

# The program GAPP and $\alpha_s$



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# Outline

- GAPP
- $\alpha_s$  from Z decays
- $\alpha_s$  from  $\tau$  decays
- $\alpha_s$  from the global EW fit
- $M_H$

# GAPP features

- GAPP = Global Analysis of Particle Properties
- special purpose FORTRAN package to compute pseudo-observables and perform least- $\chi^2$  fits
- SM parameters:  $\bar{m}_t$ ,  $\bar{m}_b$ ,  $\bar{m}_c$ ,  $M_Z$ ,  $M_H$ ,  $\alpha(m_T)$ ,  $\alpha_s$
- BSM parameters: STU, 4th family,  $Z'$  physics, ...
- full analytical expressions (expansions) when possible (not for non-enhanced 2-loop contributions to  $M_W$  and  $\sin^2\theta_W$ )
- $\overline{\text{MS}}$  scheme

# GAPP observables

- Z pole:  $M_Z, \Gamma_Z, \sigma_{\text{had}}, R_l, R_q, A_{\text{FB}}, A_{\text{LR}}$
- APV & lepton scattering:  $\sin^2\theta_W (\mu \ll M_Z)$
- low energy:  $g_\mu - 2$  and  $T_T$  (new physics)
- other:  $M_W, \Gamma_W, \text{Higgs BRs}, m_t^{\text{pole}}$ , sumrules for  $\bar{m}_b$  &  $\bar{m}_c$
- uncertainties: statistical, systematic, parametric and theoretical errors & correlations estimated and included

# $\alpha_s$ from Z decays

- determined by  $\Gamma_Z$ ,  $\sigma_{\text{had}}$ ,  $R_l$ , but other measurements, SM parameters, and new physics enter indirectly
- experimental correlations: small, known, included
- parametric uncertainties: non-Gaussian ( $\sin^2 \theta_W$ ), treated exactly in fits
- theory errors (PQCD): 100% correlated (currently neglected),  $\Delta_{\text{theo}} \alpha_s = \pm 0.00009$
- unknown unknowns?

# $\alpha_s$ from Z decays: theory

- massless non-singlet QCD corrections known to  $\mathcal{O}(\alpha_s^4)$   
*Baikov, Chetyrkin, Kühn 2008*
- FOPT vs. CIPT:  $\Delta\alpha_s = \pm 0.00005$  (opposite sign from  $\tau_T$ )
- $\mathcal{O}(\alpha_s^4)$  vector singlet terms *Baikov, Chetyrkin, Kühn 2010* known up to singlet piece in *Crewther* relation  $\Rightarrow \Delta\alpha_s \approx +10^{-5}$
- axial-vector singlet:  $\mathcal{O}(\alpha_s^2)$  *Kniehl, Kühn 1990*  $\Delta\alpha_s = +0.0027$   
 $\mathcal{O}(\alpha_s^3)$  *Larin, van Ritbergen, Vermaseren 1995*  $\Delta\alpha_s = +0.00043$   
 $\mathcal{O}(\alpha_s^4) \sim \mathcal{O}(\alpha_s^3)^2 / \mathcal{O}(\alpha_s^2) \Rightarrow \Delta\alpha_s = \pm 0.00007$  (dominant)

# $\alpha_s$ from $Z$ decays: results

	$\alpha$	$\Delta\alpha$
$\Gamma$	0.1188	$\pm 0.0076$
LEP I	0.1213	$\pm 0.0028$
LEP I + SLC	0.1198	$\pm 0.0028$
$Z$ pole +	0.1190	$\pm 0.0027$
EW fit (excl.)	0.1192	$\pm 0.0027$
full EW fit	0.1192	$\pm 0.0016$

# $\alpha_s$ [Z pole]: new physics

- expect Z pole value of  $\alpha_s$  to be stronger affected by new physics than  $\tau$  decay value
- Z pole (LEP I & SLC):  
 $\alpha_s = 0.1198 \pm 0.0028$  ( $\chi^2_{\min} = 23.2/23$ )
- allowing “oblique” (universal) parameters:  
 $\alpha_s = 0.1199^{+0.0027}_{-0.0030}$  ( $\chi^2_{\min} = 23.0/20$ )
- allowing special new physics corrections to Zbb-vertex:  
 $\alpha_s = 0.1167 \pm 0.0038$  ( $\chi^2_{\min} = 16.3/21$ )

