

# Electroweak corrections in the determination of $\alpha_s$

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Based on:

A.Denner, S.Dittmaier, T.Gehrmann, C.Kurz, Phys.Lett. B679 (2009) 219 [arXiv:0906.0372]

A.Denner, S.Dittmaier, T.Gehrmann, C.Kurz, Nucl.Phys. B836 (2010) 37 [arXiv:1003.0986]

S.Dittmaier, A.Huss, C.Speckner, JHEP 1211 (2012) 095 [arXiv:1210.0438]

S.Dittmaier, A.Huss, K.Rabbertz, to appear in the Les Houches Proceedings soon

## Features of and issues in EW precision calculations

### Relevance and size of EW corrections

generic size  $\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_s^2)$  suggests NLO EW  $\sim$  NNLO QCD  
but systematic enhancements possible, e.g.

- **by photon emission**  
↪ kinematical effects, mass-singular log's  $\propto \alpha \ln(m_\mu/Q)$  for bare muons, etc.
- **at high energies**  
↪ EW Sudakov log's  $\propto (\alpha/s_W^2) \ln^2(M_W/Q)$  and subleading log's

### EW corrections to PDFs at hadron colliders

induced by factorization of collinear initial-state singularities, new: **photon PDF**

### Instability of W and Z bosons

- realistic observables have to be defined via decay products (leptons,  $\gamma$ 's, jets)
- off-shell effects  $\sim \mathcal{O}(\Gamma/M) \sim \mathcal{O}(\alpha)$  are part of the NLO EW corrections

### Combining QCD and EW corrections in predictions

- how to merge results from different calculations
- reweighting procedures in MC's

## Issue of this talk

- EW corrections to two process types important for  $\alpha_s$  determination:
  - ◇ jet event-shape observables at  $e^+e^-$  colliders
  - ◇ jet production at hadron colliders
- review of the situation
  - ◇ EW corrections under control ?
  - ◇ future homework ?



# Jet event-shape observables at $e^+e^-$ colliders



## Frequently used event-shape observables $y$

- **Thrust:**  $T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$
- **Normalized heavy jet mass:**  $\rho = \max\{M_1^2, M_2^2\}/s$   
( $M_i$  = inv. mass flowing into hemispheres  $H_i$  defined by plane perpendicular to thrust axis)
- **Wide / total jet broadenings:**  
 $B_W = \max\{B_1, B_2\}, \quad B_T = B_1 + B_2, \quad B_i = \frac{\sum_{j \in H_i} |\vec{p}_j \times \vec{n}|}{2 \sum_k |\vec{p}_k|}$
- **$C$  parameter:**  
 $C = 3(\lambda_1 \lambda_2 + \lambda_2 \lambda_3 + \lambda_3 \lambda_1), \quad \{\lambda_i\} = \text{eigenvalues of } \Theta = \frac{1}{\sum_i |\vec{p}_i|} \sum_j \frac{\vec{p}_j \otimes \vec{p}_j}{|\vec{p}_j|}$
- **Jet transition variable:**  
 $Y_3 = \text{value of } y_{\text{cut}} \text{ at which the event turns from 3-jet to 2-jet type}$

Note: 2-jet configuration appears at an endpoint of  $\frac{d\sigma(y)}{dy}$  (e.g. at  $T \rightarrow 1$ )

↪ shapes of distributions sensitive to 3 and more jets, and thus to  $\alpha_s$

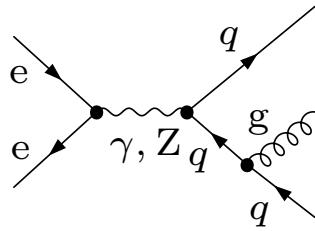
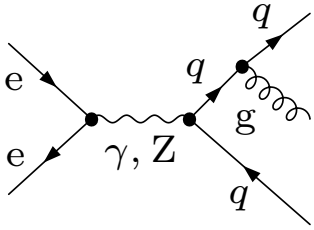
# Theory prediction for jet event shapes ( $e^+e^- \rightarrow n \text{ jets}, n \geq 3$ )

$$\begin{aligned}
 \frac{1}{\sigma_{\text{had}}} \frac{d\sigma(y)}{dy} = & \underbrace{\alpha_s C_{\text{LO}}^{\text{QCD}} + \alpha_s^2 C_{\text{NLO}}^{\text{QCD}}}_{\text{R.K.Ellis, Ross, Terrano '81; Kunszt '81; Vermaseren, Gaemers, Oldham '81; Giele, Glover '92; Catani, Seymour '96}} + \underbrace{\alpha_s^3 C_{\text{NNLO}}^{\text{QCD}}}_{\text{Gehrmann-DeRidder, Gehrmann, Glover, Heinrich '07-'09; Weinzierl '08,'09}} \\
 & + \underbrace{\text{NLL resummation}}_{\text{Catani, Turnock, Webber, Trentadue '91,'93}} + \underbrace{\text{NLL/NNLO matching}}_{\text{Gehrmann, Luisoni, Stenzel '08}} \left( + \underbrace{\text{NNLL resummation for } T}_{\text{Becher, Schwartz '08}} \right) \\
 & + \underbrace{\text{non-perturbative hadronization effects}}_{\text{Korchensky, Sterman '95; Dokshitzer, Webber '95,'97; Dokshitzer, Lucenti, Marchesini, Salam '98}} \\
 & + \underbrace{\alpha C_{\text{LO}}^{\text{EW}} + \alpha\alpha_s C_{\text{NLO}}^{\text{EW}} + \alpha^2\alpha_s C_{\text{LL}}^{\text{ISR}}}_{\text{Denner, S.D., Gehrmann, Kurz '09,'10}}
 \end{aligned}$$

- Recent NNLO QCD results already included in  $\alpha_s$  fit to event shapes  
Gehrmann, Luisoni, Stenzel '08; Dissertori et al. '08; Bethke et al. '08; Davison, Webber '08
- **NLO EW** corrections potentially of same size as **NNLO QCD**, since  $\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_s^2)$

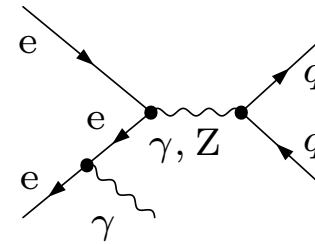
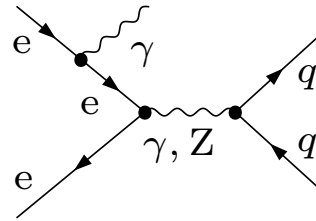
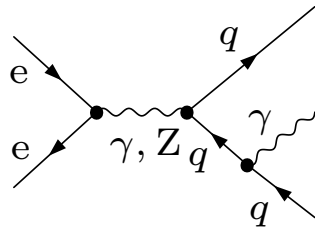
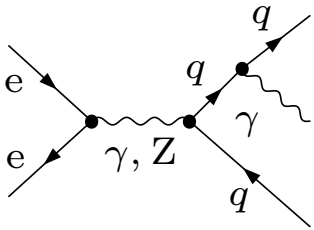
## Calculation of NLO corrections

LO diagrams for  $e^+e^- \rightarrow q\bar{q}g$ :  $\mathcal{O}(\alpha^2\alpha_s)$



$q = u, d, s, c, b$

LO diagrams for  $e^+e^- \rightarrow q\bar{q}\gamma$ :  $\mathcal{O}(\alpha^3)$

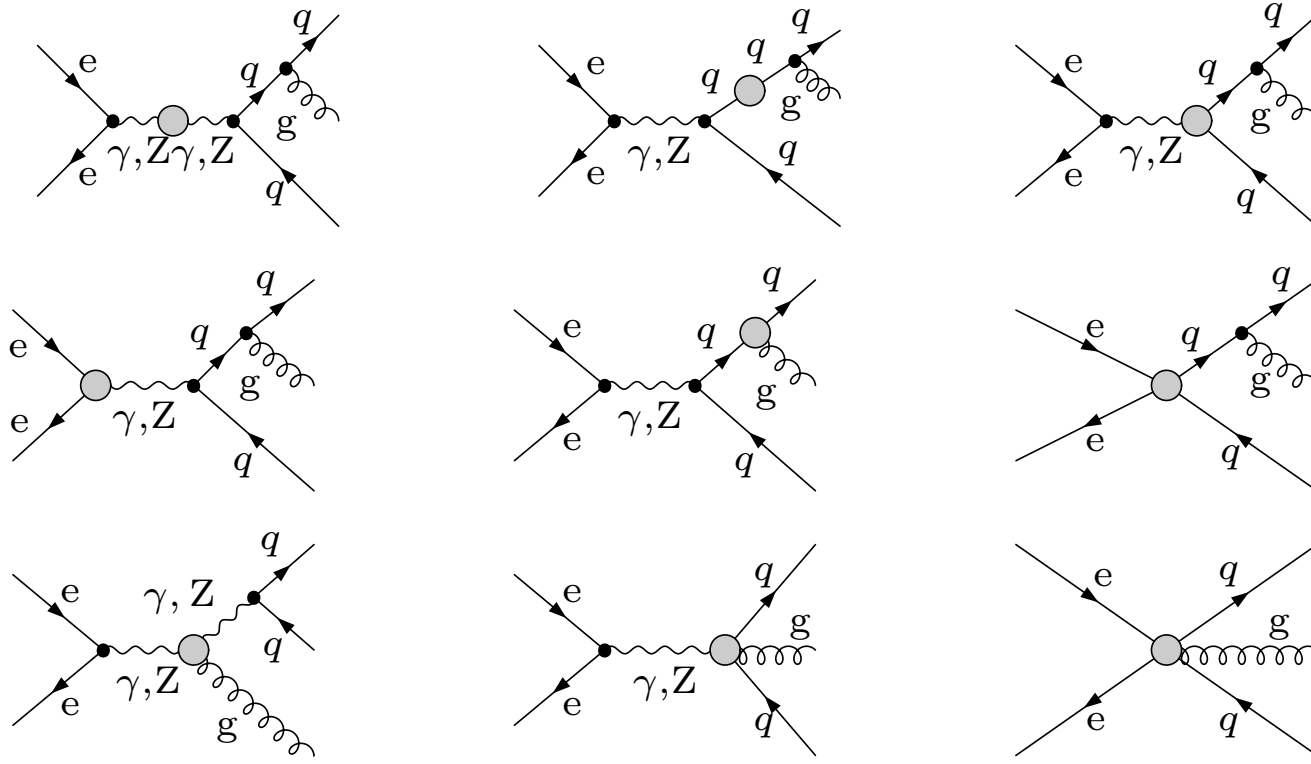


## Comments:

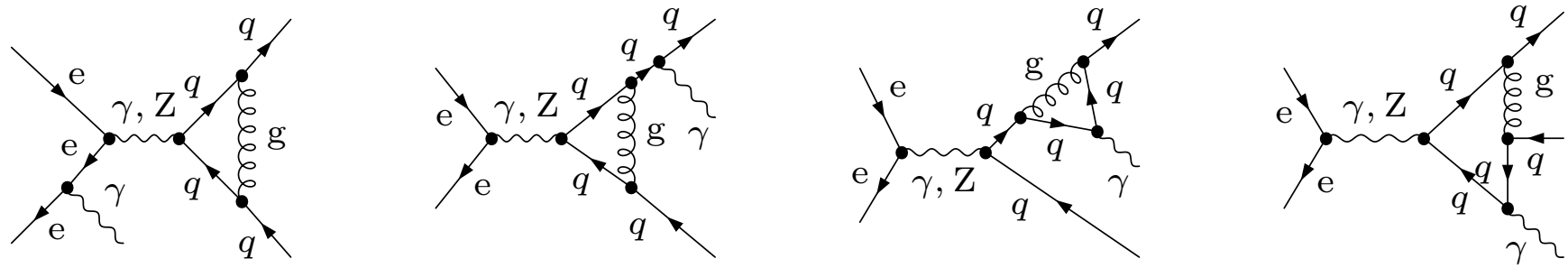
- $q\bar{q}\gamma$  final states in LO deliver contributions if  $\gamma$  is merged with  $q/\bar{q}$ , i.e. near 2-jet configurations
- focus of our calculation:  $\mathcal{O}(\alpha^3\alpha_s) =$  NLO EW correction to  $q\bar{q}g$  production  
 $=$  NLO QCD correction to  $q\bar{q}\gamma$  production

# 1PI loop insertions in EW one-loop corrections to $e^+e^- \rightarrow q\bar{q}g$

$\mathcal{O}(200)$  diagrams



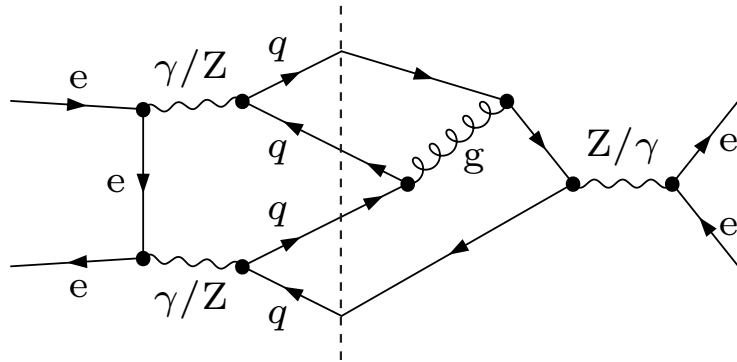
# Sample QCD one-loop diagrams for $e^+e^- \rightarrow q\bar{q}\gamma$





## Real emission corrections at $\mathcal{O}(\alpha^3\alpha_s)$ and beyond

- $e^+e^- \rightarrow q\bar{q}g\gamma$  = photon bremsstrahlung to  $q\bar{q}g$  production  
= gluon bremsstrahlung to  $q\bar{q}\gamma$  production
- QCD–EW interferences for  $e^+e^- \rightarrow q\bar{q}q\bar{q}$



↪ non-singular contributions of  $\mathcal{O}(\alpha^3\alpha_s)$  = same order as NLO EW

Interferences included in our calculation

↪ effect phenomenologically negligible ( $< 0.1\%$ )

- higher-order photonic ISR included via leading-log structure functions up to order  $\text{LO} \times \mathcal{O}(\alpha^3)$

## Definition of jet observables

**Event selection:** (closely following the procedure employed by ALEPH)

1. Discard particles too close to the beams, i.e. if  $|\cos \theta_i| > \cos \theta_{\text{cut}} = 0.965$ .
2. Reject event if  $M_{\text{visible}} < 0.9E_{\text{CM}}$ .
3. Boost to CM system of observed final-state particles.
4. Apply Durham jet algorithm with  $E$  recombination and  $y_{\text{cut}} = 0.002$  to  $q, \bar{q}, g, \gamma$   
 $\hookrightarrow$  photons appear inside jets
5. Reject “photonic events” where photon energy fraction  $z > z_{\text{cut}} = 0.9$  in a jet.

### Subtleties arising at NLO EW level:

- **Step 3 minimizes boost effects from collinear ISR photons**  
(otherwise two-jet configurations do not always appear at event-shape endpoints)

**But:** at LEP two-jet events were shifted to endpoints “by hand”

$\hookrightarrow$  renders confrontation between theory and LEP results difficult

- **Step 5 is not collinear safe**

$\hookrightarrow$  perturbative result is plagued by quark-mass singularities  $\propto \alpha \ln m_q$

**Solution:** include photon fragmentation function with non-perturbative input

Why does a naive  $\gamma$ -jet separation by a jet algorithm not work ?

