Pixel Detector

Principle: hybrid structure

- Segment p+ side (~ 50-100 $\mu m^2$)
  ➩ diode matrix
- Readout electronics chip with same geometry
- Flip-chip technique: “bump bonding”




[E. Heijne et al., NIM A384, 399 (1994)]

Challenges:

- highly integrated readout electronics
- number of electronic channels

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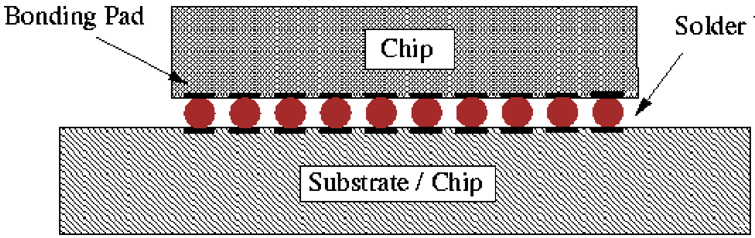


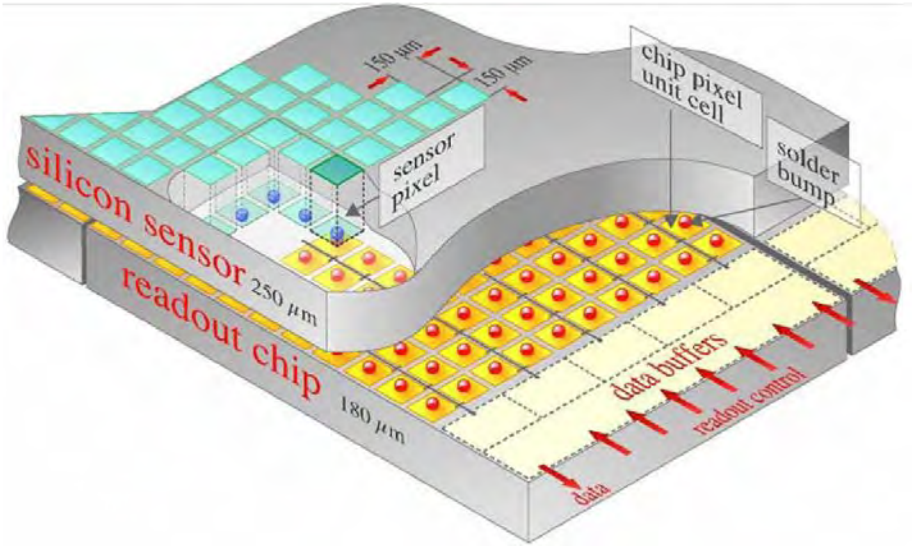


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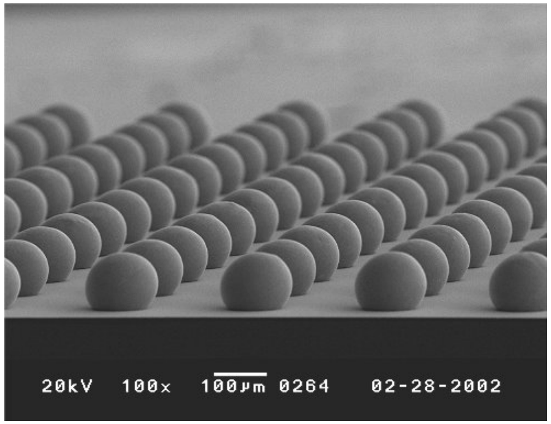



[http://microscale.en.ec21.com/]

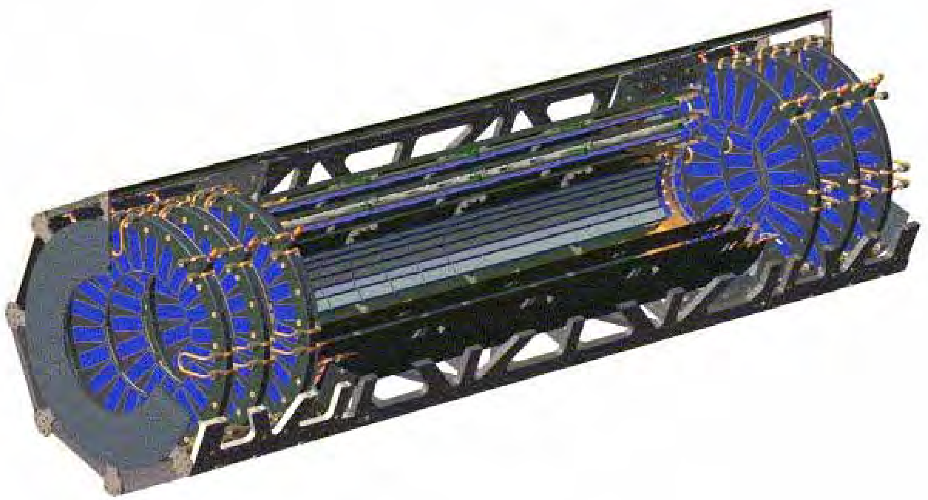
Challenges:

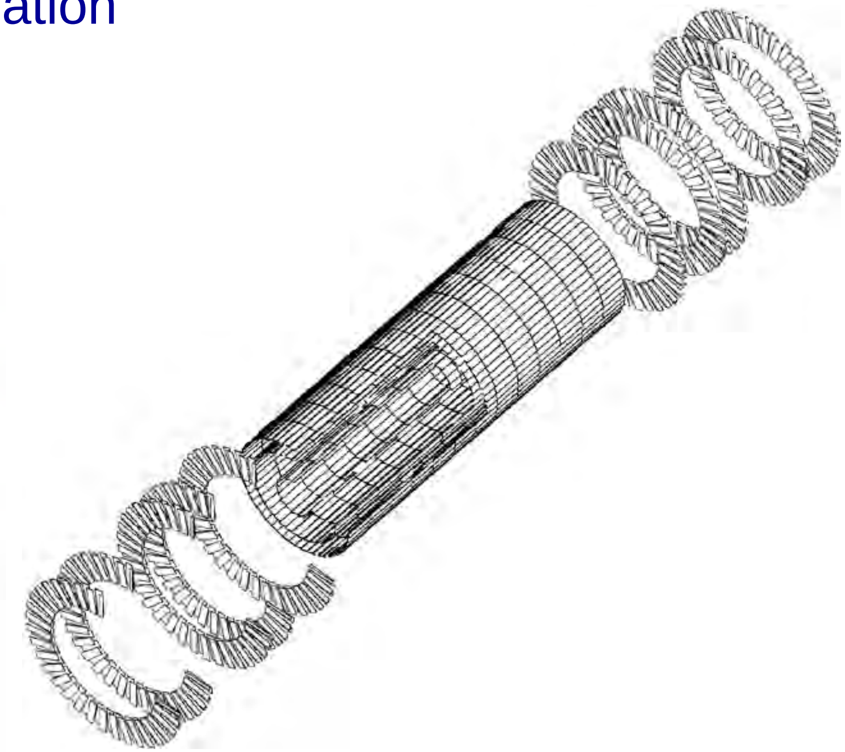
- highly integrated readout electronics
- number of electronic channels

# ATLAS Pixel Detector

## Central Vertex detector:

- Pixel size: 50 x 400 $\mu m^2$
- Pixel sensor: 16.4 x 60.8 $mm^2$ wafer with 46080 Pixels
- 16 FE Chips / Sensor, Flip-Chip configuration
- 80363520 channels
- Dimensions: $L$ = 1.4 m, $\Phi$ = 0.5 m





