α_s from the QCD static energy

Xavier Garcia i Tormo
Universität Bern

Based on:

A. Bazavov, N. Brambilla, XGT, P. Petreczky, J. Soto and A. Vairo, Phys. Rev. D **86**, 114031 (2012) [arXiv:1205.6155 [hep-ph]].

+ work in progress

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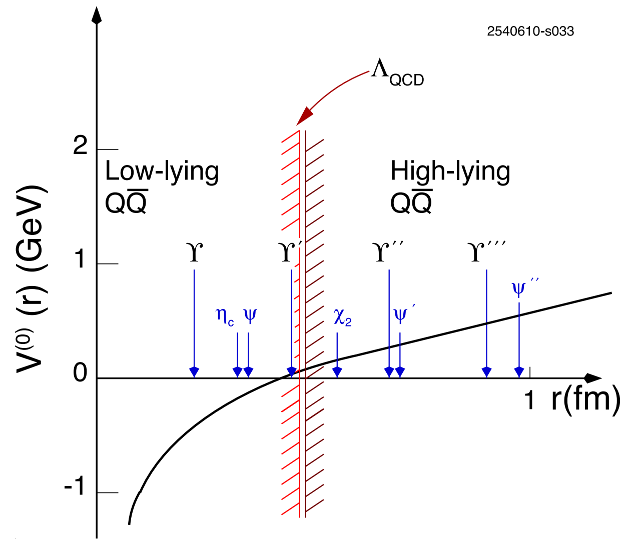

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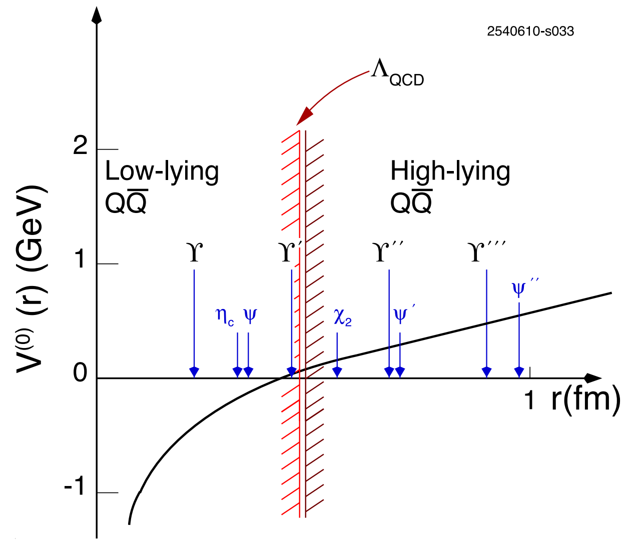

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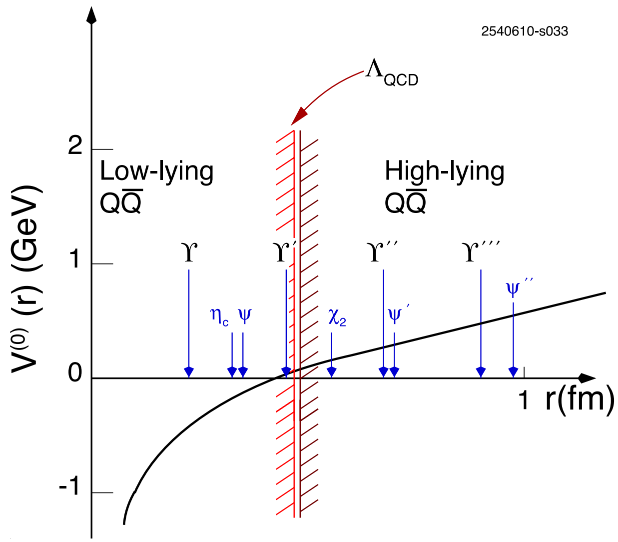

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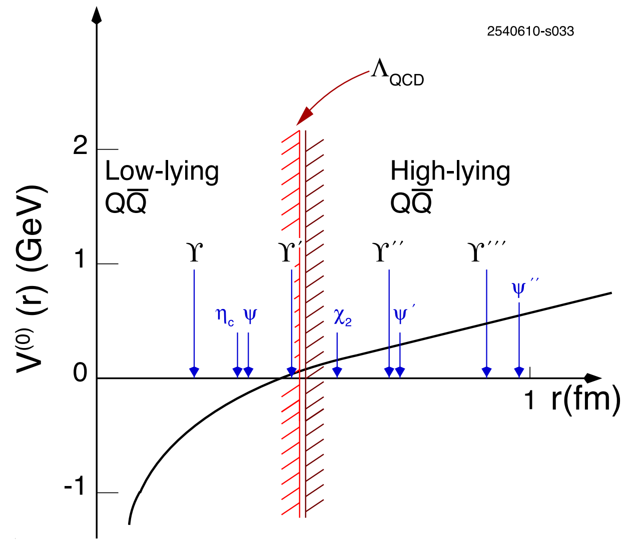

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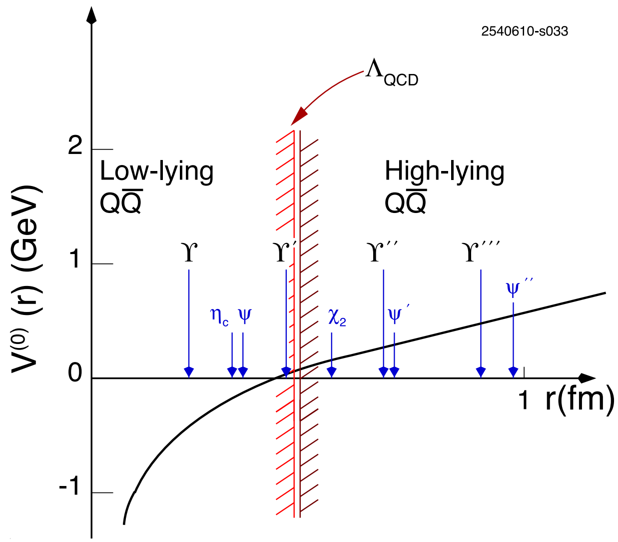

