QED radiative corrections for the P2 experiment

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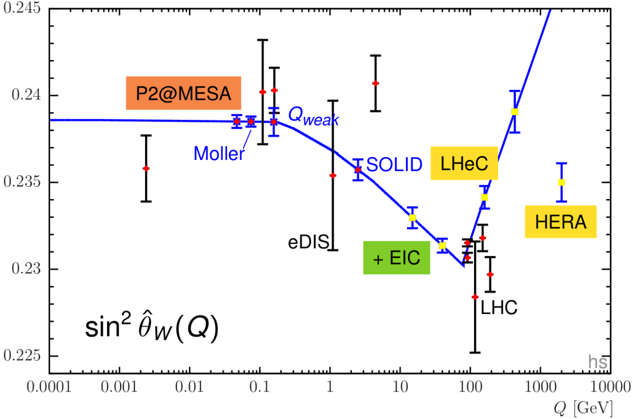
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Overview

(1) Introduction — The running of the weak mixing angle
(2) How to measure the weak mixing angle?
(3) Electroweak radiative corrections to Q_W^p.
(4) The shift in Q^2 due to photon radiation.
(5) $\mathcal{O}(\alpha^2)$ QED corrections

$\sin^2\theta_W$ is scale dependent

$$\sin^2\hat{\theta}_W(Q)_{\overline{\mathrm{MS}}} = \kappa(Q)_{\overline{\mathrm{MS}}}\sin^2\theta_W(M_Z)_{\overline{\mathrm{MS}}}$$



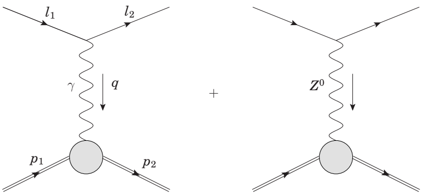
See the lectures of Krishna Kumar from last year for a more detailed introduction.

How to measure $\sin^2\theta_W$?

$\rightarrow$ Extract Q_W^p in ep scattering with polarized e^- beam and unpolarized target. (P2 and Qweak approach)

$\rightarrow$ Measure the small asymmetry $\approx 10^{-8}$ between cross sections for electrons with $+$ and $-$ helicities

$$A_{\mathrm{PV}} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}$$



In the Born approximation

$$A_{PV} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\frac{\epsilon G_E^\gamma G_E^Z + \tau G_M^\gamma G_M^Z - g_V^e\sqrt{1-\epsilon^2}\sqrt{\tau(1+\tau)}G_M^\gamma G_A^Z}{\epsilon G_E^2 + \tau G_M^2}.$$

$$\xrightarrow{\text{low } Q^2,\ \epsilon\to 1} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\left[Q_W^p - F(Q^2)\right].$$

SM at tree level $Q_W^{p,\text{tree}} \rightarrow 1 - 4\sin^2\theta_W^{\text{tree}} \approx 0.07$ (good canditate for NP search).

Why measure at low Q^2? $\rightarrow$ small contribution from $F(Q^2)$ and $\gamma - Z$ box correction.

Radiative Corrections

P2 accuracy: $\frac{\Delta A_{\mathrm{PV}}}{A_{\mathrm{PV}}} = 1.7\%$ $\rightarrow$ $\frac{\Delta \sin^2\theta_W}{\sin^2\theta_W} = 0.15\%$

$\rightarrow$ Include full treatment of radiative corrections at $\mathcal{O}(\alpha^2)$ to match experimental precision.

$$A_{PV} = -\frac{G_F Q'^2}{4\sqrt{2}\pi\alpha}\left[\widetilde{Q}^p_W - F(Q'^2)\right].$$

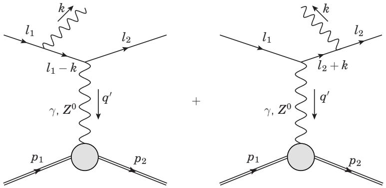
$$\widetilde{Q}^p_W = (\rho + \Delta_e)(1 - 4\kappa(Q'^2)\sin^2\hat{\theta}_W(0) + \Delta_{e'}) + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z},$$

Universal corrections $\rightarrow$ $\rho = 1 + \Delta\rho$ and $\kappa = 1 + \Delta\kappa$ from loop diagrams

Non-universal corrections $\rightarrow$ Δ_e and $\Delta_{e'}$ from vertex and external leg corrections.

