Modern Particle Detectors

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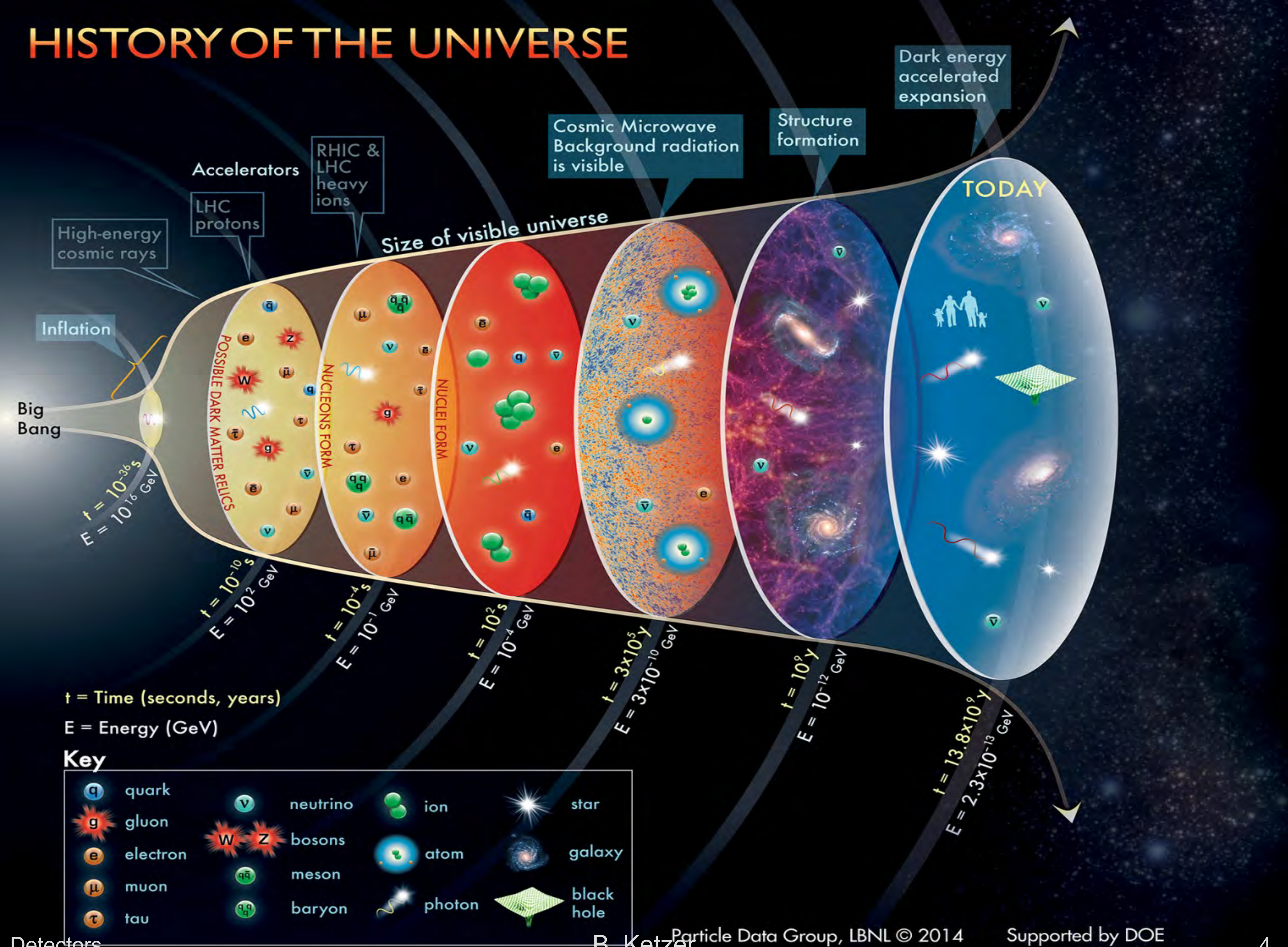
SFB 1044 School 2016
Boppard

Plan of the Lecture

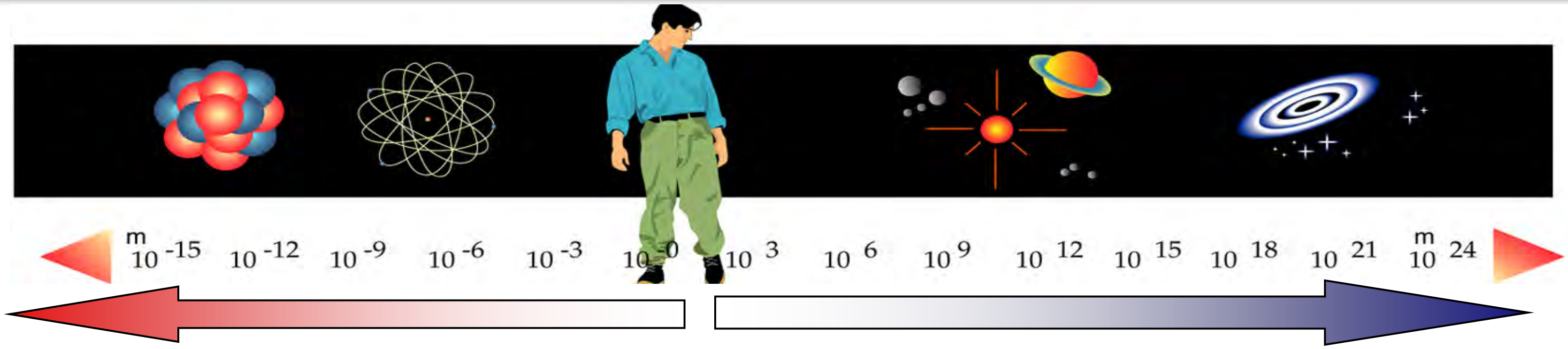
1. Introduction
2. Interaction of charged particles with matter
3. Ionization detectors
4. Position and momentum measurement / track reconstruction
5. Photon detection
6. Calorimetry
7. Detector systems

1 Introduction

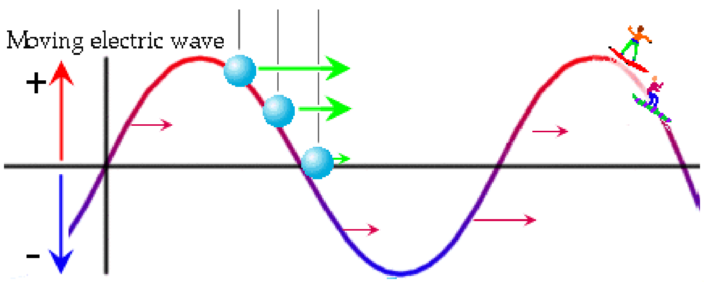


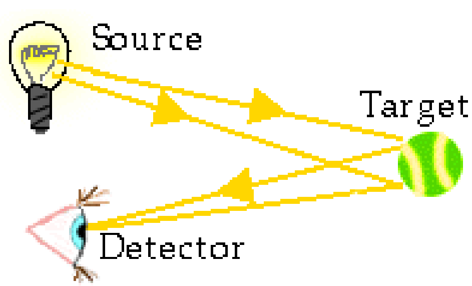
HISTORY OF THE UNIVERSE
Dark energy accelerated expansion
Cosmic Microwave Background radiation is visible
Structure formation
RHIC & LHC heavy ions
Accelerators
LHC protons
High-energy cosmic rays
Size of visible universe
TODAY
Inflation
Big Bang
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
$t = 10^{-36}$ s
$E = 10^{16}$ GeV
$t = 10^{-10}$ s
$E = 10^{2}$ GeV
$t = 10^{-4}$ s
$E = 10^{-1}$ GeV
$t = 10^{2}$ s
$E = 10^{-4}$ GeV
$t = 3x10^{5}$ y
$E = 3x10^{-10}$ GeV
$t = 10^{9}$ y
$E = 10^{-12}$ GeV
$t = 13.8x10^{9}$ y
$E = 2.3x10^{-13}$ GeV
t = Time (seconds, years)
E = Energy (GeV)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL © 2014
Supported by DOE

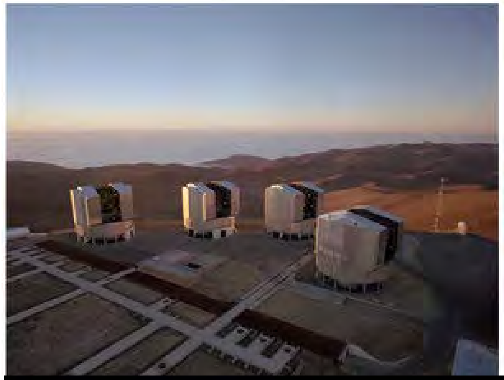
How to observe this?



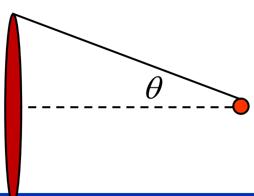
Accelerators | Microscopes | Binoculars | Optical and radio telescopes







Resolution: $\Delta r \sim \frac{\lambda}{\sin\theta}$



Old Particle Detectors

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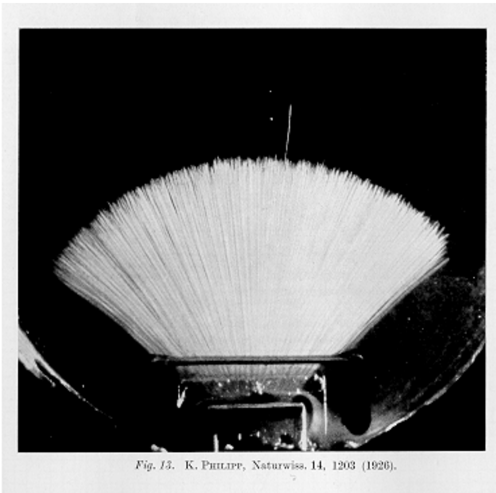
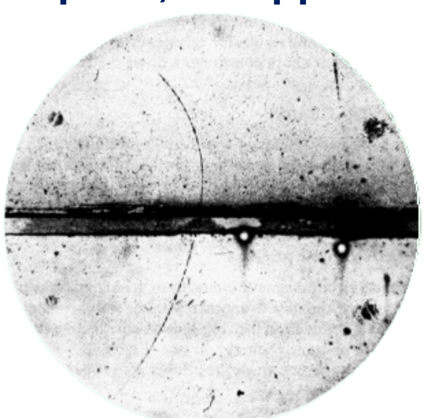
Cloud Chamber

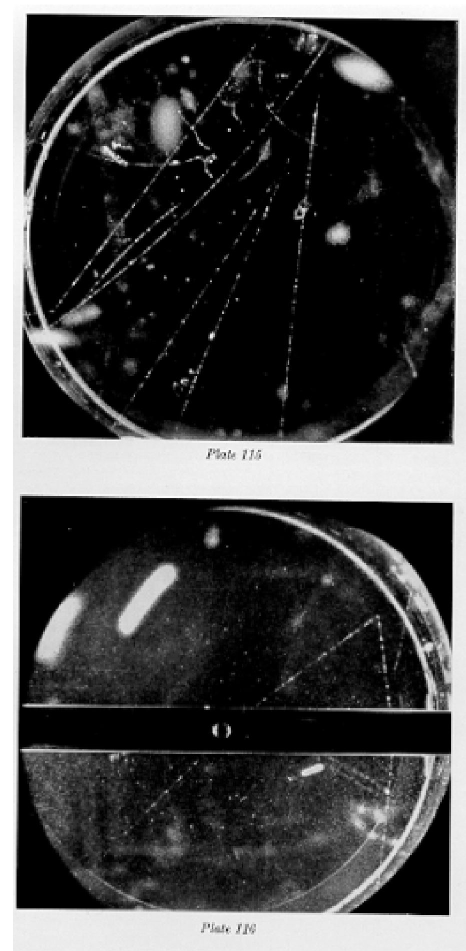
C.T.R. Wilson (1910): Charges act as condensation nuclei in supersaturated water vapor (later: alcohol vapor ⇨ diffusion cloud chamber)



Alphas, Philipp 1926



Positron discovery, Carl Andersen 1933

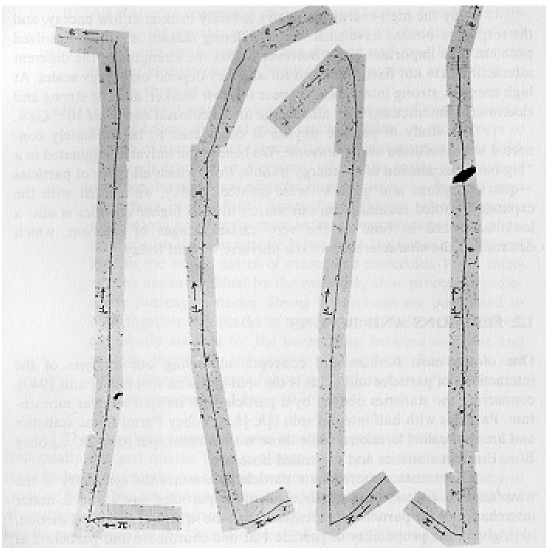


V-particles, Rochester and Wilson, 1940ies

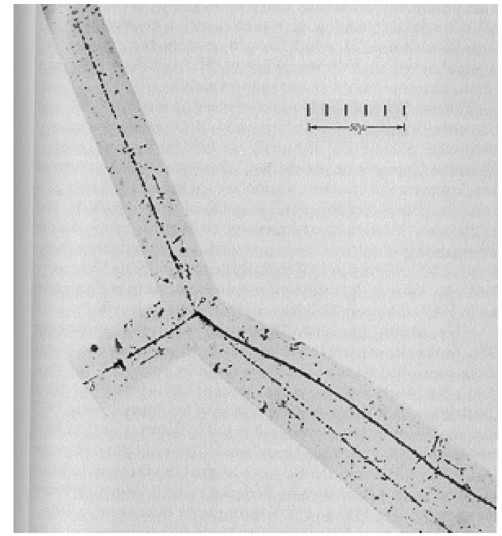
Nuclear Emulsion

universitätbonn

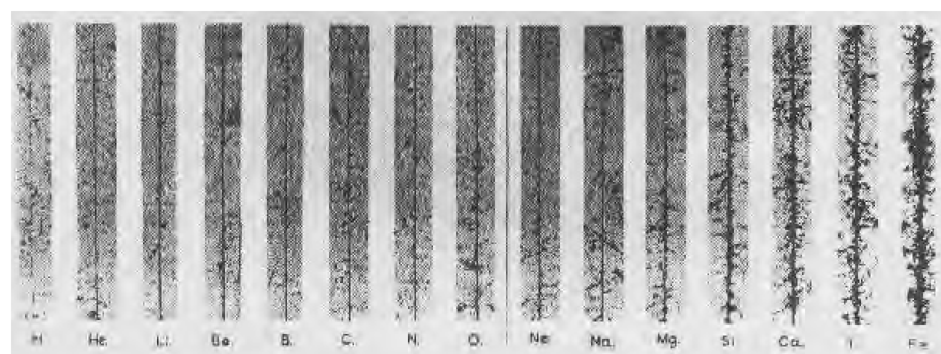
M. Blau (1930s): Charges initiate a chemical reaction that blackens the emulsion (film made of Ag-halide, e.g. AgBr)



C. Powell, Discovery of muon and pion, 1947



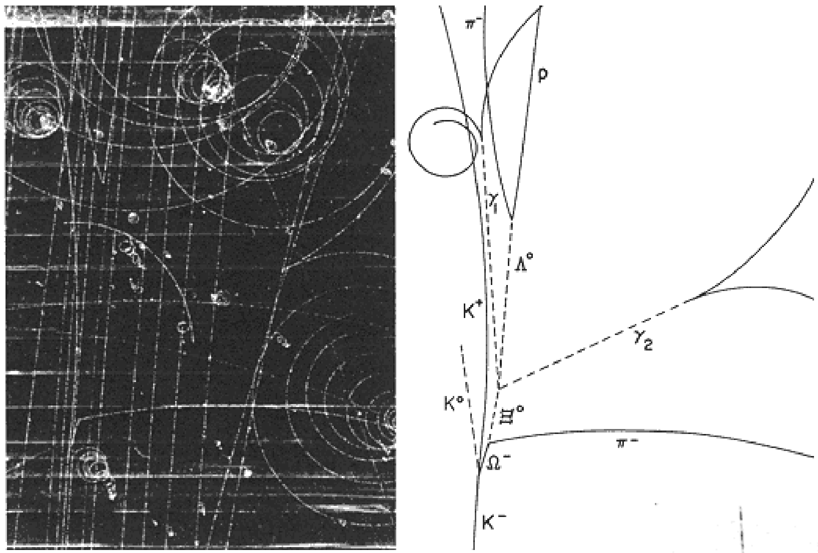
Kaon Decay into 3 pions, 1949



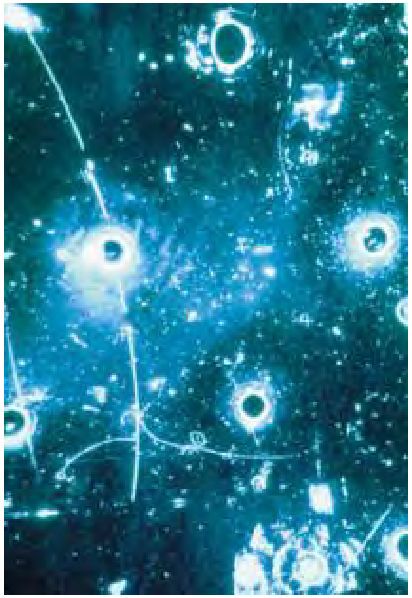
Cosmic Ray Composition

Bubble Chamber

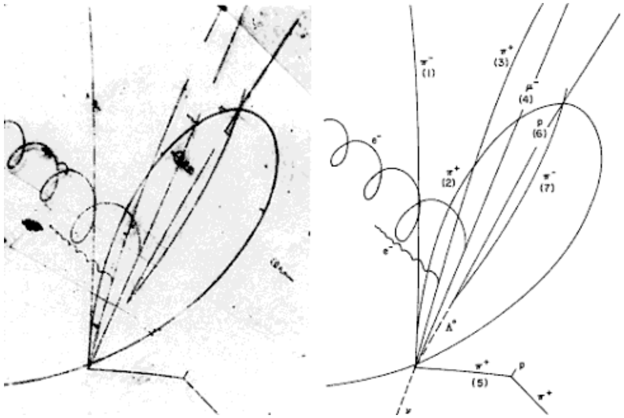
D. Glaser (1952): Charges create bubbles in superheated liquid, e.g. propane or Hydrogen (Alvarez)