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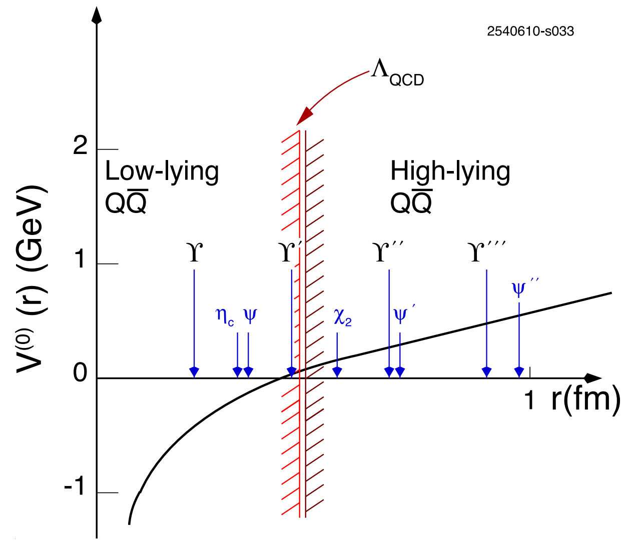

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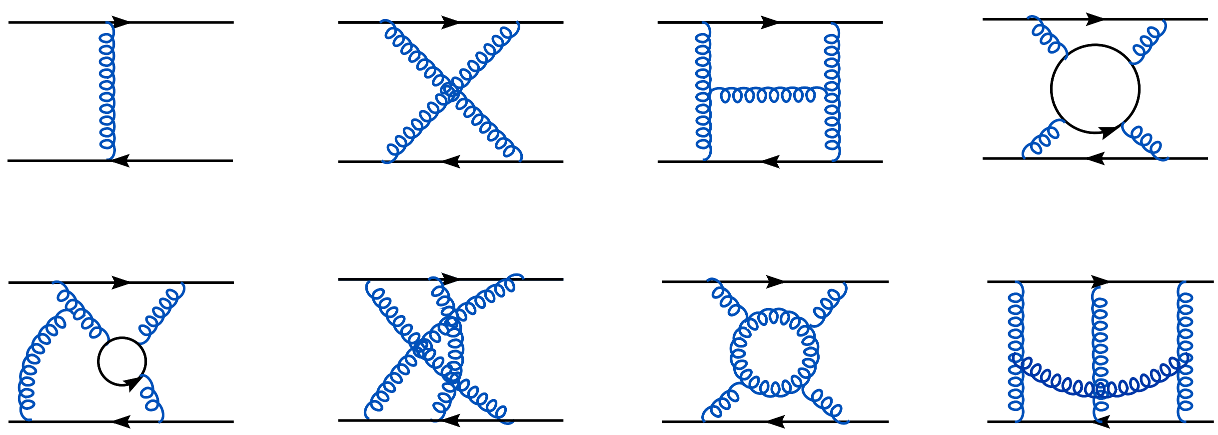

(Picture from A. V. Smirnov, V. A. Smirnov and M. Steinhauser, Phys.Rev.Lett. **104** (2010) 112002

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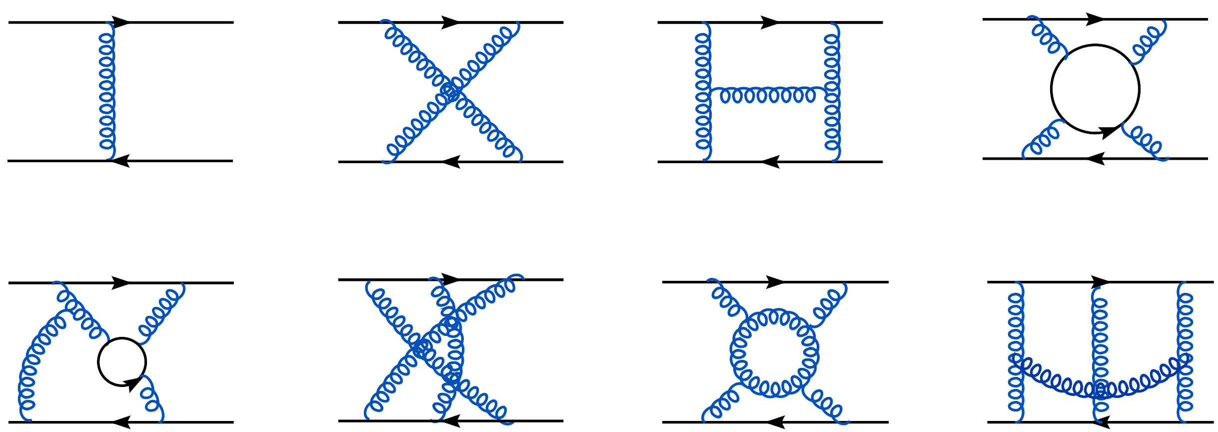

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Virtual emissions that change the color state of the pair (*Ultrasoft gluons*) Appelquist Dine Muzinich'78

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$E_0(r)$ is currently known at *3 loop +sub-leading ultrasoft log res.* (N^3LL) accuracy: $\alpha_s^{1+[3+n]} \ln^n \alpha_s$ with $n \geq 0$

