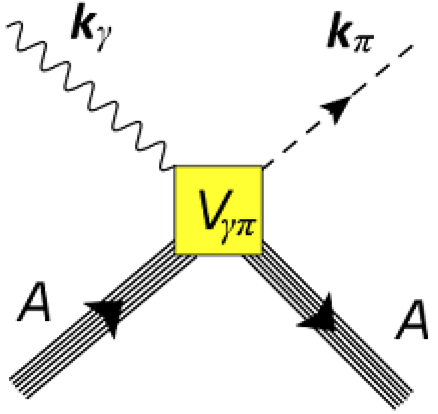
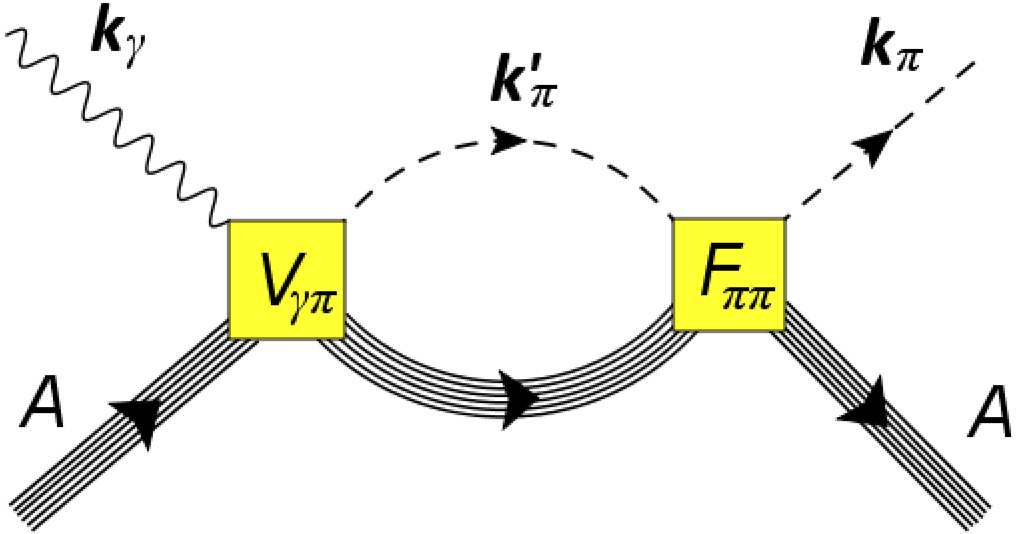
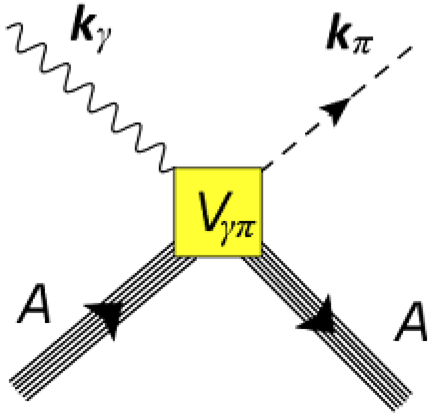
Coherent π^0 photoproduction on spin-zero nuclei