- collinear quarks and photons have to be recombined  $\rightarrow (q\gamma) = \text{jet}$   
otherwise corrections  $\propto \ln(m_q^2/Q^2) \rightarrow$  perturbative “IR instability”
- quark and gluon jets cannot be distinguished event by event  
 $\hookrightarrow$  common recombination required for quarks/gluons with photons

$$\Rightarrow \underbrace{(\mathbf{g}_{\text{hard}} + \boldsymbol{\gamma}_{\text{soft}})}_{\text{EW corr. to 3 jets}} \text{ and } \underbrace{(\mathbf{g}_{\text{soft}} + \boldsymbol{\gamma}_{\text{hard}})}_{\text{QCD corr. to 2 jets} + \boldsymbol{\gamma}} \text{ both appear as 3 jets}$$

Solution:

- exclude events with photon energy fraction  $z_\gamma = \frac{E_\gamma}{E_{\text{jet}} + E_\gamma} > z_0$   
for (jet +  $\gamma$ ) quasiparticles

- subtract convolution of LO cross section with

$$D_{q \rightarrow \gamma}^{\overline{\text{MS}}}(z_\gamma, \mu_{\text{fact}}) \Big|_{\text{mass.reg.}} = P_{q \rightarrow \gamma}(z_\gamma) \left[ \ln \frac{m_q^2}{\mu_{\text{fact}}^2} + 2 \ln z_\gamma + 1 \right] \leftarrow \text{cancels coll. singularities}$$

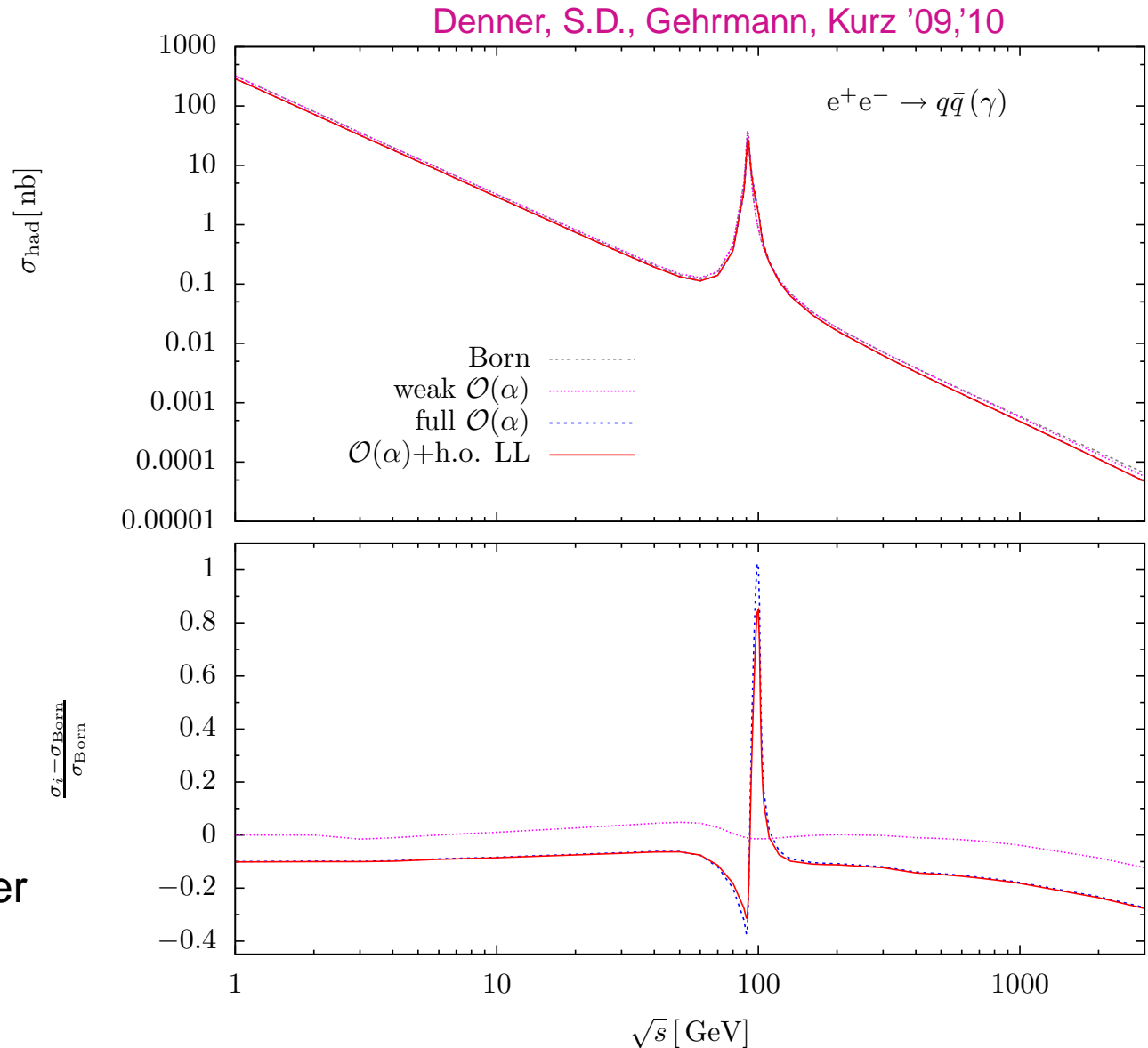
$$+ D_{q \rightarrow \gamma}^{\text{ALEPH}}(z_\gamma, \mu_{\text{fact}}) \leftarrow \text{non-perturbative part fitted to ALEPH data on } e^+e^- \rightarrow \text{jet} + \gamma$$

where  $P_{q \rightarrow \gamma}(z_\gamma) = \frac{1+(1-z_\gamma)^2}{z_\gamma} =$  quark-to-photon splitting function

# Numerical results

## Total hadronic cross section

- largest EW corrections due to ISR  
(radiative return cut off by cut  $M_{\text{visible}} < 0.9\sqrt{s}$ )
- ISR beyond one loop relevant (some %) for  $\sqrt{s} \sim M_Z$
- weak corrs. of  $\mathcal{O}(5\%)$ , increasingly negative for large  $\sqrt{s}$
- Note:  $\sigma_{\text{had}}$  calculated to same perturbative order as  $d\sigma/dy$  to obtain a proper normalization

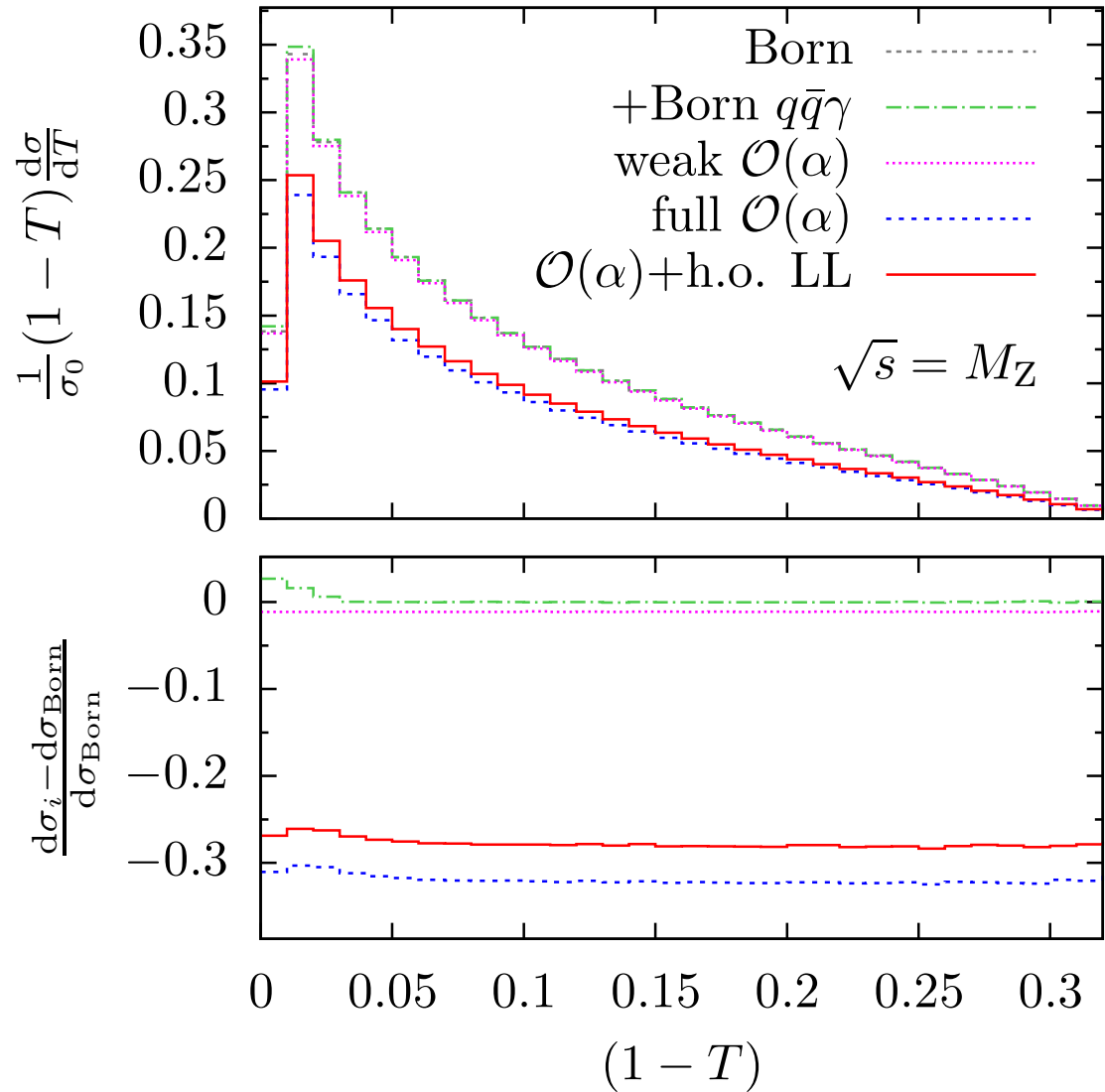


# Thrust distribution — normalized to LO hadronic cross section $\sigma_0$

$$\sqrt{s} = M_Z$$

- large ISR effects as for  $\sigma_{\text{had}}$
- h.o. ISR  $\sim$  some %
- genuine weak effects of few %, but flat
- $q\bar{q}\gamma$  final states visible for  $T \rightarrow 1$

Denner, S.D., Gehrmann, Kurz '09,'10

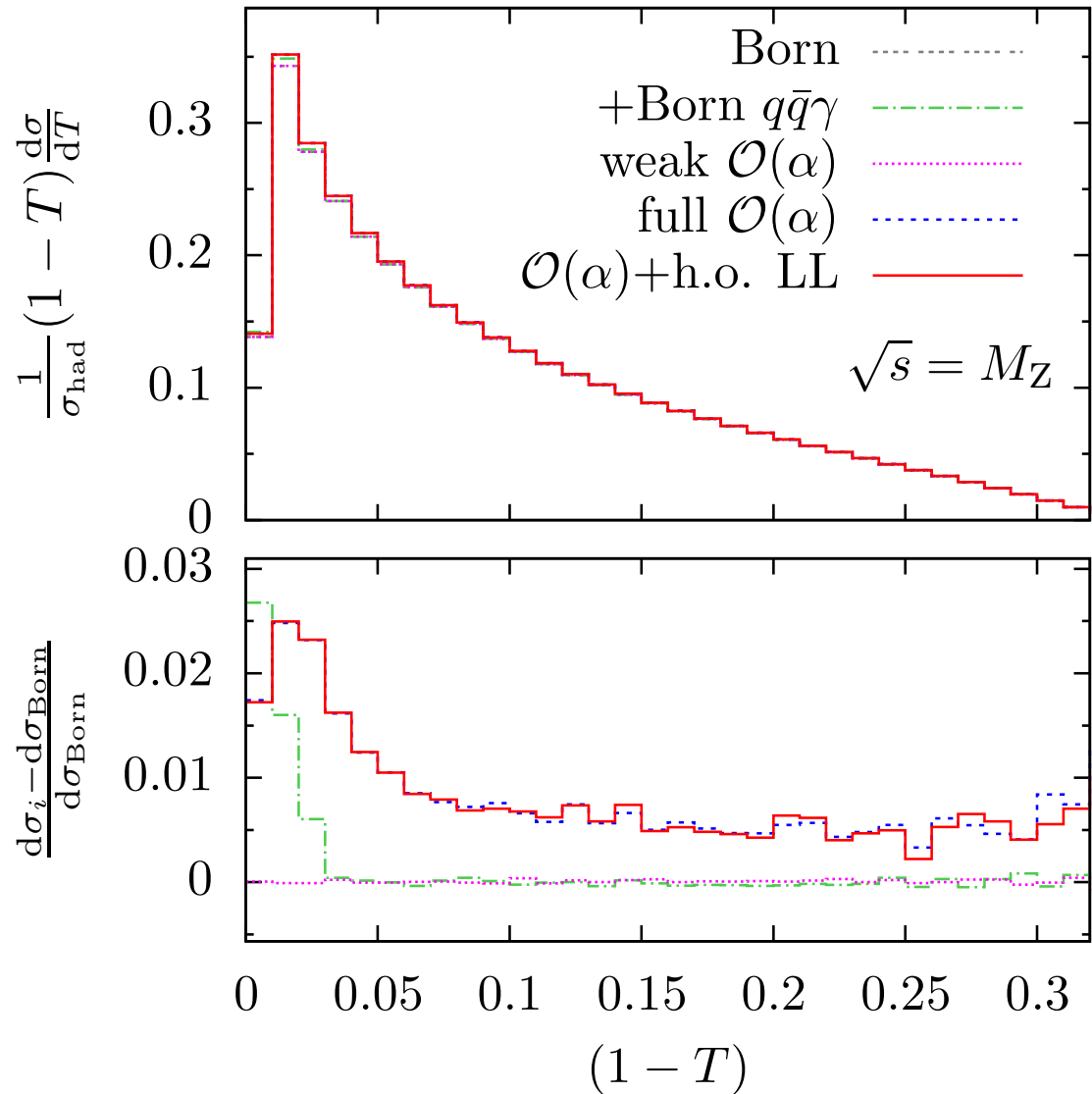


# Thrust distribution — normalized to full hadronic cross section $\sigma_{\text{had}}$

$$\sqrt{s} = M_Z$$

- large ISR effects in  $\frac{d\sigma}{dT}$  cancel against  $\sigma_{\text{had}}$   
 $\hookrightarrow$  % effects
- h.o. ISR irrelevant  
 (proper normalization important)
- genuine weak effects below 0.1%
- $q\bar{q}\gamma$  final states  $\sim \mathcal{O}(2\%)$  for  $T \gtrsim 0.97$