Lattice

$E_0(r)$ recently calculated on the lattice in 2+1 flavor QCD

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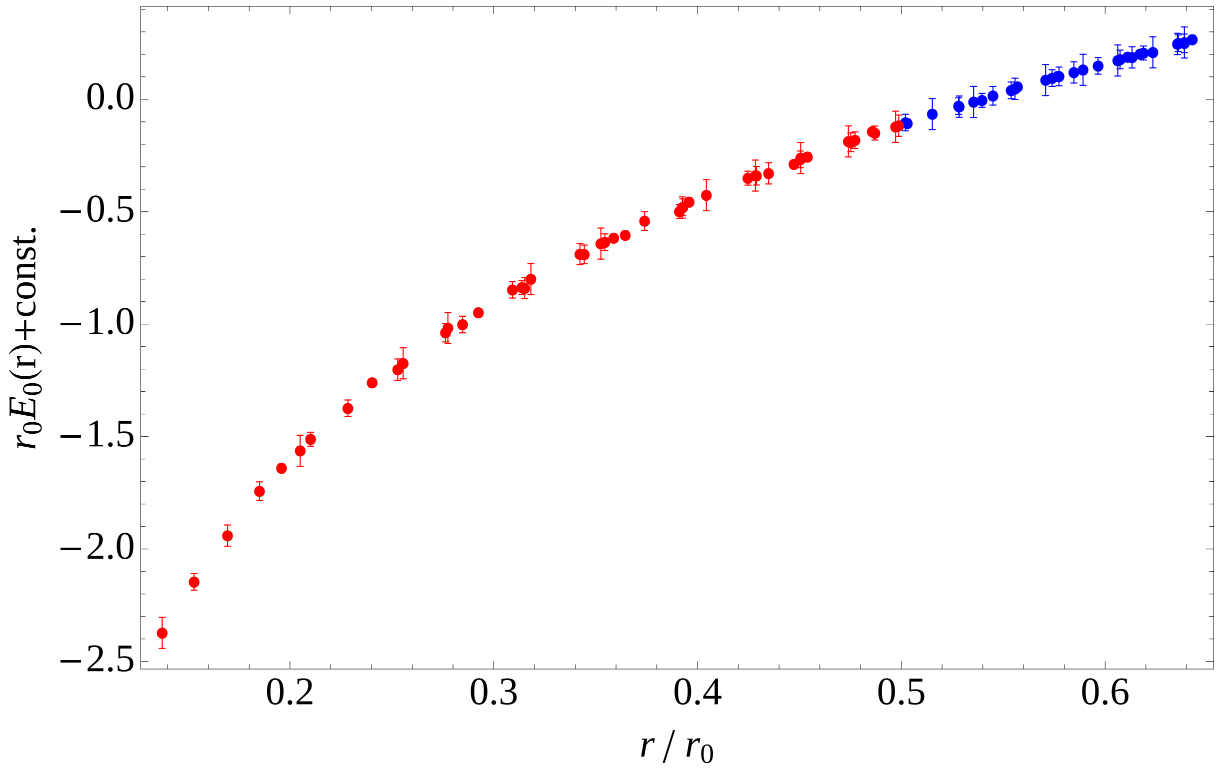

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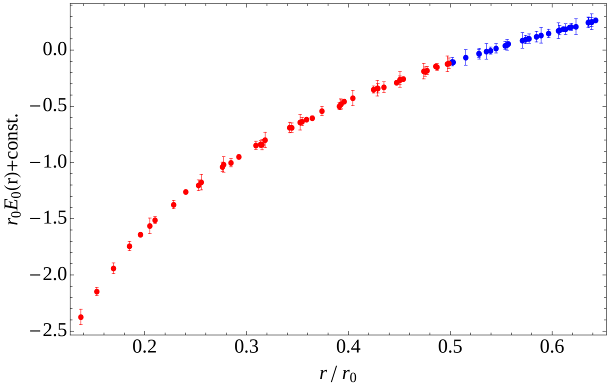

Combination of tree-level improved gauge action and HISQ action

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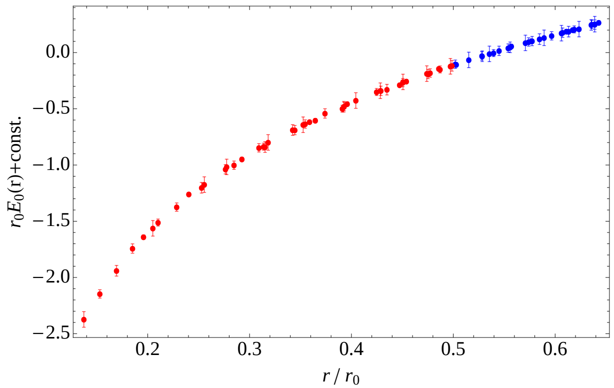

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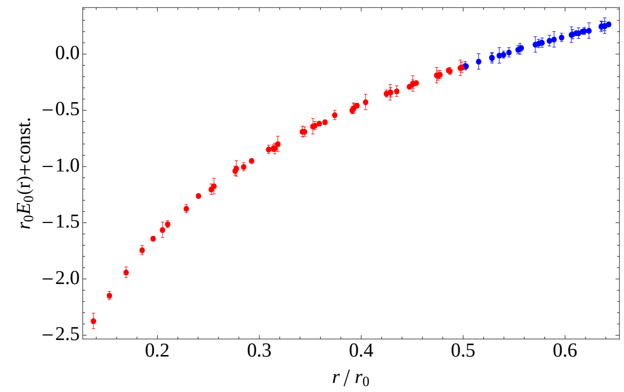