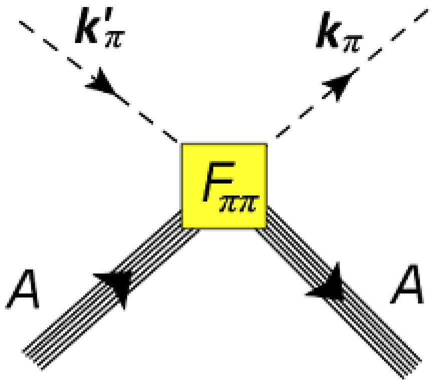


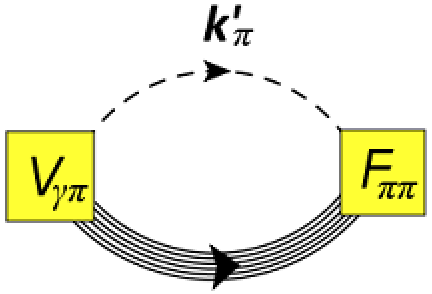
Slava Tsaran

Pion photoproduction: DWA

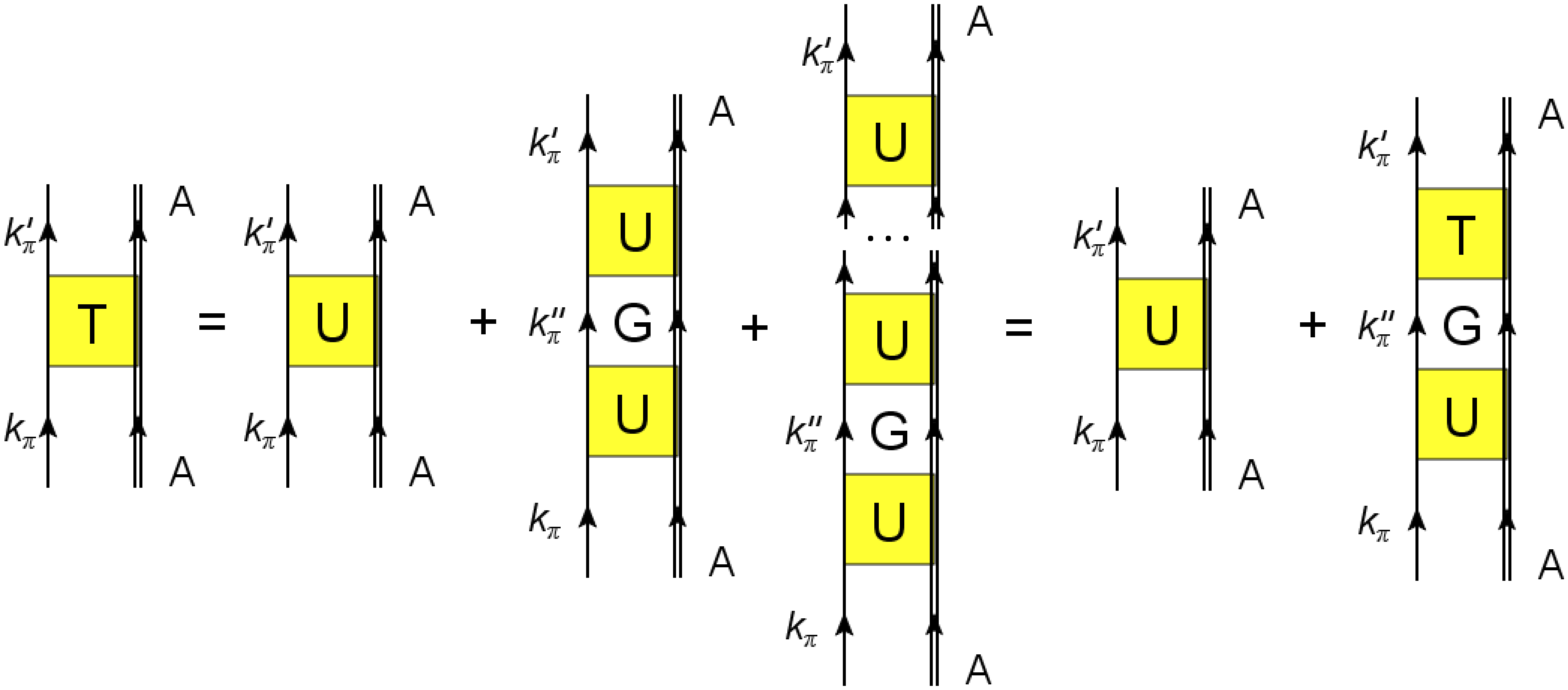






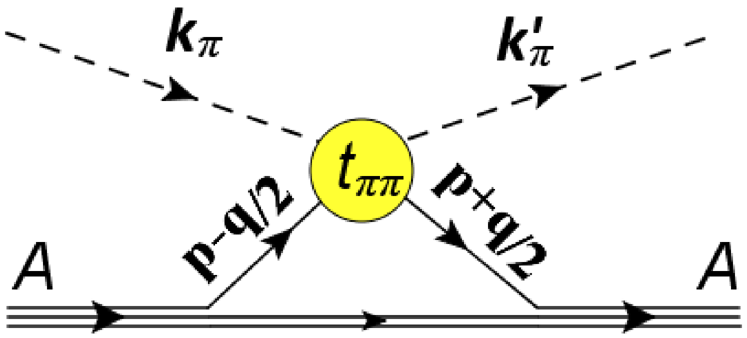


Pion-nucleus scattering: Lippmann-Schwinger equation



Pion-nucleus scattering: tρ potential

Impulse approximation:



$$T(\mathbf{k}',\mathbf{k}) = \langle \mathbf{k}'|T_{\pi A}|\mathbf{k}\rangle_{\rm IA} = \sum_{\alpha\beta} \langle \mathbf{k}',\alpha|t_{\pi N}^{\pi A\ \rm c.m.}|\mathbf{k},\beta\rangle\langle\Phi_A|\hat{c}^\dagger_\alpha \hat{c}_\beta \Phi_B\rangle =$$

$$\sum_{\alpha<F}\int \frac{d^3\mathbf{p}}{(2\pi)^3}\phi^*_\alpha(\mathbf{p}+\mathbf{q}/2)\phi_\alpha(\mathbf{p}-\mathbf{q}/2)t_{\pi N}^{\pi A\ \rm c.m.}(\mathbf{k}',\mathbf{p}+\mathbf{q}/2;\mathbf{k},\mathbf{p}-\mathbf{q}/2) \quad (1)$$

Factorization approximation:

$$\langle \mathbf{k}'|T_{\pi A}|\mathbf{k}\rangle_{\rm IA,\ Fact} = \rho(q)t_{\pi N}^{\pi A\ \rm c.m.}(\mathbf{k}',\mathbf{k};\mathbf{q}), \qquad \rho(q) = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\phi^*_\alpha(\mathbf{p}+\mathbf{q}/2)\phi_\alpha(\mathbf{p}-\mathbf{q}/2), \qquad \rho(0) = A. \quad (2)$$

Spin and isospin averaged *s*- and *p*-wave parts:

$$t_{\pi N}^{\pi N \rm c.m.}(\mathbf{k}'_{\rm c.m.},\mathbf{k}_{\rm c.m.};\mathbf{q}_{\rm c.m.}) = -4\pi(b_o + c_o\mathbf{k}'_{\rm c.m.}\cdot\mathbf{k}_{\rm c.m.}) \quad (3)$$

Kinematic correction:

$$\mathbf{k}_{\rm c.m.}\cdot\mathbf{k}'_{\rm c.m.} \cong \frac{1}{(1+\varepsilon)^2}\left[\mathbf{k}\cdot\mathbf{k}' - \frac{\varepsilon}{2}\mathbf{q}^2\right], \quad \varepsilon = \omega/m_N \quad (4)$$

$$\langle \mathbf{k}'|T_{\pi A}|\mathbf{k}\rangle_{\rm IA,\ Fact} = -4\pi\rho(q)\left((1+\varepsilon)b_o + \frac{c_o}{1+\varepsilon}\left[\mathbf{k}\cdot\mathbf{k}' - \frac{\varepsilon}{2}\mathbf{q}^2\right]\right) \quad (5)$$

Pion-nucleus scattering: optical potential

Klein-Gordon equation:

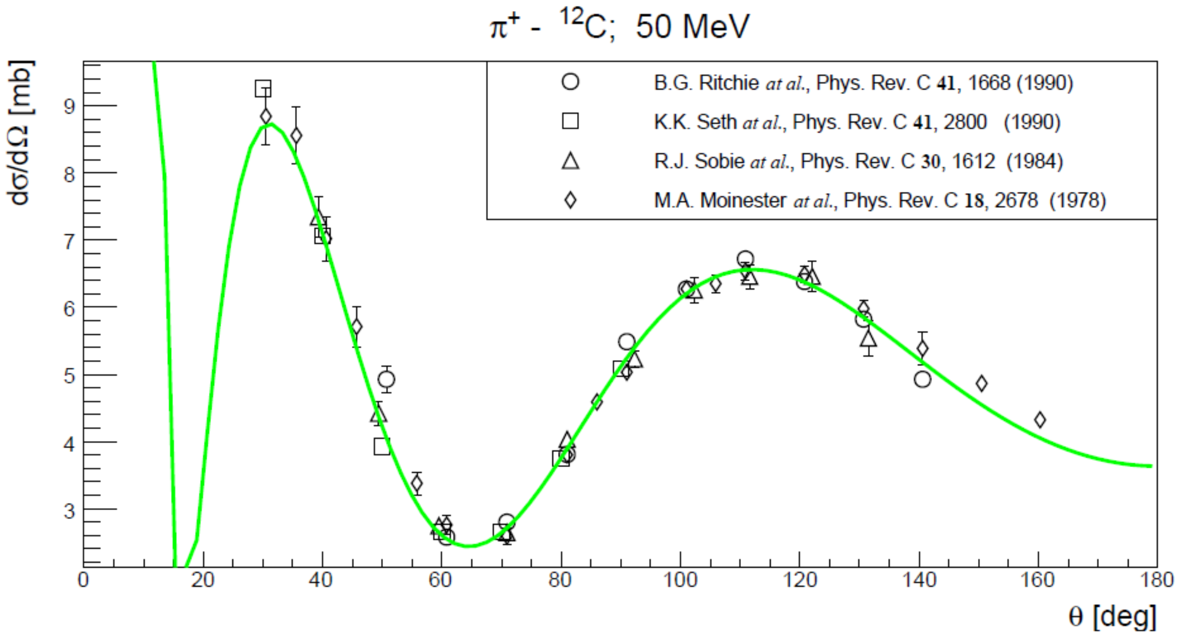
$$(\omega - \hat{V}_C)^2\Phi = (\hat{\mathbf{p}}^2 + m^2)\Phi + \tilde{U}\Phi \qquad (1)$$

$$\tilde{U}(\omega, \mathbf{r})\Phi(\mathbf{r}) = \int d^3\mathbf{r}' U(\mathbf{r}, \mathbf{r}')\Phi(\mathbf{r}') \qquad (2)$$