Denner, S.D., Gehrmann, Kurz '09,'10

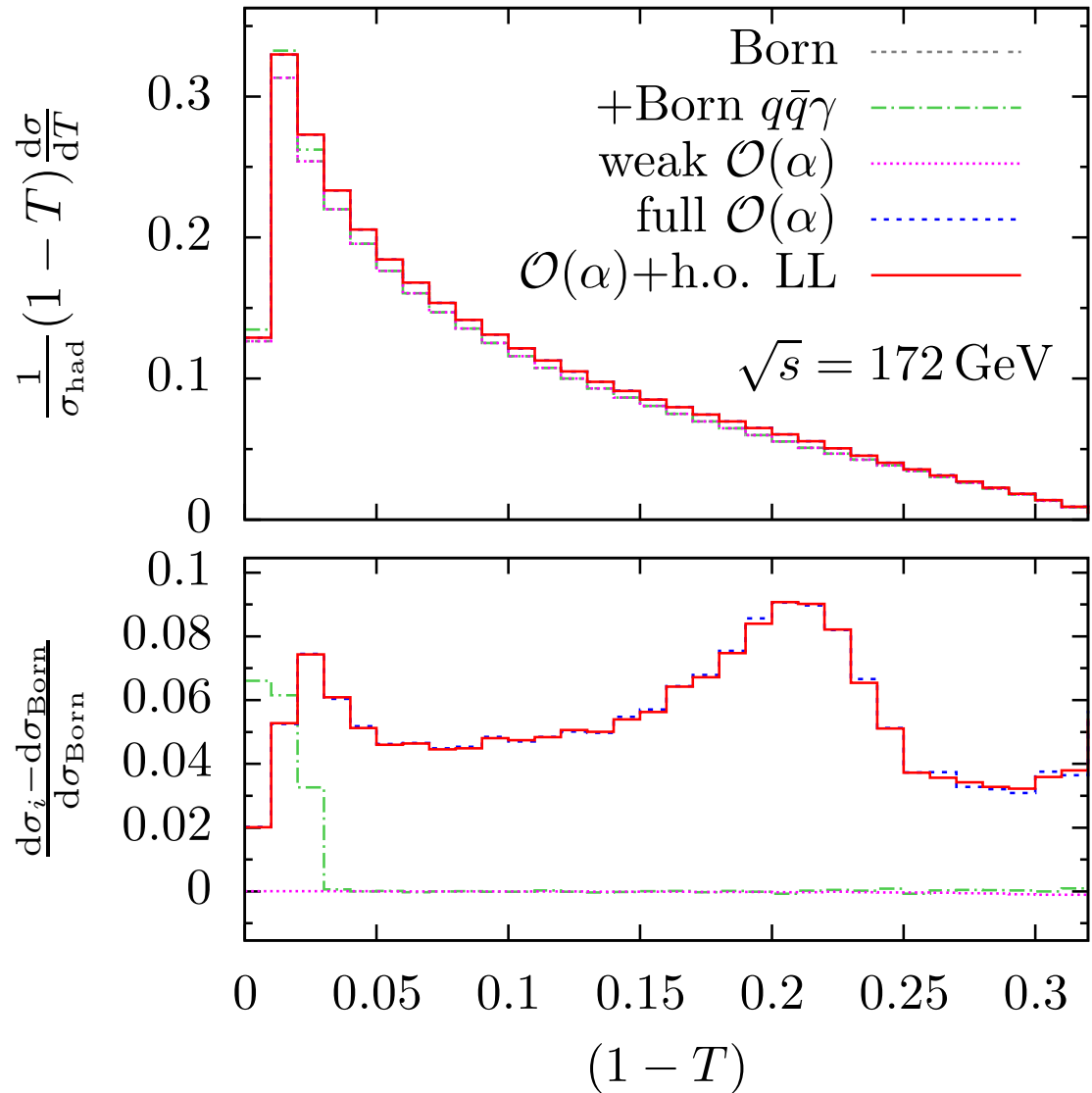


# Normalized thrust distribution — increasing CM energy

$$\sqrt{s} = 172 \text{ GeV}$$

- ISR develops structures that depend on  $y_{\text{cut}}$  and on event selection
- h.o. ISR irrelevant
- genuine weak effects remain small ( $\sim 1\%$  at  $\sqrt{s} = 500 \text{ GeV}$ )
- $q\bar{q}\gamma$  final states  $\sim$  some % for  $T \gtrsim 0.97$

Denner, S.D., Gehrmann, Kurz '09,'10



# Normalized thrust distribution — increasing CM energy

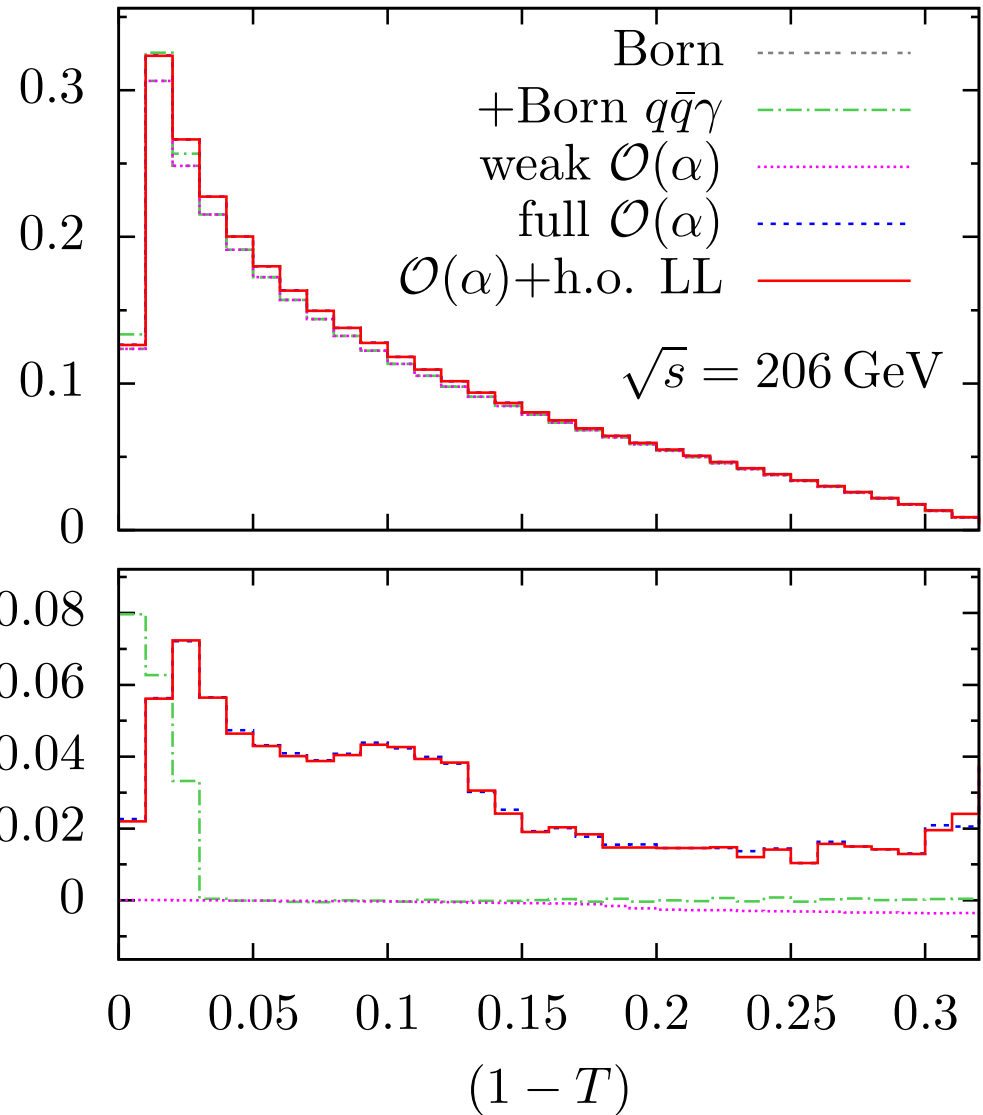
$$\sqrt{s} = 206 \text{ GeV}$$

- ISR develops structures that depend on  $y_{\text{cut}}$  and on event selection
- h.o. ISR irrelevant
- genuine weak effects remain small ( $\sim 1\%$  at  $\sqrt{s} = 500 \text{ GeV}$ )
- $q\bar{q}\gamma$  final states  $\sim$  some % for  $T \gtrsim 0.97$

$$\frac{1}{\sigma_{\text{had}}} (1 - T) \frac{d\sigma}{dT}$$

$$\frac{d\sigma_i - d\sigma_{\text{Born}}}{d\sigma_{\text{Born}}}$$

Denner, S.D., Gehrmann, Kurz '09,'10





# Normalized thrust distribution — increasing CM energy

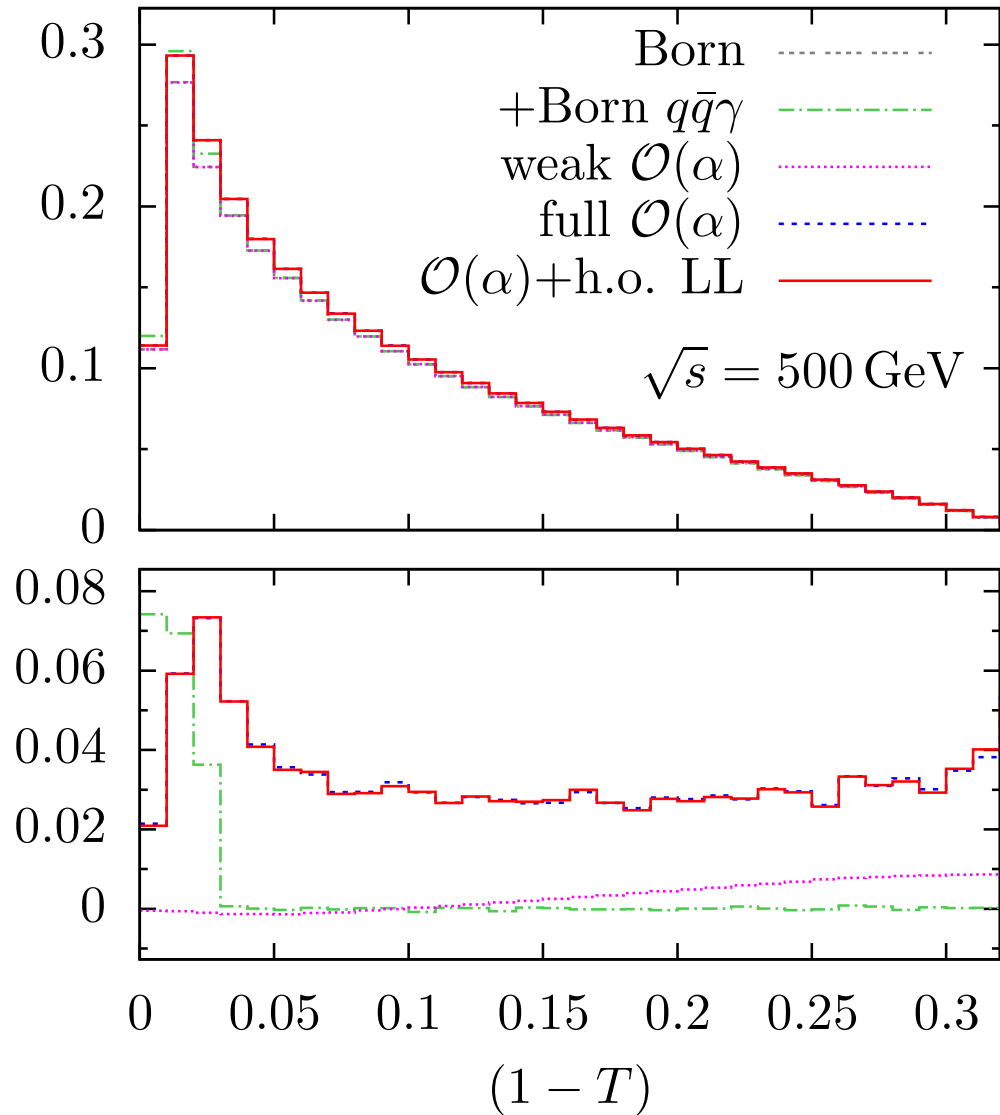
$$\sqrt{s} = 500 \text{ GeV}$$

- ISR develops structures that depend on  $y_{\text{cut}}$  and on event selection
- h.o. ISR irrelevant
- genuine weak effects remain small ( $\sim 1\%$  at  $\sqrt{s} = 500 \text{ GeV}$ )
- $q\bar{q}\gamma$  final states  $\sim$  some % for  $T \gtrsim 0.97$

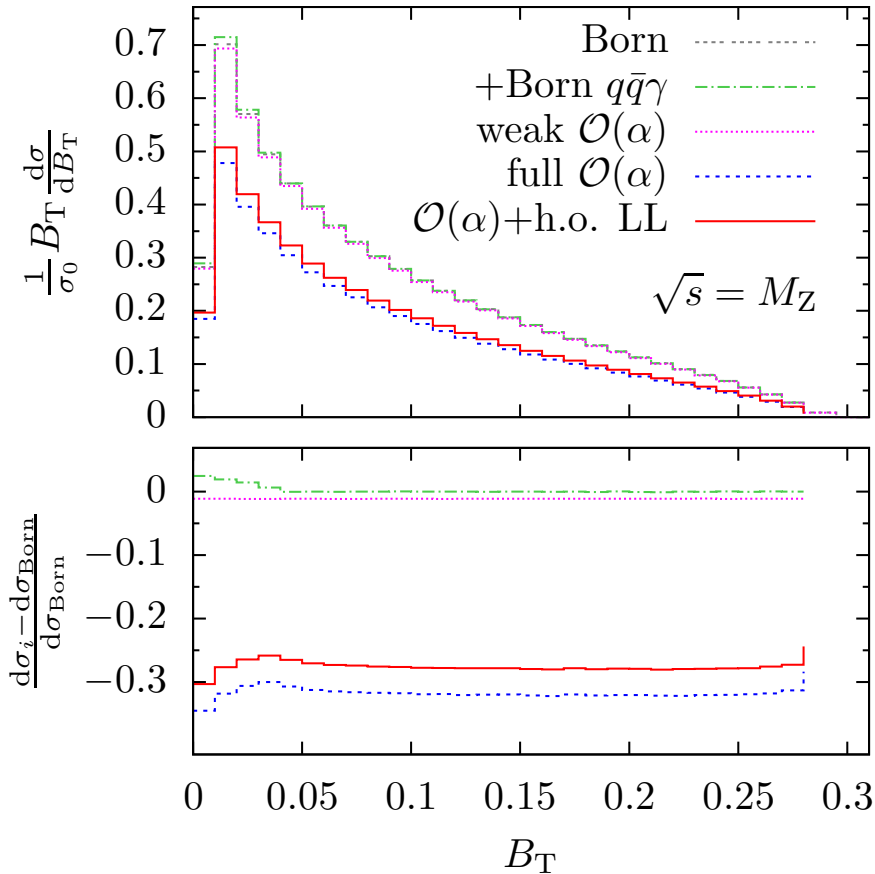
$$\frac{1}{\sigma_{\text{had}}} (1 - T) \frac{d\sigma}{dT}$$