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Energy calculated in units of r_0 (r_1) Sommer'93

$$r^2 \frac{dE_0(r)}{dr}|_{r=r_0} = 1.65 \quad ; \quad \left(r^2 \frac{dE_0(r)}{dr}|_{r=r_1} = 1 \right)$$

Calculation for a wide range of gauge couplings.
($\beta = 6.664, 6.740, 6.800, 6.880, 6.950, 7.030, 7.150, 7.280$;
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Use $r < 0.5r_0$, and we have data down to $r = 0.14r_0$, i.e.

$$0.065fm \lesssim r \lesssim 0.234fm$$

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- Replace r by improved distance $r_I = (4\pi C_L(r))^{-1}$

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$$C_L(r) = \int \frac{d^3k}{(2\pi)^3} D_{00}(k_0 = 0, \vec{k}) e^{i\vec{k}\vec{r}}$$

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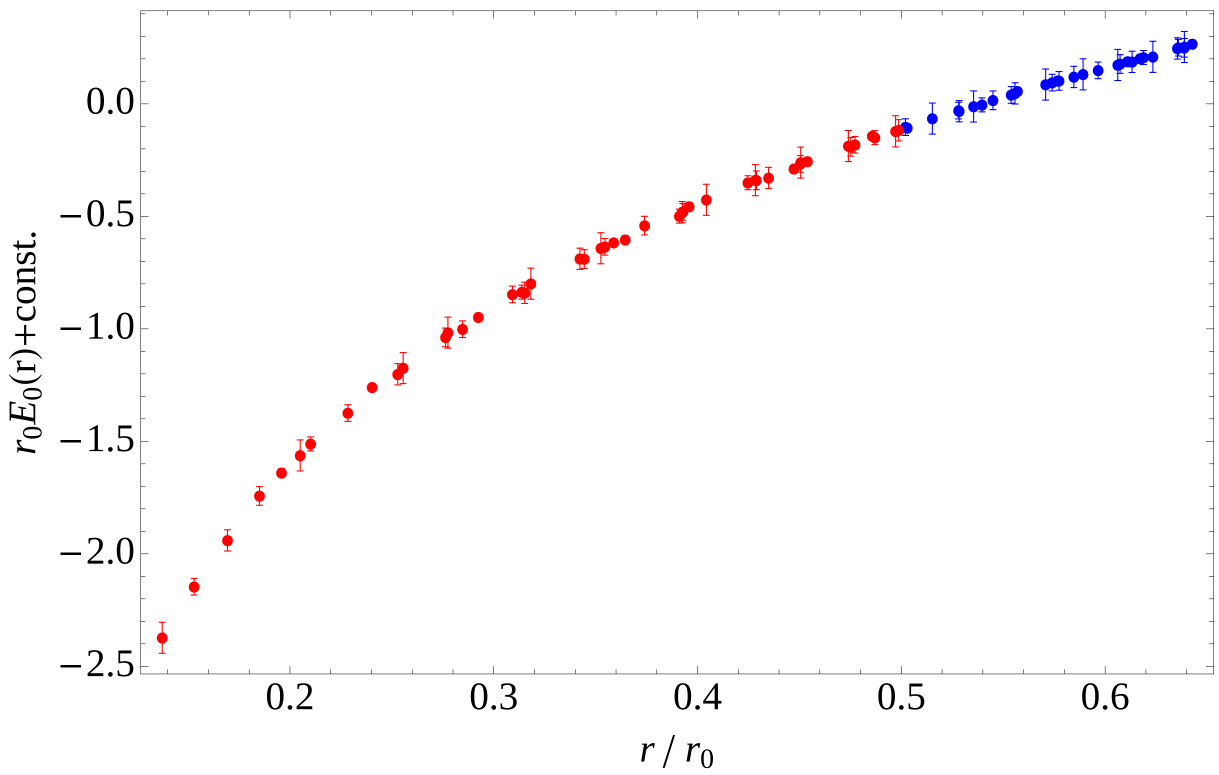
Normalization errors are larger than systematics of lattice artifacts


0.0
−0.5
−1.0
−1.5
−2.0
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0.2
0.3
0.4
0.5
0.6
$r_0 E_0(\mathrm{r})$+const.
r / r_0

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How do we implement this in practice?

Normalization is not physical, only the slope

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- Take a derivative → compare with the force

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In any case, care needs to be taken with so-called renormalon singularities in the perturbative expression. At the end no practical effect, just need to take the proper pert. expression, so that fits/comparison are not affected

- Normalization (mass) $\rightarrow$ affected by renormalon
- Slope $\rightarrow$ renormalon free

In the published analysis:
use $E_0(r) + const.$ and all lattice data together

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- Difference with weighted average at previous order
- Weighted standard deviation
- Use alternative weight assignments (p-value, constant)

Final result:

$$r_0 \Lambda_{\overline{\mathrm{MS}}} = 0.70 \pm 0.07,$$

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which corresponds to

$$\alpha_s\,(1.5\text{GeV}, n_f = 3) = 0.326 \pm 0.019$$

$$\rightarrow \quad \alpha_s\,(M_Z, n_f = 5) = 0.1156^{+0.0021}_{-0.0022}$$

Result is at 3 loop, including resummation of leading ultrasoft logs.