Form Factors $\rightarrow$ $F(Q'^2) = F_{EM}(Q'^2) + F_{axial}(Q'^2) + F_{strangeness}(Q'^2)$

Shift in momentum transfer due to photon radiation



Shifted kinematics:

$$Q^2 = -(l_1 - l_2)^2 \rightarrow Q'^2 = -(l_1 - l_2 - k)^2$$

$\rightarrow Q'^2$ can be on average much smaller than Q^2.

The average shift in momentum transfer squared due to hard-photon bremsstrahlung can be defined as

$$\langle \Delta Q^2 \rangle = \frac{1}{\sigma} \int \frac{\mathrm{d}^4 \sigma^{1\gamma}}{\mathrm{d}E' \mathrm{d}\theta_l \mathrm{d}E_\gamma \mathrm{d}\theta_\gamma} \mathrm{d}E' \mathrm{d}\theta_l \mathrm{d}E_\gamma \mathrm{d}\theta_\gamma \Delta Q^2,$$

with

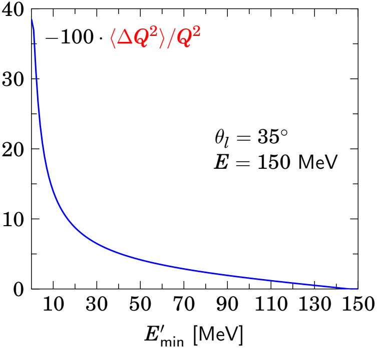
$$\Delta Q^2 = Q'^2 - Q^2,$$

$$\sigma = \sigma^{1\gamma}_{1-\text{loop}}\Big|_{E_\gamma < \Delta} + \sigma^{1\gamma}\Big|_{E_\gamma > \Delta}.$$

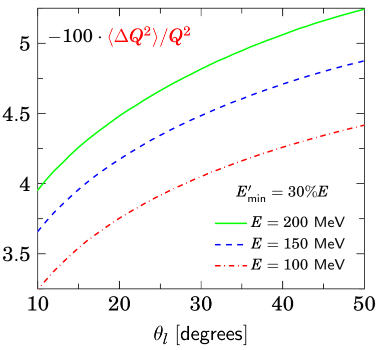
→ Strong dependence on experimental prescriptions for measuring kinematic variables

→ Need full Monte-Carlo treatment

→ Dependence on the detector acceptance

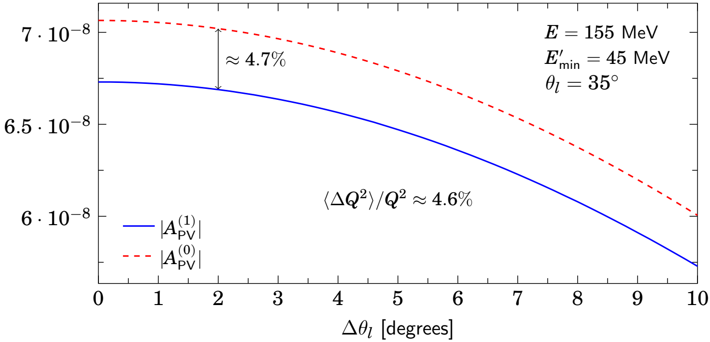


→ Dependence on the scattering angle



→ The shift in Q^2 due to photon bremsstrahlung induces a shift similar in size in the asymmetry.

→ A significant effect is also given by the bin size $\Delta\theta_l$ of the integration over the scattering angle.



The total asymmetry with first order QED corrections compared with the leading order asymmetry.

$\mathcal{O}(\alpha^2)$ QED corrections

- The cross-section with $\mathcal{O}(\alpha^2)$ QED corrections is given by

$$\mathrm{d}\sigma^{(2)} = \mathrm{d}\sigma_{0\gamma} \times \left[1 + \delta^{(1)}_{1-\text{loop}} + \delta^{(2)}_{2-\text{loop}} + \delta^{(1)}_{1\gamma}(\Delta) + \delta^{(2)}_{2\gamma}(\Delta) + \delta^{(1)}_{1-\text{loop}}\delta^{(1)}_{1\gamma}(\Delta)\right] + \int_{E_\gamma>\Delta} \mathrm{d}^4\sigma_{1\gamma}\left[1 + \delta^{(2)}_{1-\text{loop}} + \delta^{(2)}_{1\gamma}(\Delta)\right] + \int_{E_\gamma,\, E'_\gamma>\Delta} \mathrm{d}^7\sigma_{2\gamma}.$$

Δ is the cut-off energy that makes the separation between soft- and hard-photons.