1st-order optical potential:

$$U(\mathbf{k}, \mathbf{k}') = -4\pi\rho(q)\left((1+\varepsilon)b_o + \frac{c_o}{1+\varepsilon}\left[\mathbf{k}\cdot\mathbf{k}' - \frac{\varepsilon}{2}\mathbf{q}^2\right]\right) \qquad (3)$$

$$U(\omega, \mathbf{r})_{t\rho} = -4\pi\left((1+\varepsilon)b_o\rho(r) + \frac{c_o}{1+\varepsilon}\nabla\rho(r)\nabla + \frac{\varepsilon}{2}\frac{c_o}{(1+\varepsilon)}\nabla^2\rho(r)\right) \qquad (4)$$



Pion-nucleus scattering: optical potential

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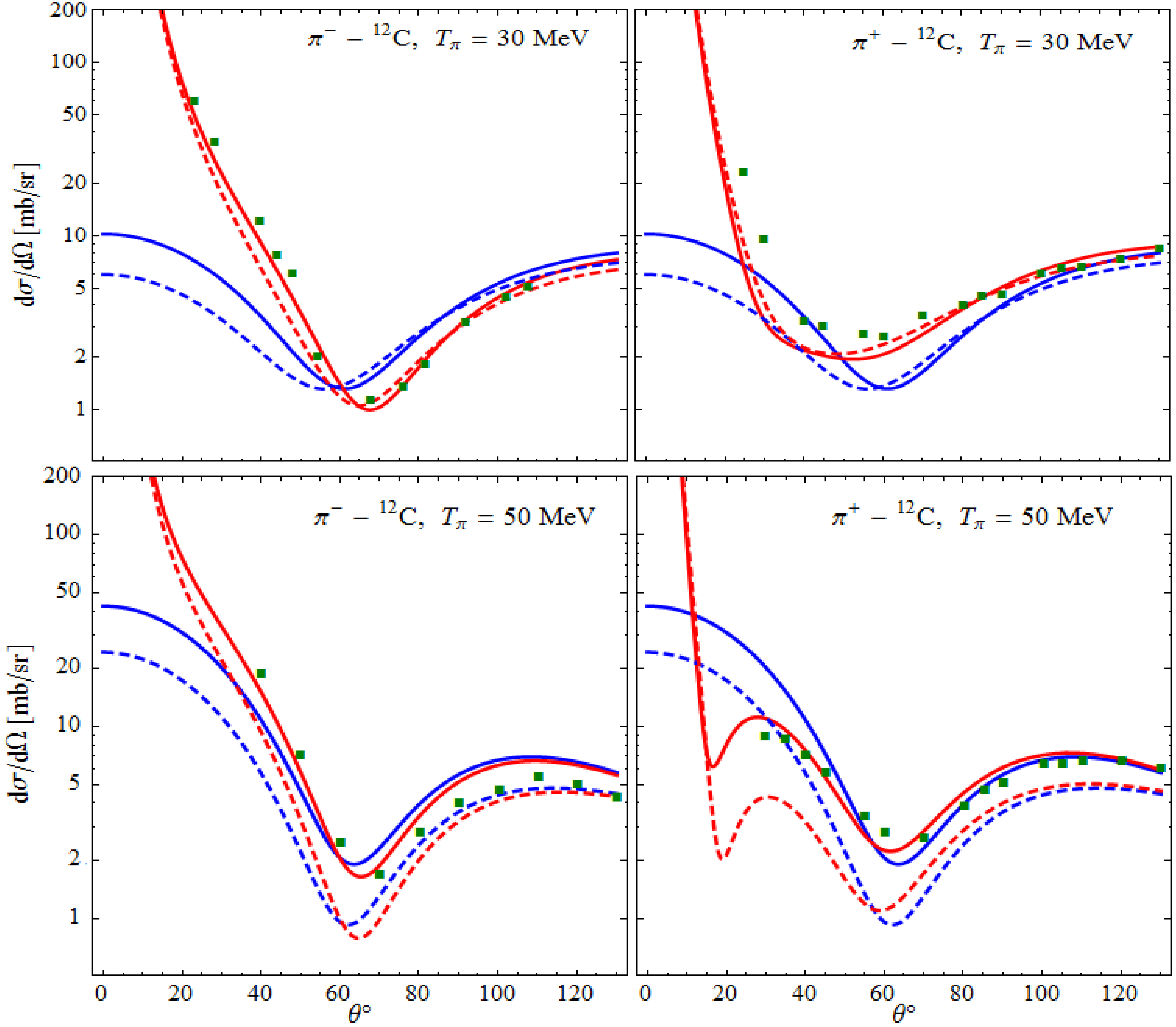
$$U(\omega,\mathbf{r})_{t\rho} = -4\pi\left((1+\varepsilon)b_o\rho(r) + \frac{c_o}{1+\varepsilon}\nabla\rho(r)\nabla + \frac{\varepsilon}{2}\frac{c_o}{(1+\varepsilon)}\nabla^2\rho(r)\right) \qquad (4)$$

2nd-order optical potential: *K. Stricker et al., Phys. Rev. C 25 (1982) 952*

$$\tilde{U}(\omega,\mathbf{r}) = -4\pi\left[p_1 b_o\rho(r) + p_2 B_o\rho^2(r) - \nabla\frac{\frac{c_o}{p_1}\rho(r) + \frac{C_o}{p_2}\rho^2(r)}{1+\frac{4\pi}{3}\lambda\left[\frac{1}{p_1}c_o\rho(r) + \frac{1}{p_2}C_o\rho^2(r)\right]}\nabla + \frac{1}{2}\left(1-\frac{1}{p_1}\right)c_o\left(\nabla^2\rho(r)\right) + \frac{1}{2}\left(1-\frac{1}{p_2}\right)C_o\left(\nabla^2\rho^2(r)\right)\right] \qquad (5)$$

$$p_1 = \frac{1+\varepsilon}{1+\varepsilon/A} \qquad\qquad p_2 = \frac{1+\varepsilon/2}{1+\varepsilon/(2A)}$$