# Radiation Damage

Crucial for LHC, but also important for Tevatron, B factories, SPS fixed target

Mechanisms:

- Displacement damage: dislocation of Si lattice atom by incident radiation
  - ⇨ defects (I, V), which alter electrical characteristics of detector
  - ⇨ linked to non-ionizing energy loss (NIEL)
    - – energy and momentum transfer to lattice atoms
  - ⇨ also depends on energy and mass of incident quanta !
    i.e. NIEL scaling violated e.g. for low proton energies

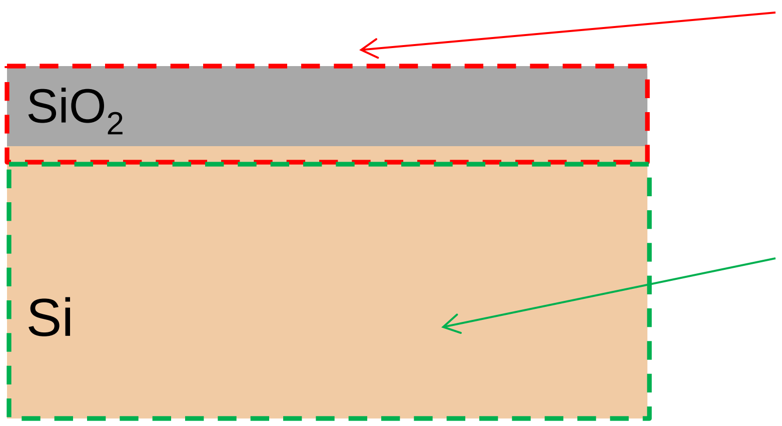
⇨ **bulk (crystal)** damage

- Ionization damage: energy absorbed by ionization in insulating layers ($SiO_2$)
  - ⇨ charge carriers drift and diffuse to other locations
  - ⇨ trapped
  - ⇨ unintended concentration of charge, parasitic fields

⇨ **surface** damage, e.g. interstrip resistance decreases

# Radiation Damage






Region affected by ionizing energy loss  - surface damage

Region affected by non-ionizing energy loss  - bulk damage

# Radiation Damage



**Quantify the amount of radiation:**

- fluence (integrated flux): $\phi = \frac{N}{A}, \quad [\phi] = \mathrm{cm}^{-2}$

  ⇨ displacement damage

  Radiation damage is known to depend on particle type / energy

  ⇨ normalization to 1 MeV neutrons (NIEL scaling)

  For ordinary Si detectors problems arise for $\phi \gtrsim 10^{-14}\mathrm{cm}^{-2}\ n_{\mathrm{eq}}$

- absorbed dose: $D = \frac{E}{m}, \quad [D] = \mathrm{Gy} = \mathrm{J/kg}$

  ⇨ ionization damage

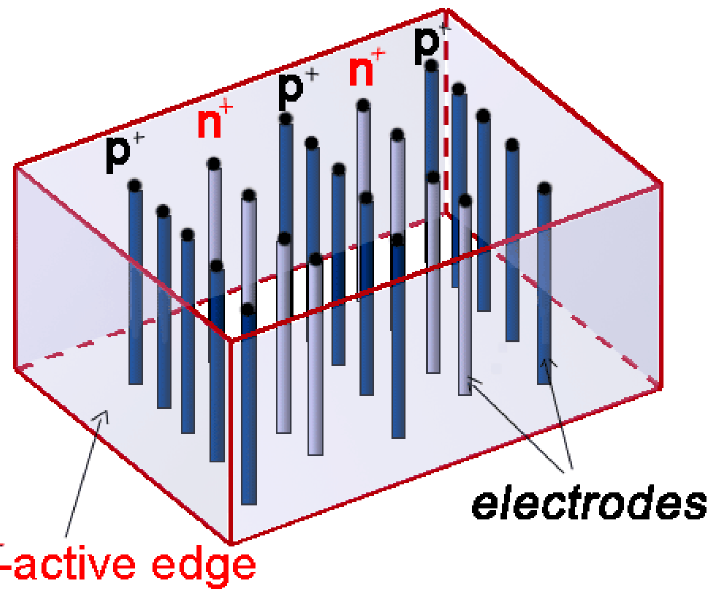
# Displacement Damage

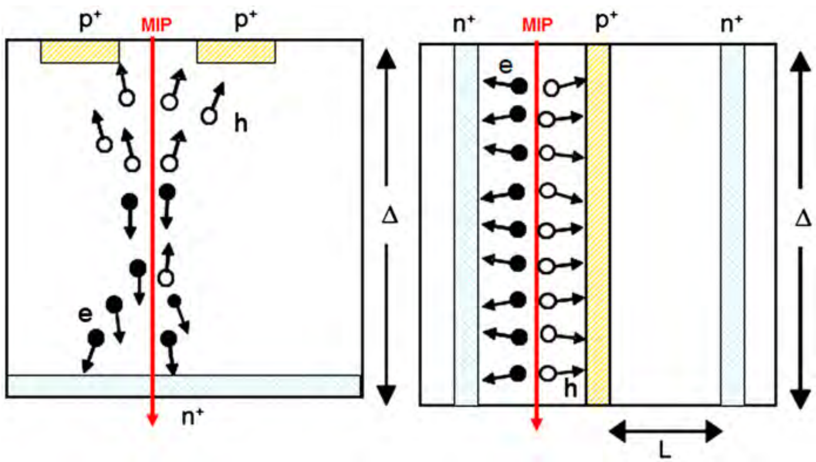
Most important for diodes!

**Manifestations:**

- Formation of mid-gap states
  - ⇨ facilitate transitions of electrons from VB to CB
  - ⇨ increase of leakage current of a reverse-biased pn junction
  - ⇨ increase of noise $\propto \sqrt{I}$
  - ⇨ increased recombination in non-depleted regions, i.e. charge loss
- Formation of states close to band edges
  - ⇨ charge trapping + delayed release
  - ⇨ loss of charge collection efficiency
- Change in doping characteristics: type inversion n → p
  - ⇨ change in depletion voltage
  - ⇨ reverse annealing

Quantity to watch: Signal-to-Noise ratio




[S. Parker et al., ICFA Instr. Bull. 14, 30 (1997),
C. Da Via et al., NIM A 694, 321 (2012), NIM A 765, 151 (2014)]

- 3D array of p+ and n+ electrodes
- particle path (signal) different from drift path
- high field with low voltages