Discovery of the Ω^- in 1964

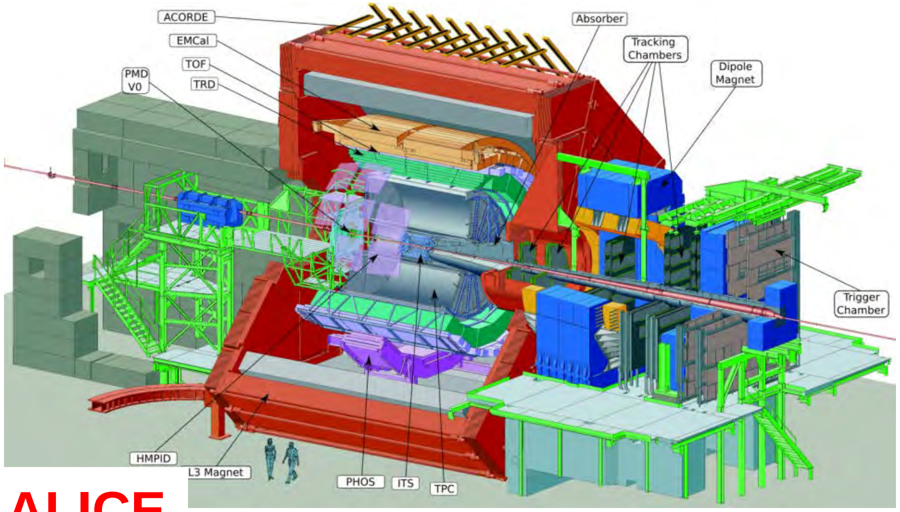


Neutral Currents 1973



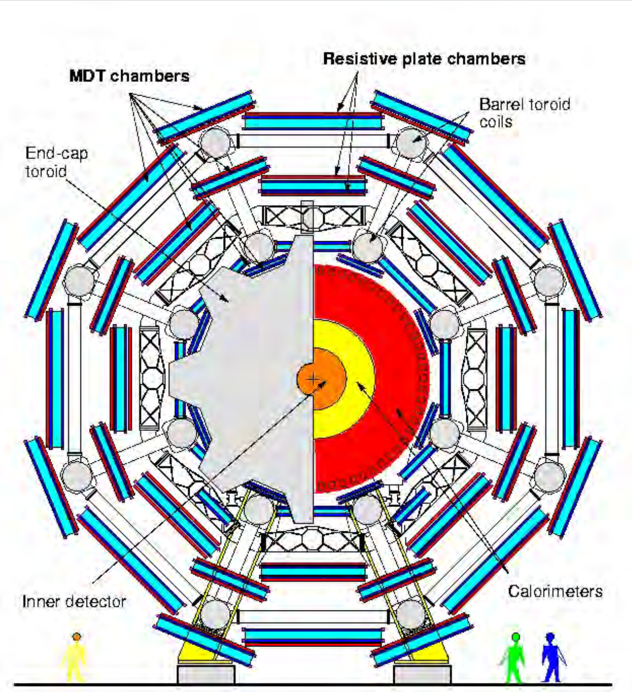
Charmed Baryon, 1975

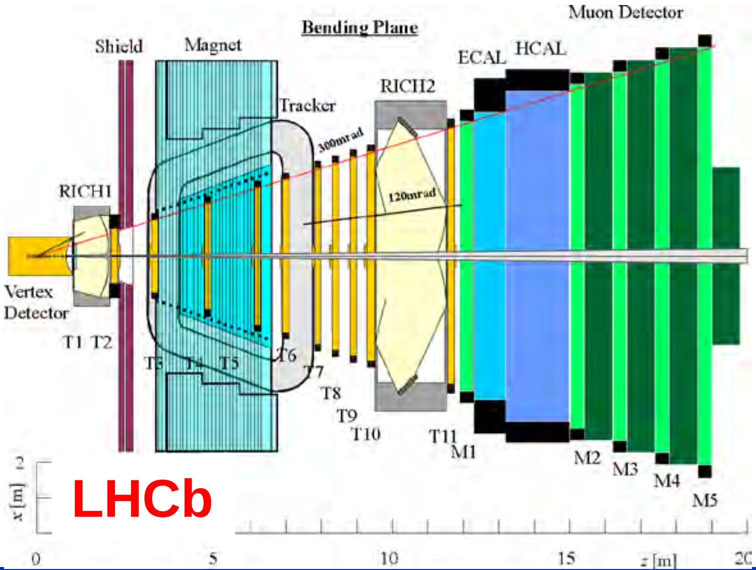
The Giants



ALICE

ATLAS



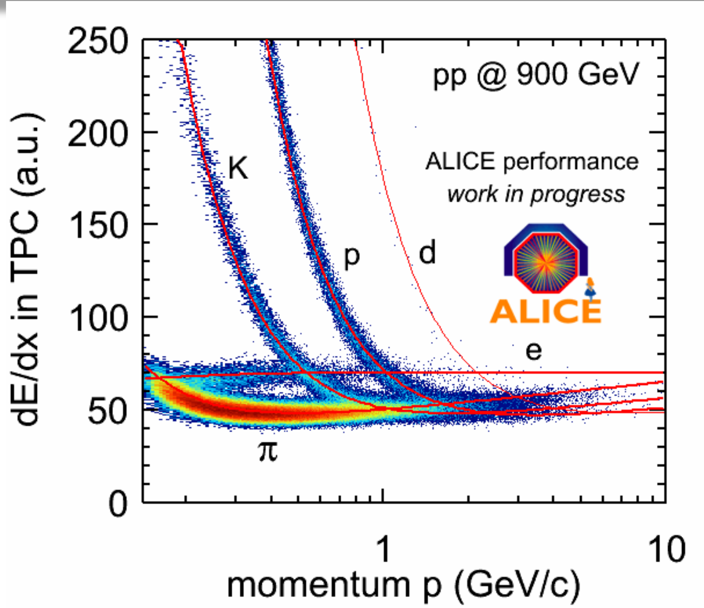


LHCb

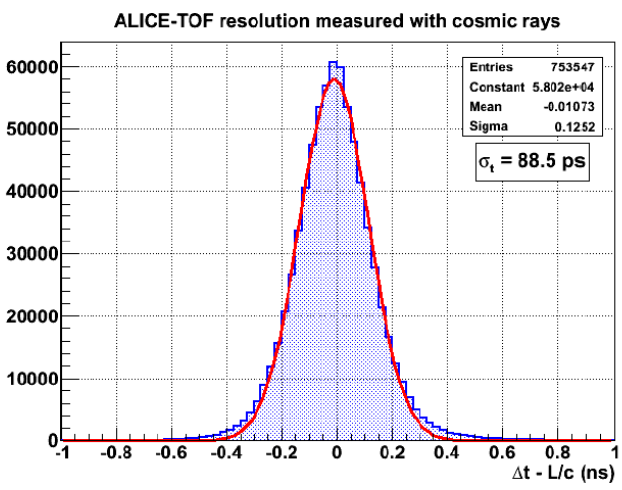
Very Large Structures

- Engineering, Services, Cooling
- Electronics

But in the end:
resolution limits are still defined by the fundamental detector physics processes …



dE/dx particle ID resolution is defined by the fundamental properties of EM interactions of charged particles with matter, 'Bethe Bloch' curve + 'Landau' distribution



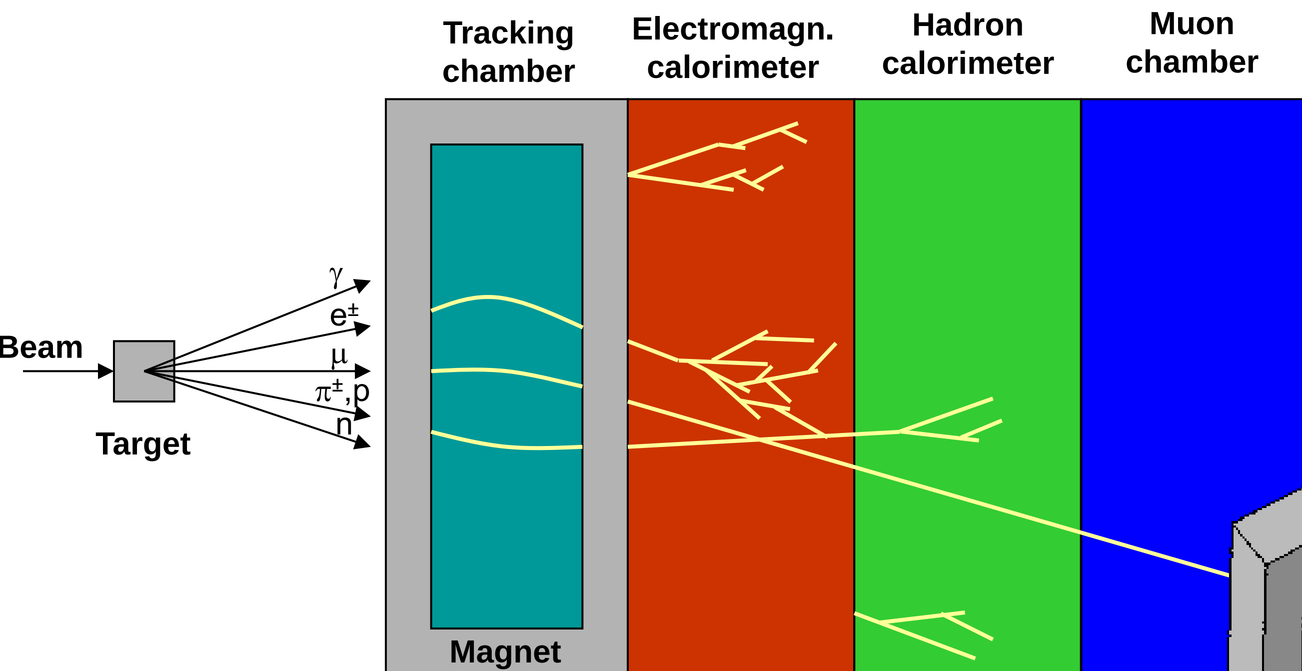
Time of Flight Resolution with Resistive Plate Chambers (ALICE) is defined by the electron avalanche fluctuations together with the drift-velocity.

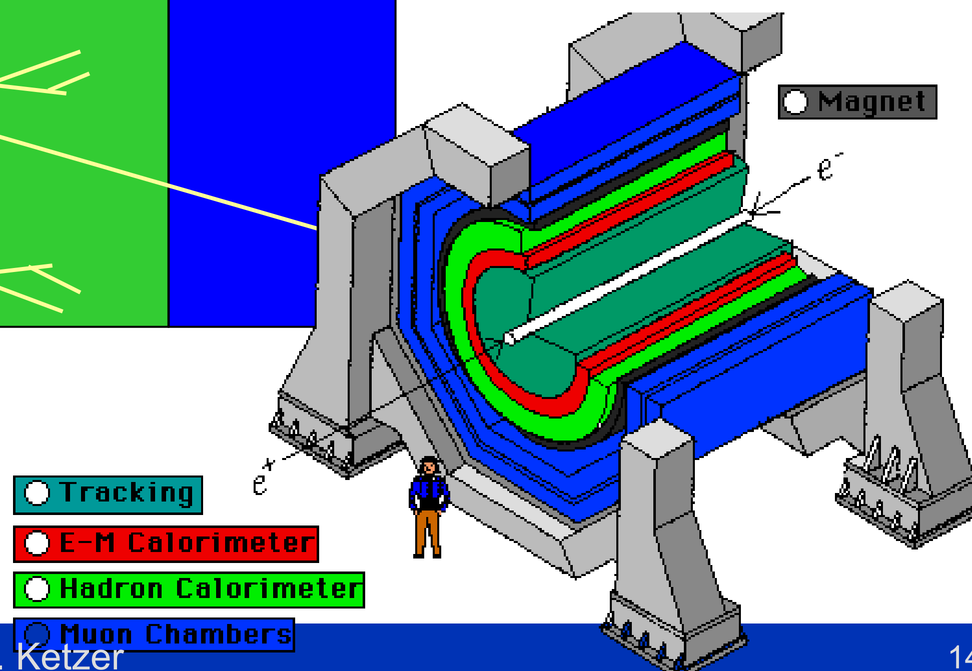
[W. Riegler, priv. comm.]

Alpha Mass Spectrometer

A Typical Detector Setup

Different **components**, measuring different **aspects** of reaction products: track, charge, energy, momentum, particle type, ...





Detection of Radiation

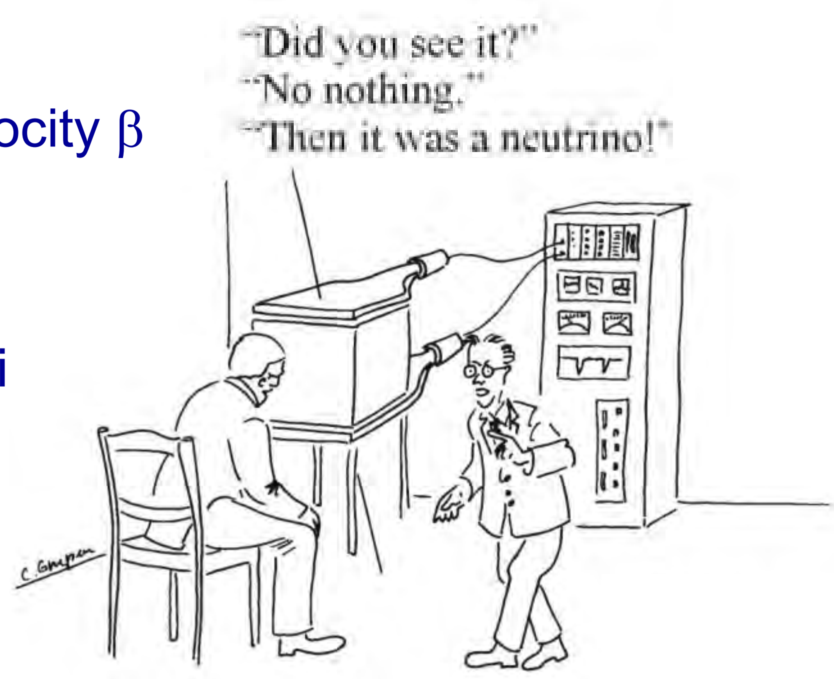
Goal: Measurement of 4-momentum and position in space of particles

Methods:

- Position-sensitive detectors ⇨ direction and position of momentum vector
- Bending in magnetic field ⇨ magnitude of p
- Absorption in calorimeter ⇨ energy
- Cherenkov radiation, time of flight ⇨ velocity β
- Transition radiation ⇨ γ
- Energy loss ⇨ β, γ
- Characteristic decay of a particle, detecti

Detection by interaction with detector

- electromagnetic interaction with $\Delta E \ll E$
- interaction with $\Delta E \sim E$ (calorimtery)



[Claus Grupen, Particle Detectors, Cambridge, 1996]

Interactions of Charged Particles

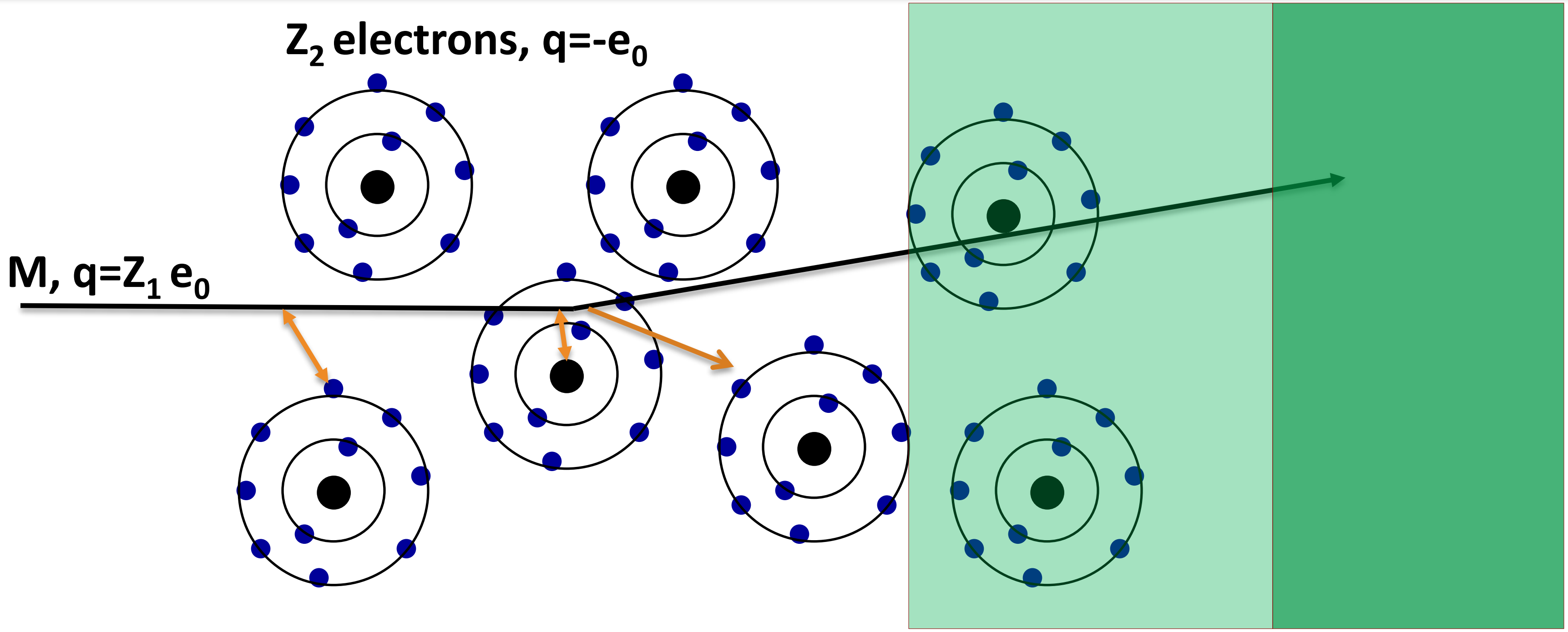
Processes for a charged particle passing through matter:

1. Inelastic collisions with atomic electrons
   ⇨ energy loss
   ⇨ excitation (soft collision) or ionization (hard collision) of hit atom
   ⇨ deflection: small
2. Elastic collisions with nuclei
   ⇨ deflection
   ⇨ energy loss: negligible, since normally $m_a \ll m_b$
   ⇨ no excitation of hit atom
3. Emission of Bremsstrahlung ⇨ important for $e^\pm$
4. Emission of Cherenkov radiation / transition radiation in inhom. materials
5. Nuclear interactions

Moderately relativistic heavy charged particles: μ, π, p, α, … ($m_a > m_e$)

⇨ energy loss almost entirely through process 1.: $\sigma \sim 10^{-17}$-10^{-16} cm^2

Electromagnetic Interaction of Particles with Matter



Interaction with the atomic electrons. The incoming particle loses energy and the atoms are excited or ionized.

Interaction with the atomic nucleus. The particle is deflected (scattered) causing multiple scattering of the particle in the material. During this scattering a Bremsstrahlung photon can be emitted.

In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as Cherenkov Radiation. When the particle crosses the boundary between two media, there is a probability of the order of 1% to produce an X ray photon, called Transition radiation.

[W. Riegler, priv. comm.]

Warm Up

1. What is the general relation between energy and momentum?
2. What approximations can be used?
3. What are β and γ? How are they calculated from E, p, m?
4. How large are the fluctuations in radioactive decay?
5. What is a cross section?
6. What are typical values of cross sections?
7. How is it related to luminosity?
8. How do charged particles interact with matter?