$$\frac{d\sigma_i - d\sigma_{\text{Born}}}{d\sigma_{\text{Born}}}$$

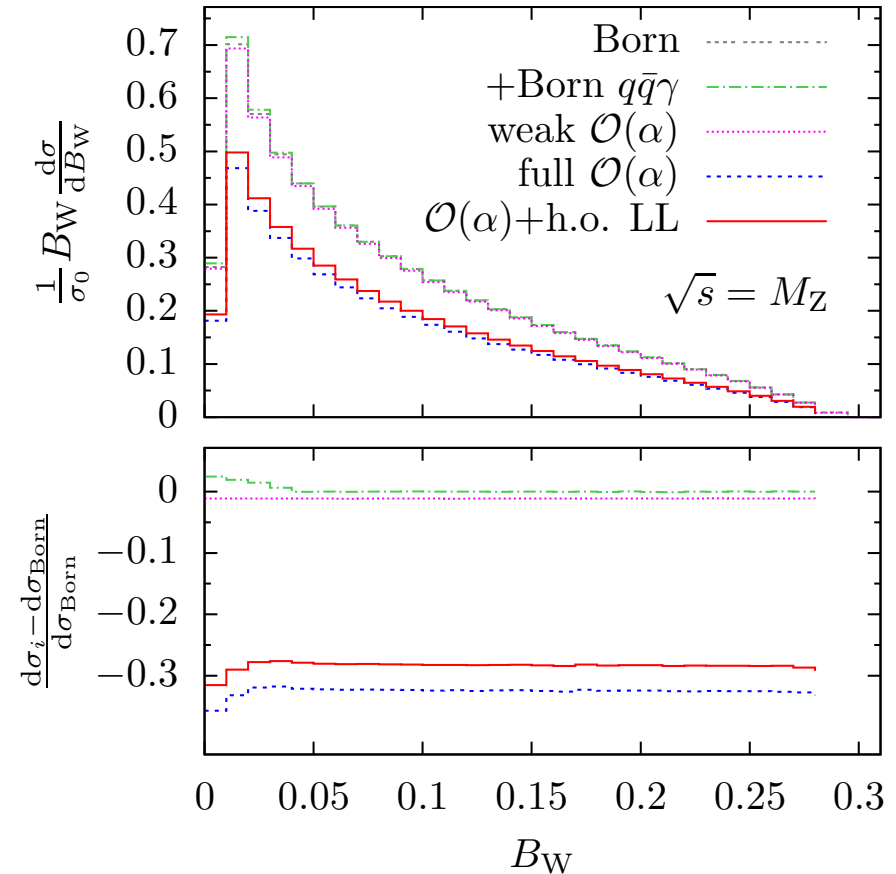
Denner, S.D., Gehrmann, Kurz '09,'10



# Jet broadenings — yet another example

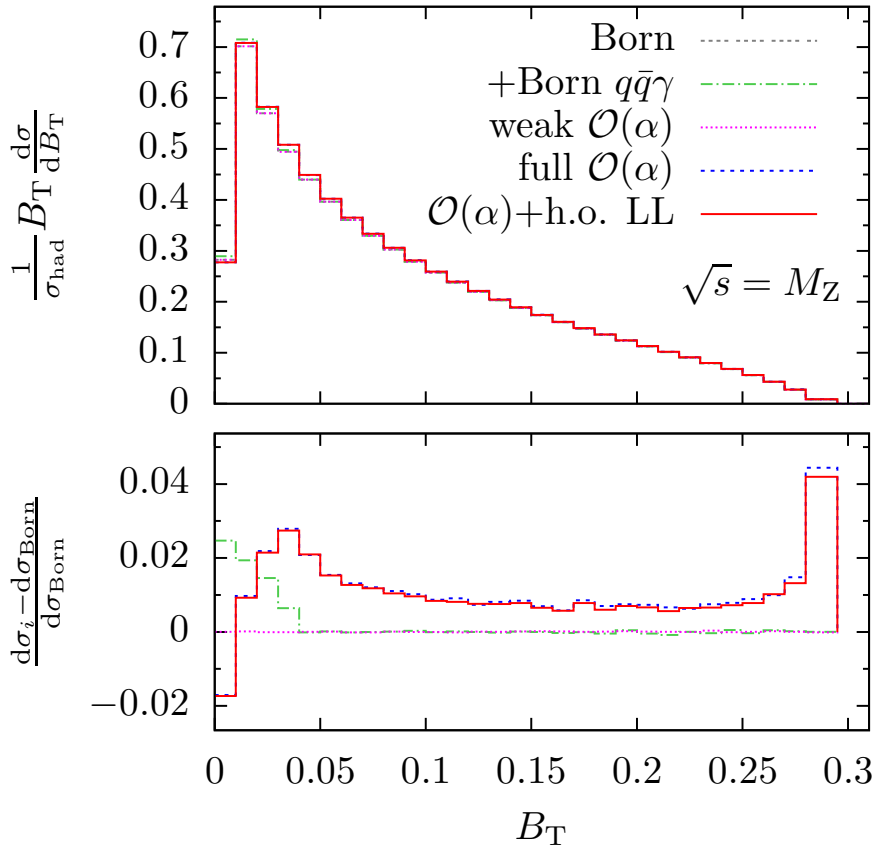


Denner, S.D., Gehrmann, Kurz '09,'10

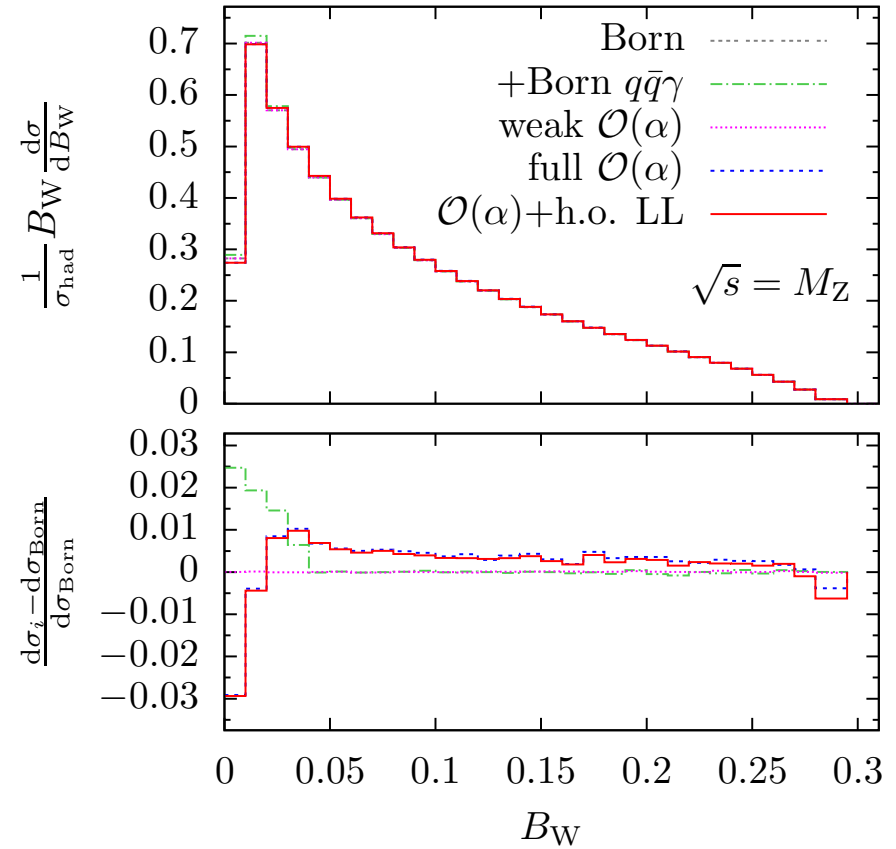


Overall size, energy dependence, and qualitative features of EW effects similar for all event shapes.

# Jet broadenings — yet another example

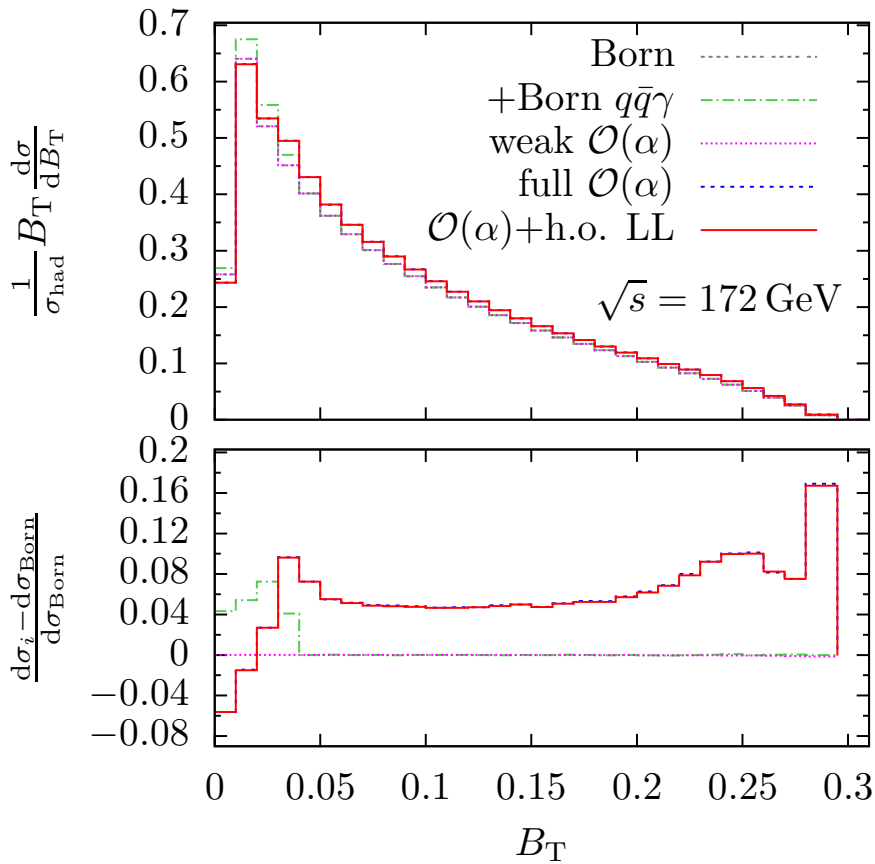


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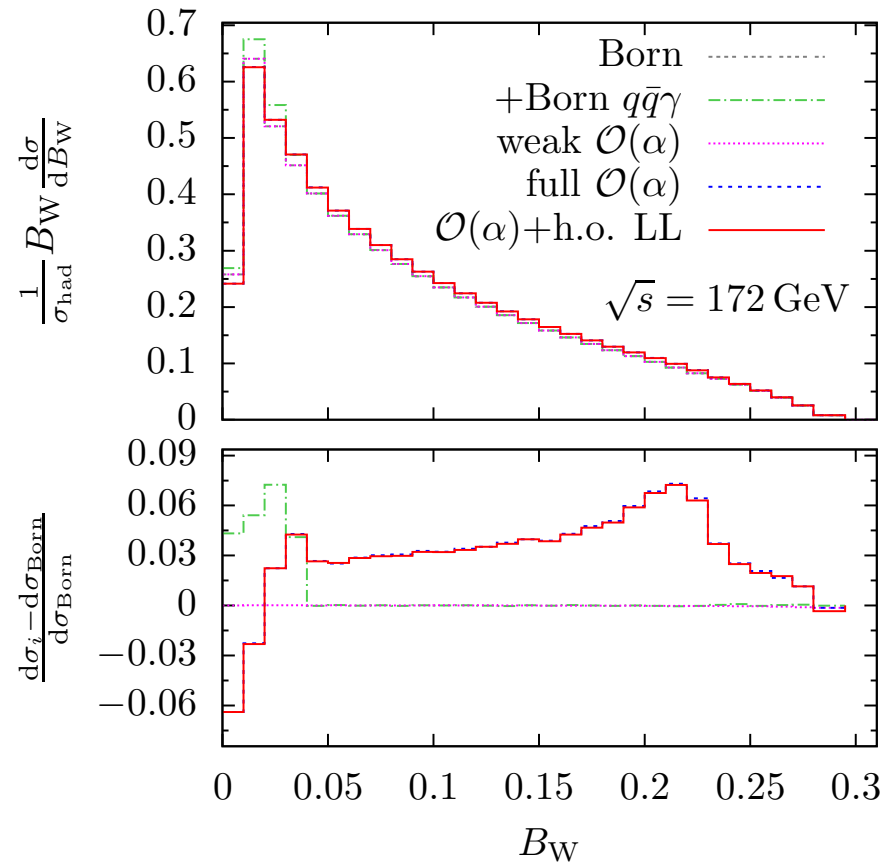


Overall size, energy dependence, and qualitative features of EW effects similar for all event shapes.

# Jet broadenings — yet another example

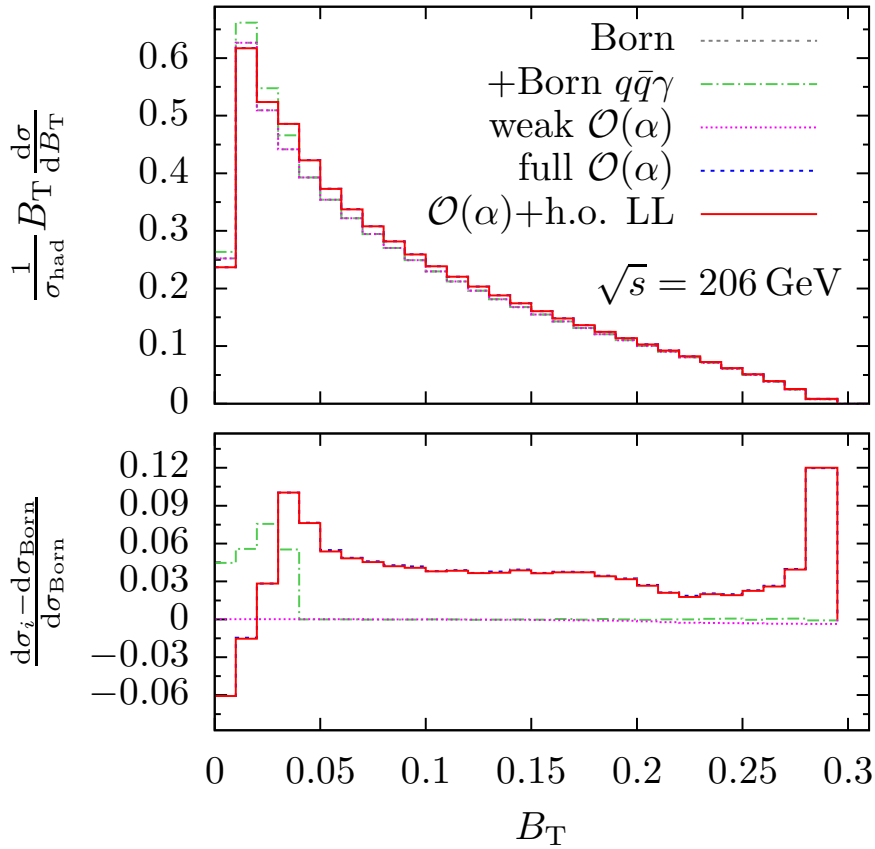


Denner, S.D., Gehrmann, Kurz '09,'10

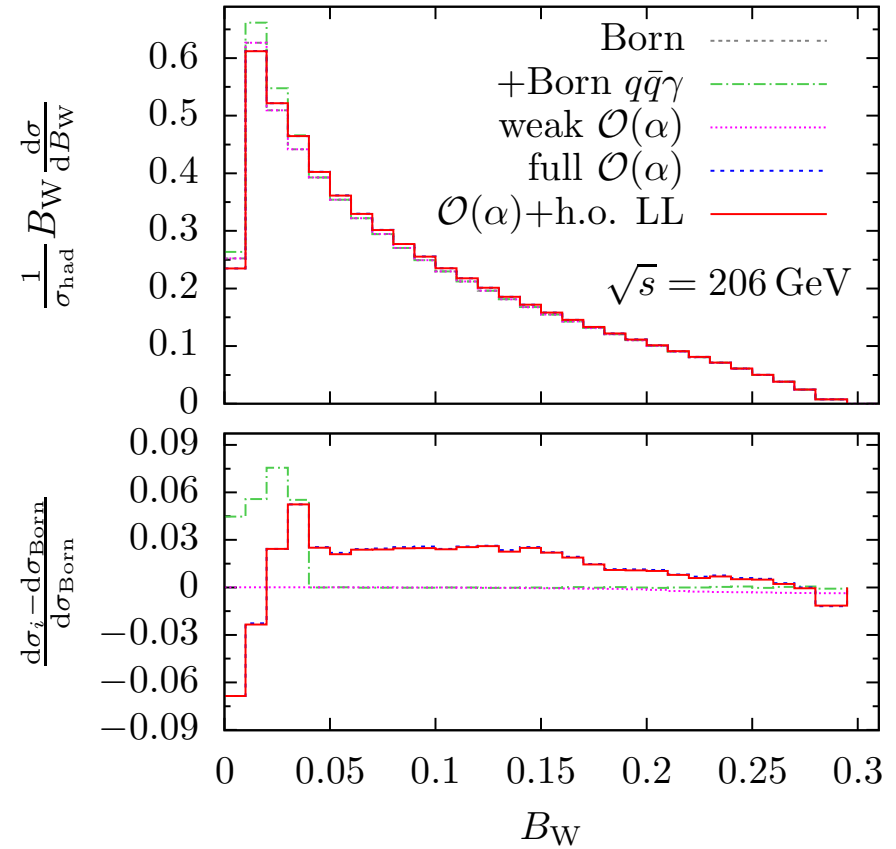


Overall size, energy dependence, and qualitative features of EW effects similar for all event shapes.