⇨ radiation tolerant: 50% charge @ $10^{16}$ cm$^{-2}$

⇨ good for inclined tracks

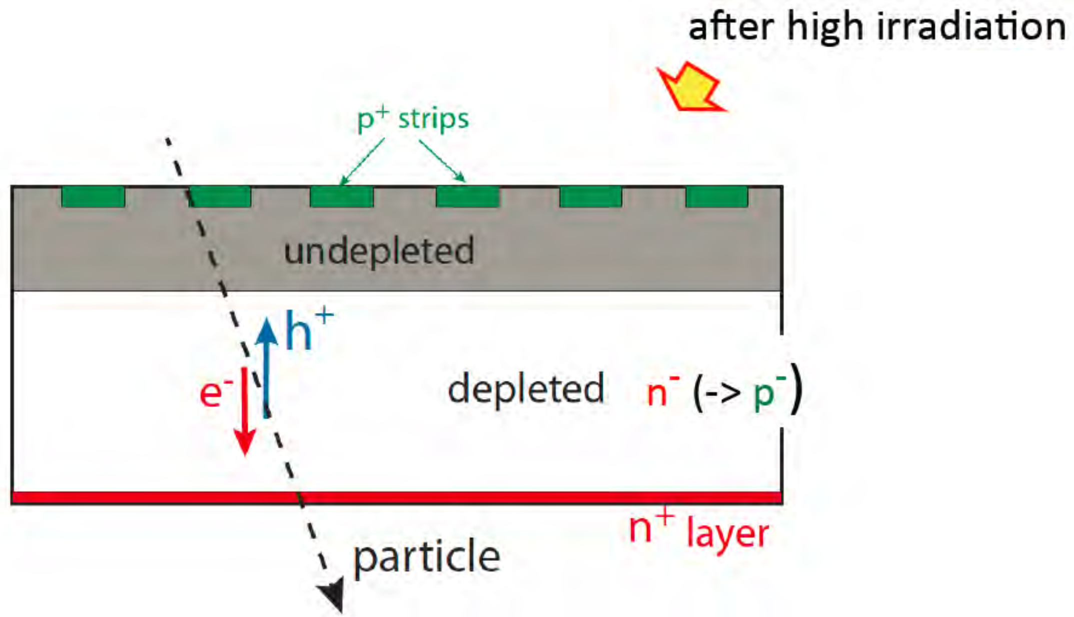
⇨ slightly larger C (noise)

⇨ now also in diamond, CdTe

⇨ operating successfully in ATLAS IBL since 1 year!

**Classic strip choice:**

$p^+$ in n




Consequences:

- signal loss
- resolution degradation (charge spreading)

[N. Wermes, VCI Vienna, 2016]

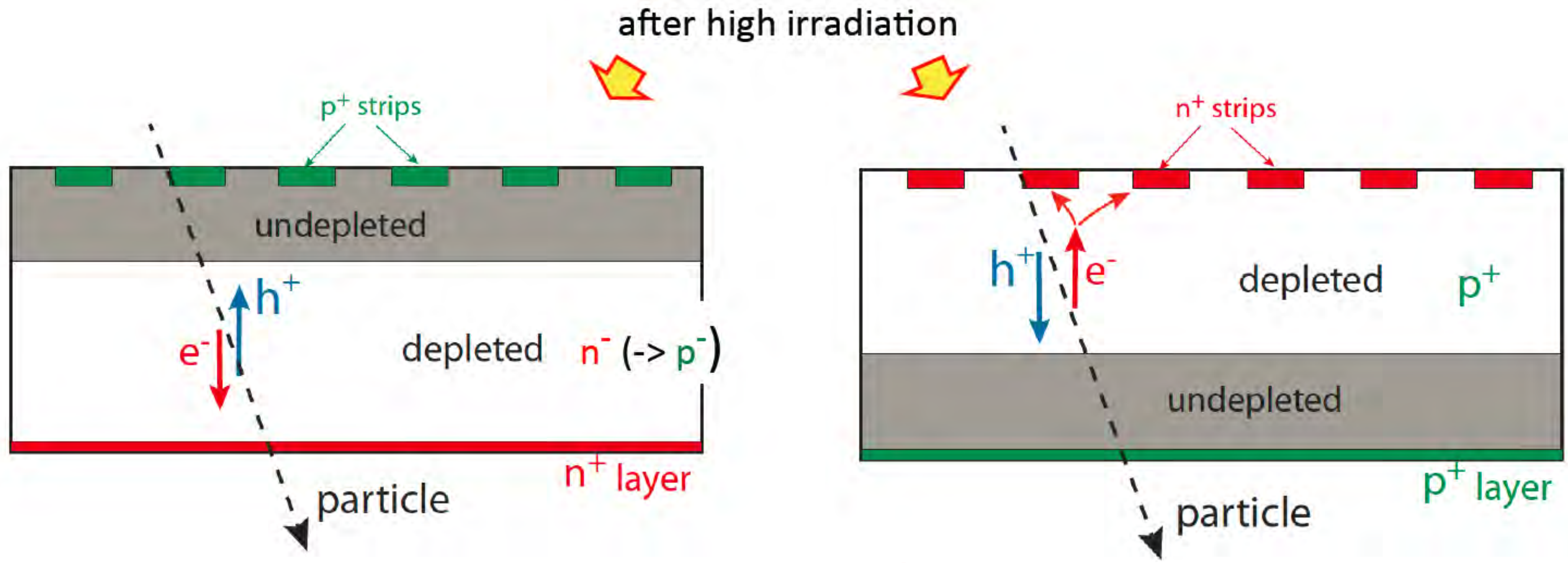
# p-Type Substrates

**Classic strip choice:**

$p^+$ in n

**For HL-LHC upgrade:**

$n^+$ in p or $n^+$ in n (→p)




Consequences:

- signal loss
- resolution degradation (charge spreading)

Advantages:

- faster charge collection (electrons)
- signal and CCE degradation less and smoother

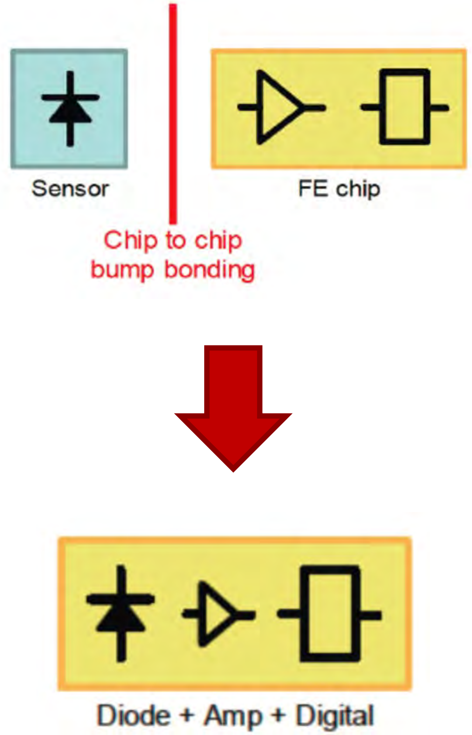
[N. Wermes, VCI Vienna, 2016]

⇨ p-type substrates favoured for LHC upgrades

# Monolithic Devices

Integration of large-scale sensor with readout electronics

- No need for bump bonding
- Single device: only power and data readout as external connections
  - Higher granularity
  - Smaller material budget
  - Low noise
  - Low power consumption
  - Smaller cost, less technological overhead



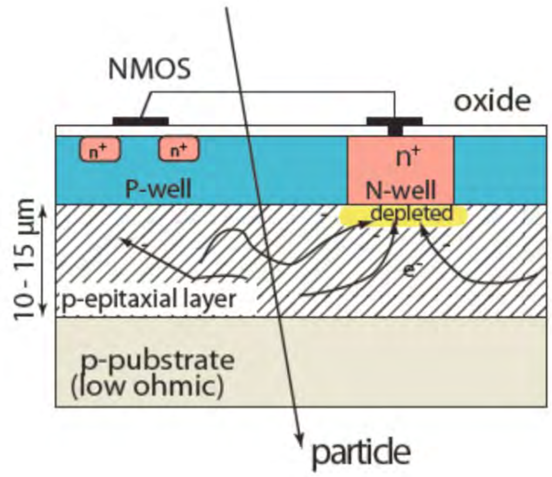

Examples:

- CCD
- Silicon drift chamber
- Monolithic active pixel sensors (MAPS)
- Depleted p-channel FET (DEPFET)