2 Electromagnetic Interactions of Charged Particles with Matter

2.1 Collisions

Interactions of a fast charged particle with speed $\beta = v/c$ and momentum $p = Mc\beta\gamma$ with matter

- ⇨ Occurrence of random individual collisions
- ⇨ In each collision the particle loses a random amount of energy E

Characterization by mean free path λ and collision cross section σ:

$$\lambda = \frac{1}{n_e \sigma} = \frac{1}{n_p}$$

n_e number density of electrons
n_p number of (primary) collisions per unit length

Number of encounters in length L described by Poisson distribution

$$P(k;\mu) = \frac{\mu^k}{k!}\, e^{-\mu} \qquad \mu = \frac{L}{\lambda} = L\, n_p$$

Collisions

Probability distribution $f(l)\mathrm{d}l$ of free flight paths l between collisions:

$$f(l)\mathrm{d}l = P\left(0;\frac{l}{\lambda}\right)\cdot P\left(1;\frac{\mathrm{d}l}{\lambda}\right) = e^{-\frac{l}{\lambda}}\cdot\frac{\mathrm{d}l}{\lambda}$$

single exponential

Probability of having zero encounters along track length L:

$$P\left(0;\frac{L}{\lambda}\right) = e^{-L/\lambda}$$

➪ inefficiency of a perfect detector, which is capable of detecting even single electrons

➪ method to measure λ, n_p

Gas	1 cm/λ	γ
H_2	5.32 ± 0.06	4.0
	4.55 ± 0.35	3.2
	5.1 ± 0.8	3.2
He	5.02 ± 0.06	4.0
	3.83 ± 0.11	3.4
	3.5 ± 0.2[a]	3.6
Ne	12.4 ± 0.13	4.0
	11.6 ± 0.3[a]	3.6
Ar	27.8 ± 0.3	4.0
	28.6 ± 0.5	3.5
	26.4 ± 1.8	3.5
Xe	44	4.0
N_2	19.3	4.9
O_2	22.2 ± 2.3	4.3
Air	25.4	9.4
	18.5 ± 1.3	3.5

Number of ionizing collisions per cm track length, measured at given value of γ

[V.K. Ermilova et al., Sov. Phys.-JETP 29, 861 (1969)]
[K. Söchting, Phys. Rev. A, 20, 1359 (1979)]

2.2 Mean energy loss

$$\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x}\right\rangle = \frac{4\pi}{(4\pi\varepsilon_0)^2}\frac{z^2e^4n_\mathrm{e}}{mc^2\beta^2}\left[\frac{1}{2}\ln\frac{2mc^2\beta^2\gamma^2T_{\max}}{I^2}-\beta^2-\frac{\delta}{2}\right]$$

"Bethe equation"

with

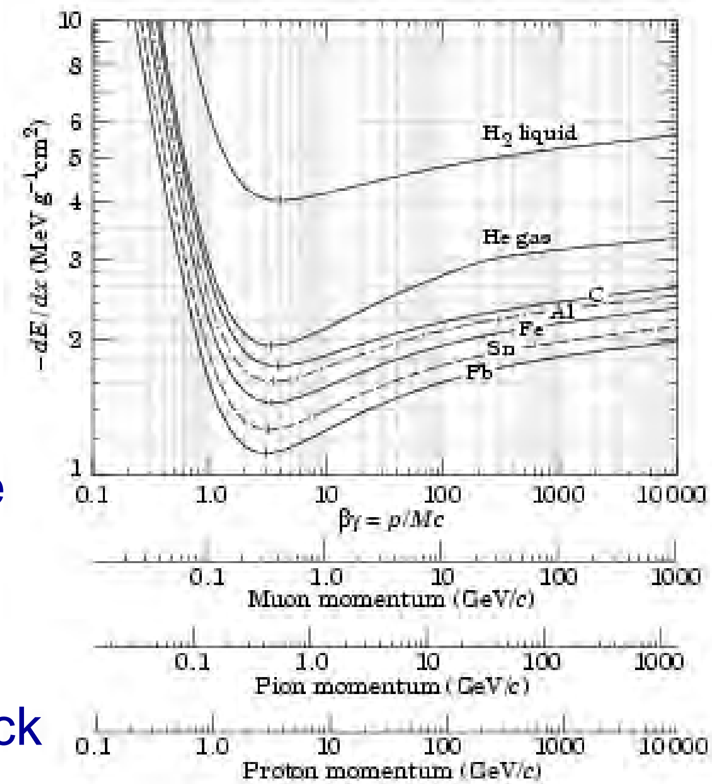
ze	charge of incoming particle
n_e	electron number density of material
m	electron mass
$\beta=v/c$	velocity of incoming particle
γ	relativistic factor
$T_{\max}$	maximum kinetic energy imparted to electron in single collision
I	mean excitation energy
δ	density effect correction

$$n_\mathrm{e} = \frac{Z}{A}N_\mathrm{A}\rho$$

Mean Energy Loss

$$\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = \frac{4\pi}{(4\pi\varepsilon_0)^2} \frac{z^2 e^4 n_e}{mc^2\beta^2} \left[\frac{1}{2} \ln \frac{2mc^2\beta^2\gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta}{2} \right]$$

- independent of mass of incident particle
- depends only on velocity of inc. particle and on I ⇨ main parameter
- low energies ⇨ $\left\langle -\mathrm{d}E/\mathrm{d}x \right\rangle \propto 1/\beta^2$
- minimum at $\beta\gamma \approx 3$: "MIP"
- high energies ⇨ $\left\langle -\mathrm{d}E/\mathrm{d}x \right\rangle \propto \ln \beta^2\gamma^2$: relativistic rise
- mass stopping power: $\left\langle -\mathrm{d}E/\rho\mathrm{d}x \right\rangle \propto z^2 \left(Z/A \right) \cdot f(\beta, I)$
  ⇨ almost independent of material
- density effect: polarization of atoms along track
  ⇨ partly compensates relativistic rise



Calculation of Energy Loss

Quantum picture: energy loss caused by a number of discrete collisions per unit length, each with energy transfer E

$$\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x}\right\rangle = \int_0^\infty E' f(E')\mathrm{d}E'$$

$f(E)\,\mathrm{d}E$ probability of energy loss per unit path length between E and $E+\mathrm{d}E$

and with $f(E) = n_e\,\mathrm{d}\sigma(E,\beta)/\mathrm{d}E$

$$\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x}\right\rangle = n_e \int_0^\infty E' \frac{\mathrm{d}\sigma}{\mathrm{d}E'}\mathrm{d}E'$$

n_e electron density

E energy transfer in single collision

$\mathrm{d}\sigma/\mathrm{d}E$ collision cross section differential in transferred energy

Mean free path: $n_p = \frac{1}{\lambda} = \int_0^\infty f(E')\mathrm{d}E'$

n_p number of primary collisions per unit path length

Spectrum of energy transfer $F(E)\mathrm{d}E = \frac{f(E)\mathrm{d}E}{n_p}$

probability of energy loss in $[E, E+\mathrm{d}E]$ per collision

⇨ need a model for collision cross section!

[H. Bichsel, NIM A 562, 154 (2006)]

Rutherford - Mott Model

Simplest ansatz: hard collisions

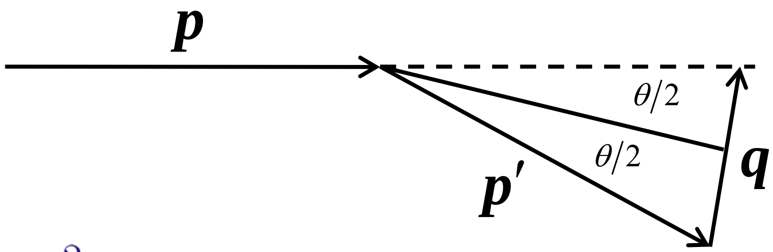
- Coulomb scattering of projectile with charge ze off free electrons
- only valid for energy transfers $\gg$ typical atomic binding energies I
- in rest frame of projectile: electron scattering off heavy particle at rest

⇨ Mott cross section:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{\star}_{\mathrm{Mott}} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Rutherford}} \cdot \left(1 - \beta^2 \sin^2 \frac{\theta}{2}\right)$$

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Rutherford}} = \left(\frac{z\alpha\hbar c}{2\,|\boldsymbol{p}|\,|\boldsymbol{v}|}\right)^2 \frac{1}{\sin^4 \frac{\theta}{2}}$$ for static potential (no recoil)

With $\boldsymbol{q} = \boldsymbol{p} - \boldsymbol{p}'$, $\sin\frac{\theta}{2} = \frac{|\boldsymbol{q}|}{2\,|\boldsymbol{p}|}$, $\boldsymbol{p} = \gamma m \boldsymbol{v}$



follows the cross section
differential in transferred energy $E = \frac{|\boldsymbol{q}|^2}{2m}$

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}E}\right)^{\star}_{\mathrm{Mott}} = \frac{2\pi}{m}\left(\frac{z\alpha\hbar}{E\beta}\right)^2\left[1 - \frac{E}{2mc^2}\left(1-\beta^2\right)\right]$$

Exercise: show this…

Rutherford - Mott

Evaluation of integral $$\int_{T_{\min}}^{T_{\max}} E' \left(\frac{\mathrm{d}\sigma(E',\beta)}{\mathrm{d}E'}\right)_{\mathrm{Mott}} \mathrm{d}E'$$

Validity range of Mott CCS: $T_{\min} < E < T_{\max}$

$$T_{\max} = \frac{2\gamma^2\beta^2 mc^2}{1 + 2\gamma(m/M) + (m/M)^2} \underset{M \gg 2m\gamma}{\simeq} 2mv^2\gamma^2 \qquad T_{\min} = \epsilon \gg I$$

I: mean excitation energy

Therefore we arrive at

$$\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle_{\mathrm{R}} = n_e \cdot \frac{2\pi}{m} \cdot \left(\frac{z\alpha\hbar}{\beta}\right)^2 \left[\ln \frac{2mv^2\gamma^2}{\epsilon} - \beta^2\right]$$

Yields Bethe equation, except

- Factor 2
- ϵ instead of I

Contribution from hard scattering!

Bethe – Fano Model

Bethe, 1930:

[H. Bethe, Ann. Phys. 5, 325 (1930)]

- drop assumption of free electrons
- derive expression for cross section double-differential in energy loss E and momentum transfer $\boldsymbol{q}$ for inelastic scattering on free atoms
- use first Born approximation

$$\frac{\mathrm{d}\sigma(E,Q)}{\mathrm{d}E\mathrm{d}Q} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}E}\right)^{*}_{\mathrm{Mott}} \cdot \frac{E^2}{Q^2} \cdot |F(\mathbf{q})|^2 \qquad Q = \frac{q^2}{2m}$$

with $$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}E}\right)^{\star}_{\mathrm{Mott}} = \frac{2\pi}{m}\left(\frac{z\alpha\hbar}{E\beta}\right)^2 \left[1-\beta^2\frac{E}{T_{\mathrm{max}}}\right]$$

Fano, 1963:

[U. Fano, Ann. Rev. Nucl. Sci. 13, 1 (1963)]

- extend method for solids
- no calculations exist for gases

Bethe – Fano Model

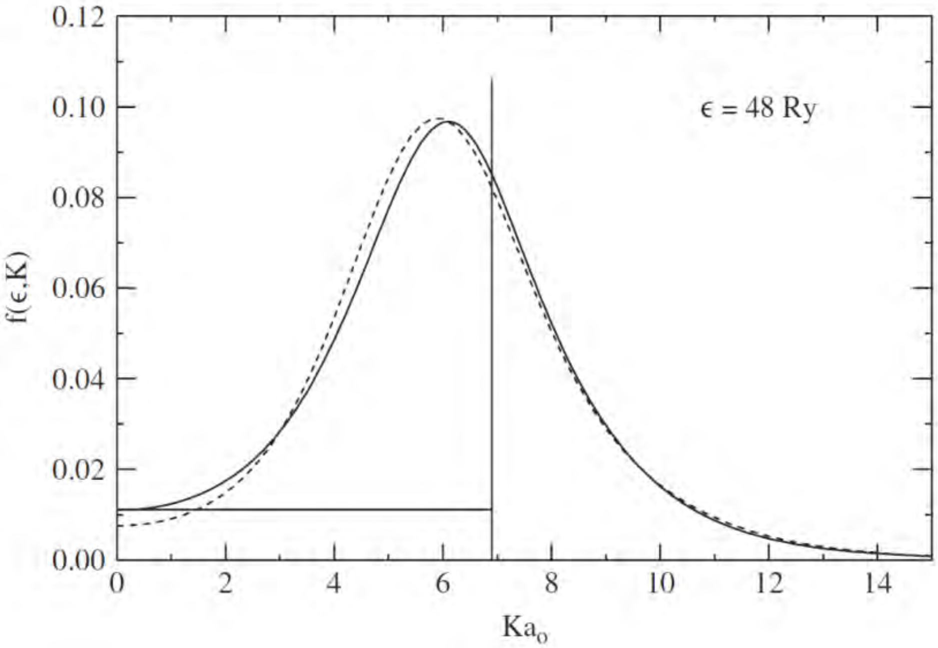


Fig. 4. Generalized oscillator strength (GOS) for Si for an energy transfer $\varepsilon = 48Ry$ $(Ry = 13.6\,\mathrm{eV})$ to the 2p-shell electrons [18]. Solid line: calculated with Herman–Skilman potential, dashed line: hydrogenic approximation. The horizontal and vertical line define the FVP approximation (Section 2.3).

$$f_n(E,k) = \frac{E}{Q} \cdot |F_n(k)|^2$$

[H. Bichsel, NIM A 562, 154 (2006)]

Bethe – Fano Model

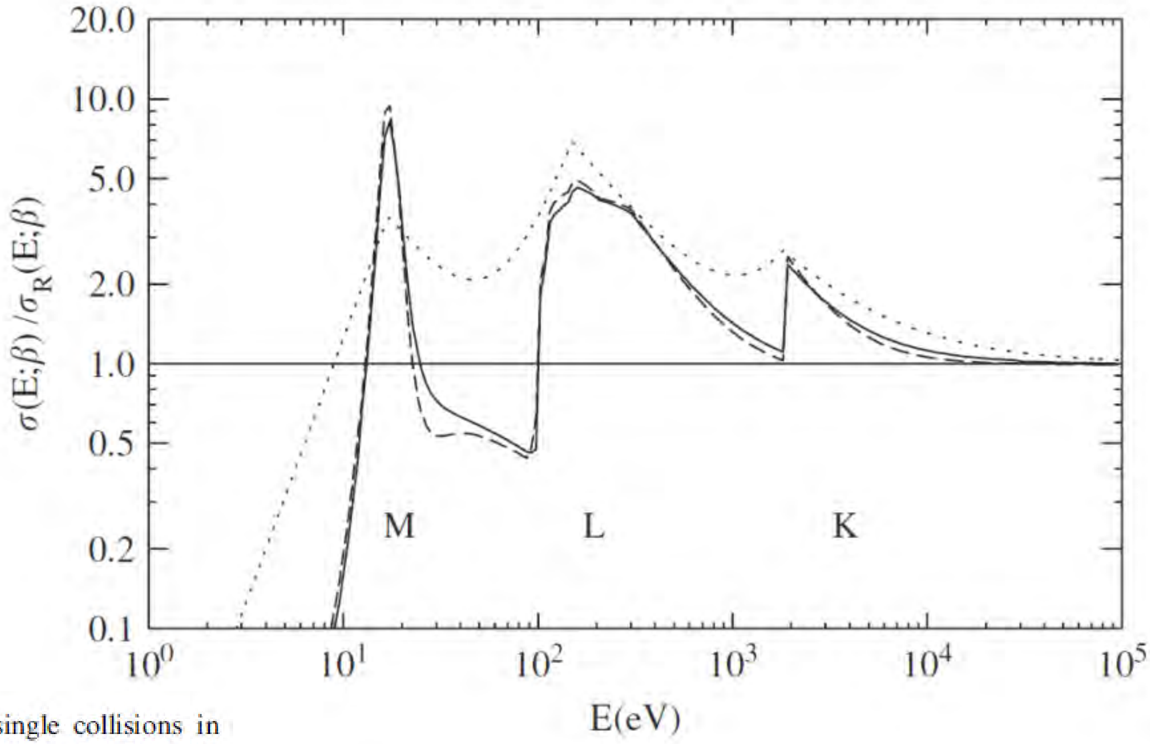


Fig. 5. Inelastic collision cross-sections $\sigma(E;\beta)$ for single collisions in silicon by particles with $\beta\gamma = 4$, calculated with different theories. In order to show the structure of the functions clearly, the ordinate is $\sigma(E;\beta)/\sigma_R(E;\beta)$. The abscissa is the energy loss E in a single collision. The Rutherford cross-section Eq. (1) is represented by the horizontal line at 1.0. The solid line was obtained with the relativistic version of Eq. (5) of the Bethe–Fano theory [18]. The cross-section calculated with FVP (Eq. (7)) is shown by the dashed line. The dashed line is calculated with a binary encounter approximation [35,36]. The functions all extend to E_{max}~16 MeV; see Eq. (1). The moments (Section 3) are $M_0 =$ 4 collisions/μm and $M_1 = 386$ eV/μm. The atomic shells are indicated by the letters M, L, K.

[H. Bichsel, NIM A 562, 154 (2006)]

Total Energy Loss

Total energy loss: Bethe-Bloch formula

$$\left\langle -\frac{dE}{dx} \right\rangle = \left\langle -\frac{dE}{dx} \right\rangle_{T>\varepsilon} + \left\langle -\frac{dE}{dx} \right\rangle_{T<\varepsilon} = n_e \frac{4\pi}{m} \left(\frac{z\alpha\hbar}{\beta} \right)^2 \left[\frac{1}{2} \ln \frac{2mc^2\beta^2\gamma^2 T_{\max}}{I^2} - \beta^2 \right]$$

Hard: Mott

Soft

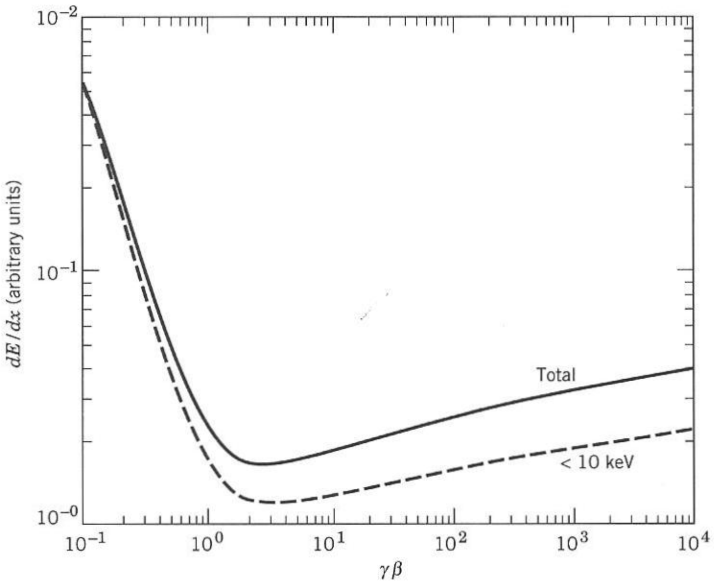
$$n_e = \frac{Z}{A} \cdot N_A \cdot \rho$$

with

independent of ε

$$\left\langle -\frac{dE}{dx} \right\rangle_{T>\varepsilon} = n_e \frac{2\pi}{m} \left(\frac{z\alpha\hbar}{\beta} \right)^2 \left[\ln \frac{T_{\max}}{\varepsilon} - \beta^2 \right]$$

$$\left\langle -\frac{dE}{dx} \right\rangle_{T<\varepsilon} = n_e \frac{2\pi}{m} \left(\frac{z\alpha\hbar}{\beta} \right)^2 \left[\ln \frac{2mc^2\beta^2\gamma^2\varepsilon}{I^2} - \beta^2 \right]$$



Total Energy Loss

In principle, mean excitation energy I can be calculated from atomic theory:

$$Z \cdot \ln(I) \propto \sum_n f_n \ln(\hbar\omega_n)$$

⇨ models needed for all but lightest atoms

⇨ often used in practice: I as phenomenological constant

Goal: Simplify cross section expression based on measured photo-absorption cross sections

⇨ Photoabsorption Ionization Model

… also called Fermi virtual photon (FVP) model

Classical Calculation of Energy Loss

Idea: Calculate $\langle \mathrm{d}E/\mathrm{d}x \rangle$ of a moving charged particle (other than $e^{\pm}$) in a polarizable medium