# Jet broadenings — yet another example

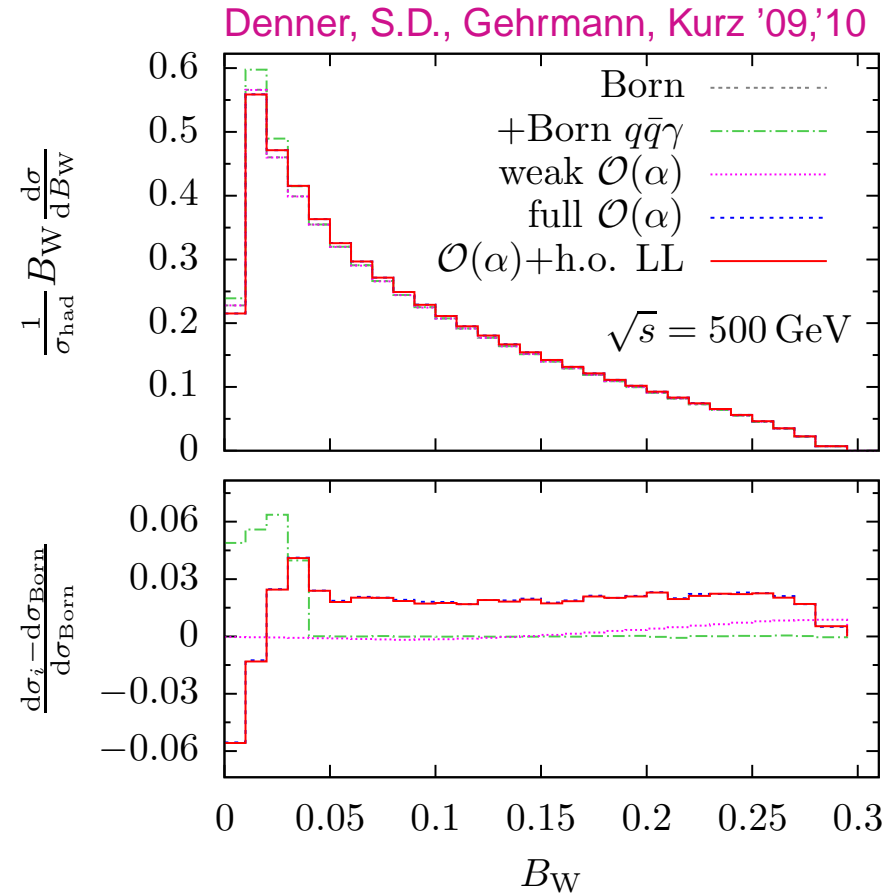
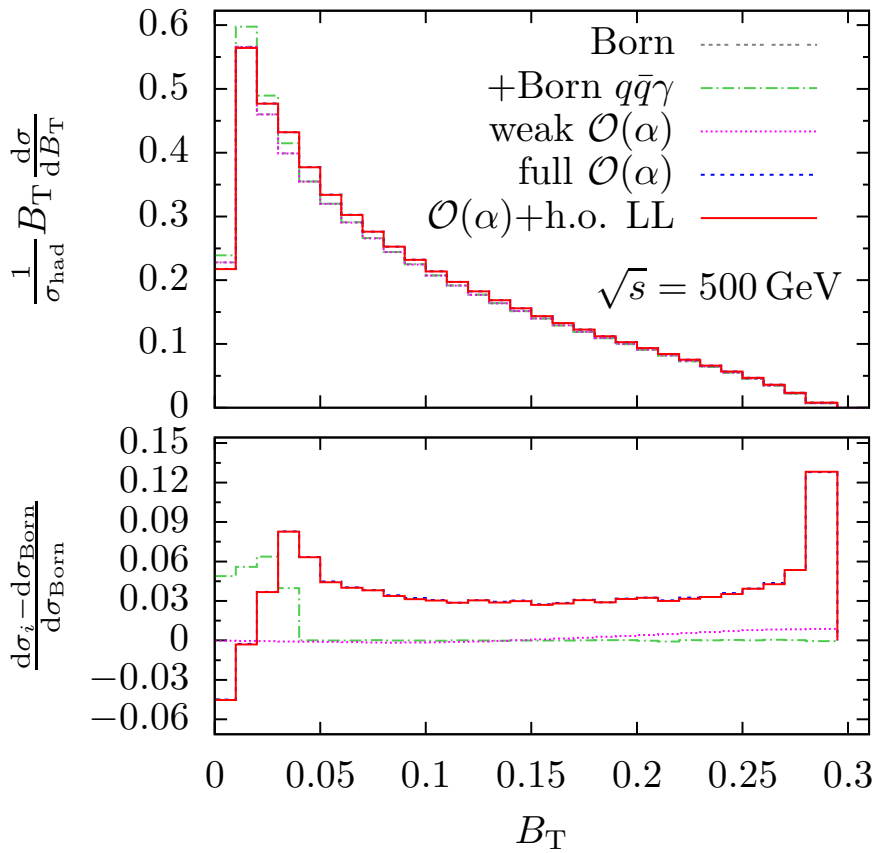


Denner, S.D., Gehrmann, Kurz '09,'10



Overall size, energy dependence, and qualitative features of EW effects similar for all event shapes.

# Jet broadenings — yet another example



Overall size, energy dependence, and qualitative features of EW effects similar for all event shapes.

# Jet production at hadron colliders



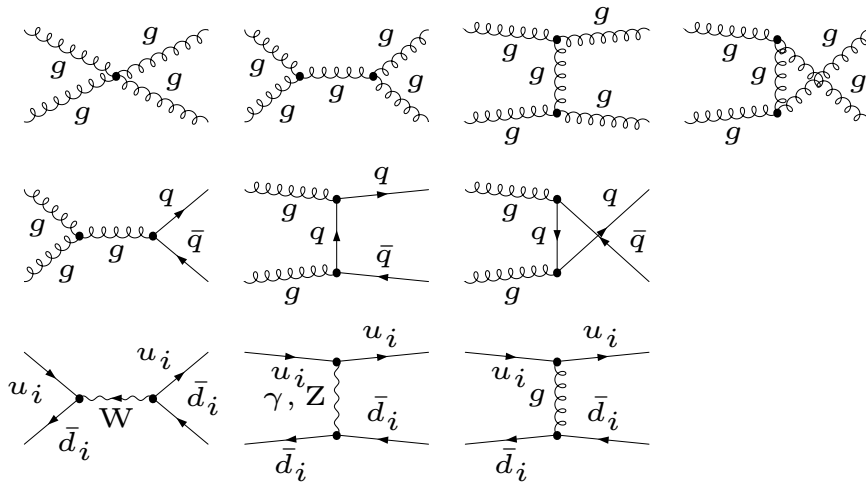
# NLO EW corrections to jet and heavy-quark production

- $pp \rightarrow 2 \text{ jets}$ 
  - ◇ purely weak NLO corrections known Moretti, Nolten, Ross '05,'06 (disagree with others); Scharf (prelim.) '09; S.D., Huss, Speckner '12
  - ◇ photonic NLO corrections unknown (but expected to be  $\lesssim 1\%$ )
  - ◇ no EW corrections available for  $pp \rightarrow \geq 3 \text{ jets}$
  
- $pp \rightarrow t\bar{t}$ 
  - ◇ SM correction Beenakker et al. '94; Moretti, Nolten, Ross '06; Kühn, Scharf, Uwer '06,'13; Bernreuther, Fückler, Si '08; Hollik, Kollar '08
  - ◇ THDM and MSSM Hollik, Möhle, Wackerath '97
  - ◇ no EW corrections with top-quark decays yet
  
- $pp \rightarrow b\bar{b}$  Maina, Moretti, Nolten, Ross '03; Kühn, Scharf, Uwer '09



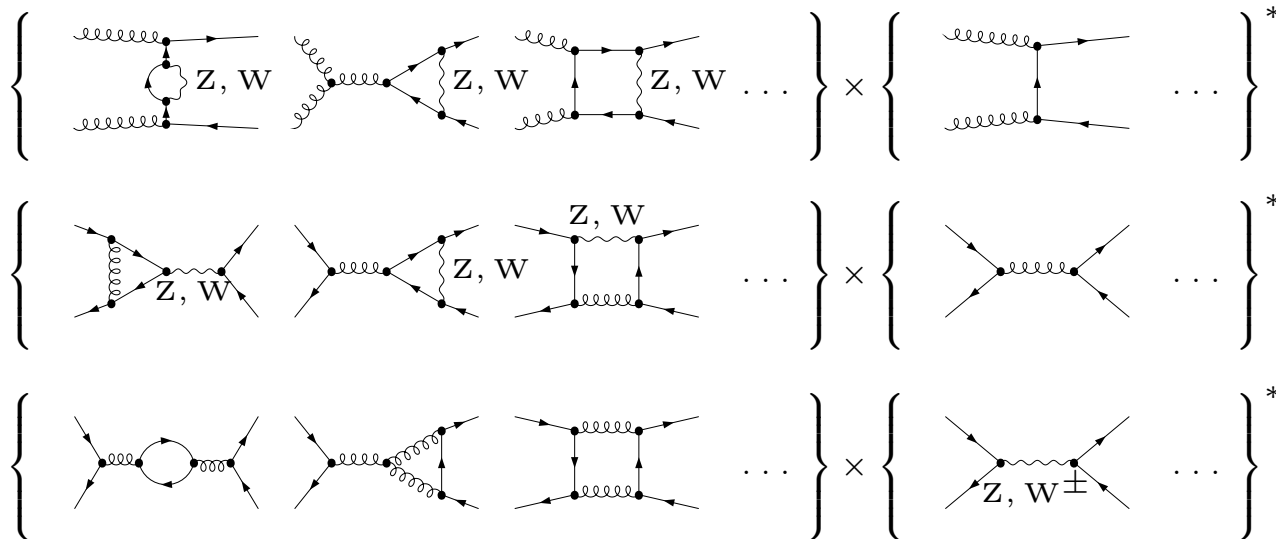
# EW corrections to dijet production – typical contributions

Tree contributions:  $\mathcal{O}(\alpha_s^2)$ ,  $\mathcal{O}(\alpha_s\alpha)$ ,  $\mathcal{O}(\alpha^2)$



Loop contributions:  $\mathcal{O}(\alpha_s^2\alpha)$

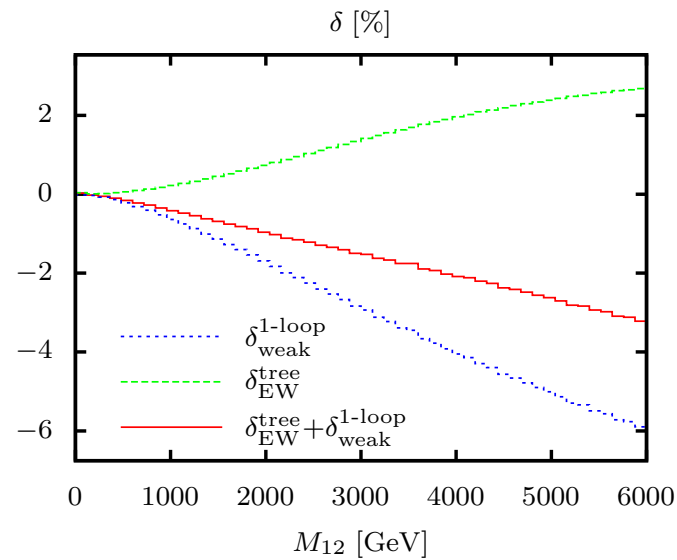
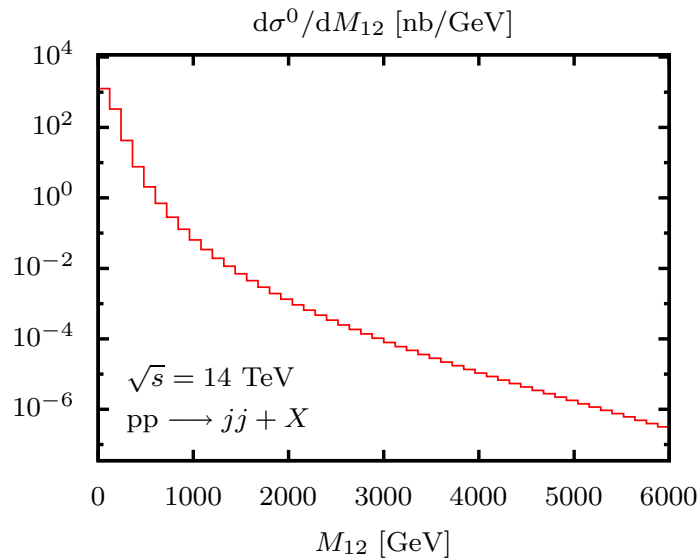
Note: involves 3-jet final states as well !