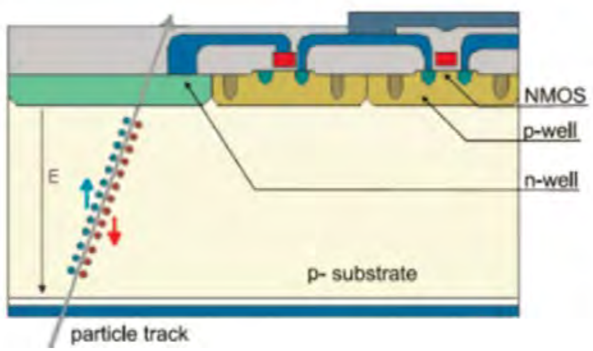
[B. Dierickx et al., SPIE 3410, 68 (1998), R. Turchetta et al., NIM A 458, 677 (2001)]

## Monolithic Active Pixel Sensors:

- Fabricated in standard IC process (CMOS)
- Active volume: thin p-epitaxial layer
  - ➪ charge collection by diffusion
  - ➪ small signal charge: ~ 1000 e-
  - ➪ long collection time: ~ 100 ns
- Small readout electrode ➪ low noise level
- STAR PXL since 2014
  - readout speed ~ 100 μs, row-wise zero supp.
  - radiation tolerance up to $2 \cdot 10^{12}$ $cm^{-2}$ (1 MeV n)
- DMAPS (CMOS pixels)
  - HR substrate and HV to create some depletion
  - CMOS on SOI




STAR + ALICE



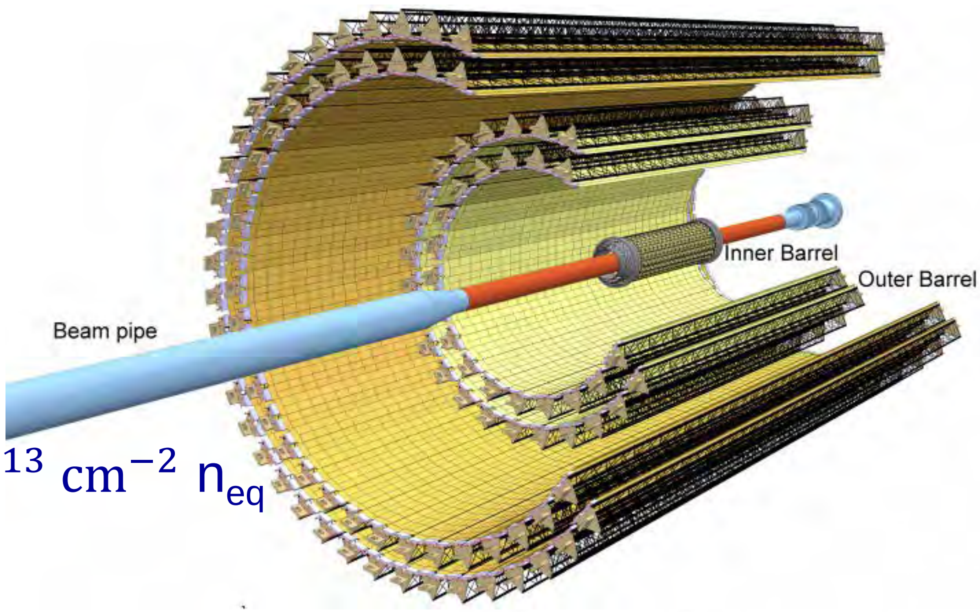

HL-LHC, PANDA

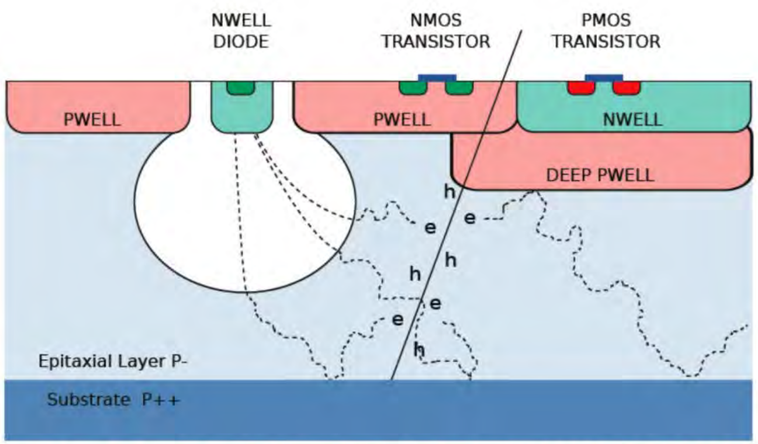
# ALICE ITS

Big step forward:

- 7 layers (3 IB, 4 OB)
- 12.5 Gpixel
- readout speed 100 kHz
- $10\ \mathrm{m}^2$ of MAPS detectors
- radiation tolerance up to $1.7 \cdot 10^{13}\ \mathrm{cm}^{-2}$ $n_{eq}$
- HR (> 1k$\Omega$cm) p-type epi-layer
- quadruple well process ⇨ shield Nwell with PMOS transistors (pixel FEE)
- very small Nwell collecting diode
- moderate reverse bias
- 180 nm TowerJazz CMOS