- ⇨ classical calculation: medium treated as continuum with $\varepsilon = \varepsilon_1 + \mathrm{i}\varepsilon_2$
- ⇨ later: quantum mechanical interpretation

$\langle \mathrm{d}E/\mathrm{d}x \rangle \Leftrightarrow$ longitudinal component of electric field $\boldsymbol{E}(\boldsymbol{r},t)$ generated by the moving particle in the medium at its own position $\boldsymbol{r} = \boldsymbol{v}t$

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = eE_{\mathrm{long}}$$

[L. Landau, E.M. Lifshitz, Electrodynamics of continuous media, 1960]

[W.W.M Allison, J.H. Cobb, Ann. Rev. Nucl. Part. Sci. 30, 253 (1980)]

[W. Blum, W. Riegler, L. Rolandi, Particle Detection with Drift Chambers, Springer 2008]

Classical Calculation of Energy Loss

Solve Maxwell equations for isotropic, homogeneous medium with

$$\rho(\boldsymbol{r},t) = e\delta^3(\boldsymbol{r}-\boldsymbol{v}t), \quad \boldsymbol{j} = \rho(\boldsymbol{r},t)\cdot\boldsymbol{v}(\boldsymbol{r},t)$$

Work in Coulomb gauge, solution by Fourier transform

$$\boldsymbol{E}(\boldsymbol{r},t) = \frac{1}{(2\pi)^2}\int \mathrm{d}^3k\,\mathrm{d}\omega\left[i\omega\tilde{\boldsymbol{A}}(\boldsymbol{k},\omega) - i\boldsymbol{k}\tilde{\varphi}(\boldsymbol{k},\omega)\right]e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$$

with Fourier transforms of $\boldsymbol{A}(\boldsymbol{r},t), \varphi(\boldsymbol{r},t)$

$$\tilde{\varphi}(\boldsymbol{k},\omega) = \frac{e}{2\pi\varepsilon_0\varepsilon k^2}\delta(\omega-\boldsymbol{k}\cdot\boldsymbol{v})$$

$$\tilde{\boldsymbol{A}}(\boldsymbol{k},\omega) = \frac{e}{2\pi\varepsilon_0 c^2}\frac{\omega\boldsymbol{k}/k^2-\boldsymbol{v}}{\left(-k^2+\varepsilon\omega^2/c^2\right)}\delta(\omega-\boldsymbol{k}\cdot\boldsymbol{v})$$

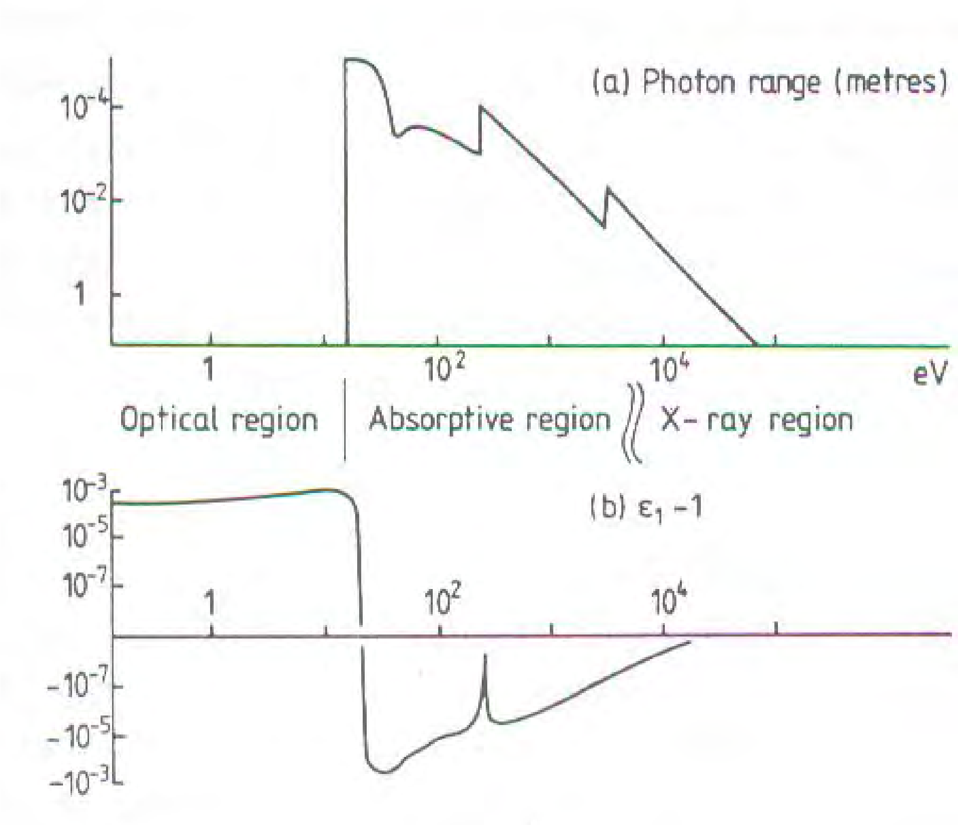
Classical Calculation of Energy Loss

⇨ Mean energy loss per unit path length:

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = e\boldsymbol{E}\left(\boldsymbol{v}t, t\right) \cdot \frac{\boldsymbol{v}}{v}$$

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = -\frac{2e^2}{4\pi\varepsilon_0\beta^2\pi} \int_0^\infty \mathrm{d}\omega \int_{\omega/\beta c}^{\infty} \mathrm{d}k \left[\omega k \left(\beta^2 - \frac{\omega^2}{k^2c^2} \right) \mathrm{Im}\left(\frac{1}{-k^2c^2 + \varepsilon\omega^2} \right) - \frac{\omega}{kc^2} \mathrm{Im}\left(\frac{1}{\varepsilon} \right) \right]$$

- Integration over direction of k assuming isotropic medium
- Time dependence drops out, because field in the medium is travelling with the particle
- Use $\varepsilon(-\omega) = \varepsilon^*(\omega)$ to combine positive and negative ω
- Lower limit for integration over k corresponds to minimum momentum transfer for a given energy transfer $\hbar\omega$
- Energy loss determined by $\varepsilon(\boldsymbol{k}, \omega)$ ⇨ atomic structure of medium



[W.W.M. Allison, P.R.S. Wright, in: Experimental Techniques in High Energy Physics, ed. T. Ferbel, Addison-Wesley (1987)]

optical region → Cherenkov-Rad. (β>1/n)

abs. region → dE/dx

X-ray region → dE/dx (Tail)
Transition Radiation

PAI Model

Photo-absorption ionization model: [W.W.M Allison, J.H. Cobb, Ann. Rev. Nucl. Part. Sci. 30, 253 (1980)]

Model of $\varepsilon(\mathbf{k},\omega)$ based on measured photo-absorption cross section $\sigma_\gamma(\omega)$

Plane light-wave travelling along x in medium (real photons):

$$k = \frac{\omega}{c}\sqrt{\varepsilon}\,, \quad \varepsilon = \varepsilon_1 + i\varepsilon_2 \quad \Rightarrow \quad I = I_0 e^{-\alpha x}\,, \quad \alpha = 2\frac{\omega}{c}\operatorname{Im}\sqrt{\varepsilon}$$

Relation to photo-absorption cross section for free (real) photons:

$$\alpha = \sigma_\gamma n = \sigma_\gamma \frac{N}{Z} \quad \Rightarrow \quad \sigma_\gamma = \frac{Z\omega}{Nc}\frac{\varepsilon_2}{\sqrt{\varepsilon_1}} \approx \frac{Z\omega}{Nc}\varepsilon_2(\omega)$$

$|\varepsilon_2| \ll |\varepsilon_1|$ $\qquad$ $\varepsilon_1 \approx 1$

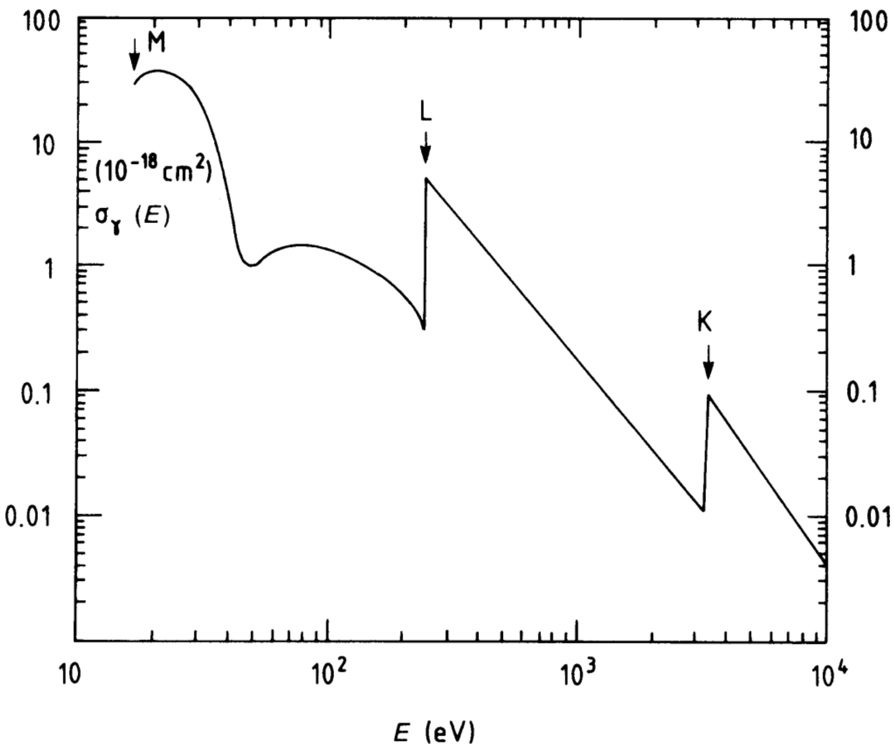
n = density of atoms
N = density of electrons
Z = atomic charge

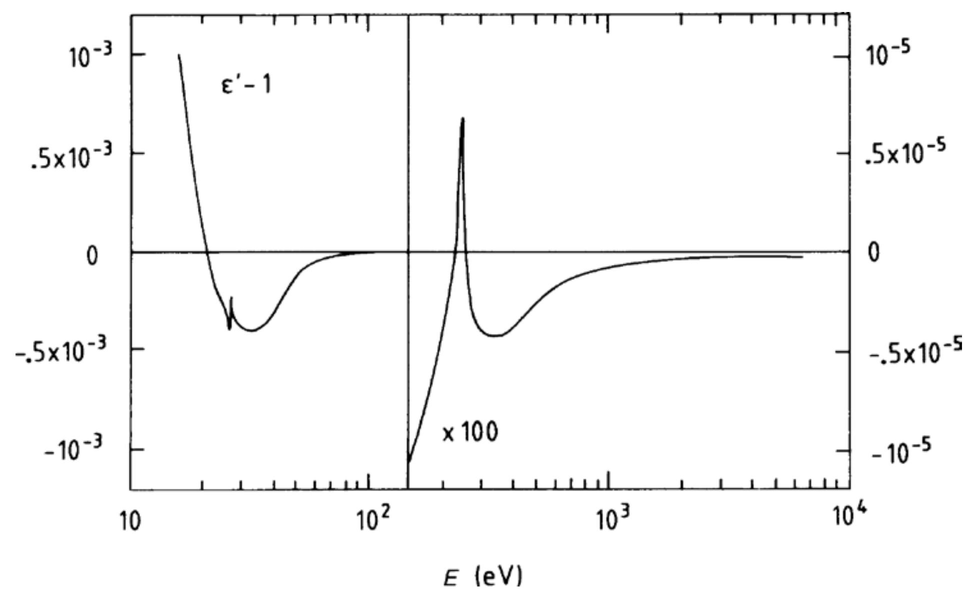
Cross section $\sigma_\gamma(\omega)$, and therefore $\varepsilon_2(\omega)$ is known, e.g. from measurements with synchrotron radiation

Real part $\varepsilon_1(\omega)$ from Kramers-Kronig relation: $\varepsilon_1(\omega) - 1 = \frac{2}{\pi}\mathrm{P}\int_0^\infty \frac{x\,\varepsilon_2(x)}{x^2-\omega^2}\mathrm{d}x$

Cauchy principal value (→ P)

Example: Argon





Total photo-absorption cross section

Real part of ε, calculated from σ_γ using Kramers-Kronig relation

[G.V. Marr, J.B. West, At. Data and Nucl. Data Tables 18, 497 (1976)]

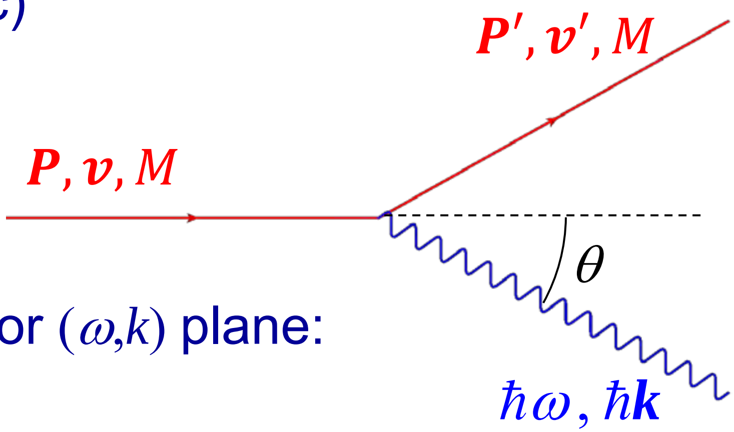
[F. Lapique et al., Nucl. Instr. Meth. 175, 297 (1978)]

PAI Model

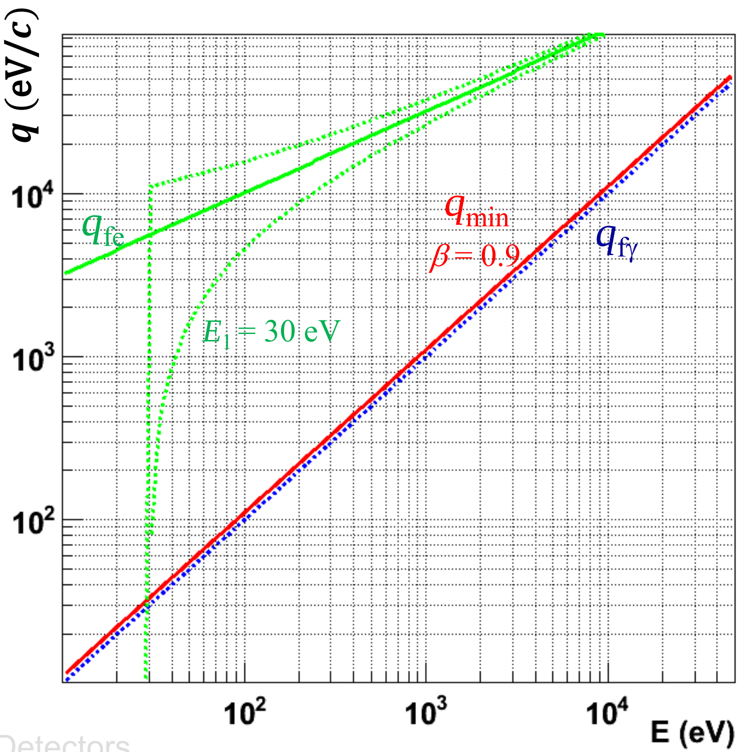
Charged particle traversing medium (dielectric)

⇨ interaction via virtual photon exchange

$$E = \hbar\omega, \quad \boldsymbol{q} = \hbar\boldsymbol{k}$$



Virtual photons: kinematic constraints in (E,q) or (ω,k) plane:



Free photons in vacuum: $q_{\rm f\gamma} = E/c$

Minimum momentum transfer: $q_{\rm min} = E/(\beta c)$

Collision with free electron at rest: $q_{\rm fe} = \sqrt{2mE}$

Collision with bound electron

($B.E. = E_1$, momentum $p_1 \sim \sqrt{2mE_1}$):

$$E = E_1 + (\boldsymbol{q} + \boldsymbol{p}_1)^2/(2m)$$

⇨ smeared band around $q_{\rm fe}$

PAI Model

Experiment: $\varepsilon = \varepsilon_1 + i\varepsilon_2$ known only for free photons, i.e. on $q_{f\gamma}$ line

PAI model: extend into the kinematic domain of virtual photons

- Below free-electron line q_{fe} (resonance region): dipole approximation

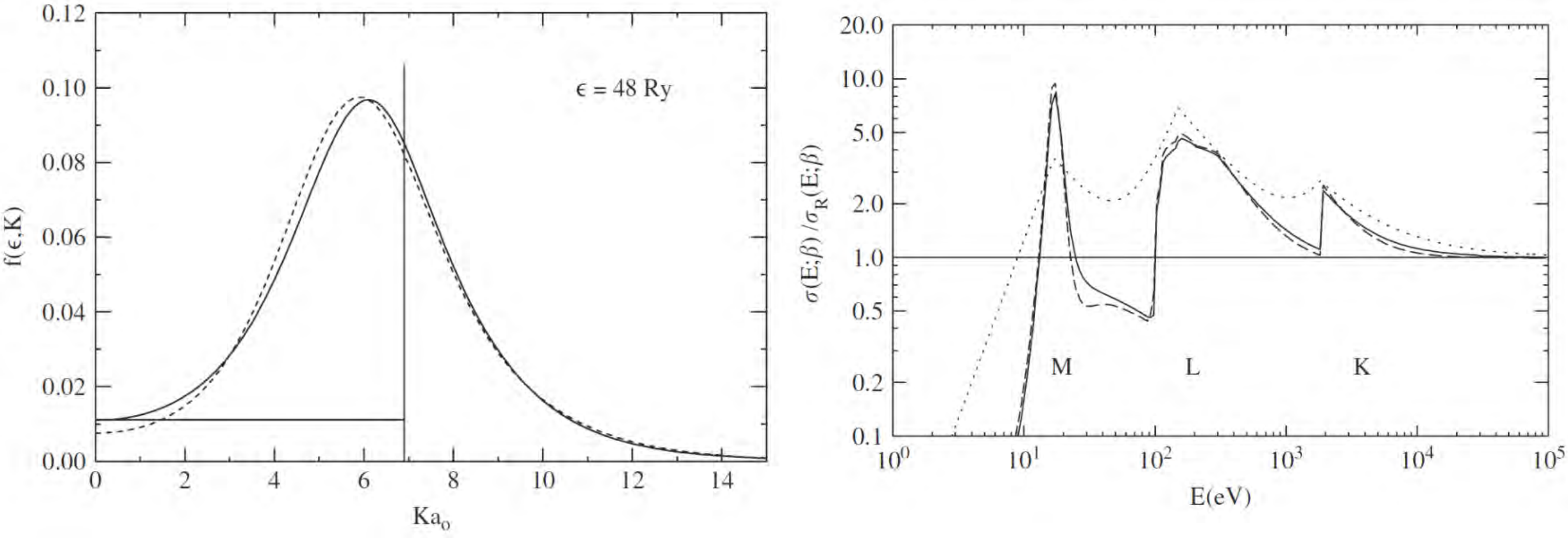
  $\varepsilon(k,\omega) = \varepsilon(\omega)$ independent of k, as for free photons

- On free-electron line q_{fe} :

  $$\varepsilon_2(k,\omega) = C\,\delta\left(\omega - \hbar k^2/(2m)\right), \quad \varepsilon_1 = 1$$

  Normalization C chosen such that total coupling strength satisfies

  $$\int_0^\infty f(k,\omega)\,\mathrm{d}\omega = 1, \quad \varepsilon_2(k,\omega) = \frac{\pi N e^2}{2\varepsilon_0 m\omega} f(k,\omega)$$ Bethe sum rule



[H. Bichsel, NIM A 562, 154 (2006)]

Optical dipole oscillator strength

$$\lim_{\boldsymbol{q}\to 0} f_n(E,k) = f_n(E)$$

$$f_n(E) = \frac{E}{Q}\left|\langle n|\sum_{i=1}^{Z}\boldsymbol{r_i}|0\rangle\right|$$

PAI Model

Integration over k:

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = -\frac{e^2}{4\pi\varepsilon_0 \beta^2 c^2 \pi} \int_0^\infty \mathrm{d}\omega \left[\frac{Nc}{Z} \sigma_\gamma(\omega) \ln\left[\left(1-\beta^2\varepsilon_1\right)^2 + \beta^4 \varepsilon_2^2 \right]^{-1/2} \right.$$

$$\left. + \frac{Nc}{Z} \sigma_\gamma(\omega) \ln\left(\frac{2mc^2\beta^2}{\hbar\omega} \right) + \frac{Nc}{Z\omega} \int_0^\omega \sigma_\gamma(\omega') \mathrm{d}\omega' + \omega \left(\beta^2 - \frac{\varepsilon_1}{|\varepsilon|^2} \right) \Theta \right]$$

N electron density

$E = \hbar\omega$ energy transfer in single collision

$q = \hbar k$

$\Theta = \arg\left(1 - \varepsilon_1 \beta^2 + i\varepsilon_2 \beta^2\right)$

Energy loss per unit path length obtained in the framework of electrodynamics of a continuous medium, using a model for $\varepsilon(k,\omega)$ inspired by a picture of photon collision and absorption.