3loop + sub. lead. us. res. (N^3LL) also known, but depends on additional (scheme dependent) constant $K_2 \sim \Lambda_{\overline{\rm MS}}$ (to be fit to the data). χ^2 as a function of $r_0\Lambda_{\overline{\rm MS}}$ is very flat, cannot improve extraction with current data

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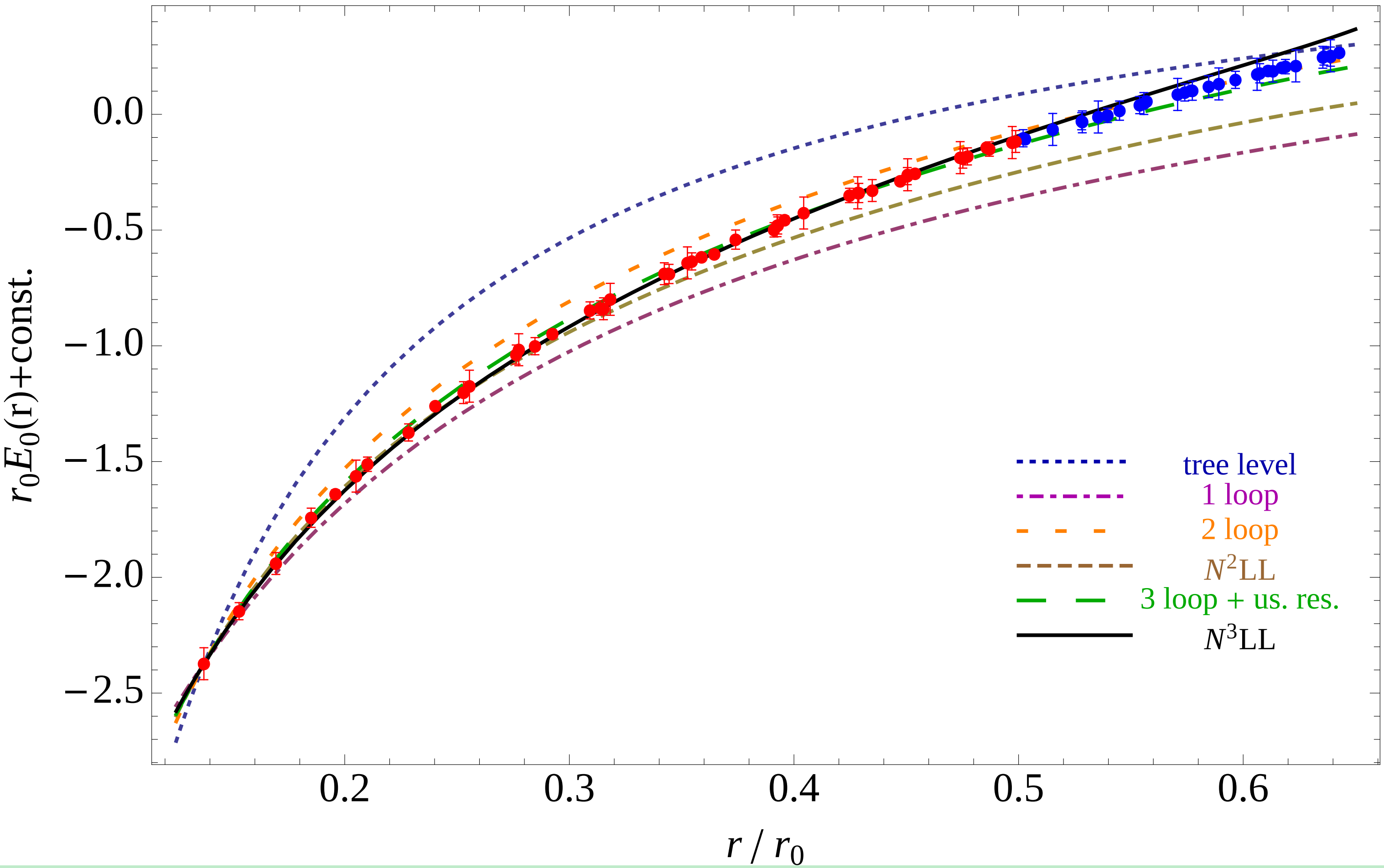

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Careful assessment of systematic errors for shorter-distance points

Other studies (in the 2 flavor case) Knechtli, Leder '11 concluded that finer lattice spacings are needed for the extraction

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- Perturbative expression better suited for the comparison: use pert. expression for the force. Avoid all logs, more important now that one goes to shorter distances

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- Further assessment of systematic errors for shortest distance points. Check effect of excluding them from the analysis

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No new final number yet, analyses of the points mentioned before still in progress. So far, we are obtaining results which are compatible with our previous number. Basically, the new preliminary results cover the upper half of our old range.

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Determination of α_s by comparing lattice data for the short-distance part of the QCD static energy with perturbation theory (3 loop + resummation of ultrasoft logs accuracy)

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Determination of α_s by comparing lattice data for the short-distance part of the QCD static energy with perturbation theory (3 loop + resummation of ultrasoft logs accuracy)