# $\alpha_s$ from $\tau$ decays

- at least one low-energy  $\alpha_s$ -value needed to promote Z-width to a SM test (constraint on BSM physics)

- incredibly shrinking error:

$$\Delta\alpha_s(M_Z) \sim [\alpha_s(M_Z)/\alpha_s(m_\tau)]^2 \Delta\alpha_s(m_\tau)$$

- OPE can be applied and tested

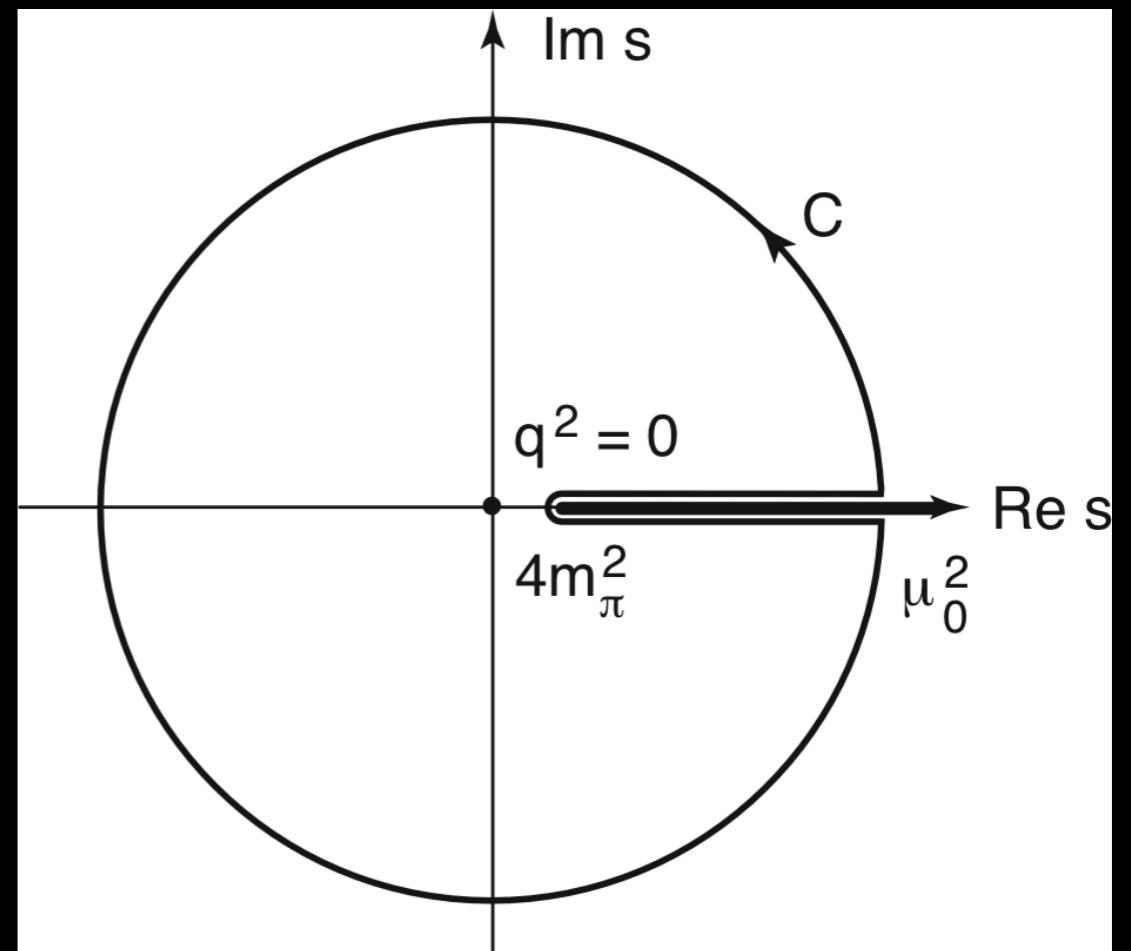
- NNNLO *Baikov, Chetyrkin, Kühn 2008*

- fully inclusive

- double zero near branch cut:

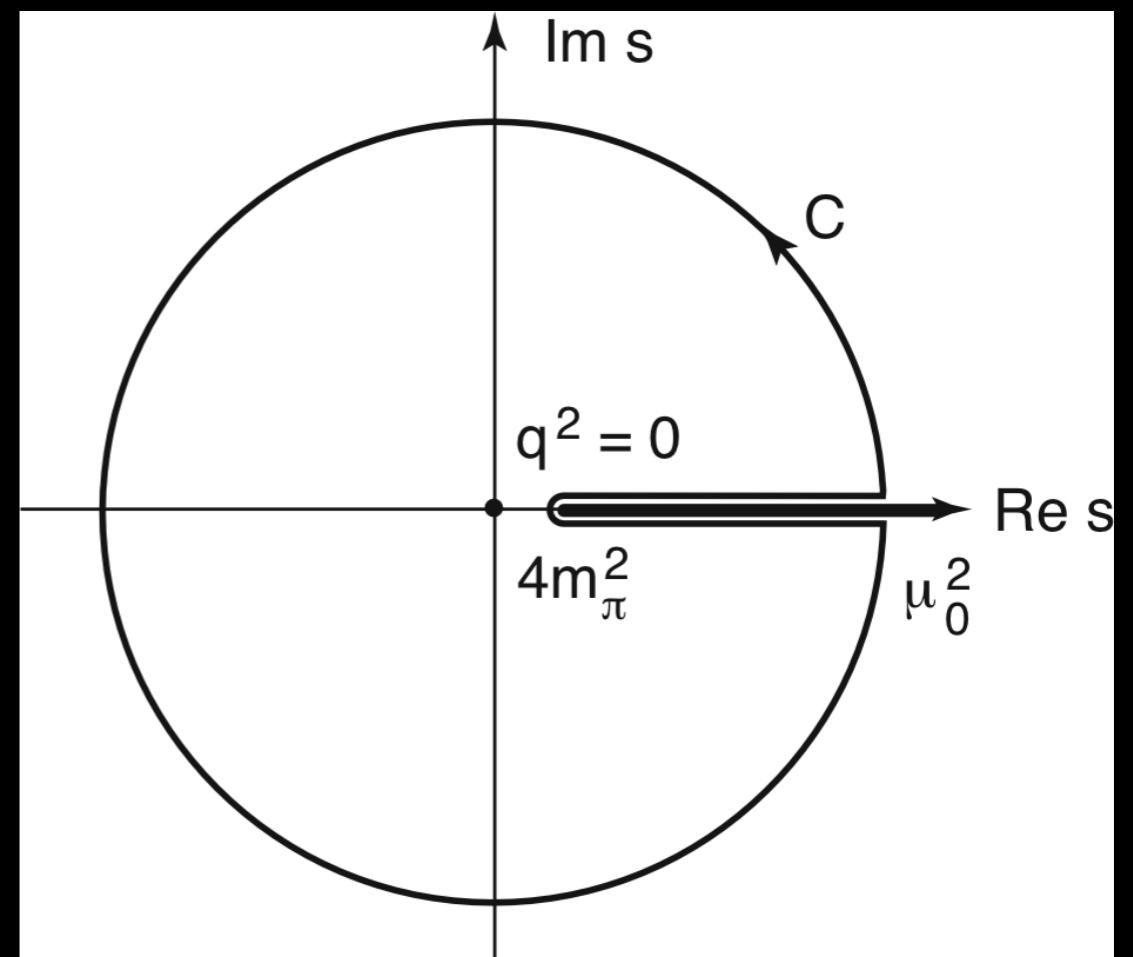
$$R_\tau = 6\pi i \oint ds/m_\tau^2 (1-s/m_\tau^2)^2 [(1 + 2 s/m_\tau^2) \operatorname{Im} \Pi(s) + O(m_q^2/m_\tau^2)]$$

- non-perturbative effects constrained from data



# $\alpha_s$ from $\tau$ decays

- 4-loop PQCD coefficient  
*Baikov, Chetyrkin, Kühn 2008*
- FOPT vs. CIPT controversy  
*Le Diberder, Pich 1992;  
Beneke, Jamin 2008*
- CIPT: expansion coefficients identical to those in Adler-function
- fits to condensate terms  
*Davier et al. 2008; Boito et al. 2012*