# Numerical results

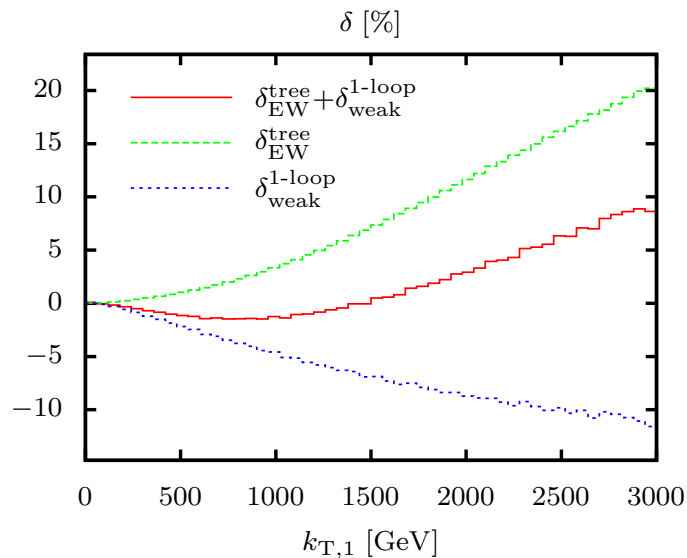
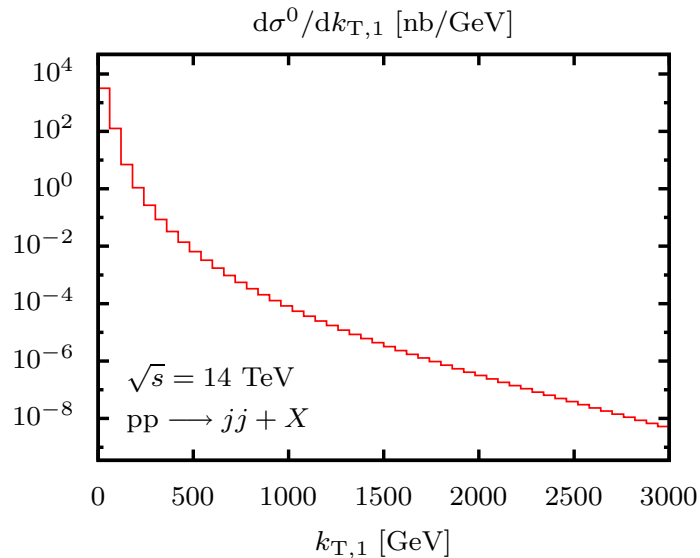
## Weak corrections to dijet production – differential cross sections

S.D., Huss, Speckner '12



### Weak corrections

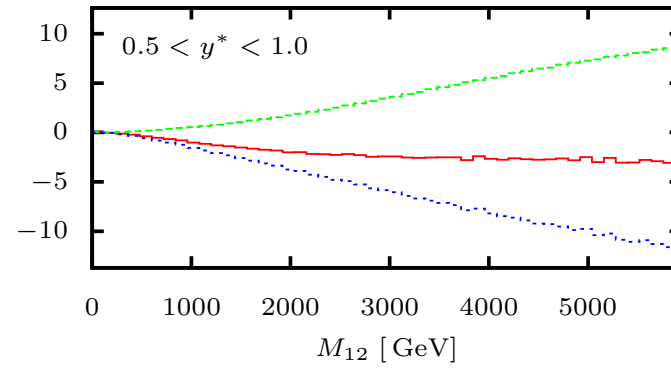
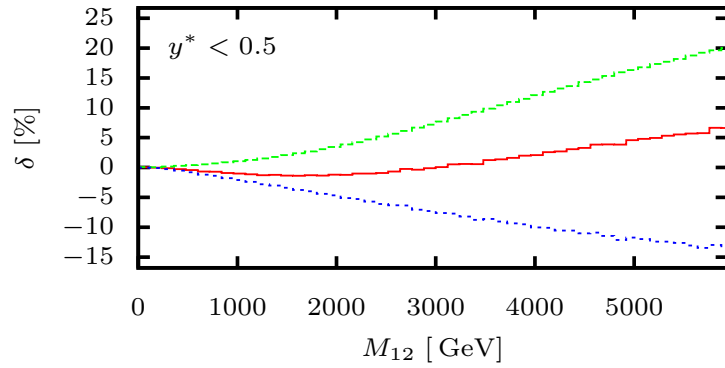
- small for integrated XS
- growing in distributions for larger scales



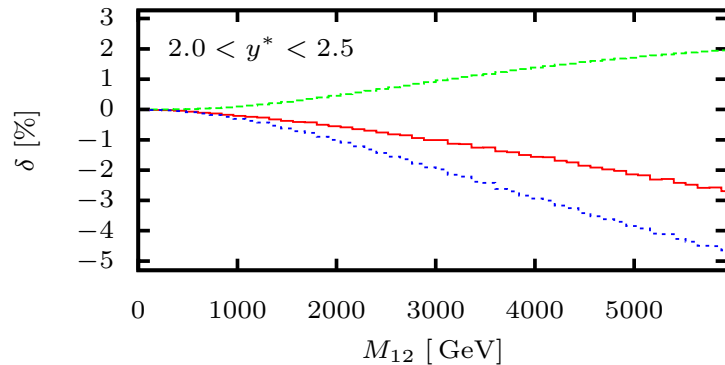
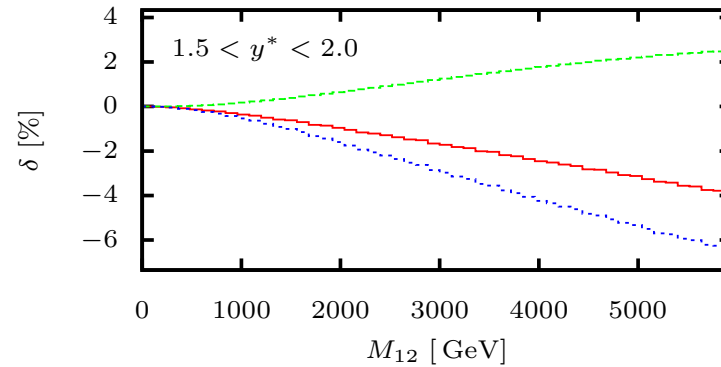
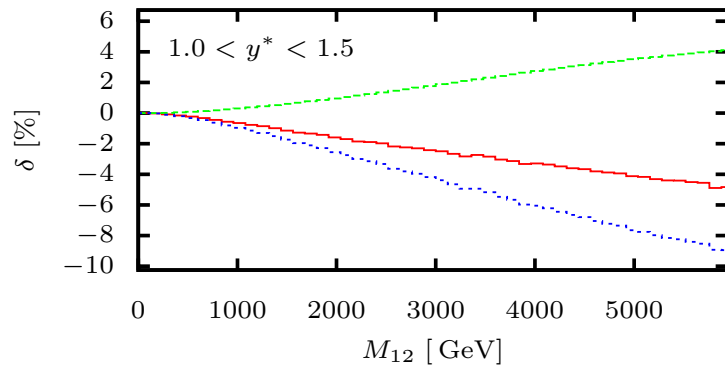
Cancellations between tree and loop corrections (cut-sensitive!)

# Weak corrections to dijet production – double-differential cross sections

S.D., Huss, Speckner '12



Sudakov-like regime  
( $M_{12}$  large,  $y^*$  small)  
→ larger corrections



pp → jj + X at  $\sqrt{s} = 14$  TeV

- $\delta_{\text{weak}}^{1\text{-loop}}$
- $\delta_{\text{EW}}^{\text{tree}}$
- $\delta_{\text{EW}}^{\text{tree}} + \delta_{\text{weak}}^{1\text{-loop}}$

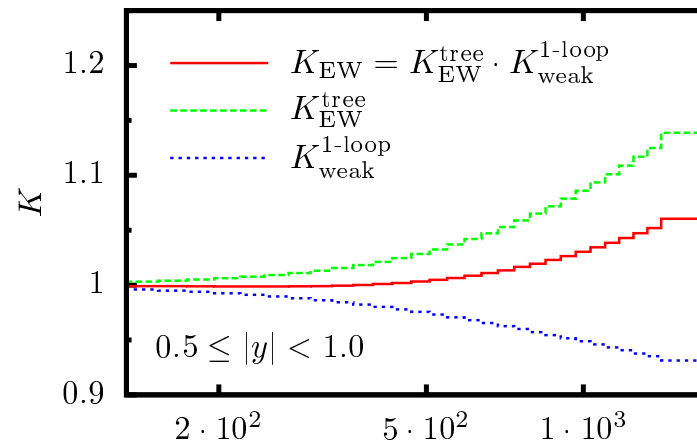
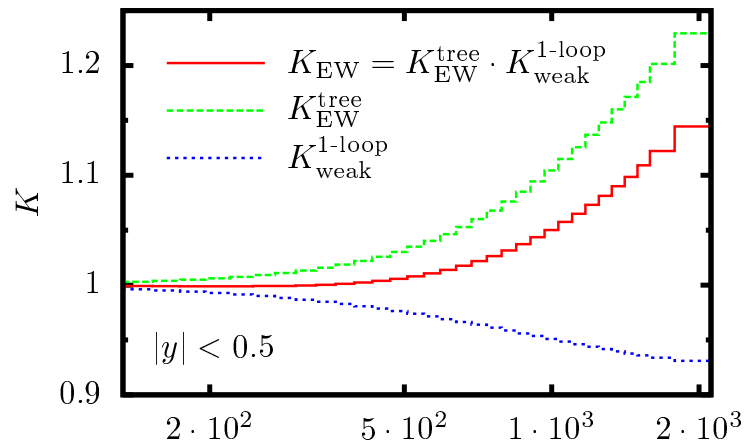
Regge-like regime  
( $M_{12}, |y^*|$  large)

# Inclusive jet production @ LHC7 – CMS data and EW corrections

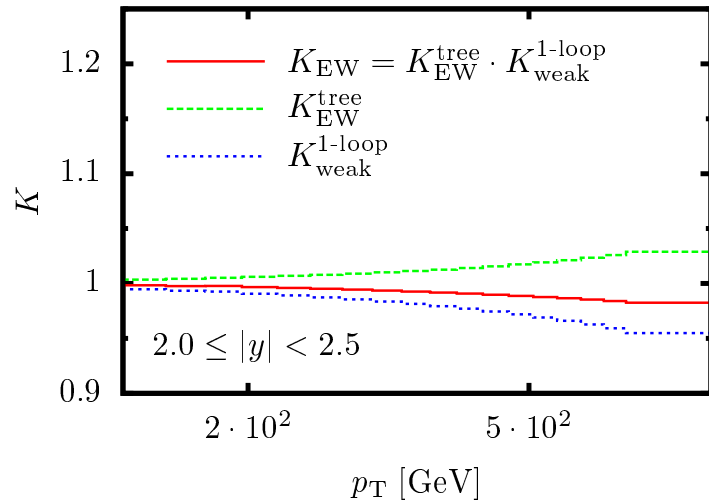
## Weak corrections

S.D., Huss, Rabbertz '14

Inclusive Jets    Anti- $k_T$   $R = 0.7$     CT10-NLO     $\sqrt{s} = 7$  TeV

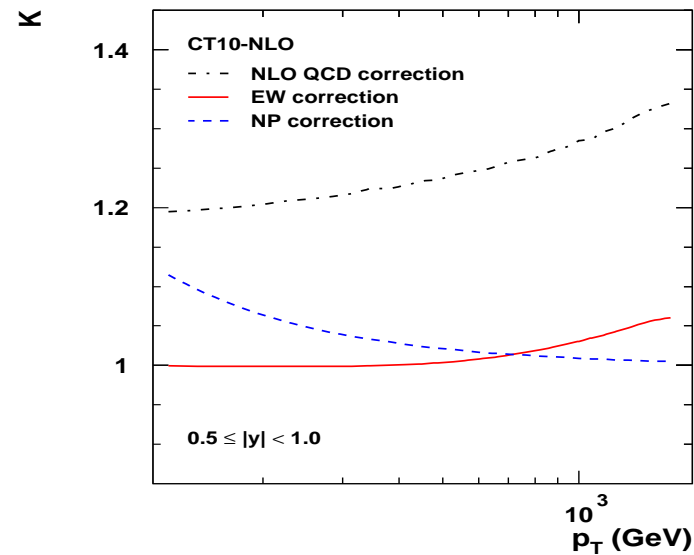
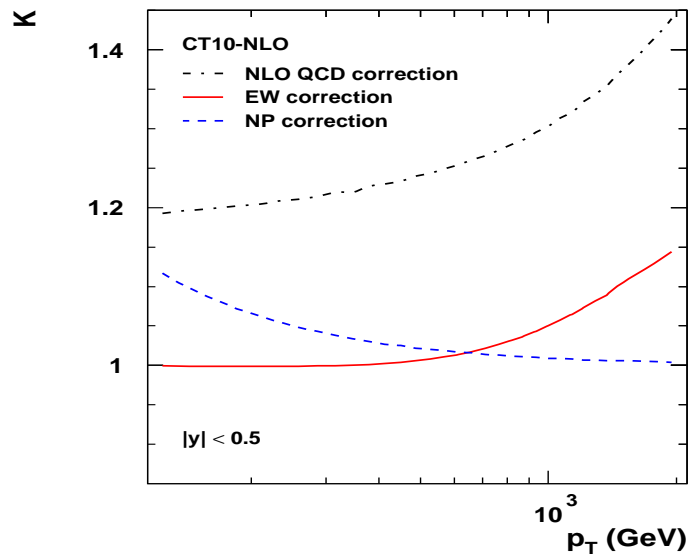


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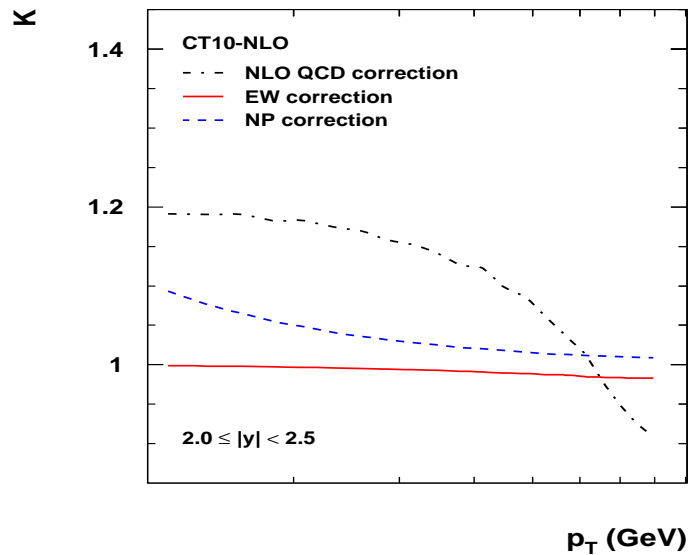


# Inclusive jet production @ LHC7 – CMS data and EW corrections

NLO weak, NLO QCD, and non-perturbative corrections S.D., Huss, Rabbertz '14



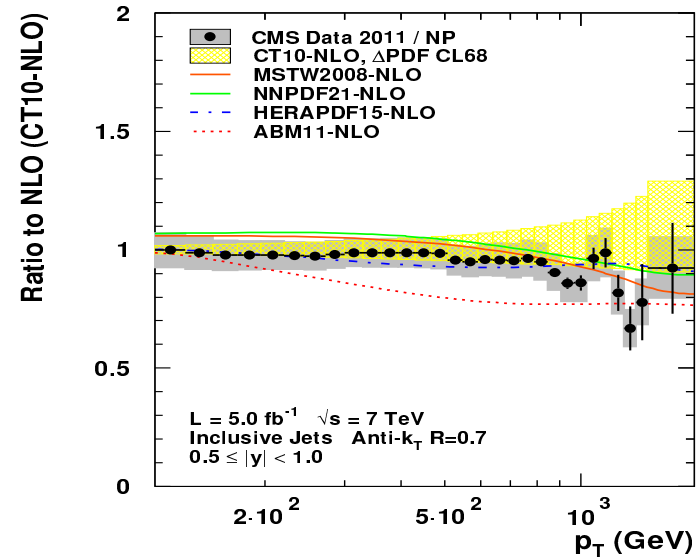
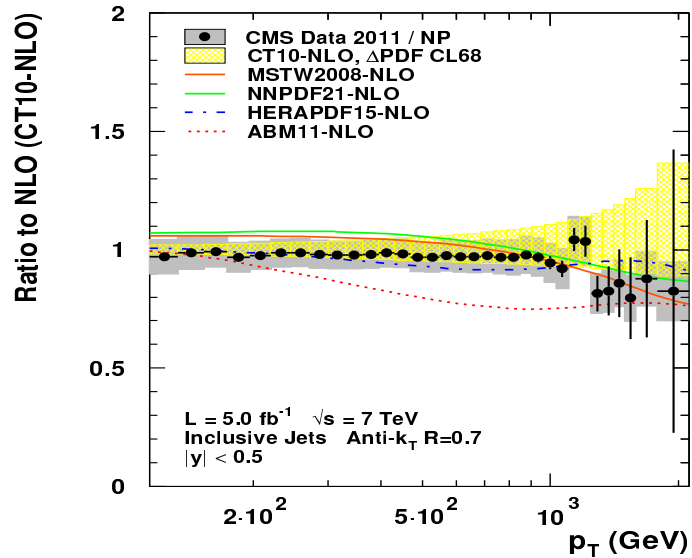
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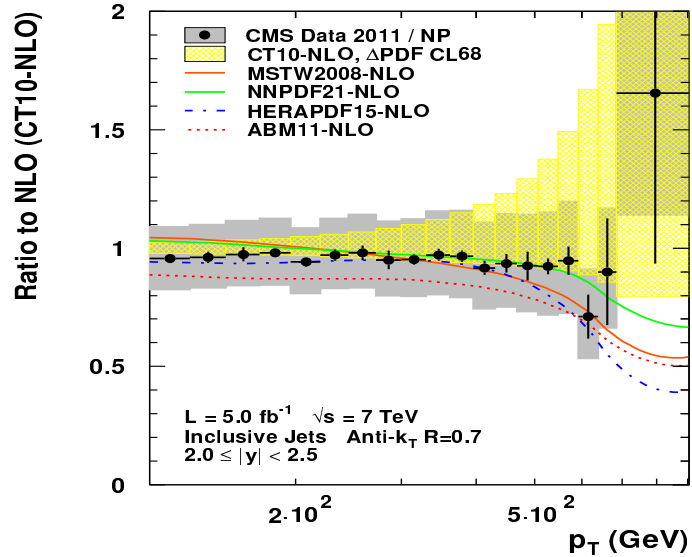
# Inclusive jet production @ LHC7 – CMS data and EW corrections