$$\alpha_s\left(M_Z\right) = 0.1156^{+0.0021}_{-0.0022}$$

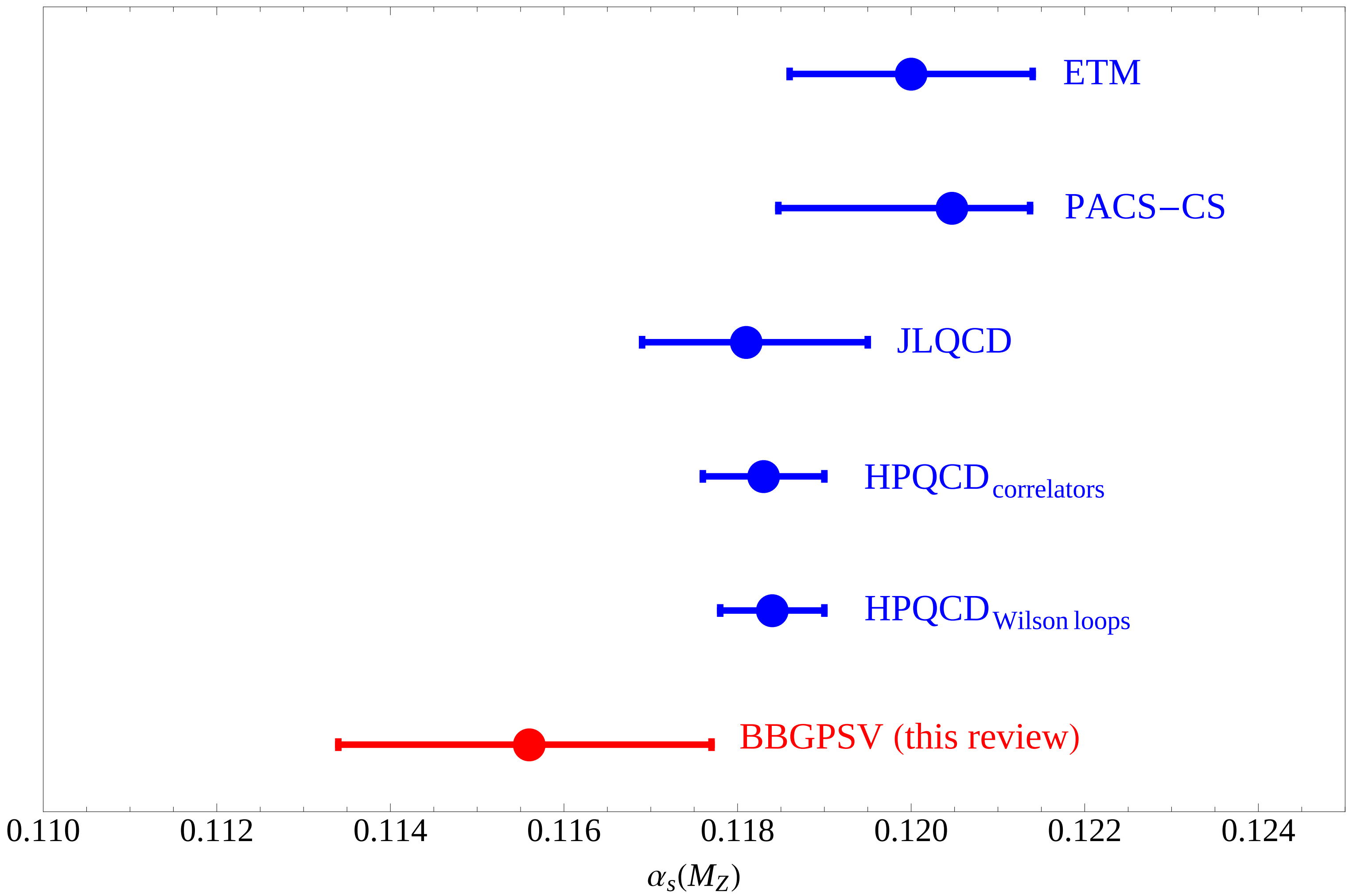
Updated analysis in progress: New lattice data with finer lattice spacings. Further analyses to verify that one is in the perturbative region. Further scrutiny of discretization errors. Take numerical derivative of data and compare also directly with the force (slope)