Pion-nucleus elastic scattering: testing of potential

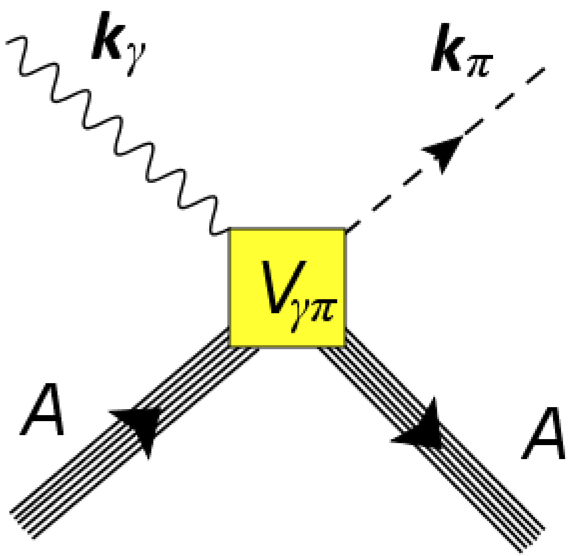


Red curves are for $\pi^\pm$
Blue curves are for π^0

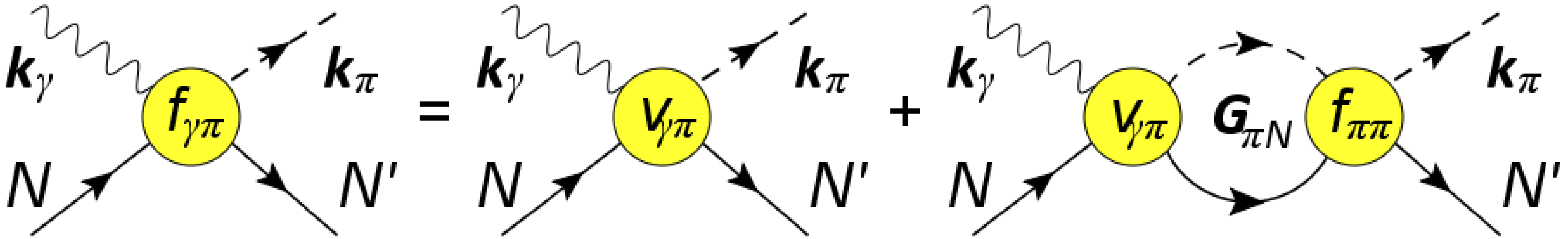
Solid curves are solutions of L.-SH.
Dashed curves are for Born approx.

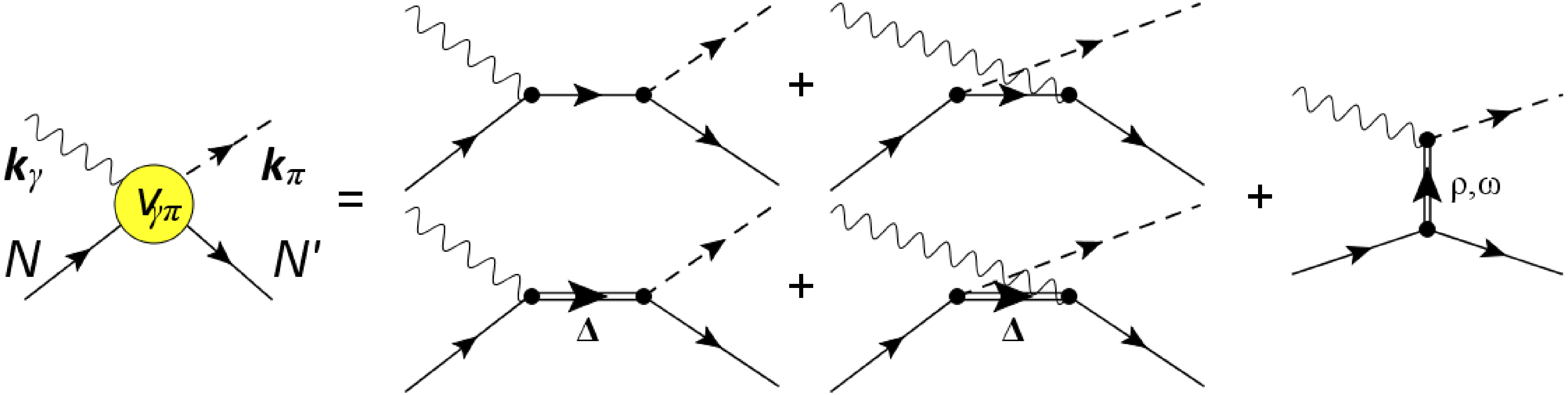
Potential parameters was taken from the $0 \le T_\pi \le 50$ MeV fit by *J.A.Carr et al., PRC 25 (1982) 952*

Pion photoproduction: PWIA



π^0 photoproduction on single nucleon





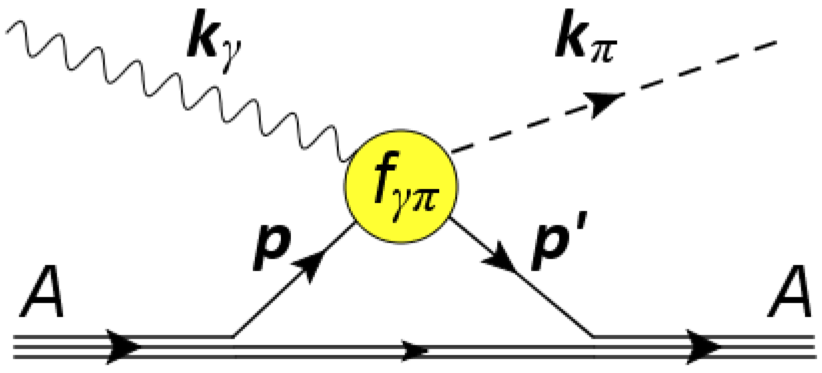
CGLN decomposition:

$$f_{\gamma\pi} = \frac{4\pi W}{m_N}\chi_f^\dagger \mathcal{F}_{\gamma\pi}\chi_i \quad (1)$$

MAID2007: $$\mathcal{F}_{\gamma\pi} = i\tilde{\boldsymbol{\sigma}}\cdot\boldsymbol{\epsilon}F_1 + (\boldsymbol{\sigma}\cdot\hat{\mathrm{k}}_\pi)(\boldsymbol{\sigma}\cdot[\hat{\mathrm{k}}_\gamma\times\boldsymbol{\epsilon}])F_2 + i(\boldsymbol{\sigma}\cdot\hat{\mathrm{k}}_\gamma)(\tilde{\mathrm{k}}_\pi\cdot\boldsymbol{\epsilon})F_3 + i(\boldsymbol{\sigma}\cdot\hat{\mathrm{k}}_\pi)(\tilde{\mathrm{k}}_\pi\cdot\boldsymbol{\epsilon})F_4 \quad (2)$$

Pion photoproduction: PWIA

Impulse approximation:



$$V^{\lambda}_{\gamma\pi}(\mathbf{k}_\gamma, \mathbf{k}_\pi) = W_A \langle 0 | \sum_{0}^{A} e^{i(\mathbf{k}-\mathbf{q})\mathbf{r}_j} f_{\gamma\pi}(\mathbf{k}_\gamma, \mathbf{k}_\pi, \mathbf{p}_j) | 0 \rangle \quad (1)$$

$$f_{\gamma\pi} = \frac{1}{2}\left(f^{p}_{\gamma\pi} + f^{n}_{\gamma\pi}\right) \quad (2)$$

Factorization approximation:

$$\mathbf{p} = -\frac{1}{A}\mathbf{k}_\gamma - \frac{A-1}{2A}\mathbf{q}, \quad \mathbf{p}' = -\frac{1}{A}\mathbf{k}_\pi + \frac{A-1}{2A}\mathbf{q} \quad (3)$$

MAID2007: $$\mathcal{F}_{\gamma\pi} = i\tilde{\boldsymbol{\sigma}}\cdot\boldsymbol{\epsilon}F_1 + (\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}_\pi)(\boldsymbol{\sigma}\cdot[\hat{\mathbf{k}}_\gamma\times\boldsymbol{\epsilon}])F_2 + i(\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}_\gamma)(\tilde{\mathbf{k}}_\pi\cdot\boldsymbol{\epsilon})F_3 + i(\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}_\pi)(\tilde{\mathbf{k}}_\pi\cdot\boldsymbol{\epsilon})F_4 \quad (4)$$