⇨ to be installed during LS2

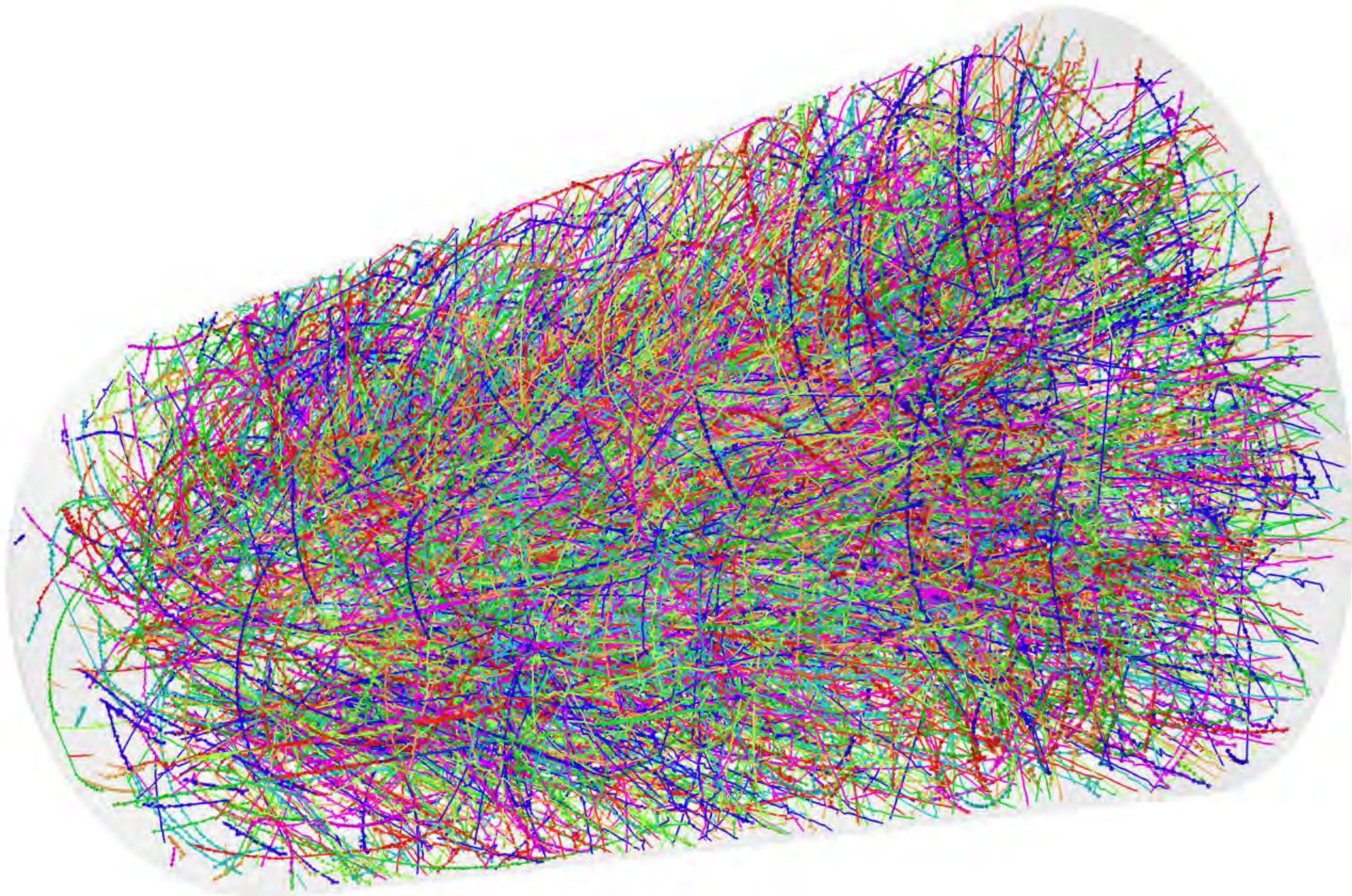



J.P. Crooks, ..., R. Turchetta et al. IEEE TNS 2007 & Sensors (2008), ISSN 1424-8820

# 5 Track Reconstruction

# Track Reconstruction



# 5.1 Overview

Tracking of particles is performed with a combination of different detectors measuring

- particle passage along one axis in a detector plane, e.g. silicon strip, MWPC
- 2-D penetration point of a particle through a plane, e.g. silicon pixel
- drift time relative to a wire position, i.e. surface of constant drift time around wire, e.g. drift chambers, straw tubes
- 3-D space points on particle trajectories, e.g. TPC
- higher dimensioinal information, e.g. 2-D position + 2-D direction, e.g. EMC

Very often, tracking software resorts to simplifications, e.g.

- projection of TPC data onto planes defined by pad rows
- projection of surfaces of constant drift time onto predefined planes

# 5.1 Overview

**From raw data to detector hits:** example of a TPC

- raw data: amplitudes for each pad (~500k) sampled at ~10 MHz, 1024 samples per event, 8 bit per sample
  - ⇨ 0.5 GByte/event
  - ⇨ 5 TByte/s for continuous data stream
- pedestal subtraction
- common mode correction
- zero suppression
- combination of samples into a pad hit (common signal pulse): pulse shape analysis (PSA)
- amplitude correction / equalization (calibration)
- clusterization of pad hits into TPC hits / clusters (different from ionization clusters!)
- TPC hits are then fed into track reconstruction algorithm, including covariance matrix

# Overview

**Track reconstruction** is traditionally divided into 2 subtasks:

1. Track finding, pattern recognition
   - division of a set of hits in a tracking detector into subsets
   - each subset contains hits believed to originate from the same particle

   ⇨ track candidates (also possible: noise hits, background hits)
2. Track fitting
   - starts with hits inside one subset as provided by the track finder
   - aims to optimally estimate a set of track parameters from the hits information
   - compute covariance matrix of the estimate

   ⇨ has to be robust against errors in 1. and wrong assumptions on errors,

   ⇨ has to be numerically stable

Boundary between 1. and 2. is fuzzy

[A. Strandlie, ACAT2005, DESY Zeuthen, 2005]

# 5.2 Track Parameterization

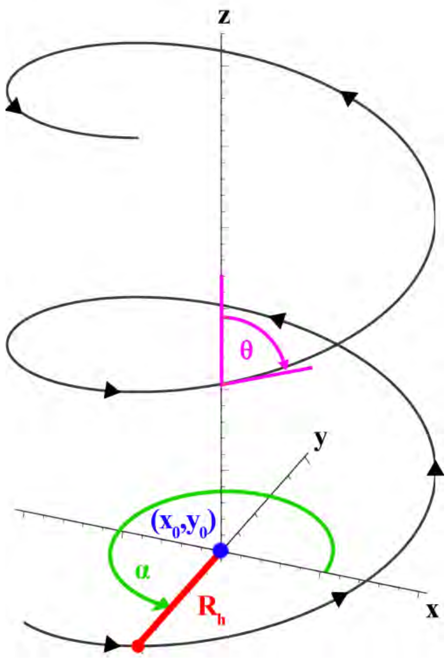
Track parameterization: describes charged particle trajectory

- in stationary field in vacuum: motion of stable particle completely described by initial position and momentum ⇨ 6 parameters
- tracking detectors do not measure where particle started
  - ⇨can be chosen anywhere along trajectory
  - ⇨5 parameters
- more difficult if multiple scattering is included
- parameterization should meet some basic requirements:
  - continuity with respect to small changes
  - errors close to Gaussian
  - correlations between parameters small
- optimal parameterization depends on detector geometry

# Track Parameterization

Examples:

- DELPHI barrel: $x = (\Phi, z, \theta, \beta = \phi - \Phi, 1/R)$
- DELPHI forward: $x = (x, y, \theta, \phi, 1/R)$
- CMS global: *position, momentum, charge*
- CMS local: $x = (q/p, dx/dz, dy/dz, x, y)$



# 5.3 Track Finding

**Two classes of methods:**

1. **Sequential or local track finding**: track follower
   - grow a track starting from a seed
   - pick up hits as you go along
   - stop when no more hits can be found
   - remove assigned hits from the pool and start again

**Examples:**

- Road method: mostly used in projections and for constant field
  - select 3 hits (or 2 hits + vertex) ⇨ seed
  - compute road around circle or straight line
  - use conformal mapping for linearization, e.g. Riemann transformation
  - pick up hits in road
- Combinatorial Kalman filter (later)

# Track Finding

**2. Parallel or global track finding**

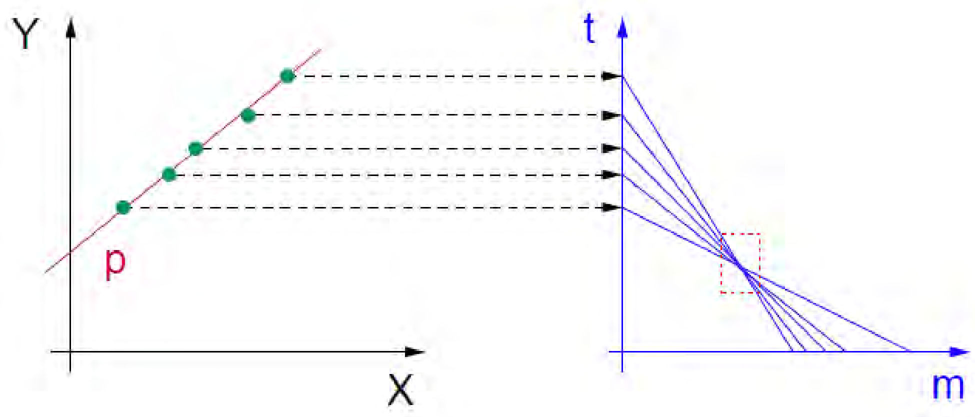
- define suitable feature space
- find clusters in feature space

**Examples:**

- conformal mapping + histogramming: Hough transform
- cellular automaton

- Assume a set of data points $\boldsymbol{X}_i$ follows a pattern with parameterization $p$
- Map each data point $\boldsymbol{X}_i$ into $N$-dimensional parameter space $\mathcal{H}$
- Look for maxima in hypersurface spanned by parameters