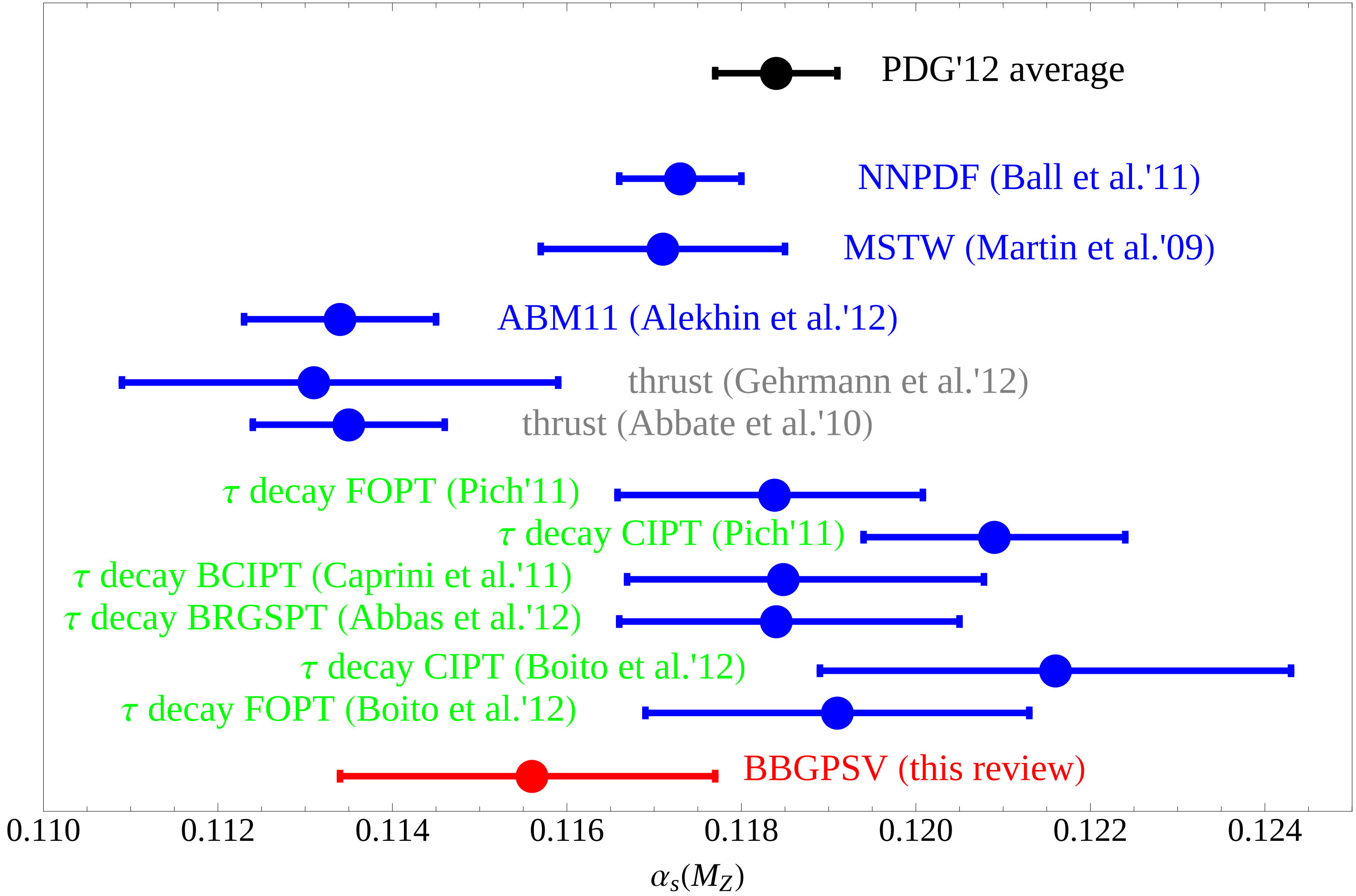
Backup slides

Accuracy	$r_0\Lambda_{\overline{\mathrm{MS}}}$
tree level	0.395
1 loop	0.848
2 loop	0.636
$\mathrm{N}^2\mathrm{LL}$	0.756
3 loop	0.690
3 loop + lead. us. res.	0.702

$\mathrm{N}^3\mathrm{LL}$ (3loop +sub-lead. us. res.) also known, but depends on additional constant $K_2 \sim \Lambda_{\overline{\mathrm{MS}}}$ (to be fit to the data). χ^2 as a function of $r_0\Lambda_{\overline{\mathrm{MS}}}$ is very flat, cannot improve extraction. Data not accurate enough to be sensitive to sub-leading us logs

Take 3 loop + lead. us. res. as our best result


ETM
PACS−CS
JLQCD
HPQCD correlators
HPQCD Wilson loops
BBGPSV (this review)
0.110
0.112
0.114
0.116
0.118
0.120
0.122
0.124
$\alpha_s(M_Z)$