# $\alpha_s$ from $\tau$ decays

- $\tau^{\text{expt}} = \hbar (I - B_s^{\text{expt}}) / (\Gamma_e^{\text{theo}} + \Gamma_\mu^{\text{theo}} + \Gamma_{ud}^{\text{theo}})$  *JE, Luo 2003*
- $\Gamma_{ud}^{\text{theo}} = G_F^2 m_\tau^5 |V_{ud}|^2 / 64\pi^3 S(m_\tau, M_Z) (I + 3/5 m_\tau^2/M_W^2) (I + a + 5.202 a^2 + 26.37 a^3 + 127.1 a^4 - 1.393 \alpha/\pi + \delta_q)$
- $\Delta S = I$  decays:  $B_s^{\text{expt}} = 0.0287 \pm 0.0007$  from data, since  $\bar{m}_s(m_\tau)$  uncertain & QCD series  $\propto \bar{m}_s^2$  poorly converging
- log enhanced EW:  $S(m_\tau, M_Z) = 1.01907 \pm 0.0003$   
*Marciano, Sirlin 1988; JE 2002*
- $\delta_q$ : quark condensates *Boito et al. 2012* and finite  $m_c, m_b$  &  $m_s$  effects *Chetyrkin 1993; Larin, van Ritbergen, Vermaseren 1995*

# $\tau$ lifetime average

$$\tau[B_e] = \hbar B_e^{\text{expt}} / \Gamma_e^{\text{theo}}$$

$$B_e^{\text{expt}}: 0.1783 \pm 0.0004 \Rightarrow \tau[B_e^{\text{expt}}] = 291.15 \pm 0.65 \text{ fs}$$

$$B_{\mu}^{\text{expt}}: 0.1741 \pm 0.0004 \Rightarrow \tau[B_{\mu}^{\text{expt}}] = 291.85 \pm 0.67 \text{ fs}$$

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$$B_{e,\mu}^{\text{expt}} (\rho_{e\mu} = 0.13) \Rightarrow \tau[B_{e,\mu}^{\text{expt}}] = 291.49 \pm 0.50 \text{ fs}$$

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$$\tau_{\text{direct}}^{\text{expt}} = 290.6 \pm 1.0 \text{ fs}$$

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$$\tau^{\text{expt}} \equiv \tau[B_{e,\mu}^{\text{expt}}, \tau_{\text{direct}}^{\text{expt}}] = 291.31 \pm 0.45 \text{ fs} \Rightarrow$$

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$$R \equiv \Gamma_{ud}/\Gamma_e = 3.471 \pm 0.009 \Rightarrow \delta_{QCD} = 0.1948 \pm 0.0032$$

# $\alpha_s [\tau_\tau]$ : experimental errors

source	uncertainty	$\Delta\tau$	$\Delta\alpha$
$\Delta\tau$	$\pm 0.45 \text{ fs}$	$\pm 0.45$	$\mp$
$\Delta$	$\pm 0.0007$	$\mp$	$\mp$
$\Delta$	$\pm 0.00015$	$\mp$	$\mp$
$\Delta$	$\pm 0.16 \text{ MeV}$	$\mp$	$\mp$
total		0.50	0.000 37

# $\alpha_s [\tau_\tau]$ : theoretical errors

source	uncertainty	based on	$\Delta\alpha$
PQCD	$\mp$	$\alpha$	+0.00180 -0.00145
RGE	$\beta$	GAPP	0.0004
$\delta$	0.012	Boito et al. 2012	0.0013
OPE	0.0008	't Hooft 1976, Davier et al. 2005	0.00012
$S(m)$	0.0003	JE 2002	0.00004
total			+0.0022 -0.0020

# Padé

- many Padé approximants, summations, or predictions (for the next term) can be defined. For illustration only:
- Padé  $[1/1]^* \equiv 1 + (a + 0.133 a^2)/(1 - 5.069 a) - 6.6 a^4$   
higher order coefficients positive
- Padé  $[2/2] \equiv (1 - 13.6 a + 40 a^2)/(1 - 14.6 a + 49.5 a^2)$   
 $\Rightarrow \alpha_s(M_Z) = 0.11725$   
fails!

# $\alpha_s [\tau_\tau]$ : summation schemes

	FOPT	CIPT	Padé [I/I]*
$\alpha$	0.1192	0.1215	0.1177
$\Delta\delta$	±	±	±
$\Delta$	+0.00180 -0.001	+0.00133 -0.00119	±0.00007
$\Delta$	±0.0004	±0.0006	±0.0004

# $\alpha_s$ from the global EW fit

Z decays:  $\alpha_s = 0.1198 \pm 0.0028$

$\tau$  decays:  $\alpha_s = 0.1192^{+0.0022}_{-0.0020}$

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Z &  $\tau$  decays:  $\alpha_s = 0.1194 \pm 0.0017$

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global fit:  $\alpha_s = 0.1192 \pm 0.0016$