Example 1: points on straight lin




$$\boldsymbol{X}_i = \{x_i, y_i\}$$

Parameterization of pattern:

$$p(x_i) = y_i = m \cdot x_i + t$$

Hough space (parameter space): $\{m, t\}$

Transformation rule: e.g. solve parameterization for Hough parameters

$$t(m) = -m \cdot x_i + y_i$$

i.e. straight lines in parameter space with slope $-x_i$ and offset $y_i$

## Weighted Least Squares Fitting:

Measurement of variable $y_i$ at $n$ points $x_i$ with error $\sigma_i$

Fit with function $g(x; a_1 \ldots a_m)$ with parameters $a_j$

⇨ best values for $a_j$ are those which minimize

$$S = \sum_{i=1}^{n} \frac{[y_i - g(x_i; a)]^2}{\sigma_i^2} = \sum_{i=1}^{n} W_{ii} r_i^2, \quad r_i = y_i - g(x_i; a)$$

Condition for minimum: $$\frac{\partial S}{\partial a_j} = 2 \sum_{i=1}^{n} W_{ii} \frac{\partial r_i}{\partial a_j} r_i \stackrel{!}{=} 0, \quad j = 1, \ldots, m$$

$$\Rightarrow -2 \sum_{i} W_{ii} \frac{\partial g(x_i; a)}{\partial a_j} r_i = 0, \quad j = 1, \ldots, m$$

# Track Fitting

Linear regression model, i.e. linear combination of parameters

$$g(x;a) = \sum_{j=1}^{m} a_j \phi_j(x)$$

With $X_{ij} \equiv \frac{\partial g(x_i;a)}{\partial a_j} = \phi_j(x_i)$ Design matrix

⇨ Normal equations $\sum_{i=1}^{n} \sum_{k=1}^{m} X_{ij} W_{ii} X_{ik} \hat{a}_k = \sum_{i=1}^{n} X_{ij} W_{ii} y_i, \quad j = 1, \ldots, m$

In matrix form: $\left(X^T W X\right) \hat{a} = X^T W y$ Normal equations

Resulting estimator $\hat{a}$ is the best linear unbiased estimator if

$W = C^{-1}$ inverse covariance matrix

# A Simple Example

- Combine 2 measurements of the same quantity with different errors:

$$x_1 \pm s_1, \quad x_2 \pm s_2$$

⇨ weighted mean: $$\mu_2 = \frac{\frac{x_1}{s_1^2} + \frac{x_2}{s_2^2}}{\frac{1}{s_1^2} + \frac{1}{s_2^2}} = \frac{x_1 s_2^2 + x_2 s_1^2}{s_1^2 + s_2^2}$$

# A Simple Example

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$$= x_1 + \underbrace{\frac{s_1^2}{s_1^2 + s_2^2}}_{\text{"Kalman gain"}} \underbrace{(x_2 - x_1)}_{\text{residual}}$$

standard deviation: $$\sigma_2^2 = \left(\frac{1}{s_1^2} + \frac{1}{s_2^2}\right)^{-1} = \frac{s_1^2 s_2^2}{s_1^2 + s_2^2} = \left(1 - \frac{s_1^2}{s_1^2 + s_2^2}\right) s_1^2$$

- Add 3rd measurement $x_3 \pm s_3$

$$\mu_3 = \mu_2 + \frac{\sigma_2^2}{\sigma_2^2 + s_3^2}(x_3 - \mu_2) \quad \ldots$$

# Track Fitting

Traditional track reconstruction algorithms:

- Full pattern recognition (association of hits in a track)
- Global fit to get track parameters
- Hit rejection / association based on hit contribution to $\chi^2$

⇨ Many iterations necessary: computationally expensive!

Track reconstruction based on Kalman Filter:

- First approximation of track parameters from pattern reco
- One pass with simultaneous
  - hit association (based on $\chi^2$ increment)
  - track fit (recursive update of track parameters)

Kalman Filter: [R.E. Kalman, Trans. ASME 82, 35 (1960)]

- Recursive solution of the discrete-data linear filtering problem
- Extensively applied in the area of assisted or autonomous navigation
  e.g. radar measurements of ballistic object trajectories, drones
- Mathematical structure allows for real-time applications
- Widely used in High Energy Physics

For track fitting:

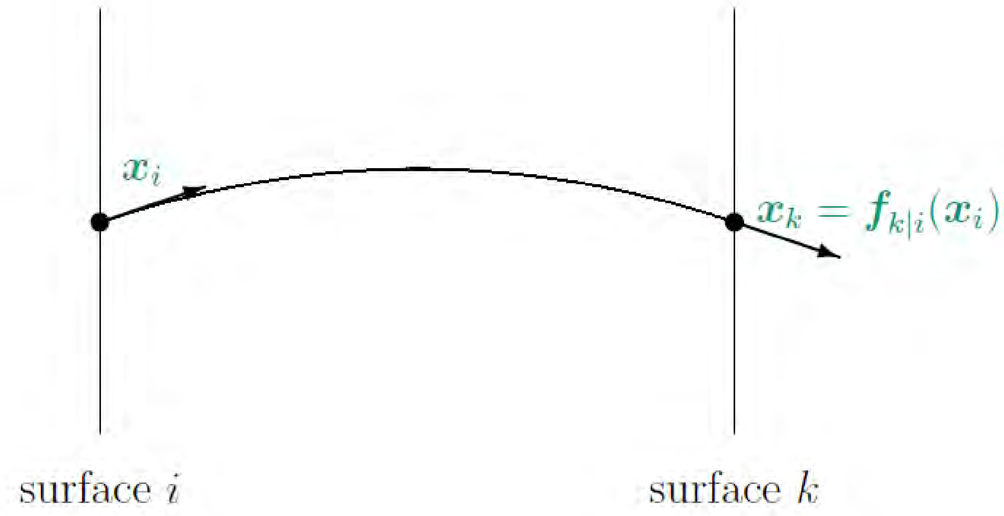
- **State vector** $x$ is a vector with track parameters
- **Measurement** $m$ is a hit in a detector
- **Measurement transformation** $H$ is a rotation
- **Measurement noise** $\nu$ is the detector resolution
- **Process noise** $Q$ includes multiple scattering, energy losses, etc.

[R.E. Kalman, Trans. ASME 82, 35 (1960)]

# Track Fitting

Track model:

- describes how the state vector $x_k$ at a given surface $k$ depends on the state vector at a different surface $i$




- linear evolution:

$$x_{k|i} = F_{k|i} \cdot x_i$$

- Non-linear evolution (B-field!)

$$x_{k|i} = f_{k|i}(x_i)$$

$f_{k|i}$ = track propagator from surface $i$ to surface $k$

"extended Kalman filter"