PDG'12 average
NNPDF (Ball et al.'11)
MSTW (Martin et al.'09)
ABM11 (Alekhin et al.'12)
thrust (Gehrmann et al.'12)
thrust (Abbate et al.'10)
τ decay FOPT (Pich'11)
τ decay CIPT (Pich'11)
τ decay BCIPT (Caprini et al.'11)
τ decay BRGSPT (Abbas et al.'12)
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BBGPSV (this review)
0.110
0.112
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0.120
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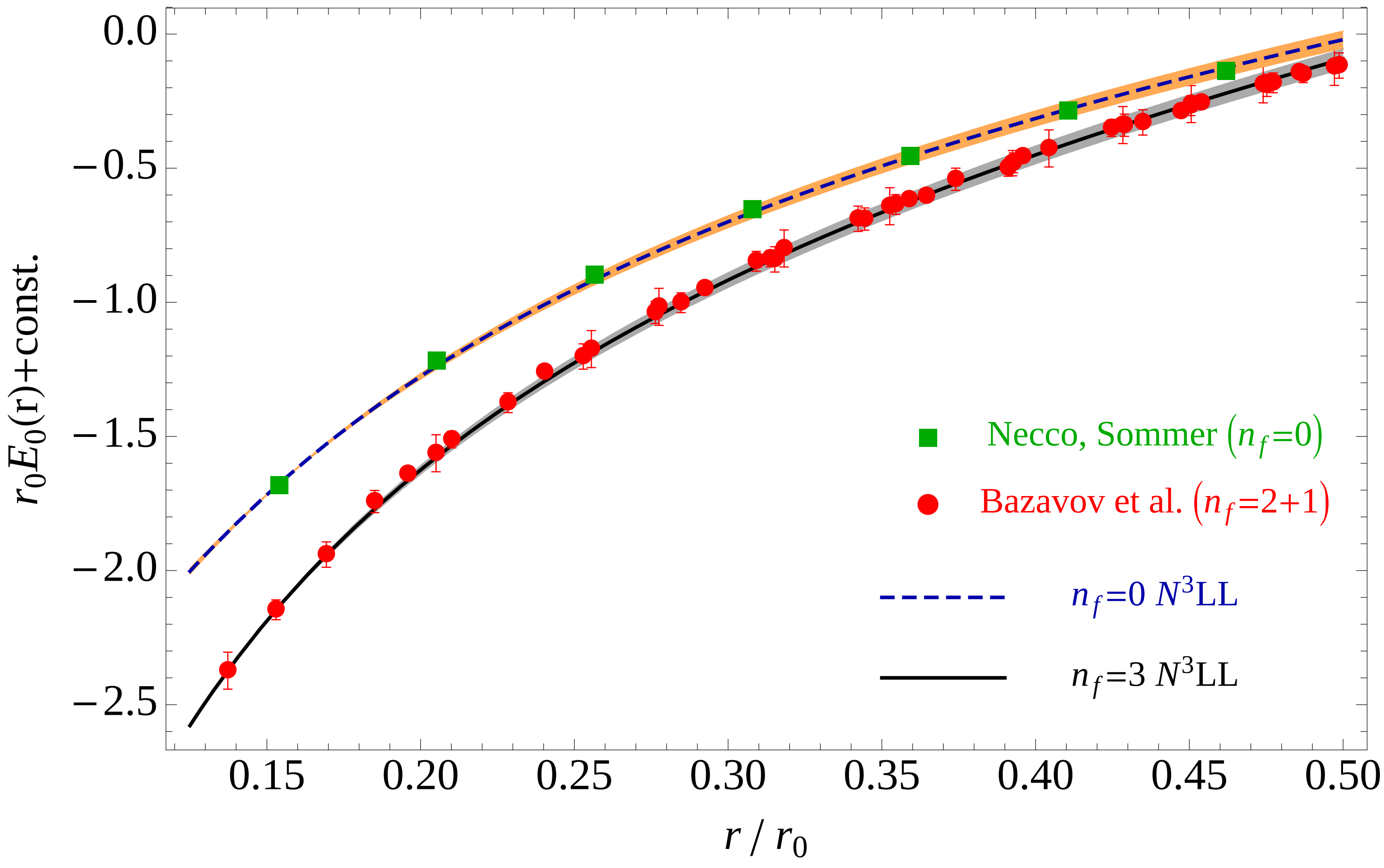
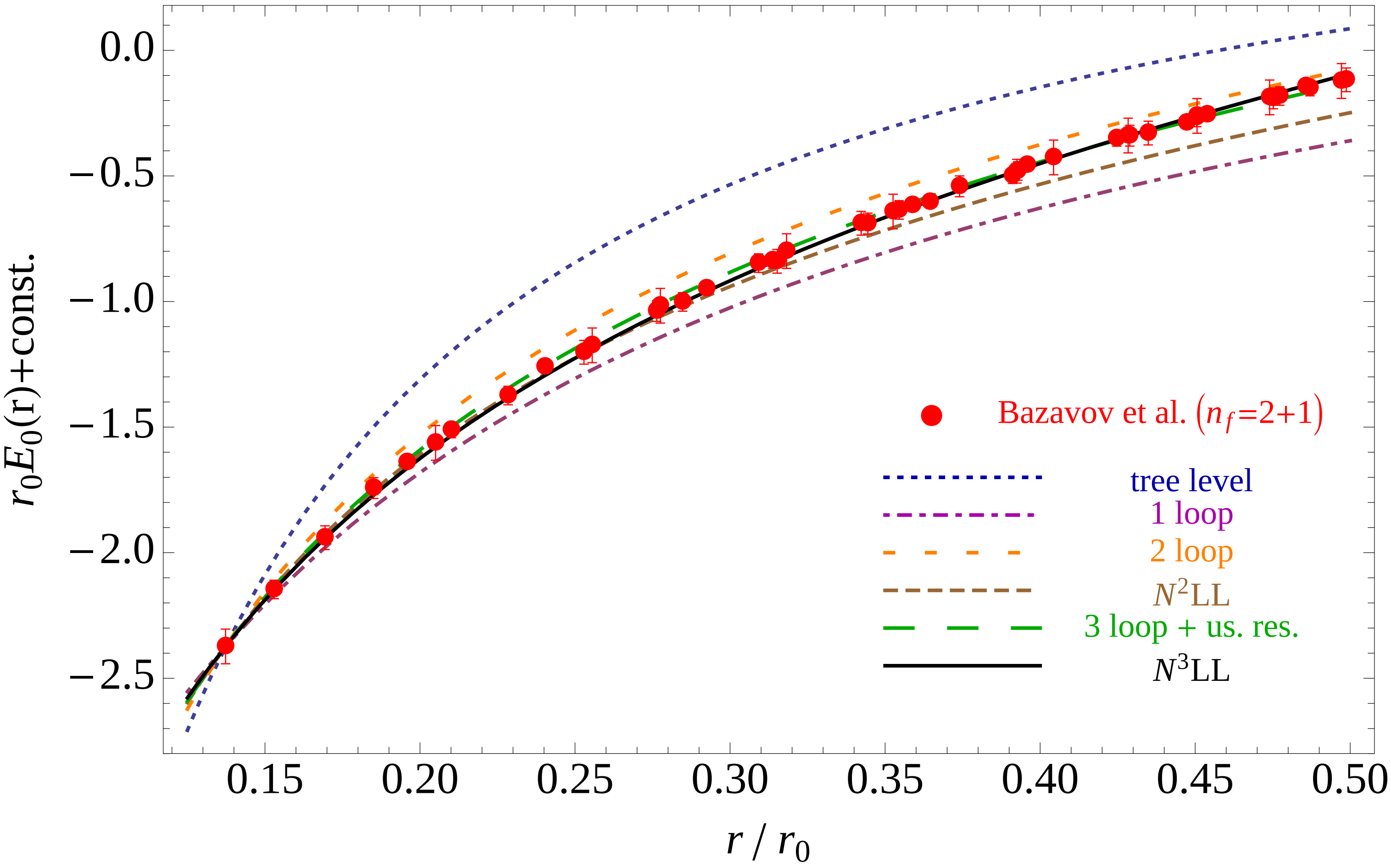
αs(MZ)


0.0
−0.5
−1.0
−1.5
−2.0
−2.5
$r_0 E_0(r)$+const.
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
r / r_0
Necco, Sommer $(n_f=0)$
Bazavov et al. $(n_f=2+1)$
$n_f=0$ N^3LL
$n_f=3$ N^3LL


0.0
−0.5
−1.0
−1.5
−2.0
−2.5
$r_0 E_0$(r)+const.
0.15
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0.25
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0.45
0.50
r / r_0
Bazavov et al. (n_f=2+1)
tree level
1 loop
2 loop
N^2LL
3 loop + us. res.
N^3LL