Only the spin-independent part contribute:

$$\tilde{f}^{\lambda}_{\gamma\pi} = F_2(W(\tilde{\mathbf{k}}_\pi, \tilde{\mathbf{k}}_\gamma), \tilde{\theta})\left[\hat{\tilde{\mathbf{k}}}_\gamma \times \hat{\tilde{\mathbf{k}}}_\pi\right]\cdot\mathbf{e}_\lambda \quad (5)$$

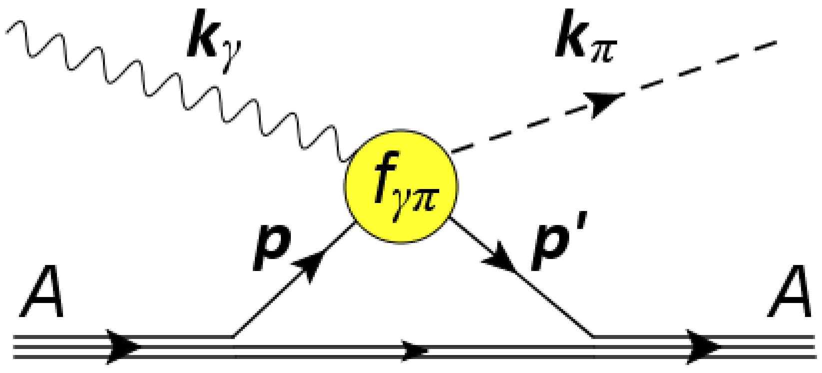
$$W = \sqrt{(k_\gamma + E_N(p))^2 - (\mathbf{k}_\gamma + \mathbf{p})^2} \quad (6)$$

$$V^{\lambda}_{\gamma\pi}(\mathbf{k}_\gamma, \mathbf{k}_\pi) = W_A F_A(q) f_2(\mathbf{k}_\gamma, \mathbf{k}_\pi) \quad (7)$$

$$F^{\mathrm{ch}}_A(q) = F_A(q) F^{\mathrm{ch}}_p(q) \quad (8)$$

Pion photoproduction: PWIA

Impulse approximation:



$$V^{\lambda}_{\gamma\pi}(\mathbf{k}_\gamma,\mathbf{k}_\pi) = W_A\langle 0|\sum_{0}^{A} e^{i(\mathbf{k}-\mathbf{q})\mathbf{r}_j} f_{\gamma\pi}(\mathbf{k}_\gamma,\mathbf{k}_\pi,\mathbf{p}_j)|0\rangle \quad (1)$$

$$f_{\gamma\pi} = \frac{1}{2}\left(f^{p}_{\gamma\pi} + f^{n}_{\gamma\pi}\right) \quad (2)$$

Factorization approximation:

$$\mathbf{p} = -\frac{1}{A}\mathbf{k}_\gamma - \frac{A-1}{2A}\mathbf{q}, \quad \mathbf{p}' = -\frac{1}{A}\mathbf{k}_\pi + \frac{A-1}{2A}\mathbf{q} \quad (3)$$

MAID2007: $$\mathcal{F}_{\gamma\pi} = i\tilde{\boldsymbol{\sigma}}\cdot\boldsymbol{\epsilon}F_1 + (\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}_\pi)(\boldsymbol{\sigma}\cdot[\hat{\mathbf{k}}_\gamma\times\boldsymbol{\epsilon}])F_2 + i(\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}_\gamma)(\tilde{\mathbf{k}}_\pi\cdot\boldsymbol{\epsilon})F_3 + i(\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}_\pi)(\tilde{\mathbf{k}}_\pi\cdot\boldsymbol{\epsilon})F_4 \quad (4)$$

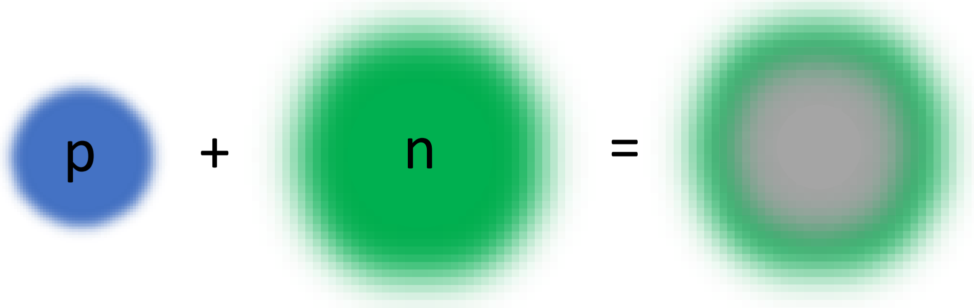
Only the spin-independent part contribute:

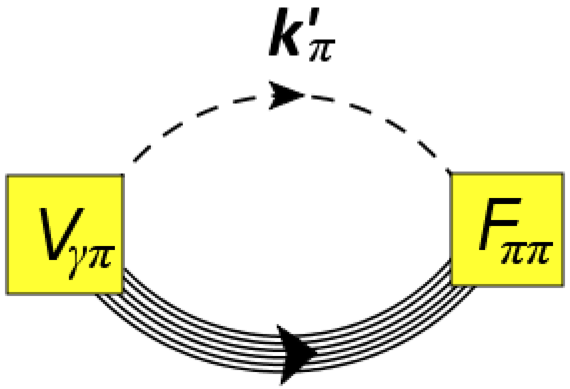
$$\tilde{f}^{\lambda}_{\gamma\pi} = F_2(W(\tilde{\mathbf{k}}_\pi,\tilde{\mathbf{k}}_\gamma),\tilde{\theta})\left[\hat{\tilde{\mathbf{k}}}_\gamma\times\hat{\tilde{\mathbf{k}}}_\pi\right]\cdot\mathbf{e}_\lambda \quad (5)$$

$$W = \sqrt{(k_\gamma + E_N(p))^2 - (\mathbf{k}_\gamma+\mathbf{p})^2} \quad (6)$$

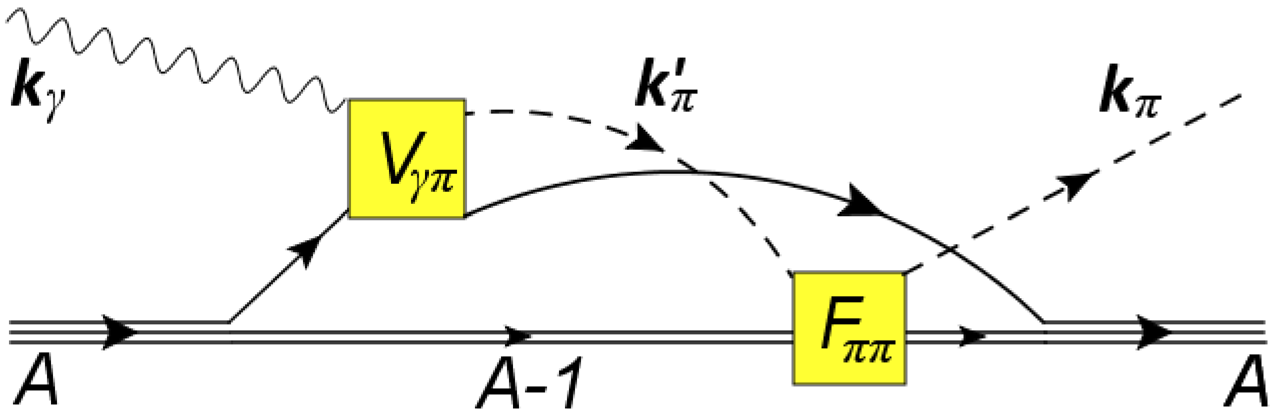
$$V^{\lambda}_{\gamma\pi}(\mathbf{k}_\gamma,\mathbf{k}_\pi) = W_A F_A(q) f_{\gamma\pi}(\mathbf{k}_\gamma,\mathbf{k}_\pi) \quad (7)$$