## Data w/o EW corrections

S.D., Huss, Rabbertz '14



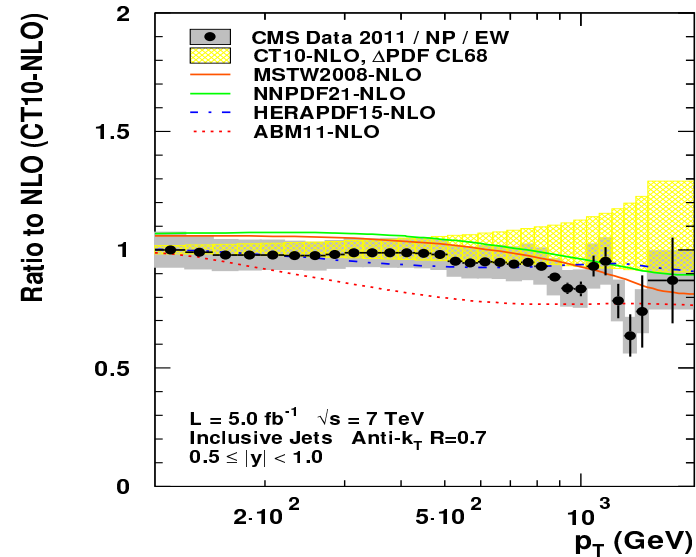
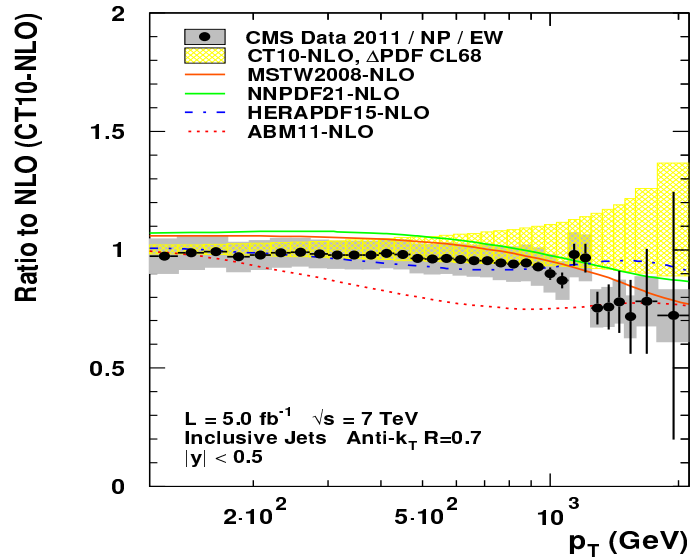
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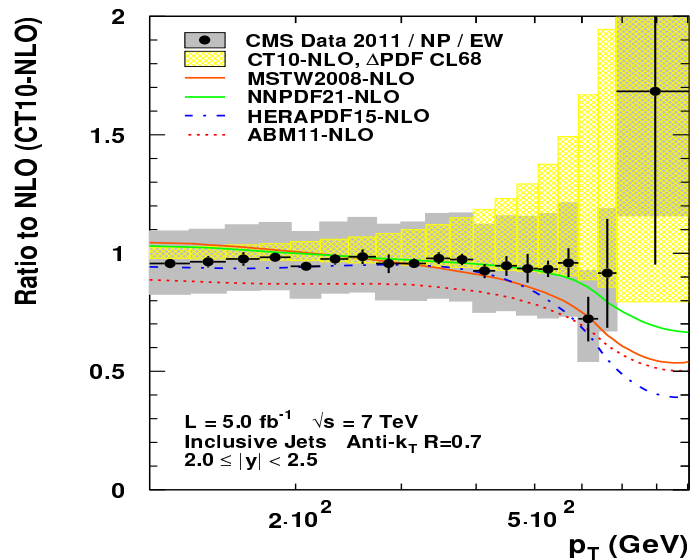
# Inclusive jet production @ LHC7 – CMS data and EW corrections

## Data w/ EW corrections

S.D., Huss, Rabbertz '14



...



EW effects hardly visible @ 7 TeV,

but certainly relevant @ 13–14 TeV  
with higher luminosity

# Conclusions





## EW corrections to hadronic event shapes at $e^+e^-$ colliders

- largest EW corrections due to ISR, but reduction to % effects by normalization to  $\sigma_{\text{had}}$
  - genuine weak effects negligible for LEP energies, at % level for 500 GeV ILC
  - $q\bar{q}\gamma$  final states separated from 3-jet events via photon fragmentation function  $\hookrightarrow$  % effects near 2-jet endpoints of event shapes
- $\Rightarrow$  NLO EW corrections completely known and sufficient  
(minor relevance for JADE/LEP, but relevant at ILC)

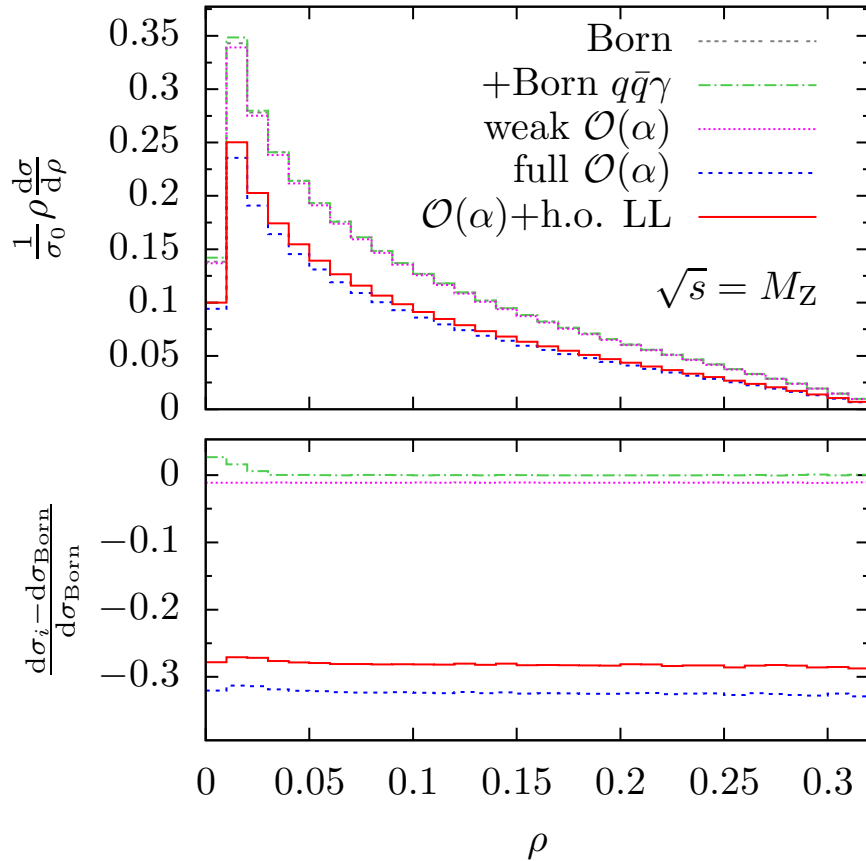
## EW corrections to jet production at hadron colliders

- weak corrections to dijet and inclusive-jet production  $\sim 5-10\%$  in TeV range (significant cancellations between EW tree and loop contributions)
  - **yet unknown:**
    - ◇ photonic corrections ( $\lesssim 1\%$  expected)
    - ◇ EW corrections to (3-jet)/(2-jet) cross-section ratio  $\hookrightarrow$  significant cancellations expected (but certainly less dramatic than in  $e^+e^-$ )
- $\Rightarrow$  Further work required !

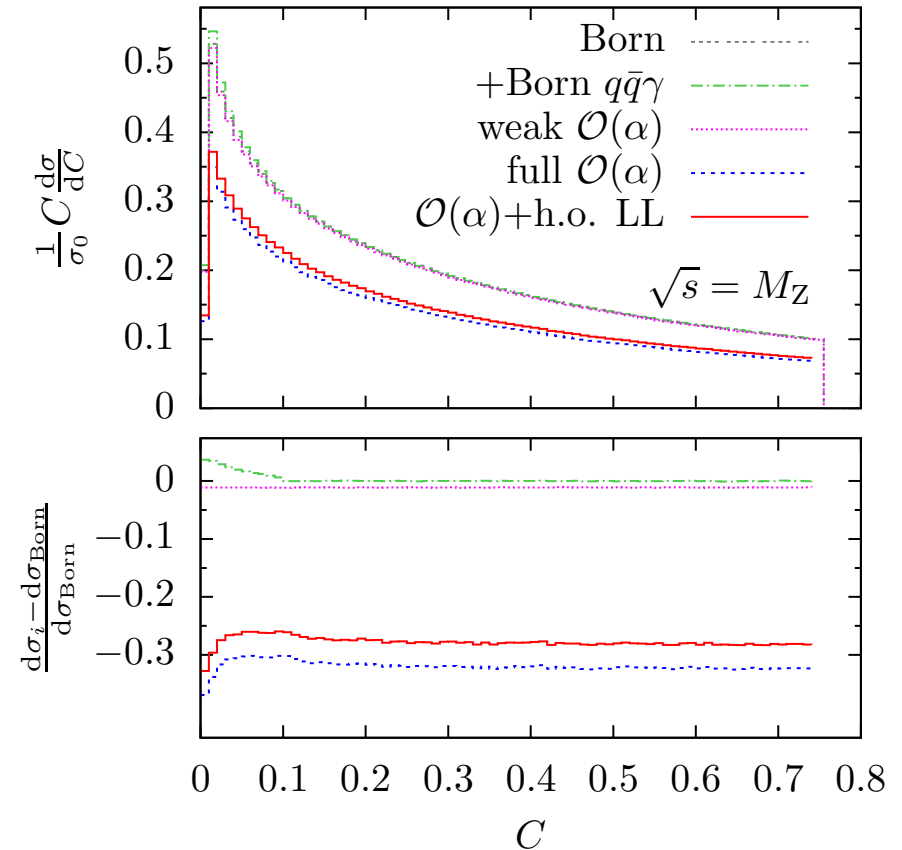
# Backup slides



# Heavy jet mass and $C$ parameter



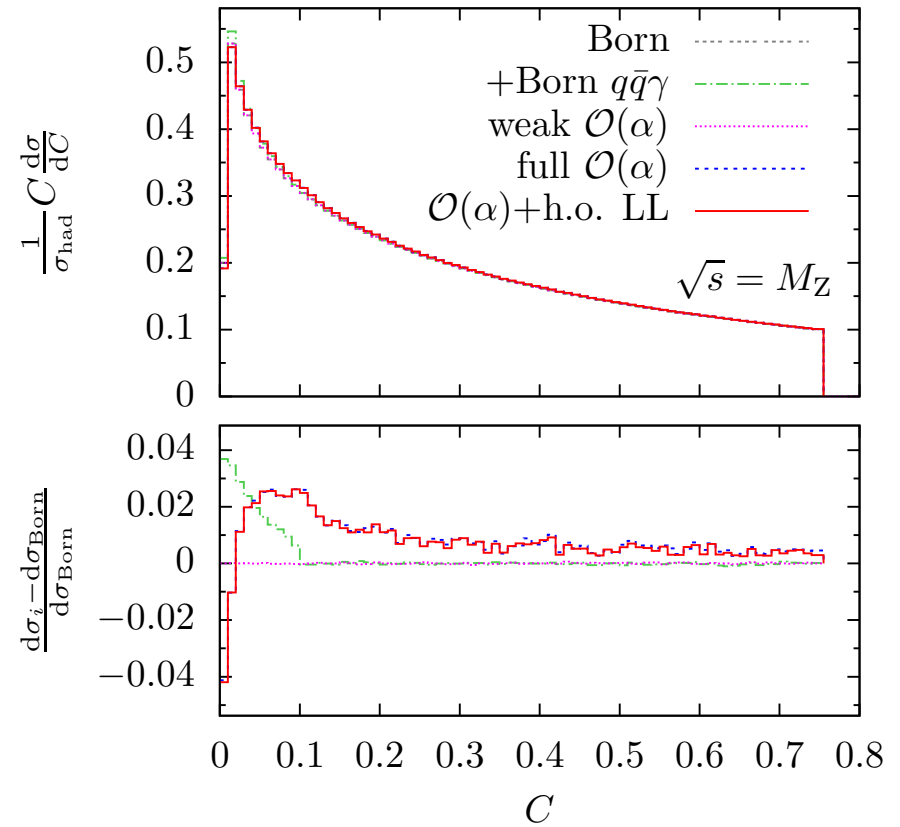
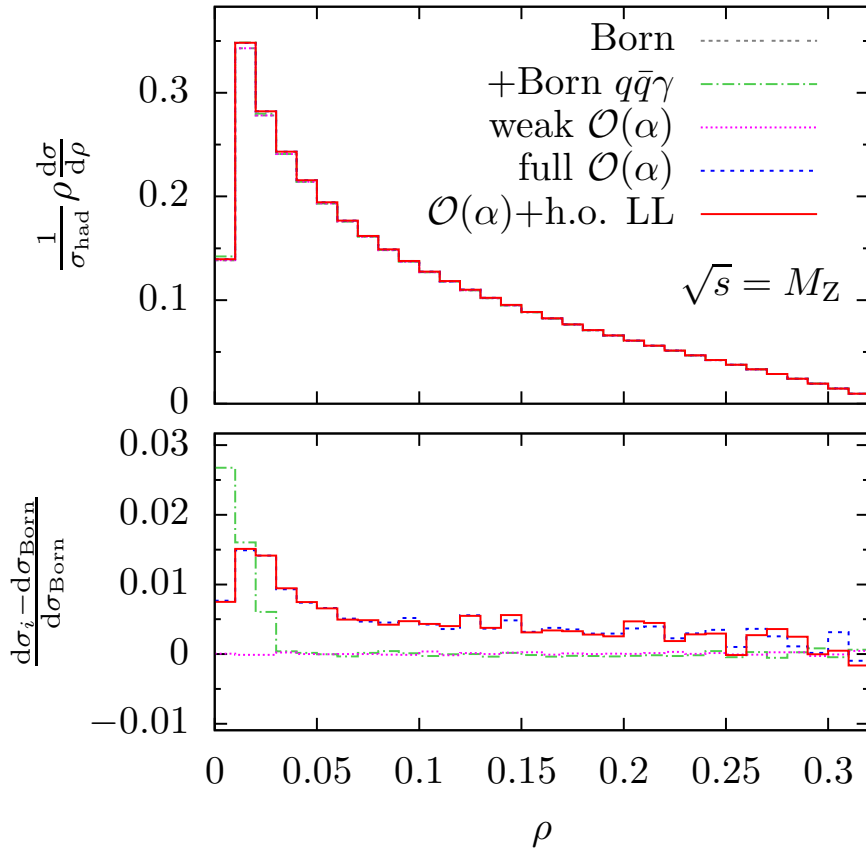
Denner, S.D., Gehrmann, Kurz '09,'10



Overall size, energy dependence, and qualitative features of EW effects similar for all event shapes.