PAI Model

Quantum picture: energy loss caused by a number of discrete collisions per unit length, each with energy transfer $E = \hbar\omega$ (single photon exchange)

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = -\int_0^\infty E\, f(E)\, \mathrm{d}E$$

$f(E)\,\mathrm{d}E$ probability of energy transfer per unit path between E and E+dE

and with $f(E) = N\,\mathrm{d}\sigma/\mathrm{d}E$

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = -\int_0^\infty E N \frac{\mathrm{d}\sigma}{\mathrm{d}E} \hbar\, \mathrm{d}\omega$$

N electron density

$E = \hbar\omega$ energy transfer in single collision

$q = \hbar k$

Therefore: differential cross section per electron

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E} = \frac{\alpha}{\beta^2\pi}\frac{\sigma_\gamma(E)}{EZ}\ln\left[\left(1-\beta^2\varepsilon_1\right)^2+\beta^4\varepsilon_2^2\right]^{-1/2}$$
$$+\frac{\alpha}{\beta^2\pi}\frac{\sigma_\gamma(E)}{EZ}\ln\left(\frac{2mc^2\beta^2}{E}\right)$$
$$+\frac{\alpha}{\beta^2\pi}\frac{1}{E^2Z}\int_0^E\sigma_\gamma(E')\mathrm{d}E'$$
$$+\frac{\alpha}{\beta^2\pi}\frac{1}{N\hbar c}\left(\beta^2-\frac{\varepsilon_1}{|\varepsilon|^2}\right)\Theta$$

Energy loss by ionization (first two terms)

Rutherford scattering (for $E >> E_K$) ⇨ δ electrons

- Optical region: $\sigma_\gamma = 0$ ⇨ Cherenkov radiation
- Transition radiation for thin radiators

with $\varepsilon_1, \varepsilon_2$: real and imaginary part of dielectric constant (for real photons)

$\Theta = \arg\left(1-\varepsilon_1\beta^2+i\varepsilon_2\beta^2\right)$ angle in pointer representation of complex number

σ_γ: atomic cross section of medium for absorption of photon with energy E

N: electron density in the medium

Energy Loss by Ionization

Described by first three terms of $\frac{\mathrm{d}\sigma}{\mathrm{d}E}$

- Large energy transfers $E \gg E_K$ ⇨ only third term survives ($\sigma_\gamma(E)$ small)

$$\frac{\alpha}{\beta^2\pi}\frac{1}{E^2}\int_0^E \frac{\sigma_\gamma(E')}{Z}\mathrm{d}E' \xrightarrow[\text{Bethe sum rule}]{E \gg E_K} \left(\frac{\mathrm{d}\sigma}{\mathrm{d}E}\right)_{\mathrm{R}} = \frac{2\pi r_e^2 mc^2}{\beta^2 E^2}, \quad r_e = \frac{e^2}{4\pi\varepsilon_0 mc^2}$$

Rutherford cross section: elastic scattering on free electron

⇨ extremely long tail of energy loss distribution due to δ electrons

⇨ ill-defined average energy loss! (log. divergence)

⇨ better: most probable value

⇨ in practice: upper limit for E depending on detector: restricted energy loss

Energy Loss by Ionization

- Remaining two terms: $\frac{d\sigma}{dE} = \frac{a}{\beta^2}\left[b + \ln \frac{\beta^2}{\left[\left(1-\beta^2\varepsilon_1\right)^2 + \beta^4\varepsilon_2^2\right]^{1/2}} \right]$

  $a, b = f(E, \sigma_\gamma)$

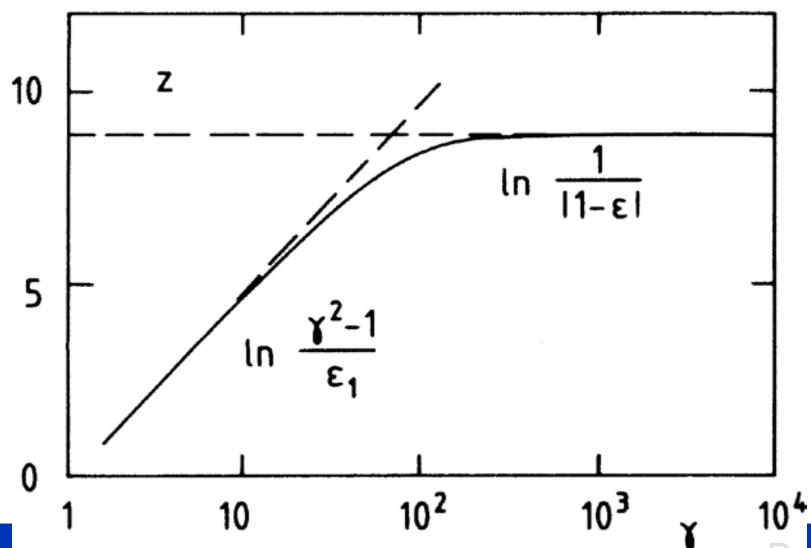
  – small β: factor $\frac{1}{\beta^2}$ dominates

  – $\beta \to 1$: logarithmic term dominates $\to$

$$\begin{cases} \ln \frac{\gamma^2 - 1}{\varepsilon_1} & \text{for} \quad \gamma^2 \ll 1/\left|1-\varepsilon\right| \\ \ln \frac{1}{\left|1-\varepsilon\right|} & \text{for} \quad \gamma^2 \gg 1/\left|1-\varepsilon\right| \end{cases}$$

relativistic rise

plateau



Plateau due to density of medium!

$\varepsilon_1 - 1, \varepsilon_2 \propto N$ e[-] density

$\Rightarrow \varepsilon_1 = 1, \varepsilon_2 = 0 \quad \text{for} \quad N \to 0$

$\Rightarrow \frac{d\sigma}{dE}$ continues to rise for $N \to 0$!

Cherenkov Radiation

Term $\dfrac{\alpha}{\beta^2\pi}\dfrac{1}{N\hbar c}\left(\beta^2-\dfrac{\varepsilon_1}{|\varepsilon|^2}\right)\Theta\,, \quad \Theta=\arg\left(1-\varepsilon_1\beta^2+i\varepsilon_2\beta^2\right)$

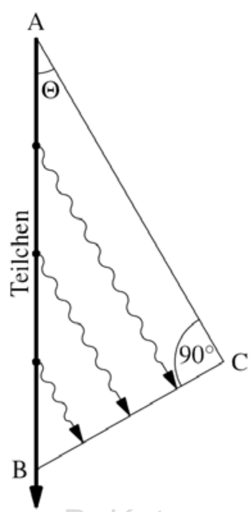
- Only remaining term for photon energies below excitation energy of atom (optical region), where $\sigma_\gamma=0\,,\varepsilon_2=0\,,\varepsilon=\varepsilon_1$
- $\Theta=\arg\left(1-\varepsilon_1\beta^2\right)$ jumps from 0 to π at $\beta_0^2=\dfrac{1}{\varepsilon_1}$ Cherenkov threshold

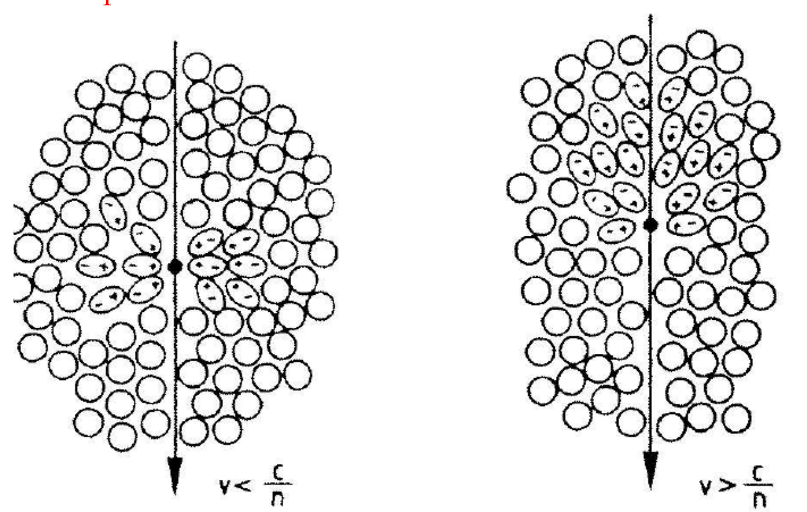
⇨ Emission of radiation if

$$\varepsilon\beta^2>1 \Leftrightarrow v>\frac{c}{n}\,, \quad n=\sqrt{\varepsilon}$$

Emission angle:

$$\cos\theta=\frac{1}{\beta n}$$





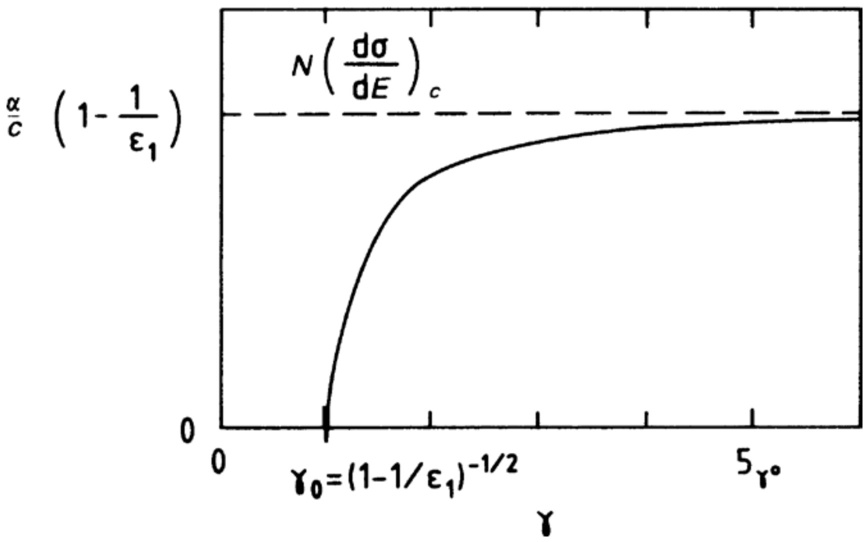
destructive constructive

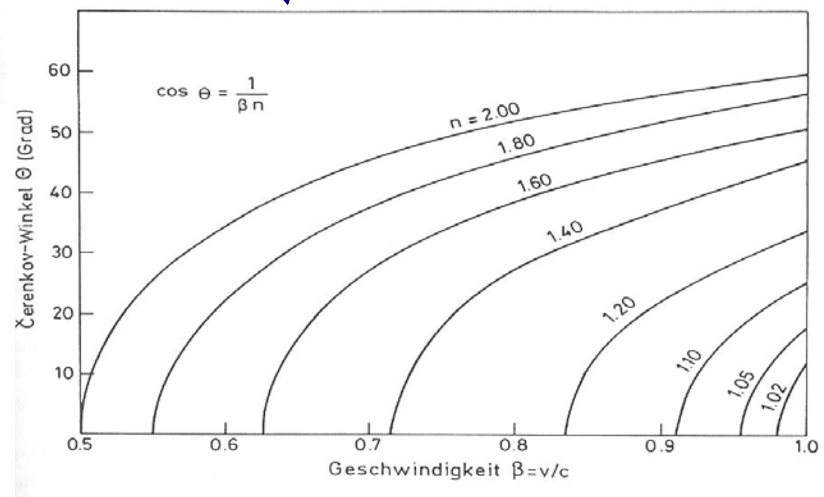
interference

Cherenkov Radiation

- Photon flux per interval of photon energy per unit path length (above thr.)

$$N\frac{\mathrm{d}\sigma}{\mathrm{d}E}=\frac{\mathrm{d}^2N_\gamma}{\hbar\,\mathrm{d}\omega\,\mathrm{d}x}=\frac{\alpha}{\hbar c}\left(1-\frac{1}{\beta^2\varepsilon}\right)=\frac{\alpha}{\hbar c}\sin^2\theta_C\,,\quad \cos\theta_C=\frac{1}{\beta\sqrt{\varepsilon}}$$





- Photon flux per interval of photon energy emitted in $\mathrm{d}\Omega$:

$$\frac{\mathrm{d}^2N_\gamma}{\mathrm{d}\omega\,\mathrm{d}\Omega}=\frac{\alpha}{2\pi c}\sin^2\theta\cdot\delta\left(\cos\theta-\frac{1}{\beta\sqrt{\varepsilon}}\right)\cdot L$$

$$\Leftrightarrow\int\frac{\mathrm{d}^2N_\gamma}{\mathrm{d}\omega\,\mathrm{d}\Omega}\mathrm{d}\cos\theta\,\mathrm{d}\varphi=\frac{\alpha}{c}\sin^2\theta\cdot L\quad\text{with}\quad\cos\theta=\frac{1}{\beta\sqrt{\varepsilon}}$$

Transition Radiation

Thin radiator (L small)

⇨ diffraction, i.e. broadening of Cherenkov emission

⇨ interference of Cherenkov emission at both boundary surfaces

Photon flux of X-ray transition radiation for small angles θ_0 and $\beta \sim 1$:

$$\frac{d^2 N_\gamma}{d\omega\, d\Omega} = \frac{\alpha}{\pi^2 \omega}\theta_0^2 \cdot 4\sin^2\left[\frac{\omega L}{4c}\left(\frac{\omega_p^2}{\omega} + \theta_0^2 + \frac{1}{\gamma^2}\right)\right] \cdot \left[\frac{1}{1/\gamma^2 + \omega_p^2/\omega^2 + \theta_0^2} - \frac{1}{1/\gamma^2 + \theta_0^2}\right]^2$$

$$\omega_p = \left(\frac{nZe^2}{\varepsilon_0 m}\right)^{1/2}$$

⇨ Maximum at $\theta_0 \sim 1/\gamma$

Neglect interference term, integrate over $d\Omega$:

$$\frac{dN_\gamma}{d\omega} \approx \frac{2\alpha}{\pi\omega}\ln\left(\frac{\gamma\omega_p}{\omega}\right) \quad \text{for} \quad \omega \ll \gamma\omega_p$$

⇨ Total energy flux $\frac{\alpha}{3}\gamma\hbar\omega_p$

[W.W.M.Allison, P.R.S. Wright, in: Experimental Techniques in High-Energy Nuclear and Particle Physics, T. Ferbel ed., 1999]

Collision Cross Section

$$\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x}\right\rangle = n_e \int_0^{\infty} E' \frac{\mathrm{d}\sigma}{\mathrm{d}E'}\mathrm{d}E'$$

Models:

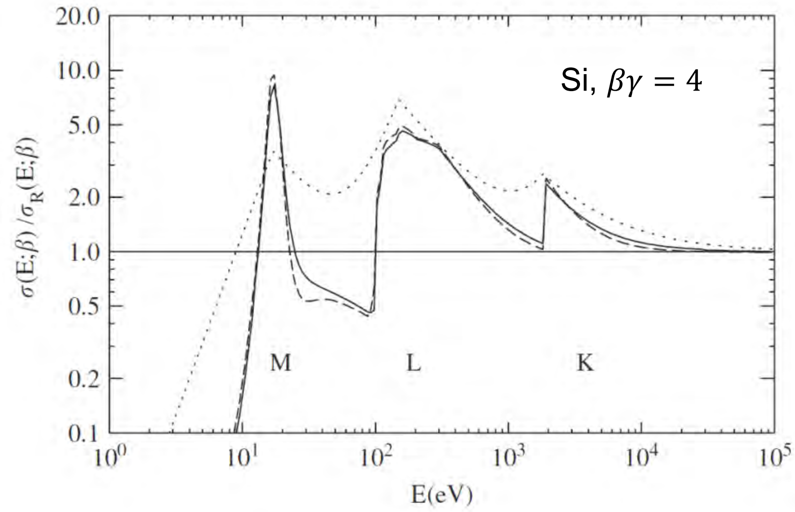
- Rutherford – Mott

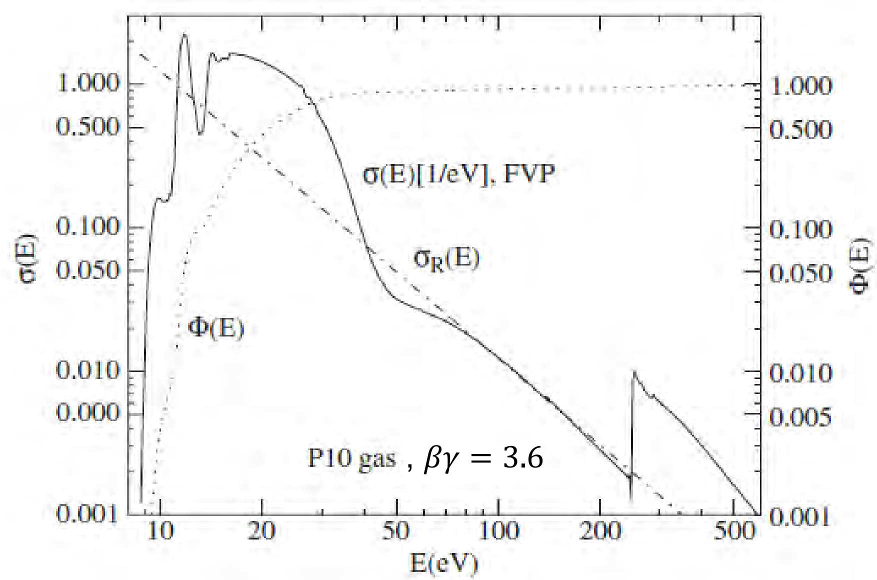
$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}E}\right)^{\star}_{\mathrm{Mott}} = \frac{2\pi}{m}\left(\frac{z\alpha\hbar}{E\beta}\right)^2 \left[1-\frac{E}{2mc^2}\left(1-\beta^2\right)\right]$$

- Bethe – Fano

$$\frac{\mathrm{d}\sigma(E,Q)}{\mathrm{d}E\mathrm{d}Q} = \left(\frac{\mathrm{d}\sigma(E;v)}{\mathrm{d}E}\right)^{*}_{\mathrm{Mott}} \cdot \frac{E}{Q} \cdot f(k,\omega)$$