$$F^{\rm ch}_A(q) = F_A(q)F^{\rm ch}_p(q) \quad (8)$$




k'_π
$V_{\gamma\pi}$
$F_{\pi\pi}$

Pion photoproduction: DWIA



$$F^\lambda_{\gamma\pi}(\mathbf{k}_\gamma, \mathbf{k}_\pi) = V^\lambda_{\gamma\pi}(\mathbf{k}_\gamma, \mathbf{k}_\pi) - \frac{\alpha}{(2\pi)^2} \int \frac{d\mathbf{k}'_\pi}{\mathcal{M}(k'_\pi)} \frac{F_{\pi\pi}(\mathbf{k}_\gamma, \mathbf{k}'_\pi) V^\lambda_{\gamma\pi}(\mathbf{k}'_\pi, \mathbf{k}_\gamma)}{E(k_\pi) - E(k'_\pi) + i\varepsilon} \qquad (1)$$

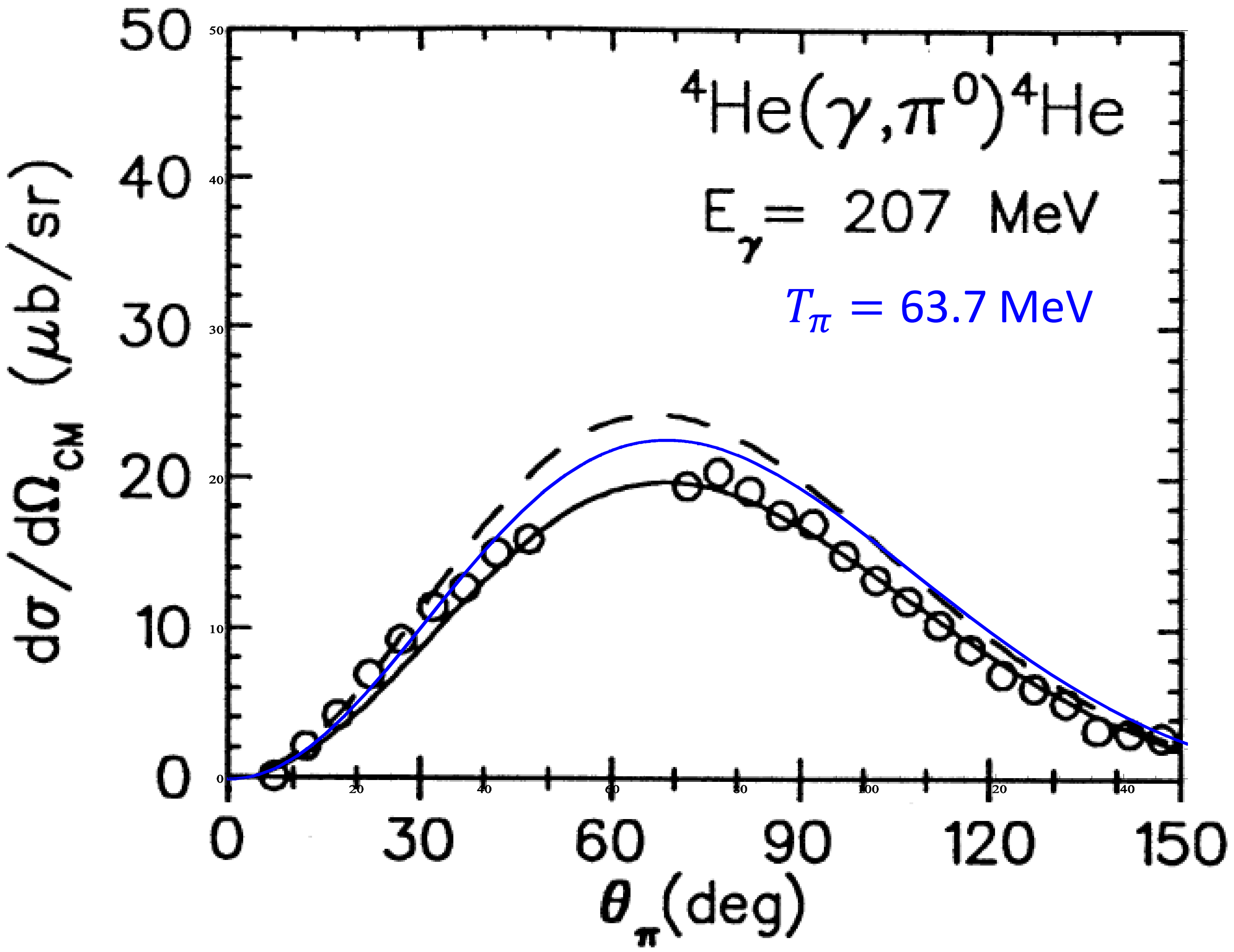
$$\boxed{\alpha = \frac{A-1}{A}} \qquad \mathcal{M}(k_\pi) = \omega(k_\pi) E_A(k_\pi)/E(k_\pi)$$

$$F_2^\Delta\left(\tilde{k}'_\pi, \tilde{k}_\pi\right) \Rightarrow \frac{\tilde{k}'_\pi}{\tilde{k}_\pi} g(\tilde{k}'_\pi, \tilde{k}_\pi) F_2^\Delta\left(W(\tilde{\mathbf{k}}_\gamma, \tilde{\mathbf{k}}_\pi), \tilde{\theta}\right) \quad (2) \qquad g(\tilde{k}'_\pi, \tilde{k}_\pi) = \left(\frac{\Lambda^2 + \tilde{k}_\pi^2}{\Lambda^2 + \tilde{k}'^2_\pi}\right), \quad \Lambda = 450\mathrm{MeV} \quad (3)$$

$$F_2\left(\tilde{k}'_\pi, \tilde{k}_\pi\right) - F_2^\Delta\left(\tilde{k}'_\pi, \tilde{k}_\pi\right) \Rightarrow g(\tilde{k}'_\pi, \tilde{k}_\pi)\left[F_2\left(\tilde{k}'_\pi, \tilde{k}_\pi\right) - F_2^\Delta\left(\tilde{k}'_\pi, \tilde{k}_\pi\right)\right] \qquad (4)$$