- The second order soft real correction can be written as

$$\delta_{2\gamma}(\Delta) = \frac{1}{2}\left[\delta_{1\gamma}(\Delta)\right]^2 - \frac{\alpha^2}{3}(L-1)^2$$

Not captured by a simple exponentiation.

Richard Hill 2016

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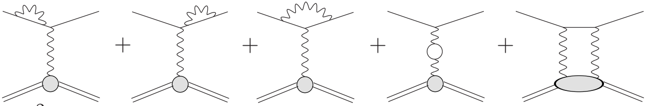
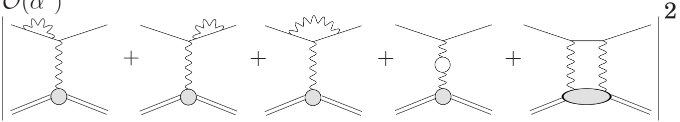
QED virtual corrections

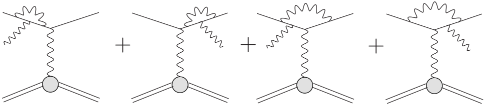
$\mathcal{O}(\alpha)$



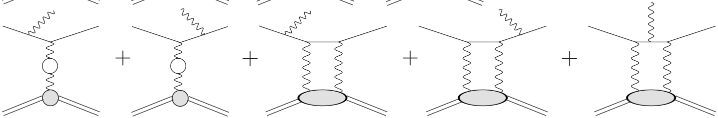
$\mathcal{O}(\alpha^2)$



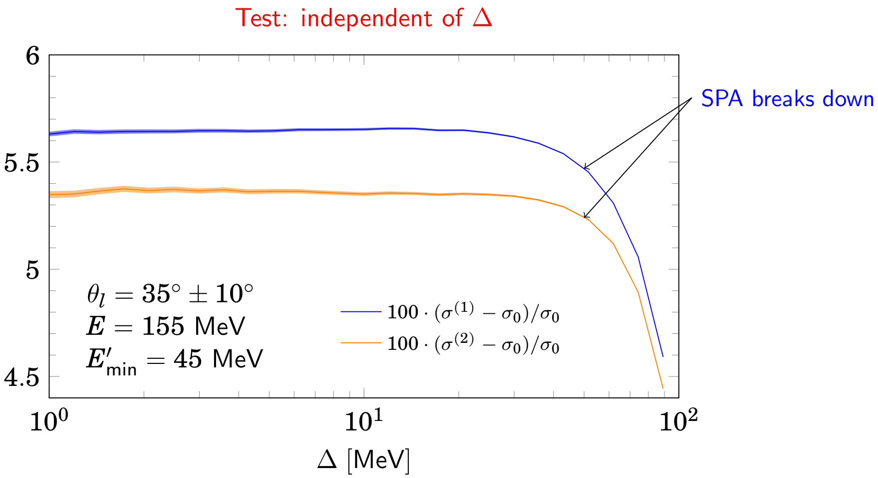
+ 2-loop



+ photon emitted from on-shell line

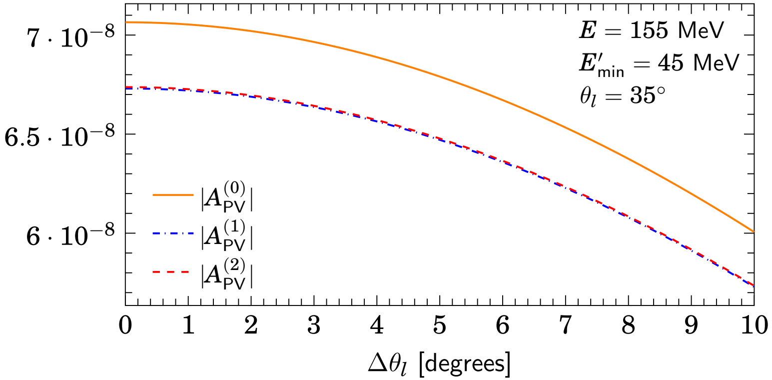


$\mathcal{O}(\alpha^2)$ QED corrections to the unpolarized cross section



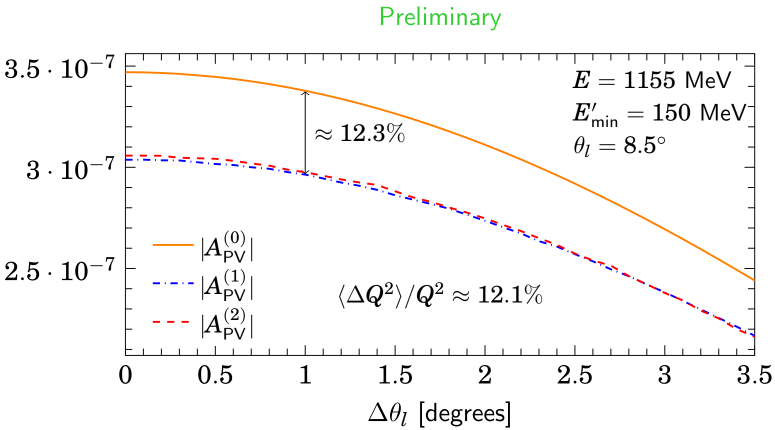
Can be used to decide which is the best value for Δ.

$\mathcal{O}(\alpha^2)$ QED corrections to the asymmetry (P2 kinematics)



The shift in Q^2 is a kinematical effect included in 1γ radiation $\rightarrow$ very small $\mathcal{O}(\alpha^2)$ corrections to the asymmetry.