- PAI (FVP)





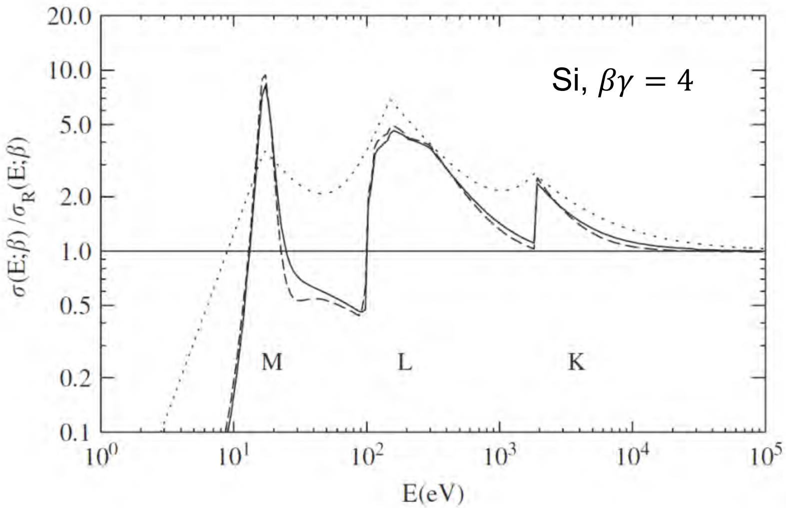
[H. Bichsel, NIM A 562, 154 (2006)]

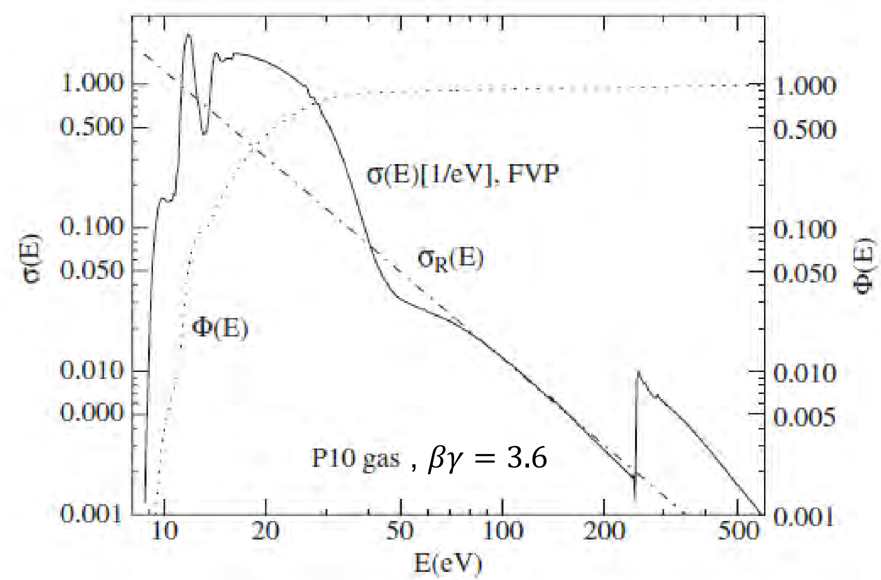
$$\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = n_e \int_0^\infty E' \frac{\mathrm{d}\sigma}{\mathrm{d}E'} \mathrm{d}E'$$

Models:

- PAI (FVP)

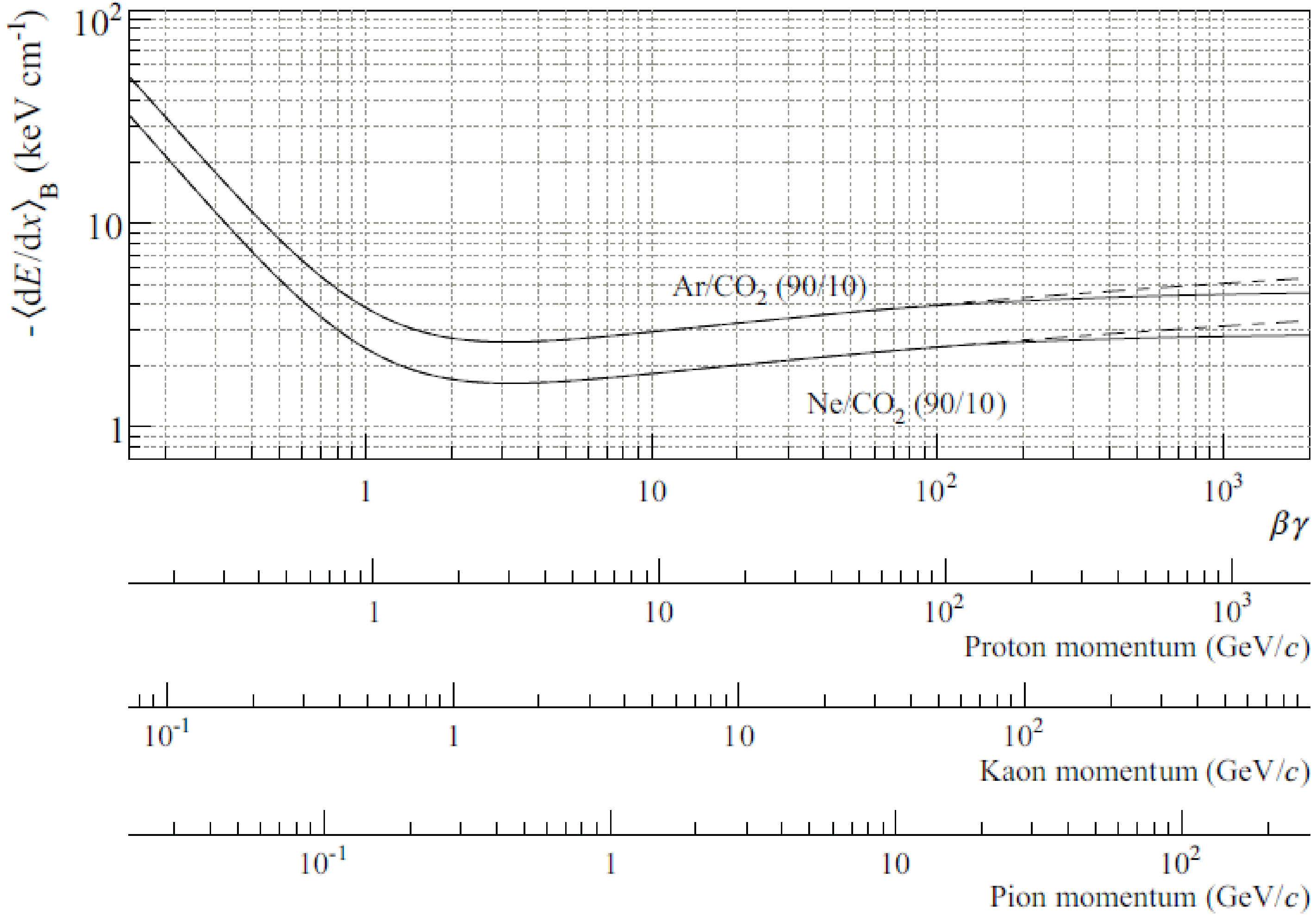
$$\frac{\mathrm{d}\sigma}{\mathrm{d}E} = \frac{\alpha}{\beta^2\pi} \frac{\sigma_\gamma(E)}{EZ} \ln\left[(1-\beta^2\varepsilon_1)^2 + \beta^4\varepsilon_2^2\right]^{-1/2}$$
$$+ \frac{\alpha}{\beta^2\pi} \frac{\sigma_\gamma(E)}{EZ} \ln\left(\frac{2mc^2\beta^2}{E}\right)$$
$$+ \frac{\alpha}{\beta^2\pi} \frac{1}{E^2 Z} \int_0^E \sigma_\gamma(E')\mathrm{d}E'$$
$$+ \frac{\alpha}{\beta^2\pi} \frac{1}{N\hbar c} \left(\beta^2 - \frac{\varepsilon_1}{|\varepsilon|^2}\right) \Theta$$





[H. Bichsel, NIM A 562, 154 (2006)]

Mean Energy Loss



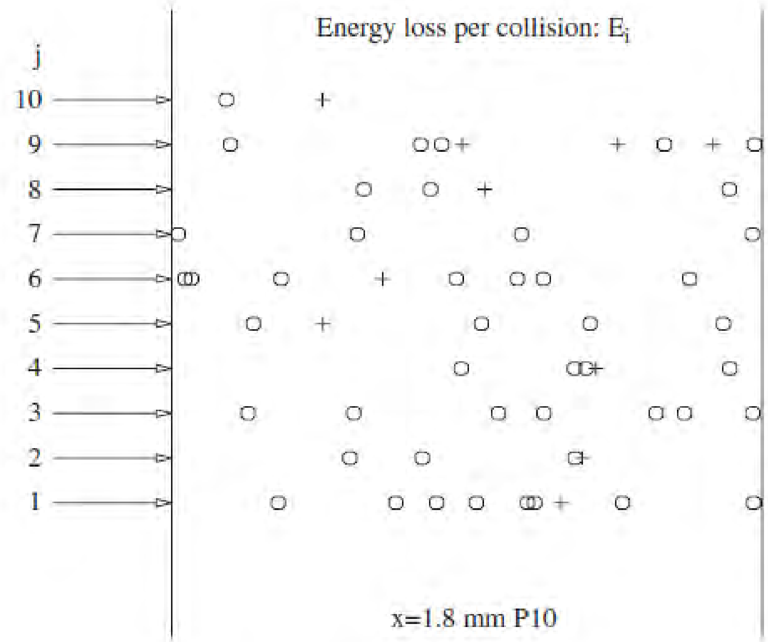
[F. Böhmer, PhD thesis, TUM]

Consider single particle: statistical fluctuations of

- number of collisions
- energy transfer in each collision

⇨ range straggling (if stopped in medium)

⇨ energy straggling (if traversing medium)



n_j	Δ_j(eV)	E_t(eV)
2	56	37
8	703	559
4	126	68
4	82	32
8	419	292
5	565	502
5	95	35
7	146	26
4	105	34
9	930	774

$\Delta = \Sigma E_i$

Fig. 3. Monte Carlo simulation of the passage of 10 particles (index j) with $\beta\gamma = 3.6$ through segments of P10 gas. The thickness of the gas layer (at 1 atm and 25 °C) is $x = 1.8$ mm. The direction of travel is given by the arrows. Inside the gas, the tracks are defined by the symbols showing the location of a collision. The mean free path between collisions is $\lambda = 0.3$ mm (see Fig. 7 or Table 2), thus the *average* number of collisions per track is six. At each collision point a random energy loss E_i is selected from the distribution function $\Phi(E;\beta\gamma)$, Fig. 9. Two symbols are used to represent energy losses: ∘ for $E_i < 33$ eV, + for $E_i > 33$ eV; the mean free path between collisions with $E_i > 33$ eV is 2 mm. Segment statistics are shown to the right: the total number of collisions for each track is given by n_j, with a nominal mean value $\langle n \rangle = x/\lambda = 6$ and the total energy loss is $\Delta_j = \sum E_i$, with the nominal mean value $\langle \Delta \rangle = x\,dE/dx = 440$ eV, where dE/dx is the Bethe–Bloch *stopping power*, M_1 in Table 2. The largest energy loss E_t on each track is also given. The mean value of the Δ_j is 325 ± 314 eV, much less than $\langle \Delta \rangle$. Note that the largest possible energy loss in a single collision is $E_{max} = 13$ MeV, while the probability for $E > 50,000$ eV is 0.002 per cm, Eq. (12) or Figs. 9 and 10.

[H. Bichsel, NIM A 562, 154 (2006)]

Fluctuations of Energy Loss

Important quantity in order to understand response of detector:

$f(\Delta; x)$ probability density function for energy loss Δ in material of thickness x,

determined by

- collision cross section $\mathrm{d}\sigma/\mathrm{d}E$
- $n_e x$

Straggling functions

Calculation of energy loss distribution: two approaches

- Convolution method
- Laplace transform method

[Allison, Cobb, Ann. Rev. Nucl. Part. Sc., 253 (1980)]

[H. Bichsel, NIM A 562, 154 (2006)]

2.3.1 Convolution Method

In each collision, the probability to transfer an energy E is given by

$$F(E) = \lambda n_e \frac{\mathrm{d}\sigma(E;\beta)}{\mathrm{d}E} = \frac{1}{\sigma}\frac{\mathrm{d}\sigma(E;\beta)}{\mathrm{d}E}$$

Energy loss Δ for exactly N_c collisions ⇨ N_c-fold convolution of $F(E)$

$$\tilde{F}_{N_c}(\Delta) = \int_0^{\Delta} \tilde{F}_1(E) \cdot \tilde{F}_{N_c-1}(\Delta - E)\,\mathrm{d}E$$

with $\tilde{F}_0(\Delta) = \delta(\Delta)$ and $\tilde{F}_1(\Delta) = \frac{1}{\sigma}\frac{\mathrm{d}\sigma(\Delta;\beta)}{\mathrm{d}E} = F(\Delta)$

Convolution Method

Number of collisions N_c in layer of thickness x

$$P(N_c; m_c) = \frac{m_c^{N_c}}{N_c!} \exp\left(-m_c\right) \qquad m_c = \frac{x}{\lambda}$$

⇨ Linked to CCS through mean free path

$$\lambda = \lambda(\beta) = \frac{1}{n_e \sigma} \qquad \sigma = \int_0^\infty \frac{\mathrm{d}\sigma(E';\beta)}{\mathrm{d}E'} \mathrm{d}E'$$

Mean value $\langle P(N_c; m_c) \rangle = m_c$

Standard deviation $s_c = \sqrt{m_c}$

Relative width $s_c / m_c = 1/\sqrt{m_c}$

Convolution Method

⇨ Pdf for total ionization energy loss Δ in material slice of thickness x = sum of all $\tilde{F}_{N_c}(\Delta)$, weighted by their Poissonian probability for exactly N_c collisions

$$f(\Delta; x) = \sum_{N_c=0}^{\infty} P(N_c; m_c)\, \tilde{F}_{N_c}(\Delta)$$

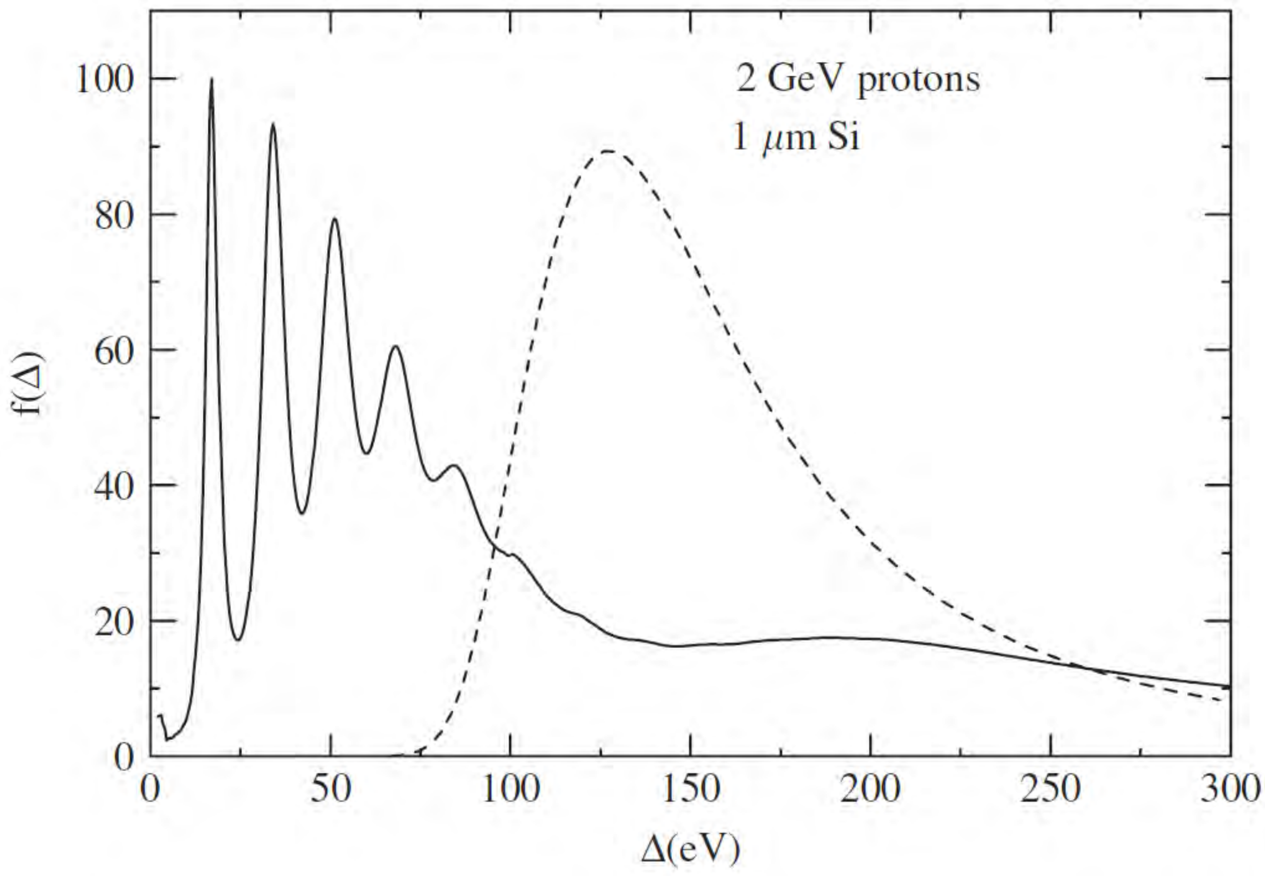
Straggling functions

- Poissonian contribution dominant for very small number of collisions (very thin absorbers)
- Peak structure vanishing for larger N_c

Convolution Method

Solution for thickness x:

- Iterative application of convolution integral (numerical)
  [Bichsel et al., Phys. Rev. A 11, 1286 (1975)]
- Monte-Carlo method [Cobb et al., Nucl. Instr. Meth. 133, 315 (1976)]
  - calculate mean number of collisions m_c from integrated cross section
  - for each trial (particle penetration) choose actual number of collisions from Poisson distribution with mean m_c
  - total energy loss = sum of energy losses in single collisions, taken from normalized $\mathrm{d}\sigma/\mathrm{d}E$ distribution $F(E)$



Bethe-Bloch mean energy loss: $\langle\Delta\rangle = 400\ \mathrm{eV}$

[H. Bichsel, NIM A 562, 154 (2006)]

Straggling Functions

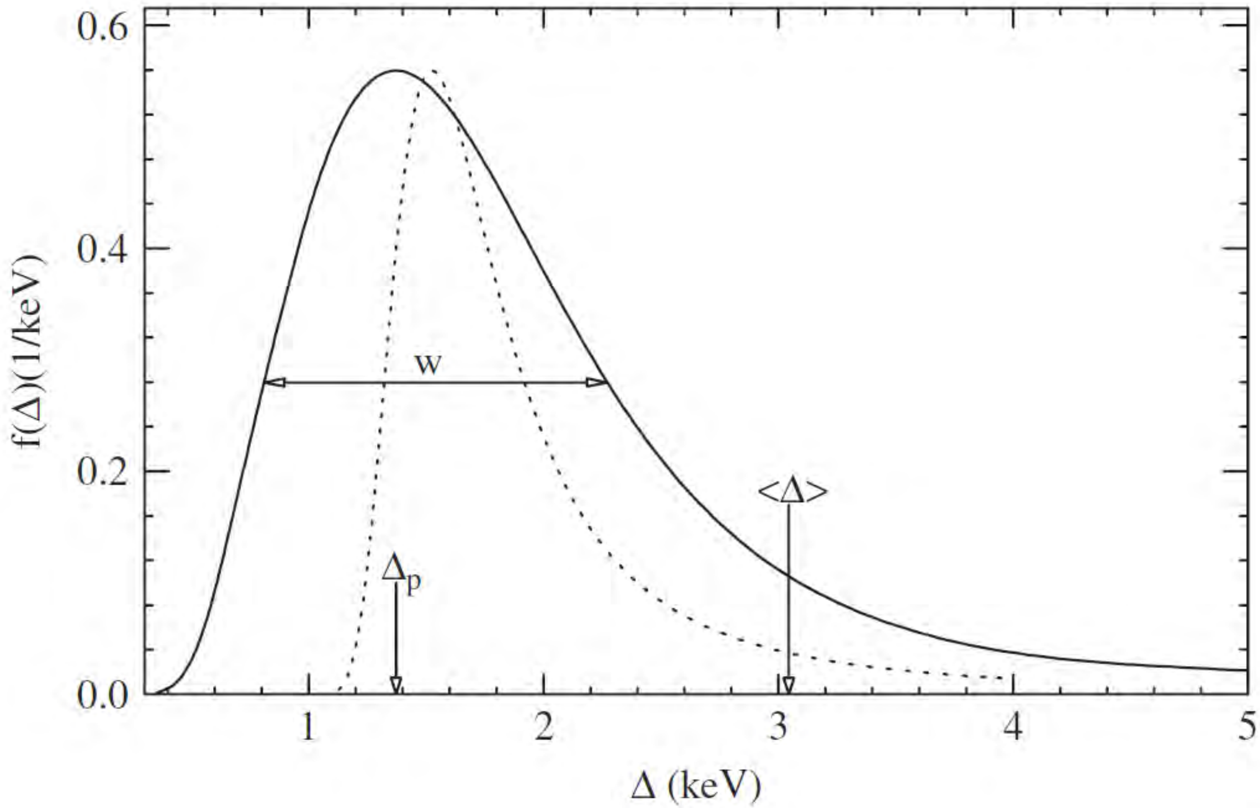


Fig. 1. The straggling function $f(\Delta)$ for particles with $\beta\gamma = 3.6$ traversing 1.2 cm of Ar gas is given by the solid line. It extends beyond $E_{max} \sim 2\,mc^2\beta^2\gamma^2 = 13\,\text{MeV}$. The original Landau function [2,3] is given by the dotted line. Parameters describing $f(\Delta)$ are the most probable energy loss $\Delta_p(x;\beta\gamma)$, i.e. the position of the maximum of the straggling function, at 1371 eV, and the full-width-at-half-maximum (FWHM) $w(x;\beta\gamma) = 1463\,\text{eV}$. The mean energy loss is $\langle\Delta\rangle = 3044\,\text{eV}$.