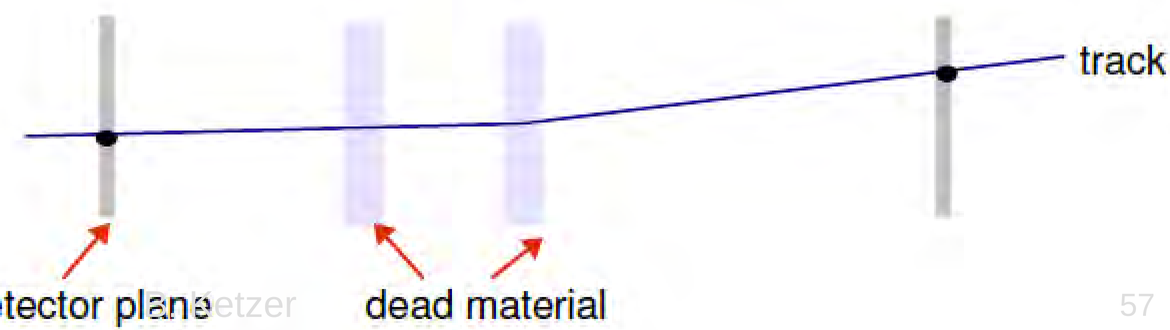
- Error propagation: Jacobians

$$F_{k|i} = \left.\frac{\partial x_k}{\partial x_i}\right|_{x_{i,0}}$$

$x_{i,0}$ expansion point

# Track Fitting

- In some cases the track model is analytical
  - no magnetic field: straight line
  - constant magnetic field: helix
    - only for cylindrical surfaces ∥ $z$ or
    - planes perpendicular to $z$
- More complicated field or detector geometry:
  - helix can still be used, but parameters depend on coordinate along trajectory
  - numerical integration e.g. by Runge-Kutta method
  - approximation by polynomials, splines (training needed!)
- Propagation also includes material effects



# Track Fitting

Material effects: treated at various levels of complexity

- Multiple Coulomb scattering (MS) ➪ Gaussian approximation + tails
- Energy loss (EL) ➪ Bethe-Bloch for average energy loss
- Bremsstrahlung loss (BL, for e and very high energy μ) ➪ Bethe-Heitler

Track state vector propagation between 2 adjacent surfaces:

$$x_{k|k-1} = f_{k|k-1}(x_{k-1|k-1} + \underbrace{\sigma_{k-1}}_{\text{MS}} + \underbrace{\delta_{k-1}}_{\text{EL}} + \underbrace{\beta_{k-1}}_{\text{BL}}) \approx \underbrace{F_{k|k-1}}_{\text{Transition matrix: dynamics}} \cdot x_{k-1|k-1} + \underbrace{w_{k|k-1}}_{\text{Process noise: MS}}$$

- MS and BL stochastic
- EL deterministic

Covariance matrix propagation:

$$C_{k|k-1} = F_{k|k-1} C_{k-1|k-1} F_{k|k-1}{}^{T} + Q_{k|k-1}$$

$Q_{k|k-1}$ accounts for random noise in extrapolation: MS and energy loss fluctuations

# Track Fitting

Measurement model:

- transforms state vector $x_k$ at detector surface into coordinate space of measurement $m_k$

$$m_k = h_k(x_k)$$

- vector of measurements $m_k$ contains measured detector coordinates but may include also other information, e.g. direction, momentum
  - pixel detector: $m_k$ is 2-dimensional
  - strip detector: $m_k$ is 1-dimensional
  - TPC: $m_k$ is 3-dimensional
- Linearization: Jacobians

$$H_k = \left.\frac{\partial m_k}{\partial x_k}\right|_{x_{k,0}}$$

- Include measurement errors:

$$m_k = H_k x_k + \nu_k$$

- Calibration: determination of mean $\mu_k$ and covariance matrix $V_k$ of $\nu_k$

**Kalman Filter:** [R.E. Kalman, Trans. ASME 82, 35 (1960)]

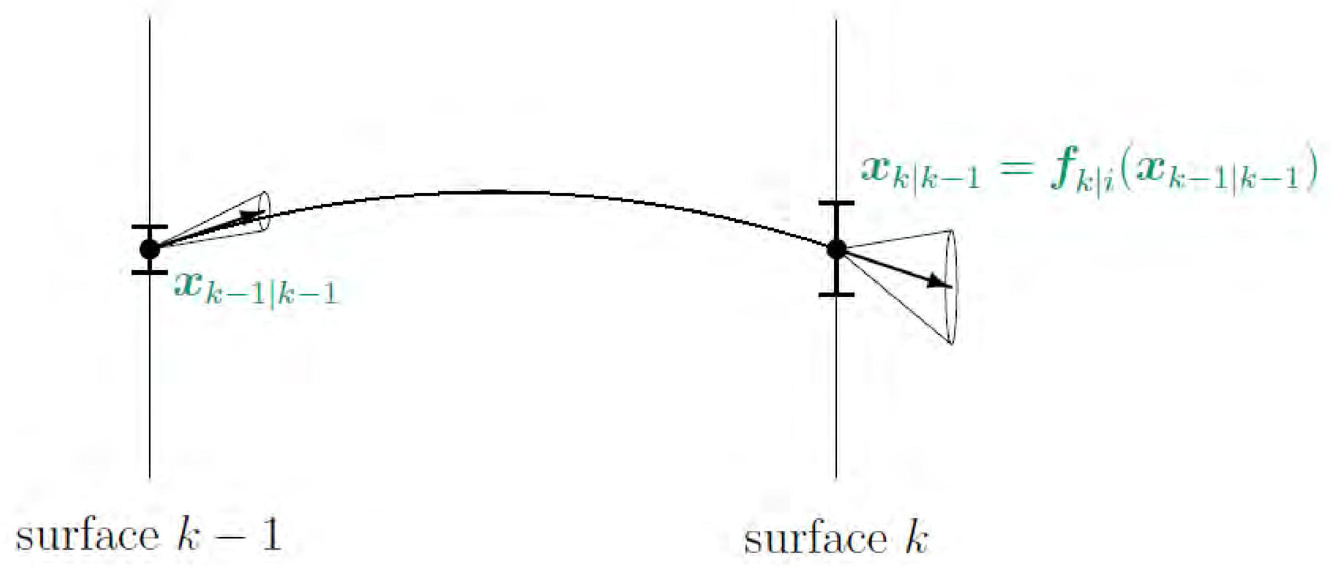
- Iterative least-squares estimation of state vectors and covariance matrix from a series of (noisy) measurements in all measurement layers
- Repetition of 2 steps:
  - Prediction: extrapolate the state to the next layer $k$ based on all information up to layer $k\text{-}1$
    add up multiple scattering, subtract energy loss
    (process model and process noise model)
  - Update: combine the predicted state with the current measurement in layer $k$
    (measurement equation and accuracy, i.e. measurement noise)

**Prediction step:**

Predicted state $x_{k|k-1} = f_{k|k-1}(x_{k-1|k-1} + \sigma_{k-1} + \delta_{k-1} + \beta_{k-1})$

Predicted covariance matrix $C_{k|k-1} = F_{k|k-1} C_{k-1|k-1} F_{k|k-1}{}^T + Q_{k-1}$

⇨ inflates covariance matrix



# Track Fitting

**Update (filter) step** using gain matrix $K_k$ (weighted mean):

Updated state $x_{k|k} = x_{k|k-1} + K_k r_{k|k-1}$

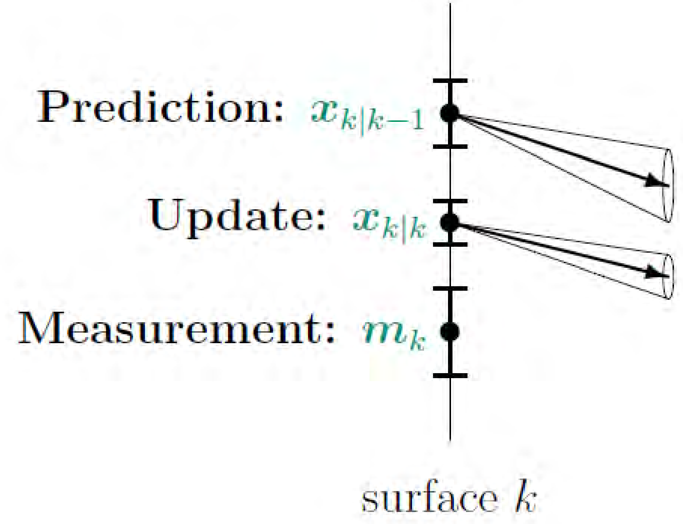
with Kalman gain $K_k$

and residual $r_{k|k-1} = m_k - H_k x_{k|k-1}$

compare with

$$\mu_3 = \mu_2 + \frac{\sigma_2^2}{\sigma_2^2 + s_3^2}(x_3 - \mu_2)$$

- $K_k = 0$ ⇨ new measurement is ignored
- $K_k \neq 0$ ⇨ new state will be pulled towards $H_k^{-1} m_k$