$\mathcal{O}(\alpha^2)$ QED corrections to the asymmetry (Qweak kinematics)

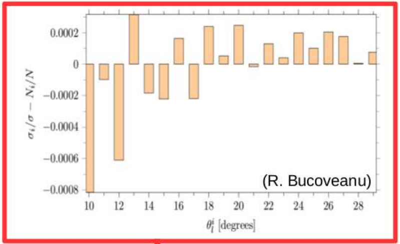


Strongly dependent on experimental prescriptions. More details about detector acceptance are needed.

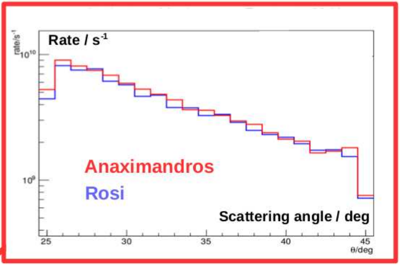
Dominik Becker

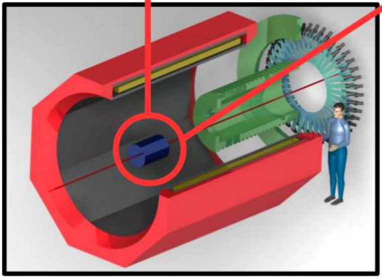
Final state generation for simulation of elastic e-p scattering

“Anaximandros” vs. Theory



“Anaximandros” vs. “Rosi”





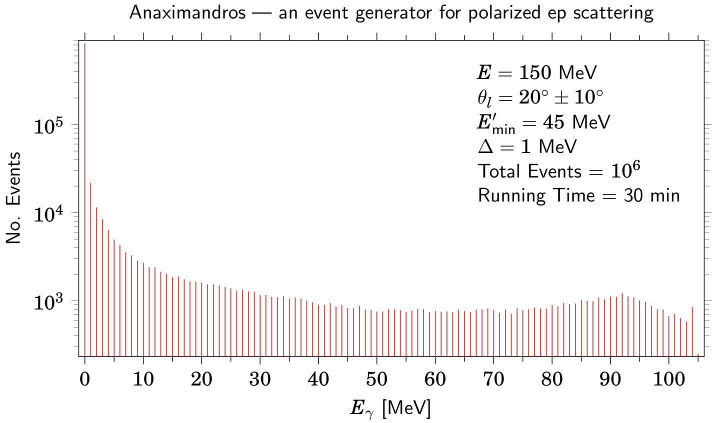
Use initial state distribution with final state generator...

Two final state generators available:

- **“Rosi”:**
  Tree-level, based on the Rosenbluth formula
- **“Anaximandros”: (R. Bucoveanu)**
  Includes radiative corrections, allows for a photon in the final state.
  Implementation is work in progress.

Conclusion

→ It is important to include full treatment of radiative corrections starting at the level of the event generator.