Testing of DWIA result

D. Drechsel et al., Nuclear Physics A 660 (1999) 423-438

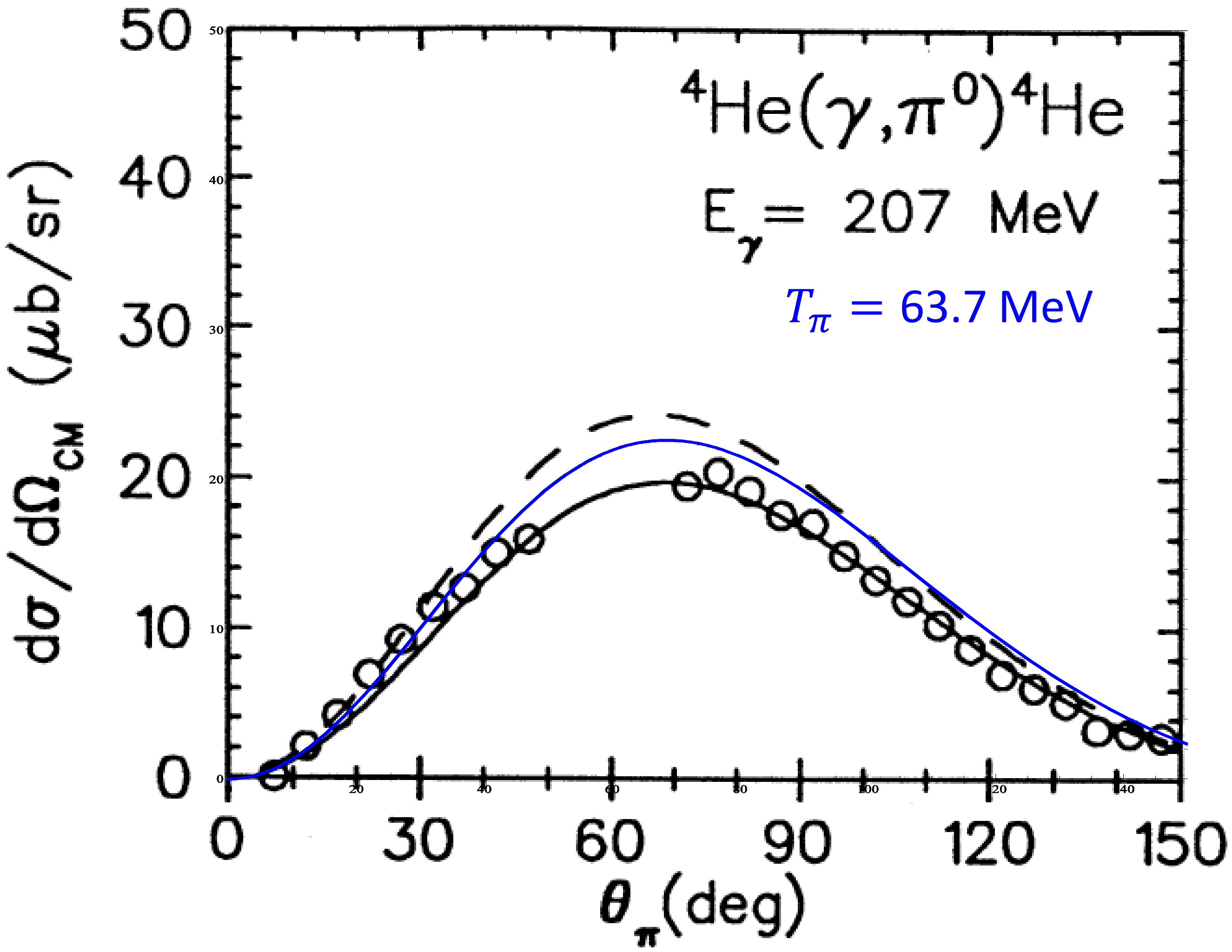


Dashed curves are the DWIA result.
Solid curves are obtained with parameterization of Δ self-energy. Experimental data are from *F. Rambo et al., Nucl. Phys. A 660 (1999) 69*

Blue curve is our DWIA result.

Testing of DWIA result

D. Drechsel et al., Nuclear Physics A 660 (1999) 423-438

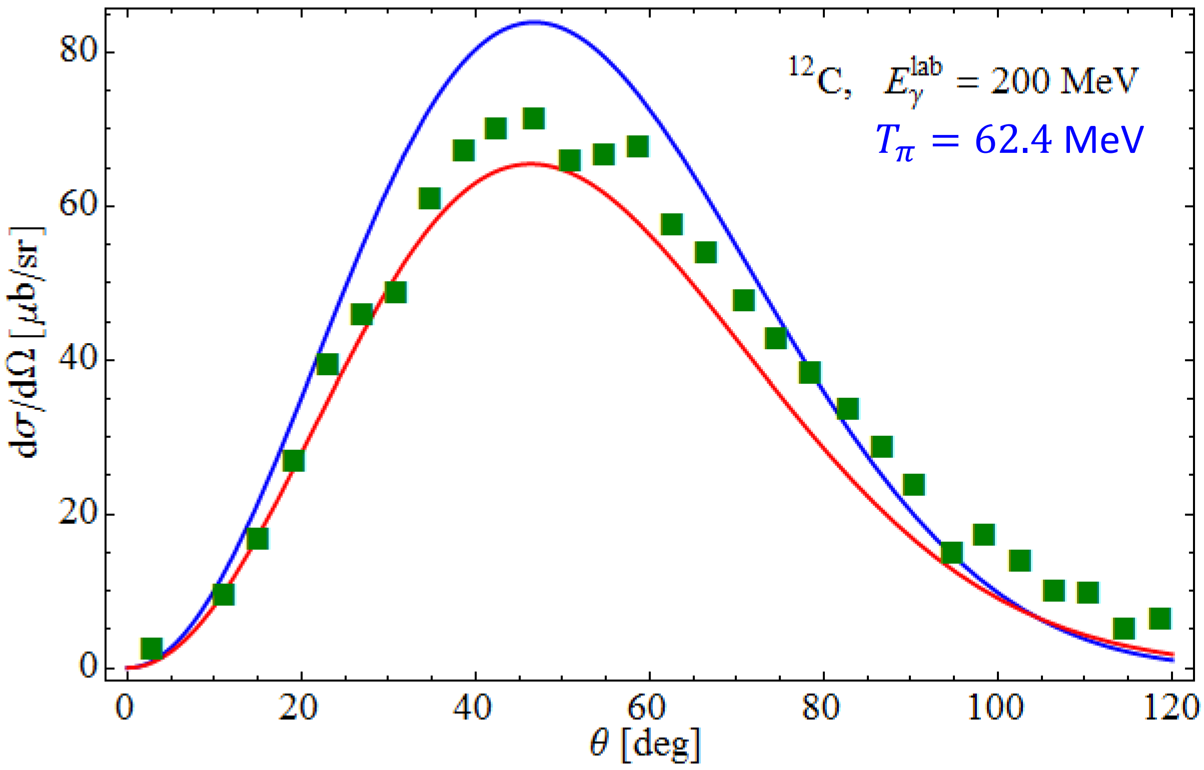


Dashed curves are the DWIA result.
Solid curves are obtained with parameterization of Δ self-energy. Experimental data are from *F. Rambo et al., Nucl. Phys. A 660 (1999) 69*

Blue curve is our DWIA result.

$$\frac{1}{W-\overline{M}_\Delta+i\overline{\Gamma}_\Delta/2} \Rightarrow \frac{1}{W-\overline{M}_\Delta+i\overline{\Gamma}_\Delta/2-\Sigma_\Delta}$$