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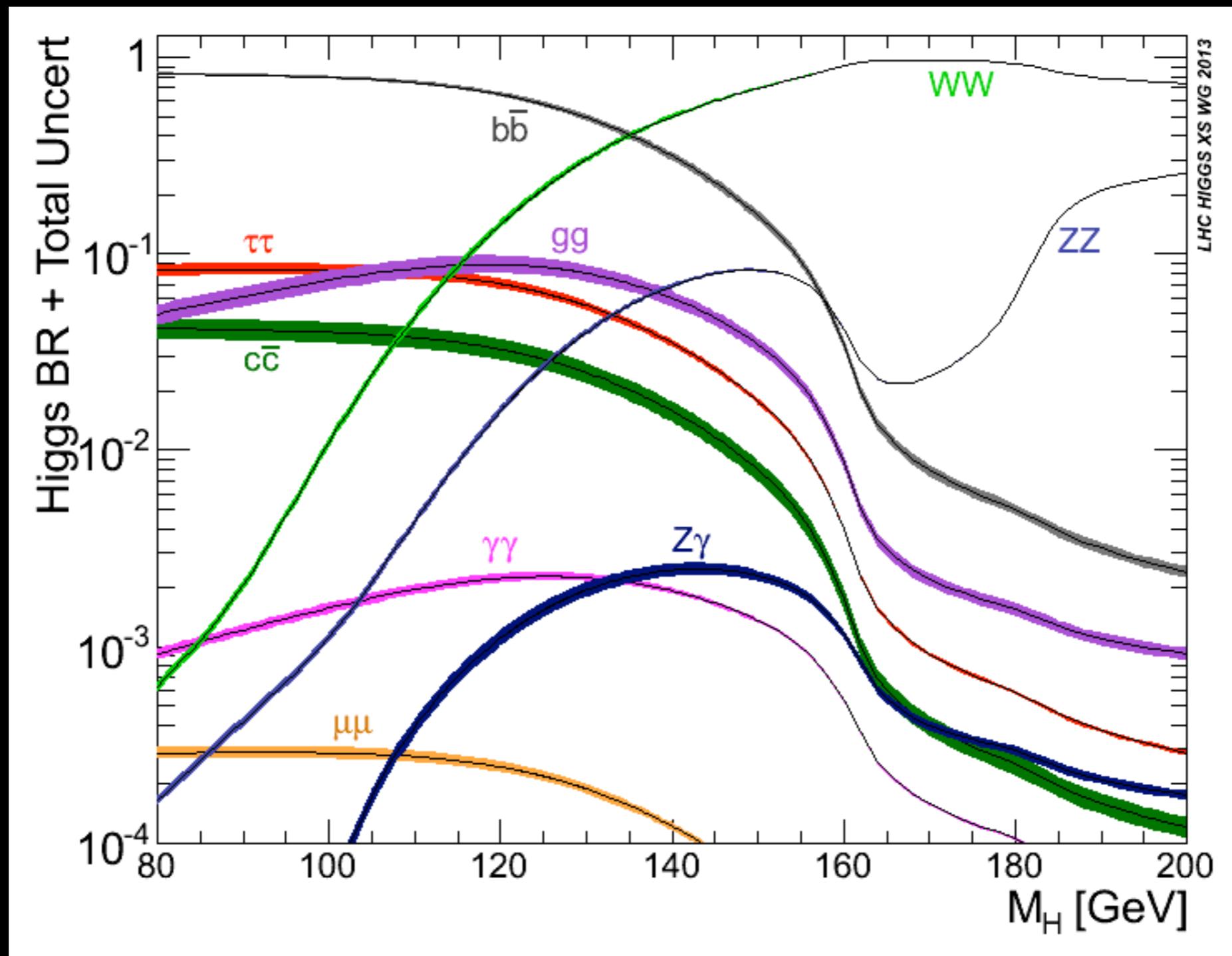
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# $M_H$ [GeV]