[H. Bichsel, NIM A 562, 154 (2006)]

2.3.2 Laplace Transform Method

[L. Landau, J. Phys. USSR 8, 201 (1944)]

Change of energy-loss distribution $f(\Delta; x)$ as a result of the particle passing through a thin elemental layer δx:

$$
\begin{aligned}
f(\Delta; x+\delta x) - f(\Delta; x) &= +n_e \delta x \int_0^{\Delta} \frac{\mathrm{d}\sigma(E;\beta)}{\mathrm{d}E} f(\Delta - E; x)\,\mathrm{d}E \\
&\quad -n_e \delta x \int_0^{\infty} \frac{\mathrm{d}\sigma(E;\beta)}{\mathrm{d}E} f(\Delta; x)\,\mathrm{d}E
\end{aligned}
$$

- 1st term: probability that the energy loss in x was $(\Delta - E)$, and a collision with energy transfer E occurred in δx, which makes the total energy loss equal to Δ (particle scattered into Δ)
- 2nd term: probability that the energy loss in x was already equal to Δ before entering δx, where a further collision increased the energy loss beyond Δ (particle scattered out of Δ)

Laplace Transform Method

Put in form of a transport equation:

$$\frac{\partial f(\Delta;x)}{\partial x} = \int_0^\infty n_e \frac{\mathrm{d}\sigma(E)}{\mathrm{d}E}\left[f(\Delta - E;x) - f(\Delta;x)\right]\mathrm{d}E$$

upper integration limit $E \to \infty$ for 1st term ok, since $f(x,\Delta) = 0$ for $\Delta < 0$

Solution: Laplace transform of both sides

+ solve for $\bar{f}(s;x)$

+ inverse Laplace transform

$\Delta \quad \circ\!\!-\!\!\bullet \quad s$

$$\mathcal{L}\{f(\Delta;x)\} = \bar{f}(s;x)$$

$$f(\Delta;x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \mathrm{d}s\, \exp\left[s\Delta - x\int_0^\infty n_e \frac{\mathrm{d}\sigma(E)}{\mathrm{d}E}\left(1 - e^{-sE}\right)\mathrm{d}E\right]$$

$0 < c \ll 1$

Exact solution, but numerical integration necessary in most cases!

Remarks to both methods:

- result determined by $\mathrm{d}\sigma/\mathrm{d}E$
- given the same cross section $\mathrm{d}\sigma/\mathrm{d}E$, both methods are equivalent

Straggling Functions

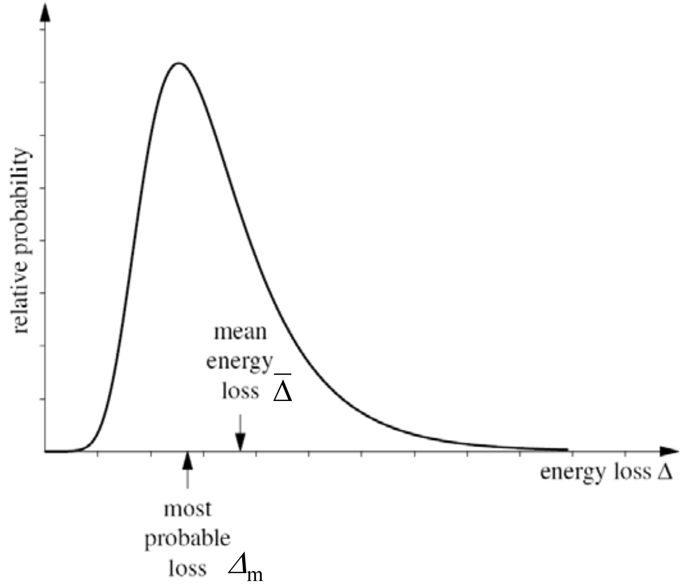
Different approximations, depending on thickness of absorber

Characteristic parameter: $\kappa = \frac{\xi}{T_{\max}}$, $\xi = \frac{2\pi}{m}\left(\frac{z\alpha\hbar}{\beta}\right)^2 n_e \cdot x$

ξ = scaling parameter
(1st term of Bethe-Bloch eq.)

Thin absorbers: $\kappa \leq 10$

- possibility of large energy transfer in single collisions: δ-electrons
- long tail on high-energy side, strongly asymmetric shape



Landau Distribution

Very thin absorbers: $\kappa \rightarrow 0$ (i.e. $T_{max} \rightarrow \infty$)

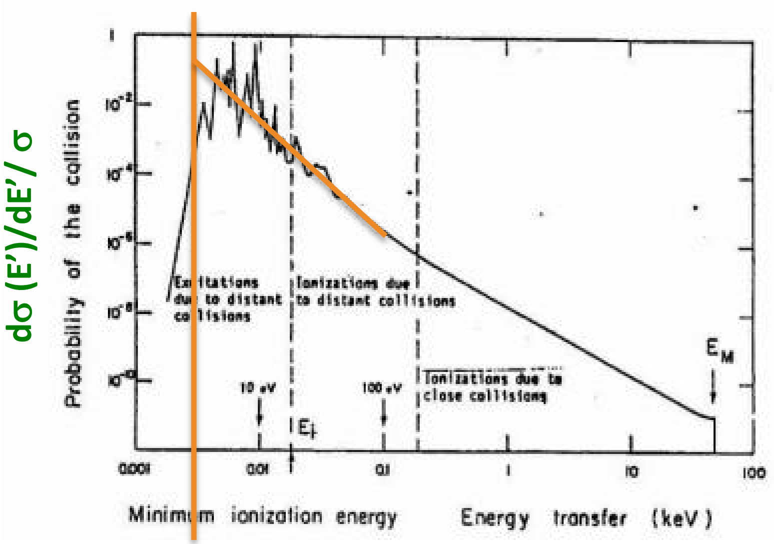
- single energy transfers sufficiently large to consider e$^-$ as free

⇨ Rutherford

$$\left(\frac{d\sigma}{dE}\right)^{\star}_{R} = \frac{2\pi}{m}\left(\frac{z\alpha\hbar}{E\beta}\right)^{2} = \frac{\xi}{n_e x}\frac{1}{E^2}$$

- particle velocity remains constant

Landau distribution [Landau 1944]



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- particle velocity remains constant

Landau distribution [Landau, J. Phys. USSR 8, 201 (1944)]

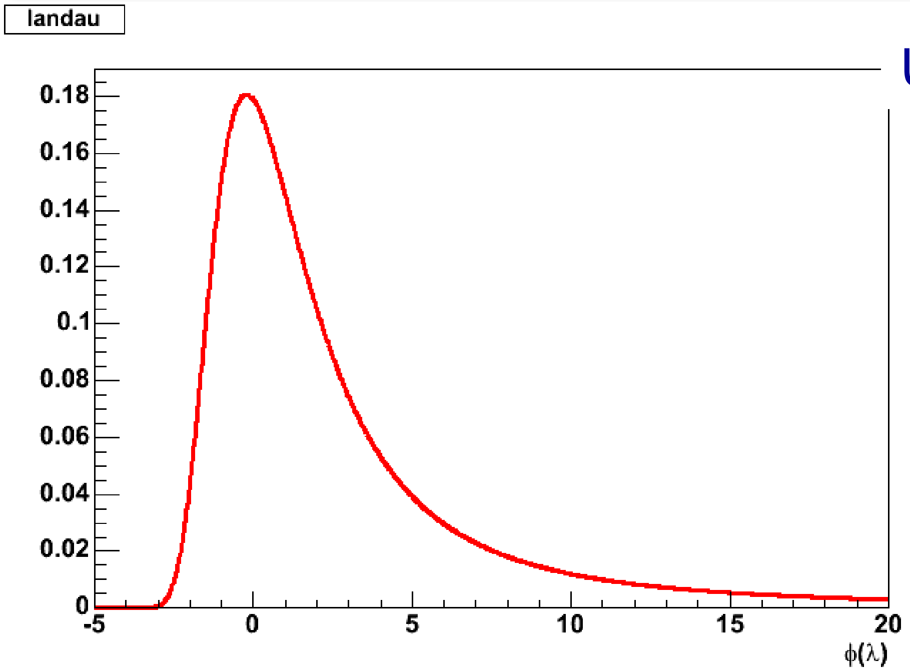
$$f_L(x,\Delta) = \frac{1}{\xi}\phi(\lambda)$$

λ = universal parameter, see next page for relation to Δ_m and ξ

Analytical approximation: Moyal distribution [Moyal, Phil. Mag. 46, 263 (1955)]:

$$f_M(x,\Delta) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\lambda + e^{-\lambda})}, \quad \lambda = \frac{\Delta - \Delta_m}{\xi}$$

Note: λ different from parameter in Landau distr. above



Universal Landau distribution:

$$\phi(\lambda) = \frac{1}{\pi}\int_0^\infty e^{-\pi u/2}\cos(u\ln u + \lambda u)\,du \quad ,$$

mit $\lambda = \frac{\Delta - \overline{\Delta}}{\xi} - (1+\beta^2 - C) - \ln\kappa \quad ,$

$$\xi = \frac{2\pi}{(4\pi\varepsilon_0)^2}\cdot\frac{z^2e^4}{mv^2}\cdot n_e x \approx \overline{\Delta} \quad , \kappa = \frac{\xi}{T_{\max}} \quad ,$$

$C = 0.5772\ldots$ (Euler-Konstante) ,

$$\overline{\Delta} = 2\xi\left[\frac{1}{2}\ln\left(\frac{2mc^2\beta^2\gamma^2T_{\max}}{I^2}\right) - \beta^2 - \frac{\delta}{2}\right]$$

Properties of $\phi(\lambda)$:

- asymmetric: tail up to $T_{max} \rightarrow \infty$
- Maximum at λ=-0.223
- FWHM=4.02•λ
- numerical evaluation

Landau distribution in ROOT:

$$f(\Delta|p_1,p_2,p_3) = p_1\times\phi\left(\frac{\Delta - p_2}{p_3}\right) \quad ,$$

mit p_1 Normierung (Integral)

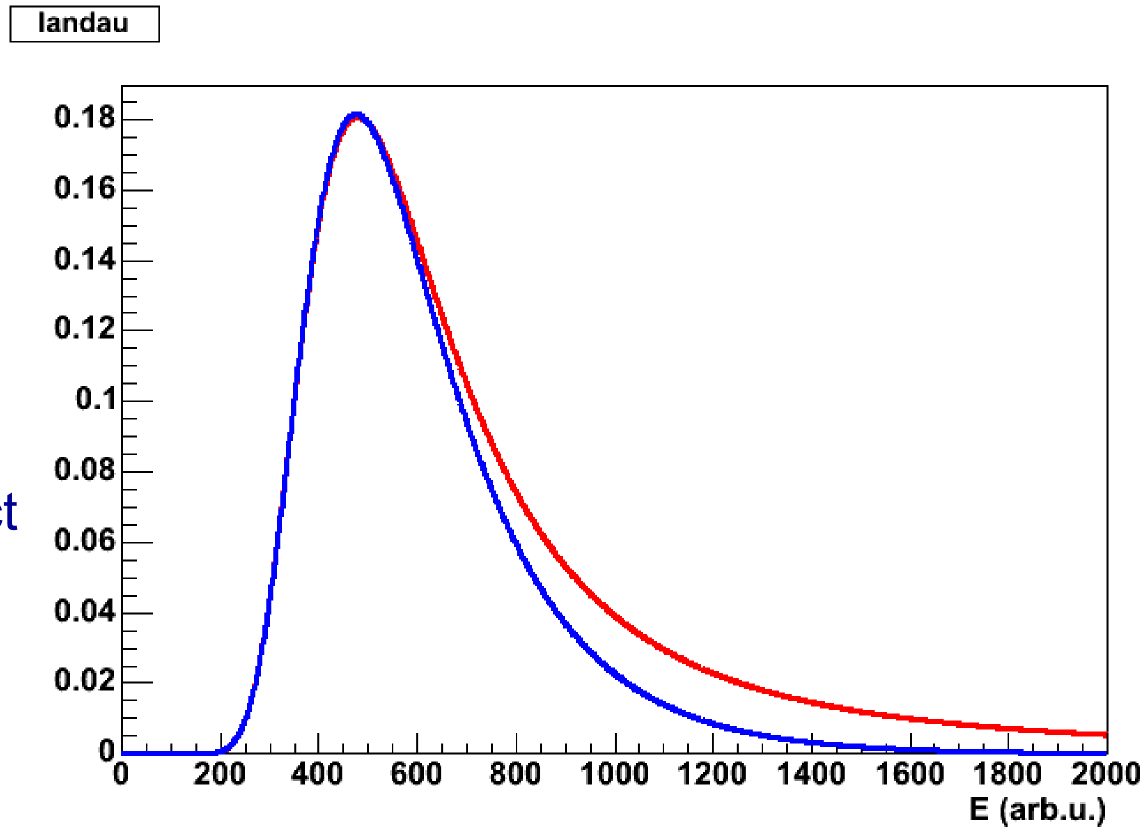
$\Delta_m = p_2 - 0.22278\times p_3 \quad ,$

$\text{FWHM} = 2\sqrt{2\ln 2}\times\frac{p_3}{0.5860} \quad .$

Comparison Landau - Moyal

Landau:
p_1=1.
p_2=500.
p_3=100.

Moyal:
- coarse shape correct
- tail too low!



But: tails are important for detector resolution!

Landau Distribution

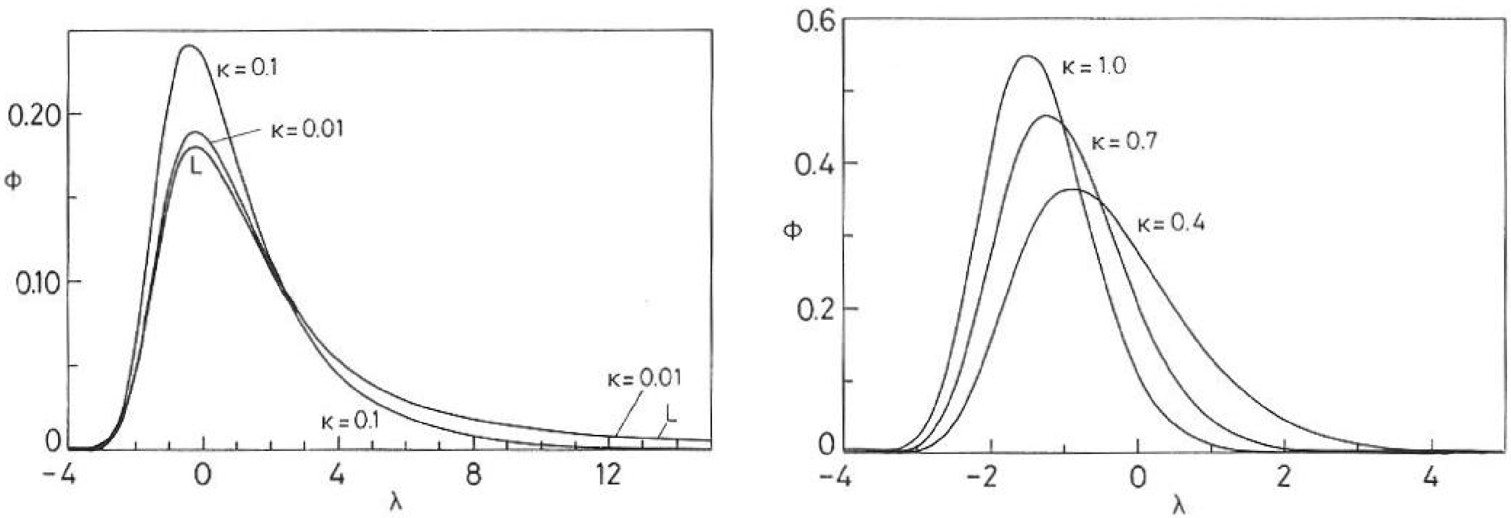
Landau distribution:

- Uses Rutherford cross section
- Does not reproduce straggling functions based on more realistic models for CCS for very small N_c
- Narrower width also for higher N_c ⇨ related to mean free path λ
  - Rutherford CCS underestimates λ (overestimates N_c)
  - Poisson contribution to straggling function leads to broadening