# Heavy jet mass and $C$ parameter

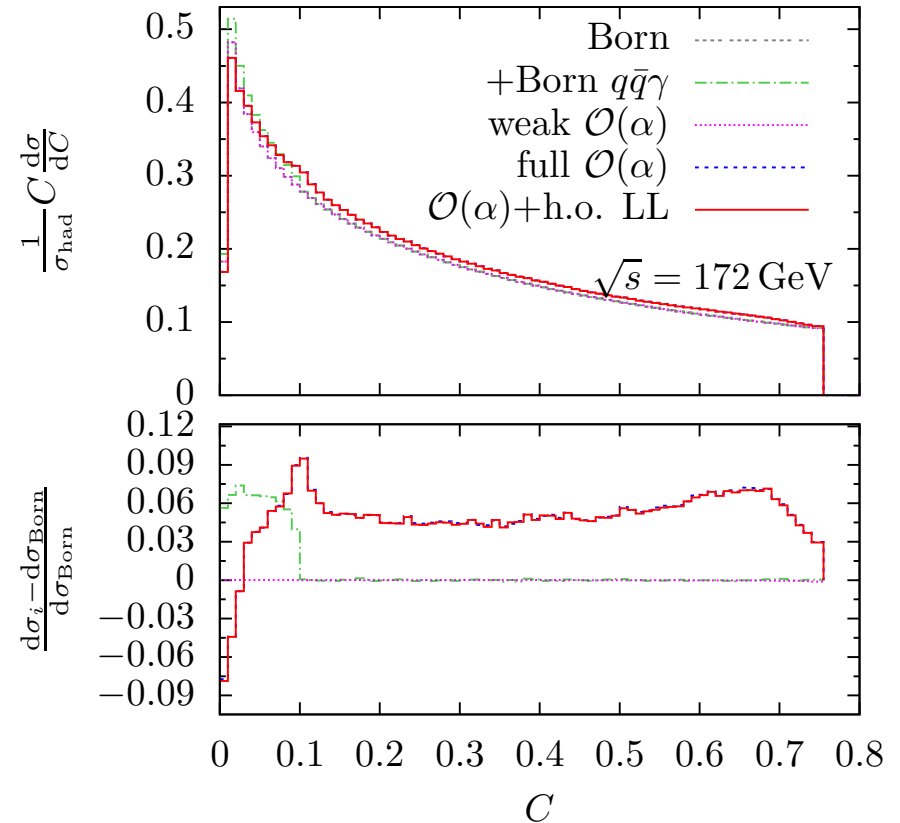
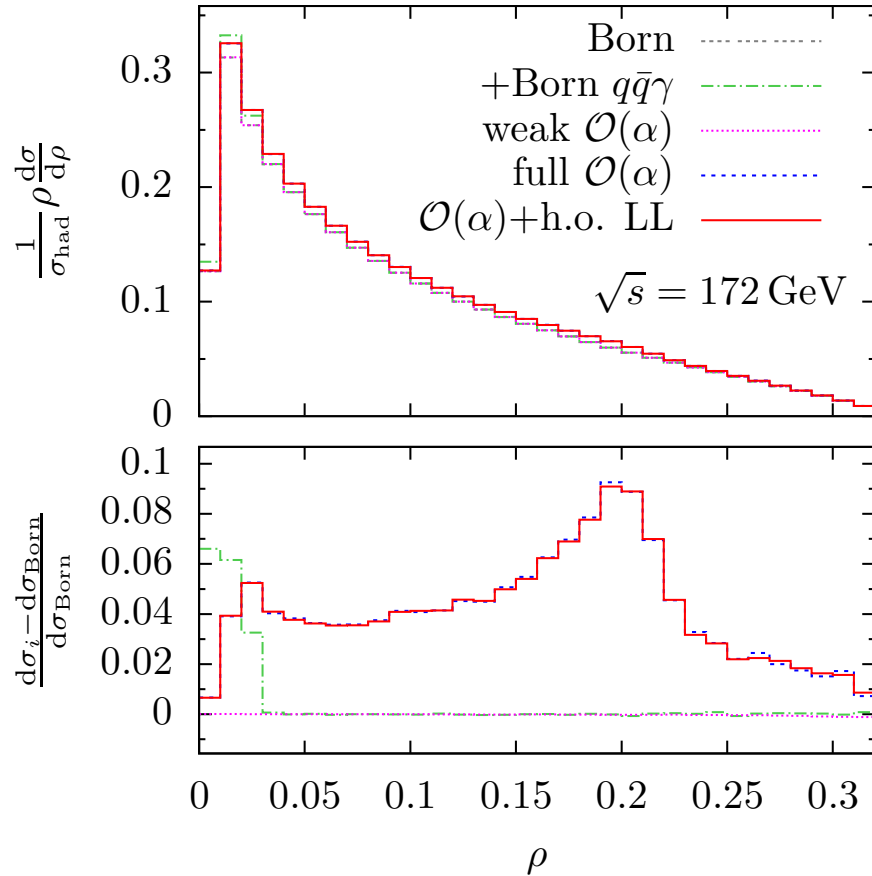
Denner, S.D., Gehrmann, Kurz '09,'10



Overall size, energy dependence, and qualitative features of EW effects similar for all event shapes.

# Heavy jet mass and $C$ parameter

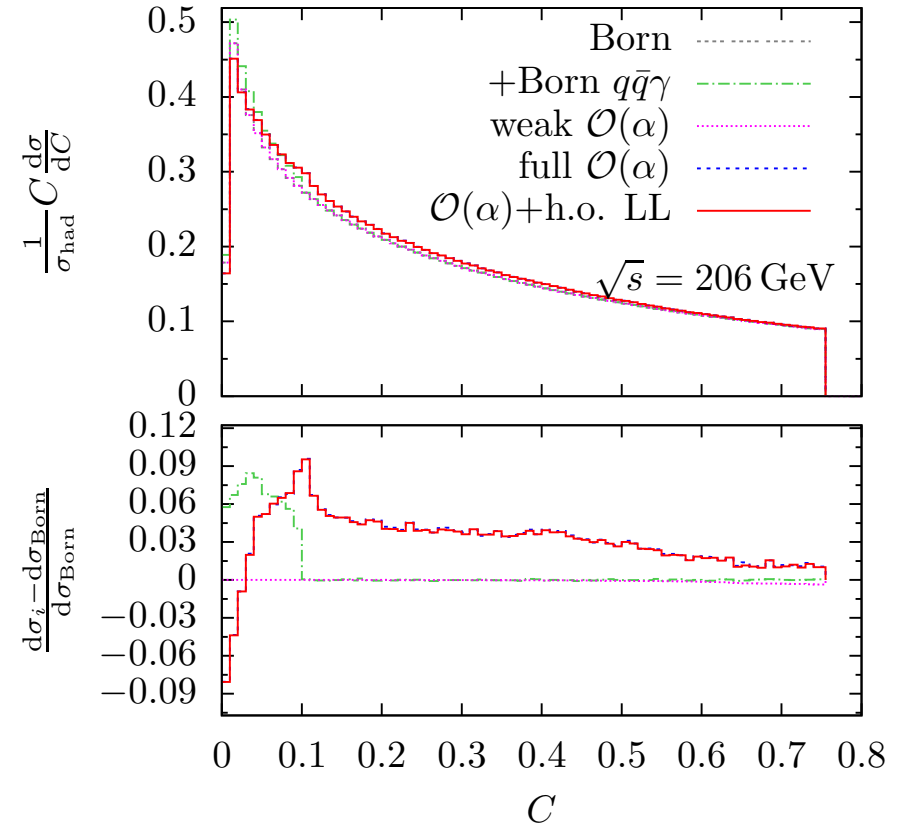
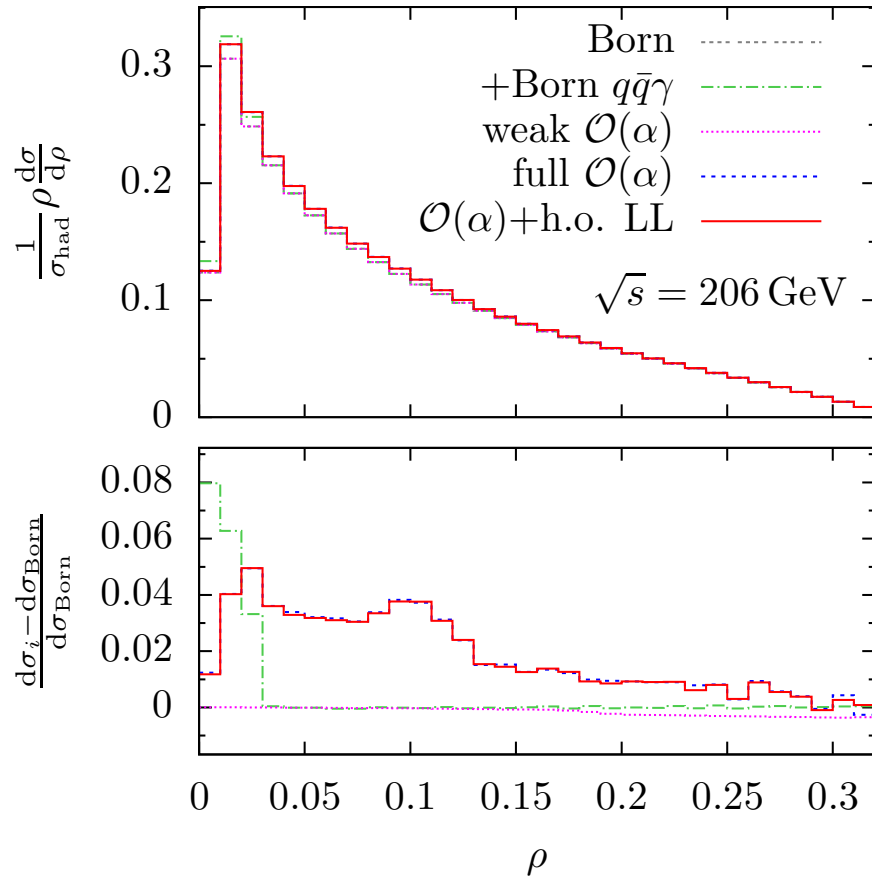
Denner, S.D., Gehrmann, Kurz '09,'10



Overall size, energy dependence, and qualitative features of EW effects similar for all event shapes.

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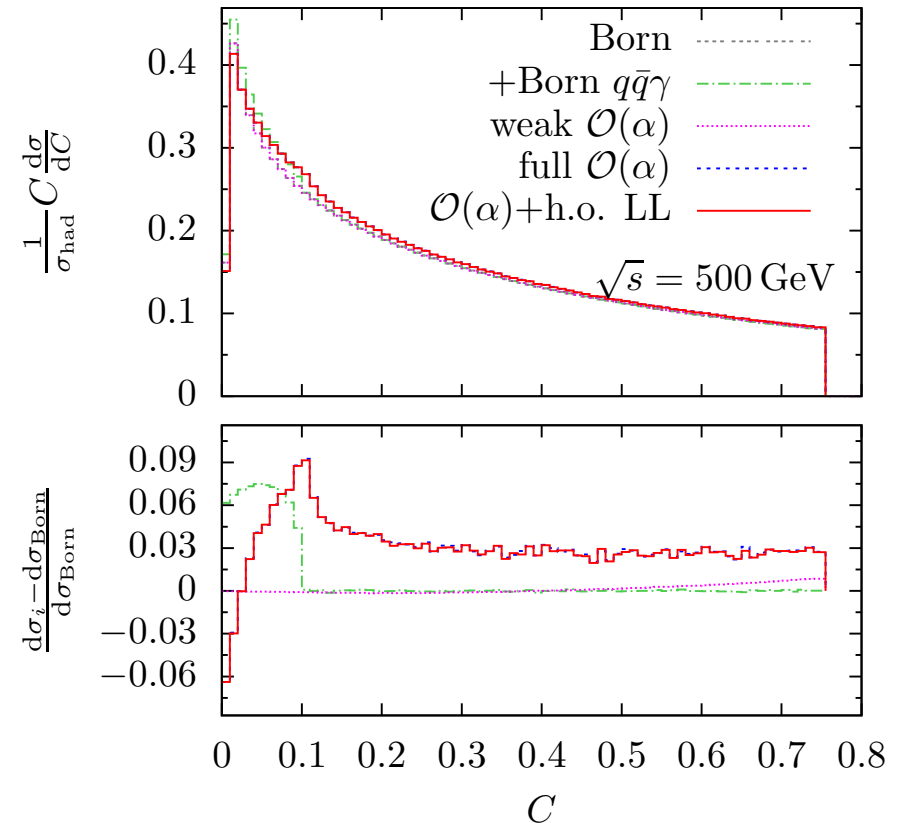
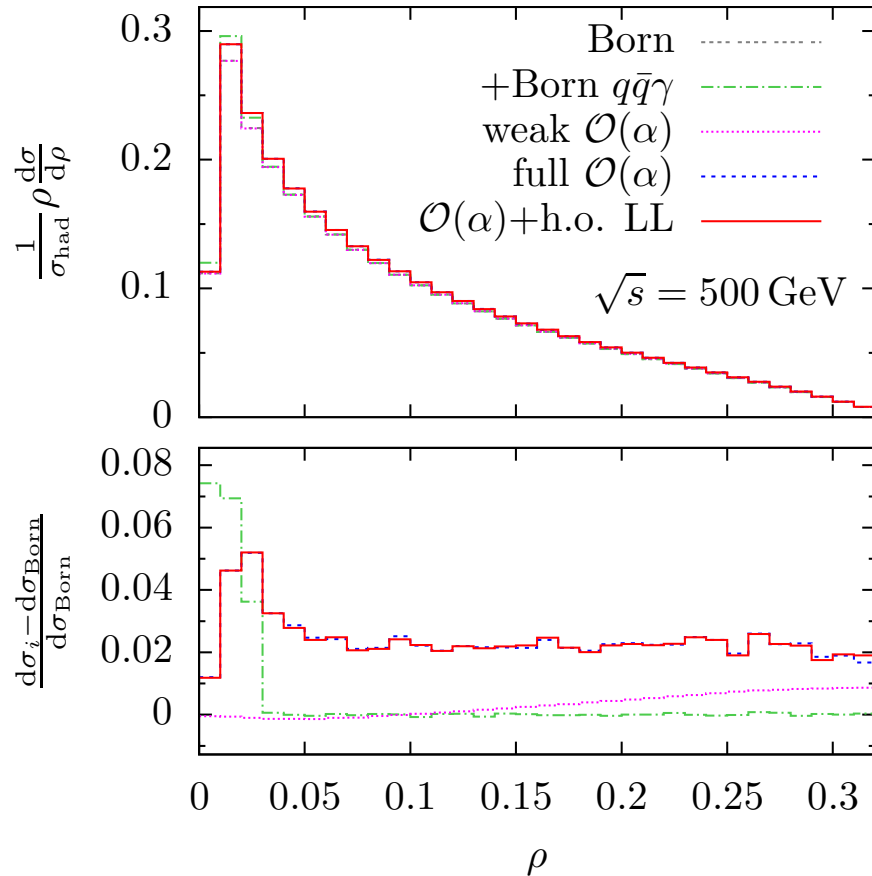
Denner, S.D., Gehrmann, Kurz '09,'10



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Denner, S.D., Gehrmann, Kurz '09,'10



Overall size, energy dependence, and qualitative features of EW effects similar for all event shapes.