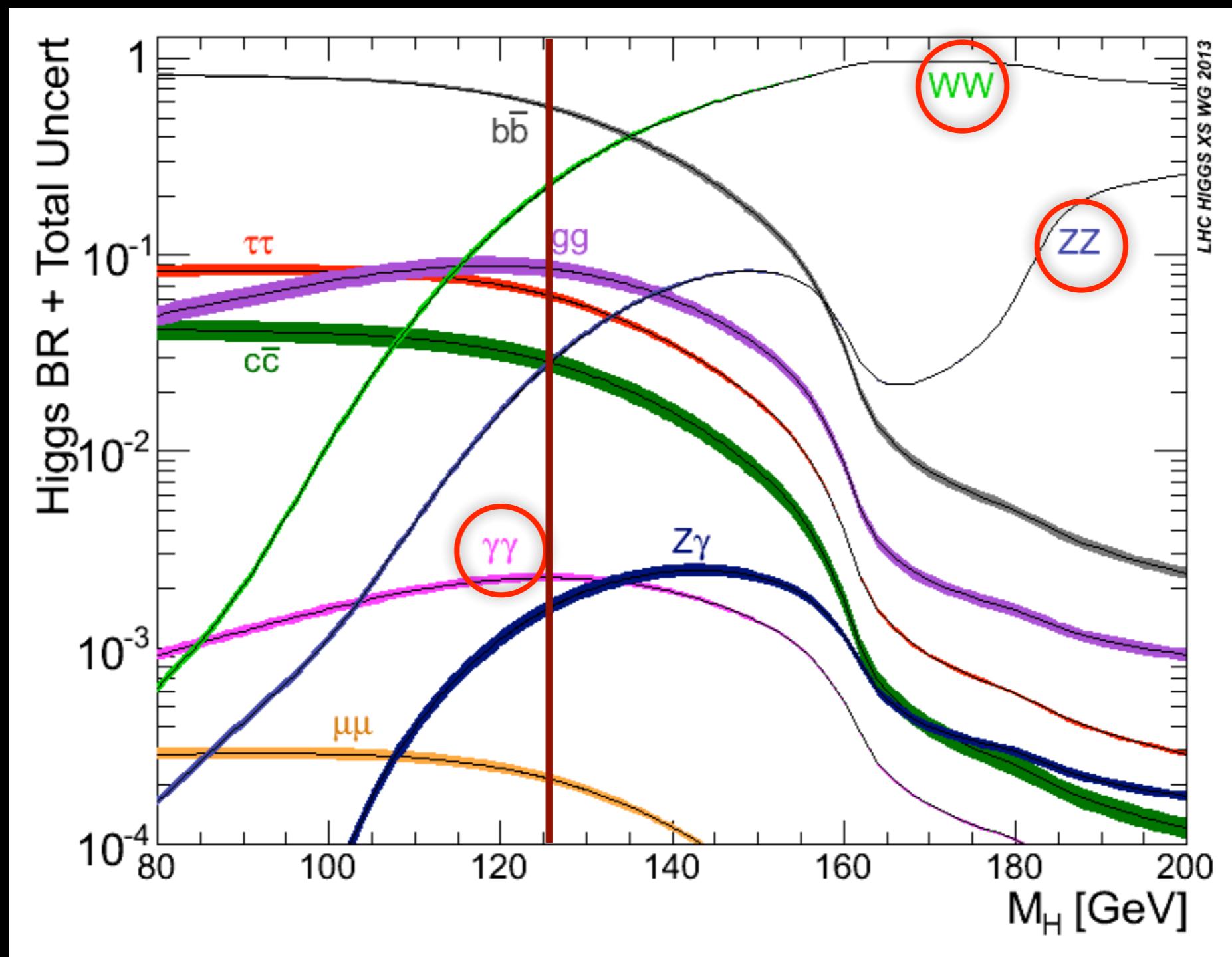
source	$M$	uncertainty
radiative corrections	89	+22
LHC	123.7	2.3
ATLAS	125.5	0.6
CMS	125.7	0.4
global fit	125.5	0.4

*JE, Ayres 2013  
PDG 2014*

# $M_H$ from Higgs branching ratios?



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# $M_H$ [GeV]

source	$M$	uncertainty
radiative corrections	89	+22
LHC	123.7	2.3
ATLAS	125.5	0.6
CMS	125.7	0.4
global fit	125.5	0.4

*JE, Ayres 2013  
PDG 2014*

# Summary

- global EW fit:  $\alpha_s = 0.1192 \pm 0.0016$
- new physics: negligible (significant) sensitivity of Z-pole value to (non-)universal corrections
- $\alpha_s [\tau_\tau]$ : theory uncertainty itself  $\alpha_s$  dependent  $\Rightarrow$  asymmetric error (re-calculated in each call in the fits)

FOPT vs CIPT vs Padé (geometric progression)

power corrections need improvement