# Track Fitting



Kalman gain: $K_k = C_{k|k-1} {H_k}^T \left( H_k C_{k|k-1} {H_k}^T + V_k \right)^{-1}$

compare with

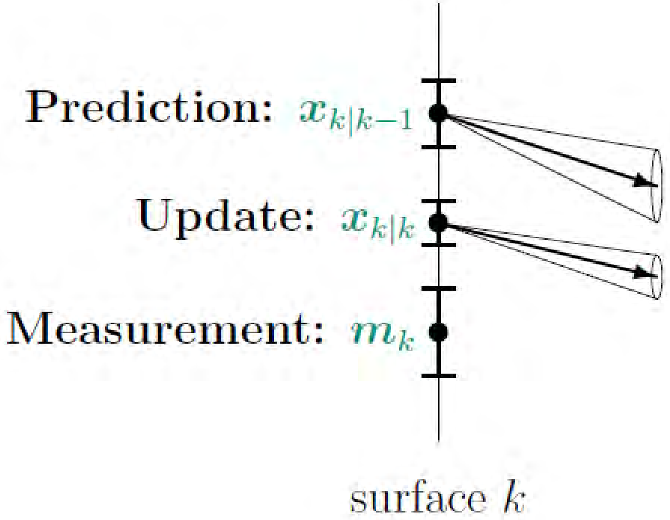
$$\mu_3 = \mu_2 + \frac{\sigma_2^2}{\sigma_2^2 + s_3^2}(x_3 - \mu_2)$$

Updated covariance matrix: $C_{k|k} = (1 - K_k H_k)\, C_{k|k-1}$

$$\sigma_3 = \left(1 - \frac{\sigma_2^2}{\sigma_2^2 + s_3^2}\right)\sigma_2^2$$

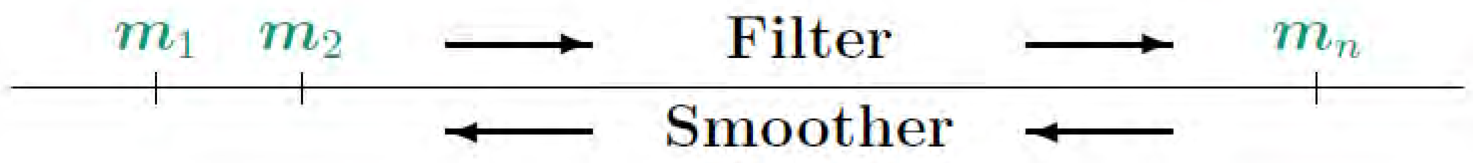
⇨ shrinks covariance matrix because of additional information from measurement:

$C_{k|k} < C_{k|k-1}$



Smoothing:

- The last estimate $x_{n|n}$ of the fitter contains the full information from all hits
- The full information can be propagated back to all previous estimates
- This can be done by another iterative procedure: smoothing
- Runs in opposite direction to filter



# Track Fitting

- Kalman filter can also be used for track finding and vertexing
- Kalman filter weighs all hits according to their covariances
- Accidental inclusion of noise hits or outliers in PR may bias the fit

  ⇨ Deterministic Annealing Filter (DAF):

  update weights by distance to track

  [R. Frühwirth et al., Comp. Phys. Comm. 120, 197 (1999)]
- In case of non-Gaussian noise: non-linear filter, e.g.

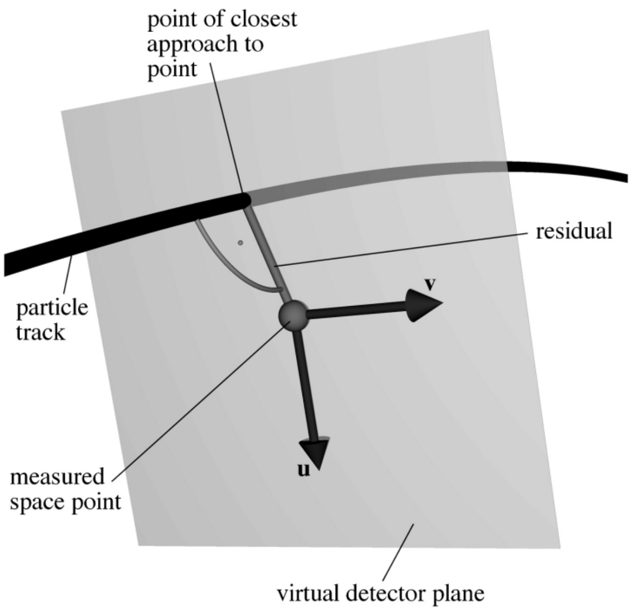
  Gaussian Sum Filter (GSF)

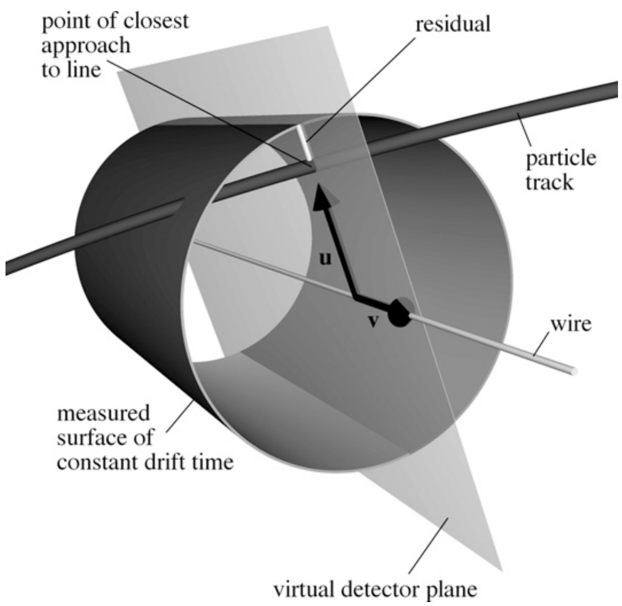
# GENFIT



- Generic framework for track fitting: http://sourceforge.net/projects/genfit
- Completely modular design
  - Fitting algorithms: Kalman filter, Gaussian sum, DAF
  - Track representation & extrapolation routines: GEANE, RKTrackRep
  - Detector hits: strip, pixel, wires, 3-D space points

[C. Höppner et al., Nucl.Instrum.Meth. A620 (2010) 518-525]



# Theory and Practice…

**Many novel detector concepts** for nuclear and particle physics experiments:

- tracking
- photon detection
- calorimetry…

**Main challenges for the future:**

- resolution
- rate capability
- power consumption
- radiation hardness
- trigger-less readout
- data reduction at front-end level

**Basic understanding of underlying processes indispensable…**

*"Now, this is just a simulation of what the blocks will look like once they're assembled."*

“New directions in science are launched by new tools much more often than by new concepts.

The effect of a concept-driven revolution is to explain old things in new ways.

The effect of a tool-driven revolution is to discover new things that have to be explained.”

Freeman Dyson, Imagined Worlds