Pion photoproduction on ^{12}C: DWIA

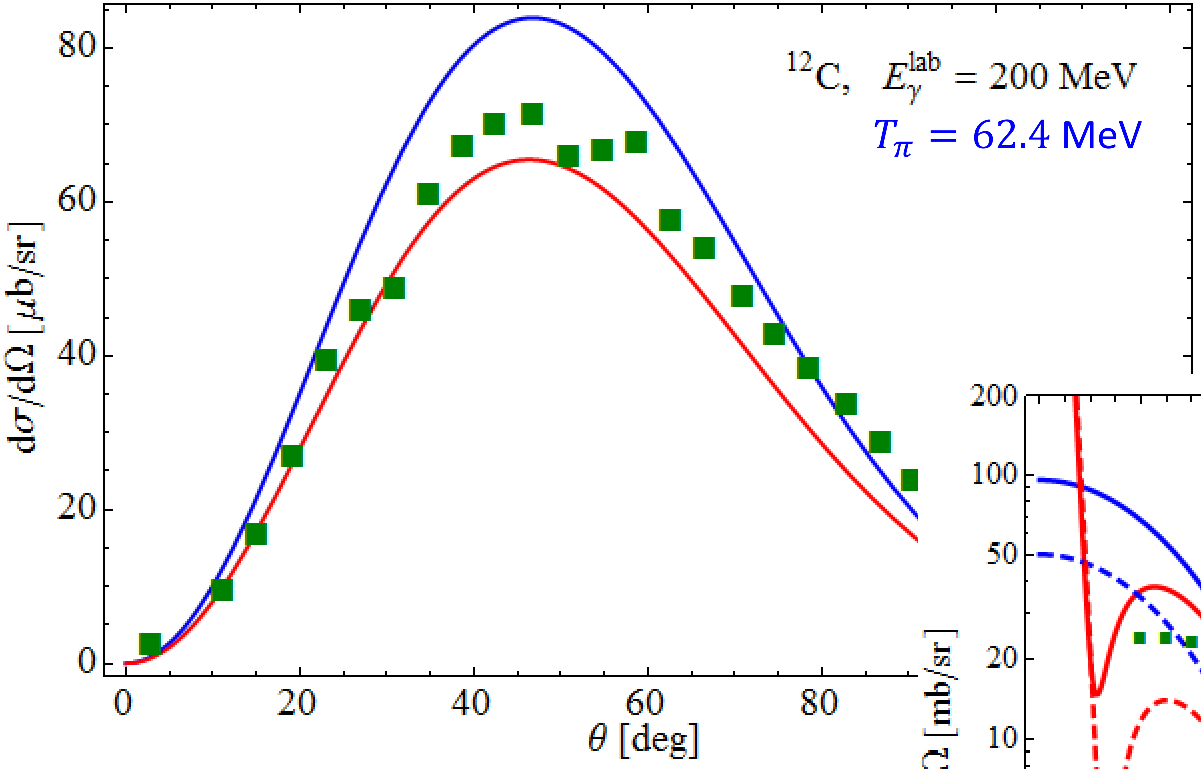


Blue curve is differential cross section in DWIA approx.
Red curves is the same, but without Δ contribution.

Data from

B. Krusche et al., Physics Letters B 526 (2002) 287-294

Pion photoproduction on ^{12}C: DWIA



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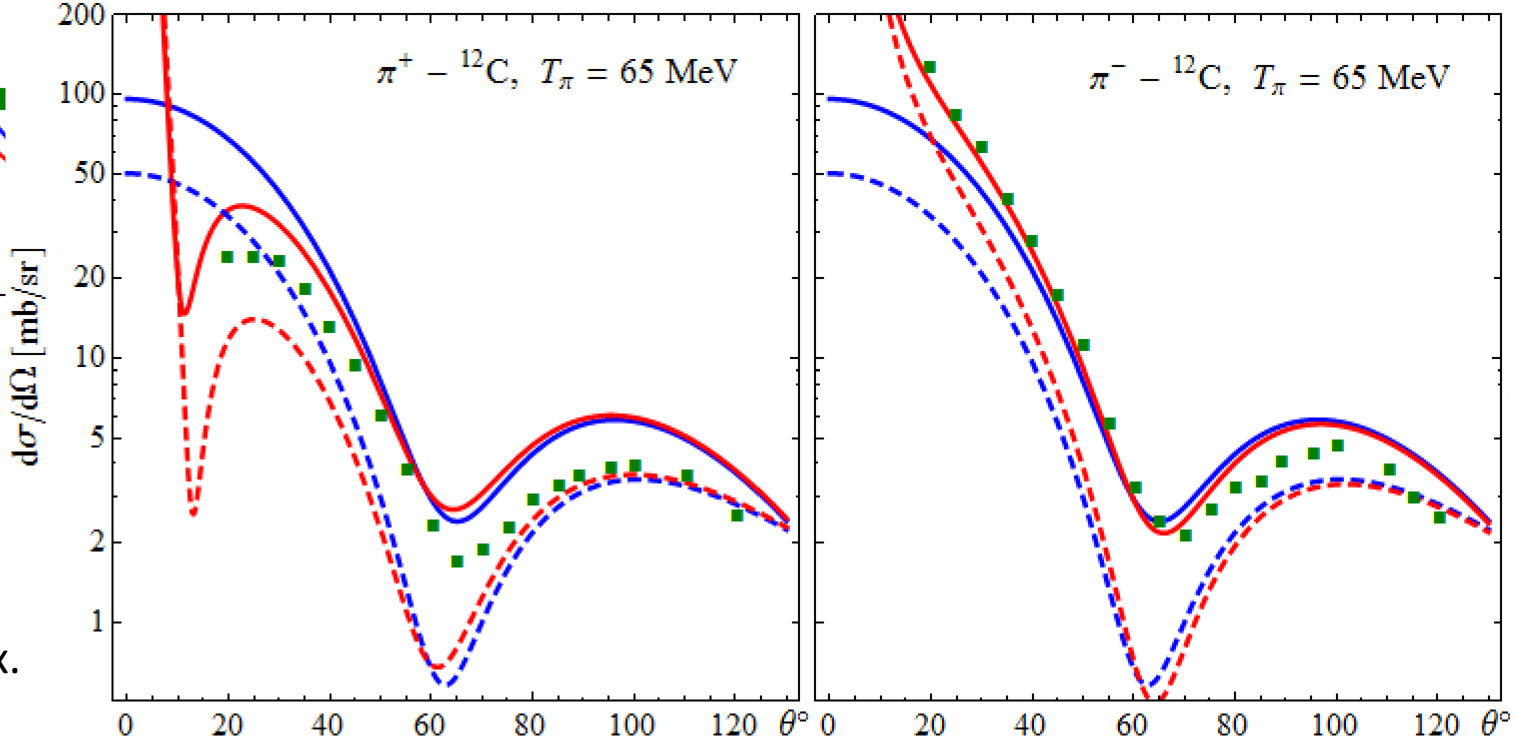

Red curves are for $\pi^\pm$
Blue curves are for π^0

Solid curves are solutions of L.-SH.
Dashed curves are for Born approx.

Work to be done

- Perform fits of optical potential for ^{4}He and ^{12}C
- Extend fit to other nuclei i.e. ^{40}Ca,^{208}Pb …
- Study of model sensitivities:
  - — Δ resonance suppression
  - — deviations from tρ
  - — investigate size of effects beyond the impulse approximation
- Theoretical error estimate