Overview

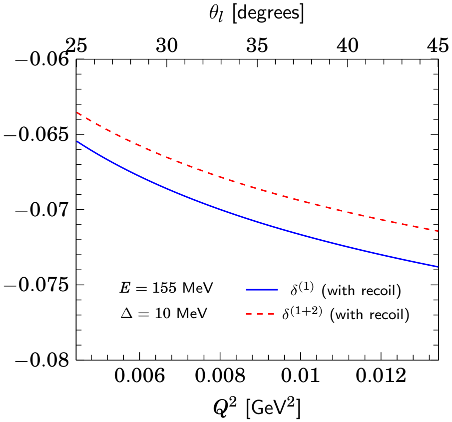
- Significant $\mathcal{O}(\alpha)$ QED corrections to the asymmetry were found
  $\rightarrow$ need full Monte Carlo treatment

- Very small $\mathcal{O}(\alpha^{\mathbf{2}})$ to the asymmetry.

- A modern, flexible, easy to use event generator was developed that will include complete $\mathcal{O}(\alpha^{\mathbf{2}})$ electroweak corrections.

Thank you for your attention!

Appendix

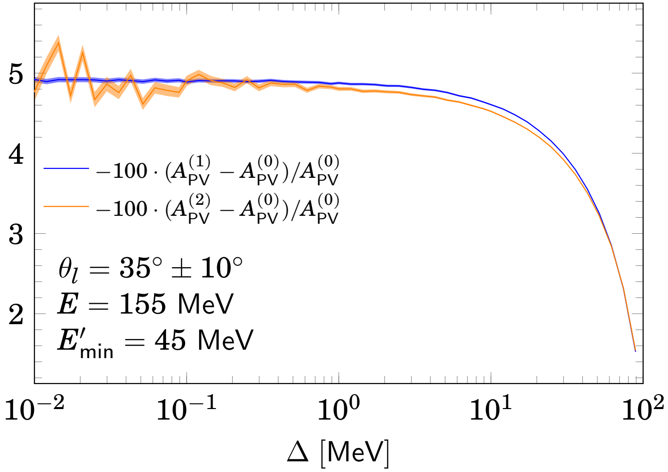


$$\delta^{(1)} = \delta^{(1)}_{1-\text{loop}} + \delta^{(1)}_{1\gamma}$$

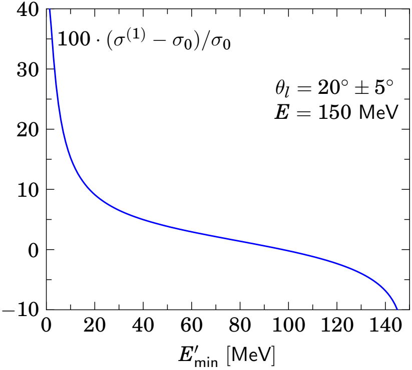
$$\delta^{(2)} = \delta^{(2)}_{2-\text{loop}} + \delta^{(2)}_{2\gamma} + \delta^{(1)}_{1-\text{loop}}\delta^{(1)}_{1\gamma}$$

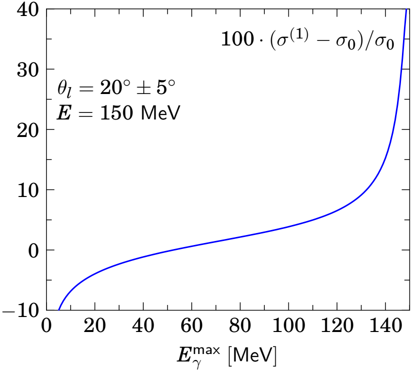
$$\delta^{(1+2)} = \delta^{(1)} + \delta^{(2)}.$$

Richard J. Hill 2016



5
4
3
2
$-100 \cdot (A_{\mathsf{PV}}^{(1)} - A_{\mathsf{PV}}^{(0)})/A_{\mathsf{PV}}^{(0)}$
$-100 \cdot (A_{\mathsf{PV}}^{(2)} - A_{\mathsf{PV}}^{(0)})/A_{\mathsf{PV}}^{(0)}$
$\theta_l = 35^\circ \pm 10^\circ$
$E = 155$ MeV
$E'_{\mathsf{min}} = 45$ MeV
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Δ [MeV]


40
30
20
10
0
−10
0
20
40
60
80
100
120
140
$100 \cdot (\sigma^{(1)} - \sigma_0)/\sigma_0$
$\theta_l = 20^\circ \pm 5^\circ$
$E = 150$ MeV
E'_{min} [MeV]


40
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E_γ^{max} [MeV]