Symon-Vavilov Distribution

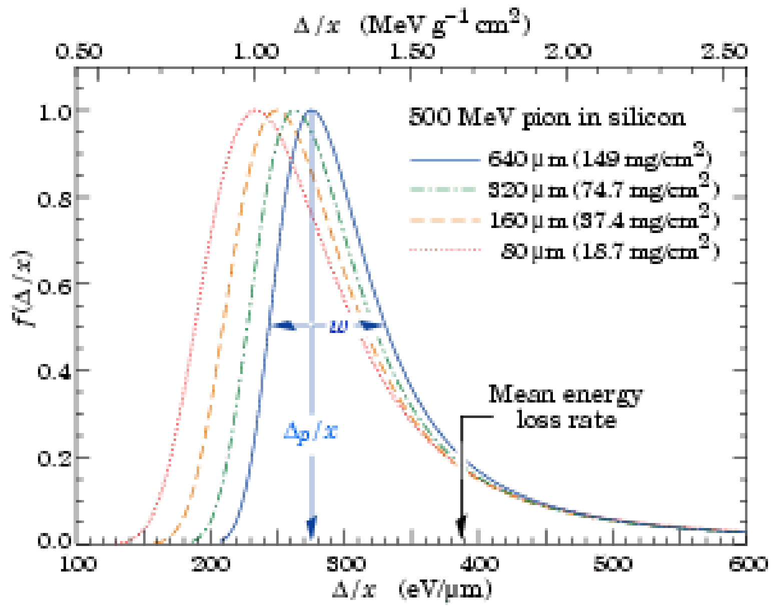
Thin absorbers: $0.01 < \kappa < 10$

- use correct expression for $T_{\max}$
- use Mott cross section instead of Rutherford
- reduces to Landau distribution for very small κ
- less asymmetric shape for larger κ



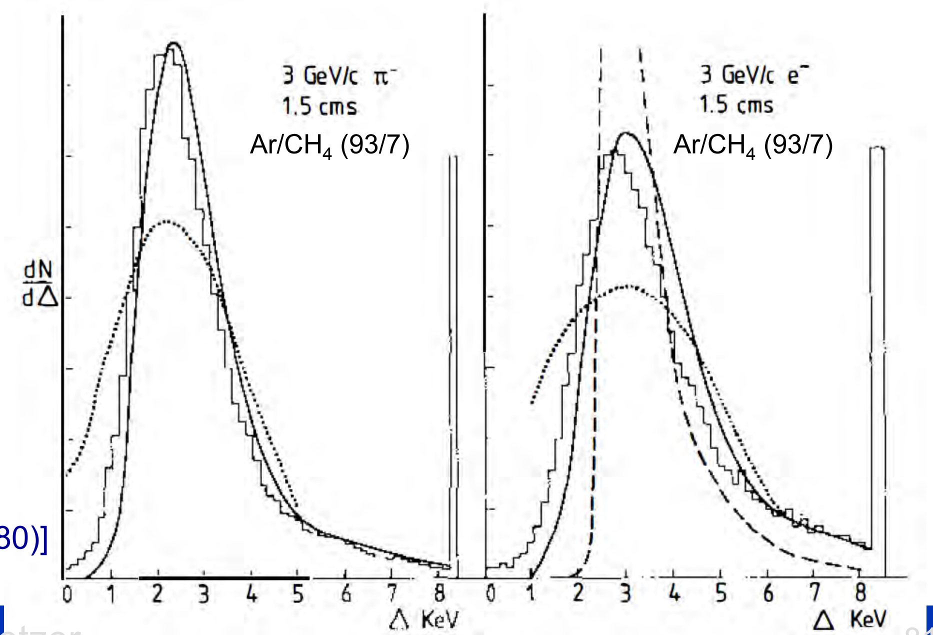
[S.M. Seltzer, M.J. Berger,
Nucl. Sc. Ser. Rep. No. 39 (1964)]

Realistic Straggling Functions



Iterative Solution of convolution integral

[H. Bichsel, Rev. Mod. Phys. 60, 663 (1988)]



PAI Model vs Landau

- Histogram: data
- Landau + corrections

– – – – [Maccabee, Papworth, Phys. Lett. A 30, 241 (1969)]

.... [Blunck, Leisegang, Z. Phys, 128, 500 (1950)]

- PAI model

——— [Allison, Cobb, Ann. Rev. Nucl. Part. Sci. 30, 253 (1980)]

Energy Loss Distributions

Medium-thick absorbers:

- number of collisions large
- total energy loss $\Delta \ll E_0$ of incident particle
  - ⇨ velocity $v \approx \text{const.}$
  - ⇨ single collisions statistically independent, i.e. probability distribution the same, with a well-defined expectation value and variance
  - ⇨ $f(\Delta; x)$ approaches Gaussian form with mean $\bar{\Delta}$ and $\sigma^2 = \xi \cdot T_{\max}$

Follows directly from Laplace transform or from Central Limit Theorem:

The sum of N random variables, which all follow the same statistical distribution, is of Gaussian shape for large N, provided the individual processes are statistically independent

Energy Loss Distributions

Medium-thick absorbers:

- number of collisions large
- total energy loss $\Delta \ll E_0$ of incident particle

  ⇨ velocity $v \approx \text{const.}$

  ⇨ single collisions statistically independent,
  i.e. probability distribution the same, with a well-defined expectation value and variance

  ⇨ $f(\Delta; x)$ approaches Gaussian form with mean $\overline{\Delta}$ and $\sigma^2 = \xi \cdot T_{max}$

Even in thick detectors, the distribution never becomes Gaussian!

- due to the condition $\Delta \ll E_0$, i.e. insignificant energy loss of the particle
- average energy loss per collision and its variance are very large (even infinite for Rutherford cross section: $d\sigma/dE \propto 1/E^2$)

2.5 Measurement of Energy Loss

Restricted energy loss:

$$\left\langle -\frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle_{T<T_{\text{cut}}} = \frac{4\pi}{(4\pi\varepsilon_0)^2} \frac{z^2 e^4 n_e}{mc^2\beta^2} \left[\frac{1}{2} \ln \frac{2mc^2\beta^2\gamma^2 T_{\text{cut}}}{I^2} - \frac{\beta^2}{2}\left(1 + \frac{T_{\text{cut}}}{T_{\max}}\right) - \frac{\delta}{2} \right]$$

approaches normal Bethe-Bloch equation for $T_{\text{cut}} \to T_{\max}$

Transforms into average total number of e$^-$ ion pairs n_{T} along path length x:

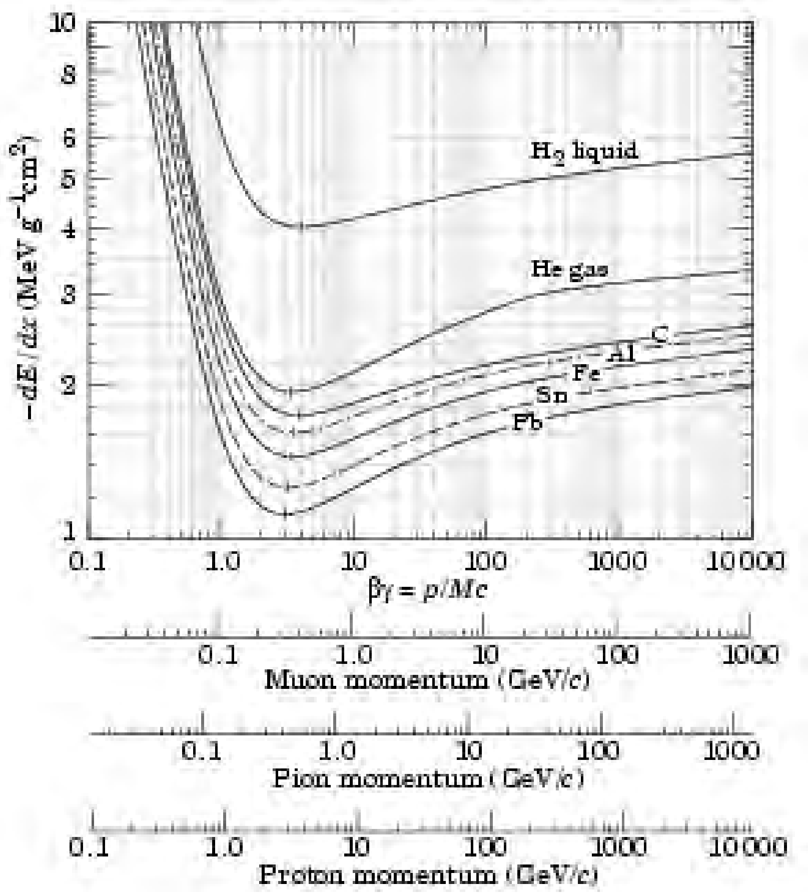
$$x\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = n_{\mathrm{T}} W$$

But: actual energy loss fluctuates with a long tail (Landau distribution)

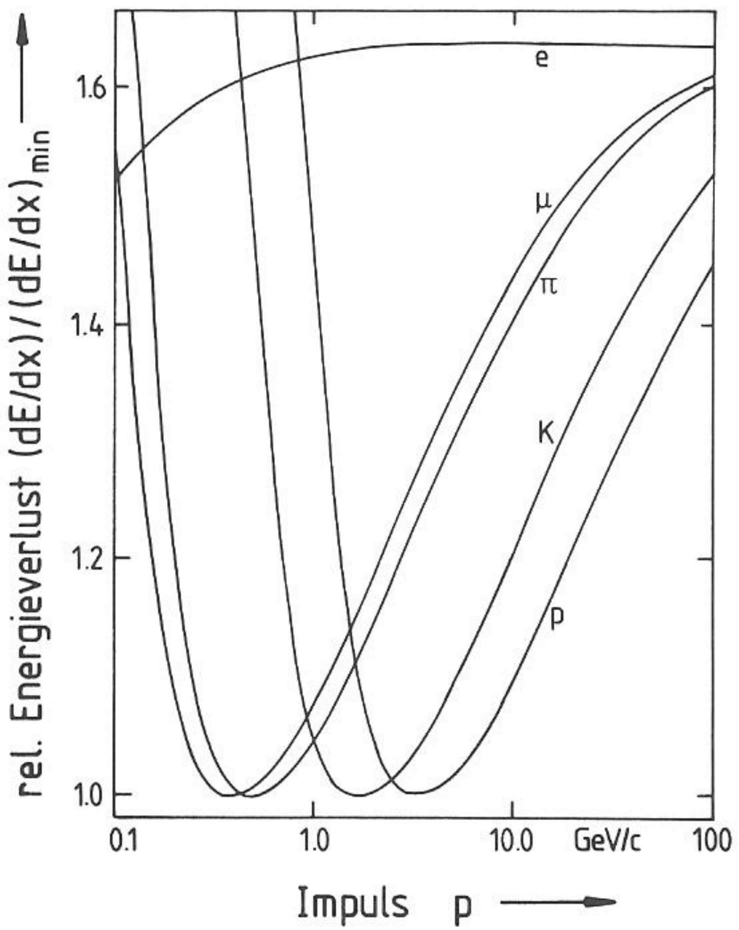
⇨mean value of energy loss is a bad estimator

⇨ use truncated mean of N pulse height measurements along the track:

$$\langle A \rangle_t = \frac{1}{N_t} \sum_{i=1}^{N_t} A_i \qquad \begin{array}{l} A_i \le A_{i+1} \quad \text{for} \quad i = 1, \ldots, N \\ N_t = t \cdot N, \quad t \in [0,1] \end{array}$$

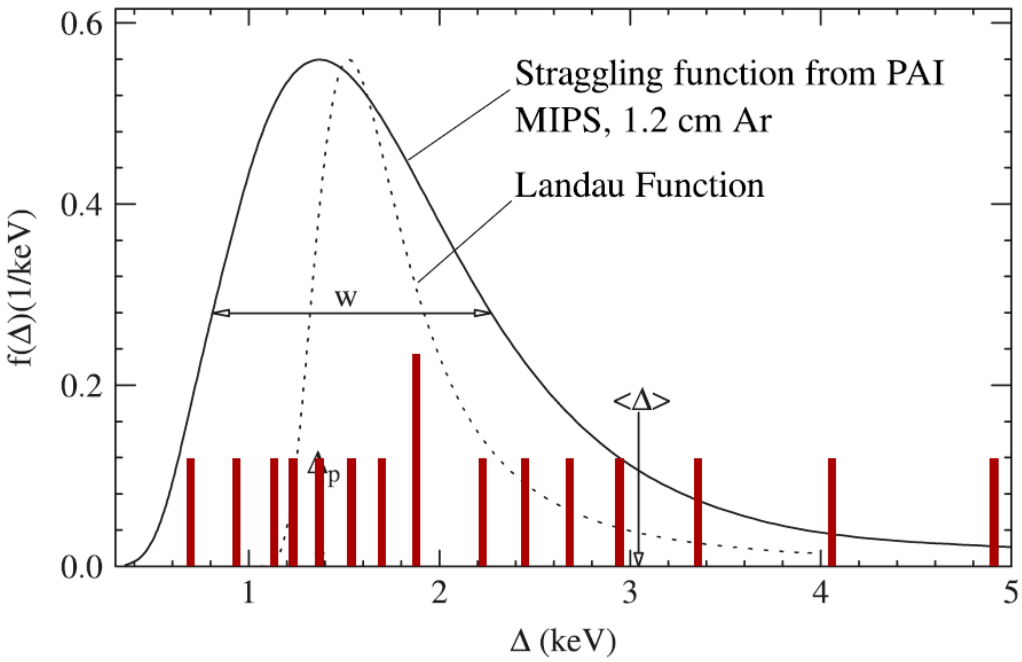


$-dE/dx$ (MeV g^{-1}cm^2)
H$_2$ liquid
He gas
C
Al
Fe
Sn
Pb
$\beta\gamma = p/Mc$
Muon momentum (GeV/c)
Pion momentum (GeV/c)
Proton momentum (GeV/c)

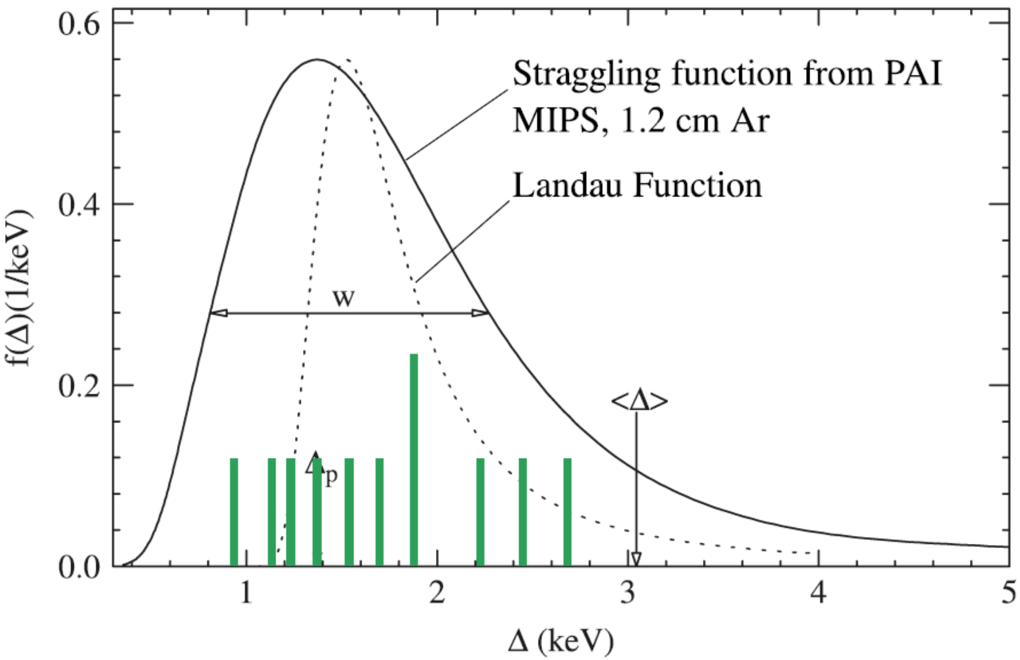


rel. Energieverlust (dE/dx) / (dE/dx)$_{min}$
e
μ
π
K
p
GeV/c
Impuls p

Measurement of Energy Loss



Measurement of Energy Loss

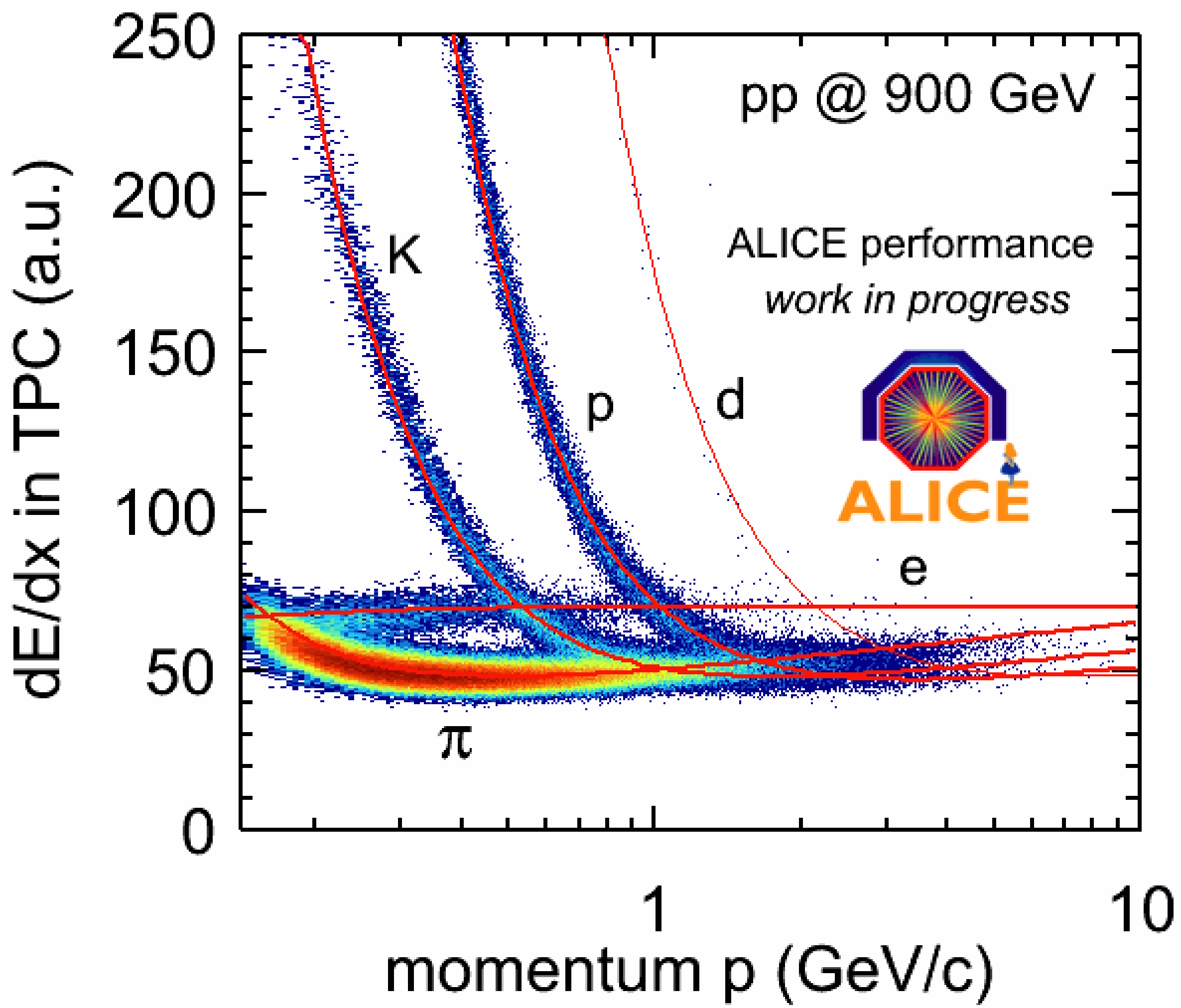


Resolution (empirical):

$$\Delta E/E = 0.96 \cdot N^{-0.46} \cdot (\Delta x \cdot p)^{-0.32}$$

N = number of samples
Δx = sample length (cm)
p = gas pressure (atm)

Example: ALICE TPC



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